Krengel-Lin decomposition for probability measures on hypergroups

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Abstract

A Markov operator P on a σ-finite measure space (X, Σ, m) with invariant measure m is said to have Krengel-Lin decomposition if $L^2(X) = E_0 \oplus L^2(X, \Sigma_d)$ where $E_0 = \{f \in L^2(X) \mid \|P^n(f)\| \to 0\}$ and Σ_d is the deterministic σ-field of P. We consider convolution operators and we show that a measure λ on a hypergroup has Krengel-Lin decomposition if and only if the sequence $(\check{\lambda}^n * \lambda^n)$ converges to an idempotent or λ is scattered. We verify this condition for probabilities on Tortrat groups, on commutative hypergroups and on central hypergroups. We give a counter-example to show that the decomposition is not true for measures on discrete hypergroups.

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1 Introduction and generalities

Let K be a locally compact space. Let $\mathcal{P}(K)$ be the space of regular Borel probability measures on K with the weak topology that is the smallest topology for which the functions $f \mapsto \mu(f)$ from $\mathcal{P}(K)$ to $\mathbb{R}$ are continuous for any bounded continuous function on K. For any $x \in K$, let δ_x be the probability measure concentrated at the point x and for any $\mu \in \mathcal{P}(K)$, supp(μ) is the support of μ (which is smallest closed subset C of K for which $\mu(C) = 1$).

A locally compact space K with a binary operation $*$ on the space $M^b(K)$ of bounded measures on K is called a *hypergroup* if

1. $*$ is bilinear and separately continuous from $M^b(K) \times M^b(K) \to M^b(K)$ so that $(M^b(K), +, *)$ is an associative algebra where $+$ is the usual additive operation,

2. the mapping $(\mu, \lambda) \mapsto \mu * \lambda$ from $\mathcal{P}(K) \times \mathcal{P}(K)$ into $\mathcal{P}(K)$ is continuous,

3. for every $x, y \in K$, the support of $\delta_x * \delta_y$ is compact and the mapping $(x, y) \mapsto \text{supp}(\delta_x * \delta_y)$ from $K \times K$ to $\mathcal{A}(K)$, the space of compact subsets of K with the Michael topology is continuous,

4. there exists an element $e \in K$ such that $\delta_e * \delta_x = \delta_x * \delta_e = \delta_x$ for all $x \in K$,

5. there exists an involution $x \mapsto \check{x}$ of K such that e is contained in the support of $\delta_x * \delta_y$ if and only if $x = \check{y}$ and $\mu \mapsto \check{\mu}$ is an anti-homomorphism of $\mathcal{P}(K)$.

We assume that K is a σ-compact hypergroup with a right-invariant Haar measure m. It is known that locally compact groups, commutative hypergroups, discrete hypergroups and central hypergroups admit invariant measures. For any locally compact group G and any compact subgroup M of G, the double coset space $G//M$ with the quotient topology and the convolution induced by the group operation in G is a hypergroup with an invariant measure (which is induced by the Haar measure on G): see [3] for results on hypergroups.

Let $L^2(K)$ be the space of square integrable function on K with respect to m. For $f \in L^2(K)$ and for $x, y \in K$, let

$$f(x * y) = \int f(z) d(\delta_x * \delta_y)(z).$$

For any $\lambda \in M^b(K)$ let P_λ be the convolution operator on $L^2(K)$ defined by

$$P_\lambda(f)(x) = \int f(x * y) d\lambda(y)$$

for all $x \in K$ and for all $f \in L^2(K)$. It is easy to see that for μ and λ in $M^b(K)$, $P_{\mu * \lambda} = P_\mu P_\lambda$ and for $\mu \in \mathcal{P}(K)$, P_μ is a contraction and $P_{\check{\mu}} = \check{P}_\mu$ where $\check{P}$ is the adjoint of P for any operator P (see [3] for more on convolution operators, for $f * \lambda$ in 1.2.15 of [3], $P_\lambda(f) = f * \check{\lambda}$).

Remark 1 Let $M^{(1)}(K)$ be the space all non-negative measures in $M^b(K)$ such that $0 \leq \mu(K) \leq 1$. We would like to remark following well-known facts regarding convolution operators on $L^2(K)$.

