Velocity and concentration profiles in uniform sediment-laden flow

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Abstract

This paper presents a theoretical model for computing the velocity and sediment concentration profiles in a uniform sediment-laden flow carrying all fine, medium and coarse sediments. The proposed model essentially includes the effect of sediment concentration in total turbulent shear stress and eddy diffusivity in addition to the modified mixing length derived by Umeyama and Gerritsen [J. Hydr. Engng., ASCE, 118 (2) (1992) 229–245] applied to Hunt's diffusion equation. Numerical solution of coupled differential equations for velocity and sediment concentration is carried out. The theoretical results show quite good agreement with the experimental data.

Keywords: Velocity and sediment concentration profiles; Turbulent shear stress; Eddy diffusivity; Mixing length; Hindered settling

1. Introduction

Numerous investigations related to sediment-laden turbulent flows in open channels have been undertaken to examine the vertical distributions of flow velocity and suspended sediment concentration. The problems of sediment-laden flows are of direct concern to river engineers and

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Nomenclature

A	relative density
c_i	computed velocity at ith height in a run
C	volumetric concentration of suspended sediment
C_{ξ_a}	sediment concentration at $\xi = \xi_a$
D_{50}	mean grain size
d	water depth
g	acceleration due to gravity
l	mixing length
n	exponent of reduction of fall velocity
o_i	observed velocity at ith height in a run
R_g	grains Reynolds number
s_β^2	a measure of variability of estimated β's
u	horizontal time-average velocity
u_*	shear velocity
W	particle settling velocity in sediment-laden flow
w_0	particle settling velocity in clear water
y	vertical distance from bed
z_*	Rouse number
α	exponent in mixing length
β	coefficient for mixing length
β^*	estimated initial value of β
ϵ_s	sediment diffusion coefficient
ϵ_m	momentum diffusion coefficient
γ	ratio of sediment diffusion to momentum diffusion coefficients
κ	von-Karman constant
ρ	density of mixed fluid
ρ_f	density of clear water
ρ_s	density of sediment
τ_t	total turbulent shear stress
ξ	dimensionless depth
ξ_a	reference level at $\xi = \xi_a$

geomorphologists, but also relevant to the field such as coastal sediment transport and transport of solids in pipelines. The study of vertical concentration distribution is related to the logarithmic velocity distribution based on the Prandtl mixing-length hypothesis [1] and on von-Karman similarity hypothesis [1]. In the hydraulic open-channel flow over a smooth bed surface without sediments, the vertical velocity profile is usually described by a 'log-law'. The proper measurements in the whole turbulent boundary layer have indicated that the 'log-law' can be applied strictly only to the near wall region with the von-Karman constant $\kappa = 0.4$ and the integrating constant to be 5.29; and a semi-empirical equation based on wake-strength function developed by Coles [2], which modifies the log-law, is applied to the whole region. Several researchers [2–4] suggested that the measured velocity cannot be fitted only by log-law, the deviation from the log-law have to be accounted by taking a suitable function known as 'wake function'.

In classic papers, Vanoni [5] and Einstein and Chien [6] (notation EC will be used to denote Einstein and Chien in the paper) indicated that the logarithmic velocity distribution with a smaller value of κ than the universal one ($\kappa = 0.4$), agreed with their experimental data of velocity and suspension distributions throughout the flow depth, except near the bottom. Their results led to a view that an increase in suspension concentration reduces the von-Karman coefficient κ. Itakura and Kishi [7] and Coleman [8] proposed models for velocity distribution to study the vertical sediment concentration in which the value of κ remains unchanged. Itakura and Kishi [7] deduced a log-linear velocity equation based on the hypothesis of Monin–Oboukhov length. They argued that, instead of usual log-law, the log-linear law should give a better description of the velocity defect law based on the characteristic of Monin–Oboukhov theory of weakly stably stratified flows. Coleman [8] derived the concentration equation using the velocity distribution for sediment-laden flow based on the log-law and the law of wake for clear water flow. He also analyzed the data obtained by Vanoni [5] and EC [6] and revealed that von-Karman coefficient κ is independent of sediment concentration. If the density of the fluid is increased by suspended sediments, buoyancy force is increased, and hence the substantial reduction of particle fall velocity (known as hindered settling) in suspension occurs. According to Maude and Whitemore [9], the term 'hindered settling' is used to designate the fall velocity of sediment in suspension resulting from an increase in sediment concentration. Thacker and Lavelle [10] have shown that the reduction of fall velocity may be taken into account by a factor $(1 - C)^n$ multiplied by the single fall velocity w_0, where C is the concentration per unit volume and n is the exponent of reduction of fall velocity. Subsequently, Lavelle and Thacker [11] studied the effects of hindered settling on sediment concentration profiles. Woo et al. [12] considered the effect of hindered settling of sediment to estimate the vertical distribution of large concentration of sand particles, using the log law and power law velocity profiles. The derivation of Woo et al.'s model includes the effect of viscous shear stress which is a function of concentration in suspension.

