Quasiprobability Distribution and Phase Distribution in Interacting Fock Space

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In this paper we study quasiprobability distribution and phase distribution for coherent states, squeezed states, and Kerr states in one-mode interacting Fock space.

KEY WORDS: quasiprobability distribution; phase distribution.

1. INTRODUCTION

To deal with fluctuating fields we introduce a distribution for the complex field amplitude in classical coherence theory. By integrating over the strength of the field we then obtain the phase distribution. But to define a Hermitian phase operator in the quantum mechanical description of phase goes back to the work of Dirac (1927). Dirac defined a phase operator by a polar decomposition of the annihilation operator. Thereafter, Susskind and Glogower (1964), Carruthers and Nieto (1968), Pegg and Barnett (1989), and Shapiro and Shepard (1991) contributed significantly. Dirac's phase operator was modified by Susskind and Glogower to a one-sided unitary operator. Nevertheless, their phase operator has been used in quantum optics extensively. Phase measurement statistics was introduced by Shapiro and Shepard through quantum estimation theory (Helstrom, 1976).

Keeping the ideas of Susskind and Glogower in mind we describe here a phase operator in interacting Fock space (Accardi and Bozejko, 1998) and study phase distribution of coherent states, squeezed states (Das, 2002), and Kerr states.

The work is organized as follows. In Section 2, we give preliminaries and notations. In Section 3, we introduce Kerr states in interacting Fock space. In Section 4, we describe coherent state representation of Kerr state. In Section 5, we calculate quasiprobability distribution of squeezed states and Kerr states. In Section 6, we give a description of phase distribution that we would like to associate to a given density operator. In Section 7, we give a few illustrative examples. In fact, we describe how the phase distribution will look like when we take coherent states, squeezed states, and Kerr states in interacting Fock space. And finally in Section 8 we give a conclusion.

2. PRELIMINARIES AND NOTATIONS

As a vector space one-mode interacting Fock space $\Gamma(\mathbb{C})$ is defined by

$$\Gamma(\mathbb{C}) = \bigoplus_{n=0}^{\infty} \mathbb{C}|n\rangle \tag{1}$$

where $\mathbb{C}|n\rangle$ is called the n-particle subspace. The different n-particle subspaces are orthogonal, that is, the sum in (1) is orthogonal. The norm of the vector $|n\rangle$ is given by

$$\langle n \mid n\rangle = \lambda_n \tag{2}$$

where $\{\lambda_n\}\rangle 0$. The norm introduced in (2) makes $\Gamma(\mathbb{C})$ a Hilbert space.

An arbitrary vector f in $\Gamma(\mathbb{C})$ is given by

$$f \equiv c_0|0\rangle + c_1|1\rangle + c_2|2\rangle + \cdots + c_n|n\rangle + \dots \tag{3}$$

with $\| f \| = (\sum_{n=0}^{\infty} |c_n|^2 \lambda_n)^{1/2} \langle \infty$.

We now consider the following actions on $\Gamma(\mathbb{C})$:

$$a^+|n\rangle = |n+1\rangle$$

$$a|n+1\rangle = \frac{\lambda_{n+1}}{\lambda_n}|n\rangle \tag{4}$$

a^+ is called the *creation operator* and its adjoint a is called the *annihilation operator*. To define the annihilation operator we have taken the convention $0/0 = 0$.

We observe that

$$\langle n \mid n\rangle = \langle a^+(n-1), n\rangle = \langle (n-1), an\rangle = \frac{\lambda_n}{\lambda_{n-1}} \langle n-1, n-1\rangle = \cdots \tag{5}$$

and

$$\| |n\rangle \|^2 = \frac{\lambda_n}{\lambda_{n-1}} . \frac{\lambda_{n-1}}{\lambda_{n-2}} \cdots \frac{\lambda_1}{\lambda_0} = \frac{\lambda_n}{\lambda_0} \tag{6}$$

By (2) we observe from (6) that $\lambda_0 = 1$.

The commutation relation takes the form

$$[a, a^+] = \frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} \tag{7}$$

where N is the number operator defined by $N|n\rangle = n|n\rangle$.

