Quality Engineering
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/lqen20

Design Evaluation of Air Gap in a Ceiling Fan—A Simulation Case Study

Tridib K Dutta [a]

[a] SQC and OR Unit, Indian Statistical Institute, Kolkata, India

Available online: 15 Mar 2007

Design Evaluation of Air Gap in a Ceiling Fan—A Simulation Case Study

Tridib K. Dutta
SQC and OR Unit, Indian Statistical Institute, Kolkata, India

Lower air gap in ceiling fan motors leads to higher rework at an assembly. It is suspected that inappropriate component design specifications may be the root cause. To resolve the issue and prevent worker dissatisfaction, a study is undertaken to evaluate the extent of air gap variation when components are produced as per design. First air gap is modeled as a function of different parameters of a fan assembly. The theoretical probability distribution of this modeled air gap function, though not impossible to obtain, is quite complex to derive. Thus this model is then used to generate data on air gap using Monte Carlo simulation. Analysis of the data shows that lack of process capability, not inappropriate design specifications, is mainly responsible for higher rework. The findings were discussed with management and were accepted. As a sequel to the study, a cost-benefit analysis is being undertaken for procuring new and improved machines.

Keywords Chi-square test; Distribution function; Goodness-of-fit; Normal distribution; Probability density function; Simulation; Uniform distribution.

INTRODUCTION

A particular model of ceiling fan, which is a flagship product of a reputed fan manufacturer in India, has 30% rework level at the assembly stage. This high rework percentage not only reflects poorly on the process performance but also reduces productivity considerably. The records indicate that more than 50% of the total reworks are due to friction sound between the stator and the rotor of the fan motor or due to fan motor getting jammed at a lower voltage of 190 V (henceforth referred to as 190 V jam). It may be noted that the normal household supply in India is 230 V AC.

These two defects, originate mainly from insufficient clearance between the rotor and the stator. The assembly operators are highly skeptical of the existing design specifications. They believe that being less than adequate, these supposedly erroneous specifications are mainly responsible for high rework and their subsequent loss of production incentives due to lower productivity. So the management wanted the dispute to be resolved for higher employee satisfaction.

The model of the fan mentioned above accounts for more than 50% of the company's production and is a natural choice for resolving the dispute. The air gap is defined as the distance between the rotor inner diameter (ID) and the stator outer diameter (OD). The diametrical clearance specification between rotor and stator assembly, as per the design for this model, is 0.3302 mm–0.4572 mm. (0.013 in.–0.018 in.)

In a perfect assembly, the rotor ID and the stator OD are expected to be concentric circles with rotor ID being the outer one. So the difference between these two diameter measurements is defined as the diametrical clearance, and the difference between the radii is called radial clearance. However, it may be noted that there is no design specification for radial clearance, but the notion is in vogue because of the acute problems faced at assembly. Operational specification followed at assembly for radial clearance is half of the design specification for diametrical clearance, 0.1651 mm–0.2286 mm (0.0065 in.–0.0090 in.). It has been observed that although the diametrical clearance conforms to specification, insufficient radial clearance due to cover spigot eccentricity leads to friction sound and jam on assembling the fan. The rotor is placed on the cover spigot.

In view of the above condition, the objective of the study is to theoretically evaluate the expected clearance between the rotor-stator assembly based on the design specifications and also to examine the possibility of rotor-stator rubbing. The engineering drawing being a confidential document cannot be reproduced and hence a rough sketch of the fan cross-section is given in Figures 1 and 2 for a better understanding of the subsequent discussions.

