BRIEF COMMUNICATIONS

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Cylindrical and spherical dust–ion acoustic shock waves

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The effect of the bounded nonplanar geometry on dust–ion acoustic shock wave (DIASW) in unmagnetized dusty plasmas is investigated for the first time. By using the standard reductive perturbation method, a cylindrical/spherical Korteweg–de Vries–Burgers (KdV–Burgers) equation is obtained. The change of the DIASW structure due to the effect of the geometry, dust density, and ion temperature is studied by numerical calculation of the cylindrical/spherical KdV–Burgers equation.

Propagation of ion acoustic shock waves in dusty plasmas has received considerable attention, and has been extensively studied both experimentally and theoretically. Luo *et al.*[1] observed the steeping of the leading edge of a ramp signal propagating in a Q-machine plasma with dust particles. The dust–ion acoustic shock wave (DIASW) was observed by Nakamura *et al.*[2,3] in a collisional dominated dusty plasma. The collision due to the dust–ion interaction produces a kinematic viscosity which is responsible for the formation of the DIASW. The results show that both monotonic and oscillatory shock structures exist and the dust density has significant effects on shock structures and phase velocity of the wave. The detailed theoretical models for the DIASW are given by Shukla[4] in a weakly correlated dusty plasma. The studying of the dust acoustic shock wave (DASW) in a strongly coupled dusty plasma is also presented by Shukla and Mamun.[5] The results show that the propagation of DIASW and DASW can be described by the Korteweg–de Vries–Burgers (KdV–Burgers) equation. The equation has a dissipation term in addition to the nonlinear and normal dispersion terms. When the wave breaking due to nonlinearity is balanced by the combined action of dispersion and dissipation, a monotonic or oscillatory dispersive shock wave is generated in plasma. Recently, several authors[6–11] show that, when ion viscosity or Landau damping effects are not important in dusty plasmas, the nonadiabatic dust charge variation provides an alternate physical mechanism causing dissipation, and as a consequence this gives rise to shocks for which both monotonic and oscillatory structures are possible. It is also seen that such shocks are described by the KdV Burgers equation. A critical review of shock wave phenomena in dusty plasmas can be found in several review papers (see, Shukla,[12] and the referenced papers) and books.[13,14] However, all of those studies for DIASW or DASW are limited to the unbounded planar geometry, which may not be a realistic situation in laboratory devices and space. Recent theoretical studies for dust–ion acoustic (DIA) and dust acoustic (DA) solitary waves in nonplanar geometry show[15–20] that the properties of solitary waves in bounded nonplanar cylindrical/spherical geometry are very different from that in unbounded planar geometry, and a stationary propagation of cylindrical/spherical soliton no longer exists. The waves in nonplanar geometry are described by the cylindrical/spherical KdV equations. But up to now, there is no investigation about the nonplanar geometry effect on the shock wave structures. Therefore, in this Brief Communication, the effect of the nonplanar geometry on DIASW is considered. By using the standard reductive perturbation method, a cylindrical/spherical KdV–Burgers equation is obtained. The effects of nonplanar geometry, dust density, and ion temperature on DIASW structures is studied by numerical calculation of the cylindrical/spherical KdV–Burgers equation.

To study the DIASW in the nonplanar cylindrical and spherical geometry, we assume that the DIASW propagate in an axial symmetry cylindrical geometry filled with the viscous unmagnetized collisionless dusty plasma whose constituents are warm inertial ions, Boltzmann distributed electrons, and negatively charged immobile dust particles. In equilibrium, the charge neutrality condition is $n_{e0}-n_{i0}+Z_d n_{d0}=0$, where n_{e0}, n_{i0}, and n_{d0} are the unperturbed electron, ion, and dust number densities, respectively, and Z_d is the number of electrons residing on the dust grains. The DIASW propagating in cylindrical and spherical geometry is governed by the following usual ion fluid equations:

$$\frac{\partial n_i}{\partial t}+\frac{1}{r^m}\frac{\partial(r^m n_i u_i)}{\partial r}=0,$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial r}=-\frac{\partial \phi}{\partial r}-3\sigma n_i\frac{\partial n_i}{\partial r}+\eta\left[\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial u_i}{\partial r}\right)-\frac{mu_i}{r^2}\right],$$

