Measuring Human Poverty: A Generalized Index and an Application Using Basic Dimensions of Life and Some Anthropometric Indicators

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Abstract The Human Poverty Index (HPI) is a composite index of poverty that focuses on deprivations in human lives, aimed at measuring poverty as a failure in capabilities in multiple dimensions, in contrast to the conventional headcount measure focused on low incomes. The HPI was introduced in the United Nations Development Programme *Human Development Report* 1997 and concentrates on deprivations in basic dimensions of life. This paper develops an axiomatic characterization of a family of global deprivation indices using an arbitrary number of dimensions of human life. When we consider only the three basic dimensions, a member of this family becomes ordinally equivalent to HPI. The general index allows the calculation of percentage contributions of deprivations in different dimensions, and hence to identify the dimensions whose failures affect the overall deprivation more. This is important from a policy perspective. We also provide an empirical illustration of the characterized indices using cross-country data for the three basic dimensions and the anthropometric indicators birth weight, height for age and nourishment.

Key words: Deprivation, Human poverty, Axioms, Characterization, General index, Basic dimensions of life, Anthropometric indicators

Introduction

Poverty has been in existence for many years and continues to exist in a large number of countries. Poverty alleviation therefore remains an important policy issue. Poverty of a population, which is a manifestation of insufficient well-being, has often been measured in terms of income (see Kolm, 1969; Atkinson, 1970; Sen, 1973, 1976, 1997; Kakwani, 1980; Shorrocks, 1983; Chakravarty, 1990, 1997). While it is true that with higher income a person may be able to improve his/her position in some non-monetary attributes of well-being, it may be the case that markets for certain non-monetary attributes do not exist. One example is a public good like a flood control programme in an underdeveloped country. Income as the sole attribute of well-being is therefore inappropriate and should be supplemented by other attributes; for example, housing, literacy, life expectancy at birth, nutritional status, provision of public goods, and so on. The basic needs approach advocated by development economists regards development as an improvement in an array of human needs, not just as a growth of income (Streeten, 1981). Thus, the construction of a composite index of well-being is deemed to be a worthwhile exercise (Kolm, 1977; Atkinson and Bourguignon, 1982; Tsui, 1995, 1999, 2002; Ravallion, 1996; Chakravarty *et al*., 1998; Bourguignon and Chakravarty, 1999, 2003; Atkinson, 2003; Weymark, 2003).

Sen (1985, 1987, 1997) defined standard of living in terms of: (i) functioning, which indicates attainments of different attributes; and (ii) capability, which is the ability to achieve. The capability approach emphasizes what a person can do and not what he/she can purchase as the ultimate metric of well-being. An example of an achievement index is the Human Development Index (HDI) suggested by the United Nations Development Programme (UNDP, 1990–2005). It is a composite index that measures average achievements in three basic dimensions of human development, aggregating country level functioning attainments of three attributes: a decent living standard measured by gross domestic product per capita (purchasing power parity in US dollars), a long and healthy life measured by life expectancy at birth, and knowledge measured by the educational attainment rate (adult literacy rate and combined primary, secondary and tertiary gross enrolment rates).

The HDI has its shortcomings, but its widespread use indicates that it has greater advantages. More than a decade after its introduction, the HDI has established itself as a prominent development index. By focusing on basic capabilities it has shown applicability to both developing and developed countries, although the selection of three attributes is quite restrictive, especially when people value other types of capabilities, e.g. human rights (Anand and Sen, 1993). It cannot reflect the inequality and deprivational aspect of development. Different countries may attach a different value to indicators so that assigning the same weight to the three functionings generates differences in respect of how societies view development (Srinivasan, 1994). It fails to provide information on how development is distributed among the population (Foster *et al*., 2005). Simple averaging, which is one of the many aggregation rules (Kelley, 1991), implies perfect substitutability between any two of the three attributes of development. But "people do not just want to be alive. They want to be knowledgeable; and they certainly may want a decent life, one that is not considerably undermined by extreme poverty and constant worry about sheer physical survival" (UNDP, 1991, p. 88). Perfect substitutability between attributes does not reflect this view. Chakravarty (2003) suggested a generalization of the HDI using an arbitrary number of components of well-being, where the marginal rate of substitution between any two attributes diminishes along an indifference curve as the first attribute rises and the second falls. This is a natural assumption, given the nature of attributes.

While the HDI measures average achievement, in 1997 the UNDP proposed a composite index, the Human Poverty Index (HPI), for measuring deprivations in the three basic dimensions of human development captured in the HDI (see the fourth section for an analytical form and discussion). The HPI aims to measure poverty as a failure in capabilities in multiple dimensions, in contrast to the conventional headcount measure focused on low incomes.

By focusing attention on the most deprived people in a country, the HPI highlights the *presence of poverty* in a country rather than the *average national achievement*. Since the HPI formalizes poverty in terms of functioning failures in a multi-attribute framework, it explicitly shows that even among the richest industrialized countries there is human poverty (UNDP, 2001). This demonstrates that poverty is not only a 'southern' phenomenon or one pertaining only to developing countries. Furthermore, a high HDI value does not automatically imply low levels of human poverty (UNDP, 1999). Therefore, monitoring human poverty is important for assessment, policy formulation and making lives of millions better.

Now, human poverty should also include important components other than the three UNDP components. For instance, tuberculosis incidence, and prevalence of anaemia among women and preschool children can be significant dimensions of human poverty. Another crucial dimension can be malnutrition. In spite of the debate on the importance of low income as a cause of undernutrition (Lipton and Ravallion, 1995), because poverty and malnutrition often go hand in hand, it is natural to measure poverty by the extent of a population's failure to achieve desirable nutritional status (Osmani, 1992). Malnutrition is an important cause of low birth weight, brain damage and other brain defects, contributing to physical and cognitive retardation, an increase in infection and death, as well as other health problems in infants and children.

One approach to studying malnutrition is to assess its status based on anthropometric indicators. Malnutrition can be viewed as the output of an ill-health production function, whose inputs are grades of nutritional deficiency based on anthropometric indicators. For example, three of the most common indicators for infants and children are weight-for-height, height-for-age and weight-for-age. Grades of malnutrition based on these indicators are constructed by comparing indicators with reference data for healthy population (see World Health Organisation [WHO], 1995; Alderman, 2000). Another anthropometric indicator that can be used is the body mass index (BMI) for adult population. The BMI, a measure of obesity or emaciation, is defined as the weight in kilograms divided by the square of height in meters (WHO, 1995).

