A generalized index of employment segregation

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Abstract

This article axiomatically derives a class of numerical indices of integration (equality) in the distribution of male–female workers across occupations. The associated segregation (inequality) indices parallel the multidimensional Atkinson inequality indices. Two members of the class of segregation indices are monotonically related to the Hutchens [Hutchens, R.M., 2004. One measure of segregation. International Economic Review 45, 555–578.] square root index and the Theil–Finizza (1967) index. A numerical illustration of the family of indices is also provided using U.S. occupational data.

Keywords: Occupational segregation; Axiomatic derivation; Multidimensional Atkinson index; Square root index; Theil–Finizza index

JEL classification: C43; J16; J71

1. Introduction

Occupational segregation by sex, that is, inequality in the distribution of male and female workers across occupational groups, exists in almost all countries of the World. Several reasons may be given for this phenomenon. But one of the most important reasons for this is unequal access to jobs.

In contrast to segregation, integration refers to equality in the occupational distribution across groups (see Hutchens, 2004). Complete or perfect integration occurs if employees are allocated to occupations in proportions to their shares in the population. Segregation arises if this condition does not hold. In other words, segregation refers to the comparison of actual distributions of types of workers among occupations with the distribution that would arise if types had been allocated in proportions to their shares.

The increasing consciousness about occupational segregation has naturally raised the question about the measurement of segregation. Since the seminal contribution of Duncan and Duncan (1955), several numerical indices of employment segregation have been suggested in the literature. (See Flückiger and Silber, 1999 for a survey and also Hutchens, 1991, 2001.) In a recent paper Hutchens (2004) characterized an index of segregation, the square root index, using subgroup decomposability.

According to subgroup decomposability, if we subdivide the occupational structure into, say two subgroups 1 and 2, then the overall segregation can be decomposed into a component for subgroup 1, a component for subgroup 2, and a between group component. This enables us to assess how the different components changed through time. Hutchens' characterization relies on the concepts of aggregation and additive decomposability presented in the analysis of income inequality (Shorrocks, 1980, 1984).

In this paper we provide a characterization of a parametric family of integration indices. The corresponding segregation indices parallel the multidimensional Atkinson (1970) indices of inequality considered by Tsui (1995). Two members of the family of segregation indices are monotonically related to the square root index and the Theil-Finizza (see Theil, 1967) logarithmic index. Thus, our characterization interprets the square root and the logarithmic indices from an alternative perspective.

The next section of the paper presents the formal framework and the characterization theorem. In Section 3 we give a numerical illustration of several indices of segregation using U.S. occupational data. Finally, Section 4 concludes.

2. Formal framework

We consider a society with two types of people distributed over $T>1$ groups or occupations. For concreteness, we may suppose that the two types of people are male and female. Let x_{ij} be the number of persons of type i in occupation j ($i=1,2$; $j=1,2,\ldots,T$). We may also regard x_{ij} as the number of hours spent by workers of type i in occupation j. Each x_{ij} is a positive real number. Non-integer values of x_{ij} are possible because part-time workers and child labor can be regarded as fractional workers. For example, a child labor can be counted as $\sqrt{.5}$. Let X be the $2\times T$ matrix of these x_{ij} values. We denote the set of all $2\times T$ matrices with positive entries as M. For any $X\in M$, the numbers of type 1 and type 2 people are denoted by $n_1(X)$ and $n_2(X)$ (or, simply by n_1 and n_2) respectively, that is, $n_i = \sum_{j=1}^{T} x_{ij}, i = 1,2$. For any $X\in M$, the kth column of X, x_k, represents the distribution of the two types of persons in occupation k.

An index of employment segregation examines the inequality in the distribution of people across occupations. A dis-segregation or integration index is concerned with the opposite feature, that is, it can be regarded as a measure of equality in the occupation-wise distribution of the population that exists in X between the two types of employees in different occupations. Under ceteris paribus assumptions, an increase in the value of a segregation index is equivalent to a reduction in integration and vice-versa. This correspondence is similar in nature to a relationship found in the literature on income inequality between an index of inequality and its associated equality or welfare index.