$$\delta\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial\phi}{\partial r}\right)=\exp(\phi)-\delta n_i+(\delta-1), \tag{1}$$

where $m=0$ for a one-dimensional geometry and $m=1$ (2) for a nonplanar cylindrical (spherical) geometry. The variables of time (t), space (r), ion number density (n_i), ion fluid velocity (u_i), and electrostatic wave potential (ϕ) are normalized to the reciprocal ion plasma frequency $\omega_{pi}^{-1}(\omega_{pi}=\sqrt{4\pi n_{i0}e^2/m_i})$, Debye radius λ_D $(=\sqrt{k_BT_e/4\pi n_{i0}e^2})$, unperturbed equilibrium plasma density n_{i0}, effective ion acoustic velocity $C_i=(k_BT_e/m_i)^{1/2}$, and k_BT_e/e, respectively, and $\eta=\mu/\omega_{pi}\lambda_D^2$, in which μ is the ion kinematic viscosity.[2–4] We have set $\sigma=T_i/T_e,\delta=n_{i0}/n_{e0}$, where T_i and T_e are ion and electron temperature, respectively.

In order to investigate the DIASW in the plasma, we employ the standard reductive perturbation technique to obtain the modified KdV equation. Here, we are interested in the incoming waves in nonplanar geometry, so, the independent variables are stretched as[15,16] $\xi=-\varepsilon^{1/2}(r+v_0t)$ and $\tau=\varepsilon^{3/2}t$, where ε is a small parameter and v_0 is the wave phase velocity. The dependent variables are expanded as

$$n_i=1+\varepsilon n_1+\varepsilon^2n_2+\cdots,$$

$$u=\varepsilon u_1+\varepsilon^2u_2+\cdots,$$

$$\phi=\varepsilon\phi_1+\varepsilon^2\phi_2+\cdots. \tag{2}$$

In many experimental situations the value of η is small, so we set

$$\eta=\varepsilon^{1/2}\eta_0, \tag{3}$$

where η_0 is a finite quantity of the order of unity. If η is not small we can still use the same substitution but now η_0 should be large. This does not present any hurdle to the subsequent theoretical analysis. Substituting Eqs. (2) and (3) into Eqs. (1) and collecting the terms in the different powers of ε, to lowest order in ε, we obtain $n_1=(1/\delta)\phi_1$, $u_1=-(v_0/\delta)\phi_1$, and $v_0^2=3\sigma+\delta$. For the next higher order, we can obtain

$$\frac{\partial n_1}{\partial\tau}-v_0\frac{\partial n_2}{\partial\xi}-\frac{\partial u_2}{\partial\xi}-\frac{\partial(n_1u_1)}{\partial\xi}-\frac{m}{v_0\tau}u_1=0,$$

$$\frac{\partial u_1}{\partial\tau}-v_0\frac{\partial u_2}{\partial\xi}-u_1\frac{\partial u_1}{\partial\xi}-\frac{\partial\phi_2}{\partial\xi}-3\sigma\left(\frac{\partial n_2}{\partial\xi}+n_1\frac{\partial n_1}{\partial\xi}\right)-\eta_0\frac{\partial^2u_1}{\partial\xi^2}=0,$$

$$\delta\frac{\partial^2\phi_1}{\partial\xi^2}=\frac{1}{2}\phi_1^2+\phi_2-\delta n_2. \tag{4}$$

