Production, Manufacturing and Logistics

Vickrey–Dutch procurement auction for multiple items

Debasis Mishra [a], Dharmaraj Veeramani [b,*]

[a] Department of Computer Science and Automation, Indian Institute of Science, Bangalore 560012, India
[b] Department of Industrial and Systems Engineering, University of Wisconsin-Madison, 266 B, Mechanical Engineering Building, 1513 University Avenue, Madison, WI 53706, USA

Abstract

We consider a setting where there is a manufacturer who wants to procure multiple items from a set of suppliers each of whom can supply one or more of these items (bundles). We design an ascending price auction for such a setting which implements the Vickrey–Clarke–Groves outcome and truthful bidding is an ex post Nash equilibrium. Our auction maintains non-linear and non-anonymous prices throughout the auction. This auction has a simple price adjustment step and is easy to implement in practice. As offshoots of this auction, we also suggest other simple auctions (in which truthful bidding is not an equilibrium by suppliers) which may be suitable where incentives to suppliers are not a big concern. Computer simulations of our auction show that it is scalable for the multi-unit case, and has better information revelation properties than its descending auction counterpart.

Keywords: Iterative auctions; Procurement auctions; Vickrey–Dutch auctions; Winner determination problem

1. Introduction

Procurement is an integral part of supply chain operation for many companies. The traditional procurement process involves sending *request for quotation* (RFQ) to the suppliers and receiving *quotes* for the RFQ from the suppliers. This is also termed as *sealed-bid* auction procedure in the auction literature. The other method, popularized by the Internet, is the *reverse* auction. In a reverse auction, suppliers iteratively lower the price on an item till the price reaches a level where there is only one supplier interested in supplying the item at that price. The popularity of reverse auctions is substantiated by research that shows that iterative auctions can be preferred over sealed-bid auctions due to better transparency, preference elicitation etc. (Cramton, 1998; Parkes, 2005).

The multi-item setting is very different from a single item setting because items can be complements or substitutes to the suppliers. So, the auction design for procuring multiple items is no more a trivial generalization of the single item reverse auction. The descending price reverse auctions for procuring multiple items are already known (Demange et al., 1986; Ausubel, 2004; Ausubel and Milgrom, 2002; de Vries et al., forthcoming; Mishra and

* Corresponding author. Tel.: +1 608 262 0861; fax: +1 608 262 8454.

E-mail addresses: mishra@csa.iisc.ernet.in (D. Mishra), raj@cae.wisc.edu (D. Veeramani).

Parkes, forthcoming).[1] In this work, we propose an ascending price auction for procurement that implements the Vickrey–Clarke–Groves (VCG) outcome. We call our auction the Vickrey–Dutch auction (VDA) for procurement. This generalizes our prior work on Vickrey–Dutch auctions for special settings (Mishra and Veeramani, 2006; Mishra and Parkes, 2004a,b).[2]

Our Vickrey–Dutch auction is designed on the foundations of *universal competitive equilibrium* (UCE) prices introduced in Mishra and Parkes (forthcoming). By giving *bonus* to every supplier from the final price of the auction, we implement the VCG outcome. These bonuses, which are essentially marginal contributions of the suppliers to the price of procurement, give suppliers incentive to participate truthfully in the auction. In particular, bidding truthfully in an ex post Nash equilibrium for suppliers in our auction. As offshoots of this auction, we also suggest other simple auctions (in which truthful bidding is not an equilibrium by suppliers), which may be suitable where incentives to suppliers are not a big concern.

The Vickrey–Dutch auction design is different from the descending reverse auction design. This is because the prices in the descending reverse auction starts at a high price where many suppliers are willing to supply the items, i.e., supply is more than demand. The price dynamics are designed to decrease supply to balance the supply and demand. Contrast this with the Vickrey–Dutch auction where prices start low with suppliers not willing to supply items (i.e., supply is less than demand). Thus, the price dynamics should be designed to increase supply to balance supply and demand. Thus, the design of a Vickrey–Dutch auction is not a trivial generalization of the design of a descending reverse auction. In fact, a careful look at our auction and some of the standard descending reverse auctions (Mishra and Parkes, forthcoming) reveal that their underlying price adjustment rules are very different.

The single-item Vickrey–Dutch auction appeared in Vickrey (1961) in the single-seller setting, where a seller is selling an item to multiple buyers. The appropriate method to generalize Vickrey's idea of Vickrey–Dutch auction to multiple items case remains a puzzle in the literature. For instance, in their work on the design of an ascending price auction for the homogeneous items case in single-seller setting, Bikhchandani and Ostroy (2006) observe the following while interpreting their auction as a primal–dual algorithm: "*The primal–dual algorithm we describe starts at a low price where there is excess demand. One could start the primal–dual algorithm at a high price at which there would be excess supply, but it is unlikely that this would converge to a marginal pricing equilibrium*''.[3] In that sense, this work, along with Mishra and Veeramani (2006), Mishra and Parkes (2004b,a), fill a void in the literature of iterative auctions.

We highlight several benefits of a (reverse) Vickrey–Dutch auction, e.g., speed, better preference elicitation, privacy etc. Elmaghraby (2004) discuss how such issues are even more important in procurement setting. In particular, we discuss the information revelation properties of our Vickrey–Dutch auction and compare it with a (descending price) reverse auction. Using simulation, we show that our Vickrey–Dutch auction has better preference elicitation properties than its descending price auction counterpart. An inherent drawback of iterative auctions for multiple items is that they need to maintain an exponential-sized price vector in every iteration to implement the VCG outcome (this is necessary in general—Mishra, 2004). But, for a special case, when all the items are of the same type, the size of the price vector in these auctions are manageable. For this special case, we show, using simulation, that our Vickrey–Dutch auction is computationally scalable—our auction has an average running time of under 2 minutes for 30 suppliers and 150 units.

The rest of the paper is organized as follows. Section 2 defines our model and introduces some preliminaries. In Section 3, we introduce our Vickrey–Dutch auction and prove its theoretical properties. We discuss some practical issues of our auction in Section 4 and its information revelation properties in Section 5. We conclude with a summary and discussion in Section 6.

[1] This literature is for the single-seller model where a single seller is selling multiple items to buyers. But this can be easily adapted to our procurement setting, a single-buyer model.

[2] Research literature in iterative auction dealing exclusively in procurement setting is scarce. An exception is Parkes and Kalagnanam (2005) who design a descending price reverse auction that allows bidding for multiple attributes.

[3] A marginal pricing equilibrium is a price where all bidders get their respective marginal products as payoffs.

