SOME STUDIES ON NON RECURSIVE EVALUATION OF NETWORK RELIABILITY

DISSERTATION SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIRMENTS FOR THE M. TECH (COMPUTER SCIENCE) DEGREE OF THE INDIANSTATISTICAL INSTITUTE

BY

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A C K N O W L E D G E M E N T S

I cannot find words to express my gratitude to my guide Prof B.P.Sinha, but for whom nothing would have been possible. I would like to thank Mr.K.Mukhopadhyaya for his helps and moral support. I would also like to thank Dr.B.B.Bhattacharyya, Dr.J.Dutta Gupta and Mrs N.Das for their interest and moral support. Last but not the least , I thank my friends Arani, Debashree, Bhaskar, Saurav, Gouri, Milind, Somnath Da, and Tarun Da.

C O N T E N T S

CHAPTER I

1.1. Introduction :

During recent years, design of suitable topologies for computer communication networks has become an important area of research . A network is modeled by a graph , where the edges of the graph represent the communication links and the nodes stand for the processing elements. It is very important to compare the performance of a newly designed network with the existing networks. A good network structure should have a number of desirable properties , e.g. (1) low degree of vertices, (2)small number of edges , (3) low diameter , (4) high degree of fault tolerance i.e., high connectivity of the network graph , (5) regular structure ,(6) simple routing algorithms in both faulty and fault-free conditions. Some of these properties are mutually conflicting.

An integrated design approach, simultaneously optimizing all of these aspects is very difficult to achive. The usual practice is to consider one or some of them, but not all at a time , to obtain an optimal or near-optimal design.

Fault-tolerance of a computer network topology is a fundamental consideration . The faults to be considered may be processor failure or link failures. A network topology is said to be fault-tolerant if it remains "operational", in the presence of faults. It is, however , the topological requrements set by the application environment that essentially determine when a network is considered "operational". For special purpose networks , the requirement may be that the induced subgraph on the live or non-faulty nodes satisfies some particular property. This property may be the presence of some particular structure or may be anything else . For general purpose networks, usually it is considered operational as long as the induced subgraph on the live nodes is connected.

Though there is no universally accepted measure of reliability, a simple measure for general topology may be connectivity (i.e. the minimum number of nodes that has to be removed to make the graph disconnected [3]) of the network-graph. A more intricate approach may be to use a stochastic model [5]. Here, with each node and each link we assign a probability of failure. The failures are assumed to occur independently. Under this model our aim is to find the probability that the network is connected. Some work [1,2] has been done on finding the reliability when the links have positive failure probability only.

(1.2) Basic Concepts :

The reliability of a network is given by the probability that the network is connected under the assumption that only the nodes and not the links have positive rates of failure and failure of one node does not affect the failure of the other nodes.

We formally define network reliability under the following model :

Model : For a network graph G=(V,E), where V is the set of nodes and E is the set of links,

(1) with every node we assign a probability of failure.

(2) node faults are assumed to occur independently. When two nodes are alive (non-faulty) we assign a probability that the link between them if any is alive. We assume this probability to be equal to 1. When one of the terminal nodes of a link is dead the link automatically dies.

Under above assumption the reliability of any network G (denoted by R(G)) will be given by R(G)=P(A) [P(.) denotes probability] , where A is the event that the induced subgraph [4] on the live nodes are connected.

Definitions:

Let G = (V,E) be a network of n nodes and v ($\in$ V) be any node of G.

R(G)= reliability of the network G.

(G-{v})= the network graph formed by deleting the node {v} from G & its associated links i.e., the links which come out from or go into the node {v}.

V2= the induced subgraph formed by deleting the node {v} and also the nodes adjacent to node {v}.

V1= the set of nodes which are adjacent to {v}.

G_v = the network-graph formed by deleting the node {v} and its associated links and forming a complete graph with the set of nodes u such that $u \in V1$.

$P(L_v)$= probability that the node {v} is alive.

$P(L_v^c)$= probability that the node {v} is not alive.

Example: Consider the graph G as given below.

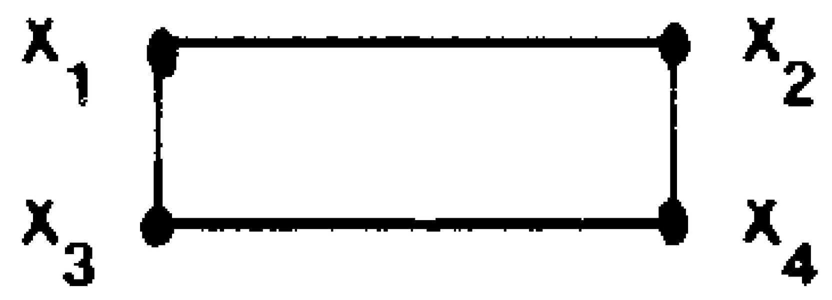

fig : 1

Let $\{v\}=\{X_1\}$. Then $G-\{v\}$, G_v, V_1, V_2 are as follows :

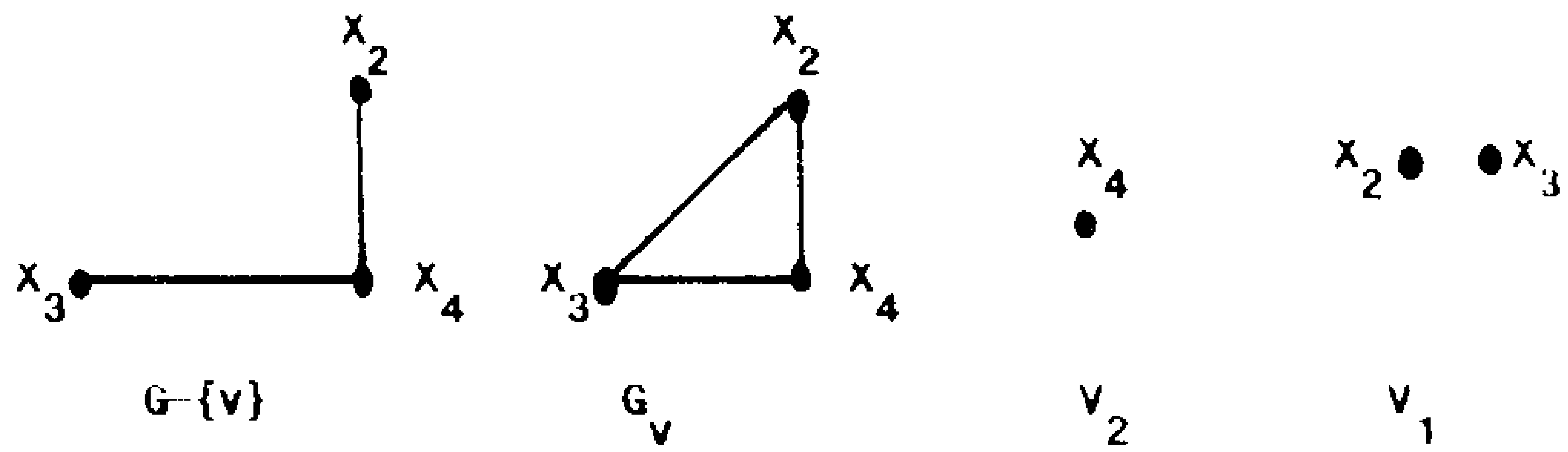

fig : 2

We state the following result due to Mukhopadhyaya and Sinha [5]

THEOREM 1 :

For a network G=(V,E),if the links are fault free,then for any $v \in V$

$$R(G)=R(G-\{v\})*P(L_v^c) + P(L_v)*[R(G_v) - [R(\langle V2 \rangle) - \prod_{u \in V2} P(L_u^c)] \prod_{u \in V1} P(L_u^c)]$$

Remarks : Theorem 1 provides us a recursive way of computing the reliability of a network in terms of the reliability of network with fewer nodes.

1.3. Motivation And Problem Definition :

We developed a Pascal program to determine the reliability of any network where failure rates of each node are given ,by the direct application of theorem 1. We used a very simple data structure (adjacency matrix) to represent the network graph.This programis efficient enough to determine the reliability in numeric form.

But the expression for reliability in theorem 1, is recursive in nature. For only one pass of the recursion using this formula, reliabilities of three graphs have to be computed.Even for a simple graph like cycle,with only 25 nodes,the recursive execution leads to stack overflow in 8650, VAX-VMS system.

Theorem 1 helps us to find an analytical form for the reliability of a few common network graphs [5] like Path, Cycle, Star, Wheel etc. But if we are to find the reliability of an arbitrary network structure for which an analytical expression for the reliability is difficult to find out, we can still find the reliability by writing a recursive program as a direct application of theorem 1. However the problem of writing such a recursive program are (1) exponential time requirment and (2) stack overflow.

The result of time required to evaluate reliability for some simple, common networks is shown in the following table.

Type Of Network	# Of Nodes	Time Required In Cpu Unit	Remarks
Path	5	10	
Path	10	180	
Path	15	6400	
Path	20	195540	
Path	30		Stack overflows.
Cycle	5	20	
Cycle	15	123000	
Cycle	25		Stack overflows.
Star	5	10	
Star	10	150	
Star	15	3450	
Star	20	15700	
Star	25	45900	
Wheel	5	1260	
Wheel	10	125980	
Wheel	15		Stack Overflows.

The objective of this dissertation work is to remove recursion in the formula given in theorem 1. Here we have tried to develop an analytical formula for the reliability of a family of network graphs Though we have failed to determine an analytic expression for a general network graph still we have given five structural forms,for which we have found out the analytic forms.It is evident from these general forms that with the help of these forms and with some pre-processing we can evaluate the reliability of a large class of network directly.

Lastly we have outlined some results regarding the increase or decrease in the reliability of a network due to addition or deletion of one or more links. This incrmental information about the reliability would help one in designing a network with a given number of nodes within a given reliability range.

2.1. Notations :

$r(i)$ = Number of nodes in K_n adjacent to the node x_i.

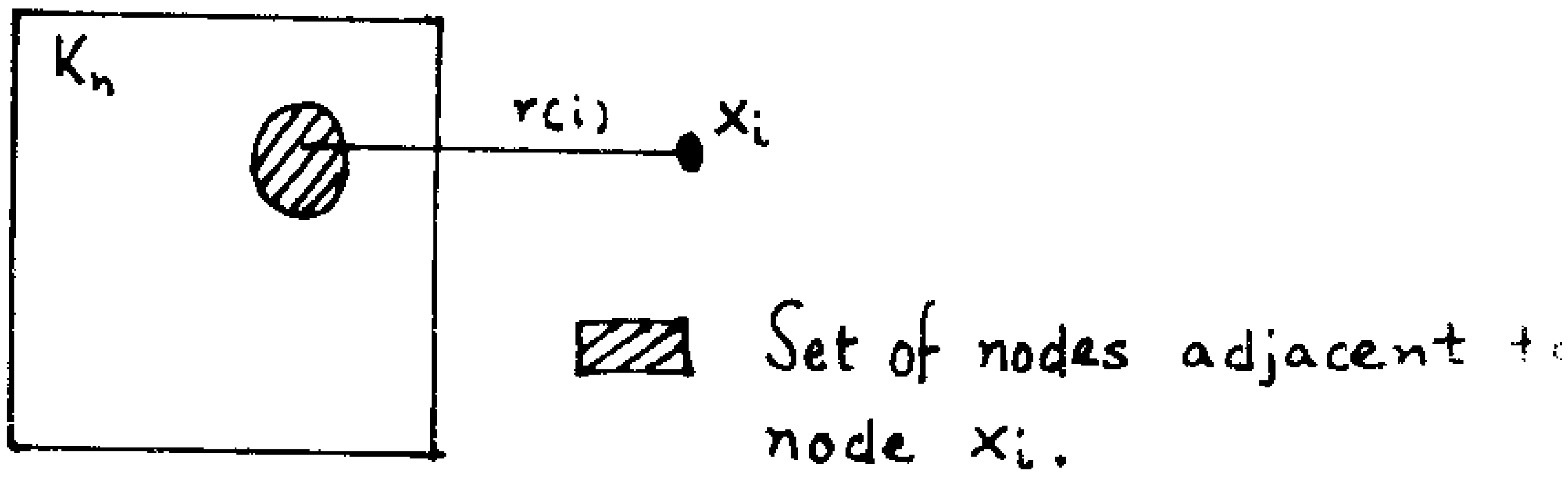

fig: 3

$r(i,j)$ = Number of nodes in K_n adjacent to both the node x_i & x_j

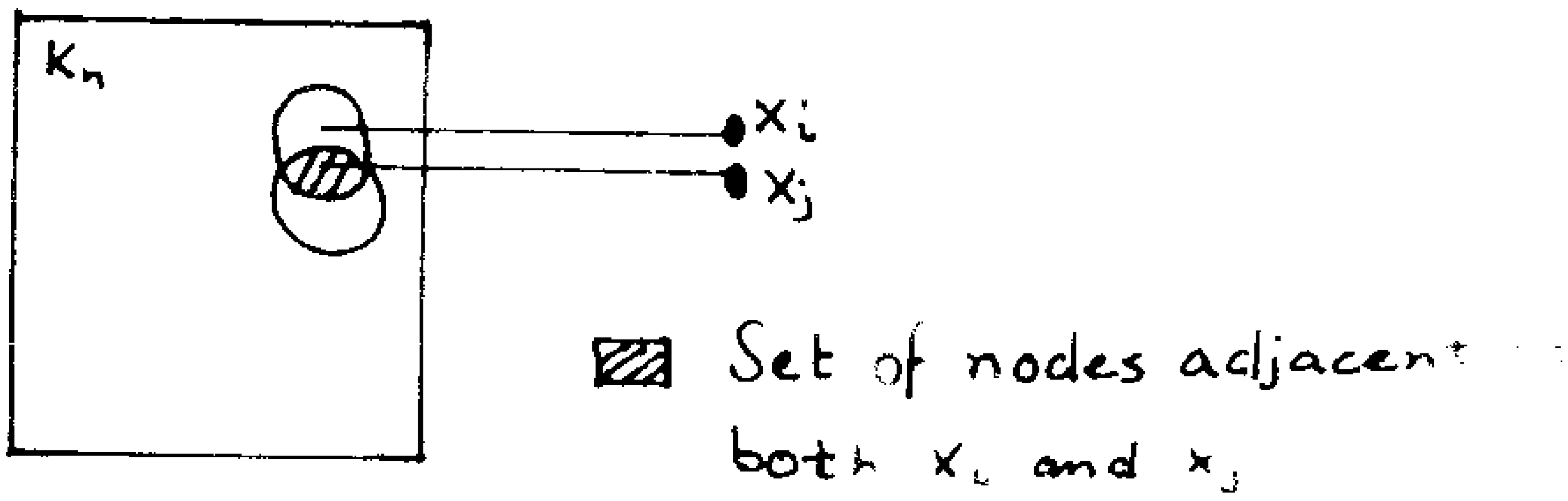

fig: 4

$\bar{r}(i,j)$ = Number of nodes in K_n adjacent to the node x_i but not adjacent to node x_j.