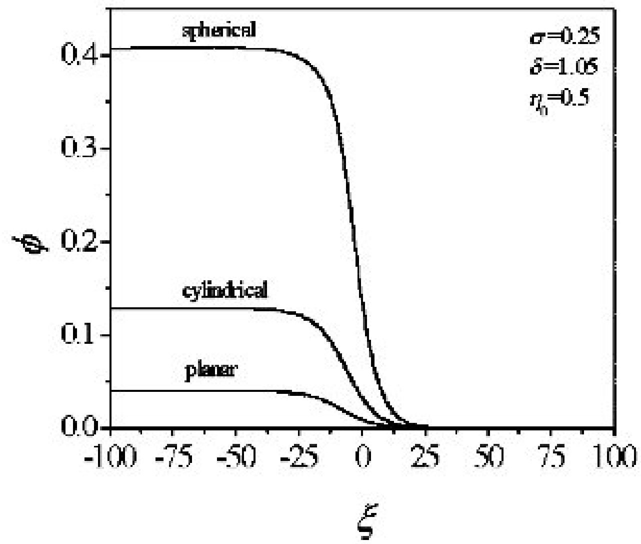

FIG. 1. Shock wave structures in different geometry at $\tau=-10$.

Now, using the derived expression of n_1, u_1 and eliminating n_2, u_2, and ϕ_2 from Eq. (4), one obtains the cylindrical and spherical KdV–Burgers equation

$$\frac{\partial\phi}{\partial\tau}+A\phi\frac{\partial\phi}{\partial\xi}+B\frac{\partial^3\phi}{\partial\xi^3}-C\frac{\partial^2\phi}{\partial\xi^2}+\frac{m}{2\tau}\phi=0 \tag{5}$$

where $\phi\equiv\phi_1$, $A=(12\sigma+3\delta-\delta^2)/2v_0\delta$, $B=\delta^2/2v_0$, and $C=\eta_0/2$. The fourth and fifth term on the left-hand side of Eq. (5) represents the dissipation and geometry effects, respectively. It is clear from Eq. (5) that the nonplanar geometrical effect is significant when $\tau\to0$ and weaker for larger value of $|\tau|$.

For one-dimensional geometry $(m=0)$, a stationary propagation of the DIASW governed by Eq. (5) has the following form:

$$\phi=\frac{3C^2}{25AB}\left[1+\tanh\frac{C}{10B}\left(\xi-\frac{V}{A}\tau-\xi_0\right)\right]^2, \tag{6}$$

where ξ_0 is a real constant and represents the initial wave position. When the geometrical effect is taken into account $(m\neq0)$, an exact analytical solution of Eq. (5) is not possible. Therefore, we have integrated Eq. (5) numerically by finite difference method and have studied the geometrical effects on the propagation of DIASW. The results are displayed in Figs. 1–6. The initial condition that we have used in all our numerical results is the form of the stationary solution of Eq. (6) without the geometry term at $\tau_0=-100$ (at this stage the geometry effect is weaker, so we can take this stage as the initial stage of evolution). Figure 1 shows the shock wave structure evolved at $\tau=-10$ in different geometry. It is clear that the developed shock height and shock steepness in different geometry are different from each other. The spherical shock wave, with higher height and larger steepness, is the strongest one. The height and steepness of cylindrical shock wave are larger than that of the one-dimensional shock wave but smaller than that of the spherical shock wave. The variation of shock height against time is plotted in Fig. 2. We can see that, as time goes on, the increasing rate of the shock wave height in spherical geometry

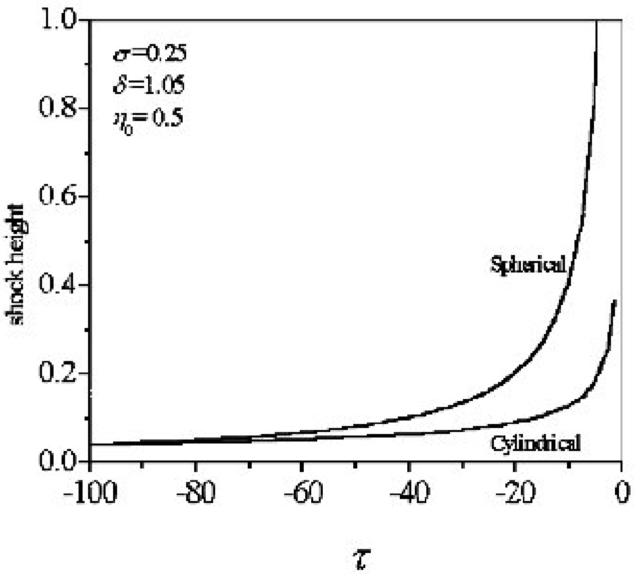

FIG. 2. Variation of shock height against time τ.