2. The model and preliminaries

2.1. The model

Let $A = \{1, \ldots, n\}$ be the set of items that the manufacturer needs to procure. Let $\Omega = \{S : S \subseteq A\}$ be the set of bundles of items. There are m suppliers who can supply the items to the manufacturer. Let $B = \{0, 1, \ldots, m\}$ be the set of suppliers. Supplier 0 is a dummy supplier who supplies the items that cannot be supplied by other suppliers. In a sense, the dummy supplier represents the manufacturer who can be thought to have "in-house" manufacturing capability.

The cost of supplying a set of item $S \in \Omega$ by a supplier $i \in B$ is denoted by $c_i(S)$. So, the cost of in-house production of a bundle of items S is $c_0(S)$. We assume all costs to be non-negative integers. The payoff of a supplier $i \in B_{-0}$ from supplying a bundle S at price p is $p - c_i(S)$. If the manufacturer procures items in S at price p from suppliers in B_{-0} and manufacturers the remaining items in-house, then his payoff is $-p - c_0(A \setminus S)$. An allocation $X = (X_0, X_1, \ldots, X_m)$ is a vector on suppliers with X_i denoting the bundle to be supplied by supplier i and $\cup_{i \in B} X_i = A$. The total cost of procurement from an allocation X is given by $\mathbb{C}(X) = \sum_{i \in B} c_i(X_i)$. Observe that the total cost of procurement equals the total payoff of suppliers in B. An *efficient allocation* is an allocation X such that $\mathbb{C}(X) = \min_Y \mathbb{C}(Y)$, i.e., an allocation that minimizes the total cost of procurement.

We will denote $B \setminus \{i\}$ as B_{-i}. Let $\mathbb{B} = \{B, B_{-1}, \ldots, B_{-m}\}$. We will often be interested in "marginal economies" where a single supplier is absent from B_{-0}. We will denote an economy with suppliers from $M \in \mathbb{B}$ as $E(M)$. Notice that every economy $E(M)$ contains the dummy supplier. We will denote the total cost of procurement in an efficient allocation in economy $E(M)$ as $\mathbb{C}(M)$. We will call economy $E(B)$ the main economy. We will call economy $E(M)$ for $M \in (\mathbb{B} \setminus \{B\})$ a marginal economy.

2.2. The VCG mechanism

In this research, we are interested in implementing an efficient allocation. Mechanism design literature points to the VCG mechanism for implementing an efficient allocation. In its natural form, the VCG mechanism is a sealed-bid auction in which bidders (suppliers in this case) are asked to submit their entire cost function. Based on this, the efficient allocation is determined, and payments are given to suppliers such that their payoff equals their respective "marginal product". Specifically, the payoff of supplier $i \in B$ in the VCG mechanism is $\pi_i^{vcg} = \mathbb{C}(B) - \mathbb{C}(B_{-i})$. This makes the VCG mechanism *strategy-proof*.[4] The payment of a supplier associated with the VCG mechanism will be referred to as the Vickrey payment of that supplier.

The VCG mechanism is not easy to implement in practice. Its computational and preference elicitation problems has motivated researchers to design iterative auctions that overcome these problems (Parkes, 2005; Cramton, 1998). The fundamental idea behind iterative auctions is that of prices. In every iteration, a price vector is announced and bidders are asked to report their "bids" against this price. The prices are adjusted given the current bids. The auction ends when there is no need to adjust prices. The natural question is how to find a price vector that gives enough information to implement the VCG mechanism. Using a modified notion of "competitive equilibrium", Mishra and Parkes (forthcoming) characterized such prices.

A price vector p belongs to $\mathbb{R}_+^{|\Omega| \times (m+1)}$, i.e., every supplier sees a personalized price on every bundle.[5] Given a price vector p and an economy $E(M)$ for $M \in \mathbb{B}$, components of p corresponding to suppliers in M only are considered. We will always assume that $p_0(S) = c_0(S)$ for all $S \in \Omega$ and for all p. Also, $p_i(\emptyset) = 0$ for all $i \in B$ and for all p. Given a price vector p, define the payoff of supplier $i \in B$ as $\pi_i(p) := \max_{S \in \Omega}[p_i\ (S) - v_i(S)]$, and his *supply set* as $L_i(p) := \{S \in \Omega : p_i(S) - c_i(S)\}$. Notice that $\pi_0(p) = 0$ and $L_0(p) = \Omega$. Let $\mathbb{X}(M)$ denote all the feasible allocations of economy $E(M)$. Given a price vector p, for every economy $E(M) (M \in \mathbb{B})$ define the *price of procurement* of the manufacturer as $\pi^m(M, p) := \min_{X \in \mathbb{X}(M)} \sum_{i \in M} p_i(X_i)$, and the *demand set* as $D(M, p) := \{X \in \mathbb{X}(M) : \sum_{i \in M} p_i(X_i) = \pi^m(M, p)\}$. Using these notions, we define a competitive equilibrium.

[4] We note that we do not consider the dummy supplier as a strategic agent. The dummy supplier (the manufacturer) acts as a "social planner" whose sole objective is to implement an efficient allocation. There are inherent difficulties, in terms of impossibility results, in considering the manufacturer (along with all the suppliers) as a strategic agent (see any standard textbook in microeconomics, e.g., Mas-Colell et al., 1995).

[5] Such complex prices are necessary to implement the VCG mechanism using iterative auctions (Mishra, 2004).

Definition 1. A price vector $p \in \mathbb{R}_+^{|\Omega|\times(m+1)}$ and an allocation $X \in \mathbb{X}(M)$ are a *competitive equilibrium* (CE) of economy $E(M)$ if $X \in D(M,p)$ and $X_i \in L_i(p)$ for every $i \in M$. If (p,X) is a CE of economy $E(M)$, then p is called a CE price vector of economy $E(M)$. p is a *universal CE* (UCE) price vector if it is a CE price vector of economy $E(M)$ for every $M \in \mathbb{B}$.

The UCE price concept is central to the design of iterative auctions. This was shown in Mishra and Parkes (forthcoming) who proved the following result.[6]

Theorem 1 Mishra and Parkes, forthcoming. *Let (p,X) be a CE of main economy. The Vickrey payments of every supplier in B_{-0} can be calculated from (p,X) if and only if p is a UCE price vector. Moreover, if p is a UCE price vector then for every supplier $i \in B_{-0}$, the Vickrey payment of i is $p_i^{vcg} = p_i(X_i) + [\pi^m(B,p) - \pi^m(B_{-i},p)]$.*

We will refer to the term $[\pi^m(B,p) - \pi^m(B_{-i},p)]$ as the *bonus* of supplier i at the UCE price vector p.