In turbulent boundary layer flow Prandtl [1] suggested a concept of mixing length which represents a transverse distance of fluid elements exchanging momentum and it is related to the velocity fluctuations. He assumed that the mixing length is proportional to the distance y from the bed as

$$l = \kappa y, \tag{1}$$

where κ is the von-Karman coefficient. In sediment-laden flow, the mixing length concept was examined by Umeyama and Gerritsen [13] (notation UG will be used to denote Umeyama and Gerritsen in the paper). They proposed an excellent theoretical model for velocity distribution in sediment-laden flow based on new mixing length hypothesis, that depends on sediment concentration. They verified that the mixing length in highly concentrated flow is smaller than that for lower concentrated flow or clear water flow. Umeyama [14] studied the vertical distribution of fine suspended sediment in uniform open-channel flow using the new mixing length hypothesis developed by UG [13], assuming the Rouse diffusion equation in a steady state condition as

$$\epsilon_s(y)\frac{dC}{dy} + w_0 C(y) = 0, \tag{2}$$

where w_0 is the fall velocity of sediment in clear water and $\epsilon_s(y)$ is the sediment diffusion coefficient at y. In fact, at large concentration in suspension, it is evident that the Rouse equation (2) does not properly hold [12,15]. Moreover, Umeyama considered the momentum diffusion coefficient $\epsilon_m(y)$ according to Boussinesq's concept for clear water flow as

$$\tau = \rho_f \epsilon_m \frac{du}{dy}, \tag{3}$$

where ρ_f is the density of clear water, τ the turbulent shear stress and u the mean velocity. Later, Umeyama [16] extended his model in a flow carrying medium and coarse sands using hindered settling effect due to the increased concentration and compared his results with the experimental data of EC [6]. However the above mentioned models are not concerned with the effects of suspended sediment concentration on total shear stress and eddy diffusivity even at large sediment concentrated flows despite the fact that such effects have greater importance in sediment laden flow to develop the models.

The purpose of this work is to develop simple realistic models for velocity from Prandtl's momentum transfer theory and suspension concentration taking into account:

(1) the diffusion equation satisfies the continuity of sediment and water,
(2) the turbulent shear stress due to Boussinesq's formula and eddy diffusivity are modified in the sediment-laden flow, and
(3) the fall velocity of sand particle decreases with increase in suspension concentration.

Using the modified mixing length as a function of concentration, it is necessary to develop a more realistic model for large concentrated flows, in which the velocity and sediment suspension profiles are sensitive for the above effects. The efficiency of the present model has been tested by comparing the computed velocity and sediment concentration with the observed historical data of Vanoni [5], EC [6], Coleman [8] and Lyn [17]. Comparison shows quite a good agreement.

2. Theoretical models

In Prandtl's momentum transfer theory, replacing the density ρ_f of clear-water flow with the density ρ of fluid-sediment mixture and letting density ρ be a function of concentration, the turbulent shear stress as modified by EC [6], is given by

$$\tau = \rho_f (1 + AC) l^2 \left(\frac{du}{dy}\right) \left|\frac{du}{dy}\right|, \tag{4}$$

where l is the mixing length of the eddy representing the transverse distance of fluid elements exchanging momentum, u the mean velocity in the direction of flow, $A(= \frac{\rho_s}{\rho_f} - 1)$ the relative density which may be treated as a correction factor, ρ_s the density of sediments, and C the volume concentration of suspended sediments. In clear water ($C = 0$), this equation reduces to Prandtl's momentum transfer formula. The presence of concentration will inevitably change the flow characteristics and hence, for sediment-laden flow total shear stress τ_t depends on the unit weight of the suspension (ρg) and energy gradient of the flow; and can be written as

$$\tau_t = \rho_f u_*^2 \left(1 - \xi + A \int_\xi^1 C \mathrm{d}\xi'\right), \tag{5}$$

where $\xi = y/d$, the dimensionless vertical distance, and u_* is the friction velocity. This equation is in general valid for both low and high suspended sediment concentration, because for low concentration the term $A \int_\xi^1 C \mathrm{d}\xi'$ is negligible. For sediment-laden flow, UG [13] suggested a modified mixing length hypothesis as a function of concentration which was assumed as

$$l = \kappa y (1 - \xi)^\alpha \tag{6}$$

with $\alpha = 1/2(1 + \beta C/C_{\xi_a})$, where C_{ξ_a} is the reference concentration at the level $\xi = \xi_a$ and β is a constant to be determined from the experimental data at sediment-laden flow. Combining (4)–(6), the velocity gradient $\frac{\mathrm{d}u}{\mathrm{d}\xi}$ may be extended as

$$\frac{\mathrm{d}u}{\mathrm{d}\xi} = \frac{u_*(1 - \xi + A \int_\xi^1 C \mathrm{d}\xi')^{1/2}}{\kappa \xi (1 + AC)^{1/2} (1 - \xi)^{1/2(1+\beta C/C_{\xi_a})}}. \tag{7}$$