An integration index I is a real valued function defined on M, that is, I: $M\rightarrow R^1$, where R^1 is the real line. For any $X\in M$, $I(X)$ indicates the extent of integration that exists in X between the two types of employees in different occupations.

We will now propose a set of axioms for an integration index. They are stated in terms of an arbitrary index I.

Symmetry in Occupations (SYO): For any $X \in M$; if Y is obtained from X by a permutation of the columns of X, then $I(X)=I(Y)$.
Continuity (CON): I is continuous on M.
Strict Separability (SSP): For any non-singleton subset Q of $\{1,2,\ldots,T\}$; for any $X \in M$; $I(X)=I(\phi(X^{Q}),X^{C})$, where X^{Q} is the submatrix of X including the vectors of people in Q, X^{C} is the complement of X^{Q} and ϕ is some continuous function.
Scaling Consistency (SCC): For any $X,Y \in M$; $I(X)=I(Y)$ implies and is implied by $I(\Omega X)=I(\Omega Y)$, where Ω is the 2×2 diagonal matrix (ω_1,ω_2), $\omega_i>0, i=1,2$.

Before we proceed to state the remaining axioms, let us explain the ones already proposed. SYO, which is taken from Hutchens (1991, 2004), means that if the number of persons in occupation k trade places with those in occupation j, then integration remains unaffected. In other words, integration does not depend on the labeling of the occupations. What is important is the number of persons in different occupations. Continuity means that minor changes in the numbers of persons in different occupations will not produce an abrupt change in the integration index. Thus, a continuous integration index will not be oversensitive to minor observational errors in the number of persons in an occupation. Strict Separability shows how we can calculate the overall integration when we subdivide the occupations in two subgroups, say with respect to the region (see Blackorby et al., 1981). SCC means that if any two occupational distribution matrices X and Y of type 1 and type 2 people have the same level of integration, then the level of integration remains unchanged for any rescaling of the matrices using the same scale transformation. This condition certainly holds if the integration index is homogeneous of some arbitrary degree $k \geq 0$. It is a generalization of a function being homothetic (see Tsui, 1995).

The next postulate, which was considered by Hutchens (1991, 2004), is concerned with movement between occupations. To understand this, let us consider the case of perfect integration, that is, $X \in M$ is such that

$$\frac{x_{1h}}{x_{2h}} = \frac{n_1}{n_2} \text{ for all } h = 1, 2, \ldots\ldots, T. \tag{1}$$

Segregation arises if the above equality is violated and hence

$$\frac{x_{1h}}{x_{2h}} < \frac{n_1}{n_2} < \frac{x_{1j}}{x_{2j}} \text{ for some } h \neq j. \tag{2}$$

Since in this case the share of labor force of type 2 is higher than that of type 1 in occupation $h\left(\frac{x_{1h}}{n_1}<\frac{x_{2h}}{n_2}\right)$, we say that it is type 2 dominated. Conversely, occupation j is type 1 dominated. A transfer of labor force that strengthens the dominating positions of type 2 in occupation h and type 1 in occupation j should increase segregation. Formally, such a transformation will be of the type

$$\frac{y_{1h}}{y_{2h}} \equiv \frac{x_{1h-\delta}}{x_{2h}} < \frac{x_{1h}}{x_{2h}} < \frac{x_{1j}}{x_{2j}} < \frac{x_{1j}+\delta}{x_{2j}} \equiv \frac{y_{1j}}{y_{2j}}, \tag{3}$$

where '$a \equiv b$' means that 'a is defined to be equal to b', $0<\delta \leq x_{1h}$ and $y_{ik}=x_{ik}$ for all $k \neq h,j$. Thus, Y is more segregated than X. (It may be noted that replacement of the central strict inequality by a weak one does not change the situation.) This is the essential idea underlying this postulate.[1]

More formally, for any $X \in M$; we say that $Y \in M$ is obtained from X by a disequalizing movement of type 1 people if, for l and j, (i) $x_{2l}=x_{2j}=y_{2l}=y_{2j}>0$, (ii) $(x_{1l}/x_{2l})<(x_{1j}/x_{2j})$, (iii) $y_{1l}=x_{1l}-q$ and $y_{1j}=x_{1j}+q$ for $0<q\le x_{1l}$ and (iv) $x_{hk}=y_{hk}$, $h=1,2;k\neq l,j$.

