Price Interventions in Cournot Oligopoly with a Dominant Firm

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Abstract

We study a dominant firm Cournot oligopoly, with one low-cost firm and one or more high-cost firms. If equilibrium is interior, with all firms producing positive quantity, a reallocation of production relative to the equilibrium point, such that the low-cost firm produces more, while the high-cost firms produce less, can increase consumers' surplus, as well as joint firm profit. A price intervention (either a price floor or a fixed price) may help achieve such an improvement.

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1 Introduction

A significant part of total supply in many industries comes from a small number of large firms, with the balance coming from a large number of small firms. A variety of "dominant firm" models have been constructed to understand firm behaviour in such settings. These models investigate asymmetric market structures in which one advantaged firm, typically with lower cost, interacts with a number of disadvantaged firms, with higher cost, and have long been used to provide policy guidance in relation to several issues such as antitrust, vertical and horizontal integration and differentiation, licensing, innovation, foreign direct investment, etc.[1]

This paper studies a Cournot oligopoly with one low-cost firm (the dominant firm), and one or more identical high-cost firms, and investigates the impacts of some simple price interventions in such a setting. While many different price interventions are commonly seen in a wide variety of imperfectly competitive industries, there have been few analyses of their effects, especially in environments where producers are not identical to each other. The essay attempts to take a step in this direction.

Our baseline model (see Section 2) examines a duopoly where firms have constant marginal costs and face a linear demand curve. Consider the following notion of efficiency: a production vector is efficient if there exists no other production vector simultaneously yielding higher consumers' surplus, as well as joint firm profit, at least one strictly. We show in Sections 3.1 and 3.2 that if equilibrium is interior, with all firms producing a positive amount, it is inefficient.[2] With a linear demand curve, interior equilibrium exists whenever the market size is sufficiently large relative to the high-cost firm's marginal cost. To see the nature of the inefficiency, consider a reallocation of production, relative to the equilibrium vector, whereby total output is unchanged, yet the low-cost firm produces relatively more. Since total output is unaffected, equilibrium price, consumers' surplus, as well as industry revenue are unchanged. However, joint firm costs are lower, as a relatively efficient firm produces more while a relatively inefficient firm produces less.

[1] There is a large body of work using dominant firm models in a variety of contexts; Schmalensee (1987), George and Jacquemin (1992) and Viscusi, Vernon and Harrington (2000) provide partial reviews.

[2] Versions of this property of asymmetric Cournot oligopoly are well-known: see, for example, Farrell and Shapiro (1990).

We further show in Sections 3.3 and 3.4 that a fixed price intervention can induce a more efficient outcome. With such an intervention, all trade takes place at the set price. To understand the intuition, notice that at any fixed price, a firm can always increase production without fearing a reduction in the price. Thus, compared to the unconstrained environment, a fixed price acts as an implicit production subsidy. This subsidy effect gives all firms an incentive to increase output and, as long as the set price is not too low, leads to equilibrium excess supply. In such a situation, however, increased production by any firm leads to lower probability of sale for other firms. There is thus a substitution effect present in this environment which, as in the unconstrained case, is discriminatory, and relatively benefits the low-cost firm, causing it to produce more than the high-cost firm. The subsidy effect is itself discriminatory, with benefit to a firm decreasing in cost. The relative advantage to the low-cost firm arising from the differential nature of the substitution effect is therefore accentuated by the discriminatory aspect of the subsidy effect. So, compared to the unconstrained outcome, total production can be expanded, and also reallocated. In addition, if the degree of asymmetry between firms is sufficiently high, i.e., if the marginal cost of the low-cost firm as a fraction of the marginal cost of the high-cost firm is sufficiently low, the high cost firm will produce less than at the unconstrained equilibrium outcome. If such an effect arises, and is strong, the intervention can serve to significantly reduce output coming from the high-cost firm and increase that coming from the low-cost firm, yielding efficiency improvements.

Suppose then firms are symmetric with unknown marginal costs at a stage prior to production, and suppose also that one firm always develops low marginal cost, relative to the other, at the production stage. Innovation races, for example, often have such a characteristic, with the firm discovering the better innovation developing a significant competitive and cost advantage. A price intervention at the earlier stage then can lead to an ex ante Pareto-improvement, with higher expected consumers' surplus as well as firm profit.[3]

We show in Section 4 that some of the simplifying assumptions in the baseline model (linear demand, constant marginal cost and single high-cost firm) are unnecessary for the main result, that efficiency enhancing price interventions can exist if the firms are 'sufficiently asymmetric'. The simpler structure is used for the development of the results because of

[3] Efficiency benefits of price interventions have been previously noted, in a variety of asymmetric information environments, by Hellmann, Murdock and Stiglitz (2000), Pesendorfer and Wolinsky (2003) and Frisell and Lagerlöf (2005). Informational issues play no role in the current analysis.

expositional transparency and because it allows the presentation of some conditions in terms of parameters, rather than endogenous variables.

The intuition presented above suggests that efficiency enhancements may be feasible because a fixed price prevents the price from falling, and so differentially encourages the low-cost firm to expand production. In this sense, the floor aspect of a fixed price, rather than the ceiling aspect, is the driver behind the result. The analysis is less tractable with a pure price floor however, and optimisation problems may not be well-behaved; we present a brief discussion in Section 5.

Our work is related to some previous analyses of asymmetric Cournot oligopoly. It is well-known that in asymmetric Cournot oligopoly with linear demand and total cost functions, firm marginal costs affect industry output, and therefore equilibrium price, consumers' surplus and industry revenue, given interior equilibrium, only through their sum. Bergstrom and Varian (1985a, b), Salant and Shaffer (1999), Long and Soubeyran (2001), Février and Linnemer (2004) etc., then show that if the variance of marginal costs increases, keeping the sum constant, aggregate industry cost decreases, thereby enhancing efficiency in the sense discussed above. The reason is that this change causes high-cost firms to reduce production, and low-cost firms to increase production. We discuss this result further in Section 3.4.[4] The current article does not address changes in firm costs, and instead focusses on the role of price interventions in achieving equilibrium production reallocations.

The paper is organised as follows. Section 2 presents the benchmark model and Section 3 analyses it. Sections 4 and 5 discuss the main assumptions. Section 6 concludes. Some proofs are collected in an Appendix.

2 An asymmetric Cournot duopoly model

There are two firms, 1 and 2, and a Regulator ($\mathcal{R}$). There is a single, homogeneous, infinitely divisible good. The firms compete à la Cournot. Let firm i's output be denoted q_i. The

[4] See also Lahiri and Ono (1988) who show that equilibrium production reorganisations contingent on the exit of a high-cost firm can lead to increased aggregate welfare. A result with a similar flavour can be found in Bulow, Geanakoplos and Klemperer (1985), who show that an increase in the number of high-cost competitors can cause the dominant firm's output to increase.

demand curve is linear.

Assumption 1 : The inverse demand function is $p = a - bq$, where a, $b > 0$, and $q = q_1 + q_2$.

Firms face no fixed costs. The marginal cost is constant for any firm. Assume without loss of generality that firm 1 has lower cost than firm 2.

Assumption 2 : Firm i's constant marginal cost is c_i, with $c_2 > c_1 \geq 0$.

Unless otherwise mentioned, we carry out the analysis assuming $c_1 > 0$. The case where $c_1 = 0$ is discussed at the end of Section 3.3.

$\mathcal{R}$ can impose a fixed price p^0, if he wishes. We assume that p^0 is less than a and more than c_2.[5] With such an intervention, any sale takes place at the set price p^0. We assume that this regulation can be costlessly enforced, so black markets, aftermarkets, secret price-cutting, etc., cannot arise.

As we shall see below, surplus production may emerge with a fixed price. Since total industry output then exceeds demand, some units will be unsold. We assume in that case that disposal costs are 0. We further assume the probability that a unit of a firm's output will be sold, given surplus production, is the same for both firms.