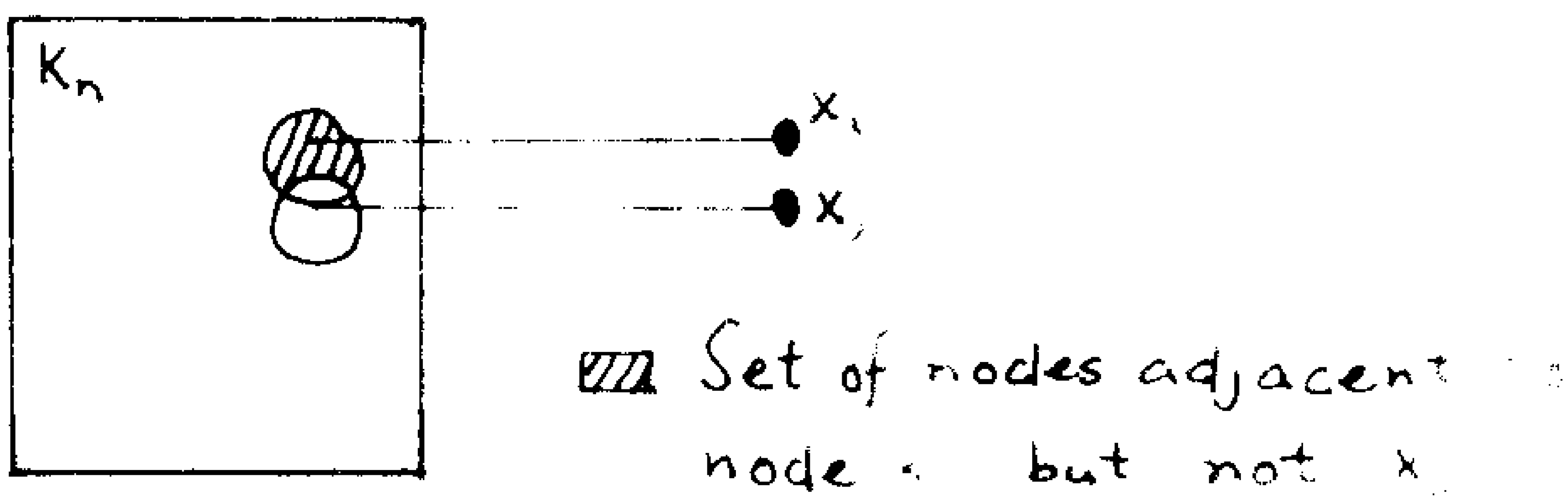

fig : 5

$S_{i,j}$ = Number of nodes in K_n adjacent to nodes X_i or X_j or both.

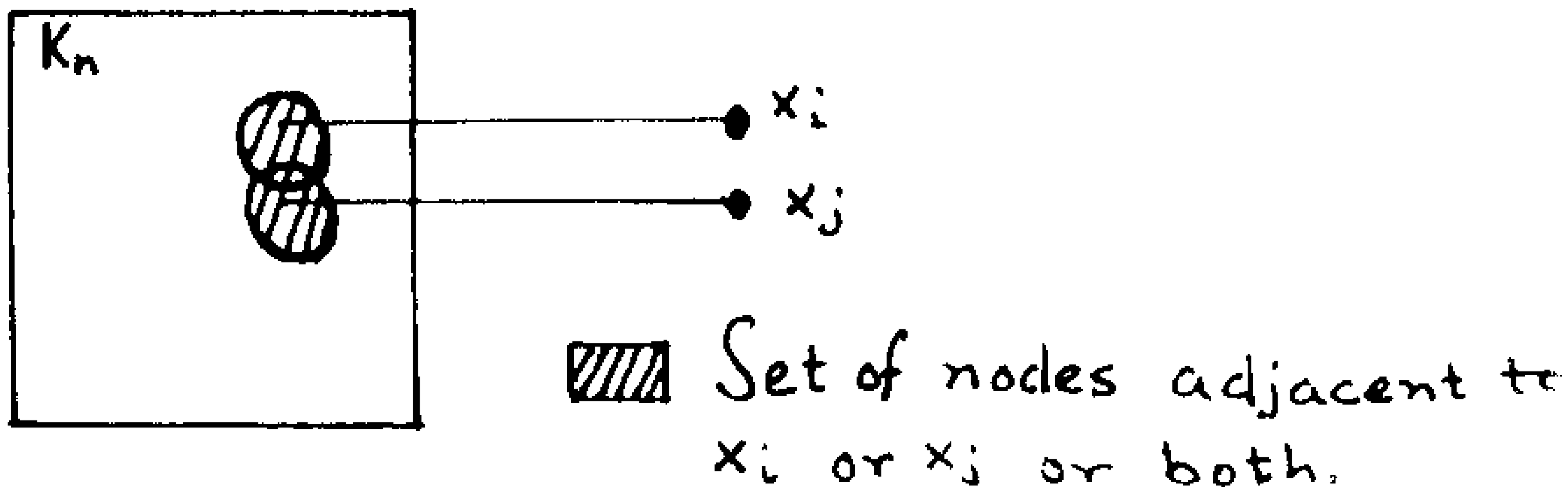

fig : 6

$S_{i,j,\bar{k}}$ = Number of nodes in K_n adjecent to nodes X_i or X_j or bothbut not adjacent to node X_k.

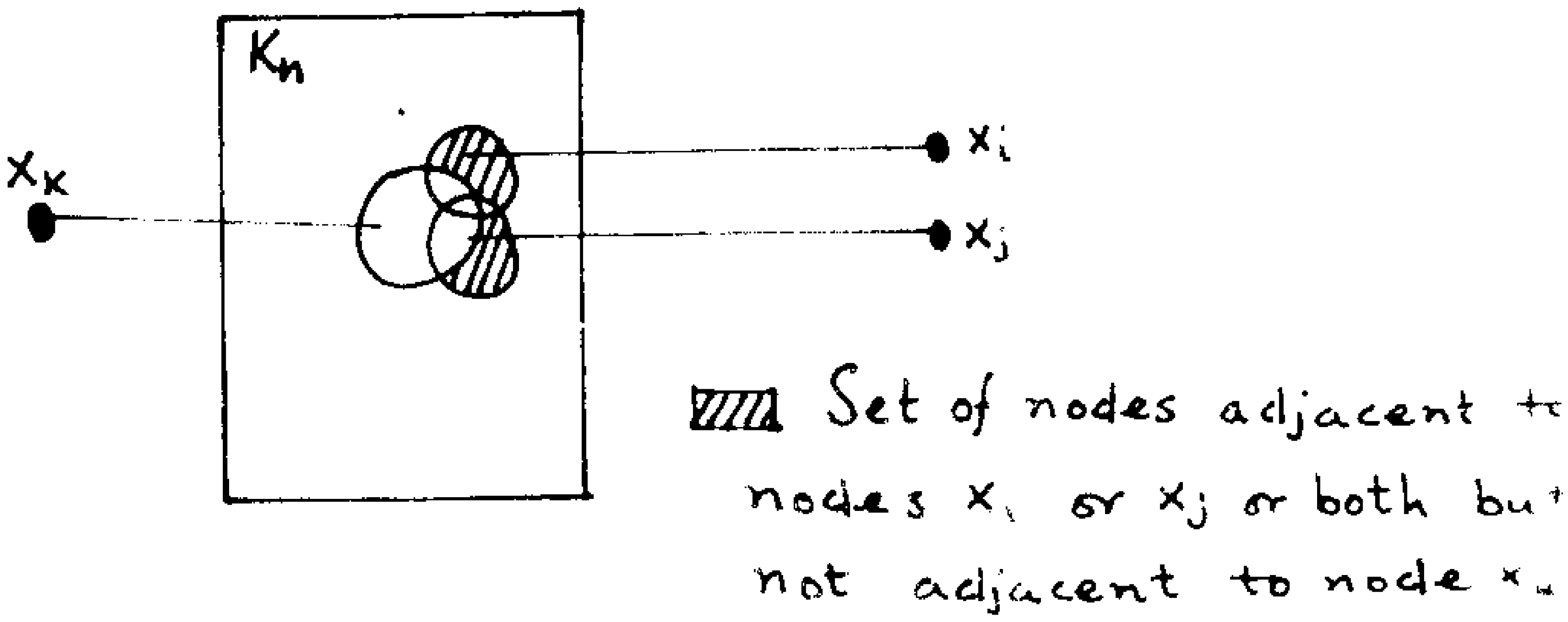

fig : /

2.2. Steps Towards Generalization :

In the subsequent chapter we try to generalize the theorem(1) step by step.

Case 1

First we give the general formula of a network consisting of a complete graph say K_n and some m number of nodes outside this K_n. These m nodes do not have links in between them but they can have links with the nodes of K_n. The maximum degree of these m no. of nodes will be n.

Case 2

In this case we consider the case same as above the case (1) The only difference is that the m number of nodes that lie outside the complete graph K_n form itself a complete graph K_m.

Case 3

In this case we consider another subgraph of say t number of nodes along with the structure as in case (2). There are no links in between these t number of nodes,but there may be links with these t number of nodes and the nodes in the subgraph K_n.

Case 4

In this case we try to re-generalize the case(2). Here we consider the same network-graph as in case(2) with an additional node say X_m outside those two complete graph. There may be links in between the nodes X_m with the nodes of K_n and K_m.

Case 5

Lastly we re-generalize the case(4). Here we consider three complete sub graph K_n, K_m, K_t.There may be links in between these three complete sub-graph. The maximum degree for any node in K_n, K_m, K_t will be (n+m+t). Also the minimum degree of any node in K_n will be [illegible] and similarly for other.

2.3. Structure 1 :

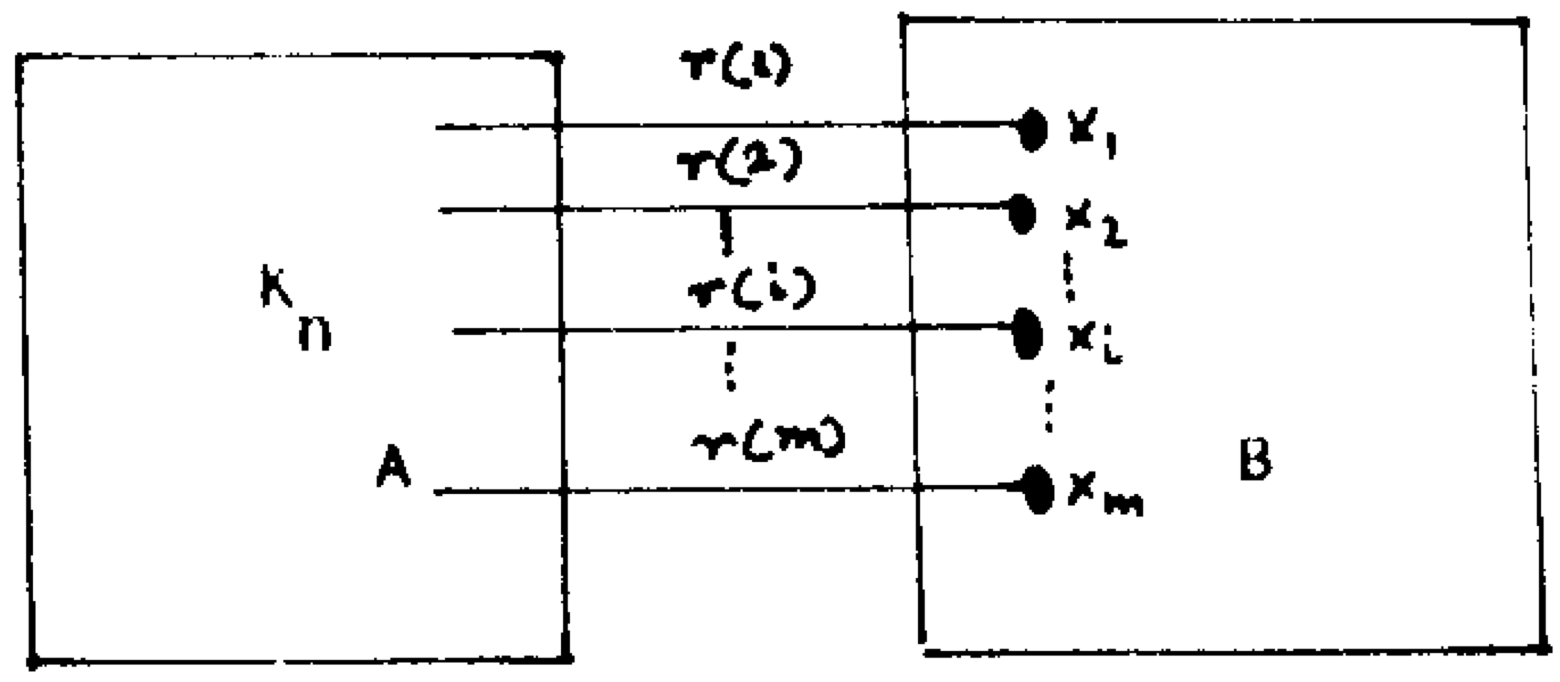

fig : 8

Description : "A" is a complete graph of n number of nodes. Subgraph "B" has m number of nodes and there are no links in between these m number of nodes. The nodes in "B" are numbered from 1 to m. Node i has r_i number of adjacent nodes in subgraph "A". Maximum value of r_i will be equal to n and its minimum value is 0.

Notation : The Reliabilty of the network is denoted by ⇒

$$R(G^n(r_1, r_2, \ldots, r_m))$$

Theorem 2 :

$$R(G^n(r_1, r_2, \ldots, r_m)) = 1 + mp(q)^{n+(m-1)} \quad [illegible] \quad p\sum_{i=1}^{m}(q)^{r_i}$$

$$+ p^2 \sum_{\substack{i,j=1 \\ i<j}}^{m}(q)^{S_{i,j}} \ldots + (-1)^m p^m q^{S_{1,2,\ldots,m}}$$

Proof :

Proof by induction on **m** ⇒

$$R(G^n(r_1, r_2, \ldots, r_{m+1}))$$

$$= q[R(G^n(r_1, r_2, \ldots, r_m))] + p[R(G^n(r_1, r_2, \ldots, r_m))]$$

$$+ p(q)^{m+n} \quad p(q)^{r_{m+1}} \{R(G^{r_{m+1}}(r_{1,m+1}, r_{2,m+1}, \ldots, r_{m,m+1}))\}$$

.... by Theorem(1).

$$= \{1 + mp(q)^{n+(m-1)} - p \sum_{i=1}^{m}(q)^{n(i)}$$

$$+ p^2 \sum_{\substack{i,j=1 \\ i<j}}^{m} (q)^{S(i,j)} \ldots + (-1)^m p^m q^{S(1,2,3,\ldots,m)} \}$$

$$+ p(q)^{m+n}$$

$$-p(q)^{n(m+1)} [1 + mp(q)^{n(m+1)+(m-1)} - \{p\sum(q)^{n(i,m+1)} - p^2 \sum_{i=1}(q)^{S(i,j,m+1)}$$

$$+(-1)^m p^m (q)^{S(1,2,\ldots,m,m+1)} \}]$$

$$= 1 + [mp(q)^{n+(m-1)} + p(q)^{m+n}] - p \sum_{i=1}^{m}(q)^{n(i)}$$

$$+ p^2 \sum_{\substack{i,j=1 \\ i<j}}^{m} (q)^{S(i,j)} \ldots\ldots\ldots\ldots + (-1)^m p^m q^{S(1,2,3,\ldots,m)}$$

$$-[p(q)^{n(m+1)} + mp^2(q)^{n+(m-1)} \{p^2 \sum_{i=1}^{m}(q)^{S(i,m+1)} - p^3 \sum_{\substack{i,j=1 \\ i<j}}^{m}(q)^{S(i,j,m+1)}$$

$$+ \ldots\ldots + (-1)^m p^m (q)^{S(1,2,\ldots,m,m+1)} \}]$$

$$= 1 + (m+1)p(q)^{n+m} - p \sum_{i=1}^{m+1} (q)^{n(i)}$$

$$+ p^2 \sum_{\substack{i,j=1 \\ i<j}}^{m+1} (q)^{S(i,j)} + (-1)^{m+1} p^{m+1} q^{S(1,2,3,\ldots,m+1)}$$

Examples :

(1)