3. GENERATION OF KERR STATE

The Kerr vectors in $\Gamma(\mathbb{C})$ are defined by

$$\phi_K(\alpha) = e^{\frac{i}{2}\gamma a^+ a(a^+ a - 1)} f_\alpha \tag{8}$$

where $f_\alpha \in \Gamma(\mathbb{C})$ is a coherent vector given by (6), γ is a constant, and $a^+ a$ is defined by

$$a^+ a|n\rangle = \frac{\lambda_n}{\lambda_{n-1}}|n\rangle$$

Now,

$$\begin{aligned}
\phi_\alpha^K &= e^{\frac{i}{2}\gamma a^+ a(a^+ a - 1)} f_\alpha \\
&= e^{\frac{i}{2}\gamma a^+ a(a^+ a - 1)} \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n}|n\rangle \\
&= \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)}|n\rangle \\
&= \sum_{n=0}^{\infty} \left[\psi(|\alpha|^2)^{-1/2} \frac{\alpha^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)}\right]|n\rangle = \sum_{n=0}^{\infty} q_n|n\rangle
\end{aligned} \tag{9}$$

where

$$q_n = \psi(|\alpha|^2)^{-1/2} \frac{\alpha^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)} \tag{10}$$

The photon number distribution

$$P_n = \left|\langle n \mid \phi_\alpha^K\rangle\right|^2 = |q_n \lambda_n|^2$$

for the Kerr state is identical to that of the coherent state because the probability amplitude q_n and r_n differ only by a phase factor.

4. COHERENT STATE REPRESENTATION

To obtain the coherent state representation of Kerr state Φ_α^K we try to calculate the matrix element $(f_{\alpha'}, \Phi_\alpha^K)$, which contains all important information about the state Φ_α^K.

The matrix element is obtained by the following method. We utilize the completeness relation of coherent state in $\Gamma(\mathbb{C})$

$$I = \int_{\alpha\in\mathbb{C}} d\mu(\alpha)|f_\alpha\rangle\langle f_\alpha|$$

where

$$d\mu(\alpha) = \psi(|\alpha|^2)\sigma(|\alpha|^2) r\, dr\, d\theta \tag{11}$$

with $\alpha = r\, e^{i\theta}$ and $\sigma(x)$ is some weight function.

Now,

$$\begin{aligned}(f_{\alpha'}, \phi_\alpha^K) &= (f_{\alpha'}, U f_\alpha) \\ &= \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha'}, U | f_{\alpha_1}\rangle\langle f_{\alpha_1} | f_\alpha) \\ &= \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha_1}, f_\alpha)(f_{\alpha'}, U f_{\alpha_1})\end{aligned} \tag{12}$$

where $U \equiv e^{\frac{i}{2}\gamma a^+ a(a^+ a - 1)}$.

Now,

$$(f_{\alpha_1}, f_\alpha) = \psi(|\alpha|^2)^{-1/2}\psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{(\bar{\alpha}_1)^n (\alpha)^n}{\lambda_n} \tag{13}$$

and

$$(f_{\alpha'}, U f_{\alpha_1}) = \psi(|\alpha'|^2)^{-1/2}\psi(|\alpha_1|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{(\bar{\alpha}')^n (\alpha_1)^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)} \tag{14}$$

Hence we have

$$\begin{aligned}(f_{\alpha_1}, f_\alpha)(f_{\alpha'}, U f_{\alpha_1}) &= \psi(|\alpha_1|^2)^{-1}\psi(|\alpha|^2)^{-1/2}\psi(|\alpha'|^2)^{-1/2} \\ &\quad \times \sum_{m,n=0}^{\infty} \frac{(\bar{\alpha}_1)^n(\alpha)^n}{\lambda_n} \cdot \frac{(\bar{\alpha}')^m(\alpha_1)^m}{\lambda_m} e^{\frac{i}{2}\gamma \frac{\lambda_m}{\lambda_{m-1}}(\frac{\lambda_m}{\lambda_{m-1}} - 1)}\end{aligned} \tag{15}$$