3. Vickrey–Dutch procurement auction

In this section, we describe our Vickrey–Dutch procurement auction. To do so, we define some concepts first. Define the *compatible demand set* of the manufacturer in economy $E(M) (M \in \mathbb{B})$ at a price vector p as $D^*(M,p) := \{X \in D(M,p) : X_i \in L_i \cup A\}$. Every allocation in the compatible demand set belongs to the demand set, and thus minimizes the price of procurement. Also, every allocation in the compatible demand set is such that every supplier is either allocated a bundle from his supply set or the entire set of items. The motive behind defining such a notion will become clear as we analyze the properties of the auction.

Definition 2. A price vector p is a *restricted CE price vector* of economy $E(M)$ $(M \in \mathbb{B})$ if the compatible demand set $D^*(M,p)$ is not empty.

Observe that a CE price vector is also a restricted CE price vector.

Definition 3. *Undersupply* holds in economy $E(M)$ $(M \in \mathbb{B})$ at a price vector p if p is a restricted CE price vector but not a CE price vector of economy $E(M)$.

Using the notions of restricted CE price vector and undersupply, we define our Vickrey–Dutch auction for procurement.

Definition 4. The *Vickrey–Dutch auction* (*VDA*) *for procurement* is an iterative auction with the following steps:

- S0 Initialize the price vector to the zero price vector.[7]
- S1 Collect supply sets of suppliers at the current price vector p.
- S2 If undersupply does not hold in economy $E(M)$ for every $M \in \mathbb{B}$ at the current price vector p, go to Step (S3). Else, select an economy $E(M)$ in which undersupply holds at the current price vector p and do the following price adjustment.
  - S21 For every $i \in B_{-0}$ and $S \in \Omega$, if $S \notin L_i(p)$ then increase price $p_i(S)$ by 1,[8] else do not change the price $p_i(S)$. Go to Step (S1).
- S3 The auction ends. If p^* is the final price vector in the auction, then a final allocation $X \in D(B,p^*)$ is chosen such that the number of suppliers who get a bundle from their supply set is maximized. The final payment of supplier i is $p_i^*(X_i) + [\pi^m(B_{-i},p^*) - \pi^m(B,p^*)]$.

Since prices of bundles in the supply set of a supplier is never increased and prices of bundles not in the supply set are increased by 1, the payoff of a supplier is unchanged by price adjustment. Since initial payoff is zero, it remains zero throughout the auction. Also, since the price of bundles in the supply sets never increase, once a bundle enters the supply set it never leaves. This gives us the following lemma directly.

Lemma 1. *The payoff of a supplier remains zero throughout the auction and the supply sets of the supplier (weakly) grows from iteration to iteration.*

[6] The idea of giving bonuses to bidders can also be found in Parkes and Kalagnanam (2005).

[7] Technically, we require that the price vector be initialized to a low price vector where every supplier has zero payoff and it is a restricted CE price vector of economy $E(M)$ for every $M \in \mathbb{B}$.

[8] We can choose any arbitrary $\epsilon > 0$ as price increment and all our results will hold with some error terms, which will depend on ϵ. The choice of 1 as ϵ makes the analysis less cumbersome.

An analogous lemma corresponding to the manufacturer is the following.

Lemma 2. *Suppose undersupply holds in economy $E(M)$. Then, after a price adjustment the price of procurement increases by 1 in economy $E(M)$.*

Proof. Since undersupply holds in economy $E(M)$, in every allocation $X \in D(M,p)$, where p is the price vector before price adjustment, there is a supplier $i \in M$ such that $X_i \notin L_i(p)$. By the price adjustment rule, price of $X_i \notin L_i(p)$ will increase by 1. This implies that the price of procurement is increased by at least 1 on every $X \in D\ (M,p)$. Consider $X \in D^*(M,p)$ ($D^*(M,p)$ is non-empty since p is a restricted CE price vector). Since undersupply holds, there is only one supplier $k \in M$ such that $X_k = A \notin L_i(p)$ and for every other supplier $i \neq k$, $X_i = \emptyset \in L_i(p)$ (this follows from Lemma 1 and the fact that $\emptyset$ is in the supply set of every supplier initially). Again, by the price adjustment rule, the price of procurement from allocation X is increased by exactly 1. The payoff from any allocation $X \notin D(M,p)$ is at least 1 above the payoff from any allocation $X \in D(M,p)$. This implies that the price of procurement increases by 1 in economy $E(M)$ by price adjustment if undersupply holds in economy $E(M)$. □

The following theorem says that we maintain a restricted CE price vector in every iteration of our auction.

Theorem 2. *Consider any iteration t in the auction where undersupply holds in some economy. Let the price in iteration t be p^t in the auction. If p^t is a restricted CE price vector in economy $E(M)$ for any $M \in \mathbb{B}$, then p^{t+1} is a restricted CE price vector of economy $E(M)$.*

Proof. Since p^t is a restricted CE price vector of economy $E(M)$, there are two cases:

Case 1: Undersupply holds in economy $E(M)$. Consider the allocation $X \in D^*(M,p^t)$. From Lemma 2, the price of procurement is increased by 1 in economy $E(M)$. By price adjustment, price of procurement from X is increased by 1. This implies that $X \in D(M,p^{t+1})$ after the price adjustment.

From Lemma 1, the payoff of all suppliers remain zero and the supply sets weakly increase in each iteration. This implies that if $X_i \in L_i(p^t)$ then $X_i \in L_i(p^{t+1})$ for every $i \in M$. So, $X \in D^*(M,p^{t+1})$ after the price adjustment. So, p^{t+1} is a restricted CE price vector of economy $E(M)$.

Case 2: Undersupply does not hold in economy $E(M)$. This implies that p^t is a CE price of economy $E(M)$. Let $X \in D^*(M,p^t)$ be an allocation such that (p^t, X) is a CE of economy $E(M)$. For every $i \in M$, $X_i \in L_i(p^t)$. So, prices of bundles in X do not increase, and X remains in $D^*(M,p^{t+1})$. From Lemma 1, a supplier i will continue to have X_i in his supply set throughout the auction. This implies that (p^{t+1}, X) is a CE of economy $E(M)$ in iteration $t+1$, and p^{t+1} is a CE price vector of economy $E(M)$. □

Theorem 2 shows how we maintain a restricted CE price vector for economy $E(M)$ for every $M \in \mathbb{B}$ in every iteration of our auction. The following theorem shows that the auction achieves the VCG outcome.