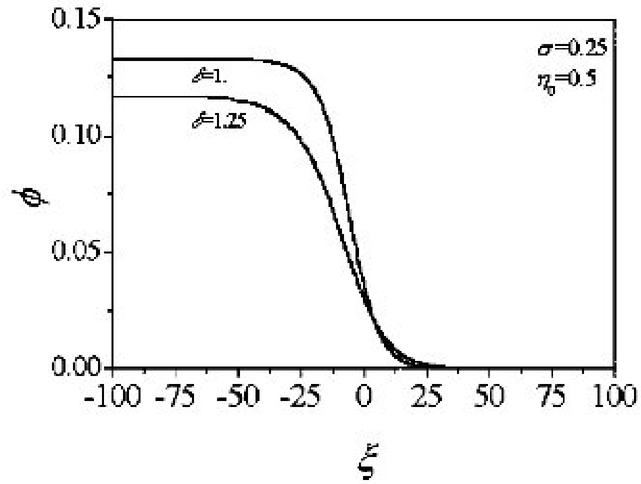

FIG. 4. Cylindrical shock structures for different δ at $\tau=-10$.

is larger than that in cylindrical geometry. We also can see from Fig. 2 that the cylindrical and spherical shock waves are similar to one-dimensional shock wave for large value of $|\tau|$. This is because for large value of $|\tau|$ the nonplanar geometrical effect is no longer dominant. However, as the value of $|\tau|$ decreases, the nonplanar geometrical effect, represented by $(m/2\tau)\phi$, will become dominant and the cylindrical, spherical, and one-dimensional shock waves differ from each other.

The effects of dust density and ion temperature on shock wave are also studied. The dust density can be expressed by the variation of δ because of $\delta-1=Z_d n_{d0}/n_{e0}$. In Fig. 3 the cylindrical shock height is plotted against τ for different values of δ. This figure shows that the shock height decreases with δ. That is, the shock height decreases with the dust density and the ion–acoustic shock wave ($\delta=1$) has the largest height. Figure 4 shows that the steepness of the shock wave is also modified by dust density and it decreases with dust density. The time variation of the shock height for different ion temperature σ is displayed in Fig. 5. It is clear that the shock height decreases with ion temperature and the cold ion–acoustic shock wave ($\sigma=0$) has the largest height. But we can see from Fig. 6 that the ion temperature has a weak effect on the shock steepness.

In conclusion, we have investigated the nonplanar cylindrical and spherical DIASW which is governed by the modified KdV–Burgers equation. The nonplanar geometry effect for DIASW is very strong for a small value of $|\tau|$ and there are obvious differences between the cylindrical DIASW, spherical DIASW, and one-dimensional DIASW. The height of DIASW decreases with increasing dust density and ion temperature. Physically, the phase velocity of the wave, the nonlinearity and the wave amplitude are modified by the ion temperature and the constituents of the dust grains, ions, and electrons. Because the dusty plasma always supports low-frequency DIA waves with phase speed much smaller (larger) than electron (ion) thermal speed,[13,21] the normalized dispersion relation of the linear DIA waves (without dissipation effect) is[13,21] $\omega/k=[3\sigma+\delta/(1+\delta k^2)]^{1/2}$. It is clear that the phase velocity of DIA wave increases with dust density (δ) and ion temperature (σ). When the kinematic viscosity (η) is considered, the dispersion relation will be modified,