Thus,

$$\begin{aligned}\left(f_{\alpha'}, \phi_\alpha^K\right) &= \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)\, (f_{\alpha_1}, f_\alpha)(f_{\alpha'}, U f_{\alpha_1}) \\ &= \sum_{m=0, n=0}^{\infty} \psi(|\alpha_1|^2)^{-1}\psi(|\alpha|^2)^{-1/2}\psi(|\alpha'|^2)^{-1/2} \frac{(\bar{\alpha}')^m(\alpha)^n}{\lambda_m \lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_m}{\lambda_{m-1}}(\frac{\lambda_m}{\lambda_{m-1}} - 1)} \\ &\quad \times \int_{\alpha_1 \in \mathbb{C}} d\mu(\alpha_1)(\bar{\alpha}_1)^n(\alpha_1)^m \\ &= \sum_{n=0}^{\infty} \psi(|\alpha|^2)^{-1/2}\psi(|\alpha'|^2)^{-1/2} \frac{(\bar{\alpha}')^n(\alpha)^n}{\lambda_n} e^{\frac{i}{2}\gamma \frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}} - 1)}\end{aligned} \tag{16}$$

where we have utilized the fact $\int_0^\infty dx\, \sigma(x) x^n = \frac{\lambda_n}{\pi}$ (Das, 2002).

5. QUASIPROBABILITY DISTRIBUTION

The *quasiprobability distribution*, known as the Q function, is the diagonal matrix elements of the density operator in a pure coherent state

$$Q(\alpha) = \frac{(\alpha|\rho|\alpha)}{\pi} \tag{17}$$

We now calculate the quasiprobability distribution for the following states:

5.1. Squeezed States

For the squeezed states f (Das, 2002),

$$f = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{(\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1})^2}{(\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n-1})^2\lambda_{2n}}\right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \frac{\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1}}{\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n}}|2n\rangle \tag{18}$$

we take the density operator to be

$$\rho = |f\rangle\langle f|, \quad \alpha = |\alpha|e^{i\theta_0} \tag{19}$$

and calculate the quasiprobability distribution $Q(\alpha')$ as

$$\begin{aligned} Q(\alpha') &= \frac{1}{\pi}(f'_\alpha, \rho f'_\alpha) \\ &= \frac{1}{\pi}(f_{\alpha'}, |f\rangle\langle f|f_{\alpha'}) \\ &= \frac{1}{\pi}|(f_{\alpha'}, f)|^2 \\ &= \frac{1}{\pi}\left|\sum_{n=0}^{\infty} \alpha'^{2n}\alpha^n \frac{\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1}}{\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n-2}\lambda_{2n}}\psi(|\alpha'|^2)^{-1/2}\right. \\ &\quad \left.\times\left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{(\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1})^2}{(\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n-1})^2\lambda_{2n}}\right]^{-1/2}\right|^2 \end{aligned} \tag{20}$$

5.2. Kerr States

For the Kerr states ϕ_α^K (9),

$$\Phi_\alpha^K = \sum_{n=0}^{\infty} q_n|n\rangle \tag{21}$$

where

$$q_n = \psi(|\alpha|^2)^{-1/2}\frac{\alpha^n}{\lambda_n}e^{\frac{i}{2}\gamma\frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}}-1)} \tag{22}$$

we take the density operator to be

$$\rho = |\phi_\alpha^K\rangle\langle\phi_\alpha^K|, \quad \alpha = |\alpha|\, e^{i\theta_0} \tag{23}$$

and calculate the quasiprobability distribution $Q(\alpha')$ as

$$\begin{aligned} Q(\alpha') &= \frac{1}{\pi}\left(f_{\alpha'}, |\phi_\alpha^K\rangle\langle\phi_\alpha^K|f_{\alpha'}\right) \\ &= \frac{1}{\pi}\left|\left(f_{\alpha'}, \phi_\alpha^K\right)\right|^2 \\ &= \frac{1}{\pi}\left|\sum_{n=0}^{\infty}\psi(|\alpha|^2)^{-1/2}\psi(|\alpha'|^2)^{-1/2}\frac{(\alpha')^n(\alpha)^n}{\lambda_n}\, e^{\frac{i}{2}\gamma\frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}}-1)}\right|^2 \end{aligned} \tag{24}$$

6. PHASE DISTRIBUTION

To obtain phase distribution we consider first the phase operator

$$P = \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} + a^* a\right)^{-1/2} a$$

and try to find the solution of the following eigenvalue equation

$$P f_\beta = \beta f_\beta \tag{25}$$

where $f_\beta = \sum a_n|n\rangle$.