Address correspondence to T.K. Dutta, SQC and OR Unit, Indian Statistical Institute, Kolkata, India. E-mail: tridib@isical.ac.in

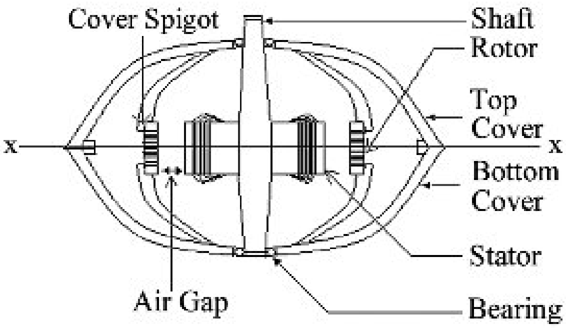

Figure 1. Cross-sectional view of fan assembly.

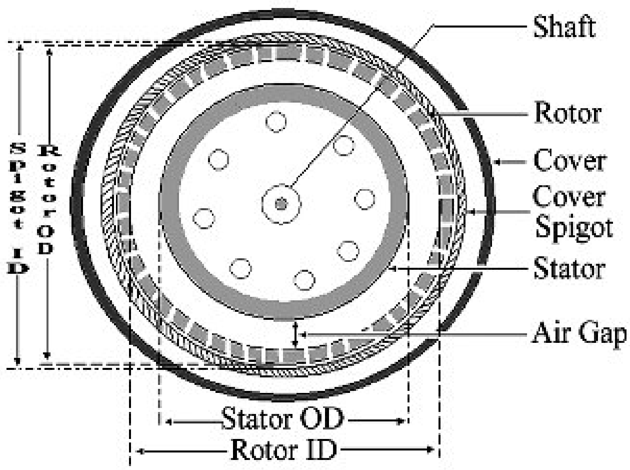

Figure 2. Section x-x.

APPROACH

The approach adopted here is to express the air gap (radial as well as diametrical clearances between rotor-stator assembly) as a function of rotor ID, stator OD, rotor and stator eccentricities, rotor OD, cover spigot ID, and cover eccentricity. The air gap values are then simulated using this relation to study its variation relative to the specification. The contribution of different parameters toward this variation is also evaluated.

MODEL BUILDING

Based on the survey of technical manuals and discussions with the technical personnel, the air gap can be expressed as a function of different parameters using the relations given below.

Let $\delta =$ Bottom cover spigot ID − Rotor OD + Stator eccentricity + Rotor eccentricity + Cover spigot eccentricity with respect to bore.

It can be shown, assuming ovality to be absent, that

Radial Clearance

$$= \begin{cases} (Rotor\ ID - Stator\ OD)/2 \\ \quad - \delta \text{ at minimum point and} \\ (Rotor\ ID - Stator\ OD)/2 \\ \quad + \delta \text{ at maximum point.} \end{cases} \tag{1}$$

Diametrical Clearance

$$= \begin{cases} \sqrt{(Rotor\ ID)^2 - (2\delta)^2} \\ \quad - Stator\ OD \text{ at minimum point and} \\ (Rotor\ ID - Stator\ OD) \text{ at maximum point.} \end{cases} \tag{2}$$

Both these types of clearances are taken as measures of air gap.

MODEL VALIDATION

To validate the model it is decided to use Monte Carlo simulation based on the existing process data. To facilitate this procedure the following assumptions are made.

Assumptions

a) The respective characteristics in the air gap function are assumed to have statistical distributions with known parameter(s) which are evaluated based on the existing process status.
b) A random assembly of components is assumed, as is the normal practice in the assembly section, to simulate data.

Data Generation and Analysis

With these assumptions, the following steps are followed to validate the developed model:

i) A total of 200 observations are collected from the past inspection records for each of the model parameters. These observations are then used to fit appropriate probability distributions using the Chi-square test for goodness-of-fit. These tests are carried out following procedures discussed in Montgomery and Runger (1994, pp. 444–449). As an example, sample data on Rotor ID measurements along with the necessary computations are given in Table 4 and Appendix B, respectively. Table 5 illustrates the necessary computations for the test of goodness-of-fit.