Eq. (7) differs significantly from that of UG [13] by the term $A \int_\xi^1 C \mathrm{d}\xi'$ in the numerator because they ignored the effect of concentration on the turbulent shear stress component. The velocity distribution can be calculated numerically from (7) except at the free surface $\xi = 1$ if a suitable function of C is chosen as a first approximation.

In a steady and uniform two-dimensional turbulent flow when the concentration C varies only with vertical co-ordinate y throughout the depth, the vertical concentration of sediments with representative settling velocity W at highly concentrated sediment-laden flow can be described by Hunt [18] as

$$\epsilon_s \frac{\mathrm{d}C}{\mathrm{d}y} + (1 - C)CW = 0, \tag{8}$$

where ϵ_s is the sediment diffusion coefficient. Eq. (8) satisfies the continuity condition of sediment and water, that is derived for volumetric concentration ranging from 3.8% to 20%. According to Richardson and Zaki [19] and Thacker and Lavelle [10], the representative settling velocity W of fluid-particle mixture varies with the volumetric concentration C as

$$W = w_0 (1 - C)^n, \tag{9}$$

where w_0 is the particle settling velocity in clear water and n is the exponent of reduction of fall velocity, which varies on particle Reynolds number R_g as: $n = 4.35 R_g^{-0.03}$ for $0.2 \leqslant R_g \leqslant 1.0$; $n = 4.45 R_g^{-0.01}$ for $1.0 \leqslant R_g \leqslant 500$; $n = 2.39$ for $500 \leqslant R_g$. Experiments on fluid sediment mixtures have shown that a substantial reduction of settling velocity of particle occurs due to the increased suspension concentration [12,20]. The sediment diffusion coefficient ϵ_s is obtained from momentum diffusion coefficient ϵ_m according to Reynolds analogy ($\epsilon_s = \gamma \epsilon_m$) in fluid-sediment mixtures and is given by EC [6] as

$$\epsilon_s = \frac{d \gamma u_*^2 (1 - \xi + A \int_\xi^1 C \mathrm{d}\xi')}{(1 + AC) \frac{\mathrm{d}u}{\mathrm{d}\xi}}, \tag{10}$$

where $\frac{du}{d\xi}$ is given by (7). In clear water flow ($C = 0$), this equation reduces to Boussinesq's formula, which was used by Umeyama. It is worthwhile to consider ϵ_s as a function of volumetric concentration C. Using (9) and (10) in (8), the expression for concentration gradient is given by

$$\frac{dC}{d\xi} = -\frac{w_0 C(1-C)^{n+1}(1+AC)\frac{du}{d\xi}}{\gamma u_*^2(1-\xi+A\int_\xi^1 C\,d\xi')}. \tag{11}$$

This equation is of general applicability in determining the concentration profiles for fine, medium and coarse sands; and it represents an integro-differential equation [12]. Due to non-linear term and the integral sign in the denominator of (11), a numerical approach (Runge–Kutta method) is adopted to solve the equation. Differentiating (11) with respect to ξ in order to remove the integral sign, one gets the following second order non-linear differential equation as

$$\frac{d^2C}{d\xi^2} = H(\xi)\frac{dC}{d\xi} - \left[\frac{\gamma u_*^2(1+AC)}{w_0 f(C)G(\xi)} - \frac{f'(C)}{f(C)}\right]\left(\frac{dC}{d\xi}\right)^2, \tag{12}$$

where

$$G(\xi) = \frac{du}{d\xi}, \quad H(\xi) = \frac{d^2u}{d\xi^2}\bigg/\frac{du}{d\xi}, \quad f(C) = C(1-C)^{n+1}(1+AC)$$

with the boundary conditions at the reference level $\xi = \xi_a$ near the bed as

$$C = C_{\xi_a} \quad \text{at } \xi = \xi_a,$$

$$\left.\frac{dC}{d\xi}\right|_{\xi=\xi_a} = -\frac{w_0 f(C_{\xi_a})G(\xi_a)}{\gamma u_*^2(1-\xi_a+A\overline{C})}, \tag{13}$$