Assumption 3*a*: If fixed price p^0 is imposed, $p^0 \in (c_2, a)$, all sales take place at p^0. Unsold units are freely disposed.

Assumption 3*b*: If at fixed price p, total demand is q_p^d, and total supply is $q_p^s > q_p^d$, then the probability a unit of any firm's output is sold is $\frac{q_p^d}{q_p^s}$.

We shall also impose the following parameter restriction, which says that demand is sufficiently high relative to cost. The exact reason behind this requirement is discussed below in Section 3.4.

Assumption 4 : $a > 2(c_1 + c_2)$.

3 Fixed price intervention

We first study the unregulated equilibrium, and then equilibrium with a fixed price intervention. We then compare outcomes across the two regimes.

[5]Implicit in this requirement is the assumption that $a > c_2$, which is guaranteed by Assumption 4 below.

3.1 The unregulated equilibrium

Firm i's problem is to choose a production level to maximise its profit. In doing so, it recognises the influence of firm j's output choice on its own revenue, via the demand curve. Formally, firm i's problem is

$$\max_{q_i} q_i(a - bq_i - bq_j) - c_i q_i$$

It is well-known that given Assumptions 1, 2 and 4, a unique, stable, interior equilibrium, with both firms producing a positive amount, exists.[6]

Proposition 1 *A unique equilibrium exists given Assumptions 1, 2 and 4. In equilibrium, firms* 1 *and* 2 *respectively produce*

$$q_1^* = \frac{(a - 3c_1) + (c_1 + c_2)}{3b}; q_2^* = \frac{(a - 3c_2) + (c_1 + c_2)}{3b}$$

The equilibrium price (p^*), consumers' surplus (S^*), industry revenue (R^*), joint firm cost (C^*) and total profit (π^*) are respectively

$$p^* = \frac{a + (c_1 + c_2)}{3}; S^* = \frac{[2a - (c_1 + c_2)]^2}{18b}$$

$$R^* = \frac{[a + (c_1 + c_2)][2a - (c_1 + c_2)]}{9b}; C^* = \frac{a(c_1 + c_2) + (c_1 + c_2)^2 - 3(c_1^2 + c_2^2)}{3b}$$

$$\pi^* = \frac{[a + (c_1 + c_2)][2a - (c_1 + c_2)]}{9b} - \frac{a(c_1 + c_2) + (c_1 + c_2)^2 - 3(c_1^2 + c_2^2)}{3b}$$

Proof. The proof is standard and is therefore omitted. ■

3.2 Efficiency

We define a simple notion of efficiency. Given any production vector $\mathbf{q} = (q_1, q_2)$, let $S(\mathbf{q})$ and $\pi(\mathbf{q})$ be respectively consumers' surplus and joint firm profits. A production vector $\mathbf{q}$ dominates another vector $\mathbf{q}'$ iff $S(\mathbf{q}) \geq S(\mathbf{q}')$ and $\pi(\mathbf{q}) \geq \pi(\mathbf{q}')$, with at least one inequality

[6] For interior equilibrium, we need that $a > 2c_2 - c_1$, which is satisfied, given $A4$.

strict. If both are strict, then we say $\mathbf{q}$ strictly dominates $\mathbf{q}^{'}$. A production vector $\mathbf{q}$ is efficient if there exists no other vector $\mathbf{q}^{'}$ which dominates $\mathbf{q}$.

Let $\mathbf{q}^*$ be the unregulated Cournot equilibrium production vector identified above. It is easy to see it is inefficient. Referring to the duopoly model, consider a production vector $\widetilde{\mathbf{q}}$ $= (q_1^* + \delta, q_2^* - \delta)$, $\delta > 0$, and δ sufficiently small so that $q_2^* - \delta \geq 0$. Since total industry output is the same across $\widetilde{\mathbf{q}}$ and $\mathbf{q}^*$, they both yield the price p^*, and consumers' surplus and joint firm revenue are also the same. Yet since $\widetilde{\mathbf{q}}$ allocates a greater share of total output to the low cost firm, joint firm cost is lower for $\widetilde{\mathbf{q}}$, compared to $\mathbf{q}^*$. Indeed, restricting attention to production vectors with total industry output equal to q^*, the industry output at the equilibrium point, the only efficient vector is $\widehat{\mathbf{q}}_1 = (q^*, 0)$, where the high cost firm produces nothing. The equilibrium vector will typically not have this property. This is because while the substitution effect discriminates in favour of the low-cost firm, allowing it to produce more than the high-cost firm in equilibrium, the discriminatory impact may not be strong enough, when the market size is large relative to the high-cost firm's marginal cost, to achieve an efficient allocation.

3.3 Equilibrium with a fixed price

We now study equilibrium given a fixed price intervention p^0. Total demand is $\frac{a-p^0}{b}$, by $A1$. Suppose an equilibrium exists where firm i produces an amount $q_i^0 \geq 0$. Given the proposed equilibrium quantities q_1^0 and q_2^0, let p^d denote the market-clearing price that would have emerged had there been no price control. Then, $p^d = a - b(q_1^0 + q_2^0)$. We have the following result.

Lemma 1 *Suppose, given a fixed price p^0, an equilibrium exists where firm i produces an amount $q_i^0 \geq 0$. Given Assumptions 1, 2 and 3, $p^d \leq p^0$, and so $q^0 = q_i^0 + q_j^0 > 0$.*

Proof. Suppose in equilibrium, $p^d > p^0$. We shall show a contradiction.

Notice that in equilibrium, a firm i sells an amount q_i^0 at price p^0, and earns profit $q_i^0(p^0 - c_i) > 0$ iff $q_i^0 > 0$. Now consider a deviation by this firm. If it deviates and produces an amount $q_i^0 + \epsilon$, with ϵ small and positive, it sells the entire amount at the same price p^0. This is because there is excess demand at the price p^0, as $p^d > p^0$. It therefore increases profit by an amount $\epsilon(p^0 - c_i)$. Therefore, (q_1^0, q_2^0) cannot be an equilibrium. We see hence that if (q_1^0, q_2^0) constitutes an equilibrium, $p^d \leq p^0$.

In turn, this implies that $0 < a - p^0 \leq b(q_1^0 + q_2^0)$, and so $q^0 = q_i^0 + q_j^0 > 0$. ■

So excess demand cannot arise in equilibrium in the presence of a fixed price. This is because if excess demand exists, a firm can always produce and sell a little bit extra at the fixed price and obtain extra payoff. We now study whether it is possible, in equilibrium, to have excess supply, i.e., $p^d < p^0$. We show below that such an excess supply equilibrium exists if and only if the interventionary price is not too low. First define

$$\widetilde{p} = c_1 + c_2 < a, \text{ where the inequality follows from } A4$$

Lemma 2 *Given Assumptions 1 through 4, a unique excess supply equilibrium exists if and only if $p^0 > \widetilde{p}$.*

Proof. See the Appendix. ■

Thus, if an equilibrium with excess supply exists, it is unique. Such an equilibrium exists if and only if the fixed price is more than the sum of the two firms' marginal costs. The intuition is as follows. The fixed price generates a subsidy effect, as discussed earlier, and encourages production. For excess production to arise as an equilibrium phenomenon, the subsidy effect needs to be strong enough, for which in turn the interventionary price needs to be high enough.

The extent of equilibrium excess supply, as a function of p^0, can easily be calculated, using Lemma 2, on the domain $(\widetilde{p}, a)$. Denoting the degree of excess supply as $E(p^0)$, we find

$$E(p^0) = \frac{(a - p^0)(p^0 - \widetilde{p})}{b\widetilde{p}}$$

Thus, excess supply is strictly concave and vanishes as p^0 approaches either a, or $\widetilde{p}$.

