## **Essays in Financial Intermediation**

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# This thesis is dedicated to Baba, Ma, Mum

and

Mausumi

### Acknowledgements

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## Chapter 1

## Introduction

In this thesis, we consider the functioning of financial intermediaries under different market setups. Such intermediaries are modelled as agencies that reduce the asymmetric information between the firm and the investors. We typically consider situations where a firm has a project and needs outside investment. In chapters 2, 3 and 4, we model the problems associated with project financing through banks. In chapter 5 we present a model of a credit rating agency.

Financial intermediaries in general provide two kinds of services-brokerage, and qualitative asset transformation (QAT) (Bhattacharya and Thakor [14]). Brokerage services include transaction services (e.g., buying/ selling of securities etc.), portfolio management, screening and certification, underwriting/issuance etc. Setting of term maturity, loan organization, credit risk management encompasses the various activities constituting qualitative asset transformation. In line with Bhattacharya and Thakor [14], we model financial intermediaries as institutions that reduce informational asymmetries among the various agents present in the economy. Bhattacharya and Thakor assert "... [financial] intermediation is a response to the inability of market mediated mechanisms to efficiently resolve informational problems," pg 14,[14].

The common feature connecting all the four chapters (chapters 2 - 5) is the presence of asymmetric information between the firm and its creditors. In chapters 2, 3 and 4, the firm has private knowledge about the project outcomes while the creditors do not. Additionally, in chapter 3, the firm also has private information about its project quality which the creditors do not possess. In chapter 5, the firm has private information regarding the riskiness

of its project which the investors do not possess.

Asymmetric information often implies that the firm is unable to undertake the project even if the project is viable. This is because the small investor, who cannot observe the private action or information available to the firm, finds it more risky to invest money in these firms. The small investors finds either gathering necessary information too costly, or the *free -rider* problem prevents them from gathering such information. Therefore, in the event that the firm cannot credibly signal its private information, information has to dissipate to the uninformed through a reliable third party. The financial intermediaries are successful in reducing this informational asymmetry between the individual investors and the managers of the projects. They do so by either diversifying project risks (Diamond [34]) or by adding small independent risks to the overall portfolio reducing the aggregate per head risks (Ramakrishna and Thakor [79]).

In chapters 2, 3 and 4, asymmetric information leads to a problem of strategic default or 'underreporting' of project realizations by the firm. Under reporting means that the firm reports a lower project realization than what actually is, in order to retain more as residual claims. In this setup, the institutional lenders or banks are the financial intermediaries that operate in a costly state verification (CSV) framework as in Townsend [92] and Moore [70]. To reduce this under reporting, the banks have to monitor the firm's claim regarding a low project realization. Here monitoring by the banks is analogous to auditing or verification of the firms' claim regarding its report of a low cash flow from the project. The framework therefore, crucially depends upon the monitoring activities of the intermediary. This is the QAT aspect of financial intermediation, where institutional lenders simultaneously fund projects with loans and at the same time monitor the borrower to reduce the default risk.

In chapter 5, we look at the role of a financial intermediary as information producer. This is the brokerage aspect of financial intermediation. The intermediary, in this case is a rating agency. The rating agency provides information to the investor about possible project qualities. The investment decision of the investor is contingent on how accurately this information is disseminated.

### 1.1 Multiple Lending and Optimal Hierarchy

### 1.1.1 Problem Description and Motivation

In this chapter, we focus on two major aspects of debt financing. One, we study why multiple banks lend to the same project, even though they are risk neutral and not constrained by availability of funds. Second, we determine the optimal hierarchical arrangement of the banks.

### 1.1.2 Framework

The model in Chapter 2 has two basic features. First, the lenders monitor only in bad states (Moore [70]). Monitoring, here is a commitment to verify or audit the reported realizations (Moore [70] and Townsend [92]). The lender monitors the firm because the firm can default even when the actual funds available with the firm do not warrant such default. Such defaults are termed *strategic*. In the absence of a credible monitoring mechanism with the creditors to verify the claims made by firms, the latter will always understate project realizations leading to such defaults.