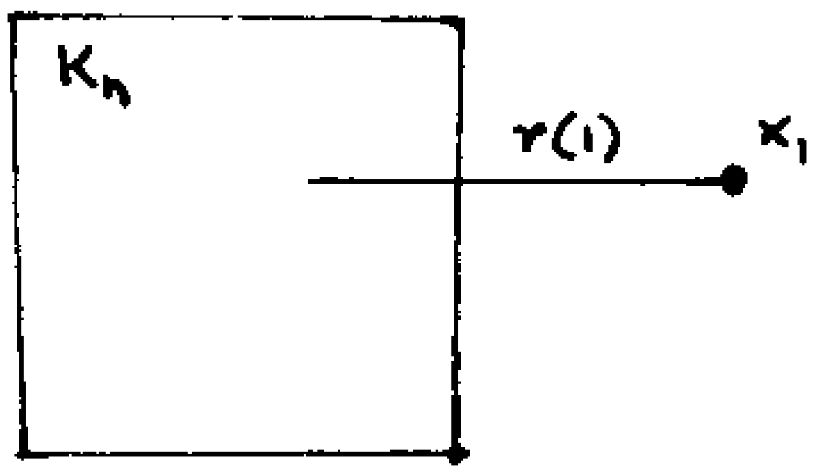

fig : 9

$$R(G^n(r_{(1)})) = 1 + p(q)^n - p(q)^{r(1)}$$

(2)

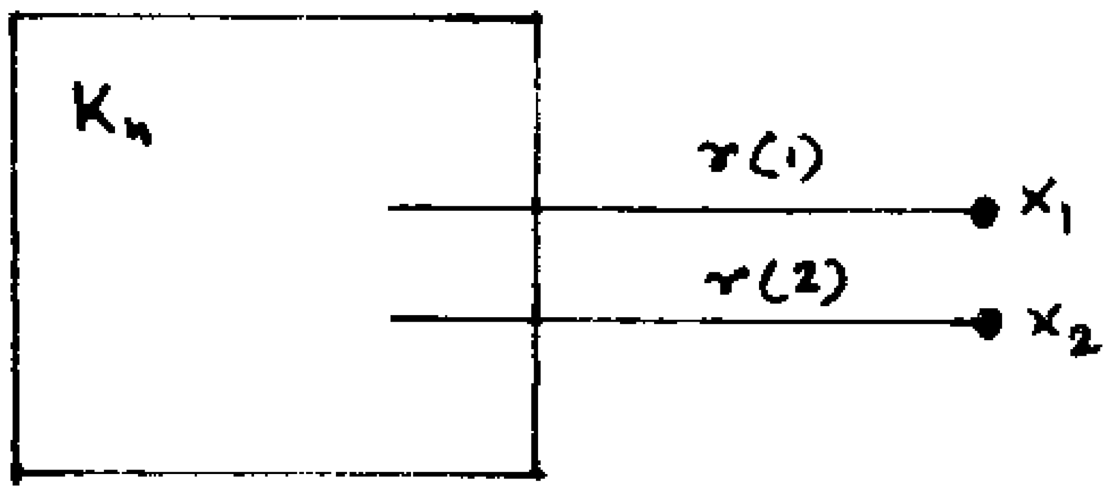

fig : 10

$$R(G^n(r_{(1)},r_{(2)})) = 1 + 2p(q)^{n+1} - [p(q)^{r(1)} + p(q)^{r(2)} - p^2(q)^{s(1,2)}]$$

2.4. Structure 2 :

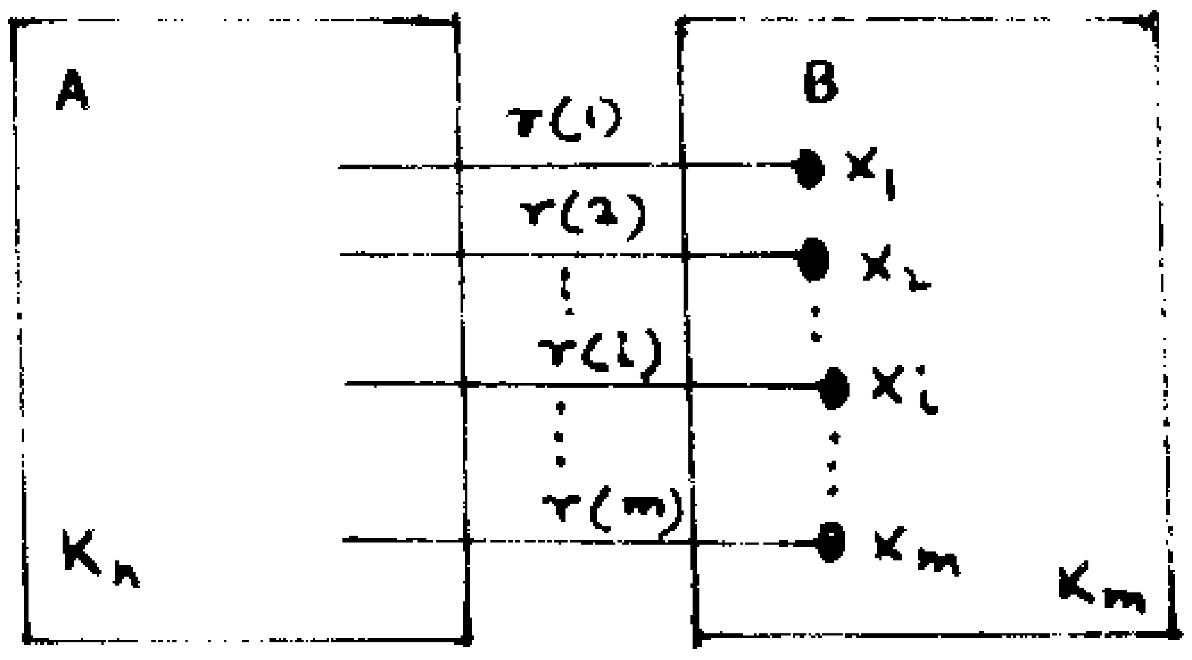

fig : 11

Description :

Graph A is a complete graph of n nodes and graph B is also a complete graph of nodes m. There are r_i nodes in A which are adjacent to node-1 of B, r_2 nodes in are A adjacent to node-2 in B, and so on.

Notation :

The Reliability of the network is denoted by ⇒

$$R(G^n[r_1, r_2, \ldots, r_m])$$

Theorem 3 :

$$R(G^n[r_1, r_2, \ldots, r_m])$$
$$= 1 + (q^n - q^{n+m})$$
$$- [\, p(q)^{m-1} \sum_{i=1}^{m} (q)^{r_i}$$
$$+ p^2 (q)^{m-2} \sum_{\substack{i,j=1 \\ i>j}}^{m} (q)^{S(i,j)} \ldots\ldots + p^m (q)^{S(1,2,3,\ldots,m)}]$$

Proof :

We shall prove the above result by induction on m.We assume that the result is true for any m and we show that it is also true for (m+1)

Proof by induction on **m** ⇒

$$R(G^n[r_1, r_2, \ldots r_{m+1}])$$
$$= q[R(G^n[r_1, r_2, \ldots r_{m+1}])]$$
$$+ p[R(G^n[r_{1\,m+1}, r_{2\,m+1}, \ldots r_{m\,m+1}])] - p[1 - (q)^{r_{m+1}}](q)^{m + r_{m+1}}$$

....By theorem (1).

$$=q[1+(q^{n}-q^{n+m}) - [p(q)^{m-1}\sum_{i=1}^{m}(q)^{r(i)}$$

$$p^{2}(q)^{m-2}\sum_{\substack{i,j=1\\ i<j}}^{m}(q)^{S(i,j)} \;......+p^{m}(q)^{S(1,2,3\ldots m)}]]$$

$$+[p+(pq^{n}-pq^{n+m}) - [p^{2}(q)^{m-1}\sum_{i=1}^{m}(q)^{S(i,m+1)}$$

$$p^{3}(q)^{m-2}\sum_{\substack{i,j=1\\ i<j}}^{m}(q)^{S(i,j,m+1)} \;......+p^{m+1}(q)^{S(1,2,3\ldots m,m+1)}]]$$

$$-p(q)^{m+r(m+1)} + p(q)^{n+m}$$

$$=1+(q^{n+1}-q^{n+m+1}+pq^{n}) - [p(q)^{m}\sum_{i=1}^{m+1}(q)^{r(i)}$$

$$+p^{2}(q)^{m-1}\sum_{\substack{i,j=1\\ i<j}}^{m+1}(q)^{S(i,j)} \;......+p^{m+1}(q)^{S(1,2,3\ldots m+1)}]$$

$$=1+(q^{n}-q^{n+m+1}) - [p(q)^{m}\sum_{i=1}^{m+1}(q)^{r(i)}$$

$$+p^{2}(q)^{m-1}\sum_{\substack{i,j=1\\ i<j}}^{m+1}(q)^{S(i,j)} \;......+p^{m+1}(q)^{S(1,2,3\ldots m+1)}]$$

.... Hence the proof.

Examples :

The following result can easily be shown to be true by applying theorem (1).

(1)

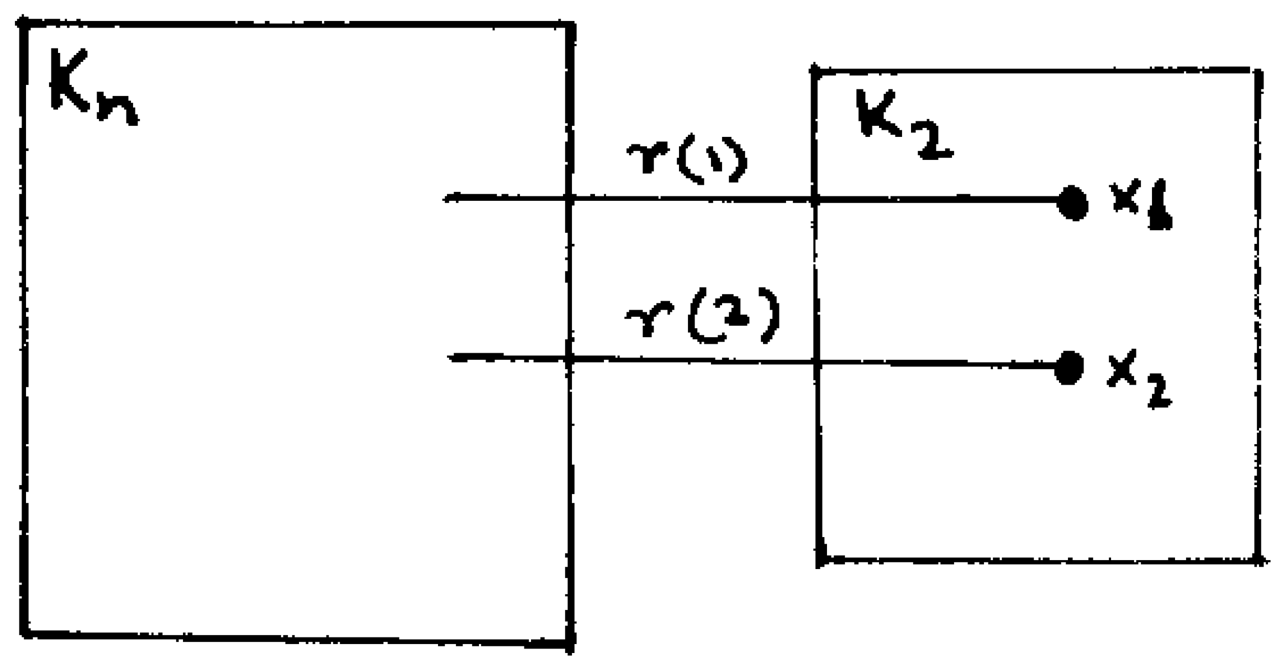

fig : 12

$$R(G^n[r(1),r(2)]) = 1 + (q^n - q^{n+2}) - [p(q)^{r(1)+1} + p(q)^{r(2)+1} + p^2(q)^{S(1,2)}]$$

(2).