Now,

$$\begin{aligned} P f_\beta &= \sum_{n=0}^{\infty} a_n \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} + a^* a\right)^{-1/2} a|n\rangle \\ &= \sum_{n=1}^{\infty} a_n \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} + a^* a\right)^{-1/2} \frac{\lambda_n}{\lambda_{n-1}}|n-1\rangle \\ &= \sum_{n=1}^{\infty} a_n \frac{\lambda_n}{\lambda_{n-1}} \left(\frac{\lambda_{N+1}}{\lambda_N} - \frac{\lambda_N}{\lambda_{N-1}} + a^* a\right)^{-1/2} |n-1\rangle \\ &= \sum_{n=1}^{\infty} a_n \frac{\lambda_n}{\lambda_{n-1}} \left(\frac{\lambda_n}{\lambda_{n-1}} - \frac{\lambda_n - 1}{\lambda_{n-2}} + \frac{\lambda_{n-1}}{\lambda_{n-2}}\right)^{-1/2} |n-1\rangle \\ &= \sum_{n=0}^{\infty} a_{n+1} \frac{\lambda_{n+1}}{\lambda_n} \left(\frac{\lambda_{n+1}}{\lambda_n}\right)^{-1/2} |n\rangle \\ &= \sum_{n=0}^{\infty} a_{n+1} \left(\frac{\lambda_{n+1}}{\lambda_n}\right)^{1/2} |n\rangle \end{aligned} \tag{26}$$

$$\beta f_\beta = \sum_{n=0}^{\infty} \beta a_n |n\rangle \tag{27}$$

From (25)–(27) we see that a_n satisfies the following difference equation:

$$a_{n+1}\left(\frac{\lambda_{n+1}}{\lambda_n}\right)^{1/2} = \beta a_n$$

That is

$$a_{n+1} = \beta\left(\frac{\lambda_{n+1}}{\lambda_n}\right)^{-1/2} a_n$$

Hence,

$$a_1 = \beta\left(\frac{\lambda_1}{\lambda_0}\right)^{-1/2} a_0$$

$$a_2 = \beta\left(\frac{\lambda_2}{\lambda_1}\right)^{-1/2} a_1 = \beta^2\left(\frac{\lambda_2}{\lambda_1}\right)^{-1/2}\left(\frac{\lambda_1}{\lambda_0}\right)^{-1/2} a_0 = \beta^2\left(\frac{\lambda_2}{\lambda_0}\right)^{-1/2} a_0$$

$$a_3 = \beta\left(\frac{\lambda_3}{\lambda_2}\right)^{-1/2} a_2 = \beta^3\left(\frac{\lambda_3}{\lambda_2}\right)^{-1/2}\left(\frac{\lambda_2}{\lambda_0}\right)^{-1/2} a_0 = \beta^3\left(\frac{\lambda_3}{\lambda_0}\right)^{-1/2} a_0$$

and so on.

Thus,

$$a_n = \beta^n\left(\frac{\lambda_n}{\lambda_0}\right)^{-1/2} a_0 = \beta^n(\lambda_n)^{-1/2} a_0$$

Hence

$$f_\beta = \sum_{n=0}^{\infty} a_n|n\rangle = a_0\sum_{n=0}^{\infty}\beta^n(\lambda_n)^{-1/2}|n\rangle$$

We take $a_0 = 1$ and $\beta = |\beta|\, e^{i\theta}$.

Then

$$f_\beta = \sum_{n=0}^{\infty} e^{in\theta}(\lambda_n)^{-1/2}|\beta|^n|n\rangle$$

Henceforth, we shall denote this vector as

$$f_\theta = \sum_{n=0}^{\infty} e^{in\theta}(\lambda_n)^{-1/2}|\beta|^n|n\rangle$$

where $0 \le \theta \le 2\pi$ and call f_θ a phase vector in $\Gamma(\mathbb{C})$.

Norm of the phase vector is given by

$$\| f_\theta \|^2 = \sum_{m,n=0}^{\infty} e^{in\theta} e^{im\theta} (\lambda_n)^{-1/2} (\lambda_m)^{1/2} |\beta|^{n+m} \langle n \mid m \rangle$$

$$= \sum_{n=0}^{\infty} \frac{1}{\lambda_n} |\beta|^{2n} \lambda_n = \sum_{n=0}^{\infty} |\beta|^{2n} \langle \infty$$

(if $|\beta| < 1$).

The phase vectors are complete. We can show that

$$I = \frac{1}{2\pi} \int_X \int_o^{2\pi} d\nu(x,\theta) | f_\theta \rangle \langle f_\theta | \tag{28}$$

where

$$d\nu(x,\theta) = d\mu(x)\, d\theta \tag{29}$$

Here we consider the set X consisting of the points $x = 0, 1, 2, \ldots$, and $\mu(x)$ is the measure on X which equals

$$\mu_n = \frac{1}{|\beta|^{2n}}$$

at the point $x = n$ and θ is the Lebesgue measure on the circle.