ii) Based on this past data analysis, it is observed that the distribution of rotor ID, rotor OD, stator OD, and cover spigot ID can be well modeled using Normal distributions with parameters equal to their respective process means and process standard deviations (See Johnson 1995, pp. 145–150, Montgomery and Runger, 1994, pp. 173–185).

iii) The distribution found to fit well for data on rotor and stator eccentricities is described by the probability density function (p.d.f.) given in Eq. (3).

$$f(x,k) = \frac{x}{k}\exp\left(-\frac{x^2}{2k}\right); \quad 0 \leq x < \infty, k > 0. \quad (3)$$

Given a random sample $x_1, x_2, \ldots, x_n$, the maximum likelihood estimator for the parameter k, which completely specifies the distribution, is obtained as $(1/2n)\sum_{i=1}^{n} x_i^2$.

iv) Based on the fitted distributions 12,500 sets of observations, representing half a month's production, are simulated for each of the model parameters. The software SYSTAT 8.0 is used to generate random observations from the respective normal distributions representing the probability distributions of rotor ID, rotor OD, stator OD, and cover spigot ID. The SYSTAT function used for this generation is *ZRN(process mean, process standard deviation)*.

v) For data on eccentricities, generation of the random variable is done using the table-look-up or inversion method; a general description of the method is given in Morgan (1984). For exact computational details, one may refer to Appendix A. Here random observations (u) are generated from uniform distribution, U (0, 1) (see Johnson, 1995, pp. 154–155, Montgomery and Runger, 1994, pp. 170–172) and then using the relation Eq. (4), these are converted to observations from the fitted distribution (x) whose p.d.f. is given in Eq. (3). The SYSTAT function used for generation of uniform variables is URN (0, 1).

$$x = \sqrt{2k\log_e\left(\frac{1}{u}\right)} \quad (4)$$

vi) Air gaps, radial as well as diametrical clearance, are computed from the observations thus generated using Eqs. (1) and (2), and assuming a random assembly of components. Based on this finding the models are cleared for further simulation. Estimation of total rework due to friction sound and 190 V jam is obtained as 28.11%, which compares well with the existing assembly rework value for the fan model under study. This estimate is obtained under the assumption that if the radial clearance is less than 0.0254 mm (0.001 in.) or the diametrical clearance is less than 0.0762 mm (0.003 in.) then rework results.

SIMULATION, DATA COLLECTION, AND ANALYSIS

Having validated the model, the next task is to simulate data to resolve the dispute on the supposedly inappropriate design specifications of the components. Monte Carlo simulation is used for simulating observations from different statistical distributions. To simulate, the following assumptions are also made, as before, but with modifications as necessary.

Assumptions

a) The respective characteristic in the air gap function is assumed to have a statistical distribution with known parameter(s) which is/(are) evaluated based on design specifications.
b) Components are produced as per design specifications from capable processes.
c) A random assembly of components is assumed in order to simulate data and to perform valid statistical analysis.
d) The probability distributions for rotor ID, stator OD, rotor OD, and cover spigot ID are assumed to be normal with mean as their respective mid-specification and standard deviation as one-sixth of the respective design tolerances. The normality assumption follows from the analysis of past data as indicated in the section on model validation.
e) The maximum specified value for eccentricity, as per the design, is assumed to be such that 99.73% of the products from a capable process is expected to meet this design tolerance.