In this paper we axiomatically characterize a family of overall social deprivation indices using an arbitrary number of attributes. In fact, we first characterize social deprivation in terms of each attribute. Then we aggregate the social deprivations of different attributes to arrive at a family of indices of the overall deprivation.[1] Thus, this approach deviates from the *ad hoc* approach to the design of the HPI. In the particular case when we consider the UNDP functionings only, the HPI becomes ordinally equivalent to a member of the family of the characterized indices. Hence, the family of suggested indices is a generalized HPI.[2]

An attractive feature of our index is that it satisfies a factor decomposability condition, which means that for any partitioning of the attributes into two or more subgroups, the overall index is a weighted average of subgroup indices. For instance, suppose that the set of functioning includes the UNDP attributes and anthropometric nutritional indicators (the selection of 'nutrition' as a development issue in supplementing the current HPI is illustrative and one can, in fact, select any other issue(s) and/or indicators in such applications). Then aggregate human poverty is simply the average of human poverty levels calculated using the sets of UNDP attributes and anthropometric indicators. Using this we can calculate the percentage contributions made by different attributes to overall deprivations. Evidently the high contributing attributes require attention from a policy perspective for reducing their contributions to obtain a lower position in the human poverty profile. This shows an important policy application of our index in diverse situations and its usefulness as a planning tool for identifying sources of concentrated poverty within a country. It helps policy formulators to judge why their country has more human poverty than another, to undertake policies to overcome relevant deficiencies and to redefine the priorities of the country. However, an assessment of overall poverty becomes contingent on the implicit valuation of the index. Since the non-welfarist approach to policy analysis is becoming quite popular (Sen, 1985), and in many situations policy is evaluated using specific forms of indices, it seems worthwhile to see what type of policy would be implied by the use of a specific HPI.

The next section characterizes an index of deprivation for an individual attribute, while the third section suggests the generalized HPI. In the fourth section we provide the empirical illustration using cross-country data for basic dimensions of life considered by the UNDP as well as several anthropometric indicators, while the final section presents our conclusions.

Characterization of individual deprivation indices

Suppose there are k attributes of well-being, each representing a dimension of human life. These can be a decent living standard, knowledge, healthy life, housing, provision of public goods and adequate nutrition.

Let $x_i \geq 0$ be the extent of failure associated with attribute i. Failure may be based on an inadequate level of the attribute or on the deviation of its range from the normal range. For instance, failure in the functioning 'knowledge' in a developing country can be measured by the fraction of illiterate adults. Failure in having a balanced nutritional status, which is manifested through the anthropometric indicator BMI being different from the normal range, can be measured using the fraction of adults over 20 whose BMI > 30 or BMI < 18.5[3] (WHO, 1995). As in these two examples, we assume that for any i, x_i is independent of the unit of measurement. Therefore, $x_i \in [0,1]$, the set of non-negative real numbers lying between zero and one.

A deprivation index for attribute i is a real valued function $d: [0.1] \rightarrow R^1_+$, which associates a value $d(x_i)$ to each $x_i \in [0,1]$, where R^1_+ is the non-negative part of the real line. More precisely, $d(x_i)$ determines the extent of deprivation corresponding to x_i. We assume that d is twice differentiable, a technical assumption that makes the analysis simple. Further, it implies continuity of d, which ensures that d will not be oversensitive to minor observational errors on failure of attribute i. We assume the same functional form of d for all attributes for computational simplicity, although one can choose different functional forms for different attributes.

We now propose the following axioms for an arbitrary index d.

Normalization (NM): $d(x_i)=0$ if $x_i=0$.

Monotonicity (MN): An increase in x_i increases d; that is, for any $x_i \in [0,1]$, $d(x_i+c) > d(x_i)$, where $c > 0$ is arbitrary and $x_i + c \in [0,1]$.

Higher Deprivation at Higher Failure Difference (HD): For any $x_i \in [0,1]$ and any $\delta > 0$, the magnitude of deprivation increase $d(x_i+\delta) - d(x_i)$ is an increasing function of x_i where $x_i + \delta \in [0,1]$.

Normalization indicates that if the level of failure for an attribute is zero, then its deprivation is zero. Monotonicity demands that an increase in the extent of failure of an attribute makes people more deprived in that attribute. Normalization and monotonicity jointly imply that the deprivation index is bounded from below by zero, the bound being attained in case of zero failure. According to HD an increase in the extent of failure of the attribute represents a greater increase in deprivation at higher levels than an identical increase at lower levels. For instance, in a developing country, an increase in the fraction of population without access to safe water from 0.6 to 0.7 shows a higher degree of deprivation increase than an increase in the fraction from 0.1 to 0.2.

HD parallels Sen's (1976) 'Ranked Relative Deprivation' axiom. Sen argued that in income poverty measurement, the poverty line can be taken as the reference point for all the poor persons. Therefore, the poverty gap of a poor person, his/her income shortfall from the poverty line, can be taken as his/her extent of deprivation. To make deprivation more sensitive to the poorer poor, Sen used a weighted form of the poverty gap as a measure of deprivation, where the weight is the rank order in the income distribution of the poor persons. This ensures that an increase in deprivation of a poor person due to a reduction in his/her income will be higher, the lower the income. Conversely, for an increase in deprivation of a poor person resulting from a decrease in his/her income to be higher — a necessary condition is to attach higher weight to a higher poverty gap (see also Chakravarty and D'Ambrosio, 2003).

Note that all the three axioms have to be imposed simultaneously for deriving our index, because they are different and none implies or is implied by a second one. In fact, this is demonstrated more rigorously in theorem 2. The index is derived in theorem 1.

We now show the relevance of these axioms in the context of the HPI. Consider the functional form of the HPI for k arbitrary attributes:

$$\mathrm{HPI} = 100\left(\frac{1}{k}\sum_{i=1}^{k} x_i^{\alpha}\right)^{\frac{1}{\alpha}}, \alpha > 1 \tag{1}$$

As x_i values are unit-free, averaging of x_i^{α}, $i=1, \ldots, k$, is well-defined in equation (1). Since for HPI failures are expressed as percentages, the formula is written in the above form.

Now consider an arbitrary component of the HPI:

$$x_i^{\alpha} \tag{2}$$

It satisfies NM, MN and HD. Therefore, the function $d_{\alpha}(x_i) = x_i^{\alpha}$, $\alpha > 1$, can be regarded as a deprivation index for attribute i. The parameter α is the deprivational aversion parameter because an increase in the value of α decreases $d_{\alpha}(x_i)$ for a given x_i.

The following theorem identifies the entire class of deprivation indices satisfying NM, MN and HD.

Theorem 1: Suppose that the deprivation index $d: [0,1] \to R_+^1$ is twice differentiable. Then it satisfies NM, MN and HD if and only if d is increasing in its argument, strictly convex and $d(0)=0$. (For proof see the Appendix).

The index identified in theorem 1 will be called regular. Given any regular d, we have a particular deprivation index. These indices will differ depending on specification of d. An alternative to d_{α} is:

$$d_r(x_i) = e^{rx_i} - 1 \tag{3}$$

where the parameter $r > 0$ determines the curvature of the deprivation curve. The restriction $r > 0$ ensures that d_r is regular. As r increases, higher weight is attached to higher failures.

We conclude this section by demonstrating that the axioms NM, MN and HD are independent. Independence means that if we drop any one of them, an index satisfying the two remaining axioms will be different from that characterized in theorem 1. We state this more formally in theorem 2 (for proof see the Appendix).

Theorem 2: Axioms NM, MN, and HD are independent.

The generalized Human Poverty Index

The index d summarizes deprivation for one attribute. The overall deprivation, which is a generalization of the HPI, has to be constructed involving all the attributes.

Assume that the overall deprivation index G is a real valued function of the single-dimensional indices. Such assumptions are made in welfare economics, where social utility is regarded as a function of individual utility levels. In the human development literature an achievement index (e.g. the HDI) is assumed to depend on individual attainment indicators (Chakravarty, 2003).