Define the operator

$$| f_\theta \rangle \langle f_\theta | \, : \, \Gamma(\mathbb{C}) \to \Gamma(\mathbb{C}) \tag{30}$$

by

$$| f_\theta \rangle \langle f_\theta | f = (f_\theta, f) f_\theta \tag{31}$$

with $f = \sum_{n=0}^{\infty} a_n |n\rangle$.

Now,

$$(f_\theta, f) = \sum_{n=0}^{\infty} e^{-in\theta} (\lambda_n)^{1/2} |\beta|^n a_n$$

and

$$(f_\theta, f) = \sum_{m,n=0}^{\infty} e^{i(m-n)\theta} (\lambda_m)^{-1/2} |\beta|^m (\lambda_n)^{1/2} |\beta|^n a_n |m\rangle$$

Hence

$$\frac{1}{2\pi} \int_X \int_0^{2\pi} d\nu(x,\theta) | f_\theta \rangle \langle f_\theta | f = \int_X d\mu(x) \sum_{m,n} (\lambda_m)^{-1/2} |\beta|^m (\lambda_n)^{1/2} |\beta|^n a_n |m\rangle$$

$$
\begin{aligned}
&\times \frac{1}{2\pi} \int_0^{2\pi} e^{i(m-n)\theta} d\theta \\
&= \int_X d\mu(x) \sum_{n=0}^{\infty} |\beta|^{2n} |n\rangle \\
&= \sum_{n=0}^{\infty} a_n |n\rangle |\beta|^{2n} \frac{1}{|\beta|^{2n}} \\
&= \sum_{n=0}^{\infty} a_n |n\rangle \\
&= f
\end{aligned}
\tag{32}
$$

We use the vectors f_θ to associate to a given density operator ρ, a phase distribution as follows:

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi} (f_\theta, \rho f_\theta) \\
&= \frac{1}{2\pi} \sum_{m,n=0}^{\infty} |\beta|^m |\beta|^n e^{i(n-m)} \left(\frac{|m\rangle}{\sqrt{\lambda m}}, \rho \frac{|n\rangle}{\sqrt{\lambda m}} \right)
\end{aligned}
\tag{33}
$$

The $P(\theta)$ as defined in (33) is positive, owing to the positivity of ρ, and is normalized

$$
\int_X \int_0^{2\pi} P(\theta)\, d\nu(x,\theta) = 1 \tag{34}
$$

where

$$
d\nu(x,\theta) = d\mu(x)\, d\theta \tag{35}
$$

for,

$$
\begin{aligned}
\int_X \int_0^{2\pi} P(\theta)\, d\nu(x,\theta) &= \int_X d\mu(x) \sum_{m,n=0}^{\infty} |\beta|^m |\beta|^n \frac{1}{2\pi} \int_0^{2\pi} \\
&\quad \times e^{i(n-m)} d\theta \left(\frac{|m\rangle}{\sqrt{\lambda m}}, \rho \frac{|n\rangle}{\sqrt{\lambda n}} \right) \\
&= \int_X d\mu(x) \sum_{n=0}^{\infty} |\beta|^{2n} \left(\frac{|n\rangle}{\sqrt{\lambda n}}, \rho \frac{|n\rangle}{\sqrt{\lambda n}} \right) \\
&= \sum_{n=0}^{\infty} \left(\frac{|n\rangle}{\sqrt{\lambda n}}, \rho \frac{|n\rangle}{\sqrt{\lambda n}} \right) = 1
\end{aligned}
\tag{36}
$$

The *phase distribution* over the window $0 \leq \theta \leq 2\pi$ for any vector f is then defined by

$$P(\theta) = \frac{1}{2\pi} |(f_\theta, f)|^2$$

7. EXAMPLES

We now consider some important states in the Hilbert space $\Gamma(\mathbb{C})$ and compute their corresponding phase distributions.