We now study the possibility of a market-clearing equilibrium, given price regulation. With market clearing, industry output is such that the market-clearing price that would have emerged in the absence of regulation is exactly equal to the regulated price. We show below that such an equilibrium exists if and only if the interventionary price is not too high.

Lemma 3 *Given Assumptions 1 through 4, a continuum of market clearing equilibria exist if and only if* $p^0 < \widetilde{p}$*, while a unique market clearing equilibrium exists if and only if* $p^0 = \widetilde{p}$*.*

Proof. See the Appendix. ■

So, if the fixed price is less than the sum of the two firms' marginal costs, a continuum of market clearing equilibria exist. The market does not clear in equilibrium if $p^0 > \widetilde{p}$.

Below, we study the efficiency implications of imposing a fixed price. We shall restrict attention to sufficiently high fixed prices in order to guarantee that a unique excess supply equilibrium exists with a fixed price. Before that, we summarise the results of this section.

Proposition 2 *Suppose Assumptions 1 through 4 hold. Given a fixed price* p^0*, equilibrium is unique if and only if* $p^0 \geq \widetilde{p}$*.*

For $p^0 > \widetilde{p}$*, there is excess supply and equilibrium output levels are given by*

$$\underline{q}_1 = \frac{p^0(a-p^0)c_2}{b(c_1+c_2)^2}; \underline{q}_2 = \frac{p^0(a-p^0)c_1}{b(c_1+c_2)^2}$$

Consumers' surplus ($\underline{S}$)*, industry revenue* ($\underline{R}$)*, joint firm cost* ($\underline{C}$) *and total profit* ($\underline{\pi}$) *are respectively*

$$\underline{S} = \frac{(a-p^0)^2}{2b}; \underline{R} = \frac{(a-p^0)p^0}{b}; \underline{C} = \frac{2(a-p^0)p^0 c_1 c_2}{b(c_1+c_2)^2}; \underline{\pi} = \frac{p^0(a-p^0)(c_1^2+c_2^2)}{b(c_1+c_2)^2}$$

Otherwise, if $p^0 \leq \widetilde{p}$*, the market clears in any equilibrium, and there is a continuum of such equilibria if and only if* $p^0 < \widetilde{p}$*.*

As c_1 *approaches* 0*, there is a unique equilibrium with excess supply for any* $p^0 \in (c_2, a)$*, as* $\lim_{c_1 \to 0} \widetilde{p} = c_2$*.*

Proof. See the Appendix. ■

The results above were derived assuming $c_1 > 0$. When $c_1 = 0$, since $\widetilde{p} = c_2$, it is clear that a market clearing equilibrium cannot arise for any fixed price intervention with $p^0 > c_2$. It can be easily be shown that for any $p^0 \in (c_2, a)$, there is excess supply in equilibrium, and the high-cost firm produces 0 output. Equilibrium is not unique: any output level q_1^0 of the low-cost firm, such that $q_1^0 \geq \frac{p^0(a-p^0)}{bc_2}$, together with output level $q_2^0 = 0$ of the high-cost firm, constitutes an equilibrium. However, the values of consumers' surplus, industry revenue and joint firm cost are independent of which equilibrium is selected. Thus the main result of the paper, arising from a comparison across the unregulated and regulated regimes, derived in the following section, is not affected by this multiple equilibria issue.

3.4 Comparing the regimes

What is the efficiency implication of the imposition of a fixed price? In particular, suppose $\mathcal{R}$ sets a fixed price p^0. Compared to imposing no such restriction, can such a fixed price intervention simultaneously increase consumers' surplus, as well as joint firm profit?

To address that question, we first need to determine the relationship between $\widetilde{p}$, the maximum regulated price such that the market clears in equilibrium, and p^*, the unregulated equilibrium price. As will be seen below, an intervention can have interesting effects if the fixed price p^0 is no more than, and sufficiently close to, p^*. In order to guarantee that a unique excess supply equilibrium exists given such a controlled price, we need, using Proposition 2, $p^* > \widetilde{p}$. We see using Proposition 1 that

$$p^* > \widetilde{p} \Leftrightarrow \frac{a + (c_1 + c_2)}{3} > (c_1 + c_2) \Leftrightarrow a > 2(c_1 + c_2)$$

which is satisfied because of $A4$. This indeed is the justification for imposing Assumption 4. It ensures that an excess supply equilibrium exists in the regulated price model if the regulated price p^0 is set at or around the same level as the unregulated equilibrium price p^*.

We show below that efficiency enhancing price interventions can exist when c_1 is sufficiently small. Proposition 2 tells us that there is a unique excess supply equilibrium for any $p^0 > c_2$ in the limit as c_1 approaches 0. For the rest of the paper, therefore, we focus, for this reason, on excess supply equilibria.

We have

Proposition 3 *Suppose Assumptions 1 through 4 are satisfied. If $\mathcal{R}$ imposes a fixed price $p^0 = p^* - \delta$, with $\delta > 0$, and small, then consumers' surplus as well as joint firm profit are strictly higher given this fixed price, compared to when no regulation is imposed, provided c_1 is sufficiently small relative to c_2. If in addition $a < \frac{7}{2}c_2$, then as c_1 approaches 0, any fixed price $p^0 \in (c_2, p^*)$ induces strict efficiency enhancement compared to the unconstrained outcome.*

Proof. See the Appendix. ■

Thus, when the degree of asymmetry between the firms is sufficiently high, given Assumptions 1 through 4, we can find fixed price interventions which lead to more efficient outcomes, i.e., which allow higher consumers' surplus, as well as higher joint firm profit, when compared to the unregulated equilibrium outcomes.

The intuition in terms of substitution and subsidy effects discussed earlier may be useful in understanding the result of Bergstrom and Varian (1985a, b), Salant and Shaffer (1999), Long and Soubeyran (2001) and Février and Linnemer (2004), outlined above. Consider the unconstrained interior equilibrium. Suppose c_2 increases and c_1 decreases such that their sum is unchanged. The relative decline in the low-cost firm's marginal cost increases the variance of marginal costs, and also changes the unconstrained Cournot substitution effect to further favour the low-cost firm. This causes the low-cost firm's output to increase, and the high-cost firm's output to decrease. As seen from Proposition 1, q_1^* increases and q_2^* decreases, while q^* remains constant, and so joint firm cost decreases, while industry revenue and consumers' surplus are unchanged.

4 Relaxing some assumptions

We showed above that a fixed price intervention in a duopoly setting where firms compete à la Cournot can increase consumers' surplus, as well as joint firm profit, when compared to the outcome at the unregulated equilibrium. The result was derived under some simplifying assumptions. We now study the impact of perturbing the assumptions. Consider first Assumption 3. The requirement that firms cannot sell with impunity at a price other than the regulated one is clearly important, as the price control needs to be reasonably effective. However, disposal costs do not need to be 0; the presence of a small disposal cost does not change our results qualitatively. The uniform rationing rule is the obvious one given the standard Cournot homogeneous good assumption, and is similarly also not critical. Any rationing rule, 'close' to the uniform one, such that a firm's expected sale is increasing and concave in its output, given excess supply, should yield similar outcomes.

We now show that our results may still hold when demand and cost are non-linear, and also when there are several high-cost firms. We first study non-linear demand.

4.1 Non-linear demand

Consider the model developed in Section 2, and leave Assumptions 2 and 3 unchanged. We replace Assumption 1 by

Assumption $1'$: The inverse demand function $p(q)$ is decreasing and differentiable. A unique interior equilibrium exists in the absence of price interventions, and is given by the first-order conditions.[7]

Denote the unconstrained equilibrium price by p^*. Replace Assumption 4 by

Assumption $4'$: $p^* > c_1 + c_2$.