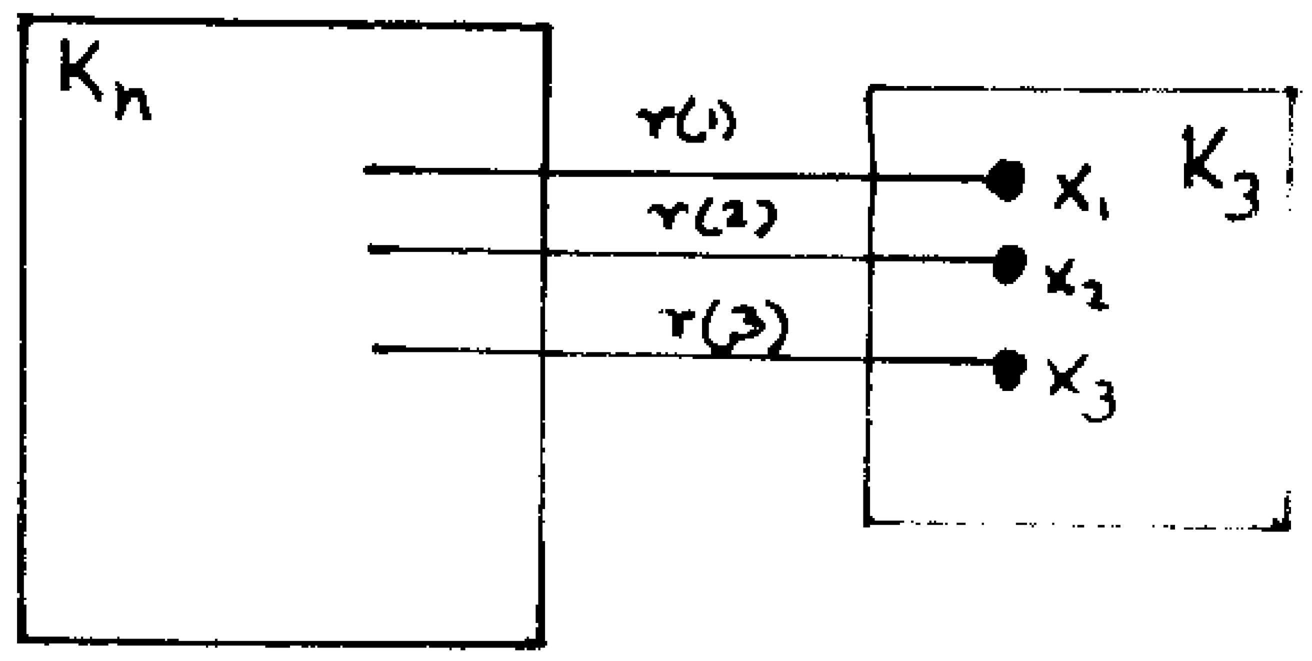

fig : 13

$$R(G^n[r(1),r(2),r(3)])$$

$$= 1 + (q^n - q^{n+2}) - [p(q)^{r(1)+2} + p(q)^{r(2)+2} + p(q)^{r(3)+2}$$

$$+ p^2(q)^{S(1,2)+1} + p^2(q)^{S(1,3)+1} + p^2(q)^{S(2,3)+1}$$

$$+ p^3(q)^{S(1,2,3)}]$$

2.5. Structure 3 :

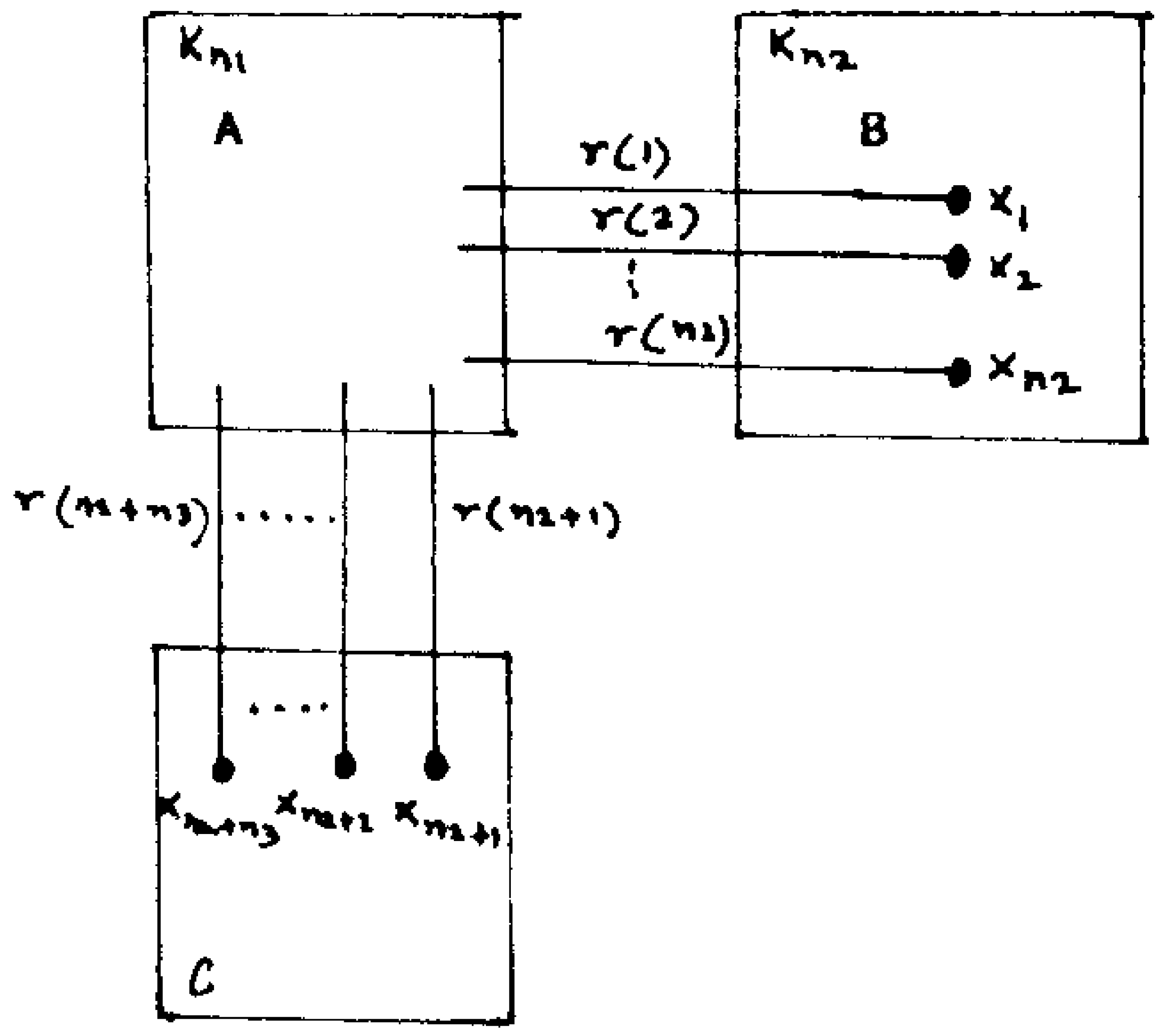

fig : 14

Description :

Graph A is a complete graph of n1 nodes and graph B is also a complete graph of nodes n2,and graph C consists of n3 nodes and no edges in between these n3 nodes.

The nodes in B is numbered from 1 to n2 and nodes in C is numbered from (n2+1) to (n2+n3).

There are r_1 nodes in A which are adjacent to node-1 of B, r_2 nodes in are A adjacent to node-2 in B, and so on.

Similarly there are r_{n2+1} nodes in A which are adjacent to node (n2+1) of C, r_{n2+2} nodes in are A adjacent to node (n2+2) in C, and so on.

There are no direct links between graph B and graph C.

Notation :

The Reliabilty of the network is denoted by →

$$R(G^{n}[r(1), r(2), \ldots, r(n2)](r(n2+1), r(n2+2), \ldots, r(n2+n3)))$$

Theorem 4 :

$$R(G^{n}[r(1), r(2), \ldots, r(n2)](r(n2+1), r(n2+2), \ldots, r(n2+n3)))$$

$$= 1 + (q^{n1+n3} - q^{n1+n2+n3}) + (n3)p(q)^{n1+n2+n3-1}$$

$$- [p \sum_{i=n2+1}^{n3} (q)^{r(i)} - p^{2} \sum_{\substack{i,j=n2+1 \\ i<j}}^{n3} (q)^{S(i,j)} \ldots$$

$$(-1)^{n3-1} p^{n3} (q)^{S(n2+1, n2+2, \ldots, n2+n3)}]$$

$$-pq^{n2-1} \sum_{i=1}^{n2} [(q)^{r(i)} - p \sum_{j=n2+1}^{n3} (q)^{S(i,j)} + p^{2} \sum_{\substack{k,j=n2+1 \\ k<j}}^{n3} (q)^{S(i,k,j)} \ldots$$

$$(-1)^{n3-1} p^{n3} (q)^{S(i, n2+1, n2+2, \ldots, n2+n3)}]$$

$$-p^{2} q^{n2-2} \sum_{\substack{j,i=1 \\ i<j}}^{n2} [(q)^{S(i,j)} - p \sum_{k=n2+1}^{n3} (q)^{S(i,j,k)} + p^{2} \sum_{\substack{k,l=n2+1 \\ k<l}}^{n3} (q)^{S(i,j,k,l)} + \ldots$$

$$+(-1)^{n3} p^{n3} (q)^{S(i,j,n2+1,n2+2, \ldots, n2+n3)}]$$

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$$-p^{n2} [(q)^{S(1,2,\ldots,n2)} - p \sum_{i=n2+1}^{n3} (q)^{S(1,2,\ldots,n2,i)} + p^{2} \sum_{i,j=n2+1}^{n3} (q)^{S(1,2,\ldots,n2,i,j)} + \ldots$$

$$+(-1)^{n3} p^{n3} (q)^{S(1,2,\ldots,n2,n2+1,n2+2,\ldots,n2+n3)}]$$

Proof :

We shall prove the above result by induction on n3.We assume that the result is true for any n3 and we show that it is also true for (n3+1)

Proof by induction on n3 ⇒

$$R(G^{n}[r(1),r(2),\dots r(n2)](r(n2+1),r(n2+2),\dots\dots,r(n2+n3+1)))$$

$$=q[\ R(G^{n}[r(1),r(2),\dots,r(n2)](r(n2+1),r(n2+2),\dots\dots,r(n2+n3)))\ ]$$

$$+p[\qquad R(G^{n}[r(1),r(2),\dots\dots,r(n2)](r(n2+1),r(n2+2),\dots\dots,r(n2+n3)))$$

$$-p(q)^{r(n3+1)}\ [R(G^{r(n3+1)}[r(1,\overline{n3+1}),r(2,\overline{n3+1}),\dots r(n2,\overline{n3+1})]$$

$$(r(n2+1,\overline{n3+1}),r(n2+2,\overline{n3+1}),\dots\dots,r(n2+n3,\overline{n3+1})))]$$

$$+p(q)^{n1+n2+n3}$$

By theorem (1).

$$=R(G^{n}[r(1),r(2),\dots r(n2)](r(n2+1),r(n2+2),\dots\dots,r(n2+n3)))$$

$$-p(q)^{r(n3+1)}\ [R(G^{r(n3+1)}[r(1,\overline{n3+1}),r(2,\overline{n3+1}),\dots r(n2,\overline{n3+1})]$$

$$(r(n2+1,\overline{n3+1}),r(n2+2,\overline{n3+1}),\dots\dots,r(n2+n3,\overline{n3+1})))]$$

$$+p(q)^{n1+n2+n3}$$

$$=1+(q^{n1+n3}-q^{n1+n2+n3})+p(q)^{n1+n2+n3}+(n3)p(q)^{n1+n2+n3-1}$$

$$-[p\sum_{i=n2+1}^{n3}(q)^{r(i)}-p^{2}\sum_{i=n2+1}^{n3}(q)^{S(i,j)}\ \dots\dots$$

$$(-1)^{n3+1}p^{n3}(q)^{S(n2+1,n2+2,\dots,n2+n3)}\ ]$$

$$-pq^{n2-1}\sum_{i=1}^{n2}[(q)^{r(i)} - p\sum_{j=n2+1}^{n3}(q)^{S(i,j)} + p^{2}\sum_{\substack{k,j=n2+1\\ k<>j}}^{n3}(q)^{S(i,j,k)}\dots$$

$$(-1)^{n3}p^{n3}(q)^{S(i,n2+1,n2+2,\dots,n2+n3)}]$$

$$-p^{2}q^{n2-2}\sum_{\substack{j,i=1\\ i<>j}}^{n2}[(q)^{S(i,j)} - p\sum_{k=n2+1}^{n3}(q)^{S(i,j,k)} + p^{2}\sum_{\substack{k,l=n2+1\\ k<>l}}^{n3}(q)^{S(i,j,k,l)} + \dots$$

$$+(-1)^{n3}p^{n3}(q)^{S(i,j,n2+1,n2+2,\dots,n2+n3)}]$$

.
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.

$$-p^{n2}[(q)^{S(1,2,\dots,n2)} - p\sum_{k=n2+1}^{n3}(q)^{S(1,2,\dots,n2,k)} + p^{2}\sum_{\substack{k,l=n2+1\\ k<>l}}^{n3}(q)^{S(1,\dots,n2,k,l)} + \dots$$

$$+(-1)^{n3}p^{n3}(q)^{S(1,\dots,n2,n2+1,n2+2,\dots,n2+n3)}]$$

$$-p(q)^{r(n3+1)}[1+(q^{r(n3+1)+n3} - q^{\overline{r(n3+1)}+n2+n3}) + (n3)p(q)^{\overline{r(n3+1)}+n2+n3}]$$

$$-[p^{2}\sum_{i=n2+1}^{n3}(q)^{S(i,n3+1)} - p^{3}\sum_{\substack{i,j=n2+1\\ i<>j}}^{n3}(q)^{S(i,j,n3+1)}\dots$$

$$(-1)^{n3-1}p^{n3+1}(q)^{S(n2+1,n2+2,\dots,n2+n3+1)}]$$

$$-pq^{n2-1}\sum_{i=1}^{n2}[p(q)^{S(i,n3+1)} - p^{2}\sum_{j=n2+1}^{n3}(q)^{S(i,j,n3+1)} + p^{3}\sum_{\substack{k,j=n2+1\\ k<>j}}^{n3}(q)^{S(i,j,k,n3+1)}$$

$$(-1)^{n3}p^{n3+1}(q)^{S(i,n2+1,n2+2,\dots,n2+n3+1)}]$$

$\vdots$

$$-p^{n2}[p(q)^{S(1,2,..n2,n3+1)} - p^{2}\sum_{k=n2+1}^{n3}(q)^{S(1,2,..n2+1,k,n3+1)} + \dots$$

$$+(-1)^{n3}p^{n3}(q)^{S(1,..n2,n2+1,n2+2,...n2+n3)}]$$

$$=1 + (q^{n1+n3+1} - q^{n1+n2+n3+1}) + (n3+1)p(q)^{n1+n2+n3}$$

$$-[p\sum_{i=n2+1}^{n3+1}(q)^{n_i} - p^{2}\sum_{\substack{i,j=n2+1\\ i\neq j}}^{n3+1}(q)^{S(i,j)} \dots$$

$$(-1)^{n3}p^{n3+1}(q)^{S(n2+1,n2+2,...n2+n3+1)}]$$

$$-pq^{n2-1}\sum_{i=1}^{n2}[(q)^{n_i} - p\sum_{j=n2+1}^{n3+1}(q)^{S(i,j)} + p^{2}\sum_{k,j=n2+1}^{n3+1}(q)^{S(i,j,k)} \dots$$

$$(-1)^{n3+1}p^{n3+1}(q)^{S(i,n2+1,n2+2,...n2+n3+1)}]$$

$$-p^{2}q^{n2-2}\sum_{\substack{j,i=1\\ i\neq j}}^{n2}[(q)^{S(i,j)} - p\sum_{k=n2+1}^{n3+1}(q)^{S(i,j,k)} + p^{2}\sum_{k,l=n2+1}^{n3+1}(q)^{S(i,j,k,l)} + \dots$$

$$+(-1)^{n3+1}p^{n3+1}(q)^{S(i,j,n2+1,n2+2,...n2+n3+1)}]$$

$\vdots$

$$-p^{n2}\left[(q)^{S(1,2\ldots n2)} - p\sum_{k=n2+1}^{n9+1}(q)^{S(1,2\ldots n2,k)} + p^2\sum_{k,l=n2+1}^{n9+1}(q)^{S(1\ldots n2,k,l)} + \ldots\right.$$

$$\left. + (-1)^{n3+1}p^{n3+1}(q)^{S(1\ldots n2,n2+1,n2+2,\ldots n2+n3+1)}\right]$$

....Hence the proof.

Examples :

(1)

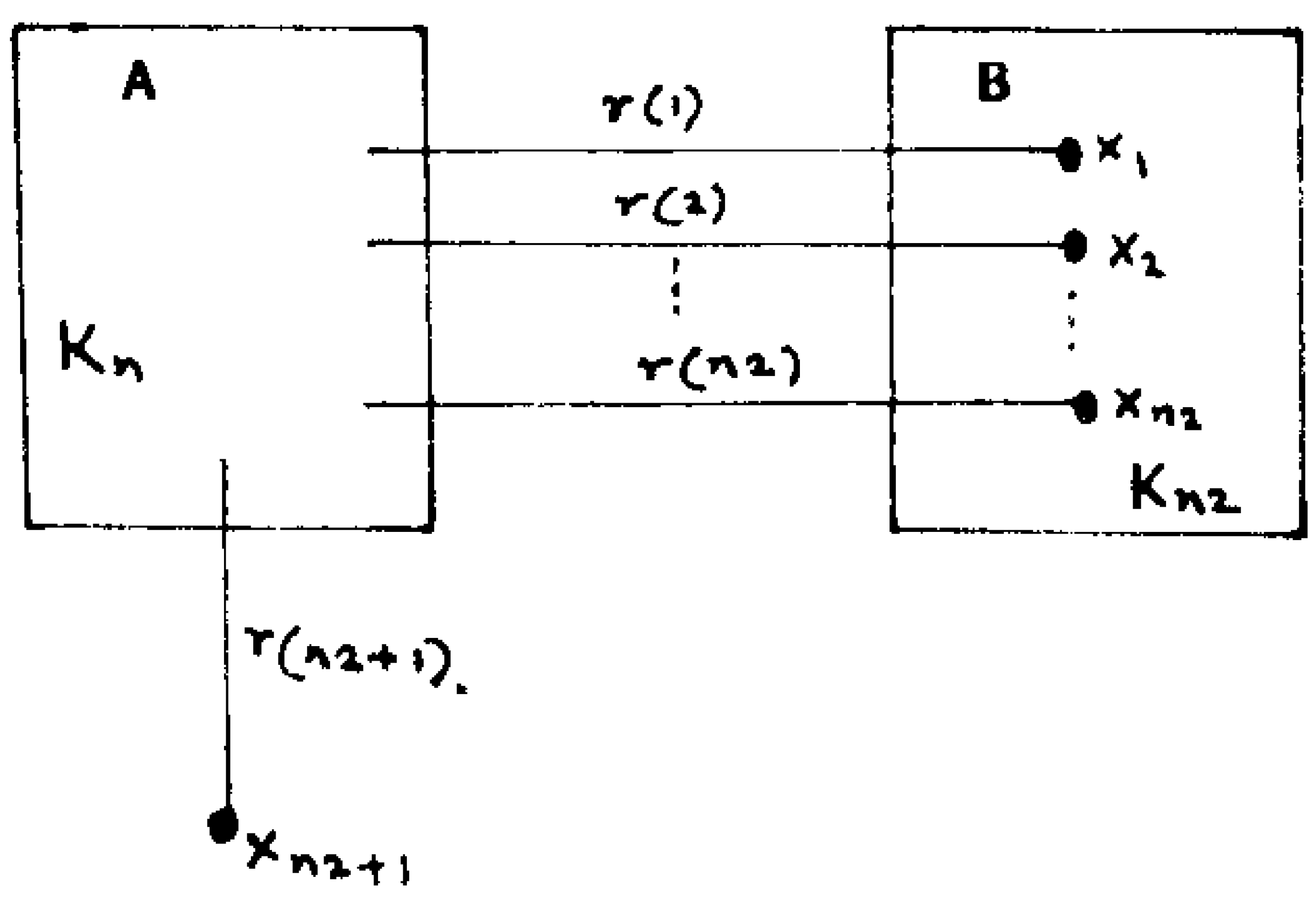


fig : 15

By applying theorem(1) it can easily shown that the reliability of the above network is given by the following analytical form ⟶

$$R(G^n[r_1, r_2, \ldots r_{n2}](r_{n2+1}))$$

$$= 1 + (q^{n+1} - q^{n+n2+1}) + p\,q^{n+n2} - pq^{r_{n2+1}}$$

$$pq^{n\ldots 1} \quad \sum_{[illegible]}^{n2} (q^{r\ldots} - pq^{S(1,n2+1)})$$

$$-p^{2}q^{n2-2}\sum_{\substack{i,j=1\\i<j}}^{n2}\left(q^{S(i,j)}-pq^{S(i,j,n2+1)}\right)$$

.

