A FACTOR ANALYSIS OF EXAMINATION MARKS

[*In India examination marks are used to select students for various courses of study. The selection of candidates is based on the average percentage of marks secured by them at some particular examination. Such a system assumes that a general ability runs predominantly through all the subjects of study. Dr. S. K. Mitra of the Indian Statistical Institute, Calcutta, examines the efficacy of this method by factor analysing examination marks. The conclusions that are drawn by him are important and should serve as a useful guide to those who are responsible for selection.*—EDITOR]

Introduction

In validity studies of tests used in the selection of students for a course of studies, an average of marks received in several examinations is frequently used as the criterion. The use of an average as the criterion raises certain problems in judging the validity of tests from the correlation of tests against the criterion.

One of the problems is about the meaning of average marks. When the average is of marks received by a group of students in the same subject, *e.g.* mathematics, at different points in time, the average marks for a student may be taken to give an estimate of the ' true ' ability of the student in that subject. But when the average is of marks received in different subjects *e.g.* mathematics, history, English at any one point in time, it is difficult to attach any meaning to such average. It is argued that if the subjects are of an allied nature *e.g.* physics and chemistry, an average has some meaning ; it means an estimate of general proficiency in a field of study *e.g.* physical sciences, involving several allied subjects. It is assumed that success in a field of study involves, very largely, a general ability, which accounts for most of the variance in the specific subjects. So, it is assumed that though each subject may involve an ability specific to that subject, its contribution to the individual differences on that subject is practically negligible. Thus, an average of marks in Indian history, English history, and World history has some justifiable meaning. But curriculum in a field of study is rarely so homogeneous ; and sometimes, it may not be even one field of study, *e.g.* the high school courses or the intermediate courses in the colleges.

When the subjects are obviously different, the justification of an average mark lies only in an assumption of a general scholastic ability which runs pre-

dominantly through all the subjects. This means that given general scholastic ability one will be *equally* proficient in all subjects of study in an examination. Actually, however, the marks for an individual are found to vary over the subjects. This variation is explained away in terms of errors of measurement and of chance. Also by the very process of averaging, one is allowed to compensate for low proficiency in one subject against high proficiency in another, on the assumption of a general ability. While this may be acceptable, if the subjects really involve *only one* general factor which accounts for most of the variance in the marks in each subject, in the absence of actual evidence, the average marks can only be taken to be measures of the largest *common* factor in the subjects with the possibility of there being more than one common factor. Consider also the fact that when marks are added across subjects, the contribution of each subject to the total is proportional to its variance. Thus, a subject, which is too easy or too difficult, contributes little to the total or the average. As the subjects vary in difficulty from time to time and from group to group, the common factor represented in the average marks tend to vary in its meaning. Thus, in one examination it may be mathematics which has the largest variance and so makes the greatest contribution to the average, in another examination it may be literature, if we think in terms of the school and college examinations. The situation is not very different in professional courses where one may have, apart from shifting levels of difficulty, such diverse subjects as chemistry, biology and mathematics, or organic chemistry and anatomy. The justification of an average, therefore, in such cases, is based on a tenuous assumption and the meaningfulness of the average marks used as a criterion for test validity is doubtful.

If the tests used in a selection happen to have one large common factor and the marks in the examination also have the same, a satisfactory validity coefficient for the tests against average marks may be obtained. If, however, there is more than one common factor in the marks, the use of average may lead to under-estimates of the validity co-efficients of tests. In a given situation, then, it will be better to analyse the tests as well as the examination marks for common factors. This may be done by factor-analysing a matrix of correlations generated by both the predictor and criterion variables. Another way may be to use the method of factor analysis separately for the predictor and criterion variables, *i.e.*, for the test scores and the examination marks in the various subjects, and then using factor scores on both sides, validity coefficients may be found. The criterion, in any case, needs to be analysed.

This paper is on the analysis of examination marks with a view to finding meaningful factor scores to be used as criteria in validating a battery of tests for selection of students.