The assumptions ensure that if a sufficiently high fixed price is imposed, industry output in equilibrium exceeds total demand given the regulated price. They further guarantee that if a fixed price is imposed at or around the level of the unconstrained equilibrium price, there is a unique excess supply equilibrium. We have

Lemma 4 *If a fixed price p^0 is imposed, given Assumptions 1', 2, 3 and 4', a unique excess supply equilibrium exists if and only if $p^0 > \widetilde{p} = c_1 + c_2$. Given $p^0 > \widetilde{p}$, market-clearing equilibria do not exist.*

Proof. Suppose p^0 is the fixed price, and Assumptions 1', 2, 3 and 4' hold. Arguments similar to those of Lemma 1 can be used to show that in any equilibrium, industry output is positive, and there is no excess demand. Next, the arguments of Lemma 2 can be easily extended to show that if an excess supply equilibrium exists, it is unique, and firms produce

$$q_1^0 = \underline{q}_1 = \frac{p^0 q^d c_2}{(c_1 + c_2)^2}; q_2^0 = \underline{q}_2 = \frac{p^0 q^d c_1}{(c_1 + c_2)^2}$$

where q^d is the total demand given p^0. In such an equilibrium, total industry output equals

$$q^0 = \frac{p^0 q^d}{(c_1 + c_2)}$$

Further, since there is excess supply, it is necessary that

[7] Many authors have derived general existence and uniqueness conditions for interior equilibrium in Cournot oligopoly: see, for example, Novshek (1985) and Gaudet and Salant (1991).

$$q^0 > q^d, \text{ or } p^0 > \widetilde{p} = c_1 + c_2$$

Also, given $p^0 > \widetilde{p}$, the same line of argument as in Lemma 2 shows that $(\underline{q}_1, \underline{q}_2)$ constitutes an excess supply equilibrium.

Finally, the arguments of Lemma 3 can be straightforwardly extended to show that market-clearing equilibria exist if and only if $p^0 \leq \widetilde{p}$. ■

Armed with this result, and Assumptions 1', 2, 3 and 4', we can now compare aggregate industry outcomes under the two regimes. Using a line of argument similar to that in the proof of Proposition 3, we can show that fixed price interventions may simultaneously increase consumers' surplus, as well as joint firm profit.

Suppose $\frac{c_1}{c_2}$ is small. Aggregate cost in the unregulated regime is positive, given interior equilibrium, and converges to a positive level as c_1 approaches 0, by $A1'$. Suppose a regulated price is set at p^*. Industry revenue and consumers' surplus are then the same across the two regimes. Further, Lemma 4 can be used to show that aggregate industry cost in the regulated regime goes to 0 as $\frac{c_1}{c_2}$ becomes vanishingly small, and so joint firm profit is higher for sufficiently small $\frac{c_1}{c_2}$. A continuity argument can then be used to show that both consumers' surplus, as well as joint firm profit, are strictly higher than in the unregulated regime, when a regulated price is set slightly below p^*. We have proved

Proposition 4 *Suppose Assumptions 1', 2, 3 and 4' hold. Then, for $\frac{c_1}{c_2}$ sufficiently small, there exist fixed price interventions yielding higher consumers' surplus as well as joint firm profits, both strictly, compared to the unregulated case.*

Another intuition behind the result arises from the first-order conditions guiding an interior Cournot equilibrium, in which

$$\frac{MR_1}{MR_2} = \frac{c_1}{c_2}$$

But since, from Lemma 2, in excess supply equilibrium, given a fixed price

$$\frac{MR_1}{MR_2} = \frac{q_2^0}{q_1^0}, \text{ we find } \frac{q_2^0}{q_1^0} = \frac{c_1}{c_2}$$

So when $\frac{c_1}{c_2}$ goes to 0, so does $\frac{q_2^0}{q_1^0}$, and hence joint firm cost. An alternative view of this point may be obtained by considering the constant elasticity inverse demand function, with $p(q) = \frac{\lambda}{q^{\kappa}}$, with $\lambda > 0$, and $\kappa \in (0,\ 2)$. Notice that the analytics when the elasticity parameter κ is unity resemble those with a fixed price. In such a situation, the ratio of marginal revenues in equilibrium simultaneously equals the ratio of marginal costs as well as the inverse production ratio. And so an increase in the degree of asymmetry causes the low-cost firm to produce more and the high-cost firm to produce less.

Considering this example further, it is easy to show that in equilibrium without intervention

$$q_1^* = [\frac{\lambda(2-\kappa)}{c_1+c_2}]^{1/\kappa}[\frac{c_2+c_1(\kappa-1)}{\kappa(c_1+c_2)}];\, q_2^* = [\frac{\lambda(2-\kappa)}{c_1+c_2}]^{1/\kappa}[\frac{c_1+c_2(\kappa-1)}{\kappa(c_1+c_2)}]$$

$A1'$ is always satisfied whenever $\kappa > 1$.[8] Moreover, since

$$p^* = \frac{c_1+c_2}{2-\kappa}, p^* > c_1+c_2 \Leftrightarrow \kappa > 1$$

Thus, $\kappa > 1$ is necessary and sufficient for $A4'$. Suppose therefore $\kappa > 1$, and a fixed price has been set at p^*. As before, industry revenue and consumers' surplus are the same across the regimes. Joint firm cost in the unregulated and regulated regimes are respectively, using Lemma 4

$$C^* = [\frac{\lambda(2-\kappa)}{c_1+c_2}]^{1/\kappa}[\frac{(c_1^2+c_2^2)(\kappa-1)+2c_1c_2}{\kappa(c_1+c_2)}];\, \underline{C} = \frac{2\lambda^{1/\kappa}c_1c_2}{(c_1+c_2)^{1+1/\kappa}(2-\kappa)^{1-1/\kappa}}$$

We have

$$C^* > \underline{C} \Leftrightarrow \frac{\lambda^{1/\kappa}(\kappa-1)[(2-\kappa)(c_1^2+c_2^2)-4c_1c_2]}{\kappa(c_1+c_2)^{1+1/\kappa}(2-\kappa)^{1-1/\kappa}} > 0$$

which is always true for c_1 small enough, as $\kappa > 1$. If $\kappa = 1$, and the fixed price is set at p^*, there is a unique market clearing equilibrium in the regulated regime, which is identical to that in the unconstrained regime. The joint firm cost levels are also therefore the same across the regimes.

The condition $p^* > c_1 + c_2$ does not always yield a ready reformulation in terms of parameters and functional forms. It does, however, have a useful reinterpretation in terms of a comparison between costs and equilibrium firm output levels.

[8] If $\kappa \leq 1$, then $q_2^* = 0$ whenever $c_1 \leq c_2(1-\kappa)$.

To see that, recall the first-order conditions driving the unconstrained equilibrium, evaluated at the equilibrium point:

$$p^* + q_1^* p'(q^*) = c_1; p^* + q_2^* p'(q^*) = c_2$$

Subtracting the second equation from the first, we find

$$(q_1^* - q_2^*)p'(q^*) = c_1 - c_2, \text{ or } q_1^* > q_2^*$$

Adding the two equations, we see

$$2p^* + q^* p'(q^*) = c_1 + c_2, \text{ so } p^* > c_1 + c_2 \Leftrightarrow p^* + q^* p'(q^*) < 0$$

where $p^* + q^* p'(q^*)$ is the industry marginal revenue evaluated at the equilibrium point. Simple algebra then yields, because $q_1^* > q_2^*$,

$$p^* + q^* p'(q^*) < 0 \Leftrightarrow \frac{p^* + q_1^* p'(q^*)}{p^* + q_2^* p'(q^*)} < \frac{q_2^*}{q_1^*}, \text{ so } p^* > c_1 + c_2 \Leftrightarrow \frac{p^* + q_1^* p'(q^*)}{p^* + q_2^* p'(q^*)} < \frac{q_2^*}{q_1^*}$$

Also, since

$$\frac{p^* + q_1^* p'(q^*)}{p^* + q_2^* p'(q^*)} = \frac{c_1}{c_2}, \text{ we find } p^* > c_1 + c_2 \Leftrightarrow \frac{c_1}{c_2} < \frac{q_2^*}{q_1^*} \Leftrightarrow c_1 q_1^* < c_2 q_2^*$$

The condition is then equivalent to requiring that the marginal cost of the low-cost firm expressed as a fraction of the marginal cost of the high-cost firm is less than the output of the high-cost firm expressed as a fraction of the output of the low-cost firm, or that total cost of the low-cost firm is less than the total cost of the high cost firm.