1. $\varphi: \lambda \mapsto P_\lambda$ is a Banach algebra representation of $M^b(K)$ into the space of bounded linear operators on $L^2(K)$.

2. $\varphi(M^{(1)}(K))$ and $\varphi(\mathcal{P}(K))$ are convex.

3. $M^{(1)}(K)$ is compact in the vague topology and hence $M^{(1)}(K)$ with vague topology is isomorphic to $(\varphi(M^{(1)}(K))$ with weak operator topology. In particular, $\varphi(M^{(1)}(K))$ is closed in the weak operator topology and hence - being convex - also in the strong operator topology.

4. On $\mathcal{P}(K)$ respectively $\varphi(\mathcal{P}(K))$ the above topologies and weak topology coincide.

A probability measure $\lambda \in \mathcal{P}(K)$ is called *scattered* if $\sup_{x\in K} \delta_x * \lambda^n(C) \to 0$ for all compact sets C in K where λ^n is the n-th convolution power of λ. It is known that for a locally compact group G, $\lambda \in \mathcal{P}(G)$ is scattered if and only if $||P_\lambda^n(f)|| \to 0$ for all $f \in L^2(G)$ (see Proposition 3.3 of [10]). It may be proved in a similar way that λ on a hypergroup is scattered if and only if $||P_\lambda^n(f)|| \to 0$ for all $f \in L^2(K)$.

For $\lambda, \mu \in M^{(1)}(K)$ and $x \in K$, $\mu\lambda$ and $x\mu$ denote $\mu * \lambda$ and $\delta_x * \mu$.

Let $E_0 = \{f \in L^2(K) \mid ||P_\lambda^n(f)|| \to 0\}$. Suppose that a probability measure $\lambda \in \mathcal{P}(K)$ is not scattered, then $L^2(K) \neq E_0$. Bartoszek and Rebowski studied all $f \in L^2(G)$ for which $||P_\lambda^n(f)|| \not\to 0$ for adapted probability measures λ (adapted probability measures are those probability measures for which the closed subgroup generated by the support is the whole group) on certain class of groups, namely it is proved in [2] that for adapted probability measures on groups G with left and right uniform structures are equivalent,

$$L^2(G) = E_0 \oplus L^2(G, \Sigma_d) \tag{1}$$

where Σ_d is the deterministic σ-field associated to the Markov operator P_λ and if λ is non-scattered, then $(L^2(G, \Sigma_d), P_\lambda)$ is isomorphic to the bilateral shift on $l^2(\mathbb{Z})$. This type of decomposition of $L^2(G)$ was first studied in [11] and for compact groups and abelian groups [11] gives an affirmative answer. The decomposition (1) of $L^2(K)$ is known as *Krengel-Lin decomposition*. Here, we are interested in proving the afore-stated Krengel-Lin decomposition for probabilities on general hypergroups.

We briefly sketch the results proved in this article. In section 2, we prove that a probability measure λ on a hypergroup K has Krengel-Lin decomposition if and only if $(\check{\lambda}^n\lambda^n)$ converges to an idempotent in $\mathcal{P}(K)$ or λ is scattered (see Theorem 2.1) and in the remaining sections we verify this condition for Tortrat groups (see Theorem 3.1), for commutative hypergroups (see Theorem 4.1) and for central hypergroups (see Theorem 4.2). In section 5, we give an example to show that Krengel-Lin decomposition does not hold for certain measures on discrete hypergroups.

2 Markov operators and measures on hypergroups

Let (X, Σ, m) be a σ-finite measure space. A *Markov operator* on (X, Σ, m) is a linear contraction $P\colon L^\infty(X) \to L^\infty(X)$ such that P preserves the cone of non-negative functions, $P(1) = 1$ and $P(f_n) \downarrow 0$ a.e. if $f_n \downarrow 0$ and $0 \leq f_n \leq 1$ in L^∞. The measure m is called *invariant* if $\int P(f)dm = \int f dm$ for all f. In that case P is also a contraction on $L^1(X)$ and therefore in all spaces $L^p(X)$ for $1 \leq p \leq \infty$ (see [11]). The convolution operators of probability measures on hypergroups (admitting a invariant measure) are examples of Markov operators.