This then leads to our next axiom:

Movement Between Occupations (MBO): For any $X \in M$ if $Y \in M$ is obtained by a disequalizing movement, then $I(Y)<I(X)$.

To understand this postulate further, let us consider the following example:

$$\begin{array}{r c} & \text{Occupation} \\ & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix}\text{Women}\\ \text{Men}\end{matrix} & \begin{bmatrix} 3 & 2 & 6 \\ 8 & 4 & 4 \end{bmatrix} \end{array} \tag{4}$$

Since occupation 2 contains two women and four men, and occupation 3 contains six women and four men, we have $x_{22}=x_{23}=4$, $x_{12}/x_{22}=2/4<6/4=x_{13}x_{23}$. Now, if one woman moves from occupation 2 to occupation 3 (so that occupation 2 contains one women and occupation 3 has 7 women), then the occupation-wise distribution of women and men is becoming more concentrated. This is because the movement increases the percentage of females in female dominated occupation 3 and the percentage of males in the male dominated occupation 2. Hence segregation should increase and integration should reduce. Likewise, integration should increase under an equalizing movement.

While MBO is concerned with a change in integration under a disequalizing movement of persons of a particular type between two occupations, we can have a monotonicity principle for integration when the size of employees in an occupation increases. Suppose that the proportion of female (male) employees increases in a male(female) dominated occupation. Then assuming that employment sizes in all other occupations remain unaltered, it is reasonable to expect that integration will increase. This is because the change reduces the gap between proportions of the two types of employees in that occupation. For instance, in the occupation distribution matrix X given by (4), if the number of female workers in occupation 1 increases to 4, integration should increase.

Formally, we have:

Monotonicity (MON): For any $X \in M$, such that $(x_{ik}/n_i)<(x_{mk}/n_m)$, $i\neq m=1,2$; we have $I(Y)>I(X)$, where $Y \in M$ is obtained from X as follows:

(i) $y_{ik}=x_{ik}+\delta$, $\delta>0$ being such that $(y_{ik}/(n_i+\delta))\le(x_{mk}/n_m)$,
(ii) $y_{ih}=x_{ih}$ for $h=1,2\ldots,T;h\neq k$,
(iii) $y_{mh}=x_{mh}$ for $h=1,2\ldots,T$.

SYO makes the occupations anonymous. Likewise, we may have a postulate that makes the types anonymous. That is, we want to make the index insensitive to whether women or men labeled types 1 or 2. This means that the integration index should remain invariant under reordering of the rows of the occupation matrix. This postulate was introduced by Chakravarty and Silber (1994). (See also Kakwani, 1994; Hutchens, 1991, 2004.) Formally,

Symmetry in Types (SYT): For any $X \in M; I(Y)=I(X)$, where $Y=\Gamma x$, with Γ being the 2×2 matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The two notions of symmetry demonstrate the difference between the measurement of welfare and segregation. While in the former, there is anonymity among individuals only, in this case we have anonymity in both occupations and types. Since no particular meaning is attached to the ways types and occupations are arranged, the variables considered in the case of segregation are purely categorical in contrast to the cardinally or ordinally measurable variables employed in the case of welfare comparisons.