4.2 Non-linear cost

Consider an extension of the basic duopoly model developed above, and assume that the inverse demand function is $a - bq$. Assume also that firm i's cost function is $\alpha_i c(q_i)$, where $c(0) = 0$ and $c'(0) > 0$. For simplicity, suppose $\alpha_2 = 1$, and $\alpha_1 \in (0, 1)$. Assume that an interior equilibrium exists in the absence of price interventions, and is given by the first-order conditions. The first-order conditions in the unregulated case are:

$$\text{Firm 1}: a - 2bq_1 - bq_2 = \alpha_1 c'(q_1); \text{Firm 2}: a - bq_1 - 2bq_2 = c'(q_2)$$

Since equilibrium is interior, joint firm cost C^* is positive, and converges to a positive level as α_1 approaches 0. Let the unregulated equilibrium price be denoted p^*. Now consider a fixed price intervention with $p^0 = p^*$. Assume that an excess supply equilibrium exists given this intervention. Let the unregulated equilibrium industry output be q^*. The first-order conditions are:

$$\text{Firm 1}: \frac{p^* q^* q_2^0}{(q_1^0 + q_2^0)^2} = \alpha_1 c'(q_1^0); \text{Firm 2}: \frac{p^* q^* q_1^0}{(q_1^0 + q_2^0)^2} = c'(q_2^0)$$

In equilibrium $q_1^0 > q_2^0$. For if not, then from the first-order conditions

$$1 \leq \frac{q_2^0}{q_1^0} = \frac{\alpha_1 c'(q_1^0)}{c'(q_2^0)} < 1$$

which yields a contradiction. Also

$$q_2^0 \geq 0 \text{ and } \alpha_1 c'(q_1^0) + c'(q_2^0) > 0$$

Then we find, using the first order conditions again

$$\text{Firm 1}: \frac{q_2^0[\alpha_1 c'(q_1^0) + c'(q_2^0)]^2}{p^* q^*} = \alpha_1 c'(q_1^0); \text{Firm 2}: \frac{q_1^0[\alpha_1 c'(q_1^0) + c'(q_2^0)]^2}{p^* q^*} = c'(q_2^0)$$

$$\text{And therefore } \lim_{\alpha_1 \to 0} q_2^0 = 0; \lim_{\alpha_1 \to 0} q_1^0 = \frac{p^* q^*}{c'(0)} > 0$$

And so joint firm cost in the unregulated case, $\underline{C}$, is less than joint firm cost in the regulated case, C^*, for sufficiently small α_1.

4.3 Many high-cost firms

Consider an extension of the basic duopoly model constructed earlier with a single low-cost firm which has constant marginal cost ρc, $\rho \in (0, 1)$ and $c > 0$, and $n - 1$ ($n > 2$) high-cost firms which have constant and common marginal cost of production c. Firms face the inverse demand function $p(q) = a - bq$. Let the low-cost firm's output be denoted as q_l, and let a high-cost firm's output be denoted as q_h. Assume

$$\frac{a}{c} > 2(1 + \frac{\rho}{n-1})$$

A unique interior equilibrium then exists in the absence of price regulation.[9] Let the unregulated equilibrium price be denoted p^*. From the first-order conditions we have in equilibrium,

$$q_l^* = \frac{a + c[n(1-\rho) - 1]}{(n+1)b}; q_h^* = \frac{a - c[(1-\rho) + 1]}{(n+1)b}; p^* = \frac{a + c[(n-1) + \rho]}{n+1}$$

Now suppose the regulator imposes a fixed price p^0. It can be readily shown, extending the arguments of Lemmata 1 through 3, and Proposition 2, that excess demand cannot arise in equilibrium, and that for $p^0 > \widetilde{p}_n = c(1 + \frac{\rho}{n-1})$, there is a unique equilibrium, with excess supply.[10] The parameter restriction above ensures that $a > \widetilde{p}_n$. In such an equilibrium, let the low-cost firm's output be denoted as q_l^0, and let a high-cost firm's output be denoted as q_h^0. Then

$$q_l^0 = \frac{(n-1)(a-p^0)p^0[\rho + (n-1)(1-\rho)]}{bc[(n-1)+\rho]^2}; q_h^0 = \frac{(n-1)(a-p^0)p^0\rho}{bc[(n-1)+\rho]^2}$$

Also,

$$p^* > \widetilde{p}_n \Leftrightarrow \frac{a}{c} > 2(1 + \frac{\rho}{n-1})$$

So under our assumptions, if the regulator sets a fixed price $p^0 = p^*$, an excess supply equilibrium exists given this intervention. Since the price, and hence the demand, is the same across the two cases, consumers' surplus as well as industry revenue are identical across regimes. The joint firm cost in the unregulated and regulated regimes are respectively

$$C^* = \frac{ac[(n-1)+\rho] - \rho^2 c^2 - (n-1)c^2[1 + (1-\rho)^2]}{(n+1)b}$$

[9] As seen from the expressions below, such an equilibrium exists if and only if $\frac{a}{c} > 2 - \rho$. The parameter restriction above guarantees this condition is satisfied.

[10] Since $\lim_{n\to\infty} \widetilde{p}_n = c$, there is a unique excess supply equilibrium for every $p^0 > c$, when the dominant firm interacts with a "competitive fringe".

$$C^0 = \frac{(n-1)(a-p^*)p^*\rho[2(n-1)-(n-2)\rho]}{b[(n-1)+\rho]^2}$$

It is immediate that, given any n, $C^0 < C^*$ for ρ sufficiently small. Hence, the efficiency enhancing effect of a fixed price intervention can persist when the dominant firm interacts with a competitive fringe. The logic is similar to the duopoly case. In an excess supply equilibrium given a fixed price intervention, any high cost firm's output is increasing in the marginal cost of the low-cost firm. Thus, when the low-cost firm's marginal cost is sufficiently low relative to that of the high-cost firms, the high-cost firms' outputs go to 0. Therefore, as ρ goes to 0, the total cost of the low-cost firm goes to 0, as does the total cost of all high-cost firms. However, as long as a is sufficiently large relative to c, the total output of all high-cost firms is positive in the absence of price regulation, and so total cost of all high-cost firms taken together is positive regardless of how small ρ is. And hence joint firm cost in the unregulated regime exceeds that in the regulated regime for sufficiently small ρ.

5 Price floors

The reason why a fixed price can induce equilibrium production vectors which yield higher joint firm profits compared to the unregulated equilibrium production point is because it gives a production subsidy to the low cost firm, by removing the possibility of price decreases. A fixed price can be thought of as a price band, with a floor and a ceiling, and the added proviso that the ceiling and the floor are the same. As indicated earlier, it is the floor component of a fixed price which drives this result. A fixed price was used for the illustration above because of expositional ease. Firm maximisation problems are typically not well-behaved with pure price floors, and multiple equilibrium issues can arise, as we now discuss.

Consider the basic duopoly model constructed earlier, and assume Assumptions 1 through 4 hold. Denote a price floor by p_f. Let the unregulated equilibrium price and production vector be denoted respectively as p^* and $\mathbf{q}^*$ (see Proposition 1), and the excess supply equilibrium production vector as $\underline{\mathbf{q}}$ (see Proposition 2).