The deterministic σ-algebra, Σ_d associated to P is defined as the σ-algebra of measurable sets A in Σ such that for each $n \geq 1$, $P^n(\chi_A) = \chi_{B_n}$ for some measurable set B_n in G. The deterministic σ-algebra was introduced to study the asymptotic behavior of the iterates of P.

We now recall the following results from [7]:

Theorem F Let P be a Markov operator on a σ-finite measure space (X, Σ, m) with invariant measure m. Then

(i) $L^2(X, \Sigma_d) = \{f \in L^2(X) \mid \check{P}^n P^n(f) = f \text{ for all } n \geq 1\}$ where $\check{P}$ is the adjoint of P on $L^2(X)$ and

(ii) if $f \perp L^2(X, \Sigma_d)$, then $P^n(f) \to 0$ in the weak topology.

It may be easily seen that the Krengel-Lin decomposition holds for P if and only if we can have strong convergence in Theorem F(ii). It is known that in general we cannot have strong convergence in Theorem F(ii) (see [11] and references cited there). Thus, the Krengel-Lin decomposition does not hold for any Markov operator.

We first state a Proposition for Markov operators on L^2-spaces.

Proposition 2.1 *Let P be a Markov operator on a σ-finite measure space (X, Σ, m) with an invariant measure m. Then*

(i) there exists a operator Q on $L^2(X)$ such that $\check{P}^n P^n \to Q$ in the strong operator topology,

(ii) if P is a normal operator, then $Q^2 = Q$ and

(iii) $\check{P}^n P^n(f) \to 0$ weakly (hence strongly in view of (i)) implies that $P^n(f) \to 0$ strongly.

Proof For a Markov operator P, (i) follows from a well-known result known as convergence of alternating sequences, here we include a proof of it. The sequence $(\check{P}^n P^n)$ is a decreasing sequence of positive contractions and hence $(\check{P}^n P^n)$ converges in the strong operator topology.

Suppose P is a normal operator, we have $\check{P}^n P^n = (\check{P}P)^n$ and hence $\check{P}PQ = Q$. This implies that $Q^2 = Q$. Thus, proving (ii) and that (iii) is easy to verify.

We now deduce from Theorem F and Proposition 2.1, a necessary and sufficient condition for a Markov operator to have Krengel-Lin decomposition: It may be mentioned that a similar result may be found in [13] Chapter IV, 4, Lemma 3.

Corollary 2.1 *Let P be a Markov operator on a σ-finite measure space (X, Σ, m) with an invariant measure m. Then P has Krengel-Lin decomposition if and only if $\check{P}^n P^n \to Q$ in the weak operator topology where Q is the projection onto $L^2(X, \Sigma_d)$.*

We now interpret the necessary and sufficient condition in Corollary 2.1 for measures on hypergroups.

Theorem 2.1 *Let K be a hypergroup and λ be a probability measure on K. Then λ has Krengel-Lin decomposition, that is $L^2(K) = E_0 \oplus L^2(K, \Sigma_d)$ if and only if either λ is scattered or $(\check{\lambda}^n \lambda^n)$ converges to an idempotent in $\mathcal{P}(K)$.*

Proof From Corollary 2.1 and Remark 1, we get that λ has Krengel-Lin decomposition if and only if $\check{\lambda}^n \lambda^n \to \rho$ in the vague topology where ρ is an idempotent in $M^{(1)}(K)$ such that $L^2(K, \Sigma_d)$ is the space of all P_ρ fixed functions. For any idempotent ρ in $M^{(1)}(K)$, either $\rho = 0$ or $\rho \in \mathcal{P}(K)$. If $\check{\lambda}^n\lambda^n \to \rho = \rho^2 \in \mathcal{P}(K)$, then by Theorem F(i) and since $\check{\lambda}^n\lambda^n\rho = \rho$, we get that $L^2(K, \Sigma_d)$ is the space of all P_ρ fixed points. This shows that λ has Krengel-Lin decomposition if and only if either λ is scattered or $(\check{\lambda}^n\lambda^n)$ converges to an idempotent.

3 Measures on Tortrat groups

In this section we prove the Krengel-Lin decomposition for probability measures on Tortrat groups. A locally compact group G is called *Tortrat* if for any sequence of the form $(x_n\lambda x_n^{-1})$ has an idempotent limit point in $\mathcal{P}(G)$ only if λ is an idempotent. Tortrat class was introduced by P. Eisele, this class contains all SIN-groups and all distal linear groups (see [6] and [12]).