The examination marks

The examination marks analysed were those obtained by a group of 35 post-graduate students of statistics. Marks were available on 18 examinations, 6 of which were periodical and 12 terminal. The subjects on which the examination papers were set are listed below:

Periodical	*Terminal*
1. Algebra	7. Algebra
2. Analysis	8. Analysis
3. Numerical mathematics	9. Probability
4. Probability	10. Numerical mathematics
5. Practical	11. Practical I
6. Economics	12. Practical II
	13. Practical III
	14. Practical IV
	15. Descriptive Statistics
	16. Economics: General
	17. Economics: Public Finance
	18. Economics: Indian

Except the ones on Analysis and Probability, the marks were expressed in per cents, as is usually done in other examinations in India. The examinations on Analysis and Probability were of the objective type.

Method of analysis

The marks on each of the 18 examinations were correlated against the marks on every other. To adjust for varying levels of difficulty, r_{tet} or the coefficient of tetrachoric correlation was found in each case, using the median of the distribution of marks on each examination to split the group into two, and then following the method given by Jenkins (2).

The matrix of inter-correlations for the periodicals, the finals and for both were factor-analysed following Thurstone's complete centroid method. The computational procedure was that recommended by Fruchter (1).

Inter-correlation of periodical marks

The inter-correlations of the 6 periodical examinations are given in the table below :

Table 1

Showing the inter-correlation of periodical examinations *

	Examination Nos.							
	1	2	3	4	5	6	Sum	
1	(391)	136	391	150	000	222	1.290	
2	136	(656)	314	656	093	222	2.077	
3	391	314	(391)	111	093	301	1.601	$\sqrt{T} = 3.24793$
4	150	656	111	(656)	467	436	2.476	
5	000	093	093	467	(467)	184	1.304	$\frac{1}{\sqrt{T}} = 0.307888$
6	222	222	301	436	184	(436)	1.801	
Sum	1.290	2.077	1.601	2.476	1.304	1.801	10.549	= T

The coefficients in Table 1 range from + 0.000 to + 0.656 with the median falling at + 0.203, which is rather low.

Factor analysis of periodical marks

Using the highest correlation in a row or column as the guessed communality (the figures in parentheses in Table 1), a centroid analysis was done. The first factor loadings are given below in order of decreasing size :

First centroid factor loadings :	.762	.639	.554	.493	.401	.397
Periodical Nos :	4	2	6	3	5	1

From the first centroid factor loadings the following product and residual matrices were obtained :

Table 2

Showing the product matrix of the first factor *

	1	2	3	4	5	6
1	158	254	196	302	159	220
2		408	315	487	256	354
3			243	376	198	273
4				581	306	422
5					161	222
6						307

* Decimal points have been omitted from the table to make printing easy.

Table 3

Showing the first residual matrix *

	1	2	3	4	5	6	Sum
1	233	−118	195	−152	−159	002	001
2	−118	248	−001	169	−163	−132	003
3	195	−001	148	−265	−105	028	000
4	−152	169	−265	075	161	014	002
5	−159	−163	−105	161	306	−038	002
6	002	−132	028	014	−038	129	003

Reflecting the variates 5, 4 and 2, in that order, and replacing the diagonal values by guessed communalities, a second centroid factor was extracted from the first residual matrix. The second factor was bipolar, as is to be expected in Thurstone's method. The positive pole was composed of the periodical nos. 3, 1 and 6 with their respective loadings as: .436, .416 and .161. The negative pole was composed of nos. 2, 5 and 4 with their loadings as :—.216, —.235 and —.506. Further extraction of factors was not done as, except one or two, the entries in the second residual matrix had a zero as the first digit. The unrotated factor matrix is given below :

Table 4

Showing the unrotated centroid factor matrix obtained from the inter-correlations of periodical marks *

Periodical No.	Unrotated Centroid Factors I	II	h^2
1	397	416	331
2	639	−216	455
3	493	436	433
4	762	−506	837
5	401	−235	216
6	554	161	333

When the points were plotted on graph-paper, it appeared clearly that with a clockwise rotation of the axes through 40° approximately, a simple structure and a positive manifold could be easily obtained. The first rotated axis would pass practically through 4 and 5 and the second through 1 and 3. On the rotated factors, 4 would have the highest loading on first factor and zero on the second, and would have the second highest loading on the second factor and

* Decimal points have been omitted.

zero on the first. There will be no negative sign. Probability and Algebra would be orthogonal to each other.

