LETTERS TO THE EDITOR

[The Editor is not responsible for the views expressed in the Letters.]

The Distribution of the Root-Mean Square of the Multiple Correlation Co-efficient on the Multivariate Normal Hypothesis

It was found by Professor Fisher in 1928 and is now well known that for samples of size n drawn from a p-variate normal population the co-efficient of multiple correlation of one variate with the $(p-1)$ other variates is distributed (under well known restrictions) in the form

$$\text{Const. } R^{p-2}(1-R^2)^{\frac{n-p-2}{2}}\, dR$$

$$\times\ {}_1F_1\left(\frac{n-1}{2}\,;\ \frac{p-1}{2}\,;\ \frac{n\beta^2}{1-\beta^2}R^2\right) \qquad (1)$$

where R is the (sample) multiple correlation of one variate on $p-1$ other variates and β is the corresponding population parameter and ${}_1F_1$ is a well known type of hyper-geometric function. It has recently been found by the authors of this note that if we consider m samples each of size n from the same p-variate normal population and if we put

$$m\bar{R}^2 = \sum_{i=1}^{m} R_i^2 \qquad (2)$$

where R_i denotes the multiple correlation co-efficient for the i th sample then $\bar{R}$ is distributed in the form,

$$\text{Const. } (\bar{R})^{m(p-1)-1}(1-\bar{R}^2)^{\frac{m(n-p-1)}{2}-1}\, d\bar{R}$$

$$\times\ {}_1F_1\left[\frac{m}{2}(n-1);\ \frac{m}{2}(p-1);\ \frac{n\beta^2}{1-\beta^2}\, m\bar{R}^2\right] \quad (3)$$

This shows that the root mean square of R is distributed in the same general manner (with only change of parameters) as R itself; that is to say, the type of the distribution is conserved when we proceed to the distribution of the mean.

In a paper[1] published in the *Proceedings* of the Science Congress held in 1939 by one of the present authors jointly with Mr R. C. Bose the distribution of the Studentised D^2-statistic for $(p-1)$ characters (when the mean differences are based on two samples of sizes n_1 and n_2 and the variances and covariances are based on k samples of sizes $n_1, n_2, \ldots, n_k$) was obtained in the form

$$\text{Const. } D^{p-2}\left(1+\frac{(p-1)\bar{n}}{2N}D^2\right)^{-(N-k+1)/2} dD$$

$${}_1F_1\left(\frac{N-k+1}{2},\ \frac{p-1}{2},\ \frac{(p-1)^2\,\bar{n}^2\Delta^2 D^2}{8N+4(p-1)\bar{n}D^2}\right) \quad (4)$$

where $N = n_1 + n_2 \ldots + n_k$, and $\bar{n}$ is the harmonic mean of n_1 and n_2. If we put

$$\left.\begin{aligned} R^2 &= \frac{(p-1)\bar{n}D^2}{2N+(p-1)\bar{n}D^2} \\ \text{and}\quad \frac{n\beta^2}{1-\beta^2} &= \frac{(p-1)\bar{n}\,\Delta^2}{4} \end{aligned}\right\} \quad (5),$$

then it is easily seen from a comparison of (1) and (4) that (4) goes over to (1) with the substitutions (5) and the substitution of n for $N-k+2$.

If we now take D^2 from m such sets and denote by D_i^2 the D^2 corresponding to the i-th set and if we put

$$m\xi^2 = (p-1)\bar{n}\sum_{i=1}^{m}\frac{D_i^2}{2N+(p-1)\bar{n}D_i^2} \qquad (6)$$

Then from (3) it is evident that is distributed as

$$\text{Const. } (\xi)^{m(p-1)-1}(1-\xi^2)^{\frac{m(N-k-p+1)}{2}-1}\, d\xi$$

$$\times\ {}_1F_1\left[\frac{m}{2}(N-k+1):\ \frac{m}{2}(p-1);\ \beta^2 m\xi^2\right] \qquad (7)$$

