Heap corrugation and hexagon formation of powder under vertical vibrations

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We report free-surface instabilities in a deep bed of fine granular material of irregular shape under vertical vibrations. At low frequency of vibration, the conical heap due to convective flow becomes unstable above a critical amplitude of vibration and acquires an azimuthal dependence which makes the heap surface corrugated. At even higher amplitude, the heap is no longer stable and splits into small heaps on a hexagonal lattice. At high frequency, we observe standing waves (stripes) at the same frequency as the driving one. The main mechanism of these instabilities can be traced back to the presence of the surrounding gas, since they vanish under vacuum conditions. [S1063-651X(99)07605-9]

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Vibrated granular media display various fluidlike properties [1]: roll convection [2], heaping [3–8], bubbling [9], free-surface excitation [10–16]. When the vessel acceleration is greater than gravity, various patterns appear on the free-surface of the grains. Pattern selection depends in particular on the dimensionless layer thickness $\mathcal{N}=h/d$ (where h is the layer thickness and d the typical size of particles). Typically when $\mathcal{N}$ is greater than 10 to 20 a heap forms spontaneously [1,15,16]. The appearance of the heap has been attributed to convective flow due either to grain friction with the container walls [8,16] or to the presence of the interstitial gas [6,16,17]. Although two-dimensional (2D) simulations account for convective motions [18] and heap formation [19] due to friction with the container sidewalls, very few simulations take the effect of air into account [20]. However, the ambient gas effect is prominent for a relatively deep bed ($\mathcal{N}\gg 1$) of small particles ($d<1$ mm) [17], and leads to a number of instabilities, as is reported here. In addition to the convection patterns in the bulk of the granular materials, its flat free-surface can exhibit different wave phenomena: parametric extended [1,10–14] or localized [21] standing surface waves (for $\mathcal{N}<10-20$), and period doubling instabilities [1,13–16] (any $\mathcal{N}$). For all these standing waves, convection due to air and to friction on the edges are negligible.

In this paper, we report qualitative observations of surface instabilities in a deep ($50\lesssim\mathcal{N}\lesssim 200$) bed of fine granular materials under vertical sinusoidal vibrations. We have used noncohesive irregularly shaped fine grains (from 40 to 120 μm) to have large intergrain friction in order to investigate the role of large dissipation on granular material under vibration. The small size of the grains makes the role of air prominent [17]. The choice of irregular shapes enhances dissipation due to inelastic collisions as well as solid friction [22]. Recently, both the effects of size and shape distributions have been shown to be linked to spontaneous stratification [22,23]. In such highly dissipative granular media, we report secondary instabilities: first, the heap corrugation which goes beyond the previous observations of heap formation (see Ref. [24] for an overview) and traveling waves on its surface [25], second, the formation of hexagons, and third, standing wave patterns (stripes) at the driving frequency. Those last two patterns are not caused by parametric instability, as was the case in previous observations on shallow layers of particles [12,13]. All these phenomena are independent of the presence of the container sidewalls and come from both the effects due to the interstitial gas and to the irregular shapes of the grains.

The experimental setup consists of a vertically vibrating vessel containing a powder (alumina powder, silica powder, or Ganga sand) with grains of irregular shape (see Fig. 1). An electromagnetic vibration exciter (BK4808) drives the container sinusoidaly, with frequency accuracy better than $\pm 0.6\%$. The vertical acceleration amplitude a is measured by means of a piezoelectric accelerometer (BK4374) glued on the top of the vessel. The horizontal component of the acceleration is measured to be less than 1.3% of the vertical one. The whole setup is fixed on a table with adjustable legs in order to ensure that it is horizontal. Our control parameters are the driving frequency f and the dimensionless acceleration amplitude $\Gamma=a/g$ ranging from 0 to 12, where g is the acceleration due to gravity. The experiments are conducted by increasing or decreasing the acceleration at a fixed value of the frequency in the range 10–300 Hz. Several containers of circular shapes (from 80 to 150 mm in diameter) or square

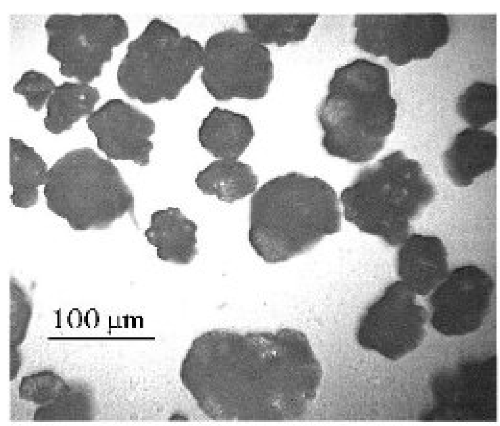

FIG. 1. Image from a reflection microscope of alumina grains used in our experiments.