Now, denote $d(x_i)$ by a_i, where $i=1, \ldots, k$. We write $a=(a_1, a_2 \ldots, a_k)$. Then the relationship can be formally stated as:

There exists a function $I: R^k_+ \to R^1$, such that for all $(x_1, x_2, \ldots x_k)$, the deprivation index $G(x_1, x_2, \ldots x_k)$ can be written as $I(a_1, a_2 \ldots, a_k)$. Since this assumption ignores all features other than individual deprivations, we call this independence of irrelevant information.

Under this assumption we state certain postulates for an arbitrary index I:

Normalization (NOM): For any $z \geqslant 0$, $I(z, z, \ldots, z)=z$.
Additivity (ADD): For any $a, b \in R^k_+$,

$$I(a_1+b_1, a_2+b_2, \ldots, a_k+b_k) \\ =I(a_1, a_2, \ldots, a_k)+I(b_1, b_2, \ldots, b_k) \quad (4)$$

Symmetry (SYM): For all $a \in R^k_+$, $I(a)=I(aP)$, where P is any $k \times k$ permutation matrix.[4]

According to NOM, if the deprivation levels for different attributes are the same, then the global deprivation index takes on this common value. Thus, the social deprivation is an average of individual deprivations. Furthermore, when there is only one attribute, the individual and global deprivations are the same. ADD can be interpreted as follows. Suppose that attribute i has two components. For instance, if attribute i is knowledge, then its two components can be the adult literacy rate, and combined primary, secondary and tertiary enrolment ratios. Let $a_i(b_i)$ be the deprivation level of attribute i based on the first (second) component. If it is not possible to split attribute i, we can attach zero value to $b_i(a_i)$ so that the first (second) component represents the extent of entire deprivation for this attribute. Then ADD says that the sum of deprivations based on the vectors $(a_1, a_2, \ldots, a_k)$ and $(b_1, b_2, \ldots, b_k)$ is the same as the deprivation based on the vector $(a_1+b_1, a_2+b_2, \ldots, a_k+b_k)$. This property, therefore, shows how to calculate deprivations when we split the attributes into components. Finally, SYM requires insensitivity of I to permutation of its arguments. That is, I remains invariant under any reordering of individual deprivations.

Theorem 3: An overall deprivation index I satisfies NOM, ADD and SYM if and only if it can written in the form:

$$I(a_1, a_2, \ldots, a_k) = \frac{1}{k}\sum_{i=1}^{k} a_i = \frac{1}{k}\sum_{i=1}^{k} d(x_i) \tag{5}$$

(For proof see the Appendix).

Theorem 3 identifies three necessary and sufficient conditions for expressing global deprivation as the arithmetic average of dimension-specific deprivations.

The deprivation index in equation (5) possesses the following interesting properties:

(a) **Minimality**: It achieves its lower bound zero in the extreme case $x_i=0$ for all i.
(b) **Increasingness**: It is increasing in individual deprivations $d(x_i)$ (hence in x_i).
(c) **Betweenness**: Min $\{d(x_1), \ldots, d(x_k)\} \leq I \leq$ Max $\{d(x_1), \ldots, d(x_k)\}$; that is, I lies between deprivations of dimensions showing minimal and maximal failures.
(d) **Linear Homogeneity**: An equiproportionate change in all deprivations changes I by the proportion itself.
(e) **Greater Sensitivity at Higher Levels of Failure**: For any component, the increase in deprivation due to a fixed increase in the extent of its failure is higher, the higher this failure extent is, given that the failure levels for other components remain unchanged.
(f) **Factor Decomposability**: Since the global index is the average of component-wise indices, it is factor decomposable. Formally, for $a^i \in R_+^{k_i}$, $i = 1, 2, \ldots, m$, we have:

$$I(a) = \sum_{i=1}^{m} \frac{k_i}{k} I(a^i) \tag{6}$$

where $k = \sum_{i=1}^{m} k_i$ and $a = (a^1, a^2, \ldots, a^m)$.

That is, for any partitioning of the k components into m subgroups, overall deprivation is a weighted average of subgroup deprivations, where the weights are the proportions of components belonging to different subgroups. The quantity $[k_i I(a^i)/k]$ may be interpreted as the contribution of subgroup i to total deprivation, the amount by which total deprivation will fall if deprivation in subgroup i were eliminated. The percentage contribution of subgroup i to overall deprivation is $(k_i I(a^i)/kI(a))100$. These contributions help us make quantitative assessment of subgroups and hence to identify those subgroups that are more afflicted by deprivation and to implement anti-deprivation policy. For instance, between two subgroups with anthropometric and non-anthropometric indicators, respectively, we can identify the subgroup that is more afflicted by overall deprivation and formulate a targeted deprivation alleviation policy.

To illustrate the formula in equation (5), suppose that d is of the form of equation (2). The corresponding global index is:

$$I_\alpha = 100\left(\frac{1}{k}\sum_{i=1}^{k} x_i^\alpha\right), \quad \alpha > 1 \tag{7}$$

where I_α is decreasing in α for all i. Note that expressing x_i values in fractional forms rather than in percentage forms prevents the average from becoming excessively large and incomprehensible. To make it comparable with HPI, this particular case of the index in equation (5) is multiplied by 100. This makes the index bounded above by 100. Note also that scaling of the index by 100 does not affect the percentage contribution made by different factors or their subgroups. Furthermore, under scaling we have ordinal equivalence of the resulting index with its original form. Thus, from the viewpoint of ranking they provide the same information.

For a given $\alpha > 1$, we consider the degree of substitutability between any two failures, say x_i and x_j, in the aggregate index I_α. The elasticity of substitution σ_{ij} between x_i and x_j along an isodeprivation curve (holding x_l, $l \neq i, j$, constant) is defined as the percentage change in (x_i/x_j) for a unit percentage change in the slope of the tangent along this curve (projected on to $x_i - x_j$ space at the given values of x_l, $l \neq i, j$). Formally,

$$\sigma_{ij} = \partial \log\left(\frac{x_i}{x_j}\right) \Big/ \partial \log y \tag{8}$$

where $y = (\partial I_\alpha/\partial x_i)/(\partial I_\alpha/\partial x_j)$. It is easy to check that $\sigma_{ij} = 1/(\alpha - 1)$. Thus, the elasticity of substitution is constant along each level set of I_α and the same for different level sets. If $\alpha = 1$, there is perfect substitutability between x_i and x_j, and the impact on I_α from a unit decrease or increase in any failure is the same, irrespective of the level of failures in different dimensions. This contradicts our postulate HD (and property (e) of I_α). The value of α also influences the ratio $(x_i/x_j)^{\alpha-1}$, the relative impact of a unit increase in x_i compared with a unit increase in x_j. For $\alpha=1$, the relative impact is 1. Given $x_i > x_j$, the relative impact increases as α is raised from 1. Note that $(\alpha-1)$ is the elasticity of the marginal deprivation function $d'(x_i)$. As $\alpha \to \infty$, there is no substitution between x_i and x_j. As α increases from 1, the elasticity of substitution decreases from ∞ to 0.

We now compare I_α with the HPI. The HPI in equation (1) satisfies NOM and SYM, and also the postulates (a)–(e), but not ADD (and hence not property (f)). It is related to I_α in an increasing way. Formally, $(HPI)^\alpha=I_\alpha$ for all $\alpha>1$. That is, I_α and the HPI are ordinally equivalent. Therefore, ranking of societies by I_α will coincide with that generated by the HPI, and hence they convey the same information. Since I_α corresponds to our general index I in equation (5) for the particular specification d_α, because of ordinal equivalence between HPI and I_α, we can say that I is a generalization of the HPI.