where $\overline{C} = \int_{\xi_a}^1 C\,d\xi'$, C_{ξ_a} is the reference concentration at $\xi = \xi_a$, and the function $f'(C)$ represents derivative of the function $f(C)$ with respect to C. Eqs. (7) and (12) together with (13) are coupled and are solved by an iterative procedure. In order to compute the velocity profile from (7), we use the solution of (2) as a first approximation of concentration C. With $C = C_{\xi_a}$ as the reference concentration at $\xi = \xi_a$, solution of the Rouse equation (2) is

$$C(\xi) = C_{\xi_a}\left(\frac{1-\xi}{\xi}.\frac{\xi_a}{1-\xi_a}\right)^{z_*}, \tag{14}$$

where $z_* = \frac{w_0}{\kappa u_*}$ is the Rouse number. Using the computed velocity gradient from (7) in (12) together with (13), the new estimates of C at any height $\xi < 1$ have been obtained and are plotted with existing experimental data.

3. Comparison with experimental data

The efficiencies of theoretical models have been tested by comparing the computed velocity and suspended sediment concentration with the most comprehensive observed data of EC [6], Vanoni [5], Coleman [8] and Lyn [17]. The sizes of the mean sediments used in the experiments of EC [6] were the fine sands ($D_{50} = 0.274$ mm) for runs S-11 to S-16, medium sands ($D_{50} = 0.94$ mm) for runs S-6 to S-10 and coarse sands ($D_{50} = 1.30$ mm) for runs S-1 to S-5. The sediment concentration near the bed was in the range 30–600 g/l. The comparison of computed and observed velocity and suspended sediment concentration profiles of all the runs S-1 to S-16 of EC [6] is depicted in Fig. 1(a) and (b) with the reference concentration C_{ξ_a} at the lowest available dimensionless height ξ_a near the bed. Using the measured velocity and concentration at the level ξ_a as initial values, (7) is solved by Runge–Kutta method taking solution (14) of Rouse equation (2) for concentration C as a first approximation. In order to improve the accuracy of predicted velocity distribution with the observed data set of EC [6], a two-layer velocity model has been chosen below and above the reference level ξ_a (Fig. 1(a)). For each data set, numerical iteration is carried out from the