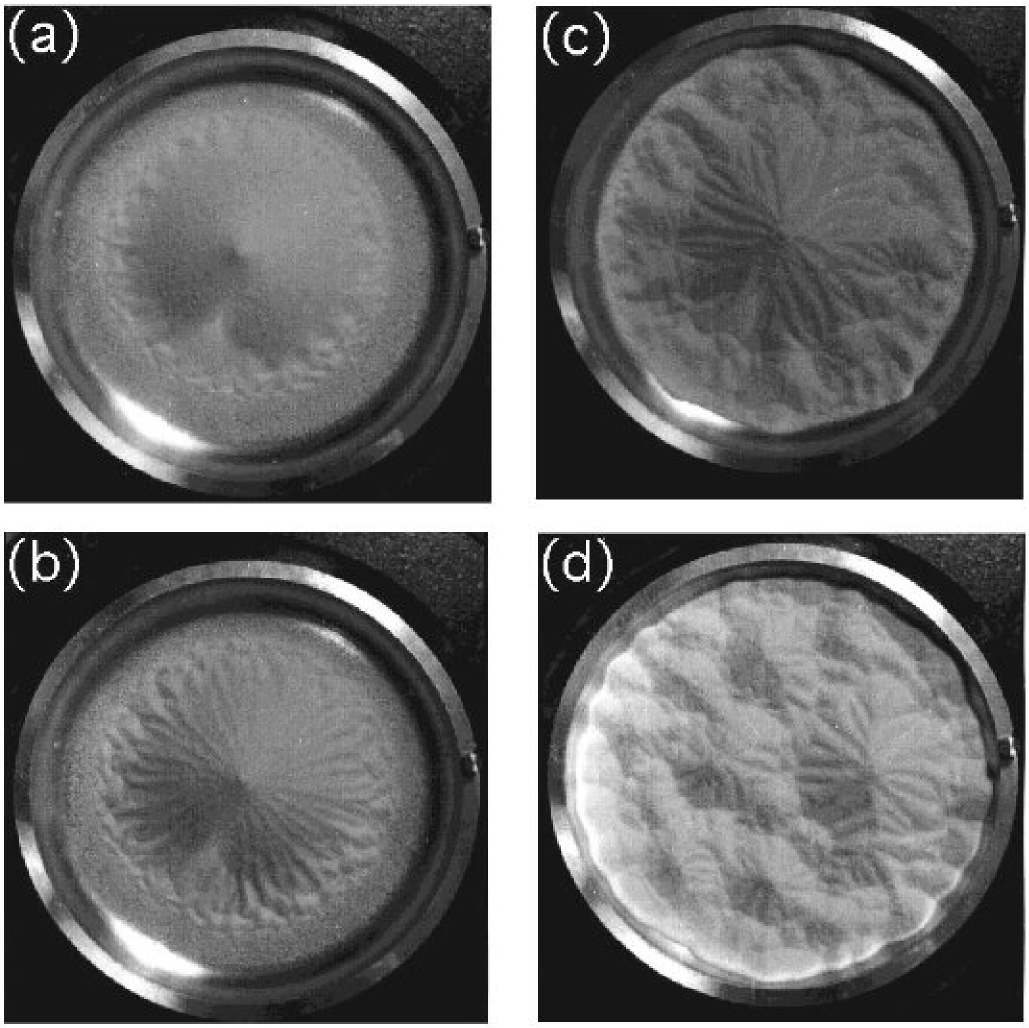

FIG. 2. Top view of free-surface instabilities of alumina powder (mass 40 g) at $f=42$ Hz. The powder surface is initially plane. (a) Spontaneous formation of a single heap due to convective motions at $\Gamma=2.87>\Gamma_{conv}$. (b) The corrugation of the free-surface of the heap ($\Gamma=3.17$). (c) Flattening of the heap at higher vibration amplitude ($\Gamma=3.75$). (d) Split of the single heap into many small corrugated heaps at much higher amplitude ($\Gamma=4.16$).

shapes (from 80 to 150 mm in side) have been used and are filled to different heights (typically from 50 to 200 layers) with a powder having a granulometry range from 40 to 120 μm. We have observed the same types of behavior whatever the vessel and the powder used, but the results reported here are from a set of experiments done in a duralumin cylindrical container, 120 mm inner diameter and 10 mm depth, filled with roughly 40 g of alumina powder. The motions of the free-surface of the granular material are visualized by a video camera or a still camera (Nikon FA) either with a direct or a strobe light. A small amount of the powder is soaked in fountain-pen ink and dried at roughly 200 °C for 1 h. This colored powder is then mixed in a small fraction, rougly 1%, to visualize motions of the grains in the heap more clearly.