The final result concerns independence of NOM, ADD and SYM.

Theorem 4: The postulates NOM, ADD and SYM are independent (for proof see the Appendix).

An empirical illustration

This section illustrates the index I_α using the three basic dimensions of life captured in the HPI and three anthropometric indicators for several countries for the year 2000. We refer to these two subgroups of dimensions as the UNDP subgroup and the anthropometric subgroup, respectively. To demonstrate that a particular case of I_α is ordinally equivalent to the HPI, we also present the HPI figures. The data have been obtained from the *Human Development Report* (UNDP, 2002). We may point out here that these dimensions of human life may not be mutually exclusive. For example, anthropometric indicators, as measures of nutrition, may overlap with living standard and healthy life. We have, however, carried out a principal component analysis and the leading eigen value (which explains 69% of the total variance) puts weights ranging between 0.56 and 0.93 to these variables, thus justifying inclusion of all in measuring the underlying latent construct 'poverty'.

The HPI, which is calculated annually by the UNDP for all countries according to the availability of statistical data, is prepared in two forms, depending on whether it is a developing country (HPI-1) or an industrialized country (HPI-2). In the HPI-1 the three dimensions that are taken are: (a) longevity failure, measured as the *percentage* of the individuals with a life expectancy less than 40 years (x_1^*); (b) failure in knowledge, as measured by the *percentage* of illiterate adults (x_2^*); and (c) failure in decent living standard (x_3^*). The indicator x_3^* is given by

$(x^*_{31}+x^*_{32}+x^*_{33})/3$, where the variables are: (i) the *percentage* of population without access to drinking water (x^*_{31}), (ii) the *percentage* of population without access to health services (x^*_{32}), and (iii) the *percentage* of children under age 5 who are moderately and severely underweight (defined as below two standard deviations from the median weight for age of the reference population)[5] (x^*_{33}). The global index, HPI-1, is then obtained by taking symmetric average of order 3 of these three dimensions:

$$\text{HPI}-1=\left(\frac{1}{3}\left(x_1^{*3}+x_2^{*3}+x_3^{*3}\right)\right)^{1/3}=100\left(\frac{1}{3}\left(x_1^3+x_2^3+x_3^3\right)\right)^{1/3} \tag{9}$$

which is the index in equation (1) for $\alpha=3$ and $k=3$.

The HPI-2 measures failures in the same dimensions as the HPI-1 but with different definitions, and also captures social exclusion (failure in employment). Thus, it reflects failures in four dimensions: (a) a long and healthy life (vulnerability to death at a relatively low age), measured by the probability (%) at birth of not surviving to age 60 (x_1^*); (b) knowledge (exclusion from the world of reading and communications), measured by the *percentage* of the adult population (aged 16–65) not possessing functional literacy skills (x_2^*); (c) a decent living standard, measured by the *percentage* of the population living below the income poverty line (50% of the median disposable household income) (x_3^*); and (d) employment (social exclusion), measured by the *percentage* of unemployment exceeding 12 months or more (x_4^*). Then the HPI-2 is given by:

$$\text{HPI}-2=\left(\frac{1}{4}\left(x_1^{*3}+x_2^{*3}+x_3^{*3}+x_4^{*3}\right)\right)^{1/3}=100\left(\frac{1}{4}\left(x_1^3+x_2^3+x_3^3+x_4^3\right)\right)^{1/3} \tag{10}$$

As in the earlier case, HPI-2 corresponds to the k-dimensional HPI in equation (1) for $\alpha=3$ and $k=4$.

The anthropometric indicators we consider are: (a) infants with low birth weight, as measured by the *proportion* of infants with a birth-weight of less than 2500 g; (b) undernourished people, as measured by the *proportion* of population whose food intake is insufficient to meet their minimum energy requirements on a chronic basic; and (c) children under height for age, as measured by the *proportion* of children under age 5 who are moderately and severely stunted, defined as below two standard deviations from the median height for age of the reference population (UNDP, 2002).

We calculate I_α for $\alpha=2, 3, 4$ and look at the percentage contributions made by individual indicators to global deprivation. The developing countries chosen are: Ethiopia, Central African Republic, Zambia, Nigeria, Bangladesh, Nepal, Ghana, India, Bolivia, China, Sri Lanka, Brazil, Thailand and Mexico. Of these, Ethiopia, Central African Republic, Zambia, Nigeria, Bangladesh and Nepal are chosen from among the low HDI ranking countries. The remaining countries are chosen from among the medium HDI ranking countries (UNDP, 2002). These countries are located in different parts of the world covering the continents Africa, Asia and Latin America. Since the nutritional dimensions can significantly contribute to human poverty and deprivation, and since mostly the low and medium HDI countries are affected by them, we have excluded the high HDI ranking countries from our analysis.[6]

The first column of Table 1 presents the names of the developing countries considered here. The next column shows their HDI rank among 175 countries considered by the UNDP. Columns 3 and 4 present, for each country, the levels of deprivations for the UNDP and the anthropometric subgroups, respectively. The weighted average of these deprivations determine the overall deprivation I_α(for $\alpha=2$), which is presented in column 5, where the weights are 3/4 and 1/4, respectively. Similar calculations for $\alpha=3$ and $\alpha=4$ are reported in columns 6, 8, 9 and 10–12, respectively. Column 7 indicates the HPI-1 rankings of the countries.

Several interesting features emerge from Table 1. Since for UNDP basic functioning I_α(with $\alpha = 3$) is ordinally equivalent to the HPI-1, columns 6 and 7 show the same ranking of the countries. However, the ranking generated by overall index values, as shown in column 9, becomes different. The reason behind this is that the anthropometric subgroup gives a different ranking from that given by the UNDP subgroup. In fact, out of $3c_2 = 3$ ranking comparisons among columns 6–9, we observe disagreement in all the three cases. To observe this in greater detail, let us consider, for example, the pair (Zambia and Bangladesh). While the latter is worse off according to the UNDP subgroup index and the overall index, we have an opposite picture for the anthropometric index. Clearly, given different directional rankings of the two countries by the subgroup indices, the direction of ranking by the overall index is dependent on the weights assigned to the two subgroups. Similar situations of non-uniform ranking occur for $\alpha = 2$ and $\alpha = 4$ as well. However, the ranking by the UNDP subgroup index is constant across values of α, and this ranking differs from the HDI ranking of the countries considered. In all cases except for the anthropometric subgroup, Ethiopia emerges as the country with maximum deprivation. In contrast, Mexico, which has the highest HDI ranking among the countries considered, emerges as the country with minimum deprivation for only the UNDP subgroup, and not for the entire set of attributes or the other subgroup. To understand the reason for this explicitly, consider the index values for Brazil and Mexico for $\alpha=2$. While for the UNDP subgroup the index value of the former is 40% higher than that of the latter, the latter has a 22% higher index value than the former for the other subgroup. Thus, a higher weight assigned to the UNDP subgroup makes the overall index higher for Brazil. Similar explanations can be provided for the other cases. We also note monotonicity of the index values with respect to α; as α rises, deprivation falls unambiguously for all countries.