Theorem 3. *The Vickrey–Dutch auction for procurement achieves the VCG outcome.*

Proof. Starting price is a restricted CE price vector of economy $E(M)$ for every $M \in \mathbb{B}$. From Theorem 2, every iteration in the auction is a restricted CE price vector of economy $E(M)$ for every $M \in \mathbb{B}$. By Lemma 1, the payoff of every supplier is zero throughout the auction. This implies that once the price of a bundle reaches its cost, it does not increase anymore. This implies that the price cannot increase forever and the auction will terminate. The terminating condition ensures that the final restricted CE price vector is a CE price vector of economy $E(M)$ for every $M \in \mathbb{B}$. Thus, the final price vector in the auction is a UCE price vector, and the final allocation is an efficient allocation (because every allocation associated with a CE allocation of the main economy is an efficient allocation). From Theorem 1, the payment of each supplier is his Vickrey payment. Thus, the Vickrey–Dutch auction for procurement achieves the VCG outcome. □

3.1. Incentives

The sealed-bid format of the VCG mechanism is strategy-proof. But in an iterative implementation (extensive form game), the VCG outcome may not support truthful bidding in a dominant strategy equilibrium (Ausubel and Milgrom, 2002; Bikhchandani and Ostroy, 2006; de Vries et al., forthcoming; Parkes and Ungar, 2000). We can still show that truthful bidding is an ex post Nash equilibrium. For ex post Nash equilibrium to exist, we require some simple bidding rules in the auction. For example, initially all suppliers should be willing to supply the $\emptyset$ bundle, the supply sets of the suppliers should weakly grow etc. These are ''consistency'' requirements in bidding and are easy to implement. For example, Parkes and Ungar (2000) propose the notion of ''proxy agents'' to implement such consistency requirements. With such consistency requirements, one can map the bidding strategy of a supplier to some cost function (may be non-truthful). Given these consistency requirements are met, the outcome of the auction will be the VCG outcome with respect to some profile of cost functions. Since the VCG mechanism is strategy-proof, truthful bidding is the best strategy for suppliers in our Vickrey–Dutch auction for procurement under consistency requirements. Truthful bidding is not a dominant strategy for suppliers because every supplier has to condition his strategy against the fact that every other supplier is playing a strategy that is consistent with some cost function. This gives us the following theorem immediately.

Theorem 4. *Truthful bidding is an ex post Nash equilibrium in the Vickrey–Dutch auction for procurement if consistency requirements are met.*

Table 1
An example

	$\emptyset$	{1}	{2}	{1, 2}
0	0	∞	∞	∞
1	0	3	3	3
2	0	2	3	6
3	0	2	4	4

3.2. An example

Consider an example with three (non-dummy) suppliers. The manufacturer needs to procure two items. The ''in-house'' costs for the manufacturer are represented by the dummy supplier. For simplicity, we have put ∞ costs for in-house production.

The progress of Vickrey–Dutch auction for procurement, for the example in Table 1, is provided in Table 2. The columns corresponding to suppliers show the personalized prices on bundles. The bundles which have prices in $(\cdot)$ are in the supply set of the suppliers. The manufacturer's price of procurement in economy $E(M)$ for every $M \in \mathbb{B}$ is shown in every iteration in the last column. The auction terminates at a UCE price vector in iteration 5. Note that bonuses are given to the winning suppliers according to the payment rule of the auction.

3.3. Alternate auctions

In this subsection, we will sidestep incentive issues (assume a CE to be a "fair'' scheme for suppliers to bid truthfully) and explore other possible auctions. These auctions can be derived from slight modification of the Vickrey–Dutch auction for procurement.

- **A1—Vickrey–Dutch auction for procurement**: This is the exact auction in Definition 4 and achieves the VCG outcome.
- **A2—Dutch auction for procurement with bonus**: In this auction, the auction in Definition 4 is stopped as soon as a CE of economy $E(B)$ is reached, but the final price vector in the auction need not be a UCE price vector. The final allocation and payment schemes remain the same as

Table 2
Progress of auction in Definition 4 for the Example in Table 1

#	Supplier 1			Supplier 2			Supplier 3			Price of procurement
	{1}	{2}	{1,2}	{1}	{2}	{1,2}	{1}	{2}	{1, 2}	$\pi^m(\cdot)$
1	0	0	0	0	0	0	0	0	0	0, 0, 0, 0
2	1	1	1	1	1	1	1	1	1	1, 1, 1, 1
3	2	2	2	(2)	2	2	(2)	2	2	2, 2, 2, 2
4	(3)	(3)	(3)	(2)	(3)	3	(2)	3	3	3, 3, 3, 3
	CE of economy $E(B)$, $E(B_{-2})$, $E(B_{-3})$ is achieved									
5	(3)	(3)	(3)	(2)	(3)	4	(2)	(4)	(4)	3, 4, 3, 3
	CE of economy $E(B_{-1})$ is achieved. A UCE price is found									
	Final allocation: Supplier 1 supplies both items									
	Final payment: Supplier 1 is paid $3 + (4 - 3) = 4$. Other suppliers are paid zero									

Step (S3) in Definition 4. So, suppliers may get bonus at the end of the auction.

- **A3—Dutch auction for procurement without bonus**: In this auction, like the auction in **A2**, the auction in Definition 4 is stopped as soon as a CE of economy $E(B)$ is reached. The final allocation remains the same as in Definition 4, but the final payment is the final price in the auction.

These auctions can be used in different scenarios. If incentive is a big concern, only auction **A1** has nice incentive properties (Theorem 4). But if incentive is not a concern and simplicity and speed is a concern, auctions **A2** and **A3** should be considered. Auction **A2** provides bonus to suppliers and thus can be viewed as more fair to the suppliers than auction **A3**.

4. Issues in practical implementation

A practical limitation of iterative auctions is the computational complexity of the *winner determination problem* (WDP) (Rothkopf et al., 1998). The WDP is not the same for every iterative auction. To define the WDP in our case, we state some corollaries of our previous results.

We state some corollaries to Lemma 2 and Theorem 2 that will help us simplify the WDP for our case. The first corollary is due to Theorem 2. We assume that the first iteration of our auction is iteration 1.

Corollary 1. *If undersupply holds in economy $E(M)$ in iteration $t > 1$ of the Vickrey–Dutch auction for procurement, then it holds in every iteration $t' < t$.*

Theorem 2 says more than Corollary 1.

Corollary 2. *If the price vector in iteration t of the Vickrey–Dutch auction is a CE price vector of economy $E(M)$, then so is the price vector in iteration $t + 1$ (if iteration $t + 1$ exists).*

As an immediate corollary to Lemma 2 using Corollaries 1 and 2 is the following result.