Given a price floor p_f, let $\mathbf{q}^e$ denote an equilibrium production vector, and p^e the corresponding equilibrium price.[11] It is then easy to show, using the same arguments as earlier, that for $p_f \geq p^*$, there is a unique equilibrium, which displays excess supply, with $\mathbf{q}^e = \underline{\mathbf{q}}$ and $p^e = p_f$, while for $p_f < \widetilde{p} = c_1 + c_2 < p^*$, there is a unique equilibrium, and the market clears, with $\mathbf{q}^e = \mathbf{q}^*$ and $p^e = p^*$.

Thus, if p_f is set equal to p^*, there is a unique excess supply equilibrium, and equilibrium consumers' surplus and industry revenue are the same as in the unregulated case. Proposition 3 can then be used to determine when joint firm costs are lower, and so efficiency is higher, with a price floor.

The problem arises for $p_f \in [\widetilde{p}, p^*)$, as both the unregulated equilibrium point, as well as the excess supply equilibrium point with a binding price floor, satisfy local necessary conditions for equilibrium. However, a firm's payoff function is typically not well-behaved, and may contain a point of non-differentiability with a change in the sign of its derivative in the neighbourhood of this point, and so the equilibrium points identified above may not satisfy global necessity conditions.

Define:

$$N_i^d : \frac{p_f(a - p_f)c_i}{(c_1 + c_2)^2} > 2(a - p_f) - (a - c_i)$$

$$\widetilde{q}_i = \frac{a - c_i}{2b} - \frac{p_f(a - p_f)c_i}{2b(c_1 + c_2)^2}$$

$$M_i^d : \widetilde{q}_i(a - b\underline{q}_j - b\widetilde{q}_i) - c_i\widetilde{q}_i < \frac{\underline{q}_i p_f(a - p_f)}{b(\underline{q}_1 + \underline{q}_2)} - c_i\underline{q}_i; i \neq j$$

So, given $p_f \in [\widetilde{p}, p^*)$, when does an excess supply equilibrium exist? It is easy to show that firm i has no incentive to choose any production level other than $\underline{q}_i$, given that firm j is producing $\underline{q}_j$, if N_i^d is satisfied. If N_i^d is not satisfied, firm i still has no incentive to deviate from the excess supply equilibrium if M_i^d is satisfied. N_i^d ensures that firm i's payoff, as a function of its production level q_i, given firm j produces $\underline{q}_j$, is monotone increasing if $q_i < \underline{q}_i$. If N_i^d is violated, firm i may have an incentive, given firm j produces $\underline{q}_j$, to produce $\widetilde{q}_i$ instead of $\underline{q}_i$. M_i^d then ensures that firm i's profit from producing $\underline{q}_i$ is higher than that from producing $\widetilde{q}_i$.

[11] With a price floor, the equilibrium price may not always equal p_f.

Thus an excess supply equilibrium exists if either (i) N_1^d and N_2^d both hold, or (ii) if N_1^d is violated, then M_1^d and N_2^d hold, or (iii) if N_2^d is violated, then N_1^d and M_2^d hold, or (iv) if N_1^d and N_2^d are both violated, then M_1^d and M_2^d hold. If this equilibrium is unique, or is selected if not unique, then the same result as derived earlier, with respect to the benefits from a price intervention, holds when a price floor is imposed.

6 Conclusion

We study a dominant firm Cournot oligopoly model with one low-cost firm, and one or more identical high-cost firms. A reallocation of production relative to an interior equilibrium point, such that the low-cost firm produces more, while the high-cost firms produce less, can increase consumers' surplus, as well as joint firm profit. The reason is that while the Cournot substitution effect relatively benefits the low-cost firm, and allows it to produce more than the high-cost rivals in equilibrium, it may not be strong enough to reach an efficient allocation.

A fixed price intervention may be able to enhance efficiency in such a setting. The regulated price behaves, by preventing a reduction in the price when firms increase output, as a production subsidy. The benefit is differential, and favours the low-cost firm more than it does the high-cost firms. At the same time, a discriminatory substitution effect, different from the well-known substitution effect in the unconstrained Cournot model, exists as well, if excess supply arises in equilibrium. The subsidy and the substitution effects may then act in conjunction in favour of the low-cost firm, and in opposition against the high-cost firms, resulting in an efficiency enhancing production reallocation under some conditions. Similar effects can exist with pure price floors. The results may help in re-evaluating the effects of price interventions in imperfectly competitive settings when firms are asymmetric.

Although industry profit is higher with an appropriately chosen price intervention, inefficient firms are worse off, as they are forced to reduce production because of the interplay of the substitution and subsidy effects. Interventions of the form discussed in this paper may therefore have beneficial consequences if it is useful to discourage production by high-cost firms. At the same time, if inefficient firms have higher cost because they are new or have untested technology whose efficacy can be increased substantially by learning through production, such interventions, by affecting the pattern of investment and entry, can have long-term deleterious effects. These issues are left for future research.

7 Appendix

Proof of Lemma 2. Suppose, given some p^0, and Assumptions 1 through 4, there exists an excess supply equilibrium. Let (q_1^0, q_2^0) constitute an equilibrium, with $a - b(q_1^0 + q_2^0) < p^0$. Since there is excess supply, no firm sells its entire output, and instead faces a probability $\frac{a-p^0}{b(q_1^0+q_2^0)}$ of selling any unit of its output. Given the good is perfectly divisible, we suppose that this is the fraction of a firm's output that is sold in equilibrium. Using Assumption 3, firm i's equilibrium payoff is given by

$$\frac{q_i^0(a-p^0)p^0}{b(q_i^0+q_j^0)} - c_i q_i^0$$

Firm i sells $\frac{q_i^0(a-p^0)}{b(q_i^0+q_j^0)}$ units. For any unit that is sold, the firm earns p^0, while unsold units are disposed off freely, and earn the firm no revenue. The cost of production is $c_i q_i^0$. We see that both firms must produce a positive amount in an excess supply equilibrium, as otherwise, the firm with positive production can always reduce its output by a small amount and raise its profit. So one necessary condition for (q_1^0, q_2^0) to be an equilibrium is that no firm has a unilateral incentive to produce less than its equilibrium level. Equally, another necessary condition is that no firm has a unilateral incentive to produce more than its equilibrium level. These conditions must be satisfied globally. Considering the local versions of these necessary conditions together, we have the first order conditions:

$$\text{Firm } i\text{: } \frac{p^0(a-p^0)q_j^0}{b(q_i^0+q_j^0)^2} - c_i = 0\text{; and Firm } j\text{: } \frac{p^0(a-p^0)q_i^0}{b(q_i^0+q_j^0)^2} - c_j = 0$$

Solving these two equations simultaneously, we see that a necessary condition for (q_1^0, q_2^0) to be an equilibrium is that

$$q_1^0 = \underline{q}_1 = \frac{p^0(a-p^0)c_2}{b(c_1+c_2)^2}; q_2^0 = \underline{q}_2 = \frac{p^0(a-p^0)c_1}{b(c_1+c_2)^2}$$

In turn, this implies that in equilibrium

$$q^0 = q_1^0 + q_2^0 = \underline{q}_1 + \underline{q}_2 = \frac{p^0(a - p^0)}{b(c_1 + c_2)}$$

And so,

$$p^d = \frac{a(c_1 + c_2) - ap^0 + (p^0)^2}{c_1 + c_2}$$

Naturally, another necessary condition that has to be satisfied is

$$p^d < p^0, \text{ or } (p^0)^2 - ap^0 + a(c_1 + c_2) < p^0(c_1 + c_2)$$

The right hand side of the above inequality is linear increasing in p^0, with slope $(c_1 + c_2)$. The left hand side is differentiable and strictly convex, attains its minimum at $p^0 = \frac{a}{2}$, and has $LHS(p^0 = 0) = LHS(p^0 = a) = a(c_1 + c_2)$. Furthermore, at $p^0 = a$, the slope of the left hand side is $a > (c_1 + c_2)$. We conclude therefore that

$$\exists! \widetilde{p} \in (0, a) \ni p^d < p^0 \Leftrightarrow p^0 \in (\widetilde{p}, a)$$

Moreover, by setting

$$(p^0)^2 - ap^0 + a(c_1 + c_2) = p^0(c_1 + c_2)$$

we see there are two solutions: a, and $c_1 + c_2$. We therefore have $\widetilde{p} = c_1 + c_2$.