Data Generation and Analysis

The basic procedure for data generation is the same as that of model validation with the parameters estimated following the new assumptions mentioned

Table 1
Air gap between stator and rotor

Sl. No.	Clearance type	Unit	Expected		
			Minimum	Average	Maximum
1	Radial (at minimum point)	mm (inch)	0.08142 (0.0032)	0.13500 (0.0053)	0.18858 (0.0074)
2	Radial (at maximum point)	mm (inch)	0.25392 (0.0100)	0.30750 (0.0121)	0.36108 (0.0142)
3	Diametrical	mm (inch)	0.39743 (0.0156)	0.44250 (0.0174)	0.48757 (0.0192)

above. Let $x^2_{\max}$ be the maximum value for eccentricity specified in the design. Then the value of the parameter k for the probability distribution of eccentricity, as given in Eq. (3), is obtained from the corresponding distribution function using the relation Eq. (5).

$$k = -\frac{x^2_{\max}}{(2\log_e(1-0.9973))} \tag{5}$$

Based on the distributions so obtained, a total of 12,500 sets of observations are simulated for each of the model parameters. Air gaps, radial and diametrical clearance, are computed as before. These values of air gaps can be well modeled with normal probability distributions. The average, minimum, and maximum values expected for these fitted distributions are computed. Estimates of total rework due to friction sound, and 190 V jam works out to be negligible. The simulated air gap values are summarized in Table 1.

The radial clearance at minimum point is due to the fact that eccentricities act together to reduce the air gap where as at maximum point, they act to increase it. Basically, these two points are complementary in nature and are diametrically opposite. Contribution to the variations in air gap due to different parameters are evaluated based on the same set of simulated air gap values (diametrical clearance) and are summarized in Table 2.

CONCLUSIONS AND RECOMMENDATIONS

Simulation revealed that though the diametrical clearance is found stable when components are assumed to have been produced as per design, there is wide variation (0.08142 mm to 0.36108 mm), in radial clearance. Efforts should be initiated to have a stricter control on radial clearance and the design must have an explicit specification for the same. Radial air gap variation is expected to be within 0.08142 mm to 0.36108 mm without considering ovality in rotor, stator, and cover spigot. Presence of ovality will only increase this spread. So if very strict controls are exercised on the processes then 190 V jam and friction may be reduced. But such a control under the prevailing situation is unlikely. Hence a review of the quality system may be initiated.

Actual variations in parameters observed during past data analysis for model validation do not compare favorably with the design specifications. So process capability studies are needed and steps will have to be initiated to increase the capabilities. Also replacing some old processes/machines may be considered to improve capability, if feasible. Table 3 is provided only to illustrate the seriousness of the situation.

Though the contention of the operators about incorrect design specification is unfounded, maintaining proper air gap to prevent friction and 190 V jam requires highly improved and capable processes. Since contribution toward uneven air gap is due primarily to diametrical clearance between spigots of rotor and cover, (Table 2), the management might consider improving these two processes. A review of design specifications is necessary, as is evident from the simulation exercise. All the dimensions are simulated assuming capable processes meeting specifications with a probability of 0.9973. But it can be seen from Table 1 that the expected maximum diametrical clearance (0.48757 mm) is more than the upper specification limit

Table 2
Contribution to air gap variation

Sl. No.	Parameter	Contribution to air gap variation (%)
1	Cover spigot diameter (ID)	34.55
2	Rotor spigot diameter (OD)	21.75
3	Cover spigot eccentricity	13.78
4	Rotor inner diameter (ID)	12.24
5	Others	17.68

Table 3
Status of rotor and stator (selected parameters)

Parameter	Process standard deviation (mm)	Tolerance (mm)	Process capability
Rotor ID	0.0264	0.075	0.4735
Stator OD	0.0281	0.051	0.3025

Table 4
Sample data on rotor ID

Rotor ID (mm)							
127.5885,	127.5679,	127.5979,	127.5418,	127.5581,	127.5736,	127.5580,	127.5501,
127.5734,	127.5369,	127.5838,	127.5544,	127.5301,	127.5739,	127.5283,	127.5655,
127.5877,	127.5334,	127.5278,	127.5197,	127.4844,	127.5572,	127.5395,	127.5574,
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–