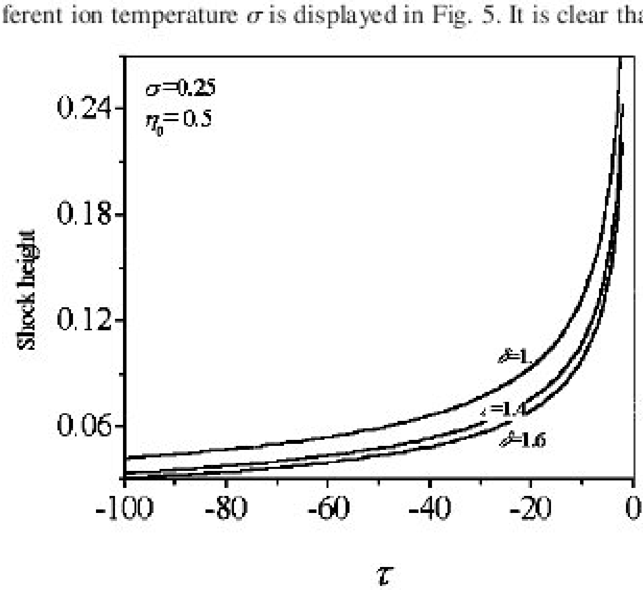

FIG. 3. Variation of cylindrical shock height against time τ for different δ.

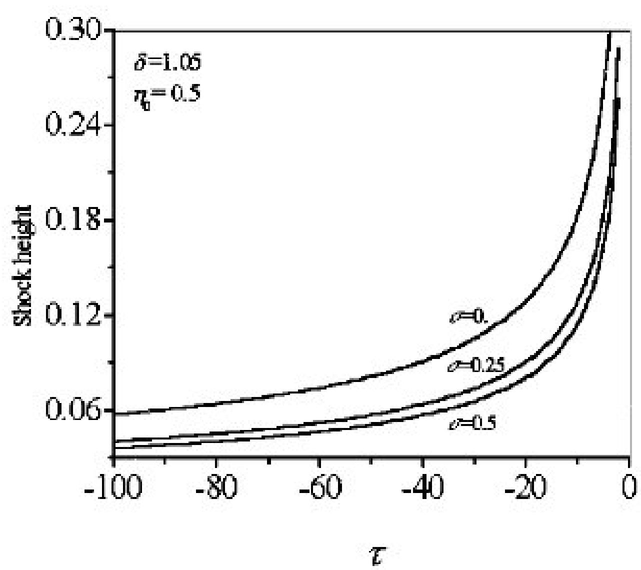

FIG. 5. Variation of cylindrical shock height against time τ for different σ.

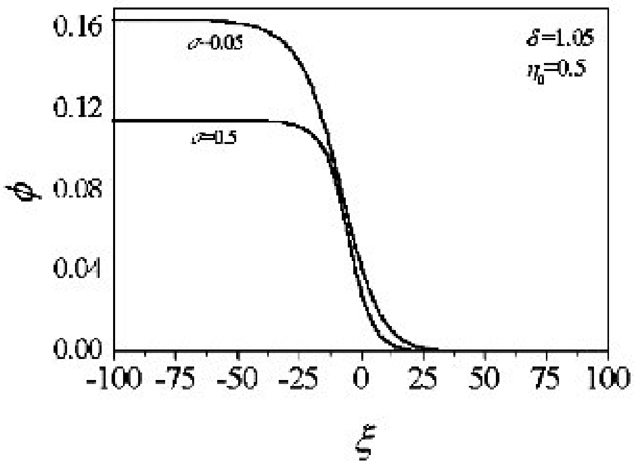

FIG. 6. Cylindrical shock structures for different σ at $\tau=-10$.

i.e., ω^2 should be replaced by $\omega(\omega+i\eta k^2)$, and the dissipation effect is introduced into the system. Hence, a shock wave can be formed when the nonlinearity is balanced by dispersion and dissipation. The dissipation caused by dust–ion collision in a collisional dusty plasma increase with dust density.[2,3] For extremely small dissipation, the shock wave will have an oscillatory profile due to higher dispersion,[2,3] although the oscillatory shock waves do not occur in our parameter range. If the dissipation is increased and it is larger than a certain critical value,[5] the shock wave will have a monotonic behavior. But further increase of the dissipation will make the shock wave more smooth and weak. So we can conclude that the ion temperature and the dust density will modify the properties of DIASW. These are also confirmed by the experiments.[2,3] The purpose of studying the dissipative nonplanar bounded plasma is to gain understanding on the propagation characteristics of the dust ion–acoustic shock waves that are of vital importance in laboratory plasmas as well as in plasma application.

ACKNOWLEDGMENT

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