When Γ is increased above some critical value Γ_{conv}, the initially flat surface of the granular material becomes unstable, and the grains self-organize themselves in the form of a single heap, as is shown in Fig. 2(a). The slope of the heap θ_d, i.e., the dynamic angle of repose [7], is determined by the dynamic equilibrium between falling particles (surface avalanches) and internal transport of fluidized grains from the base to the top of the heap (convective flow). This interpretation was first proposed by Faraday [3] and the convective regime due to air effect has been demonstrated by Laroche *et al.* [6]. In an evacuated container, i.e., at pressure lower than roughly 10 Torr, no heap formation is observed showing that convective flow due to the air is the relevant mechanism for heap formation for fine grains, i.e., $d \ll 1$ mm, in agreement with previous experiment [10,17]. Intuitively, Γ_{conv}, θ_d, and the shape of the heap must be dependent on various parameters (e.g., grain size, shape, roughness, cohesiveness, driving frequency). For all noncohesive, dry, and fine grains used here, we find that the onset of convective flow is $\Gamma_{conv}=1.17\pm0.06$ and is independent on the forcing frequency in the range 20–120 Hz. Others works [1,7–9,25] report that this value is independent on the grain shape and size. At higher frequency, Γ_{conv} slightly decreases (e.g., at 160 Hz, $\Gamma_{conv}=0.74\pm0.03$). The angle of repose is relatively high, roughly 30° at 40 Hz and $\Gamma=1.5$, which is typical of materials with nonspherical or rough grains. The angle θ_d decreases from 100 Hz to become zero near 200 Hz. The shape of the heap also depends on these parameters and varies from a cone (e.g., in sand and alumina

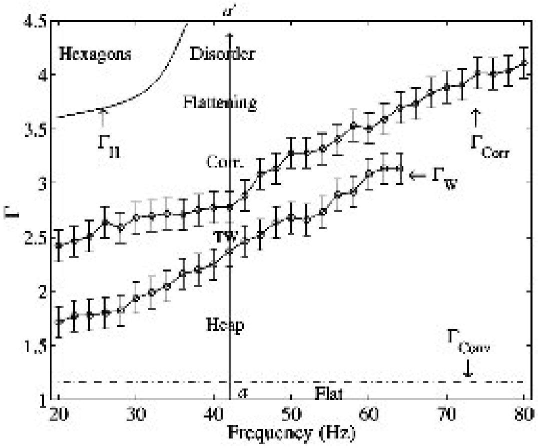

FIG. 3. Phase diagram showing several free-surface instabilities of alumina powder (mass 40 g). At a given frequency, $\Gamma_{\rm conv}$ (dashed curve), Γ_W (○), $\Gamma_{\rm corr}$ (◇), and Γ_H (upper curve) are, respectively, the onsets for the threshold of the convective flow, the traveling waves (TW), the heap corrugation regime and the hexagonal state. The transition towards hexagonal patterns is displayed here but is not measured precisely. A typical experiment in Fig. 2 consists of increasing the acceleration at fixed frequency (line aa').

at low frequencies) to nearly semispherical [e.g., in silica (alumina) at low (high) frequencies], the boundary effects being negligible as is shown in Fig. 2(b).

At higher $\Gamma > \Gamma_{\rm conv}$ and for fixed $f < 100$ Hz, we observe that the speed of the particles near the bottom of the heap becomes much higher than for those near the top. At this stage, the heap profile has two slopes, the upper one being steeper than the lower one. It seems similar to kink formed in static heap as in experiments of Makse *et al.* [22,23] except that in our case it is periodically moving upwards due to shaking. We refer to this phenomenon as *dynamic kink*. Bulges then appear where the two slopes meet. This circular corolla of bulges remains initially stable on the heap surface. At $\Gamma = \Gamma_W > \Gamma_{\rm conv}$ (see Fig. 3), it starts moving upward under the avalanche layers without interacting with the latter. This traveling wave disappears near the top of the heap and is periodically recreated where the two slopes intersect. This is similar to the observations of Pak and Behringer [25]. However, their narrow annular geometry and relatively large particles sizes (10 times ours) might have obscured the observation of dynamic kink in their experiment even though they saw bulges and traveling waves. Figure 3 shows the dependence of Γ_W on the driving frequency. This extends the result that, at fixed frequency, Γ_W is a grain size function [25]. Moreover, the interpretation of the upward direction of the traveling wave has been given in Ref. [25], but its formation due to dynamic kink of the heap was not known.