Each of the postulates considered above for an index of integration will have its segregation index counterpart. We also assume that a segregation index should be scale invariant, where scale invariance demands that if n_1 and/or n_2 are multiplied by a positive scalar and the shares of both types of people in all the T occupations remain the same, then segregation does not change. For example, suppose that the number of women in the labor force triples because of a three-fold occupation-by-occupation replication of the female labor force. Likewise, assume that the number of men doubles in the labor force under an analogous replication. Then according to scale invariance, such changes in the numbers of female and male in the labor force do not change the level of segregation. Thus, if the types have different ratio scales, rescaling them should not alter the level of segregation. More precisely, if S is any segregation index defined on M, then for any $X\in M, S(X)=S(\Omega X)$, where $\Omega=\text{diag}(\omega_1,\omega_2)$, $\omega_i>0, i=1,2$. Finally, a segregation index should be insensitive to proportional divisions of occupations. For example, if one large occupation with 80 women and 80 men is broken down into four sub-occupations, each containing 20 women and 20 men, then segregation should not change (see Hutchens, 1991, 2004).

The following theorem shows that the axioms proposed in the section are sufficient to isolate a class of integration indices uniquely.

Theorem 1. *An integration index I: $M\rightarrow R^1$ satisfies axioms SYO, CON, SSP, SCC, MBO, MON and SYT if and only if it is ordinally equivalent to*

$$aT+b\sum_{j=1}^{T}\prod_{i=1}^{2}x_{ij}^{\alpha}, \tag{5}$$

or

$$aT+\sum_{j=1}^{T}\sum_{i=1}^{2}\alpha\log x_{ij}, \tag{6}$$

where $b>0$, $0<\alpha<1$ and a are constants.

Proof. Following Blackorby et al. (1981) we can show that SSP, SYO and MON are enough to ensure that $I(\cdot)$is additively separable, that is, it is of the form $g(\sum_{j=1}^{T}d(x_j))$, where d: $R^2_{++}\rightarrow R^1$, R^2_{++} being the strictly positive part of the 2 dimensional Euclidean space and g: $R^1\rightarrow R^1$ is increasing.

Suppose that there are L occupations having distributions w and w^* and $(T-L)$ occupations having distributions v and v^* such that

$$Ld(w)+(T-L)d(v)=Ld(w^*)+(T-L)d(v^*). \tag{7}$$

We may rewrite (7) as

$$\frac{d(w)-d(w^*)}{d(v)-d(v^*)}=-\frac{(T-L)}{L}. \tag{8}$$

In view of SCC for any $\Omega=\text{diag}(\omega_1,\omega_2),\omega_i>0$ for all i, we have

$$Ld(\Omega w)+(T-L)d(\Omega v)=Ld(\Omega w^*)+(T-L)d(\Omega v^*), \tag{9}$$

which we rewrite as

$$\frac{d(\Omega w)-d(\Omega w^*)}{d(\Omega v)-d(\Omega v^*)}=-\frac{(T-L)}{L}. \tag{10}$$

From (8) and (10) it follows that

$$\frac{d(w)-d(w^*)}{d(v)-d(v^*)}=\frac{d(\Omega w)-d(\Omega w^*)}{d(\Omega v)-d(\Omega v^*)} \tag{11}$$

and hence

$$\frac{d(\Omega v)-d(\Omega v^*)}{d(v)-d(v^*)}=\frac{d(\Omega w)-d(\Omega w^*)}{d(w)-d(w^*)}. \tag{12}$$

From (12) we note that the ratio $\frac{d(\Omega w)-d(\Omega w^*)}{d(w)-d(w^*)}$ is independent of the distribution (w, w^*). Therefore, it is clear that

$$\frac{d(\Omega w)-d(\Omega w^*)}{d(w)-d(w^*)}=A(\Omega). \tag{13}$$

Assuming that w^* is fixed, we rewrite Eq. (13) as

$$d(\Omega w)=A(\Omega)d(w)+B(\Omega). \tag{14}$$

The solutions to the functional Eq. (14) are given by

$$d(x_j)=a+b\prod_{i=1}^{2}(x_{ij})^{\alpha_i} \tag{15}$$

and

$$d(x_j)=a+\sum_{i=1}^{2}\alpha_i\log x_{ij}, \tag{16}$$

(see Aczel et al., 1986).