So suppose $p^0 \in (\widetilde{p}, a)$. Is $(\underline{q}_1, \underline{q}_2)$ an equilibrium? Let $\underline{q}_1 + \underline{q}_2 = \underline{q}$. We already know that the market-clearing price that would have emerged in the absence of regulation given an industry output level $\underline{q}$, $a - b\underline{q} = p^d$, is less than p^0. Suppose now firm j produces $\underline{q}_j$. What is firm i's payoff, as a function of q_i?

We know if $a - b(q_i + \underline{q}_j) < p^0$, there is excess supply at the regulated price. For any such q_i, firm i's payoff is then

$$f_m(q_i; \underline{q}_j, p^0) = \frac{q_i(a - p^0)p^0}{b(q_i + \underline{q}_j)} - c_i q_i$$

But if $a - b(q_i + \underline{q}_j) \geq p^0$, which could happen if $\underline{q}_j$ and q_i are both relatively small, there is excess demand at the regulated price, unless the inequality is weak. For any such q_i, firm i's payoff is then

$$f_l(q_i; \underline{q}_j, p^0) = (p^0 - c_i)q_i$$

Summing up the discussion, we find that given $\underline{q}_j$, either firm i's payoff is $f_m(q_i)$ for all q_i, or it is $f_m(q_i)$ as long as q_i is not too small relative to $\underline{q}_i$, and $f_l(q_i)$ otherwise. Also recall that $\underline{q}_i$ is the unique maximiser of $f_m(q_i; \underline{q}_j)$.

Now, suppose $\exists\, \widetilde{q}_i \geq 0 \ni a - b(\widetilde{q}_i + \underline{q}_j) = p^0$. If there exists no such $\widetilde{q}_i$, we see immediately that firm i chooses $\underline{q}_i$ as its best response to $\underline{q}_j$. On the other hand, if such a $\widetilde{q}_i$ exists, then, since $f_l(q_i)$ is strictly increasing, the firm never chooses $q_i < \widetilde{q}_i$ as its best response to $\underline{q}_j$. Furthermore, we observe that (i) $f_m(\widetilde{q}_i) = f_l(\widetilde{q}_i)$, (ii) $\underline{q}_i > \widetilde{q}_i$, and (iii) $f_m(q_i)$ is strictly increasing for $q_i < \underline{q}_i$. We therefore conclude that given $\underline{q}_j$, $\underline{q}_i$ is the unique global maximiser of firm i's payoff for $p^0 \in (\widetilde{p}, a)$. Identical arguments show that that given $\underline{q}_i$, $\underline{q}_j$ is the unique global maximiser of firm j's payoff for $p^0 \in (\widetilde{p}, a)$. Hence, $(\underline{q}_1, \underline{q}_2)$ is the unique equilibrium when $p^0 \in (\widetilde{p}, a)$. And of course there is excess supply at this equilibrium. ■

Proof of Lemma 3. We first show that if a market clearing equilibrium exists, both firms must produce a positive amount.

Suppose not, and let firm i produce $q_i^0 = 0$, and firm j produce $q_j^0 = \frac{(a-p^0)}{b}$. It is clear that firm j has no incentive to deviate and produce a different amount. Suppose firm i deviates and produces amount $\epsilon > 0$. It gains from this deviation if and only if

$$\frac{p^0(a-p^0)\epsilon}{b(\frac{a-p^0}{b} + \epsilon)} - c_i\epsilon > 0 \Leftrightarrow (p^0 - c_i)(\frac{a-p^0}{b}) > c_i\epsilon$$

which is always true for ϵ small enough. We now proceed with the rest of the proof.

Suppose a market clearing equilibrium exists with both firms producing positive amounts. Let $(q_1^0,\, q_2^0)$ be the equilibrium output levels of the two firms, with

$$\text{The market-clearing condition}: \; a - b(q_1^0 + q_2^0) = p^d = p^0$$

It is clear that no firm has an incentive to deviate and produce less. At the same time, for $(q_1^0,\, q_2^0)$ to constitute an equilibrium, firms cannot have an incentive to unilaterally deviate and produce more. The local necessary conditions are

$$\text{Firm 1: } \frac{p^0(a-p^0)q_2^0}{b(q_1^0+q_2^0)^2} - c_1 \leq 0, \text{ and Firm 2: } \frac{p^0(a-p^0)q_1^0}{b(q_1^0+q_2^0)^2} - c_2 \leq 0$$

Adding, we have as a necessary condition

$$\frac{p^0(a-p^0)}{b(q_1^0+q_2^0)} \leq c_1 + c_2$$

Then, using the maket clearing condition, we find

$$p^0 \leq c_1 + c_2 = \widetilde{p}$$

So we have shown that if a market clearing equilibrium exists, we must have $p^0 \leq \widetilde{p}$. We now prove the other direction.

Suppose $p^0 \leq \widetilde{p}$. For some $\phi \in (0, 1)$, let

$$q_1^0 = \phi(\frac{a-p^0}{b}); q_2^0 = (1-\phi)(\frac{a-p^0}{b})$$

We shall show that we can always find a set of values of ϕ such that the above quantities consitute an equilibrium. Clearly, no firm has an incentive to deviate and produce less. We need to check therefore that no firm has an incentive to deviate and produce more. It is clear that it suffices to check whether the local 'no upward deviation constraint' is satisfied. Firms 1 and 2 respectively have no such incentive if

$$\begin{aligned} \text{Firm 1} \quad &: \quad p^0(1-\phi) - c_1 \leq 0, \text{ or } \frac{p^0 - c_1}{p^0} \leq \phi \\ \text{Firm 2} \quad &: \quad p^0\phi - c_2 \leq 0, \text{ or } \phi \leq \frac{c_2}{p^0} \end{aligned}$$

The proof is complete as simple algebra shows that the set of values of ϕ satisfying both these conditions is a non-empty interval if and only if $p^0 \leq \widetilde{p}$, and the set is a singleton, with $\phi = \frac{c_2}{c_1+c_2}$, if and only if $p^0 = \widetilde{p}$. ■

Proof of Proposition 2. The proof follows from Lemmata 1 through 3. For the case $p^0 \leq \widetilde{p}$, the proof is immediate from Lemmata 1 and 3.