We now prove the Krengel-Lin decomposition for certain probabilities. We first recall that for a locally compact group G, a probability measure $\lambda \in \mathcal{P}(G)$ is called *adapted* if the closed subgroup generated by the support of λ is G itself. The structure of non-scattered adapted probability measures on groups is well-studied in [9]. In [9], under some additional structural conditions on G or if λ is spread-out (a power of μ is not singular with respect to the Haar measure), it is proved that there exists a $g \in G$ such that $(g^{-n}\lambda^n)$ converges but in view of a result in [5] we are interested in studying the cases for which there is a $g \in G$ such that $(g^{-n}\lambda^n)$ converges to an idempotent.

Proposition 3.1 *Let G be a non-compact locally compact group and λ be an adapted regular Borel probability measure on G. Suppose there exist a compact normal subgroup K such that $\check{\lambda}^n\lambda^n \to \omega_K$. Then we have the following:*

1. *$L^2(G) = E_0 \oplus L^2(G, \Sigma_d)$;*

2. *Σ_d is the σ-algebra generated by $\{x^nK \mid n \in \mathbb{Z}\}$ for any x in the support of λ;*

3. *$(L^2(G, \Sigma_d), P_\lambda)$ is isomorphic to the bilateral shift on $l^2(\mathbb{Z})$.*

Proof Since $\check{\lambda}^n\lambda^n \to \omega_K$, (1) follows from Theorem 2.1.

We now claim that K is the smallest closed normal subgroup a coset of which contains support of λ. By Theorem 4.3 of [5], there exists a $x \in G$ such that $x^{-n}\lambda^n \to \omega_K$. This implies that $x^{-1}\omega_K\lambda = \omega_K$. Since K is normal,

$\omega_K x^{-1}\lambda = \omega_K$. This implies that λ is supported on Kx. It is easy to see that K is contained in any closed normal subgroup a coset of which contains the support of λ. This proves the claim.

Now, the rest of proof closely follows [2]. Since λ is adapted, K is open and hence by normalizing m, we may assume that $\omega_K(f) = \int f dm$. Now, for any x in the support of λ, we have

$$P_\lambda^n(\chi_{x^m K}) = \chi_{x^{m-n}K} \qquad (i)$$

for all m and n in $\mathbb{Z}$. This implies that the σ-algebra generated by $\{x^n K \mid n \in \mathbb{Z}\}$ is contained in Σ_d. For $f \in L^2(G, \Sigma_d)$, by Theorem F(i), $P_{\check{\lambda}^n * \lambda^n}(f) = f$ for all $n \geq 1$. Hence by assumption, $P_{\omega_K}(f) = f$. This implies that f is constant on the cosets of K. Thus, Σ_d is the σ-algebra generated by $\{x^n K \mid n \in \mathbb{Z}\}$ for any x in the support of λ. This proves (2) and (3) follows from equation (i).

We now prove the Krengel-Lin decomposition for measures on Tortrat groups.

Theorem 3.1 *Let G be a non-compact Tortrat group and λ be an adapted probability in $\mathcal{P}(G)$. Suppose λ is not scattered. Then $L^2(G) = E_0 \oplus L^2(G, \Sigma_d)$ and $(L^2(G, \Sigma_d), P_\lambda)$ is isomorphic to the bilateral shift on $l^2(\mathbb{Z})$. Also, the deterministic σ-algebra is generated by $\{g^n K \mid n \in \mathbb{Z}\}$ for any g in the support of λ and for some compact normal subgroup K of G.*

Proof Since λ is not scattered, $\check{\lambda}^n \lambda^n \to \rho \in \mathcal{P}(G)$. We first claim that $\rho^2 = \rho$. Suppose G is metrizable, by Theorem 1.1 of [5], there exists a sequence (x_n) in G such that $(x_n \lambda^n)$ converges. By Theorem 2.1 of [6], for all x in the support of λ, $x^{-n}\lambda^n \to \omega_H$ for some compact subgroup H such that $xH = Hx$. This implies that $\check{\lambda}^n \lambda^n \to \omega_H$ and $xH = Hx$ for all x in the support of λ implies that H is a normal subgroup since λ is adapted. In the general case, since λ is adapted, G is σ-compact and hence G can be approximated by metrizable groups. Then by applying standard arguments as in Theorem 3.4 of [6], we prove that ρ is an idempotent. Let K be a compact subgroup of G such that $\rho = \omega_K$. Since $\check{\lambda}^n \rho \lambda^n = \rho$ and λ is adapted we get that K is normal in G. Now the result follows from Proposition 3.1.