Inter-correlation of final marks

The inter-correlations (r_{tet}) of the 12 final examinations are given below :

Table 5

Showing the inter-correlations of 12 final examination marks *

Examination Nos.

	7	8	9	10	11	12	13	14	15	16	17	18	Sum
7	(694)	136	307	136	307	473	141	136	314	307	694	314	3.959
8	136	(473)	307	307	473	136	141	307	222	046	394	222	3.164
9	307	307	(622)	473	473	046	141	622	391	226	553	141	4.302
10	136	307	473	(633)	046	136	222	307	141	046	226	633	3.306
11	307	473	473	046	(622)	307	313	622	222	136	553	222	4.296
12	473	136	046	136	307	(473)	481	224	141	042	226	141	2.826
13	141	141	141	222	313	481	(481)	141	056	038	222	056	2.433
14	136	307	622	307	622	224	141	(693)	693	046	394	038	4.223
15	314	222	391	141	222	141	056	693	(693)	481	038	468	3.860
16	307	046	226	046	136	042	038	046	481	(553)	553	314	2.788
17	694	394	553	226	553	226	222	394	038	553	(694)	222	4.769
18	314	222	141	633	222	141	056	038	468	314	222	(633)	3.404

$$\sqrt{T} = 6.58256 \qquad \frac{1}{\sqrt{T}} = .151917 \qquad T = 43.330$$

The coefficients ranged from +0.038 to +0.694 with the median at +0.226. These values were very much like the ones obtained from the inter-correlation of periodical marks.

Factor analysis of final marks

Following the procedure described in connection with the factor analysis of periodical marks, two centroid factors were extracted from the matrix of inter-correlations given in Table 5. From the plot of points on co-ordinate axes, it was observed that a clockwise orthogonal rotation would give positive manifold though not a clear simple structure. Axis I would pass through # 18 and axis II through # 11. Approximate loadings on the rotated factors were obtained graphically by dropping perpendiculars from the points on the new axes.

* The correlation coefficients in the table have been printed without decimal points.

Table 6

Showing the product-matrix obtained from first factor loadings*

First factor loadings		7	8	9	10	11	12	13	14	15	16	17	18
.601	7	361											
.481	8	289	231										
.653	9	392	314	426									
.502	10	302	241	328	252								
.653	11	392	314	426	328	426							
.429	12	258	206	280	215	280	184						
.370	13	222	178	242	186	242	159	137					
.641	14	385	308	419	322	419	275	237	411				
.586	15	352	282	383	294	383	251	217	376	343			
.423	16	254	203	276	212	276	181	156	271	248	179		
.724	17	435	348	473	363	473	311	268	464	424	306	524	
.517	18	311	249	338	259	338	222	191	331	303	219	374	267

It was apparent from the plot that a three-dimensional representation would be more adequate. A third axis could be passed through # 7. Further extraction, however, was not done, as the picture was sufficiently clear for all practical purposes. Table 8 shows that the rotated first factor loadings are high in Economics examinations and practically zero in two of the four Practical examinations with the Mathematics papers coming in between. Orthogonal to this is the second rotated factor. But an identification of this factor is difficult, for # 9 and # 17, which are Probability and Public Finance, have about the same medium loadings on this factor. Otherwise this is a factor running largely through the Practical papers as against the first which is loaded heavily in Economics papers. Considering the medium loadings of the Mathematics papers on both the factors, it appears that a third factor (orthogonal or oblique) passing mainly through these could be extracted, leading to a greater clarification of the factor-structure. On the basis of the evidence collected thus far, it was considered reasonable to assume three common factors, *viz.*, Mathematics, Economics and Practical, which would account for most of the variance in the inter-correlations of the 18 final examination papers.

* Decimal points have not been printed and only a half of the matrix has been given for convenience.

