

On the Distribution of Fisher's Taxonomic Co-efficient and "Studentized D^2 -statistic"

In a paper published lately in the *Annals of Eugenics* (vol. VII, Part II, September 1936; pp. 178-188) under the title "The Use of Multiple Measurements in Taxonomic Problems" R. A. Fisher has obtained by the principle of maximization a certain expression based on sample readings, which he calls the taxonomic coefficient, whose object is to test, on the hypothesis of a multivariate normal population, whether two samples can be reasonably supposed to have been drawn from two populations with the same means, it being known or assumed that the two populations have the same variances and co-variances. By certain general arguments and formal analogies with partial

regression, Fisher makes the distribution of this co-efficient depend on his z -distribution, and then proceeds to numerical applications. We have now by making use of hyperspace geometry and the rectangular co-ordinates developed earlier by the authors¹ been able to derive in full the sampling distribution of this co-efficient. It is shown in conclusion that this is in entire agreement with the distribution implied in Fisher's numerical applications but not explicitly given in the *Annals of Eugenics* paper.

When the samples have been drawn from two different populations, the above distribution no longer applies. To estimate in this case the disparity between the two populations we make use of the D^2 -statistic in the "Studentized" form² with which Fisher's taxonomic coefficient can be proved to be formally identical. Denoting population value of D^2 by Δ^2 , we have now obtained the distribution of D^2 in the form

$$\text{Const} \times (1 - D^2)^{\frac{p-2}{2}} \frac{2\mu - p - 3}{(D^2)^{\frac{2}{2}}} \frac{2\mu - p - 3}{2} \frac{1}{e^{-\mu D^2}} (1 - D^2) \\ \times {}_1F_1 \left(- \left(\frac{2\mu - p - 1}{2}, \mu D^2 (1 - D^2) \right) \right) d(D^2).$$

where ${}_1F_1$ is the hypergeometric function defined in Watson's *Bessel functions*, p. 100.

Putting $A = 0$, we get the distribution of the taxonomic co-efficient considered above.

Statistical Laboratory,
Calcutta.
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Raj Chandra Bose.
Samarendra Nath Roy.

1. P. C. Mahalanobis, R. C. Bose and S. N. Roy: "Normalization of Variates and the use of Rectangular Co-ordinates in the Theory of Sampling Distributions." *Sankhya*, 3 (1), 1-40, 1937.

2. P. C. Mahalanobis: "On the Generalized Distance in Statistics" *Proc. Nat. Inst. Sci.* 2 (1), 49-55.