In addition to determining the conditions that lead to multiple lending, in chapter 2, we derive the priority structure. Absolute Priority Rule (APR) is particularly important while addressing issues in bankruptcy procedures and private reorganizations or workouts. Considerable literature has developed that addresses such issues. Frank and Torous [43] defines APR as: "..Absolute Priority denies any claimholder a stake in the securities of the reorganized firm, until more senior claims have been fully satisfied." (pg 748 [43]). In the Indian context it is widely observed that during bankruptcy, while some banks are able to recover their dues before others, some do not recover any money (Anant, Gangopadhyay and Goswami [8]). Significant literature exists regarding the importance of Absolute Priority Rules (APR) (see Franks and Torous [43], Harris and Raviv [50]. Bulow and Shoven [21]. White [95] etc). However, none of these papers explain what determines these priority structures. We derive the factors that give rise to such hierarchies.

In the model presented here, the investors have different monitoring costs and are not fund constrained. Therefore, it is not mandatory here that the firm has to approach multiple lenders to undertake the project. What determines multiple lending is efficient combination of project financing through banks having different auditing costs as well as costs of raising funds.

#### 1.1.3 Results

We obtain the following results.

- Optimal number of lenders in a project is determined by the monitoring and capital costs. The lenders must have different monitoring and capital costs for them to lend to the same project.
  - Equal priority in claims is never optimal.
- Seniority is arranged according to the ascending order of monitoring costs. That is, the lender with the highest monitoring cost is the senior most.
- Net surplus in the system is more when the lenders communicate with each other.
- Higher net surplus with communicating lenders explain the observed phenomenon that (i) *group lending* is more efficient; and (ii) within group lending, *syndicated lending* is more efficient than consortium lending.

## 1.2 Multiple Lending and Asymmetric Information

### 1.2.1 Problem Description and Motivation

This chapter is similar to chapter 2 in many ways. The firm has a project with uncertain returns that requires a fixed initiation cost. The firm decides to finance the project with a combination of debt and equity. The firm approaches identical lenders for debt financing. The lenders, who finance the project with debt, also monitors the borrower in the process. The quality of the project owned by the firm is private information. We then identify the conditions under which multiple lending is optimal.

### 1.2.2 Results

In this framework, we establish the existence of a separating equilibrium. In this equilibrium, the superior type promoter borrows from multiple lenders while, the inferior type borrows from a single lender. Therefore, under asymmetric information, the superior promoter will approach multiple lenders for project financing even though the lenders are identical.

<sup>&</sup>lt;sup>1</sup>Monitoring is similar to Moore [70], Townsend [92] and Seward [87].

Though we obtain multiple lending with identical lenders, this result is distinct from Winton [98]. In Winton, multiple lending occurs with identical lenders due to credit-constrained investors. That is, the project initiation cost is more than the funds available with any single lender. Therefore, multiple lending is assumed in that model. In our model, the lenders are not fund constrained. Our result depends upon the fact that the firm having the superior project can use the costs associated with multiple lending to signal its type.

### 1.3 Monitoring and Optimal Investment

### 1.3.1 Problem Description

In chapter 4, we present a model of banks lending in a costly state verification framework. We investigate the underinvestment versus the overinvestment problem in this framework.

### 1.3.2 Framework

The firm has a project with stochastic returns. The project does not require any fixed initiation cost. To undertake the project, the firm can either borrow from the bank with debt or can fund it with equity. The project available with the firm is characterized by first order stochastic dominance. First order stochastic dominance in this context implies that projects with higher investments have lower default probabilities. In other words, the capital raised by the firm affects the probability of success. This capital is raised through debt from the bank. In order to prevent the firm from underreporting project outcomes, the bank monitors the firm. Monitoring in this context implies auditing the firm whenever the firm defaults.