Corollary 3. *Starting from iteration 1, consider an iteration $t \geqslant 1$ of the Vickrey–Dutch procurement auction. If undersupply holds in economy $E(M)$ in iteration t, then the price of procurement in iteration $t + 1$ in economy $E(M)$ is t. Further, if t^* is the iteration in the auction where undersupply holds for the last time in economy $E(M)$, then the price of procurement in any iteration $t > t^*$ in economy $E(M)$ is t^*.*

In any WDP of an iterative auction that implements the VCG outcome, computation is done in every iteration to check two things:

W1 The demand and supply needs to be balanced in the main economy and in every marginal economy (Mishra and Parkes, forthcoming). When bidders have special class of cost functions, balance of demand and supply need to be checked only in the main economy (de Vries et al., forthcoming; Ausubel and Milgrom, 2002). But this is not true when suppliers have general cost functions.

W2 Once an imbalance in demand and supply is identified in an economy, a group of bidders are selected whose prices need to be changed. In the auction in de Vries et al. (forthcoming), these are a minimal set of "undersupplied" bidders. In the auction in Ausubel and Milgrom (2002), these are a minimal set of "losing" bidders.

In some auctions, e.g., in one of the auctions in Demange et al. (1986) and the auction in Ausubel and Milgrom (2002), the computation of (**W1**) also gives us price adjustment direction in (**W2**). This is not true for the auction in de Vries et al. (forthcoming) and in an auction in Demange et al. (1986), where explicit calculations are needed for (**W1**) and (**W2**).

We note that our Vickrey–Dutch procurement auction has a very simple price adjustment direction (See Step S21 in Definition 4) and involves no computation in (**W2**). We will now discuss how to do the computation in (**W1**) for a special case.

4.1. The multi-unit case

In this section we consider the special case where all the items are of the same type (homogeneous units). We call this the *multi-unit* case. The multi-unit case is very typical in procurement setting where multiple units of the same item is often procured simultaneously. This case is also tractable from a computation perspective because the number of bundles to consider is simply the number of units. Contrast this with the general heterogeneous items case, where the number of bundles to consider is exponential in the number of items. We will consider the winner determination problem in our auction for the multi-unit case.

For this case, we will give an integer programming formulation to check if undersupply holds in an economy. Let $A = \{1, \ldots, n\}$ be the set of units that need to be procured. Let $\Omega = \{0, 1, \ldots, n\}$ be the possible bundles of units. At a price vector p, the supply set of a supplier $i \in B$, denoted by $L_i(p)$, will include all the bundle of units that maximize his payoff at price vector p. We note that $L_i(p)$ can include 0 when the maximum payoff of supplier i is zero over all the bundles in Ω at price vector p. Let $x_i(j) \in \{0,1\}$ be a binary variable that is set to 1 if supplier i is assigned to supply $j\,(j \in L_i(p) \cup \{n\})$ units at price vector p and set to zero otherwise. We remind that $\pi'^m(M,p)$ denotes the price of procurement of economy $E(M)$ at price vector p. Now, consider the following formulation for economy $E(M)$ where $M \in \mathbb{B}$ in iteration t of our auction with price vector p^t.

$$\alpha(M,p^t) = \min \sum_{i \in M: n \notin L_i(p^t)} x_i(n) \tag{WD}$$

$$\text{s.t.} \sum_{j \in L_i(p^t) \cup \{n\}} x_i(j) = 1 \quad \forall i \in M, \tag{1}$$

$$\sum_{i \in M} \sum_{j \in L_i(p^t) \cup \{n\}} j x_i(j) = n, \tag{2}$$

$$\sum_{i \in M} \sum_{j \in L_i(p^t) \cup \{n\}} p_i^t(j) x_i(j) = \pi^m(M,p^t), \tag{3}$$

$$x_i(j) \in \{0,1\} \quad \forall i \in M, j \in L_i(p^t) \cup \{n\}. \tag{4}$$

Call a supplier unsatisfied if he is allocated a bundle of units that is not in his supply set. Formulation (WD) minimizes the number of unsatisfied suppliers in M. The first set of constraints (1) ensure that every supplier is either allocated a bundle from his supply set or all the n units. The second set of constraints (2) ensure that the total number of units allocated is exactly n. The third set of constraints (3) ensure that the price of procurement from the allocation equals the actual price of procurement. We note that the price of procurement in iteration t in economy $E(M)$, $\pi'^m(M,p^t)$, can be tracked by the auctioneer throughout our auction using Corollary 3. We will elaborate more on this after proving the following Theorem.

Theorem 5. *Undersupply holds in economy $E(M)(M \in \mathbb{B})$ in iteration t of the VDA for procurement if and only if $\alpha(M,p^t) = 1$.*

Proof. If undersupply holds in economy $E(M)$ in iteration t then by definition, p^t is a restricted CE price vector but not a CE price vector of economy $E(M)$. So, by definition $D^*(M,p^t)$ is not empty. So, there exists an allocation that gives a price of procurement $\pi^m(M,p^t)$ and allocates every supplier i a bundle of units from $L_i(p^t) \cup \{n\}$. The $x(\cdot)$ variables corresponding to this allocation is a feasible solution to (WD). Since this is not a CE price vector of economy $E(M)$, there is some supplier i that is assigned all units $n \notin L_i(p^t)$ in this allocation. By constraint (2), suppliers other than i are assigned zero unit in this allocation, which is in their respective supply sets (Lemma 1). Thus, the value of the objective function corresponding to this feasible solution is 1. Clearly, this is the minimum value of the objective function when undersupply holds, implying $\alpha(M,p^t) = 1$.

Now, consider the case when $\alpha(M,p^t) = 1$. From Theorem 2, p^t is a restricted CE price vector of economy $E(M)$. If p^t is a CE price vector then we can find an allocation in $D^*(M,p^t)$ such that every supplier is allocated bundles of units from his supply set, which implies $\alpha(M,p^t) = 0$. So, p^t is not a CE price vector, which further implies that undersupply holds in economy $E(M)$ in iteration t. □

To use Theorem 5 effectively, we need some modifications in the VDA for procurement. We provide these modifications below:

- In the VDA for procurement, we can consider economy $E(M)$ for every $M \in \mathbb{B}$ using some pre-determined order (without loss of generality, that order can be $B_{-1}, \ldots, B_{-m}, B$). Once we start considering economy $E(M)$, we continue to update prices using Step S21 in Definition 4 as long as undersupply holds in economy $E(M)$. Once undersupply does not hold in economy $E(M)$, we do not need to consider that economy again (Theorem 2).
- Using Corollary 3, we can only track the revenue of an economy whose undersupply held in the previous iteration. But, when an economy $E(M)$ is first considered for price update in an iteration t, it is not possible to say whether undersupply held in the previous iteration or not. For this iteration, i.e., the iteration in which an economy is first considered for price update using Step S21 in Definition 4, we use a formulation similar to formulation (WD) to find the price of procurement. The only differences are: (i) constraints

(3) are omitted, (ii) the objective function minimizes the price of procurement—min $\sum_{i\in M}\times\sum_{j\in L_i(p^t)\cup\{n\}} p_i^t(j)x_i(j)$. Using arguments in the proof of Theorem 5, it is easy to see that this modification finds the price of procurement of economy $E(M)$ in iteration t. Once this price of procurement is calculated, we can use formulation (WD) to determine whether undersupply holds or not. From that iteration onwards, the price of procurement of economy $E(M)$ can be tracked using Corollary 3 (or, Lemma 2) as long as undersupply holds, and we need not solve an integer program in every iteration to determine the price of procurement.