Table 7

Showing the first residual matrix and the second factor loadings *

	7	8	9	10	11	12	13	14	15	16	17	18	Sum
7	333	−153	−085	−166	−085	215	−081	−249	−038	053	259	003	.006
8	−153	242	−007	066	159	−070	−037	−001	−060	−157	046	−027	.001
9	−085	−007	196	145	047	−234	−101	203	008	−050	080	−197	.005
10	−166	066	145	381	−282	−079	036	−015	−153	−166	−137	374	.004
11	−085	159	047	−282	196	027	071	203	−161	−140	080	−116	−.001
12	215	−070	−234	−079	027	289	322	−051	−110	−139	−085	−081	.004
13	−081	−037	−101	036	071	322	344	−096	−161	−118	−046	−135	−.002
14	−249	−001	203	−015	203	−051	−096	282	317	−225	−070	−293	.005
15	−038	−060	008	−153	−161	−110	−161	317	350	233	−386	165	.004
16	053	−157	−050	−166	−140	−139	−118	−225	233	374	247	095	.007
17	259	046	080	−137	080	−085	−046	−070	−386	247	170	−152	.006
18	003	−027	−197	374	−116	−081	−135	−293	165	095	−152	366	.002
Estimated communalities	259	159	234	374	282	322	322	317	386	247	386	374	
Sum	.290	.580	.401	.529	1.653	.425	.894	.970	1.146	1.044	.560	2.012	10.504 = T
Second factor loading	.089	.179	.124	.163	.510	.131	.276	.299	.354	.322	.173	.621	3.24102 $= \sqrt{T}$ .30854 $= \frac{1}{\sqrt{T}}$

* Decimal points have been omitted.

Table 8

Showing the centroid factor matrix obtained from the inter-correlations of final examination marks*

No.	Unrotated factor loadings I	II	h^2	Rotated factor loadings I	II
7	601	-089	369	39	47
8	481	179	263	19	48
9	653	124	442	37	54
10	502	-163	279	37	35
11	653	510	686	00	84
12	429	131	201	25	36
13	370	276	213	07	42
14	641	299	500	22	66
15	586	-354	469	61	25
16	423	-322	283	51	14
17	724	173	554	52	52
18	517	-621	653	82	00

Comparison of factors in periodical and final examinations

Whereas Economics and Practical emerged as two common factors in the final, they were not so in the periodical. Practical in the periodical went with Probability and was orthogonal to Algebra; and Economics in the periodical had loadings on both factors. This difference is not surprising for it is known that specific factors can become common factors by increasing the number of tests measuring the specific factor. By increasing the number of papers in Economics and Practical, these two emerged as common factors in the final, along with a third in Mathematics.

Factors common to periodical and final examinations

Having done the analysis up to this point, a factor analysis of the inter-correlation matrix of all the 18 examination variables seemed to be indicated. The question was whether the assumption of three common factors, *viz.*, Mathematics, Economics and Practical, would stand the test of a fresh factor analysis involving all the 18 variables. There was also the question of the meaningfulness of a total or average based on both periodical and final marks. It appeared from the evidence given above that an average of periodical marks or of final marks had little meaning. Even if the average represents a measure of the first unrotated centroid factor, much information contained in the data were being thrown away by the use of average, for the first unrotated centroid factor "explains" only a part of the total variance. Hence the question of an average of both periodical and final marks became more unimportant.

The results of a centroid analysis are given in Tables 9-14.

* Decimal points have been omitted. The rotated factor loadings were obtained from a graphical solution.

Table 9

Showing the inter-correlation of periodical and final marks*

	EXAMINATIONS																		Row & Column sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	(481)	136	391	150	000	222	307	136	307	267	226	267	314	136	481	267	394	481	4.963
2		(656)	314	656	093	222	044	307	307	136	267	044	141	307	221	136	044	221	4.252
3			(693)	111	093	301	141	038	038	693	038	038	127	391	127	314	038	240	4.126
4				(656)	467	436	500	223	500	500	150	223	259	500	111	223	035	111	5.811
5					(467)	184	445	185	093	445	394	185	185	394	278	093	360	452	4.813
6						(436)	039	314	039	391	314	142	127	039	127	039	222	056	3.650
7							(694)	136	307	136	307	473	141	136	314	307	694	314	5.435
8								(473)	307	307	473	136	141	307	222	046	394	222	4.367
9									(622)	473	473	046	141	622	391	226	553	141	5.586
10										(693)	046	136	222	307	141	046	226	633	5.798
11											(622)	307	313	622	222	136	553	222	5.685
12												(481)	481	224	141	042	226	141	3.733
13													(481)	141	056	038	222	056	3.586
14														(693)	693	046	394	038	5.990
15															(693)	481	038	468	5.205
16																(553)	553	314	3.860
17																	(694)	222	5.862
18																		(633)	4.965

$\sqrt{T} = 9.36416$ $\quad \frac{1}{\sqrt{T}} = .1067901$ $\quad T = 87.687$

* Decimal points have been omitted and only half of the matrix has been printed for convenience only.