7.1. Coherent States

For the coherent states f_α (Das, 2002),

$$f_\alpha = \psi(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\lambda_n} |n\rangle \tag{37}$$

we take the density operator to be

$$\rho = |f_\alpha\rangle\langle f_\alpha|, \quad \alpha = |\alpha| e^{i\theta_0} \tag{38}$$

and calculate the phase distribution $P(\theta)$ as

$$\begin{aligned} P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\ &= \frac{1}{2\pi}(f_\theta, |f_\alpha\rangle\langle f_\alpha | f_\theta) \\ &= \frac{1}{2\pi}|(f_\theta, f_\alpha)|^2 \\ &= \frac{1}{2\pi} \left| \sum_{n=0}^{\infty} e^{in(\theta_0 - \theta)} |\beta|^n |\alpha|^n (\lambda_n)^{-1/2} \psi(|\alpha|^2)^{-1/2} \right|^2 \end{aligned} \tag{39}$$

7.2. Squeezed States

For the squeezed states f (Das, 2002),

$$f = \left[\sum_{n=0}^{\infty} |\alpha|^{2n} \frac{(\lambda_1\lambda_3\lambda_5 \cdots \lambda_{2n-1})^2}{(\lambda_2\lambda_4\lambda_6 \cdots \lambda_{2n-1})^2 \lambda_{2n}} \right]^{-1/2} \sum_{n=0}^{\infty} \alpha^n \frac{\lambda_1\lambda_3\lambda_5 \cdots \lambda_{2n-1}}{\lambda_2\lambda_4\lambda_6 \cdots \lambda_{2n}} |2n\rangle \tag{40}$$

we take the density operator to be

$$\rho = |f\rangle\langle f|, \quad \alpha = |\alpha| e^{i\theta_0} \tag{41}$$

and calculate the phase distribution $P(\theta)$ as

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\
&= \frac{1}{2\pi}(f_\theta, |f\rangle\langle f| f_\theta) \\
&= \frac{1}{2\pi}|(f_\theta, f)|^2 \\
&= \frac{1}{2\pi}\left| \sum_{n=0}^{\infty} e^{in(\theta_0-2\theta)}|\beta|^{2n}|\alpha|^n \frac{\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1}}{\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n-2}\sqrt{\lambda_{2n}}}\psi(|\alpha|^2)^{-1/2} \right. \\
&\quad \left. \times \left[\sum_{n=0}^{\infty}|\alpha|^{2n}\frac{(\lambda_1\lambda_3\lambda_5\cdots\lambda_{2n-1})^2}{(\lambda_2\lambda_4\lambda_6\cdots\lambda_{2n-1})^2\lambda_{2n}}\right]^{-1/2} \right|^2
\end{aligned}
\tag{42}
$$

7.3. Kerr States

For the Kerr states ϕ_α^K (9),

$$\phi_\alpha^K = \sum_{n=0}^{\infty} q_n|n\rangle \tag{43}$$

where

$$q_n = \psi(|\alpha|^2)^{-1/2}\frac{\alpha^n}{\lambda_n}e^{\frac{i}{2}\gamma\frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}}-1)} \tag{44}$$

we take the density operator to be

$$\rho = |\phi_\alpha^K\rangle\langle\phi_\alpha^K|, \quad \alpha = |\alpha|\, e^{i\theta_0} \tag{45}$$

and calculate the phase distribution $P(\theta)$ as

$$
\begin{aligned}
P(\theta) &= \frac{1}{2\pi}(f_\theta, \rho f_\theta) \\
&= \frac{1}{2\pi}\left(f_\theta, |\phi_\alpha^K\rangle\langle\phi_\alpha^K| f_\theta\right) \\
&= \frac{1}{2\pi}\left|\left(f_\theta, \phi_\alpha^K\right)\right|^2 \\
&= \frac{1}{2\pi}\left|\sum_{n=0}^{\infty} e^{in(\theta_0-\theta)}|\beta|^n|\alpha|^n(\lambda_n)^{-1/2}\psi(|\alpha|^2)^{-1/2}e^{\frac{i}{2}\gamma\frac{\lambda_n}{\lambda_{n-1}}(\frac{\lambda_n}{\lambda_{n-1}}-1)}\right|^2
\end{aligned}
\tag{46}
$$

8. CONCLUSION

In conclusion, we have first introduced Kerr states in the interacting Fock space and studied quasiprobability distribution of squeezed states and Kerr states in the space and then studied phase distribution in the space by defining a phase operator analogous to that studied by Susskind and Glogower and calculated specific phase distributions in the case of coherent states, squeezed states, and Kerr states.

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