Increasing again Γ, we observe a remarkable new phenomenon: at $\Gamma = \Gamma_{\rm corr} > \Gamma_W$ (see Fig. 3), equispaced ''ridges'' and ''grooves'' appear along the inclined free-surface [see Fig. 2(b)]. The conical surface of the heap is now corrugated. Locations of the ridges and grooves alternate periodically with a period much larger than the driving one. This phenomenon of corrugation is quite robust, is insensitive to any small perturbations and leveling defects, and is also observed in silica powder and Ganga sand. However, this effect disappears when the experiment is repeated either with spherical beads of glass or zirconium, from 0.5 to 3 mm in diameter. Although the convection mechanism due to air is very diminished for particles of order of milimeter in characteristic length [26], it still leads to spontaneous heap formation (see also Ref. [25]). Absence of corrugation of this heap suggests that the friction between spherical particles is lower than the one between two particles of irregular shapes. This underlines the importance of energy dissipation on pattern selection in granular materials in response to the imposed excitation. Figure 3 shows the dependence of $\Gamma_{\rm corr}$ on the driving frequency.

With further increase in Γ (see Fig. 3), the heap flattens [see Fig. 2(c) and Ref. [27]] creating many small heaps at its base. Finally, the heap splits into many smaller corrugated heaps interacting with each other randomly [see Fig. 2(d)]. Note that all these behaviors (and in particular the corrugation) are not observed when our experiment is repeated in a container evacuated to a pressure less than 10 Torr. Therefore, *both* the air flow (fluidization) effect and the irregular shapes of grains seem to be necessary for corrugation formation on the heap surface.

The origin of the corrugations seems related to the intensified convective flow beneath the free-surface. Indeed, just before the threshold of corrugation and over one oscillation period of the container, more particles are convected to the top compared to particles falling along the free-surface due to avalanches. This leads to an unstable heap. The continuous azimuthal symmetry of the heap is then broken and the angle of repose becomes (azimuthal) angle dependent. We observe that the slope is slightly lower for the ridges than for the heap just before the onset of corrugation, the contrary being true for the grooves. This is enough to increase surface avalanches, which in turn balance the enhanced convective flow. By inserting colored particles at the lower rim of the heap, we observe that the upward motions of the grains are confined in a thin conical layer including the free surface. Because of different slopes for ridges and grooves, the particles falling rate is different along them and regularly spaced mounds then appear at the base of the grooves [see Fig. 2(b)]. This in turn affects the local convective flow at the base of the heap. Ridges becomes grooves and vice versa. This is why we see periodically alternating ridges and grooves. At the instability onset, the wavelength of the corrugation, i.e., the angular separation between the middle of two successive ridges or grooves, decreases with increasing frequency of vibration. This is in qualitative agreement with pattern formation in other dissipative systems [28,29].

When we increase again $\Gamma > \Gamma_{\rm corr}$ at a fixed frequency, each small corrugated mound in Fig. 2(d) interacts more vigourously with its neighbors: tall mounds give birth to smaller ones leading, after a while, to an homogeneous height for all mounds. Nucleations and creations of mounds then take place continually and erratically. At $\Gamma = \Gamma_H > \Gamma_{\rm corr}$, mounds self-organize themselves in hexagonal lattice as shown in Fig. 4. With a direct light, hexagons appears ''fluidized'' [see Fig. 4(a)]. With a strobe light, at the driving frequency, we observe also a ''condensed'' phase [see Fig. 4(b)]. For a small height of powder, i.e., corresponding to a mass of 6 g, the wavelength of the hexagons decreases when increasing the frequency in the range 20–50 Hz. We do not observe hexagonal patterns above this upper

