Number of Repairs for a 2-Unit System

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Key Words—Cold standby, Number of repairs

Reader Aids–
Purpose: Report a derivation
Special math needed: Probability
Results useful to: Reliability theoreticians

Abstract—The number of repairs upto time t for a 2-unit cold-standby system with general repair and failure time distribution is studied. The number of failures upto time t is given as a dual case.

1. INTRODUCTION

Many mathematical models and methods to study 2-unit systems are available [1]. Using a direct approach, we give here explicit expressions for the probability that n repairs are completed during $(0, t)$ for a 2-unit cold standby system with general failure and repair time distributions.

2. MODEL

1. Units are either good (operating) or bad (in repair)
2. One repair station.
3. At time $t = 0$, one unit is put in operation and repair commences for the other unit.
4. The repair and failure times form i.i.d. r.v.'s.
5. Switching is perfect, viz, instantaneous and never does any damage.

3. NOTATION

T	failure time of a unit.
X	repair time of a unit.
$F(T), G(X)$	Cdf of failure and repair times.
$\overline{}$	implies the complement.
$H(\cdot)$	$F(\cdot)\,G(\cdot)$, Cdf of $\max\{T, X\}$
$N(t)$	number of completed repairs during $(0, t)$
$P_n(t)$	$\Pr\{N(t) = n\}$
$*$	implies convolution
$H_n^*(t)$	n-fold convolution of $H(t)$ with itself, $H_0^*(t) = 1$.

4. ANALYSIS

During $(0, t)$, these are the following different possibilities:

i) The first repair is not complete
ii) The first repair is complete but the operating unit does not fail
iii) The first repair is complete and the operating unit has failed, viz, the first replacement has taken place (other failures and/or repairs might also have occurred).

Using a total probability argument, it follows that:

$$P_0(t) = \bar{G}(t)$$

$$P_1(t) = G(t)\,\bar{F}(t) + P_0(t) * H(t)$$

$$P_n(t) = P_{n-1}(t) * H(t), n \geq 2$$

$$P_n(t) = P_1(t) * H_{n-1}^*(t), n \geq 1. \quad (1)$$

In (1) if $F(t)$ and $G(t)$ are interchanged one gets the pmf of the number of failures in $(0, t)$.

REFERENCES

[1] S. Osaki, T. Nagakawa, "Bibliography for reliability & availability of stochastic systems", *IEEE Trans. Reliability*, vol R-25, 1976 Oct, pp 284-287.

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Manuscripts Received

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"A new goodness-of-fit test for life test data based on sample entropy", Masaaki Tsujitani; College of Engineering; Dept. of Industrial Engineering; University of Osaka Prefecture; Sakai, Osaka 591 JAPAN.