Table 10

Showing the first centroid factor loadings and the residual matrix*

First factor loadings	.530	.454	.441	.621	.514	.390	.580	.466	.596	.619	.607	.399	.383	.640	.556	.412	.626	.530	Check Sum
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
1	200	−105	157	−179	−272	015	000	−111	−009	−061	−096	056	111	−203	186	049	062	200	000
2		450	114	374	−140	045	−219	095	036	−145	−009	−137	−033	016	−031	−051	−240	−020	000
3			499	−163	−134	129	−115	−167	−225	420	−230	−138	−042	109	−118	132	−238	006	−004
4				270	148	194	140	−066	130	116	−227	−025	021	103	−234	−033	−354	−218	−003
5					203	−016	147	−055	−213	127	082	−020	−012	065	−008	−119	038	180	001
6						284	−187	132	−193	150	077	−014	−022	−211	−090	−122	−022	−151	−002
7							358	−134	−039	−223	−045	242	−081	−235	−008	068	331	007	007
8								256	029	019	190	−050	−037	009	−037	−146	102	−025	004
9									267	104	111	−192	−087	241	060	−020	180	−175	005
10										310	−330	−111	−015	−089	−203	−209	−161	305	004
11											254	065	081	234	−115	−114	173	−100	001
12												322	328	−031	−081	−122	−024	−070	−002
13													334	−104	−157	−120	−018	−147	000
14														283	337	−218	−007	−301	−002
15															384	252	−310	173	000
16																383	295	096	001
17																	302	−110	−001
18																		352	002

* Decimal points have been omitted and a half of the matrix has been printed for convenience only.

Table 11

Showing the second centroid factor loadings

Examination No.	Order of reflection	Guessed communality	Column sum	Second factor loading
1	4	.272	1.354	−.299
2	8	.374	.438	−.097
3	1	.420	2.603	−.576
4		.374	.387	.086
5		.272	.834	.184
6	5	.211	.449	−.099
7		.331	1.334	.295
8		.190	.418	.092
9		.241	.823	.182
10	2	.420	1.240	−.274
11		.330	1.911	.423
12		.328	1.238	.274
13		.328	.844	.187
14		.337	1.172	.259
15	6	.337	1.059	−.234
16	7	.295	.971	−.215
17		.354	1.499	.332
18	3	.305	1.873	−.414

$$20.447 = T$$

$$4.52184 = \sqrt{T}$$

$$.221149 = \frac{1}{\sqrt{T}}$$

Table 12

Showing the second residual matrix*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Check Sum
1	183	-134	-015	153	217	-015	-088	083	-045	-143	-030	-138	-167	126	116	-015	-161	076	003
2		365	058	-382	122	035	190	-104	-054	-172	-032	110	015	-041	-054	-072	208	-060	-002
3			088	113	028	072	-055	114	120	262	-014	-020	-066	-258	-253	008	047	-232	-003
4				367	132	-202	115	-074	114	-140	-263	-049	005	081	214	015	-382	182	-001
5					238	-002	093	-072	-246	-177	004	-070	-046	017	-035	079	-023	-256	003
6						201	158	-141	175	123	-119	-013	004	185	-113	-143	-011	-192	002
7							244	-161	-093	142	-170	161	-136	-311	-061	-131	233	-129	001
8								182	012	-044	151	-075	-054	015	015	126	072	-013	002
9									208	-154	034	-242	-121	194	-103	-019	120	100	000
10										345	214	036	-036	018	-267	-268	070	192	001
11											151	-051	002	124	016	023	033	-075	-002
12												253	277	-102	017	063	-115	-043	-001
13													293	-153	113	080	-080	070	001
14														270	-398	162	-093	194	001
15															282	202	232	076	-001
16																249	-366	007	000
17																	244	-027	001
18																		134	004

* Decimal points have been omitted and half of the matrix has been printed for convenience only.