Since the maximum number of binary variables and constraints in the integer programming formulation (WD) are $m\times(n+1)$ and $m+2$ respectively, we expect the computational burden to of our auction to be reasonable for the multi-unit case (since a supplier is likely to have only a subset of bundles of units from Ω in his supply set, the actual number of binary variables will be less than this maximum amount).

The descending reverse auctions (for example, the ones derived from Mishra and Parkes (forthcoming)) do not allow for such simpler formulation of winner determination problem in the multi-unit case. The main difficulty is that it is not possible to track the price of procurement in these descending reverse auctions.[9] So, we need an explicit formulation that finds the price of procurement, and this involves more binary variables than formulation (WD).

To check the scalability of the VDA for procurement, we performed simulations where we used formulation (WD) in every iteration to determine price adjustment.

The simulations were set up as follows. The inputs to the simulation were number of suppliers and number of items. Cost values were randomly generated (using a uniform distribution) for each supplier. For every supplier and every unit, a per unit cost is drawn from a uniform distribution with

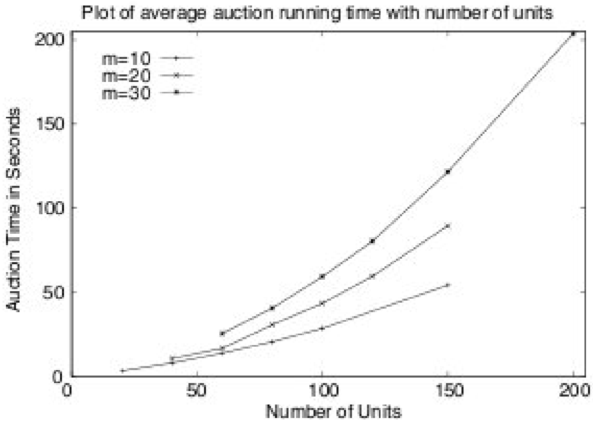

Fig. 1. Average running time with number of units.

range [10, 30]. The cost of k units is k times the per unit cost.

We measured the average run-time of our auction (average taken over 1000 runs of our auction) when the number of suppliers (m) is 10, 20, and 30. We increased the number of units in each of these cases, keeping at least twice the number of suppliers. The average run-times are shown in Fig. 1. Fig. 1 shows that the average run-times are under two minutes when the number of units are less than or equal to 150. In practice, we will expect a firm to select not more than 30 suppliers for bidding phase of procurement. Our VDA for procurement has good average running-time when the number of units are less than or equal to 150.

5. Information revelation

In this section, we will discuss (informally and experimentally) information revelation properties, defined as the cost information (of suppliers) revealed due to bidding in the auction, of the VDA for procurement and its descending reverse auction counterpart. The descending reverse auction for multi-item combinatorial procurement is derived from *i*BEA auction for single-seller setting in Mishra and Parkes (forthcoming). In this section, we will call this auction the *descending reverse auction*. Also, we will assume that the suppliers bid truthfully in both the auctions as bidding truthfully is an ex post Nash equilibrium in both the auctions (under mild consistency requirements).

We emphasize that the information revelation property of an auction is important because companies are always reluctant to reveal their internal

[9] But it is possible to track the payoff of the suppliers in these descending reverse auctions. Thus, it makes the problem of finding the supply sets of suppliers easier. Since the number of bundles involved in this setting is only $n+1$, the problem of finding the supply sets of suppliers is not that difficult even in our auction.

costs to rival suppliers due to strategic reasons. Also, Elmaghraby (2004) discusses how too much information revelation in the descending reverse auctions have discouraged participation of suppliers in E-marketplaces. Besides the strategic reason, Elmaghraby (2004) gives possibility of *collusion* due to information revelation as another reason for non-participation of suppliers in E-marketplaces. Less participation of suppliers lead to higher price of procurement. So, information revelation has effect on price of procurement of a manufacturer.

5.1. Characterizing information revelation

The VDA has the property that no supplier has payoff more than zero during the auction (Lemma 1). So, a supplier reveals his cost information on a bundle as soon as he reports that the bundle is in his supply set. Since the VDA converges to a UCE price vector, a supplier who is allocated $\emptyset$ in the efficient allocation of economy $E(M)$ for every $M \in \mathbb{B}$, will have only $\emptyset$ in his supply set throughout the auction. Call such suppliers *losing suppliers*. So, losing suppliers reveal no cost information in the VDA. Observe that if a supplier is not a losing supplier, he may not reveal his entire cost information. But he may reveal some of his cost information.

Contrast this with the information revealed in the descending reverse auction. In the descending reverse auction, once a bundle is in the supply set of a supplier, it continues to be in his supply set throughout the auction. So, once a supplier has $\emptyset$ in his supply set, he has zero payoff on all the bundles he has ever reported to be in his supply set during the auction. Since the prices of all the bundles are known, the cost information of the bundles in the supply set are revealed. A supplier who wins some bundle at positive payoff in the efficient allocation of the main economy never has $\emptyset$ in his supply set during the descending reverse auction. Call such suppliers *winning suppliers*. All winning suppliers reveal no cost information during the descending reverse auction. Again, observe that if a supplier is not a winning supplier, he may not reveal his entire cost information. But he may reveal some of his cost information.

These discussions tell us that if the number of losing suppliers are more than the number of winning suppliers, there is a good chance that the VDA will perform better in terms of information revelation. This can happen in large "competitive" economies, where the number of suppliers are more compared to the number of items.

In many situations, revealing the costs of the winning suppliers may not be considered a good idea. For example, revealing the cost information of the winning supplier may embarrass a manufacturer if he comes to know that the winning supplier is making a lot of profit and he could have got the items at a lower price. The VDA does a bad job in such situations but the descending reverse auction does not suffer from this problem. But this problem can be fixed in the VDA by imposing restrictions on information revealed during the auction. Most of the manufacturers use third party softwares for auctioning, and such restrictions on information revelation is common in those softwares (Elmaghraby, 2004). We propose some such restrictions:

Auction as a "blackbox": In such a setting, auction can be treated as a blackbox. The only information available to the suppliers and the manufacturer are the prices. Bids from suppliers are known only to the blackbox. At the end of the auction, the final allocation and payments are disclosed. Such a blackbox can be implemented for both the VDA and the descending reverse auction and neither of them reveal any information about costs of the suppliers.