Table 2 provides, for each country, the percentage contributions of the UNDP and anthropometric subgroups of indicators to global

Table 1. Deprivation indices for some developing countries for the year 2000

Country	HDI ranking				α=3				α=4		
		UNDP subgroup generalized index	Anthropometric subgroup index	Overall index, I_α	UNDP subgroup index		Anthropometric subgroup index	Overall index, I_α	UNDP subgroup generalized index	Anthropometric subgroup index	Overall index, I_α
					Generalized	HPI-1 (UNDP)					
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)	Column (9)	Column (10)	Column (11)	Column (12)
Ethiopia	168	31.31	17.15	24.23	18.05	56.51	8.40	13.22	10.56	4.18	7.37
Central African Republic	165	19.72	11.80	15.76	9.24	45.21	4.70	6.97	4.44	1.92	3.18
Zambia	153	14.28	19.37	16.82	6.43	40.06	10.35	8.39	3.12	5.67	4.39
Nigeria	148	12.21	7.49	9.85	4.27	34.96	3.28	3.78	1.50	1.50	1.50
Bangladesh	145	15.18	13.38	14.28	7.62	42.40	5.14	6.38	4.17	2.03	3.10
Nepal	142	16.61	12.95	14.78	8.15	43.35	5.96	7.06	4.31	2.99	3.65
Ghana	129	8.24	3.27	5.76	2.37	28.74	0.72	1.55	0.69	0.17	0.43
India	124	9.94	11.07	10.50	3.62	33.09	4.24	3.93	1.40	1.74	1.57
Bolivia	114	2.63	4.08	3.36	0.43	16.30	0.96	0.70	0.07	0.23	0.15
China	96	2.07	1.35	1.71	0.33	14.87	0.20	0.26	0.05	0.03	0.04
Sri Lanka	89	2.43	3.69	3.06	0.55	17.62	0.73	0.64	0.13	0.15	0.14
Brazil	73	1.46	1.01	1.23	0.18	12.27	0.10	0.14	0.02	0.01	0.02
Thailand	70	1.60	2.49	2.05	0.27	14.00	0.46	0.37	0.05	0.09	0.07
Mexico	54	0.88	1.43	1.16	0.08	9.46	0.22	0.15	0.01	0.04	0.02

Table 2. Deprivation breakdown by UNDP basic dimensions subgroup and anthropometric subgroup for some developing countries for the year 2000

Country	HDI ranking	% contribution to I_α					
		$\alpha=2$		$\alpha=3$		$\alpha=4$	
		UNDP subgroup	Anthropometric subgroup	UNDP subgroup	Anthropometric subgroup	UNDP subgroup	Anthropometric subgroup
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)
Ethiopia	168	64.60	35.40	68.23	31.77	71.62	28.38
Central African Republic	165	62.57	37.43	66.28	33.72	69.82	30.18
Zambia	153	42.43	57.57	38.31	61.69	35.47	64.53
Nigeria	148	62.00	38.00	56.57	43.43	50.01	49.99
Bangladesh	145	53.15	46.85	59.74	40.26	67.23	32.77
Nepal	142	56.18	43.82	57.74	42.26	58.99	41.01
Ghana	129	71.57	28.43	76.66	23.34	80.00	20.00
India	124	47.30	52.70	46.11	53.89	44.56	55.44
Bolivia	114	39.20	60.80	31.15	68.85	23.75	76.25
China	96	60.49	39.51	62.76	37.24	63.88	36.12
Sri Lanka	89	39.71	60.29	42.74	57.26	47.03	52.97
Brazil	73	59.13	40.87	64.43	35.57	69.89	30.11
Thailand	70	39.23	60.77	37.54	62.46	36.61	63.39
Mexico	54	38.03	61.97	27.52	72.48	18.14	81.86

deprivation for different values of α. These contributions are calculated from figures in columns 3–5, 6, 8, 9 and 10–12 of Table 1. Complete elimination of deprivation within a subgroup would lower aggregate deprivation by the percentage by which it contributes to total deprivation. We note that for all values of α, except for Zambia, India, Bolivia, Sri Lanka, Thailand and Mexico, the UNDP dimensions make more than 50% contributions. Therefore, in all such cases the UNDP functioning collectively needs more attention from an anti-deprivation policy perspective.

In order to get a closer picture of contributions made by different dimensions in the two subgroups, we decompose the subgroup deprivations by their indicators in Tables 3 and 4. Since there are three component indices in the composite index, a contribution of one-third will mean that the components are equally important. From Table 3 it follows that, for some countries in Africa (Zambia) and South America (Bolivia), at least one-third of the total contribution to their respective total deprivation comes from vulnerability to death at early age for all values of α. While some Asian countries (e.g. Nepal, Bangladesh, India and China) perform better with respect to this dimension, their performance for the functioning adult literacy rate is disturbing. Brazil's situation in literacy is disturbing as well. However, its performance in decent living standard is better than India and Ghana, but worse than Bangladesh and more or less the same as Nepal. For Sri Lanka and Thailand the situation is different. Among the countries considered in the table, they have the worst performance in relative terms with respect to decent living standard, although they perform reasonably well with respect to the other dimensions. In fact, for Mexico the contribution of this indicator also ranges from 45% to 59% and the remaining contribution is shared more or less equally by the other two factors. In the case of Ethiopia, the country with maximum deprivation, the contribution of longevity failure ranges from 11% to 21% for different values of α, but contributions of other dimensions vary within a close range. For the other two countries (Nigeria and Ghana) we have a somewhat compromise picture in terms of the three attributes.[7]

From Table 4, where the country-specific anthropometric deprivations are disaggregated into individual components, we observe that all countries except Sri Lanka and Thailand suffer from high level of deprivation in terms of 'children under height for age'. This shows the presence of a high percentage of children in these countries with height-for-age deficits indicating past or chronic inadequate nutrition and/or chronic or frequent illness among children. For Sri Lanka and Thailand the contributions of the factor are relatively low compared with the other countries. In three Asian, three African and two South American countries, for all values of α, the contribution of the fraction of undernourished people to the overall index is 12% or more. Thus, undernutrition may be regarded as an important cause of ill health in these countries. Finally, we

Table 3. Breakdown of deprivation (based on UNDP basic dimensions) by individual indicators for some developing countries for the year 2000

Country	HDI ranking	% contribution of indicators								
		α=2			α=3			α=4		
		Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living*	Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living	Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)	Column (9)	Column (10)	Column (11)
Ethiopia	168	20.24	39.49	40.27	15.31	41.72	42.97	11.41	43.43	45.16
Central African Republic	165	34.68	48.01	17.31	33.54	54.64	11.82	31.59	60.54	7.87
Zambia	153	67.08	11.20	21.72	79.84	5.45	14.71	88.28	2.46	9.26
Nigeria	148	31.00	35.57	33.43	29.86	36.70	33.45	28.73	37.84	33.43
Bangladesh	145	10.06	75.66	14.28	4.29	88.46	7.25	1.68	94.94	3.38
Nepal	142	10.16	67.98	21.86	4.66	80.64	14.70	1.98	88.83	9.18
Ghana	129	29.50	32.86	37.64	27.64	32.51	39.85	25.84	32.08	42.08
India	124	9.36	61.45	29.19	4.28	72.11	23.61	1.86	80.07	18.07
Bolivia	114	42.91	26.65	30.45	47.91	23.45	28.64	52.93	20.41	26.65
China	96	10.04	40.68	49.28	4.99	40.72	54.29	2.41	39.55	58.04
Sri Lanka	89	4.61	9.68	85.71	1.19	3.61	95.20	0.29	1.25	98.46
Brazil	73	29.22	50.13	20.65	26.04	58.49	15.47	22.51	66.24	11.25
Thailand	70	16.82	4.21	78.97	8.85	1.11	90.04	4.33	0.27	95.40
Mexico	54	26.11	28.03	45.86	22.52	25.05	52.43	19.09	22.01	58.90

*For the year 2000, owing to lack of reliable data on 'the *percentage* of population without access to health services' (x^*_{32}), this has been measured using an unweighted average of 'population not using improved water sources' (x^*_{31}) and 'underweight children under five' (x^*_{33}).