Table 13

Showing the third centroid factor loadings

Examination No.	Order of reflection	Guessed communality	Column sum	Third factor loading	Sign change
1	3	.217	1.325	−.209	−
2		.382	1.881	.297	+
3		.262	1.005	.159	+
4	1	.382	1.594	−.252	−
5	4, 11	.256	.341	.054	+
6		.202	1.133	.179	+
7		.311	2.126	.336	+
8	6	.161	.911	−.144	−
9	9	.246	1.028	−.162	−
10		.268	1.108	.175	+
11	10	.263	.376	−.059	−
12		.277	1.263	.200	+
13		.277	.433	.068	+
14	8	.398	1.603	−.253	−
15	2	.398	.957	−.151	−
16	5	.366	1.617	−.255	−
17		.382	1.283	.203	+
18	7	.256	1.480	−.234	−

$$21.464 = T$$

$$4.63293 = \sqrt{T}$$

$$.15799 = \frac{1}{\sqrt{T}}$$

Table 14

Showing the unrotated centroid factors in the 18 periodical and final examinations

Examination Nos.	Factor loadings			h^2
	I	II	III	
1	.530	−.299	−.209	.414
2	.454	−.097	.297	.304
3	.441	−.576	.159	.552
4	.621	.086	−.252	.456
5	.514	.184	.054	.301
6	.390	−.099	.179	.194
7	.580	.295	.336	.536
8	.466	.092	−.144	.246
9	.596	.182	−.162	.415
10	.619	−.274	.175	.489
11	.607	.423	−.059	.551
12	.399	.274	.200	.274
13	.383	.187	.068	.186
14	.640	.259	−.253	.541
15	.556	−.234	−.151	.387
16	.412	−.215	−.255	.281
17	.626	.332	.203	.543
18	.530	−.414	−.234	.507

Though it was difficult to identify the unrotated factors, it was clear from Table 14 that more than one common factor is involved in the examinations. The second and third common factors were bipolar and so there were a number of negative loadings. As the examinations were on knowledge and ability, negative loadings had no meaning. So the axes were rotated to achieve a positive manifold and, as far as possible, a simple structure. Several orthogonal rotations were done on standard graph paper and the rotated factor loadings were calculated from the graph. First, factors I and II were rotated. The axis for the rotated factor I′ passed through examination no. 3. The loadings on the rotated factors I′ and II′ are given in Table 15 below :

Table 15

Showing the loadings on rotated factors I′ and II′ (first rotation)

Examination No.		1	2	3	4	5	6	7	8	9
Factor loading	I′ :	.56	.35	.73	.30	.17	.15	.11	.22	.22
	II′ :	.23	.30	.00	.55	.52	.37	.64	.43	.58
Examination No.		10	11	12	13	14	15	16	17	18
Factor loading	I′ :	.59	.04	.03	.08	.19	.53	.43	.12	.65
	II′ :	.33	.74	.48	.41	.66	.30	.19	.69	.17

It should be noted that the rotated factor loadings are approximate values. Calculation of exact values through a transformation matrix was considered unnecessary, for a hypothesis about factors was at issue and *not* about the exact loadings.

Points were, then, plotted in the plane formed by the rectangular co-ordinates representing the rotated factor I′ and the unrotated factor III and also by the rotated factor II′ and unrotated factor III. An examination of the distribution of points showed that an orthogonal rotation in the plane formed by the rotated factor II′ and the unrotated factor III would be better from the point of view of positive manifold. So an orthogonal rotation of these two axes was done, axis II″ passing between the examination nos. 16 and 18 making the loadings of these two on III′ practically zero. Table 16 below gives the loadings after rotation :

Table 16

Showing the loadings on rotated factors II″ and II′ (second rotation)

Examination No.		1	2	3	4	5	6	7	8	9
Factor loadings	II″ :	.31	−.04	−.13	.54	.28	.09	.14	.37	.48
	III′ :	.05	.42	.10	.27	.44	.40	.70	.25	.35