Limited information revelation: The other option is to broadcast only limited information from the auction. For example, in a non-anonymous price setting, suppliers see their own personalized price only, but bid information of every supplier is not made public. This prevents a supplier from knowing the cost information of other suppliers.

We conclude this discussion by observing that there are several (technological) methods to overcome the information revelation disadvantages in the VDA and the descending reverse auction. But if the identity of the supplier (i.e., a winner or loser) is not an issue, the VDA will likely outperform the descending reverse auction in many settings in terms of information revelation. Combine this with faster convergence rate and possibility of minimizing collusion, the VDA can be preferred over the descending reverse auction in many settings.

5.2. Experiment results

To validate our claim about the VDA having better (overall) information revelation properties, we conducted computer based experiments. We simulated the descending reverse auction and the VDA using computer programs and observed the information revelation properties of both the auctions.

The experiments were set up as follows. The inputs to the experiment were number of suppliers, number of items, and *density*. Cost values were randomly generated (using a uniform distribution) for each supplier. Density of the system determines the fraction of the total possible bundles (2^n for a system with n items) on which the suppliers are assigned costs randomly. For every supplier, these bundles are chosen randomly and assigned costs from a uniform distribution, and the rest of the bundles are assigned high costs (indicating that the supplier cannot supply these bundles). To assign cost to a bundle, a random number is drawn from a uniform distribution and multiplied by the number of items in that bundle. We note that although the number of items are less in our simulations, the number of bundles are reasonably high.

We observed the *revelation percentage* in both the auctions. The revelation percentage of an auction is the percentage of (supplier, bundle) tuples whose costs can be exactly known (to anyone who sees the bid information), given that suppliers bid truthfully. Realize that the cost on the ∅ bundle is zero and universally known.

Density: We fixed the number of suppliers (m) at 10 and number of items (n) at 6. So, each supplier can have costs on 64 bundles. A density of 0.4 means, costs for 25 ($=0.4 \times 64$) bundles are drawn randomly from a uniform distribution for each supplier and the rest of the bundles are assigned high costs. We varied the density from 0.2 to 0.7 and plotted revelation percentages (times 0.01) of both the auctions in Fig. 2.

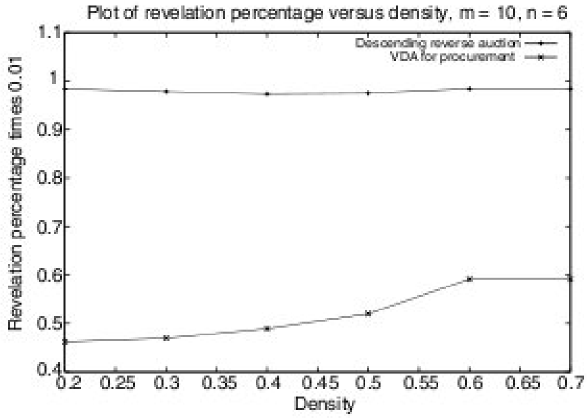

Fig. 2. Revelation percentage with density.

The VDA has lower revelation percentage than the descending reverse auction. Also, revelation percentage in the VDA shows a marginal increase with density whereas it remains almost constant in the descending reverse auction. The intuition behind this is as follows. In the VDA, *winning suppliers* (defined earlier), reveal cost information of bundles in their supply set. As the density increases, the supply sets also increase and hence the revelation percentage increases. For the descending reverse auction, the revelation percentages are already very high (almost one) for lower densities and remain that way.

Number of suppliers: We fixed the number of items at 6 (number of possible bundles 64), and varied the number of suppliers between 4 and 9. The effect of number of suppliers on revelation percentages in both the auctions is shown in Fig. 3 for density 0.5. We observed similar plots for densities 0.3 and 0.7.

The VDA has lower revelation percentage than the descending reverse auction. The revelation percentage of both the auctions increase with the increase in number of suppliers. This can be explained as follows. For the VDA, as the number of suppliers increase, the number of *winning* suppliers increase and hence the revelation percentage increases. But, after the number of suppliers reach a certain number (almost equal to number of items), the number of winning suppliers do not increase that much and hence the revelation percentage remains almost constant after that. For the descending reverse auction, when the number of suppliers are less, the competition in the economy is less and competitive equilibrium in main economy and marginal economies can be achieved without much bidding. This leads to less information revelation

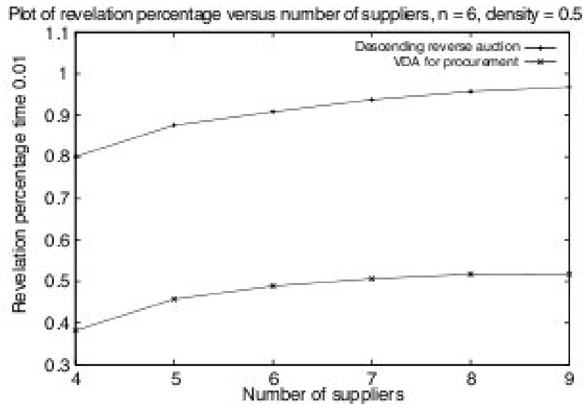

Fig. 3. Revelation percentage with number of suppliers.

when number of suppliers are less and explains the increase in revelation percentage with increase in number of suppliers.

Number of items: We fixed the number of suppliers at 6 and varied the number of items from 2 to 6. The effect of number of items on revelation percentage in both the auctions is shown in Fig. 4 for density 0.5. We observed similar plots for densities 0.3 and 0.7.

Again, the VDA does better than the descending reverse auction in terms of revelation percentage. The graph shows that the revelation percentage in the VDA decreases with the number of items. With the increase in the number of items, the number of winning suppliers increase and hence the revelation percentage should increase. But this effect is dominated by the fact that the number of bundles increase (exponentially) with the increase in number of items. Since winning suppliers do not reveal cost on all bundles, this effect reduces the revelation percentage with increase in number of items. We call this the *exponential bundle* effect. The exponential bundle effect has negative impact on the descending reverse auction as losing suppliers reveal cost information on almost all bundles in this auction. As, we will see next, to achieve a UCE price vector for larger economies, the descending reverse auction reveal cost information on almost all bundles of the losing suppliers. This continues as the number of bundles grow exponentially and hence the revelation percentage increases.