Table 4. Breakdown of deprivation (based on anthropometric dimensions) by individual indicators for some developing countries for the year 2000

Country	HDI ranking	% contribution of indicators								
		α=2			α=3			α=4		
		Infants with low birth weights	Undernourished people	Children under height for age	Infants with low birth weights	Undernourished people	Children under height for age	Infants with low birth weights	Undernourished people	Children under height for age
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)	Column (9)	Column (10)	Column (11)
Ethiopia	168	2.80	46.66	50.54	0.69	46.68	52.63	0.17	45.93	53.90
Central African Republic	165	4.78	52.25	42.98	1.56	56.38	42.06	0.50	59.35	40.16
Zambia	153	2.08	38.01	59.90	0.43	33.43	66.14	0.09	28.68	71.23
Nigeria	148	3.61	2.18	94.21	0.74	0.35	98.91	0.15	0.05	99.80
Bangladesh	145	22.42	27.13	50.45	17.53	23.33	59.15	13.29	19.45	67.26
Nepal	142	11.35	13.61	75.04	5.18	6.80	88.02	2.17	3.12	94.72
Ghana	129	8.25	22.91	68.84	3.36	15.57	81.07	1.28	9.85	88.88
India	124	20.36	15.93	63.72	13.83	9.57	76.59	8.76	5.37	85.87
Bolivia	114	5.23	39.54	55.23	1.78	37.05	61.16	0.59	33.69	65.72
China	96	8.87	19.95	71.18	3.69	12.44	83.87	1.42	7.18	91.40
Sri Lanka	89	26.11	47.79	26.11	22.34	55.32	22.34	18.69	62.62	18.69
Brazil	73	26.82	33.11	40.07	23.82	32.68	43.50	21.03	32.05	46.92
Thailand	70	6.57	59.12	34.32	2.50	67.60	29.90	0.91	74.11	24.97
Mexico	54	18.84	5.81	75.35	10.90	1.87	87.23	5.85	0.56	93.59

analyse the impact of 'infants with low birth weight' on global deprivation. In Bangladesh, Nepal, India, Sri Lanka, Brazil and Mexico, this is significant for certain values of α. Therefore, a policy of reducing the percentage contribution made by this factor will also be an attempt to combat health and nutrition problems.

One encouraging picture that emerges from Table 4 is that in some countries there is an insignificant effect of some dimension(s) on global deprivation. For instance, in Ethiopia and Zambia the impact of the indicator infant with low birth weight is rather small. In Ethiopia, the country that has maximum deprivation, the effects of the remaining two sources are quite high. Therefore, in this country these two sources deserve more or less equal importance from an anti-deprivation policy perspective. But in some cases we observe a quite disturbing picture — the influence of a factor is very high. For instance, in Nigeria and Mexico for $\alpha=2$, the indicator children under height for age characterizes, respectively, more than 94% and more than 75% of global deprivation. This contribution rises as the value of α goes up, and therefore this attribute has serious implications in terms of frequent illness and nutrition deficiency on child health. The policy-makers may, therefore, have to reformulate an appropriate policy under such circumstances.

In order to establish what an individual variable contributes to human deprivation, in Tables 5 and 6 we present the decomposition of the HPI and nutrition components. This breakdown is important when policy decisions are based on comparison of contributions across component variables. We note from Tables 5 and 6 that for any country, for any α, rankings of the percentage contributions of the UNDP and anthropometric indicators are the same as those observed in Tables 3 and 4, respectively. Next, our observation from Tables 5 and 6 reveals that, for Ethiopia, Central African Republic, Nigeria, Ghana and Brazil, for $\alpha=2,3$, each of the UNDP dimensions contributes more than any of the nutrition components. Thus, in the case of these countries, their overall situation indicates that they are in a comfortable position from the nutritional aspect. The pictures for $\alpha=4$ and the remaining countries are somewhat mixed in the sense that the contributions of the UNDP indicators and nutritional indicators cannot be separated out. For instance, in Mexico, while 'deprivation in decent standard of living' contributes the most, the next major contributor is 'children under height for age'. Positions of all other countries can be analysed analogously.

Conclusion

This paper aimed to develop a generalization of the UNDP HPI through an axiomatic approach. The index can be written as the simple average of deprivations along individual dimensions. This property permits us to

Table 5. Percentage contribution of individual indicators of UNDP basic dimensions in the overall index for some developing countries for the year 2000

Country	HDI ranking	% contribution of indicators α=2			α=3			α=4		
		Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living*	Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living	Probability at birth of not surviving to age 40	Adult illiteracy rate	Deprivation in decent standard of living
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)	Column (9)	Column (10)	Column (11)
Ethiopia	168	17.11	33.39	34.05	13.25	36.12	37.20	10.08	38.36	39.89
Central African Republic	165	28.92	40.03	14.43	28.68	46.71	10.11	27.61	52.92	6.88
Zambia	153	46.19	7.71	14.96	51.96	3.54	9.57	54.95	1.53	5.76
Nigeria	148	25.74	29.53	27.76	23.77	29.22	26.63	21.55	28.38	25.08
Bangladesh	145	7.77	58.48	11.04	3.50	72.24	5.92	1.44	81.67	2.91
Nepal	142	8.06	53.96	17.35	3.75	64.83	11.82	1.61	72.12	7.45
Ghana	129	26.05	29.02	33.24	25.10	29.52	36.18	23.85	29.61	38.84
India	124	6.82	44.81	21.29	3.08	51.89	16.99	1.31	56.60	12.77
Bolivia	114	28.28	17.56	20.07	27.59	13.50	16.49	25.57	9.86	12.87
China	96	8.25	33.40	40.47	4.17	33.99	45.32	2.03	33.28	48.83
Sri Lanka	89	3.06	6.43	56.91	0.82	2.50	65.81	0.21	0.91	71.58
Brazil	73	23.75	40.74	16.79	21.99	49.40	13.07	19.68	57.92	9.83
Thailand	70	11.09	2.77	52.08	5.69	0.71	57.92	2.74	0.17	60.49
Mexico	54	16.92	18.16	29.72	11.99	13.34	27.92	7.62	8.79	23.52

*For the year 2000, owing to lack of reliable data on 'the *percentage* of population without access to health services' (x^*_{32}), this has been measured using an unweighted average of 'population not using improved water sources' (x^*_{31}) and 'underweight children under five' (x^*_{33}).