4 Measures on hypergroups

In this section we consider probability measures on commutative hypergroups and central hypergroups. Krengel-Lin decomposition for normal probability measures on hypergroups and hence in particular, for probability measures on commutative hypergroups may be easily deduced from Proposition 2.1 (ii).

Theorem 4.1 *Let K be a hypergroup and λ be a non-scattered normal probability measure on K. Then $L^2(K) = E_0 \oplus L^2(K, \Sigma_d)$. In particular Krengel-Lin decomposition holds for all $\lambda \in \mathcal{P}(K)$ if K is a commutative hypergroup.*

We next consider central hypergroups. Let K be a hypergroup. We shall denote the maximal subgroup of K by

$$G(K) = \{x \in K \mid x * \check{x} = e\}$$

and the center of K by

$$Z(K) = \{x \in K \mid x * y = y * x \text{ for all } y \in K\}.$$

The hypergroup K is called *central* if K/Z is compact where $Z = Z(K) \cap G(K)$; we remark that K/Z is again a hypergroup. Central hypergroups arise naturally as double coset spaces of compact subgroups of central groups and central hypergroups have invariant Haar measures (see [8] for a proof of the existence of Haar measures on central hypergroups). We first recall the following result on the shift compactness of factors and see 5.1.4 of [3] for a proof.

Proposition 4.1 *Let K be a metrizable hypergroup. Let (μ_n), (λ_n) and (η_n) be sequences of probability measures on K. Suppose $\mu_n = \eta_n \lambda_n$ for all $n \geq 1$ and (μ_n) is relatively compact. Then there exists a sequence (x_n) in K such that $(x_n \lambda_n)$ is relatively compact.*

The following result may be compared with Theorem 3.1 of [4].

Proposition 4.2 *Let K be a metrizable hypergroup and $\lambda \in \mathcal{P}(K)$. Suppose λ is not scattered. Then there exists a sequence (x_n) in K such that $(x_n \lambda^n)$ is relatively compact.*

Proof By Proposition 2.1, $\check{P}_\lambda^n P_\lambda^n \to P$ in the strong operator topology. Since $P_\lambda^n(f) \not\to 0$ for some $f \in L^2(K)$, $P(f) \neq 0$. By Remark 1, there exists a $\rho \in M^{(1)}(K)$ such that $P_\rho = P$ and hence there exists a $\rho \in M^{(1)}(K)$ such that $\check{\lambda}^n \rho \lambda^n = \rho$ for all $n \geq 1$. Replacing ρ by $\rho/\rho(K)$, we may assume that there exists a $\rho \in \mathcal{P}(K)$ such that $\check{\lambda}^n \rho \lambda^n = \rho$ for all $n \geq 1$. By Proposition 4.1, there exists a sequence (x_n) in K such that $(x_n \lambda^n)$ is relatively compact.

We now prove the Krengel-Lin decomposition for measures on central hypergroups.

Theorem 4.2 *Let K be a metrizable central hypergroup and $\lambda \in \mathcal{P}(K)$ be non-scattered. Then there exists an idempotent ρ such that $\check{\lambda}^n \lambda^n \to \rho$ in $\mathcal{P}(K)$ and $L^2(K) = E_0 \oplus L^2(K, \Sigma_d)$.*

Proof Suppose λ is not scattered. By Proposition 4.2 and since K is central hypergroup, there exists a sequence (g_n) in Z such that $(g_n \lambda^n)$ is relatively compact. Then $(\check{\lambda}^n \lambda^n)$ is relatively compact. In view of Proposition 2.1 (i) and Remark 1 there exists a $\rho \in \mathcal{P}(K)$ such that $\check{\lambda}^n \lambda^n \to \rho$.