.

.

$$p^{n2}\left(q^{S(1,2,\ldots,n2)}-pq^{S(1,2,\ldots,n2,n2+1)}\right)$$

(2).

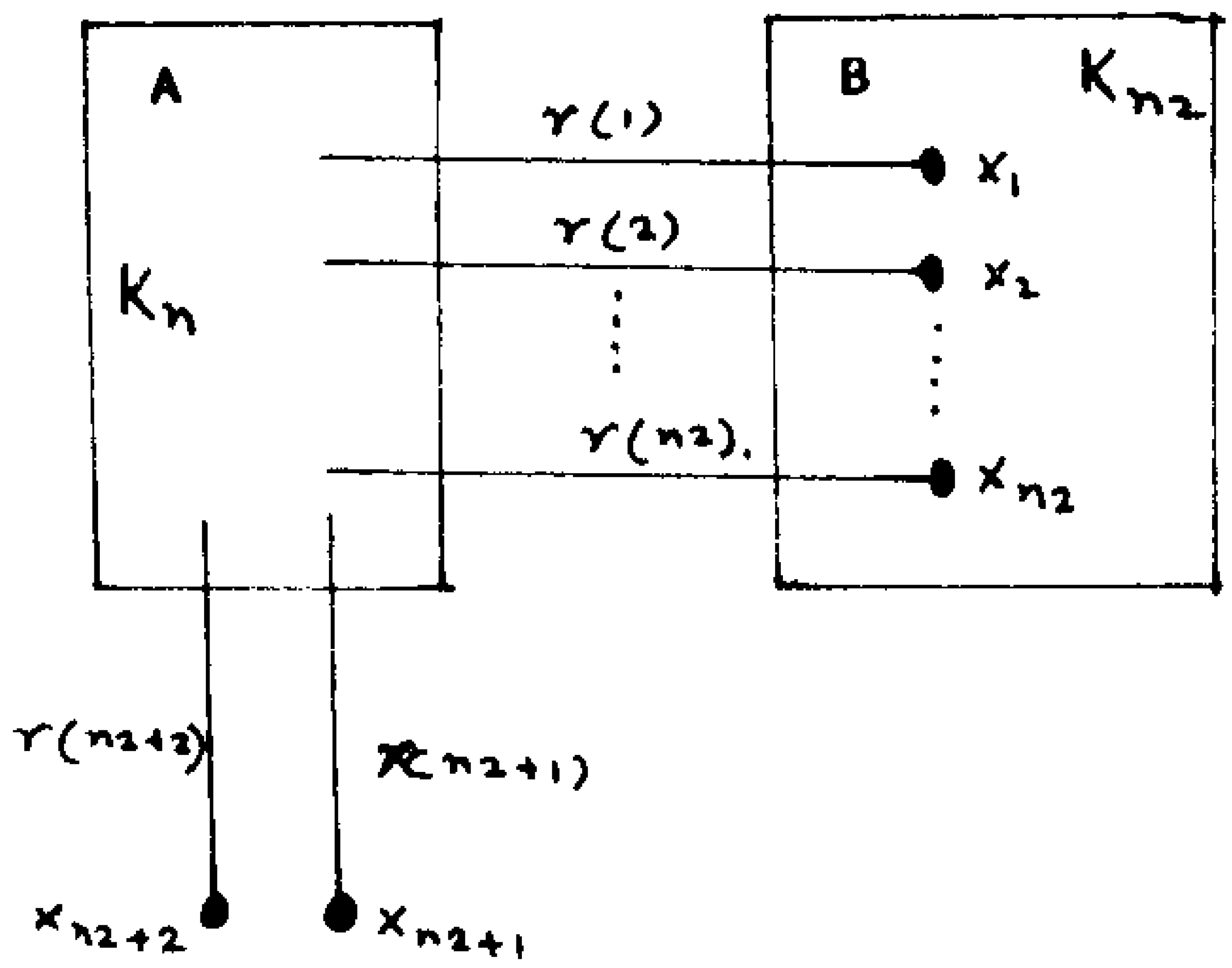


fig : 16

By applying theorem(1) it can easily shown that the reliability of the above network is given by the following analytical form ⟶

$$R(G^{n}[r(1),r(2),\ldots,r(n2)](r(n2+1),r(n2+2)))$$

$$= 1+(q^{n+2}-q^{n+n2+2})+2pq^{n+n2+1}$$

$$\left(pq^{r(n2+1)}+pq^{r(n2+2)}-p^{2}q^{S(n2+1,n2+2)}\right)$$

$$-pq^{n2-1} \sum_{i=1}^{n2} \left(q^{n(i)} - pq^{S(i,n2+1)} - pq^{S(i,n2+2)} + p^2 q^{S(i,n2+1,n2+2)}\right)$$

$$-p^2 q^{n2-2} \sum_{\substack{i,j=1 \\ i<>j}}^{n2} \left(q^{S(i,j)} - pq^{S(i,j,n2+1)} - pq^{S(i,j,n2+2)} + p^2 q^{S(i,j,n2+1,n2+2)}\right)$$

.

.

.

$$p^{n2}\left(q^{S(1..n2)} - pq^{S(1..n2,n2+1)} - pq^{S(1..n2,n2+2)} + p^2 q^{S(1..n2,n2+1,n2+2)}\right)$$

2.6. Structure 4 :

Say A is a complete subgraph of n nodes and B is also a complete subgraph of (m-1) nodes. There exists links in between A and B.The nodes in B is numbered 1 to (m-1).Other than these subgraphs A and B the network has a node X_M. The degree of this node is $r(m)$,i.e,there are $r(m)$ no of links associated with this node X_M.No of nodes adjacent to this node X_M,in the subgraph A,B is given by $r_A(m),r_B(m)$ respectively.It is obvious that $[r_A(m) + r_B(m) = r(m)]$

Following network generalizes the above :

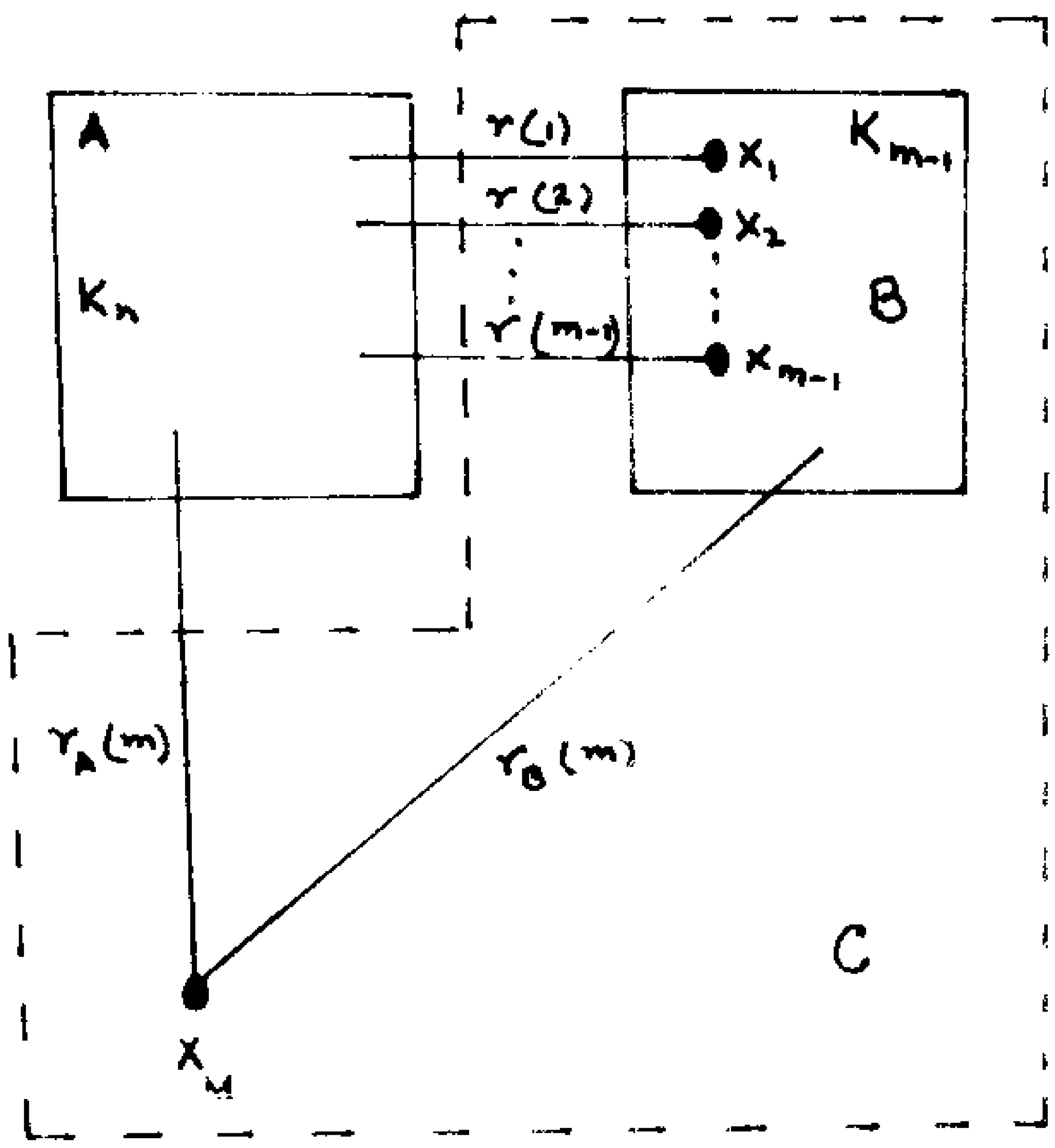


fig : 17

Notation :

$ss_A(m)$= Set of nodes in subgraph A adjacent to node X_M

$ss_{\bar{A}}(m)$= Set of nodes in subgraph A [illegible] adjacent to node X_M

$r_A(m) = |ss_A(m)|$

$\bar{r}_A(m) = |\bar{ss}_A(m)|$

$ss_B(m)$= Set of nodes in subgraph B adjacent to node X_M

$\bar{ss}_B(m)$= Set of nodes in subgraph B not adjacent to node X_M

$r_B(m) = |ss_B(m)| = k.$

$\bar{r}_B(m) = |\bar{ss}_B(m)|$

$C = B \cup \{X_m\}$

The reliabilty of the above network is denoted by ⟶

$R(G^n\ [r(1), r(2), r(3) \ldots r(m-1)]\ (r(m)\ |\ ss_A(m), ss_B(m))\ )$

Theorem 5 :

$$R(G^n\ [r(1), r(2), r(3) \ldots r(m-1)]\ (r(m)\ |\ ss_A(m), ss_B(m))\ )$$

$$= R(G^n\ [r(1), r(2), r(3) \ldots r(m)])$$

$$-[\ \sum_{i \notin ss_A(m)} (pq^{r(i)+m-2} - pq^{r(i)+m-1})$$

$$+ \sum_{\substack{i,j \notin ss_A(m) \\ i \neq j}} p(pq^{s(i,j)+m-3} - pq^{s(i,j)+m-2})$$

$$+ \ldots\ldots + p^k \sum_{\substack{i_1 \ldots i_k \notin ss_B(m) \\ i_1 \neq \ldots \neq i_k}} [q^{s(i_1 \ldots i_k)+r_B(m)} \quad q^{s(i_1 \ldots i_k)+r_B(m+1)}$$

$$]$$

$$+ 2[\ p^2 \sum_{i \notin ss_B(m)} q^{s(i,m)+m-2} + p^3 \sum_{\substack{i,j \notin ss_B(m) \\ i \neq j}} q^{s(i,j,m)+m-3}$$

$$+\ldots+ p^{k+1} q^{s(\iota_1 \ldots \iota_k, m) + r_B(m)}]$$

$$- [q^{r(m)} - q^{m-1+r_A(m)}]$$

$$- q^{n} [1 - R(C)]$$

Remarks : The proof of the above theorem follows directly from theorem(1) and theorem(8). Rigorous proof is ommited here. Also the reliability of the network "C" having the same structure as in structure (2) can easily be found out applying theorem(8) as follows--

$$R(C) = q^{r_B(m)} - q^{m-1}.$$

Examples :

(1) :