SYT implies that $\alpha_i=\alpha$ for $i=1,2$. Now, for satisfaction of MBO by $I(X)$ corresponding to the form given by (16), we need $\alpha>0$. Next, MON, will be fulfilled by $I(X)$ associated with the form (15) if b and α have the same sign. Given $b>0$, MBO will be satisfied by (15) if $0<\alpha<1$. Hence $\sum_{j=1}^{T}d(x_j)$ is given by (5) or (6). Since the integration index I is ordinally equivalent to $\sum_{j=1}^{T}d(x_j)$, this establishes the necessity part of the theorem. The sufficiency is easy to check. Δ

To derive segregation indices corresponding to the integration indices given by (5) and (6), let X_e be matrix associated with X whose (i,j)th entry is given by n_i/T. That is, in X_e for each type $i(i=1,2)$, the employees are equally distributed across occupations and occupations are of equal size. Further complete integration (or no segregation) occurs in X_e because the ratio of women to men is the same in all occupations (see Hutchens, 2004).

We define the scale factor $\theta(X)$ implicitly by

$$I(X) = I(\theta(X)X_e). \tag{17}$$

The factor θ (X) is formally similar to Debreu's (1951) coefficient of resource utilization. It has the same flavor as the equally distributed equivalent income, which is used as an index of equality in the income distribution literature (see also Kolm, 1977). Then as an index of employment segregation we suggest the use of

$$K(X) = 1 - \frac{\theta(X)}{T}. \tag{18}$$

For the forms of I given by (5) and (6) (or ordinal transforms of the respective forms), the segregation indices defined by (18) turn out to be

$$\bar{K}_\alpha(X) = 1 - \left[\frac{1}{T}\sum_{j=1}^{T} \prod_{i=1}^{2} \left(\frac{x_{ij}}{n_i}\right)^{\alpha}\right]^{1/2\alpha}, \tag{19}$$

and

$$\bar{K}(X) = 1 - \prod_{j=1}^{T} \left[\prod_{i=1}^{2} \left(\frac{x_{ij}}{n_i}\right)^{1/2}\right]^{1/T}. \tag{20}$$

The indices $\bar{K}_\alpha$ and $\bar{K}$, given respectively by (19) and (20) are continuous, bounded between zero and one, where the lower bound is achieved if there is no segregation at all. In contrast, they attain the upper bound in the case of complete segregation, that is, when all the men are in one set of occupations and all the women are in another set; no occupation contains both women and men. Further, they are symmetric in occupations and types, increasing under a disequalizing movement, scale invariant and insensitive to proportional divisions of occupations.

We can relate these indices with the ordering generated by two non-intersecting segregation curves. For any $X \in M$, its segregation curve is a plot of the cumulative proportions of type 1 people against the cumulative proportions of type 2 people both proportions being ranked in increasing order of x_{1j}/x_{2j}. Hutchens (1991) showed that for any X, $Y \in M$ if the segregation curve of X is nowhere below that of Y and at some places strictly inside, then X is regarded as more segregated than Y by all segregation indices that are symmetric in occupations, increasing under a disequalizing movement, scale invariant and insensitive to proportional divisions of occupations. Furthermore, the converse is also true. Thus, $\bar{K}_\alpha$ and $\bar{K}$ are consistent with the ordering produced by non-intersecting segregation curves.