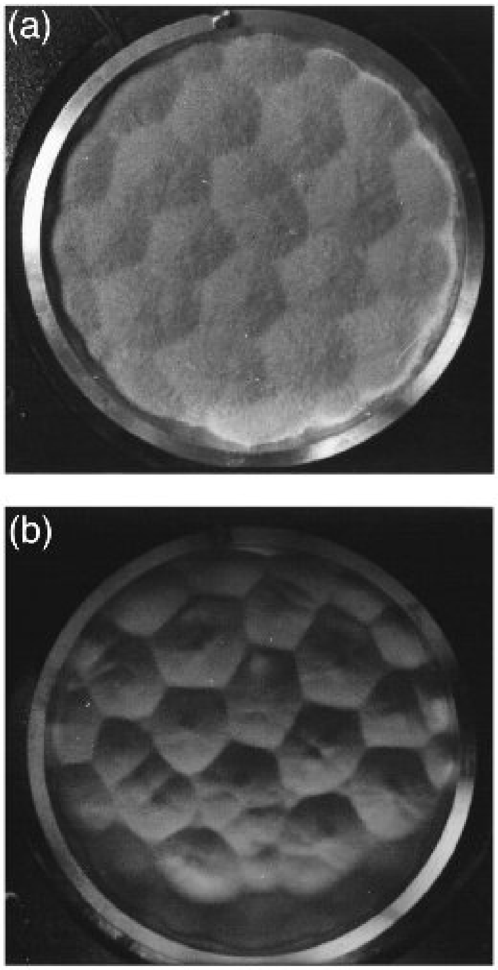

FIG. 4. Free-surface instability of alumina powder (mass 40 g) for $f=28$ Hz and $\Gamma=4.26$. (a) "Fluidized" hexagons visualized with a direct light. (b) "Condensed" hexagons visualized with a strob light (28 Hz). Note on both pictures the presence of defects (pentagons). Fluidized phase duration is 2 to 3 times longer than the condensed one which lasts roughly 5 to 6 s with a strobe light.

frequency. For a greater height of powder, i.e., corresponding to a mass of 40 g, the hexagons disappear at a lower frequency, i.e., $f\simeq 40$ Hz (see Fig. 3). At fixed f and Γ, the hexagon wavelength increases when the powder amount, inside the container, increases. Finally, for accelerations much above the onset of hexagons, the powder motion becomes completely erratic.

Let us now consider high frequencies ($f>100$ Hz) and small accelerations, just above the onset of convective instability Γ_{conv}. The dynamic angle of the heap decreases with increasing frequency, and the free-surface finally becomes flat at frequency around 200 Hz. At these higher frequencies, increasing Γ above the onset of convection, we observe standing wave stripes at the same frequency as the driving one (see Fig. 5). Increasing the acceleration, the stripes persist still the powder motion becomes completely erratic.

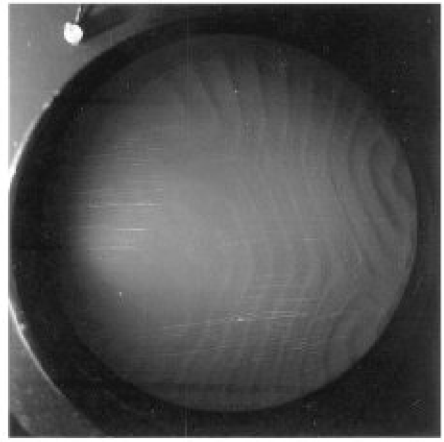

FIG. 5. Stripes on the flattened surface of alumina powder (mass 100 g) at high frequency ($f=215$ Hz) for $\Gamma=1.4>\Gamma_{conv}$. This standing waves at a vanishing frequency are visualized with a stroboscope ($f=215$ Hz). The cylindrical container is 96 mm in diameter and 48 mm in depth.

Hexagons and stripes on the free-surface of granular materials appear, at first instance, similar to those observed by Melo *et al.* [12,13]. However, our patterns are synchronous with the driving frequency, while those reported in Refs. [12,13] are subharmonically excited patterns. Our patterns are mainly due to fluidization of irregular shaped grains due to the presence of interstitial gas, and they appear in a *thick* layer of grains. Our patterns disappear when we perform our experiments in an evacuated container (e.g., with a pressure around 10 Torr). We do not observe hexagons. We observe instead subharmonically generated stripes as observed in Refs. [12,13]. Subharmonically generated patterns in *thin* layers ($\mathcal{N}<20$) of granular materials either in presence of air [12] or in vacuum [13] are similar to those observed in thin layers of viscous fluids under vertical excitation [28,29]. For thin layers of relatively large grains, the role of trapped air is, perhaps, not strong enough to influence the pattern selection.

To conclude, we report three instabilities of the free-surface of vibrated dissipative granular materials in presence of interstitial air: At fixed low frequency, the corrugation of the heap surface at low excitation amplitude and the self-organization of mounds in hexagonal lattice at relatively high excitation amplitude; standing waves (stripes) synchronous with the driving at high frequencies. They all appear due to irregular shapes of the very fine grains and fluidization due to the interstitial air. The theoretical explanation of these instabilities as well as the universal threshold Γ_{conv} of the convective motion are open and challenging problems.

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