Size of economy: We fixed the ratio of number of suppliers to number of items at $\frac{4}{3}$ and varied the number of suppliers (and the number of items). The effect of size of economy (number of suppliers and items) on revelation percentage in both the auctions is shown in Fig. 5 for density 0.5. We observed similar plots for densities 0.3 and 0.7.

The revelation percentages of the VDA got better than the descending reverse auction as the size of the economy (number of suppliers and items) increased. The decrease in the revelation percentage of the VDA and the increase in the revelation percentage of the descending reverse auction is again explained by the exponential bundle effect. In larger economies, the descending reverse auction requires almost all cost information to achieve a UCE price vector and hence the revelation percentage increases with the increase in the size of economy. On the other hand, the exponential growth in number of bundles reduces the number of bundles whose cost information is revealed by winning suppliers in the VDA.

We summarize our findings from the experiments below:

- In larger economies, the VDA has better information revelation property than the descending reverse auction. In fact, the descending reverse auction requires (almost) complete revelation of cost information to implement the VCG outcome in larger economies.
- Lower density (typical in many practical settings) favors the VDA, as revelation percentage decreases with lower density.
- With higher densities, the revelation percentage of the descending reverse auction improves (specially when the number of items or suppliers are small). But this setting is rare in practice.

These findings validate our claim that the VDA is better than the descending reverse auction in terms

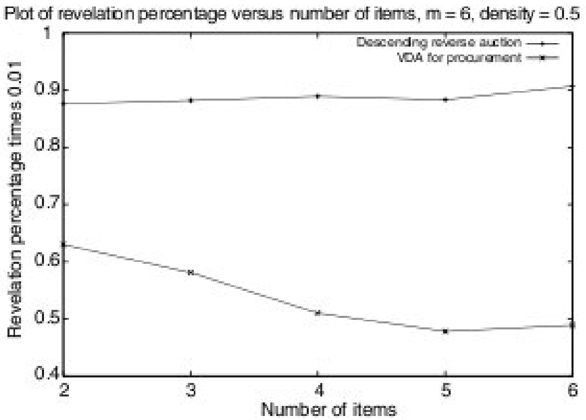

Fig. 4. Revelation percentage with number of items.

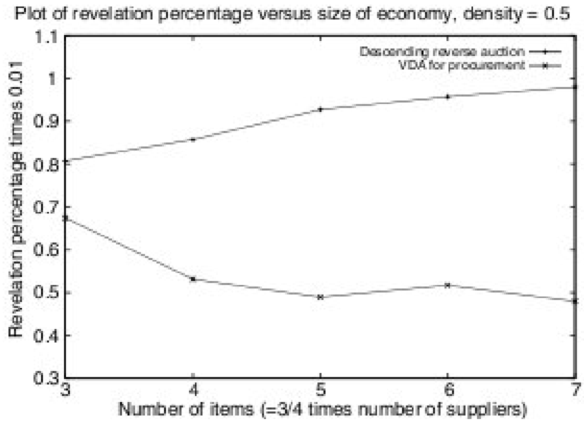

Fig. 5. Revelation percentage with size of economy.

of information revelation, and does really well in this regard in larger economies.

6. Conclusions

We have designed a Vickrey–Dutch auction, which is ascending in nature, for procuring multiple items simultaneously. The auction implements the VCG outcome and truthful bidding is an ex post Nash equilibrium. We showed that our auction is computationally scalable for the multi-unit case and has better information revelation properties than its descending price reverse auction counterpart. This, coupled with the inherent benefit of speed in the Vickrey–Dutch auctions, makes our auction suitable for practical use in procurement settings.

Acknowledgements

We thank two anonymous referees, David C. Parkes, and seminar participants at the University of Wisconsin-Madison for their comments on an earlier version of the paper. This work is based on a Chapter of the doctoral thesis of the first author, submitted at the University of Wisconsin-Madison in 2004.

References

Ausubel, L.M., 2004. An efficient ascending-bid auction for multiple objects. American Economic Review 94 (5), 1452–1475.

Ausubel, L.M., Milgrom, P.R., 2002. Ascending auctions with package bidding. Frontiers of Theoretical Economics 1 (1), 1–42.

Bikhchandani, S., Ostroy, J., 2006. Ascending price Vickrey auctions. Games and Economic Behavior 55 (2), 215–241.

Cramton, P., 1998. Ascending auctions. European Economic Review 42, 745–756.

de Vries, S., Schummer, J., Vohra, R.V., forthcoming. On ascending Vickrey auctions for heterogeneous objects. Journal of Economic Theory. DOI: http://dx.doi.org/10.1016/j.jet.2005.07.010.

Demange, G., Gale, D., Sotomayor, M., 1986. Multi-item auctions. Journal of Political Economy 94 (4), 863–872.

Elmaghraby, W., 2004. Pricing and auctions in E-marketplaces. In: Simchi-Levi, D., Wu, S. David, Max Shen, Z. (Eds.), Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era, International Series in Operations Research and Management Science. Kluwer Academic Publishers, Norwell, MA.

Mas-Colell, A., Whinston, M.D., Green, J.R., 1995. Microeconomic Theory. Oxford University Press, New York, USA.

Mishra, D. 2004. Simple primal–dual auctions are not possible. In: Proceedings of 5th ACM Conference on Electronic Commerce (EC'04), New York City, NY.

Mishra, D., Parkes, D.C., 2004a. Multi-item Vickrey–Dutch auction for unit demand preferences. Working Paper, Harvard University.

Mishra, D., Parkes, D.C. 2004b. A Vickrey–Dutch clinching auction. Working Paper, Harvard University.

Mishra, D., Parkes, D.C., forthcoming. Ascending price Vickrey auctions for general valuations. Journal of Economic Theory. DOI: http://dx.doi.org/10.1016/j.jet.2005.09.004.

Mishra, D., Veeramani, D., 2006. An ascending price procurement auction for multiple items with unit supply. IIE Transactions 38 (2), 127–140.

Parkes, D.C., 2005. Auction design with costly preference elicitation. Annals of Mathematics and AI 44, 269–302.

Parkes, D.C., Kalagnanam, J., 2005. Models for iterative multiattribute Vickrey auctions. Management Science 51, 435–451.

Parkes, D.C., Ungar, L.H., 2000. Preventing strategic manipulation in iterative auctions: Proxy agents and price-adjustment. In: Proceedings of the 17th National Conference on Artificial Intelligence (AAAI-00), pp. 82–89.

Rothkopf, M.H., Pekeč, A., Harstad, R.M., 1998. Computationally manageable combinatorial auctions. Management Science 44 (8), 1131–1147.

Vickrey, William, 1961. Counterspeculation, auctions, and competitive sealed tenders. Journal of Finance 16, 8–37.