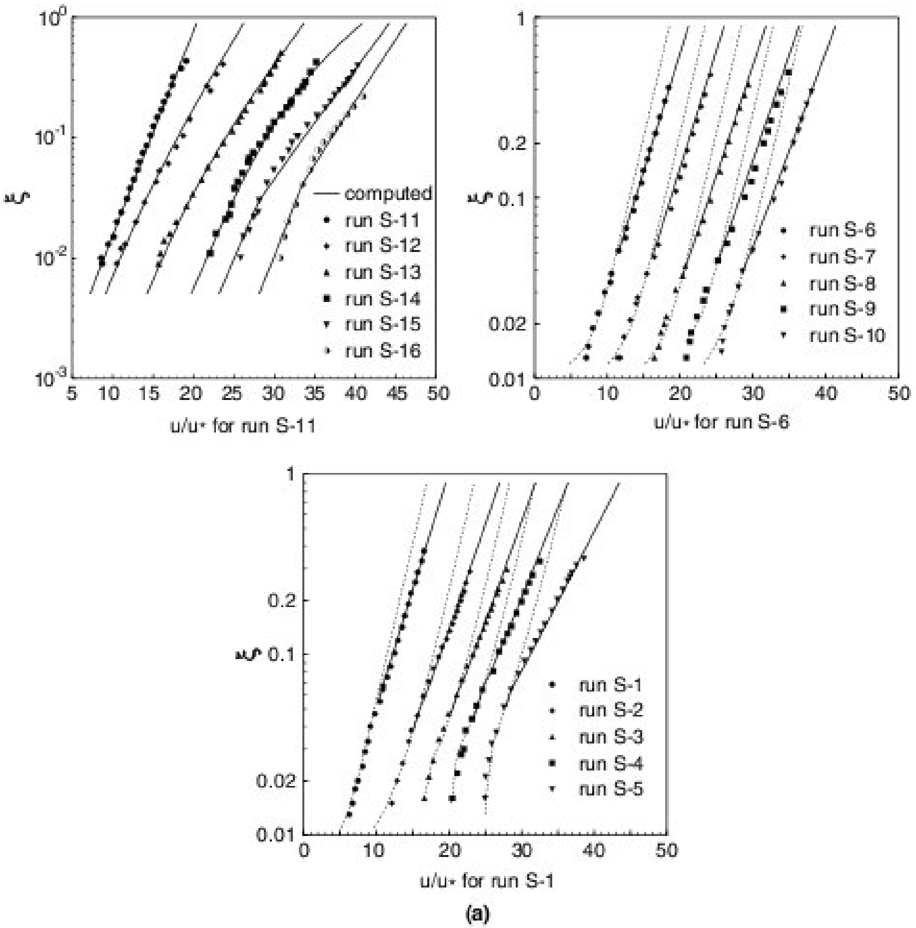
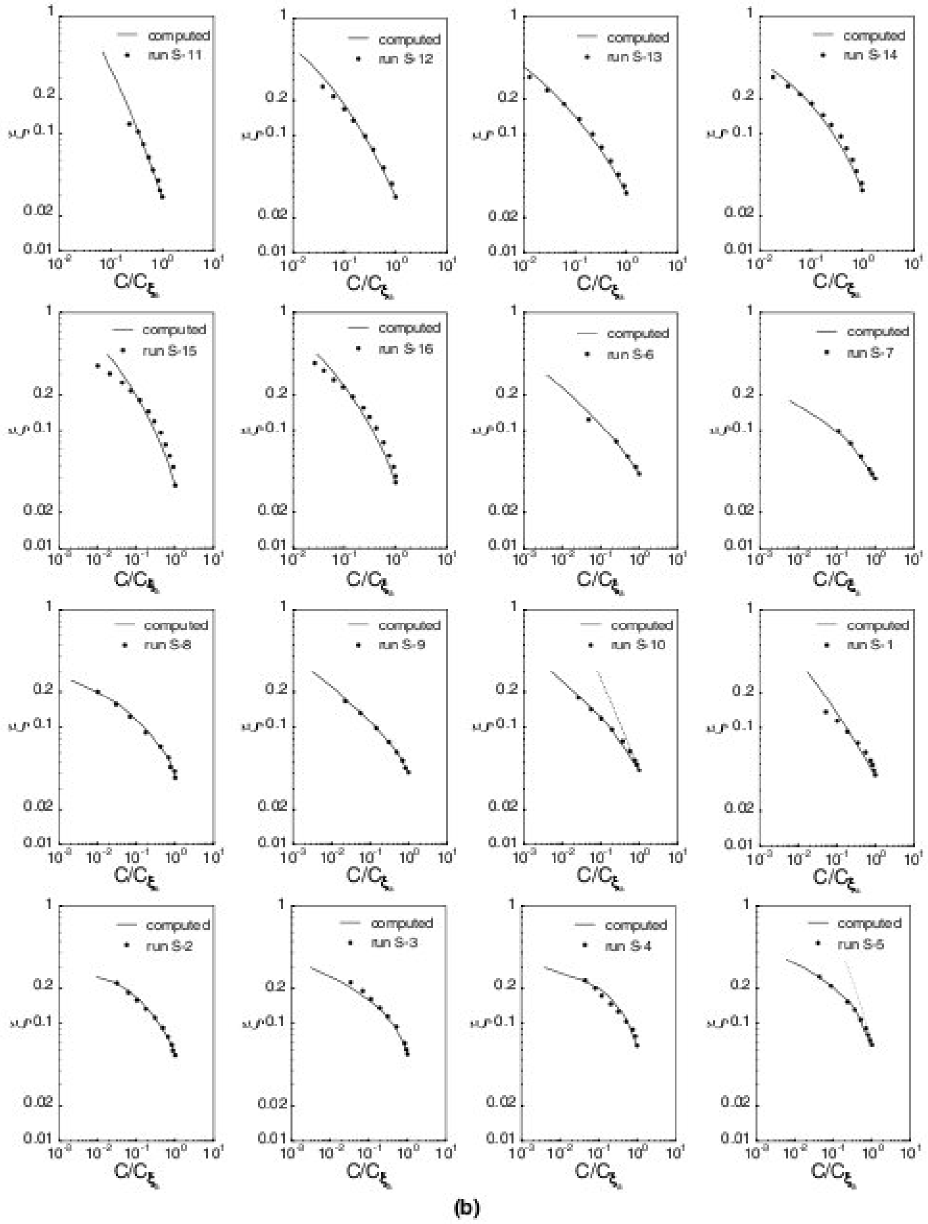


Fig. 1. (a) Calculated and measured velocity profiles for the runs S-11 to S-16 for fine sands ($D_{50} = 0.274$ mm), S-6 to S-10 for medium sands ($D_{50} = 0.94$ mm), and S-1 to S-5 for coarse sands ($D_{50} = 1.3$ mm) of Einstein and Chien [6]. Here x-axis for other runs is shifted by 5 from the preceding one. The dotted line stands for $\kappa = 0.4$ and the continuous for fitted κ. (b) Calculated and measured sediment concentration profiles for the runs S-11 to S-16 for fine sands ($D_{50} = 0.274$ mm), S-6 to S-10 for medium sands ($D_{50} = 0.94$ mm), and S-1 to S-5 for coarse sands ($D_{50} = 1.3$ mm) of Einstein and Chien [6].