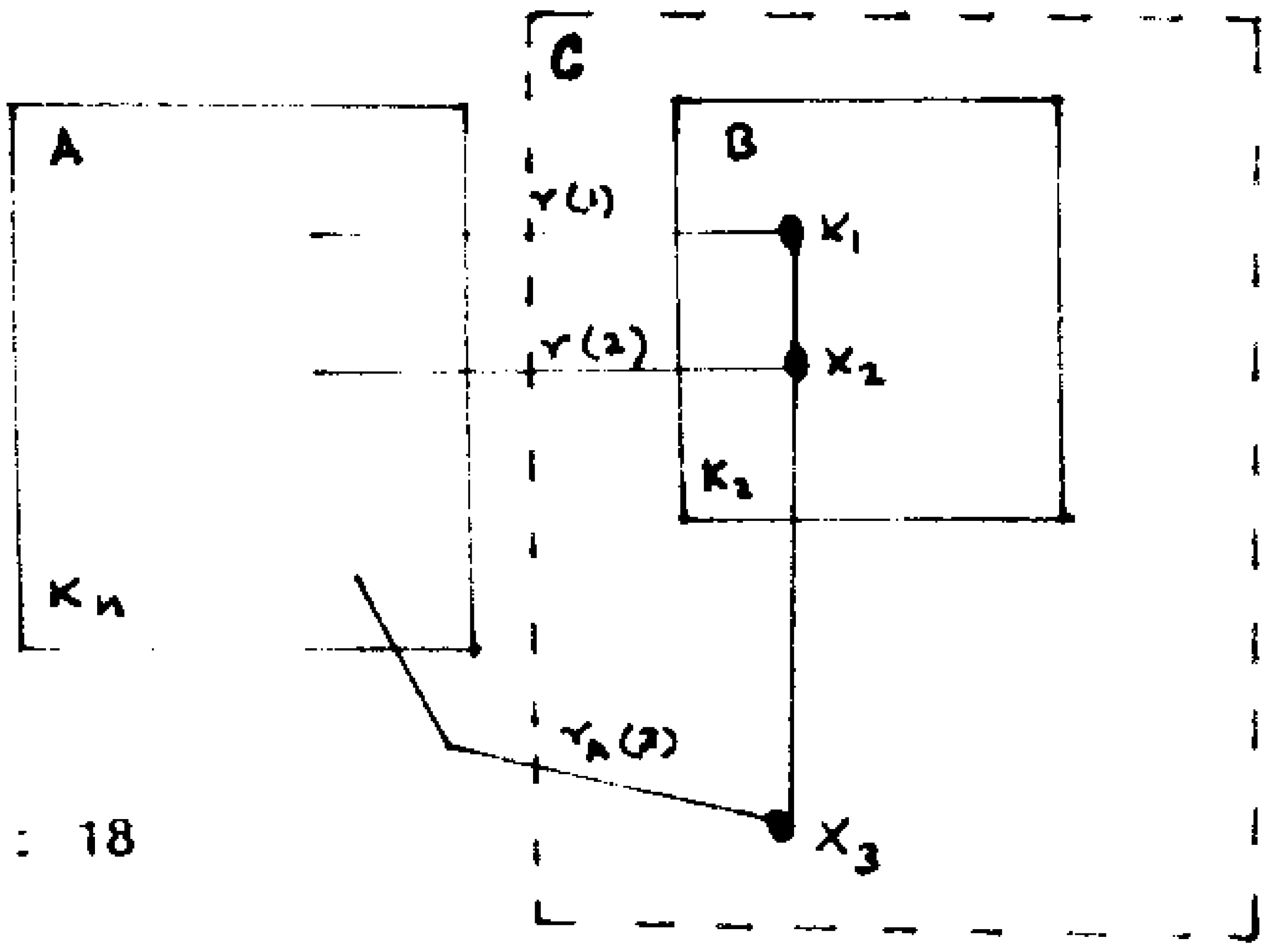


fig : 18

Result ⟶

Reliability of the above network

$$R(G^n[r(1), r(2), r(3)] - [(pq^{r(1)+1} - pq^{r(1)+2}) + (pq^{r(3)+1} - pq^{r(3)+2})$$

$$-2p^2 q^{s(1,3)+1} + (pq^{n+1} - pq^{n+2})]$$

$$= R(G^n[r(1), r(2), r(3)] - [(pq^{r(1)+1} - pq^{r(1)+2}) + (pq^{r(3)+1} - pq^{r(3)+2})$$

$$2p^2 q^{s(1,3)+1} + q^n (1 \quad R(C)]$$

(2)

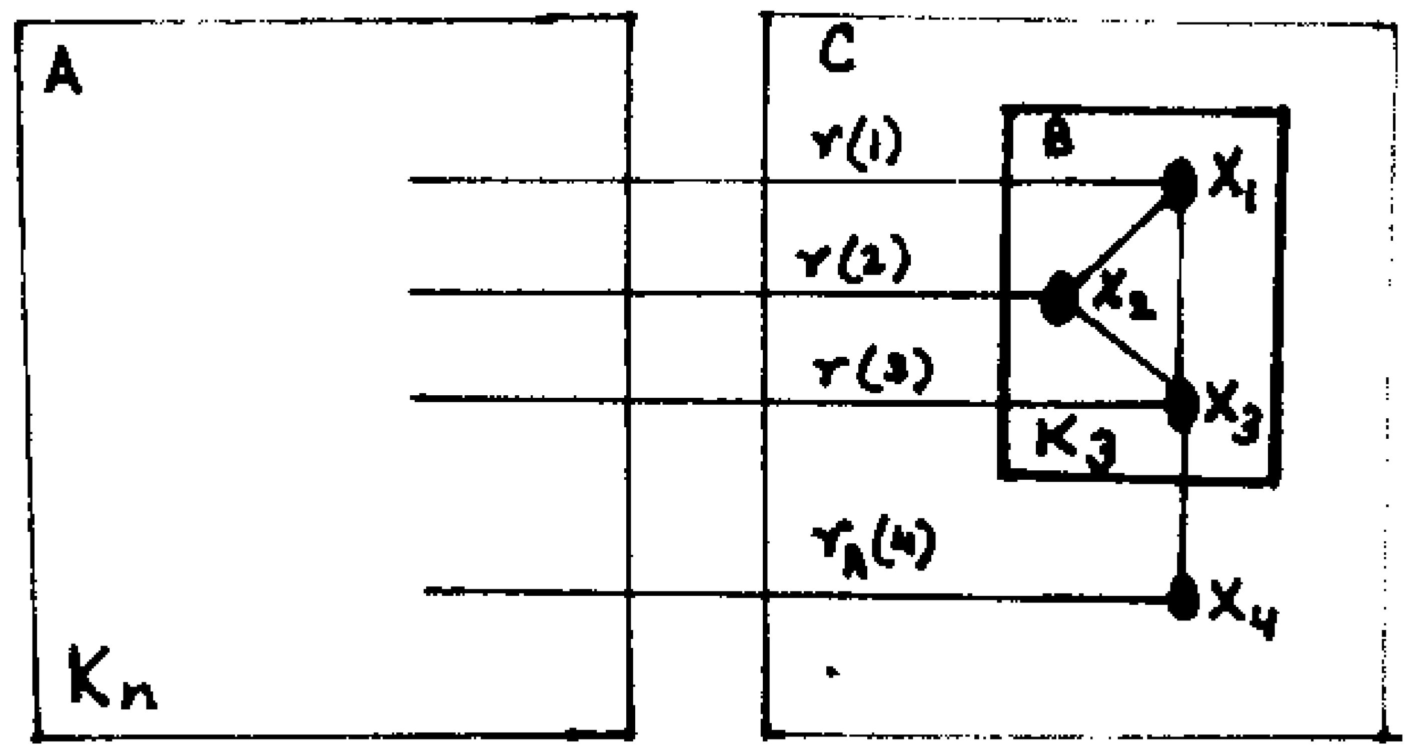


Fig : 19

Result ⟶

The relaibility of the above network is given by ⟶

$$R(G^n[r(1),r(2),r(3),r(4)]$$

$$-\ [(pq^{r(1)+2} - pq^{r(1)+3}) + (pq^{r(2)+2} - pq^{r(2)+3}) + (pq^{r(4)+1} - pq^{r(4)+3})$$

$$+p(pq^{s(1,2)+1} - pq^{s(1,2)+2})$$

$$-2p^2q^{s(1,4)+2} - 2p^2q^{s(2,4)+2} - 2p^3q^{s(1,2,4)+1}$$

$$+(pq^{n+1} - pq^{n+3})]$$

$$=R(G^n[r(1),r(2),r(3),r(4)]$$

$$-[(pq^{r(1)+2} - pq^{r(1)+3}) + (pq^{r(2)+2} - pq^{r(2)+3}$$

$$(pq^{r(4)+1} - pq^{r(4)+3}) + p(pq^{s(1,2)+1} - pq^{s(1,2)+2})$$

$$-2p^2q^{s(1,4)+2} \quad -2p^2q^{s(2,4)+2} \quad 2p^3q^{s(1,2,4)+1}$$

$$+q^n(1 - R(C))]$$

(3)

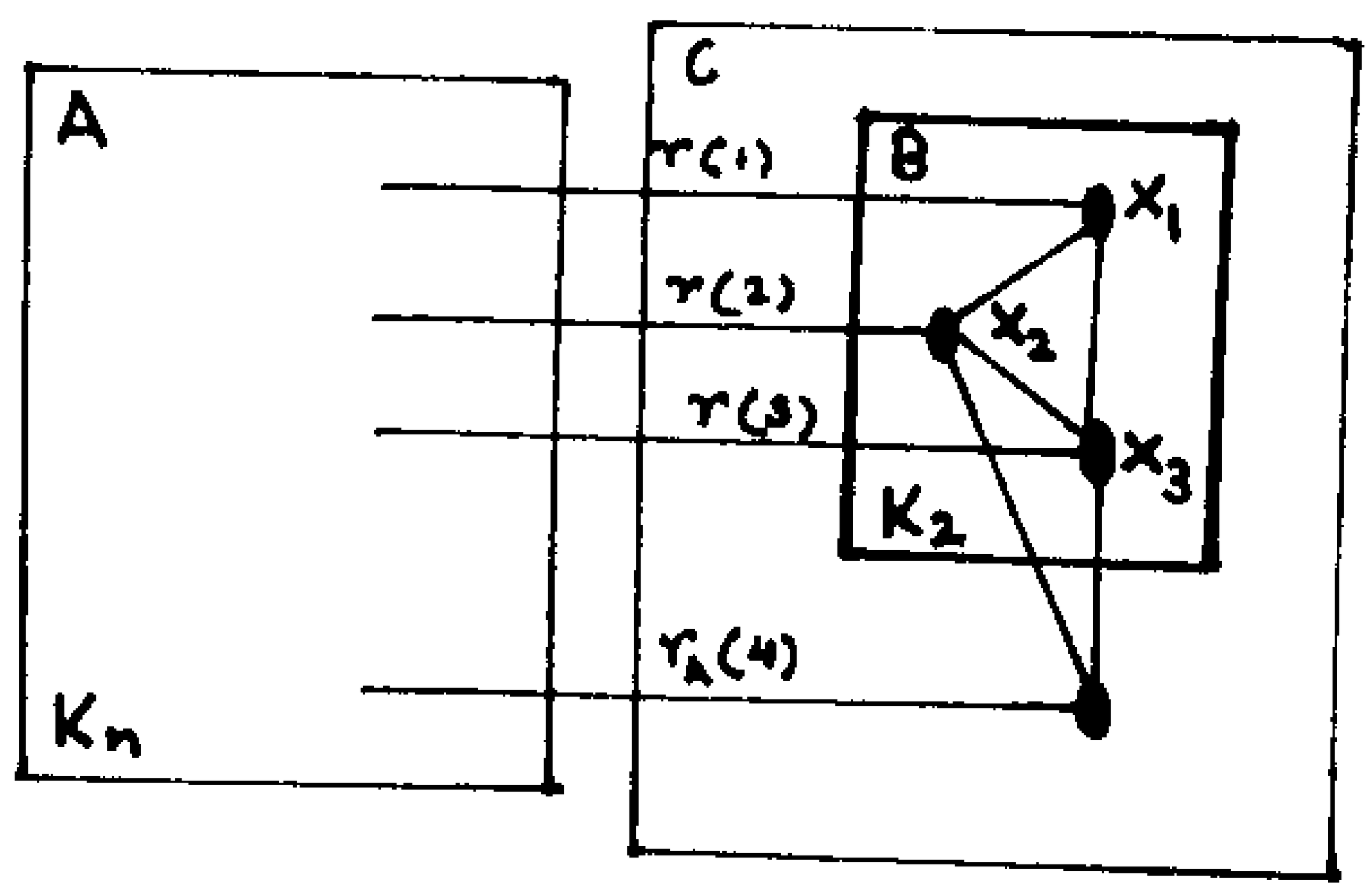


Fig : 20

Result ⟶

The relaibility of the above network is given by ⟶

$$R(G^n[r(1),r(2),r(3),r(4)]$$

$$-\ [(pq^{r(1)+2} - pq^{r(1)+3}) + (pq^{r(4)+2} - pq^{r(4)+3})$$

$$-2p^2q^{S(1,4)+2}$$

$$+(pq^{n+1} - pq^{n+3})]$$

$$= R(G^n[r(1),r(2),r(3),r(4)]$$

$$-\ [(pq^{r(1)+2} - pq^{r(1)+3}) + (pq^{r(4)+2} - pq^{r(4)+3})$$

$$-2p^2q^{S(1,4)+2}$$

$$+q^n(1 - R(C))]$$

(4)

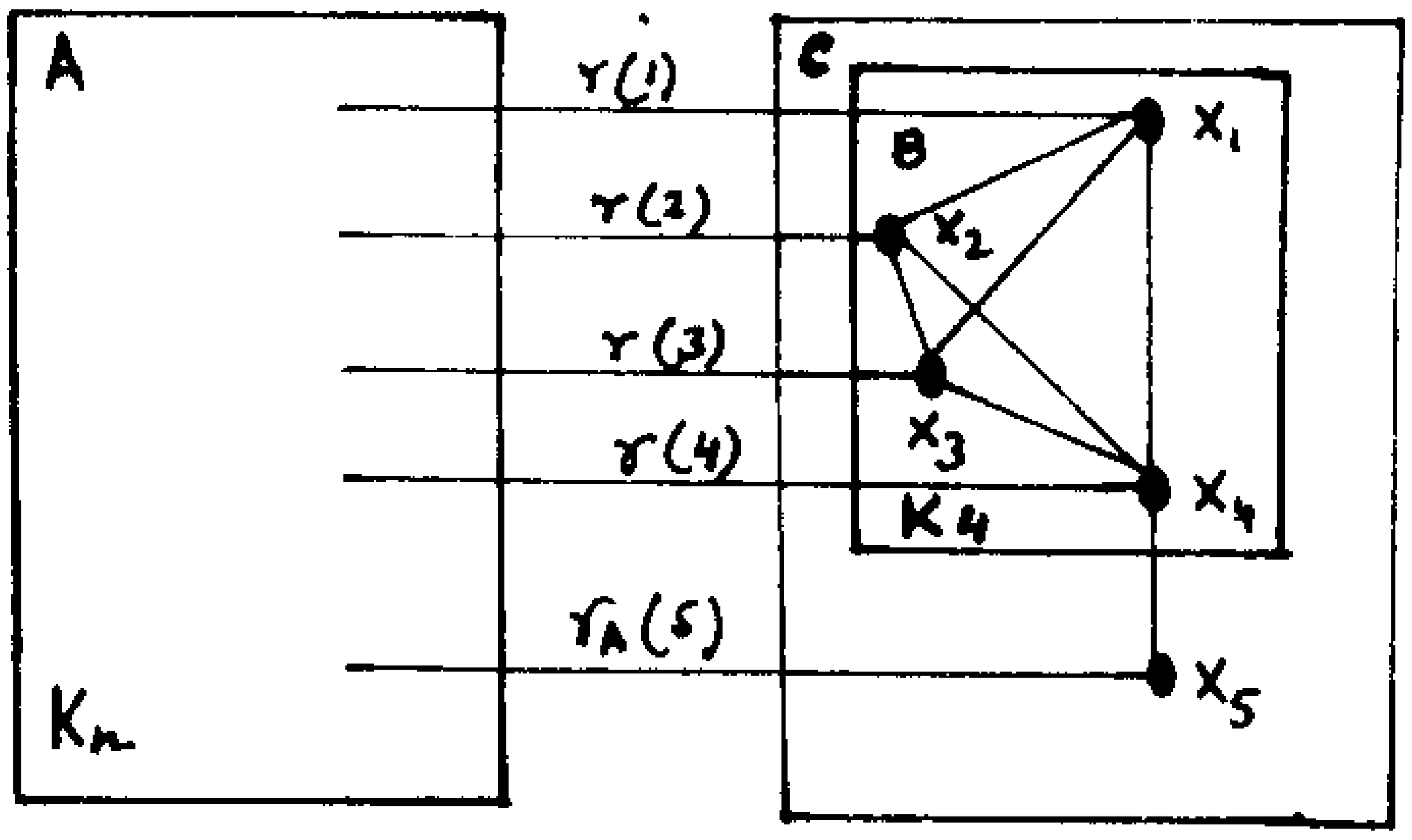


fig : 21

Result :