Examination No.		10	11	12	13	14	15	16	17	18
Factor loadings	II'' :	.07	.50	.15	.21	.60	.31	.32	.26	.26
	III' :	.36	.54	.50	.35	.35	.14	−.01	.66	.01

It should be noted that the only negative loading in factor III' was −.01, which was negligible. The question was whether II'' should be rotated again in the plane formed by the axes I' and II'', or whether I' and III' should be rotated. It was observed that the structures of the factors I' and III' were distinctly different and the loadings on both were positive and also that rotation would not improve the situation. So the decision was made in favour of rotating I' and II'' where there was a scope for improvement. In rotating the co-ordinates I' and II'', I'' passed through examination no. 3 and II''' through examination no. 12. Following this plots were made of I'' vs. III' and II''' vs. III' and it was clear that further orthogonal rotation was not possible in view of the restrictions imposed by the requirement of positive manifold. So rotation was stopped at this point and the factor matrix, given in Table 17 below, was examined for identification of factors.

Table 17

Showing the loadings (obtained graphically) of the three rotated centroid factors in periodical and final examination marks

Examination	Factor loadings I''	II'''	III'
Periodical :			
1. Algebra	.50	.40	.05
2. Analysis	.35	.02	.42
3. Numerical Mathematics	.74	.00	.10
4. Probability	.20	.58	.27
5. Practical	.11	.30	.44
6. Economics	.13	.11	.40
Final :			
7. Algebra	.08	.16	.70
8. Analysis	.40	.15	.25
9. Probability	.51	.13	.35
10. Numerical Mathematics	.57	.17	.36
11. Practical I	−.05	.50	.54
12. Practical II	.00	.15	.50
13. Practical III	.04	.22	.35
14. Practical IV	.07	.62	.35
15. Descriptive Statistics	.46	.39	.14
16. General Economics	.36	.38	−.01
17. Public Finance	.07	.27	.66
18. Indian Economics	.59	.36	.01

Looking down the columns it appears that Factor I'' runs mainly through Mathematics, Descriptive Statistics and Economics (excluding Public Finance). This factor does not enter into the Practicals and Public Finance which, along with Algebra (final), Analysis (periodical) and Economics (periodical) define Factor III'. Factor II''' appears to be defined by the two Practicals I and IV, Probability (periodical), Algebra (periodical) and Economics. Thus, the factor-structure is by no means simple and does not clearly support the assumption of three common factors: Mathematics, Economics and Practical.

On an examination row-wise of Table 17 it appears that Algebra (periodical) is different from Algebra (final), on the three factors. Whereas Algebra (periodical) has high loading on Factor I'' and practically zero loading on Factor III', it is reversed for Algebra (final).

Analysis (periodical) and Analysis (final) have the same factorial composition. The loadings for both are practically the same on Factors I'' and III'. Unlike Algebra, their loadings on Factor II''' are negligible.

The situation is not much different for Numerical Mathematics. Its loadings are, however, relatively higher on Factor I'' than on Factor III'. In this respect Analysis is equally balanced. It should be noted that Algebra (final) has the highest loading on Factor III' and Numerical Mathematics (periodical) on Factor I''.

Probability (periodical) and Probability (final) are factorially complex, having loadings on all three factors. Probability (periodical) has much higher loading than Probability (final) on Factor II''', whereas the latter has a much higher loading than the former on Factor I''.

Though it seems reasonable to assume one factor involving the knowledge and understanding of mathematics which should run through Algebra, Analysis, Numerical Mathematics and Probability, the evidence seems to suggest that the specificity of each examination is so high as to go against the assumption. At least, success in each examination on mathematics does not depend on the same set of skills or abilities. An analysis of the content of the examinations in mathematics, thus, seems to be indicated.

In Economics, Public Finance clearly does not go with the rest. Whereas its loadings are very high on Factor III', the loadings of Indian and General Economics are zero. The case is reversed for loadings on Factor I''. It is only on Factor II''' that they have about the same loadings. The Economics (periodical) seems to be loaded on the Public Finance side. The skills needed for success in Indian and General Economics are very different from the ones required in Public Finance. The difference is the same as between Numerical Mathematics and Algebra.