If we now make k (and accordingly N) large then relation (6) changes over to

$$m\xi^2 = \frac{(p-1)\bar{n}}{2N}\sum_{i=1}^{m} D_i^2$$

$$\text{or to } \xi^2 = \frac{1}{m}\sum_{i=1}^{n} \frac{(p-1)\bar{n}}{2N}\bar{D}^2, \qquad \text{where } \bar{D}^2 = \frac{1}{m}\sum_{i=1}^{m} D_i^2 \qquad (8)$$

and substituting from (8) in (7) and making, k (and N) large we get the distribution of D^2 in the form:

$$Const.\ e^{-\frac{m\bar{n}(p-1)\bar{D}^2}{4}} (\bar{D}^2)^{\frac{m(p-1)-2}{4}} d(\bar{D}^2) \times I_{\frac{m(p-1)-2}{2}}(c^2 \Delta m \bar{D}) \qquad (9)$$

$$\text{where } c^2 = \frac{n(p-1)}{2}$$

This, of course, agrees with the distribution of the root mean square of D (for the classical case) obtained by R. C. Bose[2] in 1938.

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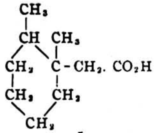
[1] S. N. Roy and R. C. Bose: The use and distribution of Studentized D^2-statistic when the variances and covariances are based on k-samples. *Sankhya*: Proceedings of the Second Indian Statistical Conference 1939.

[2] R. C. Bose: On the distribution of the means of samples drawn from a Bessel Function population. (*Sankhya*: Vol. 3, Pp. 262-264).

Synthetic Experiments in the Terpene Series

From the oil of Eremophila Mitchelli, Simonsen and co-workers[1] first isolated a crystalline ketone $C_{15}H_{22}O$ and two other closely related solid hydroxyketones $C_{15}H_{22}O_2$ and $C_{15}H_{24}O_2$ and these were named eremophilone, 2-hydroxyeremophilone and 2-hydroxy-1: 2 dihydroeremophilone respectively. By the oxidation of the benzoate of 2-hydroxy-eremophilone, Bradfield, Hellstrom Penfold and Simonsen[2] obtained a solid keto-acid $C_{10}H_{16}O_3$ and also a liquid acid $C_{10}H_{18}O_2$ by the reduction of the above keto-acid by the Clemmensen method. Formulae (I) and (II) have been suggested by Simonsen[3] as representing the probable structures of the reduced acid and the keto-acid respectively.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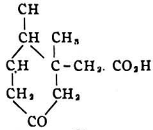
II

Following experiments have been carried out with a view to synthesising the reduced acid (I).

Ethyl- δ -acetyl-n-valerate was condensed[4] with ethylcyanoacetate to yield ethyl-l-cyano-2-methyl- Δ_1 hexane-1:6-dicarboxylate, b.p. 170-178°/6.5-7 m.m. 167-171°/4 m.m. The crude dicyanoester, obtained by the addition of potassium cyanide to the above unsaturated cyanoester, was hydrolysed to yield 2-methyl-hexane-1: 2:6-tricarboxylic acid m.p. 119-120°. The triethyl ester of the above acid b.p. 183-185°/8 m.m., on cyclisation by sodium in benzene solution gave ethyl-2-methyl-6-carbethoxy cyclohexanoe-2-acetate, b.p. 155-160°/6.5 m.m., which on hydrolysis yielded 2-methyl-cyclohexanone-2-acetic acid b.p. 150-155°/5.5 m.m. (Semicarbazone m.p. 181-181.5°). The ethyl ester of the above keto-acid b.p. 122°/7.5 m.m. (Semicarbazone m.p. 132°), on treatment with methyl-magnesium-iodide yielded the lactone of 1.2-dimethyl-2-hydroxy-cyclohexane-1-acetic acid b.p. 114-115°/6 m.m. Unusual difficulty is however being encountered in the reduction by usual methods of the above lactone which is expected to yield the acid (I) as the final product.

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[1] *J.C.S.*, 2744, 1932.
[2] *Ibid*, 767, 1938.
[3] *Ibid*, 87, 1939.
[4] Linstead *et al*, *J.C.S.*, 1977, 1937.