The relaibility of the above network is given by ⟶

$$R(G^n[r(1),r(2),r(3),r(4),r(5)])$$

$$= [(pq^{r(1)+3}-pq^{r(1)+4})+(pq^{r(2)+3}-pq^{r(2)+4})$$

$$+(pq^{r(3)+3}-pq^{r(3)+4})+(pq^{r(5)+1}-pq^{r(5)+4})$$

$$+p\{(pq^{s(1,2)+2}-pq^{s(1,2)+3})+(pq^{s(1,3)+2}-pq^{s(1,3)+3})$$

$$+(pq^{s(2,3)+2}-pq^{s(2,3)+3})\}$$

$$+p^2\{(pq^{s(1,2,3)+1}-pq^{s(1,2,3)+2})\}$$

$$-2(p^2q^{s(1,5)+3}+p^2q^{s(2,5)+3}+p^2q^{s(3,5)+3}$$

$$+p^3q^{s(1,2,5)+2}+p^3q^{s(1,3,5)+2}+p^3q^{s(2,3,5)+2}$$

$$+p^4q^{s(1,2,3,5)+1})$$

$$+(pq^{n+1}-pq^{n+4})]$$

$$= R(G^n[r(1),r(2),r(3),r(4),r(5)])$$

$$= [(pq^{r(1)+3}-pq^{r(1)+4})+(pq^{r(2)+3}-pq^{r(2)+4})$$

$$+(pq^{r(3)+3}-pq^{r(3)+4})+(pq^{r(5)+1}-pq^{r(5)+4})$$

$$+p\{(pq^{s(1,2)+2}-pq^{s(1,2)+3})+(pq^{s(1,3)+2}-pq^{s(1,3)+3})$$

$$+(pq^{S(2,3)+2}-pq^{S(2,3)+3})\}$$

$$+p^2\{(pq^{S(1,2,3)+1}-pq^{S(1,2,3)+2})\}$$

$$-2(p^2q^{S(1,5)+3} + p^2q^{S(2,5)+3} + p^2q^{S(3,5)+3}$$

$$+p^3q^{S(1,2,5)+2}+p^3q^{S(1,3,5)+2}+p^3q^{S(2,3,5)+2}$$

$$+p^4q^{S(1,2,3,5)+1})$$

$+q^n$(1- reliability of the network when all the n nodes of the complete subgraph A is dead)]

2.7. Structure 5 :

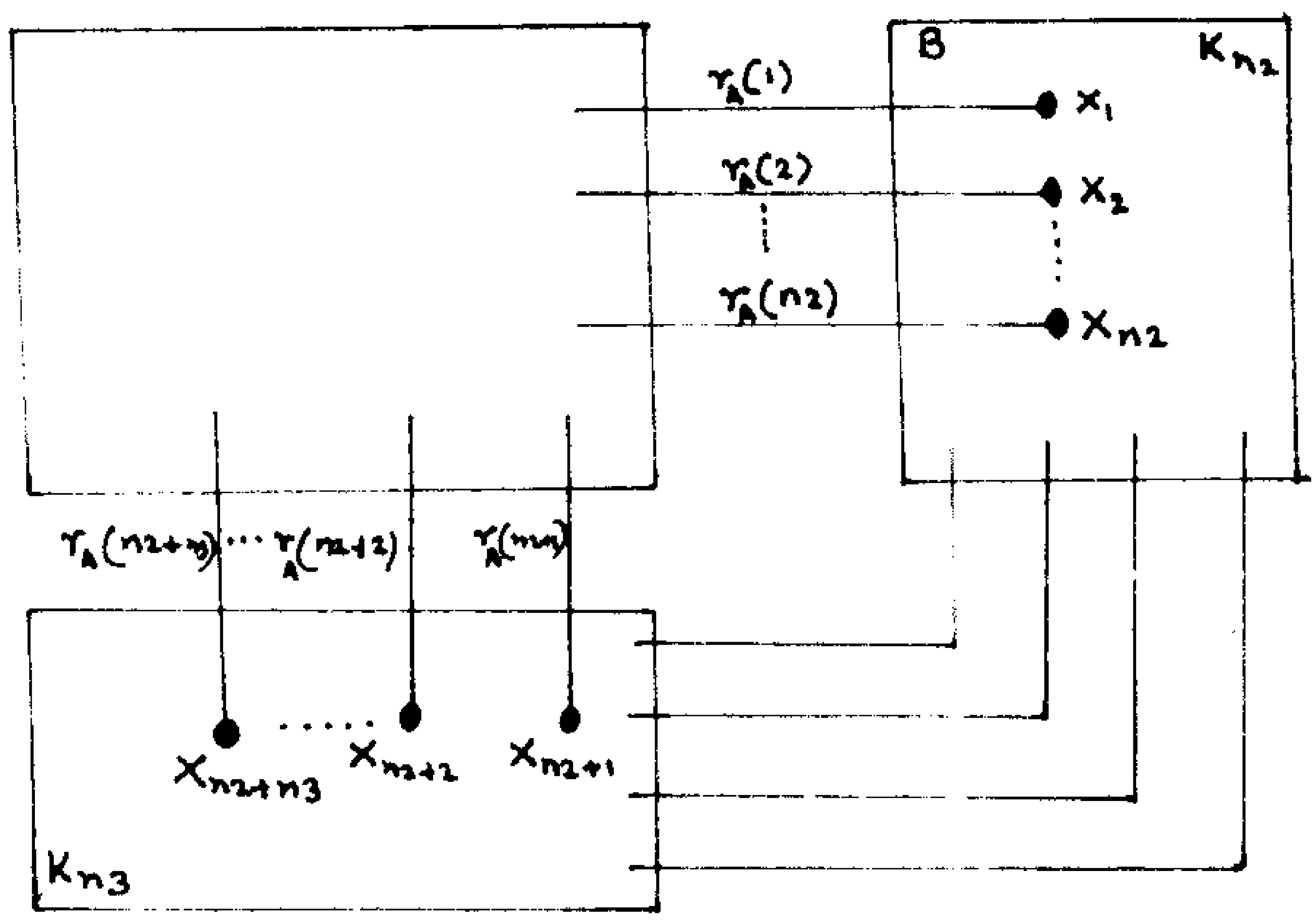


fig :22

Description : Subgraph "A", "B" and "C" all are complete graph of say n1, n2 and n3 nodes. Nodes in subgraph "B" and "C" are numbered. The first node of subgraph "B" is numbered as X_1, the second as X and so on. Similarly the first node of subgraph "C" is numbered $X_{(n2+1)}$ and last node being numbered as $X_{(n2+n3)}$.

There are $r_A^{(i)}$ nodes in "A", $r_C^{(i)}$ nodes in "C" which are adjacent to node X_i ($1 \leq i \leq n2$) in subgraph "B". Similarly there are $r_A^{(i)}$ nodes in "A", $r_B^{(i)}$ nodes in "B" which are adjacent to node X_i ($n2+1 \leq i \leq n3$) in subgraph "C".

Maximum degree of any node X_i will be $(n1+n2+n3-1)$, and the minimum degree will be $(nj-1)$ $j=1,2,3$ accordingly as X_i belongs to subgraph "A", "B", "C".

Notations :

$r_A^{(i)}$ = # of nodes in "A" adjacent to node X_i where X_i belongs to either of subgraph "B", or "C".

$r_B^{(i)}$ = # of nodes in "B" adjacent to node X_i where X_i belongs to "C".

$r_C^{(i)}$ = # of nodes in "C" adjacent to node X_i where X_i belongs to subgraph "B".

$d^{(i)}$ = degree of node X_i.

$d^{(i)} = r_A^{(i)} + r_C^{(i)} + (n2 - 1)$ if X_i belongs to subgraph "B".

$d^{(i)} = r_A^{(i)} + r_B^{(i)} + (n3 - 1)$ if X_i belongs to subgraph "C".

$S_A(i_1, i_k)$ = # of nodes in "A" adjacent node X_{i_1} or X_{i_2}, ... or X_{i_k}

$S_B(i_1 \ldots i_k)$ = # of nodes in "B" adjacent node X_{i_1} or X_{i_2}, ... or X_{i_k}, all X_i belonging to "C".

$S_C(i_1 \ldots i_k)$ = # of nodes in "C" adjacent node X_{i_1} or X_{i_2}, ... or X_{i_k}, all X_i belonging to "B".

$D = B \cup C$.

Definition :

We define a function $F(i_1, \ldots, i_k)$ as follows

$F(i_1, \ldots, i_k) = 1$ if there is a path of length $(k-1)$ consisting the nodes $X_{i_1}, \ldots, X_{i_k}$, all the nodes belonging to "D".

$F(i_1, \ldots, i_k) = 0$ otherwise.

The reliability of the above graph (fig ...) is denoted by

$$R(G^n[\,(r_A^{(1)}, r_C^{(1)}), \ldots, (r_A^{(n_2)}, r_C^{(n_2)})\,]\;[\,(r_A^{(n_2+1)}, r_B^{(n_2+1)}), \ldots, (r_A^{(n_2+n_3)}, r_B^{(n_2+n_3)})\,]\,)$$

Theorem 6 :

$$R(G^n[\,(r_A^{(1)}, r_C^{(1)}), \ldots, (r_A^{(n_2)}, r_C^{(n_2)})\,]\;[\,(r_A^{(n_2+1)}, r_B^{(n_2+1)}), \ldots, (r_A^{(n_2+n_3)}, r_B^{(n_2+n_3)})\,]\,)$$

$$= R(G^n\,[r_A^{(1)}, \ldots, r_A^{(n_2)}, r_A^{(n_2+1)}, \ldots, r_A^{(n_2+n_3)}])$$

$$- \Big[\quad p \sum_{i=1}^{n2+n3} \left(q^{d(i)} - q^{r_A(i)+n2+n3-1} \right)$$

$$+ \Big\{ p^2 \sum_{\substack{i,j=1\\ i\neq j}}^{n2} \left(q^{s_A(i,j)+s_C(i,j)} - q^{s_A(i,j)+n2+n3-2} \right)$$

$$+ p^3 \sum_{\substack{i,j,k=1\\ i\neq j}}^{n2} \left(q^{s_A(i,j,k)+s_C(i,j,k)} - q^{s_A(i,j,k)+n2+n3-3} \right)$$

$$+\dots+p^{n2} \sum_{i_1,\dots,i_{n2}=1}^{n2} \left(q^{s_A(i_1,\dots,i_{n2})+s_C(i_1,\dots,i_{n2})} - q^{s_A(i_1,\dots,i_{n2})+n3} \right) \Big\}$$

$$+ \Big\{ p^2 \sum_{\substack{i,j=1\\ i\neq j}}^{n3} \left(q^{s_A(i,j)+s_B(i,j)} - q^{s_A(i,j)+n2+n3-2} \right)$$

$$+ p^3 \sum_{\substack{i,j,k=1\\ i\neq j\neq k}}^{n3} \left(q^{s_A(i,j,k)+s_B(i,j,k)} - q^{s_A(i,j,k)+n2+n3-3} \right)$$

$$+\dots+p^{n3} \sum_{i_1,\dots,i_{n3}=1}^{n3} \left(q^{s_A(i_1,\dots,i_{n3})+s_B(i_1,\dots,i_{n3})} - q^{s_A(i_1,\dots,i_{n3})+n2} \right) \Big\}$$

$$\Big]$$

$$+ 2 \Big[$$

$$p^2 \sum_{i=1}^{n3} q^{s_A(i,j)+n2+n3-2}$$

$$+ p^{3} \sum_{\substack{i,j,k=1 \\ i\neq j\neq k \\ F(i,j,k)=0}}^{n9} \left(q^{S_A(i,j,k)+n2+n9-3} \right)$$

$$+ \ldots$$

$$+ p^{n2+n9-1} \sum_{\substack{i_1,\ldots,i_{n2+n9-1}=1 \\ F(i_1,\ldots,i_{n2+n9-1})=0}}^{n9} \left(q^{S_A(i_1,\ldots,i_{n2+n9-1})} \right)$$

$$]$$

$$- q^{n} [1- R(D)]$$

Remarks : The rigorous proof of the above theorem is ommited here.

Subgraph "D" has the same structure as (4) ,reliability of which can be computed directly from theorem **.

CHAPTER III

3.1. Change In Reliability For Links Failure :

If there exists an edge between two nodes say u & v of a network A and we delete this edge the resulting network (say B) thus formed will be obviously less reliable than network A. The degree of unreliabilty, that is the measure of how much less-realiable the network B has become than network A will be given by [R(A) - R(B)].

Consider now the case where the network-graph B will be partitioned into two and only two sub-network-graph say X and Y such that $u \in X$ and $v \in Y$, u and v both being alive.

Let

S= set of nodes such that all of its member being dead the network B is partitioned in the above mentioned way.

$x = |X|$ = number of nodes (alive) in the network X.

$y = |Y|$ = number of nodes (alive) in the network Y.

$s = |S|$ = number of nodes in the set S.

It is obvious that $|A| = x + y + s$.

Followign the definition of reliability we observe that [R(A)-R(B)] will be equal to the probability that the network B will have two and only two partitions X,Y such that $u \in X$ and $v \in Y$,u and v both being alive.This is due to the reason that for the other cases network A will be connected through the link between u and v.Hence we can state the following Theorem.

We are giving below some definitions which will help us in stating the theorem formally.

Definitions :

A cut set of a connected graph is a set of minimal number of nodes such that removal of this set from the graph will make the graph discobnected. No. of disconnected components of the resulting graph may be two or more.

(1) Two-component-cut-set : A cut set that partitions a connected network-graph into two and only two components.

(2) Super-set-of-two-component-cut-set: This set is a super set of the two component cut set, i.e. number of nodes of this cut set is not minimal in nature.

(3) Deletion-set : This is a super-set-of-two-component-cut-set such that removal of this set from a graph K will always partition the graph into two and only two components in which two particular nodes say m and n always belongs to two diferrent components both of them being always alive.

We denote this Deletion-Set by D(K,m,n).

By the definition of Deletion-Set it is obvious that the set 'S' as definied earlier is nothing but a Deletion-Set. So S = D(B,u,v), where B,u,v as definied earlier.

Let the set of all possible such deletion set D(B,u,v) for a graph B is denoted by say SD(B).

THEOREM 7 :

From a network-graph "A" if we delete a link (u,v), the resulting network-graph "B" will be less reliable than "A".

In this case

$$[R(A) - R(B)] = \sum_{S \in SD(B)} (q^{s} p^{n-s})$$

Where SD(B) = set of all deletion-set of B

$s = |S|$.

$n = |A|$.

Remark : The proof of the theorem is obvious and follows directly from the definition of the reliabilty.

Examples:

(1)

Consider the network P_3 (a path of 3 nodes).

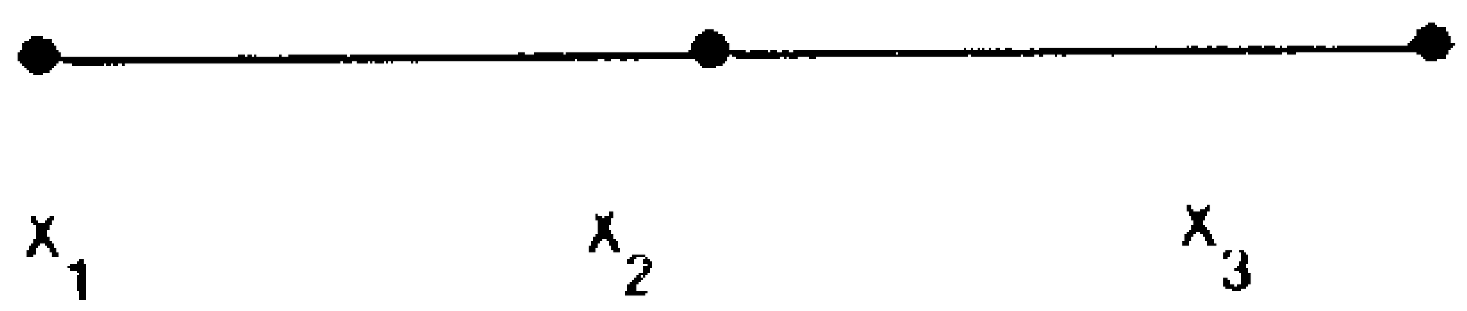


fig : 23(a)

$R(P_3) = R(A) = q^3 + 3pq^2 + 2p^2q + p^3$.