Table 6. Percentage contribution of individual indicators of anthropometric dimensions in the overall index for some developing countries for the year 2000

Country	HDI ranking	% contribution of indicators								
		$\alpha=2$			$\alpha=3$			$\alpha=4$		
		Infants with low birth weights	Under-nourished people	Children under height for age	Infants with low birth weights	Under-nourished people	Children under height for age	Infants with low birth weights	Under-nourished people	Children under height for age
Column (1)	Column (2)	Column (3)	Column (4)	Column (5)	Column (6)	Column (7)	Column (8)	Column (9)	Column (10)	Column (11)
Ethiopia	168	0.43	7.21	7.81	0.09	6.27	7.07	0.02	5.36	6.29
Central African Republic	165	0.79	8.68	7.14	0.23	8.18	6.10	0.06	7.47	5.06
Zambia	153	0.65	11.84	18.66	0.15	11.68	23.10	0.03	10.83	26.89
Nigeria	148	0.61	0.37	15.98	0.15	0.07	20.15	0.04	0.01	24.94
Bangladesh	145	5.09	6.16	11.46	3.21	4.28	10.85	1.86	2.72	9.40
Nepal	142	2.34	2.81	15.48	1.02	1.33	17.26	0.41	0.59	17.82
Ghana	129	0.96	2.68	8.05	0.31	1.43	7.47	0.10	0.76	6.84
India	124	5.51	4.31	17.25	3.88	2.68	21.47	2.57	1.57	25.17
Bolivia	114	1.78	13.48	18.82	0.76	15.72	25.95	0.30	17.42	33.98
China	96	1.59	3.57	12.73	0.61	2.06	13.85	0.22	1.14	14.50
Sri Lanka	89	8.77	16.06	8.77	6.90	17.08	6.90	5.10	17.09	5.10
Brazil	73	5.02	6.20	7.50	3.70	5.08	6.76	2.64	4.02	5.89
Thailand	70	2.24	20.13	11.69	0.89	24.11	10.67	0.33	27.12	9.14
Mexico	54	6.63	2.05	26.52	5.10	0.87	40.78	3.51	0.33	56.22

determine the percentage contributions made by the dimensions to overall deprivation. These contributions can be used to separate the dimensions according to their degrees of sensitivity. The high contributing dimensions may require the attention of policy-makers to reduce their impact in order to occupy a lower position in the human poverty profile. An empirical illustration of the generalized index using cross-country data for the dimensions of life considered by the UNDP and some anthropometric indicators is also provided in the analysis.

Finally, we present suggestions on policies that warrant attention based on application of our index. For example, an improvement in the adult literacy rate can be brought about by: (i) reduction of dropouts at different levels of education by some incentives, and (ii) having adult education programmes such as free night schools where people can study after work. In the context of anthropometric indicators, it is known that malnutrition is typically caused by a combination of insufficient food intake and infection, which impairs the body's ability to absorb or assimilate food. Therefore, the situation in this context can be improved by providing better health infrastructure, by arranging proper diets for pregnant women, by improving calorie intake for children through midday meals at schools, and so on. Contributions of the indicators to the overall index indicate on which aspect a country will need to focus its attention to formulate pertinent policies.

Acknowledgements

The authors are grateful to John Komlos for his suggestion to include anthropometric indicators in the index. They thank the seminar participants at the Centre for Development Studies, Trivandrum and two anonymous referees for comments.

Notes

1 This procedure, which Dutta *et al.* (2003) referred to as Procedure II, was adopted in Chakravarty (2003) for developing a generalization of the HDI.
2 It may be noted here that while the HDI deals with achievement of attributes, the HPI is concerned with failure of attributes. Given that achievement and failure deserve to be treated separately, the generalized HDI considered in Chakravarty (2003) has a completely different objective from the generalized index developed here.
3 While the consequences of low BMI (malnourishment, thinness) are obvious, the consequences of high BMI (obesity) are diabetes, cardiovascular disease, hypertension, stroke, and so on.
4 A non-negative square matrix of order k is called a $k \times k$ permutation matrix if its entries are 0 or 1 and each of its rows and columns has only one positive element.
5 This indicator is commonly used for monitoring growth and to assess changes in the magnitude of malnutrition over time. It confounds the effects of short-term and long-term health and nutrition problems.
6 We may mention that information on anthropometric indicators is not available for these countries. Therefore, an analysis for these countries with UNDP dimensions will only detract from the value of the paper.

7 In principle, the distribution of deprivation of living standards across the three individual components can be investigated applying a similar approach and the underlying policy implication can be examined. However, due to close similarity of the exercise with the earlier ones, the exercise has not been done here.

References

Aczél, J. (1966) *Lectures on Functional Equations and Their Applications*, Academic Press, New York.

Alderman, H. (2000) 'Anthropometry', in M. Grosh and P. Glewwe (Eds), *Designing Household Survey Questionnaires for Developing Countries: Lessons from 15 years of Living Standards Measurement Study*, The World Bank, Washington, DC.

Anand, S. and Sen, A.K. (1993) 'Human Development Index: methodology and measurement', *Human Development Report Office Occasional Paper 12*, United Nations Development Programme, New York.

Atkinson, A.B. (1970) 'On the Measurement of Inequality', *Journal of Economic Theory*, 2(3), pp. 244–263.

Atkinson, A.B. (2003) 'Multidimensional deprivation: contrasting social welfare and counting approaches', *Journal of Economic Inequality*, 1(1), pp. 51–65.

Atkinson, A.B. and Bourguignon, F. (1982) 'The Comparison of Multidimensioned Distributions of Economic Status', *Review of Economic Studies*, 49(2), pp. 183–201.

Bourguignon, F. and Chakravarty, S.R. (1999) 'A family of multidimensional poverty measures', in D.J. Slottje (Ed.), *Advances in Econometrics, Income Distribution and Scientific Methodology: Essays in Honor of C. Dagum*, Physica-Verlag, Heidelberg.

Bourguignon, F. and Chakravarty, S.R. (2003) 'The measurement of multidimensional poverty', *Journal of Economic Inequality*, 1(1), pp. 331–344.

Chakravarty, S.R. (1990) *Ethical Social Index Numbers*, Springer-Verlag, New York.

Chakravarty, S.R. (1997) 'On Shorrocks' reinvestigation of the Sen poverty index', *Econometrica*, 65(5), pp. 1241–1242.

Chakravarty, S.R. (2003) 'A generalized Human Development Index', *Review of Development Economics*, 7(1), pp. 99–104.

Chakravarty, S.R. and D'Ambrosio, C. (2003) *The Measurement of Social Exclusion*, German Institute for Economic Research, Berlin.

Chakravarty, S.R., Mukherjee, D. and Ranade, R. (1998) 'On the family of subgroup and factor decomposable measures of multidimensional poverty', *Research on Economic Inequality*, 8, pp. 175–194.

Dutta, I., Pattanaik, P.K. and Xu, Y. (2003) 'On measuring deprivation and the standard of living in a multidimensional framework on the basis of aggregate data', *Economica*, 70(2), pp. 197–221.

Foster, J.E., López-Calva, L.F. and Székely, M. (2005) 'Measuring distribution of human development: methodology and an application to Mexico', *Journal of Human Development*, 6(1), pp. 5–25.

Kakwani, N.C. (1980) *Income Inequality and Poverty: Methods of Estimation and Policy Applications*, Oxford University Press, Oxford.