### 1.3.3 Main Findings

In this setup, we find that a debt contract leads to overinvestment. However, as in the literature, costly state verification frameworks in general, leads to underinvestment with debt contract (Gale and Hellwig [44], Biais and Casamatta [15]etc). Underinvestment implies that the actual investment is

less than the first best investment level. Similarly, when the actual investment exceeds the first best level of investment, we have overinvestment. Our results differ from those in the literature because, in our model, the probability of success is positively related to the total amount of investment. However, in [15] and [44], the probability of success is independent of the investments made in the project. In other words, we obtain that, in equilibrium, higher investment by any one party induces higher investment by the other.

Our model also offers a possible explanation for different debt equity ratio across economies. The model also establishes an inverse relationship between auditing cost and the debt equity ratio. Our findings corroborate some of the empirical findings on the subject.

## 1.4 Rating Agencies- Efficiency and Regulation

### 1.4.1 Problem Description and Motivation

In this chapter we develop a model of information producing intermediariesthe credit rating agencies. We analyze whether the rating agencies enhance efficiency. In this framework, we also investigate the possibility of a regulator increasing net surplus by appropriate policies. Finally, we extend our model to include competition among rating agencies.

### 1.4.2 Framework

We consider a simple model where the economy consists of three sets of risk neutral agents - an investor, a firm and a credit rating agency. The firm has a project with uncertain returns requiring fixed initiation costs. The firm has private information regarding the default probability of her project. The firm does not have capital to initiate the project and hence, has to approach the investor. This investment is raised by offering debt. The investor is rational and has Bayesian beliefs.

The rating agency charges a fee to the firm if he comes to it for ratings. It then evaluates the firm using its screening technology. The fee is charged ex ante and is same across all the types. The precision of the screening function used by the rating agency, can be improved by the rating agency if it sets higher/stricter evaluation standards. The equilibrium involves the investors funding only those projects that are announced 'good' by the rating agency.

#### 1.4.3 Main Results

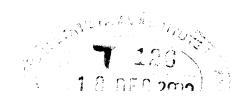
We initially assume that that the face value of debt is exogenously given. We then assume that in the absence of a rating agency, the firm is unable to raise necessary funds to undertake the particular project. The firm fails to do so because, the exogenously specified face value of debt, the investors perceive the firm to be of 'average' quality. This average quality in turn, is 'too risky' for the investors to invest.

Our results are as follows:

- In equilibrium, only a portion of the projects get funded. The projects with very high default probabilities do not come to the rating agency.
- Of those that go the rating agency, the investor invests in those that are declared to be good by the agency.
  - The agency unambiguously improves efficiency.
- Efficiency in the system cannot be increased by regulating the rating fees.

We then consider the case where the face value of debt is endogenous. By doing so, all the projects are funded even in the absence of a rating agency. The resulting equilibrium is a pooling equilibrium where all types set a common face value of debt. We then consider how the equilibrium changes with the presence of a rating agency. Our results are as follows.

- There is a separating equilibrium where different types set different debt claims.
- Projects with very high default probabilities are not funded at all while all other projects are funded.
- Depending upon the parametric values, the rating agency may actually end up reducing the net surplus. In fact, higher is the probability that the project is viable, greater is the inefficiency with the rating agency.
- In the presence of a regulator, who can set the fee, the surplus in the system can be increased. Interestingly, even in those cases where the rating agency was inefficient, regulating fees makes it efficient.
- Other regulatory policies like, compulsory ratings will in general be inefficient.
- Finally, we find that the regulated fee will in generally be lower than the profit maximizing, or unregulated, fee.



We also extend our framework to analyze competition among rating agencies. The results obtained are similar with competition than without it. Interestingly, we find that competition may actually be inefficient as compared to a monopolistic rating industry.

### 1.5 Literature Overview

Through chapters 2-4 we analyze some of the problems associated with bank lending in a costly state verification framework. In chapters 2 and 3, we focus mainly on the possibility of multiple lending. In chapter 2, we also look the existence of various forms of debt. In chapter 4 we look at problems pertaining to underinvestment and overinvestment along with the optimal leverage ratios.