Since the indices $\bar{K}_\alpha$ and $\bar{K}$ use an Atkinson (1970) type aggregation principle to the products of occupation-wise male and female employment proportions, we can refer to them as segregation-analogue to the multidimensional Atkinson inequality indices (see Tsui, 1995). Thus, $\bar{K}_\alpha$ and $\bar{K}$ are fairly natural translations of multidimensional Atkinson inequality indices to the measurement of segregation. For a given X, as α increases, $\bar{K}_\alpha$ decreases. α can therefore be interpreted as a segregation aversion parameter. The parameter α also shows relative sensitivity of $\bar{K}_\alpha$ to movement between occupations. A disequalizing movement between occupations increases $\bar{K}_\alpha$ by a larger amount, the lower is the value of α. For instance, for the example we have considered in (4), one may think that the movement of one woman from occupation 2 to occupation 3 is highly undesirable because both male and female workers are suitable for these two occupations and the movement is increasing the proportion of female workers in occupation 3 from .55 to .64 and making the male dominated occupation (occupation 2) more male dominated. Then, given two values of α, to look at the resulting increase in segregation, one should choose the smaller value of α. In contrast, if one thinks that occupation 3 is more important than occupation 2 for women, then possibly the higher value of α should be chosen.

Hutchens (2004) translated the Shorrocks (1980) generalized entropy family of inequality indices into indices of segregation. He showed that only one member H of this family lies between zero (no segregation) and one (complete segregation). For $\alpha=1/2$, $\bar{K}_\alpha$ is related to H by the increasing transformation $H=1-T(1-\bar{K}_\alpha)$. For a given T, H and $\bar{K}_\alpha$ (for $\alpha=1/2$) will rank any two occupational distribution matrices exactly in the same way. Therefore, $\bar{K}_\alpha$ (for $\alpha=1/2$) and H essentially convey us the same information. Hutchens (2004) refers to H as the square root index because it is expressed in terms of sum of the square roots of the products of occupation-wise male and female employment proportions. Note that although H is the only member of the generalized entropy family that lies on the unit interval, all members of $\bar{K}_\alpha$ and $\bar{K}$ fulfil this boundedness condition.

In a recent paper, Frankel and Volij (2005) characterized H in the case of more than two demographic groups using properties of the underlying segregation ordering. There is a major difference between their characterization and that of Hutchens (2004) in the sense that some of the axioms used by Hutchens (2004) do not have natural translation into properties of segregation ordering. Note also that our way of arriving at $\bar{K}_\alpha$ (for $\alpha=1/2$) is totally different from the approach used by Frankel and Volij (2005) for characterizing H.

Next, we note that given the total number of occupations T, the index in (20) is monotonically related to the following

$$K'(X)=\frac{1}{T}\sum_{i=1}^{2}\sum_{j=1}^{T}\log\left(\frac{1/T}{x_{ij}/n_i}\right). \tag{21}$$

K' in (21) is simply the Theil-Finizza (see Theil, 1967) index of segregation (see Mora and Ruiz Castillo, 2003). Therefore, for a given T, K' will rank occupational distribution matrices in the same way as $\bar{K}$. Hence our indices can be regarded as a generalization of the Hutchens square root index and the Theil–Finizza index.

3. An empirical illustration

The empirical illustration is based on U.S. data (see, Bureau of Labor Statistics, 2005), where the occupational classification includes 22 categories. (See the Appendix for a detailed list of these 22 occupations.) These data give on the one hand the occupational distribution of men and women, no distinction being made between the various ethnic groups, on the other hand the distribution of White, Blacks and Asian individuals, no distinction being made this time between the genders. We have, therefore, been able to compute indices giving the extent of segregation between men and women, Whites and Blacks, Whites and Asians and Blacks and Asians.

The results are presented in Table 1. Nine indices were estimated: the traditional Duncan index, the Gini segregation index, the Theil–Finizza index, Hutchens' square root index, the new index $\bar{K}$ derived in this paper as well as the new indices $\bar{K}_\alpha$ for values of α respectively equal to 0.1, 0.3, 0.5, 0.7 and 0.9.

