Erratum: Correction to Phase Operator on a Deformed Hilbert Space

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A modification of phase operator is given.

I studied a phase operator $P = (q^n + T^*T)^{-1/2}T$ in five of my papers (Das, 1999, 2000a,b, 2001a,b) to describe the phase vector and studied its various applications. It appears that the phase operator is to be modified to an appropriate one. By analogy I propose the modified phase operator to be

$$P = (q^{N+1} + T^*T)^{-1/2}T$$

where

$$Nf_n = nf_n$$

and $\{f_n\}$ is given in Das (1998).

Now the phase vector is obtained by solving the eigenvalue equation

$$Pf_\beta = \beta f_\beta \tag{1}$$

where $f_\beta(z) = \sum_{n=0}^{\infty} a_n z^n = \sum_{n=0}^{\infty} a_n \sqrt{[n]!} f_n(z)$. That is,

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!} f_n. \tag{2}$$

Then

$$\begin{aligned} Pf_\beta &= \sum_{n=0}^{\infty} a_n \sqrt{[n]!} (q^{N+1} + T^*T)^{-1/2} T f_n \\ &= \sum_{n=1}^{\infty} a_n \sqrt{[n]!} (q^{N+1} + T^*T)^{-1/2} \sqrt{[n]} f_{n-1} \end{aligned}$$

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$$= \sum_{n=1}^{\infty} a_n \sqrt{[n]!}\sqrt{[n]}(q^n + [n-1])^{-1/2} f_{n-1}$$

$$= \sum_{n=0}^{\infty} a_{n+1} \sqrt{[n+1]!}\sqrt{[n+1]}(q^{n+1} + [n])^{-1/2} f_n \tag{3}$$

and

$$\beta f_\beta = \beta \sum_{n=0}^{\infty} a_n \sqrt{[n]!} f_n. \tag{4}$$

From (1)–(4) we observe that a_n satisfies the following difference equation:

$$a_{n+1} \sqrt{[n+1]!}\sqrt{[n+1]}(q^{n+1} + [n])^{-1/2} = \beta a_n \sqrt{[n]!}. \tag{5}$$

That is,

$$a_{n+1} = \frac{\beta a_n (q^{n+1} + [n])^{1/2}}{[n+1]}. \tag{6}$$

Hence,

$$a_1 = \frac{\beta(q + [0])^{1/2} a_0}{[1]}.$$

$$a_2 = \frac{\beta a_1 (q^2 + [1])^{1/2}}{[2]} = \frac{\beta^2 a_0 \sqrt{(q + [0])(q^2 + [1])}}{[2]!}.$$

$$a_3 = \frac{\beta a_2 (q^3 + [2])^{1/2}}{[3]} = \frac{\beta^3 a_0 \sqrt{(q + [0])(q^2 + [1])(q^3 + [2])}}{[3]!}.$$

and so on. Thus,

$$a_n = \frac{\beta^n a_0 \sqrt{(q + [0])(q^2 + [1])(q^3 + [2]) \cdots (q^n + [n-1])}}{[n]!}.$$

Hence,

$$f_\beta = \sum_{n=0}^{\infty} a_n \sqrt{[n]!} f_n$$

$$= a_0 \sum_{n=0}^{\infty} \beta^n \sqrt{\frac{(q + [0])(q^2 + [1])(q^3 + [2]) \cdots (q^n + [n-1])}{[n]!}} f_n.$$

where $\beta = |\beta| e^{i\theta}$ is a complex number. These vectors are normalizable in a strict sense only for $|\beta| < 1$.

Now, if we take $a_0 = 1$ and $|\beta| = 1$ we have

$$f_\beta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n. \qquad (7)$$

Henceforth, we shall denote this vector as

$$f_\theta = \sum_{n=0}^{\infty} e^{in\theta} \sqrt{\frac{(q+[0])(q^2+[1])(q^3+[2])\cdots(q^n+[n-1])}{[n]!}}\, f_n, \qquad (8)$$

$0 \leq \theta \leq 2\pi$ and call f_θ a *phase vector* in H_q.

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