(b)

Fig. 1 (*continued*)

reference level ξ_a. For example, in the lower region, the value of κ remains equal to 0.4 and the parameter β has been adjusted to best fit with all the observed velocity data; whereas for the upper region κ has been adjusted to best fit of velocity data, keeping the same value of β (Table 1). Estimated value of β lies in the range 1–18 for fine grained sediments (S-11 to S-16) and in the range 1–11 for both coarse and medium sediments (S-1 to S-10); whereas Umeyama found the β value in

Table 1
Parameters used for computation of velocity and sediment concentration in suspension

Data	Sand	D_{50} (mm)	Run no.	ξ_a (observed)	C_{ξ_a} (observed)	β (estimated)	κ_{upper} (estimated)	γ (estimated)
Einstein and Chien [6]	Fine	0.274	S-11	0.029	0.01185	1	0.40	0.74
			S-12	0.030	0.07721	11	0.40	0.63
			S-13	0.032	0.13283	18	0.40	0.55
			S-14	0.033	0.14566	18	0.40	0.51
			S-15	0.034	0.22679	13	0.40	0.60
			S-16	0.036	0.23321	4	0.40	0.54
	Medium	0.940	S-6	0.043	0.01057	5	0.30	1.99
			S-7	0.039	0.03313	6	0.30	1.90
			S-8	0.037	0.03147	6	0.28	1.70
			S-9	0.041	0.05736	7	0.27	1.60
			S-10	0.043	0.08151	11	0.25	1.70
	Coarse	1.300	S-1	0.040	0.02189	1	0.30	2.00
			S-2	0.054	0.04566	8	0.27	1.90
			S-3	0.055	0.05679	9	0.26	1.85
			S-4	0.065	0.07321	10	0.23	2.50
			S-5	0.066	0.12377	10	0.21	2.60
Vanoni [5]	Fine	0.100	20	0.067	0.00145	1.5	0.40	1.20
		0.100	21	0.077	0.00106	4	0.40	1.70
		0.133	22	0.063	0.00218	5	0.40	1.25
Coleman [8]	Fine	0.105	2	0.035	0.00085	3	0.40	1.50
			5	0.035	0.00400	7	0.40	1.20
			9	0.035	0.00900	11	0.40	1.25
Lyn [17]	Fine	0.150	1565EQ	0.093	0.002145	7	0.40	1.00
	Fine	0.190	1965EQ	0.072	0.002347	15	0.40	1.20

the range 1–40 for fine sands and 28–160 for coarse and medium grains. The larger values of β obtained by Umeyama may be due to the ignorance of concentration effects on total shear stress in the sediment-laden flow.

From the set of estimated β values (Table 1), one β^* can be determined statistically which may be used as a starting value to compute the velocity profiles for any measured data set. If s_β^2 is a measure of variability of the estimate of β values, then s_β^2 can be estimated as

$$s_\beta^2 = \frac{\sum_{i=1}^{N}(c_i - o_i)^2}{N-2}, \tag{15}$$

where c_i and o_i are the computed and observed velocities, respectively in the ith height of a run and N is the total number of data points in that run. Let $w = \frac{1}{s_\beta^2}$. Then β^* can be estimated as

$$\beta^* = \frac{\sum_{j=1}^{M}\beta_j w_j}{\sum_{j=1}^{M} w_j} \tag{16}$$

summation being taken over all the runs under consideration. Here, the value of β^* has been obtained as 9.1 for fine sediments (runs S-11 to S-16), and β^* has been estimated as 7.6 for both medium and coarse sediments (runs S-1 to S-10). It may be noted that β^* is simply a weighted average of the individual β's, where the weights attached are inversely proportional to their inherent variability. In a similar way, κ^*_{upper} has also been estimated and it is found to be 0.27 for medium and coarse sediments.

Using the same values of the parameters κ_{upper} and β of fitted velocity profiles in the upper region, the value of γ is obtained for the best fit of desired sediment concentration profiles with reference concentration C_{ξ_a} (Fig. 1(b)). For all data set, whole region is computed with the values of n corresponding to the values of R_g. For fine grained sediments, the fitted value of γ was found to be in the range 0.5–0.75 and for coarse and medium sands the range of γ was 1.6–2.6. The values of κ_{upper} and β for velocity profiles and the values of γ for concentration profiles are depicted in Table 1. It is observed that the values of κ decreases with increase of sediment concentration