Kelley, A.C. (1991) 'The Human Development Index: handle with care', *Population and Development Review*, 17(2), pp. 315–324.

Kolm, S.C. (1969) 'The optimal production of social justice', in J. Margolis and H. Guitton (Eds), *Public Economics*, Macmillan, London.

Kolm, S.C. (1977) 'Multidimensional egalitarianism', *Quarterly Journal of Economics*, 91(1), pp. 1–13.

Lipton, M. and Ravallion, M. (1995) 'Poverty and policy', in J. Behrman and T.N. Srinivasan (Eds), *Handbook of Development Economics*, volume 1, North–Holland, Amsterdam.

Osmani, S.R. (1992) 'Introduction', in S.R. Osmani (Ed.), *Nutrition and Poverty*, Oxford University Press, Oxford.

Ravallion, M. (1996) 'Issues in measuring and modeling poverty', *Economic Journal*, 106(438), pp. 1328–1343.
Sen, A.K. (1973) *On Economic Inequality*, Clarendon Press, Oxford.
Sen, A.K. (1976) 'Poverty: on ordinal approach to measurement', *Econometrica*, 44(2), pp. 219–231.
Sen, A.K. (1985) *Commodities and Capabilities*, North–Holland, Amsterdam.
Sen, A.K. (1987) *Standard of Living*, Cambridge University Press, Cambridge.
Sen, A.K. (1997) *On Economic Inequality, with a Substantial Annexe by J. E. Foster and A. K. Sen*, Oxford University Press, Oxford.
Shorrocks, A.F. (1983) 'Ranking income distributions', *Economica*, 50(1), pp. 3–17.
Srinivasan, T.N. (1994) 'Human development: a new paradigm or reinvention of the wheel?', *American Economic Review, Papers and Proceedings*, 84(1), pp. 238–243.
Streeten, P. (1981) *First Things First: Meeting Basic Human Needs in Developing Countries*, Oxford University Press, Oxford.
Tsui, K.-Y. (1995) 'Multidimensional generalizations of the relative and absolute indices: the Atkinson–Kolm–Sen approach', *Journal of Economic Theory*, 67(1), pp. 251–265.
Tsui, K.-Y. (1999) 'Multidimensional inequality and multidimensional generalized entropy measures: an axiomatic derivation', *Social Choice and Welfare*, 16(1), pp. 145–157.
Tsui, K.-Y. (2002) 'Multidimensional poverty indices', *Social Choice and Welfare*, 19(1), pp. 69–93.
United Nations Development Programme (1990–2005) *Human Development Report*, Oxford University Press, New York.
Weymark, J.A. (2003) 'The Normative Approach to the Measurement of Multidimensional Inequality', Working Paper, Department of Economics, Vanderbilt University, Nashville, TN.
World Health Organisation (1995) *Physical Status: The Use and Interpretation of Anthropometry*, Report of a WHO Expert Committee, WHO Technical Report Series, 854, WHO, Geneva.

Appendix

Proof of Theorem 1:

MN requires increasingness of d. For NM to hold we need $d(0)=0$. Next, let $h>0$. Then given $x_i > y_i$, HD demands

$$d(x_i+h)-d(x_i) > d(y_i+h)-d(y_i). \tag{A1}$$

Dividing both sides of equation (A1) by h and letting h tend to zero, we get

$$d'(x_i) > d'(y_i), \tag{A2}$$

where d' is the derivative of d.

The argument given above shows that equation (A1) implies equation (A2). To prove the reverse implication, let us take the definite integral of the left (right)-hand side of equation (A2) from lower limit $x_i(y_i)$ to upper limit $x_i+h(y_i+h)$ to get

$$\int_{x_i}^{x_i+h} f'(z)\mathrm{d}z > \int_{y_i}^{y_i+h} f'(z)\mathrm{d}z \tag{A3}$$

which is equation (A1). Therefore, equation (A1) implies and is implied by equation (A2).

Given $x_i > y_i$, for equation (A2) to hold we require increasingness of d'; that is, strict convexity of d. In other words, we need $d''>0$, where d'' is the derivative of d'.

This establishes the necessity part of the theorem. The sufficiency is easy to verify.

Proof of Theorem 2:

Suppose that the deprivation index is of the form

$$t_1(x_i) = x_i^{\alpha} + 1,\ \alpha > 1, \tag{A4}$$

since $t_1(0)=1$, NM is violated. However, t_1 satisfies the remaining two postulates.

The index given by

$$t_2(x_i) = -\frac{x_i}{1+x_i} \tag{A5}$$

violates MN because it is decreasing in x_i. But it verifies NM and HD.

Since the index

$$t_3(x_i) = x_i^c, \quad 0<c<1, \tag{A6}$$

is strictly concave in x_i, HD is not fulfilled. However, t_3 meets NM and MN.

Proof of Theorem 3:

The idea of the proof is taken from Aczél (1966, pp. 239–240). For any i, $1 \le i \le k$, define $I\,(0, \ldots, 0, a_i, 0, 0, \ldots, 0) = t_i(a_i)$.

$$I(a_1, a_2, 0, \ldots, 0) = I(a_1, 0, \ldots, 0) + I(0, a_2, 0, \ldots, 0) = t_1(a_1) + t_2(a_2). \tag{A7}$$

Repeating this procedure we get

$$I(a_1, a_2, \ldots, a_k) = \sum_{i=1}^{k} t_i(a_i). \tag{A8}$$

Symmetry of I requires that t_i be independent of i, say $t_i=t$ for all i. Therefore,

$$I(a_1, a_2, \ldots, a_k) = \sum_{i=1}^{k} t(a_i). \tag{A9}$$

If a_i's are all identical (say $a_i=a$ to all i), then

$$I(a, a, \ldots \ldots, a) = kt(a). \tag{A10}$$

But by NOM in this extreme case

$$I(a, a, \ \dots \ \dots, a) = a. \tag{A11}$$

From equations (A10) and (A11) it follows that $t(a)=a/k$. Substituting this explicit form of t in equation (A9), we get

$$I(a_1, a_2 \ \dots, a_k) = \sum_{i=1}^{k} a_i/_k = \sum_{i=1}^{n} d(x_i)/k.$$

This completes the necessity part of the theorem. The sufficiency can be verified by noting that I in equation (5) fulfils NOM, ADD and SYM.

Proof of Theorem 4:

Consider the global index

$$G_1(a_1, a_2, \ \dots, a_k) = \sum_{i=1}^{k} a_i. \tag{A12}$$

Since NOM demands that the index should be expressed as an average of attribute-wise deprivations, G_1 does not satisfy NOM. However, it fulfils ADD and SYM.

We have noted that the k-dimensional HPI in equation (1) violates ADD but meets NOM and SYM.

Finally, suppose that the overall deprivation index is of the form

$$G_2(a_1, a_2, \ \dots, a_k) = \frac{1}{k}\sum_{i=1}^{k} a_i^{c_i}, \tag{A13}$$

where $c_i \geqslant 1$ for all i and $c_i \neq c_j$ if $a_i \neq a_j$ for $i \neq$ j, and $c_i = c_j = 1$ if $a_i = a_j$ for $i \neq$ j. By construction G_2 does not fulfil SYM, although it meets ADD and NOM.