Now if we delete any one of the links the resulting network "B" will look like

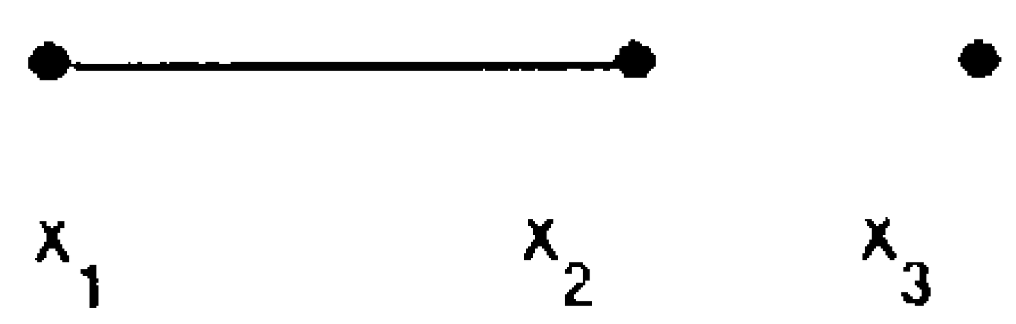


fig : 23(b)

By applying theorem(1) we get $R(B) = q + pq^2$.

$$\text{So } [R(A) - R(B)] = (q^3 + 3pq^2 + 2p^2q + p^3) - (q + pq^2)$$

$$= q^3 + 2pq^2 + 2p^2q + p^3 - q$$

$$= 1 - pq - q \quad \text{(after some simplification)}$$

$$= p^2. \quad \ldots(1)$$

Now we calculate $[R(A) - R(B)]$ considering the deletion set of graph "B".

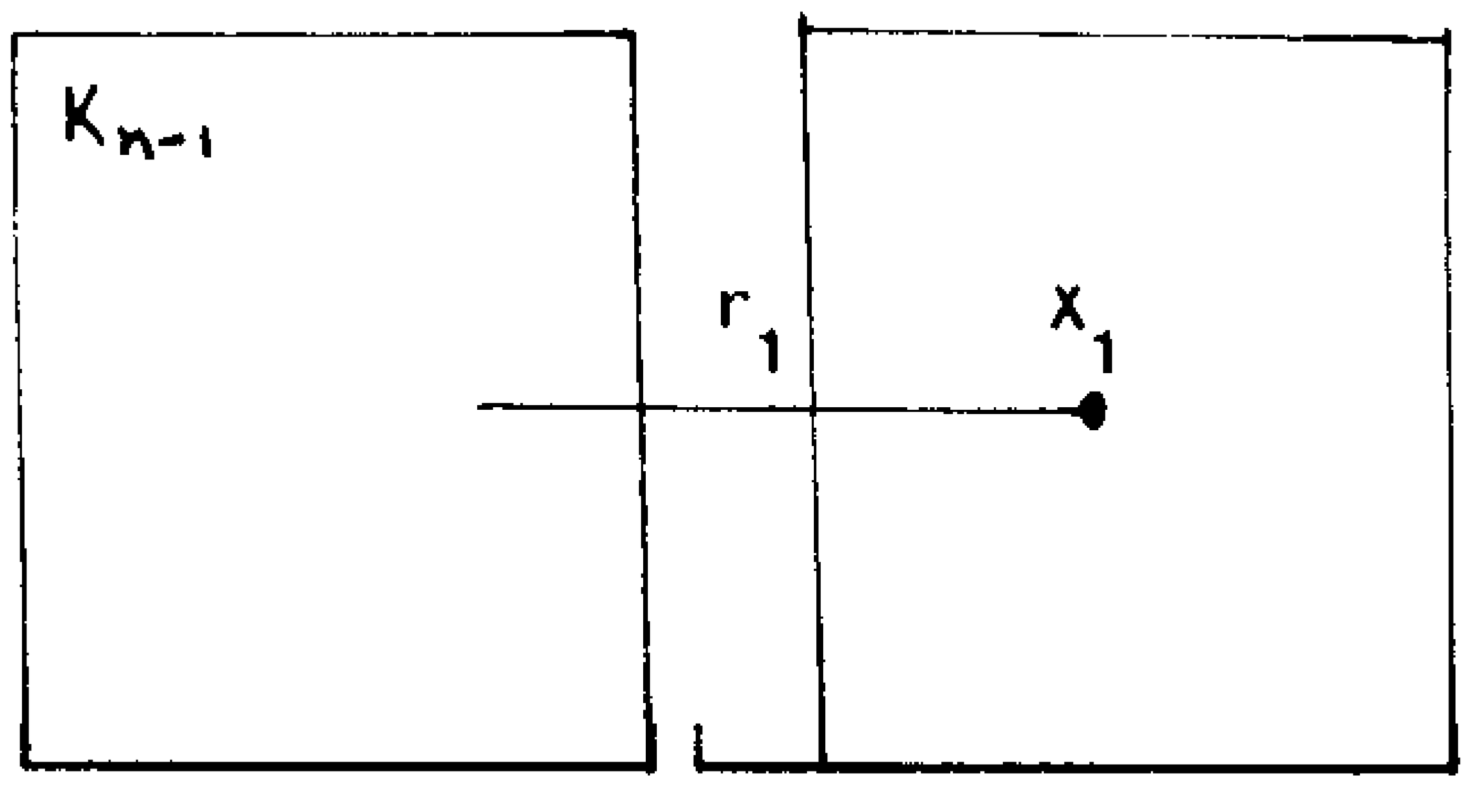
It is easy to observe that there will be only two suvh deletion-set, namely S_1, S_2 where $S_1 = \{\}$ and $S_2 = \{x_1\}$.

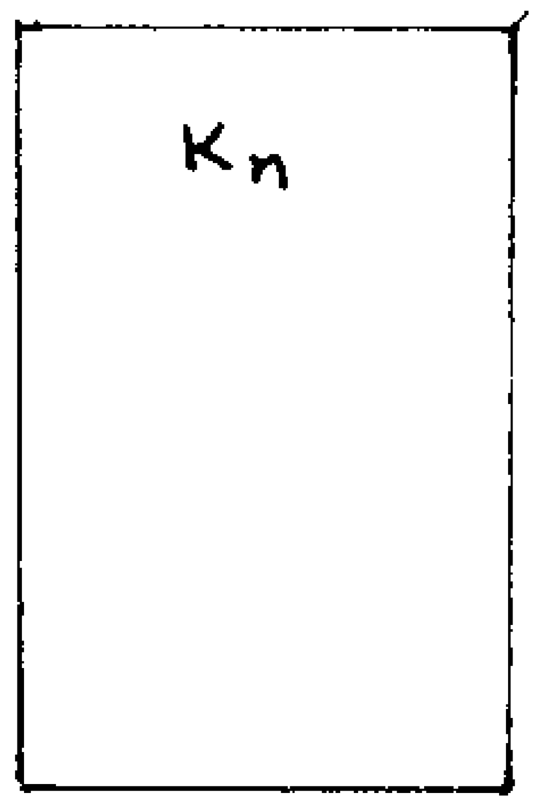
Also $|S_1| = s_1 = 0$, $|S_2| = s_2 = 1$

So according to the above theorem 13

$[R(A) - R(B)] = p^3 + qp^2 = p^2$ ⟶ Same as above result (1).

(2).





B A

fig 24(a) fig 24(b)

Let r_1 = (n-2). So only one link is deleted from "A" to obtain "B".

$R(A) = 1$ from theorem (2).

$R(B) = 1 + pq^{n-1} - pq^{n-2}$ from theorem (1).

$- 1 \quad p^2q^{n-2}$.

So $[R(A) - -R(B)] = p^2q^{n-2}$.

Now considering deletion-set of "B" we observe that there is only one such deletion-set given by S_1=set of all nodes in B adjacent to node x_1.

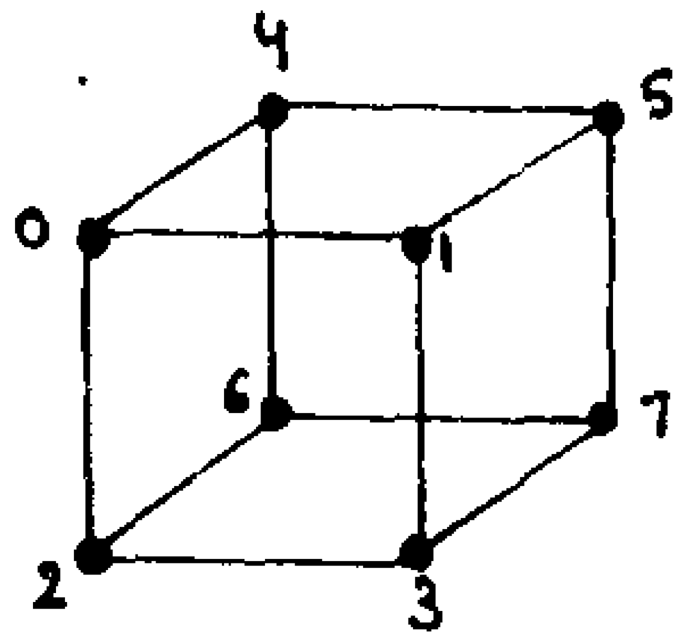
Also $|S_1|$=(n 2).

So according to theorem-13 $[R(A) - R(B)]=q^{n-2}p^2$..... same as the result obtain above.

(3).

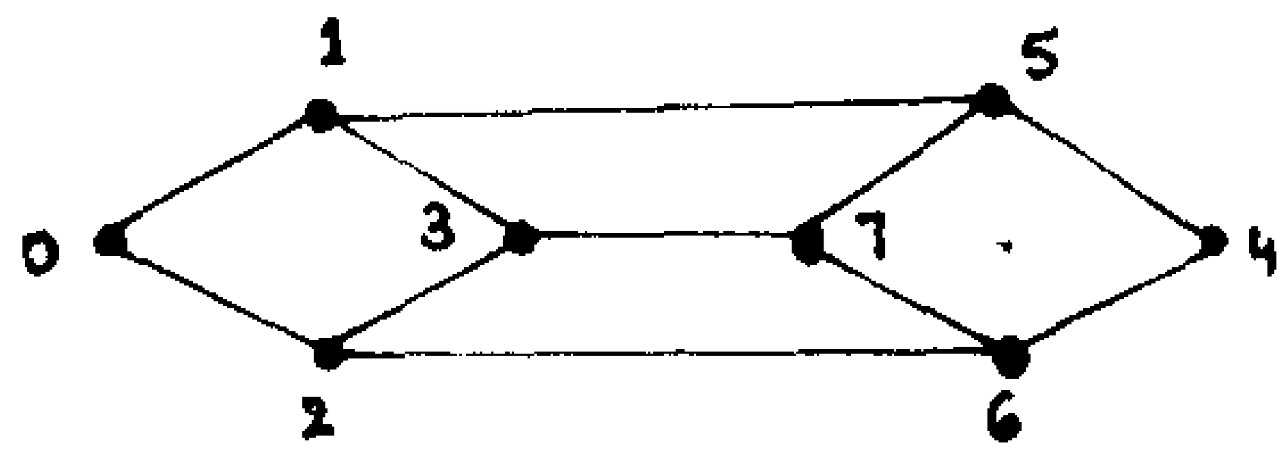
Deletion of a single link from a 3-cube.

We shall apply theorem 13 to obtain an analytical expression of the difference of reliability a 3-cube and the reliability of a 3-cube when one edge of it is deleted.



A

fig : 25(a)



B

fig : 25(b)

Let the reliability of the 3-cube is denoted by R(A) and that of the other by R(B).

Case 1:

We observe that that minimum number of nodes in the deletion-set "B" will be 2. This is because of the fact that if we delete link (0,4) of the 3-cube ,"A" then degree of the nodes (0),(4) will become 2,so to isolate one of these nodes we have to delete at least two nodes.In case we want to disconnect the node (0) from rest of the network then we can choose nodes {1,2} Similarly for node(4) we can choose {5,6} So there are two cases when number of nodes in the deletion-set of "B" is minimum.

Case 2 :

The maximum number of nodes in the deletion-set of "B" will be 6, and there are only one such case.This deletion-set of "B" willbe given by {1,2,3,4,5,6}.

Case 3 :

When the # of nodes in the deletion-set is three we get the following 10 deletion-sets.

... {1,2,3} , {1,6,3} , {1,6,7} , {5,2,3} , {5,2,7} , {5,6,7} , {1,2,5}, {1,2,6} , {1,5,6}, {2,5,6},

Case 4 :

When the # of nodes in the deletion-set is four , the list of possible 8 deletion-sets is given below ⟶

{1,2,3,5}, {1,2,3,7}, {1,2,3,6}, {5,6,7,1}, {5,6,7,2}, {5,6,7,3}, {1,3,7,6}, {2,3,7,5}

Case 5 :

When the # of nodes in the deletion-set is five , the list of possible 4 deletion-set is listed below ⟶

{1,2,3,7,6}, {1,2,3,7,5}, {5,6,7,1,3}, {5,6,7,2,3}

Result:

When we delete one edge from a 3-cube the decrease in reliability is given by the following expression,

$$R(A) - R(B) = 2q^2p^6 + 10q^3p^5 + 8q^4p^4 + 4q^5p^3 + q^6$$

3.3. Remarks :

Theorem 7 may help us in desiging a network. Say maximum degree of the nodes of a netwok-graph is specified and we want to design a network with a given number of nodes and links such that realiability of it is maximized.We may apply theorem 7 in a greedy way in the following manner.We start with a network with no links. We then try to add a link such that its reliability increases maximally.We then try to add a seccond link to this newly created network, and so on, satisfying the given constraints (i.e. degree constraint). In this way at each stage we try to maximize the reliability.

We have failed to explore any generalised way of determining the # of deletion-sets for a n-cube, but from the above discussions,it is apparent that there may be some way. So further work can be done in this field.

CHAPTER IV

4.1. Conclusion :

Reliability of a network is usually expressed in terms of connectivity of the underlying graph. Another approach is to take a probabilistic model for the failure of the different nodes. In this dissertation work we have adopted the definition of reliability as given by Mukhopadhyaya and Sinha [5] where the reliability of any network has been computed recursively using a probabilistic model. The problem of this recursive evaluation of reliability is (1) exponential time requirement , (2) stack overflow.

Here we have tried to find out the reliability of any network using the same probabilistic model used by Mukhopadhyaya and Sinha [5] in a non recursive way. We have tried to give the analytical formula of the reliability of any network using its topological properties. Though we have failed to give a formula for a general network , still we have given some structures for which we have found out the reliabilities in a non recursive way. We have also got some preliminary results for the general case.

In Chapter-III, we have outlined some results regarding the incremental reliability of a network. Through this incremental reliability we may find out the increase or decrease in reliability of a network by addition or deletion of one or more links. This approach may help us in network designing. We have also got some preliminary results, and some conjectures about this incremental reliability in case of a 3-cube which might help in finding the reliability of a general n-cube.

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