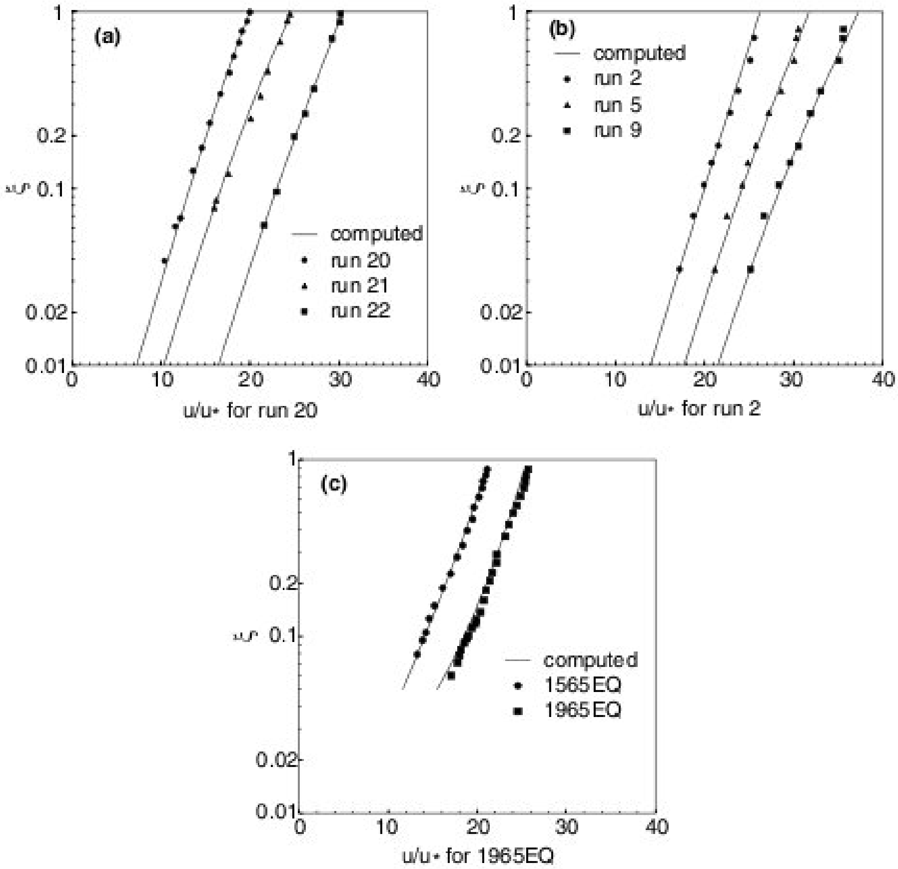


Fig. 2. Calculated and measured velocity profiles of (a) Vanoni [5] for the runs 20, 21 ($D_{50} = 0.1$ mm) and run 22 ($D_{50} = 0.133$ mm) (b) Coleman [8] for the runs 2, 5 and 9 ($D_{50} = 0.105$ mm) (c) Lyn [17] for the exp. 1565EQ and 1965EQ ($D_{50} = 0.15$ mm and 0.19 mm, respectively).

which was also obtained by Vanoni [5] and EC [6]. Furthermore, the value of γ increases consistently with increase in grain sizes—fine to coarse grained sediments [21]. The starting value of γ which may be called as γ^*, may also be approximated by the statistical method described in (15) and (16) or from the functional relationships suggested by Rijn [22] for medium and coarse grains and by Graf and Cellino [23] for fine grains.

It may be noted that for fine sands (runs S-11 to S-16) the value of κ is 0.4 throughout the region; whereas for medium and coarse sands (runs S-1 to S-10), a two-layer model has been chosen.

The velocity and sediment data of Vanoni [5], Coleman [8] and Lyn [17] are also verified with the theoretical models in a similar way. The reference level ξ_a in the case of Vanoni is chosen at the lowest available height of observed velocity data set and the same for the case of Coleman is chosen at the lowest available height of observed velocity or concentration data set (as the lowest

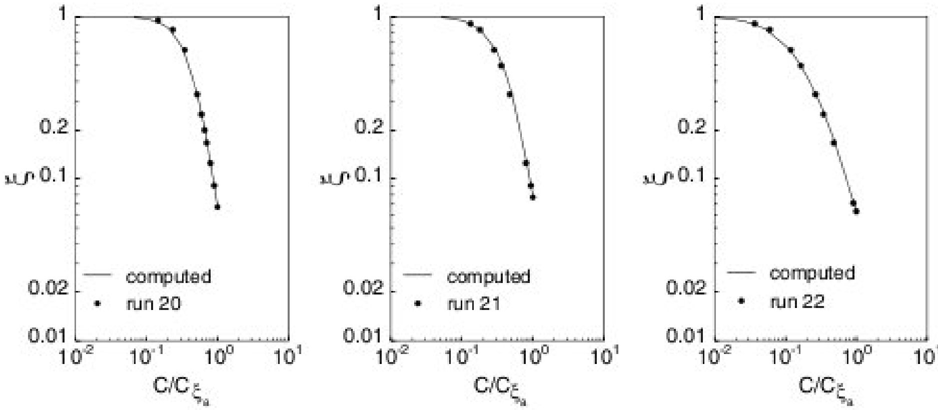


Fig. 3. Calculated and measured sediment concentration profiles for the runs 20, 21 ($D_{50} = 0.1$ mm) and run 22 ($D_{50} = 0.133$ mm) of Vanoni [5].