Table 5
Frequency distribution and goodness-of-fit of rotor ID

Class interval	Observed frequency (O_i)	Expected frequency (E_i)	$(O_i - E_i)^2/E_i$
127.48435–127.49879	2	4.7455 }	0.0013
127.49879–127.51323	13	10.3950 }	
127.51323–127.52767	25	22.3697	0.3093
127.52767–127.54211	34	35.9262	0.1033
127.54211–127.55655	45	43.0645	0.0870
127.55655–127.57099	37	38.5304	0.0608
127.57099–127.58543	23	25.7306	0.2898
127.58543–127.59987	14	12.8240	0.1078
127.59987–127.61431	5	4.7693 }	
127.61431–127.62875	2	1.6448 }	0.0535
Total	200	200	1.0128

of 0.4572 mm (0.018 in.). This may not lead to the problems of jam or friction but is sure to result in higher power consumption and lower rotational speed of fans.

The findings of the study have been discussed in a meeting with the top and senior management personnel and have been accepted by them. It was decided to shift the rotor and cover spigot machining operation to newer machines with better process capability to reduce variation. The accounts department and the quality control department have been instructed to undertake studies on cost-benefit analysis for acquiring new machines.

ABOUT THE AUTHOR

Tridib K. Dutta is a senior technical officer with the SQC and OR Division of the Indian Statistical Institute (ISI), India. Currently he is posted at Kolkata, India. He received his B.Stat. (Hons.), M.Stat., and Ph.D. degrees from the ISI. His current area of research includes statistical quality control, quality management, design of experiments, and visual cryptography. His present assignment includes teaching and industrial consultancy on behalf of ISI.

REFERENCES

Johnson, R. A. (1995). *Probability Densities. Miller & Freund's Probability and Statistics for Engineers*, 5th ed. New Delhi: Prentice-Hall of India Pvt. Ltd., pp. 145–150; 154–155.

Montgomery, D. C., Runger, G. C. (1994). Continuous Random Variables and Probability Distributions. *Applied Statistics and Probability for Engineers*. New York: John Wiley & Sons, Inc., pp. 170–172; 173–185; 444–449.

Morgan, B. J. T. (1984). General Methods for Non-uniform Random Variables. *Elements of Simulation*. London: Chapman and Hall, pp. 95–97.

APPENDIX A

Simulating Observations for Eccentricity

Let $f(x,k) = (x/k)\ \exp(-x^2/2k); 0 \leq x < \infty, k > 0$ be the p.d.f. and let the distribution function be $F(x)$. It is easy to show that

$$F(x) = \int_0^x f(t,k)dt = 1 - \exp\left(-\frac{x^2}{2k}\right)$$

Let $u \sim U$ (0,1). Then using inversion methods for nonuniform continuous random variables for simulating observations from the above distribution $f(x,k)$, the following may be written. (See Morgan, 1984, for the details of the method.)

$$u = 1 - \exp\left(-\frac{x^2}{2k}\right)$$

$$\text{or, } \log_e\left(\frac{1}{1-u}\right) = \frac{x^2}{2k}$$

But if $u \sim U(0, 1)$ then $(1 - u) \sim U(0, 1)$. So, it can be said that

$$x = \sqrt{2k \log_e\left(\frac{1}{u}\right)}$$

has the required distribution.

APPENDIX B

Fitting Normal Distribution to Rotor ID

Note

1. The variable of interest is of the measurement of Rotor ID (Table 4).
2. H_0: The form of the distribution is normal.
3. H_1: The form of the distribution is non-normal.
4. $\alpha = 0.05$.
5. Test statistic $(\chi_0{}^2)$ value $= 1.0128$ (Table 5).
6. Degrees of freedom $= 8 - 2 - 1 = 5$.
7. Tabulated value of $(\chi^2_{0.05,5}) = 11.07$.
8. Since $(\chi_0{}^2) < (\chi^2_{0.05,5})$, we are unable to reject H_0, so there is not enough evidence to believe that the distribution is non-normal. The P-value for the $\chi_0^2 = 1.1028$ is $P = 0.9615$.