The Practicals are all clearly arranged on two factors only, II''' and III''', and are also equally balanced between the two. Factor I'' can be thus regarded as a Non-Practical Factor involving something other than the ability to apply principles in solving specific problems, provided the Practicals involve this ability. A guess may be made now about the nature of Factor I''; probably it involves an ability to demonstrate knowledge of principles learnt in Mathematics and Economics (excluding Public Finance). This, however, does not help very much in solving the problem of the nature of Factors II''' and III'. It can, probably, be said that as the Practicals are equally high on both these factors two distinct abilities relating to application of principles in problem-solving are involved. An analysis of the Public Finance and Probability (periodical), in terms of skills required for success in the examinations is crucial for an identification of the two abilities. Such analysis, however, is beyond the scope of this report.

Discussion

There are several ways of looking at a work of factor analysis. In the present study, factor analysis was used as a tool for the classification of examinations from the point of view of using examination results as the concurrent validity of selection tests could be obtained. From the analysis given above, it appears that a simple meaningful classification could not be obtained with the given set of marks. Why ?

An obvious answer will be to put the blame on the method of factor analysis. Thus, it may be pointed out that when the matrix of inter-correlations of the six periodical examinations only was analysed, one factor was found to pass through Algebra and Numerical Mathematics and the other through Probability and Practical. When the matrix of inter-correlations of the twelve final examinations were analysed, a two-factor solution was found to be inadequate. It was, however, found that Economics and Practical were orthogonal to each other. When the entire set of correlations was analysed, the three-factor solution was found to be complex. It should be noted, however, that the periodical Algebra and Numerical Mathematics still define a factor in Table 17 and the periodical Probability and Practical are largely orthogonal to these. This relationship, therefore, remains invariant to a great extent, even when other variables are thrown in the matrix. Similar invariance of the relationship between final Economics and Practical may be noted in both Table 8 and Table 17. The relationship of Probability (periodical) and Practical (periodical) is maintained in Probability (final) and Practical (final). Thus, the relationships are not changed by only increasing the size of the matrix of inter-correlations. The problems are really created by the two Algebras, the two Probabilities and

Public Finance. This cannot be attributed to the method which has not affected the relationship of most of the variables from one matrix to another.

Is it due to time ? There was an interval of time between one periodical and another, and between them and the final. But unless time affects results differentially over subjects, the peculiar relationship of the two Algebras and Probabilities cannot be explained.

Could it be due to a change in the examiner or standard of examination ? Instead of viewing correlations between examinations as indicative of common ability in examinees, the correlations may be thought of as indicative of examiner consistency. In that case the factor matrix will indicate examiner classes. Considering the fact that except Probability and Analysis, the other examinations were all of the essay-type, the correlation between examinations may be affected by changes in the examiner. The changes may be between different examiners or within the same examiner over time. Changes in standards of examination may also affect results. Thus, in one examination, the question may be easy, bringing into use only a meagre part of the abilities of the examinees. In another examination on the same subject the questions may be difficult, bringing into play a major part of the abilities. Or, the actual traits or skills tested in two examinations on the same subject, may be different resulting in a situation like the one we have here.

It seems, on the whole, that the communality or common factor variance of the examinations is low (Table 14). This means that the contribution of specificity and error to the total variance is high. While specificity may be considered to be a desirable feature of an examination, error is not. Considering the fact that the examinations are mostly of the essay-type, there is a possibility of large error. While error may be less in objective examinations, specificity may be high. In order, therefore, to get a clear evidence on factor-structure, error should be reduced to a minimum, and either specificity should be increased so that there is practically no common factor or else it should be so reduced as to maximise communality. Without making the examination objective, it is difficult to see how this can be done.

Conclusion

Considering the evidence on the present set of marks, it can be said that an average over all examinations has little meaning as a criterion. As there are at least three common factors and the factors are not clearly identified, even factor scores will not be useful as criteria. It, thus, appears that, as criteria, each examination marks should be used separately. Even this is questionable in view of the high uniqueness of each examination, a major part of which is likely to be error.

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