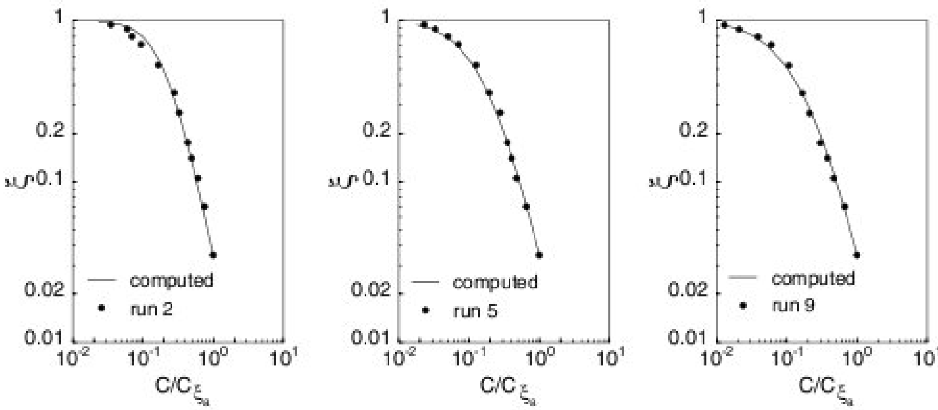


Fig. 4. Calculated and measured sediment concentration profiles for the runs 2, 5 and 9 ($D_{50} = 0.105$ mm) of Coleman [8].

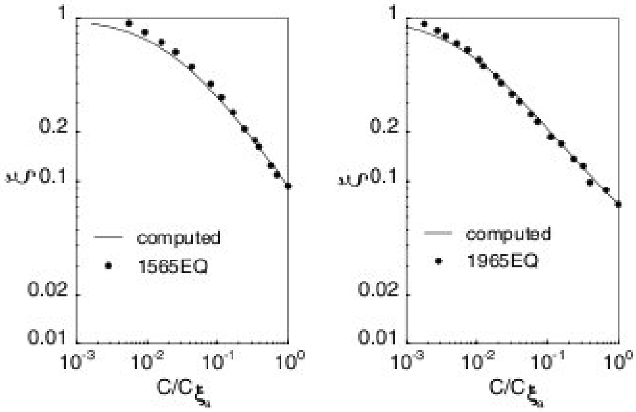


Fig. 5. Calculated and measured sediment concentration profiles for the exp. 1565EQ and 1965EQ ($D_{50} = 0.15$ mm and 0.19 mm, respectively) of Lyn [17].

heights for both velocity and concentration data are same). The reference level for Lyn [17] is chosen at the lowest available height of observed velocity or concentration data set for experiment 1965EQ and the second lowest available height of observed concentration data set for experiment 1565EQ. In order to best predict the velocity and sediment data, it is found that there are only two parameters: β for the velocity and γ for the sediment concentration. For all the data set, the whole region was predicted with $\kappa = 0.4$ and the values of n associated with R_g. The values of the parameters β and γ for Vanoni and Coleman are depicted in Table 1. The comparison of computed and observed velocity profiles for Vanoni [5] for runs 20, 21 ($D_{50} = 0.1$ mm) and run 22 ($D_{50} = 0.133$ mm), Coleman [8] for runs 2, 5 and 9 ($D_{50} = 0.105$ mm) and Lyn [17] for experiments 1565EQ and 1965EQ ($D_{50} = 0.15$ mm and 0.19 mm, respectively) are shown in Fig. 2(a)–(c). The calculated and measured sediment concentration profiles for those runs of Vanoni, Coleman and Lyn are plotted, respectively in Figs. 3–5.

4. Conclusions

The proposed model for computation of velocity and suspension concentration essentially suggests that the inclusion of the effect of concentration in total shear stress and eddy diffusivity in addition to new mixing length concept is more realistic to the sediment-laden turbulent flow containing low as well as high sediment in suspension. The vertical distributions of suspended sediment have been studied based on Hunt's diffusion equation which is more acceptable than Rouse equation. The values of the parameter β obtained by Umeyama are significantly large compared to present values, because concentration effect was not included by Umeyama in total shear stress and eddy diffusivity. The present model has been verified with the experimental data of Vanoni, EC, Coleman and Lyn; and shows quite a good agreement. Therefore, the model should be claimed to be a more general one than those of previous investigators for fine, medium and coarse grained sediments.

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