Suppose then $p^0 > \widetilde{p}$. The expressions for $\underline{q}_1$ and $\underline{q}_2$ have been derived in Lemma 2. Since total sale (demand), given an excess supply equilibrium, at $p = p^0$ is $\frac{a-p^0}{b}$, consumers' surplus is

$$\underline{S} = \frac{1}{2}(\frac{a-p^0}{b})(a-p^0) = \frac{(a-p^0)^2}{2b}$$

In the excess supply equilibrium, industry revenue is

$$\underline{R} = \frac{\underline{q}_1(a-p^0)p^0}{b(\underline{q}_1+\underline{q}_2)} + \frac{\underline{q}_2(a-p^0)p^0}{b(\underline{q}_1+\underline{q}_2)} = \frac{(a-p^0)p^0}{b}$$

Joint firm cost ($\underline{C}$) is

$$\underline{C} = c_1\underline{q}_1 + c_2\underline{q}_2 = \frac{2(a-p^0)p^0 c_1 c_2}{b(c_1+c_2)^2}$$

Finally, total profit is given by

$$\underline{\pi} = \underline{R} - \underline{C} = \frac{(a-p^0)p^0(c_1^2+c_2^2)}{b(c_1+c_2)^2}$$

Finally, there is a unique excess supply equilibrium for any $p^0 \in (c_2, a)$ when c_1 approaches 0, as $\widetilde{p}(c_1 = 0) = c_2$. ■

Proof of Proposition 3. Suppose the fixed price, if imposed, is $p^0 = p^* - \delta$, with $\delta > 0$, and small. Suppose also $c_2 = c > 0$, and $c_1 = \rho c$, with $\rho \in (0, 1)$. Since $p^0 > \widetilde{p}$, as δ is small, a unique equilibrium exists with excess supply if the fixed price is imposed (see Proposition 2). Using Proposition 2, it is easy to see that consumers' surplus $\underline{S}$ is strictly decreasing in p^0, where

$$\underline{S}(p^0) = \frac{(a-p^0)^2}{2b}$$

How does consumers' surplus in the fixed price equilibrium compare with the consumers' surplus in the unregulated equilibrium? Using Propositions 1 and 2, we find

$$\underline{S}(p^0 = p^* - \delta) - S^* = \frac{(a-p^*+\delta)^2}{2b} - \frac{(a-p^*)^2}{2b} > 0$$

Thus, consumers' surplus in the regulated environment is higher than consumers' surplus in the unregulated environment when $p^0 < p^*$. We now turn our attention to the firms. Given $p^0 = p^* - \delta$, how does joint firm profit in the regulated equilibrium, $\underline{\pi}(p^0)$, compare with joint firm profit in the unregulated equilibrium, π^*?

To answer that, we recall, using Proposition 2, for $p^0 > \widetilde{p}$, joint profit in the regulated case is

$$\underline{\pi}(p^0) = \frac{p^0(a-p^0)(1+\rho^2)}{b(1+\rho)^2}$$

How does joint firm profit in regulated equilibrium change as the fixed price changes? It is easy to see that $\underline{\pi}(p^0)$ is a concave in p^0, and achieves a maximum at $p^0 = \frac{a}{2}$. Moreover,

$$\frac{a}{2} - p^* = \frac{a - 2c(1+\rho)}{3} > 0, \text{ by } A4.$$

Therefore, since $p^* > \widetilde{p}$, by $A4$, $\underline{\pi}(p^0)$ is increasing in p^0 for $p^0 \in (\widetilde{p},\, p^*]$. We also see that $\underline{\pi}(p^0; \rho)$ is continuous in p^0 (and hence δ) and ρ. Next we see, using Propositions 1 and 2, after some straightforward algebra, that

$$\underline{\pi}(p^0 = p^*; \rho) > \pi^*(\rho)$$

$$\Leftrightarrow 2\rho[2a^2 + ac(1+\rho) - c^2(1+\rho)^2] < 3c(1+\rho)^2[a(1+\rho) + c(1+\rho)^2 - 3c(1+\rho^2))]$$

At $\rho = 1$, the above inequality becomes

$$(a-c)(a-4c) < 0$$

which is never true. Hence, if the two firms are symmetric, joint firm profit with regulated price set at p^* is lower than joint firm profit in the unregulated environment. Notice however, both the left hand and the right hand side of the expression are continuous in ρ. Moreover,

$$LHS(\rho = 0) = 0; RHS(\rho = 0) = 3c(a - 2c) > 0, \text{ by } A2 \text{ and } A4.$$

Since π^* is independent of δ, and continuous in ρ, by Proposition 1, we immediately conclude that $\exists \overline{\delta} > 0$, such that for every $\delta \in (0,\, \overline{\delta})$, if $p^0 = p^* - \delta$, $\exists \overline{\rho}(\delta) > 0$, such that $\rho \in (0,\, \overline{\rho}(\delta))$ $\Rightarrow \underline{\pi} > \pi^*$.

In other words, as long as one firm's marginal cost is sufficiently low compared to the other firm's marginal cost, i.e., when the two firms are 'sufficiently asymmetric', joint firm profits are higher in the regulated environment compared to the unregulated environment, when $p^0 = p^*$. By continuity, the same conclusion holds when $p^0 = p^* - \delta$, for δ sufficiently small. This completes the proof of the first part of the proposition.

For the second part, clearly, if $p^0 \in (c_2,\, p^*)$, consumers' surplus is higher given an intervention. Also, we see from Propositions 1 and 2 that

$$\lim_{\rho \to 0} \underline{\pi}(\widetilde{p}) = \frac{c(a-c)}{b}; \lim_{\rho \to 0} \pi^* = \frac{2a^2 - 2ac + 5c^2}{9b}$$

And so,

$$\lim_{\rho\to 0}\underline{\pi}(\widetilde{p}) > \lim_{\rho\to 0}\pi^* \Leftrightarrow (2a-7c)(a-2c)<0$$

Thus, if $a < \frac{7}{2}c_2$, given Assumptions 1 through 4, then as ρ approaches 0, any fixed price $p^0 \in (c_2,\, p^*)$ induces strict efficiency enhancement compared to the unconstrained outcome. ■

8 References

1. Bergstrom, T. and H. Varian (1985a): "Two Remarks on Cournot Equilibria", *Economics Letters* v. 19, p. 5-8.

2. Bergstrom, T. and H. Varian (1985b): "When are Nash Equilibria Independent of the Distribution of Agents' Characteristics?", *Review of Economic Studies* v. 52, p. 715-18.

3. Bulow, J., J. Geanakoplos and P. Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements", *Journal of Political Economy* v. 93, p. 488-511.

4. Farrell, J. and C. Shapiro (1990): "Horizontal Mergers: An Equilibrium Analysis", *American Economic Review* v. 80, p. 107-26.

5. Février, P. and L. Linnemer (2004): "Idiosyncratic Shocks in an Asymmetric Cournot Oligopoly", *International Journal of Industrial Organization*, v. 22, p. 835-48.

6. Frisell, L. and J. Lagerlöf (2005): "Eliciting Demand Information through Cheap Talk: An Argument in Favour of Price Regulations", *mimeo.*

7. Gaudet, G. and S. Salant (1991): "Uniqueness of Cournot Equilibrium: New Results from Old Methods", *Review of Economic Studies* v. 58, p. 399-404.

8. George, K. and A. Jacquemin (1992): "Dominant Firms and Mergers", *Economic Journal* v. 102, p. 148-57.

9. Hellmann, T., K. Murdock and J. Stiglitz (2000): "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?", *American Economic Review* v. 90, p. 147-65.

10. Lahiri, S. and Y. Ono (1988): "Helping Minor Firms Reduces Welfare", *Economic Journal* v. 98, p. 1199-1202.

11. Long, N. and A. Soubeyran (2001): "Cost Manipulation Games in Oligopoly, with Costs of Manipulating", *International Economic Review* v. 42, p. 505-33.

12. Novshek, W. (1985): "On the Existence of Cournot Equilibrium", *Review of Economic Studies* v. 52, p. 85-98.

13. Pesendorfer, W. and A. Wolinsky (2003): "Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice", *Review of Economic Studies* v. 70, p. 417-37.

14. Salant, S. and G. Shaffer (1999): "Unequal Treatment of Identical Agents in Cournot Equilibrium", *American Economic Review* v. 89, p. 585-604.

15. Schmalensee, R. (1987): "Standards for Dominant Firm Conduct", in D. Hay and J. Vickers eds., *The Economics of Market Dominance*. Oxford: Basil Blackwell.

16. Viscusi, W., J. Vernon and J. Harrington (2000): *Economics of Regulation and Antitrust*. Cambridge, MA: MIT Press.