We note the decreasingness of $\bar{K}_\alpha$ as α increases. Next, in each case, the values of $\bar{K}_\alpha$ and $\bar{K}$ are found to be much higher than the remaining index values. This is because of the specific type

Table 1
Occupational segregation by gender or nationality in the United States in 2005

Type of segregation	Duncan index	Gini-segregation Index	Theil–Finniza index	Hutchens' square root index	K	$\bar{K}_{0.1}$	$\bar{K}_{0.3}$	$\bar{K}_{0.5}$	$\bar{K}_{0.7}$	$\bar{K}_{0.9}$
Occupational segregation between Whites and Blacks	0.170	0.230	0.303	0.021	0.9679	0.965473	0.9604	0.955	0.9506	0.946
Occupational segregation between Whites and Asians	0.183	0.270	0.299	0.033	0.9678	0.965470	0.9607	0.956	0.9514	0.947
Occupational segregation between Asians and Blacks	0.248	0.360	0.343	0.057	0.969	0.967	0.9619	0.957	0.9525	0.948
Occupational segregation by gender	0.414	0.560	0.425	0.149	0.972	0.970	0.9660	0.961	0.9564	0.952

of normalization considered in (18). In fact, we have an analogous feature for the Hutchens square root index as well. This is a consequence of the relationship between the two indices.

Furthermore, it appears, as expected, that segregation by gender is higher than ethnic segregation. As far as the latter is concerned, segregation is generally the highest when Asians and Blacks are compared and the lowest when Whites and Blacks are compared. Note that the ranking of the four cases examined is the same for the following indices: the Duncan index, the Gini-segregation index, Hutchens' square root index and the indices $\bar{K}_\alpha$ when α is equal to 0.3, 0.5, 0.7 and 0.9. The ranking is slightly different for the Theil–Finizza index, the index K and the index $\bar{K}_\alpha$ when $\alpha=0.1$. The relationship mentioned previously between Hutchens' index and the index $\bar{K}_\alpha$ when$\alpha=0.5$ as well as between the Theil–Finizza index and the index K is easy to verify numerically.

4. Conclusions

Inequality in the distribution of employees in occupations by sex is an important issue in labor market analysis. An indicator of this type of inequality is called a segregation index. In this paper we have characterized a family of integration (equality) indices of the distribution of male female workers across occupations using a set of intuitively reasonable axioms. The corresponding family of segregation indices, which parallels multidimensional Atkinson inequality indices, contains monotone transformations of the Hutchens (2004) square index and the Theil (1967) index as special cases. A numerical illustration of these indices along with some other indices has been also been provided in the paper.

One limitation of the class of integration indices we have characterized is that they do not allow to rank situations where one or more occupations are not filled by one or the other type, that is, when some of the entries in a matrix $X \in M$ are zero. Chances of such a situation arising in practice are rather low. As Tsui (1995, p.264) stated this domain restriction 'helps simplify our results and renders the discussion more translucent.' It is important to note that our characterization relies on a result from Aczel et al. (1986) that invokes the assumption $x_{ij}>0$. We may mention here that in the inequality poverty literature several indices need the analogous assumption that all incomes are positive. For instance, Theil's (1972) mean logarithmic deviation index of inequality and Watts' (1968) poverty index require positivity of all incomes to be well defined. But from a mathematical point of view it will certainly be worthwhile to use the more appropriate domain where $x_{ij} \geq 0$ and derive the corresponding class of integration indices. We leave this issue as a future research program.

Note that, since employment segregation is studied mostly between men and women, we also assumed that there are only two types. But we may often be interested in studying segregation for more than two types, e.g. segregation by race. It is easy to see that our indices can be extended to the cases when there are more than two types. In future work we plan on extending our result to more than two categories.

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Appendix A. List of occupations

Management occupations
Business and financial operations occupations
Computer and mathematical occupations
Architecture and engineering occupations
Life, physical and social science occupations
Community and social services occupations
Legal occupations
Education, training and library occupations
Arts, design, entertainment, sports and media occupations
Healthcare practitioner and technical occupations
Healthcare support occupations
Protective service occupations
Food preparation and swerving related occupations
Building and ground cleaning and maintenance occupations
Personal care and service occupations
Sales and related occupations
Office and administrative support occupations
Farming, fishing and forestry occupations
Construction and extraction occupations
Installation, maintenance and repair occupations
Production occupations
Transportation and material moving occupations

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