Let $\mu_n = g_{n-1}^{-1} \lambda g_n$ and $\nu_k^n = \mu_{k+1} \cdots \mu_n$ for all $n \geq 1$ and $k < n$ where $g_0 = e$. Then (ν_0^n) is relatively compact. Arguing as in [4], we may prove that there exists a subsequence $n(i)$ such that $\lim \nu_k^{n(i)} = \tilde{\nu}_k \in \mathcal{P}(K)$ for all $k \geq 1$ and $\lim \tilde{\nu}_{n(i)} = \nu_\infty$. Also, ν_∞ is an idempotent in $\mathcal{P}(K)$. Now, for $n > 1$, $\lambda \lambda^{n-1} g_{n-1} g_{n-1}^{-1} g_n = \lambda^n g_n$. This implies that $(g_{n-1}^{-1} g_n)$ is relatively compact. Now for $n > 1$ and $k < n$, we have $\nu_k^n = \lambda^{n-k} g_{n-k} g_{n-k}^{-1} g_k^{-1} g_n$ and $\check{\nu}_k^n \nu_k^n = \check{\lambda}^{n-k} \lambda^{n-k}$ for all $n > 1$ and for all $k < n$. Thus, $\check{\tilde{\nu}}_k \tilde{\nu}_k = \rho$ for all $k \geq 1$. Thus, $\rho = \nu_\infty$ which is an idempotent. This shows that $(\check{\lambda}^n \lambda^n)$ converges to an idempotent. This proves the result.

5 Example

It is known that there exists a measure λ on certain locally compact groups such that λ is supported on a coset of a compact normal subgroup but $(\check{\lambda}^n \lambda^n)$ does not converge to an idempotent and hence λ does not have Krengel-Lin decomposition (see [5] or [9]) .

We now construct a discrete hypergroup and a measure λ such that $(\check{\lambda}^n * \lambda^n)$ does not converge to an idempotent. It may remarked that in [1] Bartoszek proved that probability measures on discrete groups admit Krengel-Lin decomposition but our examples show that shifted convolution powers on discrete hypergroups as compared to on discrete groups need not have similar behavior. Let A be an locally compact abelian group and α be an automorphism of A such that

1. $\alpha(g) \to e$ for all $g \in A$,
2. there exists a compact open subgroup L of A and $\alpha(L) \subset L$ and
3. there exists a $x \in A$ with $x^6 \notin L$ and $\alpha(x) \in L$.

Let $\mu = \frac{1}{2}(\delta_x + \delta_e)\omega_L$. Let G be the semidirect product of $\mathbb{Z}$ and A where the $\mathbb{Z}$ action is given by α. Define λ by $\check{\lambda} = (1, \mu)$. Let $g = (1, e)$. Then $\check{\lambda}^n g^{-n} = \mu\alpha(\mu)\cdots\alpha^{n-1}(\mu)$. It is easy to see that $\mu\alpha(\mu) = \mu$. Thus, $\check{\lambda}^n g^{-n} = \mu$ for all $n \geq 1$. Thus, $\check{\lambda}^n\lambda^n \to \rho$ where $\rho = (\frac{1}{2}\delta_e + \frac{1}{4}\delta_x + \frac{1}{4}\delta_{x^{-1}})\omega_L$. Suppose ρ is an idempotent, then either x^2 or x^3 is in L. This implies that $x^6 \in L$ which is a contradiction. Thus, ρ is not an idempotent. It is easy to see that λ is L-biinvariant. Let K be the hypergroup of L-double cosets in G. Then K is a discrete hypergroup and λ may be viewed as a probability measure on K. Thus, we have a discrete hypergroup K and a $\lambda \in \mathcal{P}(K)$ such that $(\check{\lambda}^n\lambda^n)$ does not converge to an idempotent.

We now provide a A, x, L and α satisfying the above conditions. Fix a prime integer p, let $\mathbb{Q}_p$ be the field of p-adic numbers and $|\cdot|$ be the p-adic absolute value. Now take $A = \mathbb{Q}_p$, x with $|x| = p^3$, $L = \{g \in \mathbb{Q}_p \mid |g| \leq 1\}$ and α is defined by $\alpha(g) = p^3 g$ for all $g \in \mathbb{Q}_p$. Then $|\alpha^n(g)| = p^{-3n}|g| \to 0$ for all $g \in G$. Also, $|6x| \geq p^2$ but $|\alpha(x)| = 1$, that is condition 3 is satisfied and it is easy to check condition 2. Using this idea one may construct many such examples.

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