One strand adopted in the literature that explains multiple lending is when firms may have different class of investors and/or securities. The different classes represent either dispersed versus concentrated debt holders or. loans with different term structure or maturities. In these models, the optimal number of creditors is obtained through exercises involving efficient liquidation/continuation decisions. Myers [72] mentioned that highly indebted firms may forego socially efficient investment projects because the creditors are the main beneficiaries from future cash flows. However, such projects would be financed with renegotiation possibilities among the various claim holders in the firm. In Gertner and Scharfstein [45] and White [96], bank loans lead to lower renegotiation costs. This is optimal for the firm. In reality, along with the bank loans, significant amount of project financing is made via public debt.<sup>2</sup> In Seward [87], the bank raises capital through bank loans as well as through dispersed debt. During bankruptcy, banks monitor the firm while the other creditors do not. Thus, monitoring by the bank prevents the manager from absconding with output in the bankruptcy states. In Detragiache [31], both public and private debt contracts play a role in the

<sup>&</sup>lt;sup>2</sup>We use the term public debt holders or dispersed debt holders interchangeably. Bank loans and private debt will be used interchangeably. The common feature that connects public debt holders/ dispersed debt holders on one hand and private debt holders/ banks on the other is, compared to private debts, public debts are costlier to renegotiate. This is because, the debtors holding public debts, find it costlier to monitor the entrepreneur. For chapters 2 and 4, we shall only consider the differences in monitoring costs across these debts. We do not incorporate renegotiation in any of our models. Chapter 3 in contrast. considers the creditors who have identical monitoring costs.

optimal capital structure because of their different renegotiation costs. The central result in [31] is that, the problems of asset substitution (Jensen and Meckling [58]) and underinvestment (Myers [72]) can be ameliorated if the firm borrows from both public and private sources.

Bolton and Scharfstein [17] and Dewatripont and Maskin [32] determine that the firm has multiple investors because multiple creditors can extract more cash flows from the firm than a single creditor during bankruptcy. Therefore, *strategic default* are less likely with multiple investors as renegotiation is likely to break down.

Hart and Moore [53], [54] obtains that multiple lending leads to an expost efficiency in renegotiations. A theoretical foundation for models that explain multiple lending to ensure efficient renegotiation outcomes is provided in Hart [51] and [52]. Other studies include Aghion and Bolton [2], Berglöf and Thadden [12], Diamond [36] and [38]. These papers establish the existence of debt instruments with different term maturities with the firms choosing more than one (class of) investors, and the investors separate their claims across time and the states of nature.

Diamond ([36] and [38]) considers a two period model and explain the existence of short term and long term debt. In [36], existence of multiple investors is assumed as the investors have different liquidation abilities. In [38], the choice is influenced by the entrepreneur's willingness to signal high quality projects as there is an asymmetry of information regarding project qualities between the entrepreneur and the investors. High quality projects are signaled by issuing short term debts. By issuing short term debts, the entrepreneur is willing to take the risk that refinancing may not be available in the case of a default.

However, Krishnaswami et-al [63] find little evidence that firms with favourable private information about future profitability choose more private debt. Evidence of contracting cost hypotheses or agency costs are also absent in empirical findings in Barclay and Smith [9], [10]. Foglia, Raviola and Reetz [42], study the effect of multiple banking vis-a-vis a single bank lending in the Italian context. They test for the hypothesis that, multiple borrowing leads to desirable sharing of risk as against the case that, with multiple banking, monitoring incentives for each bank considerably weakens. Their findings suggest that, a multiple bank arrangement is desirable provided there is a long term relationship between the firm and a lead bank who monitors the firm.

Effect of multiple lending on firm performance is also dealt in Dewatripont

and Tirole [33]. In [33], entrepreneur's effort choice is unverifiable. They find out how multiple investors induce more effort from the entrepreneur. If multiple investors hold different claims in different proportions, then tendency to liquidate the firm when it defaults, increases. Therefore, multiple investors with different claims will induce the manager to put in more effort.

The theoretical literature on multiple lenders investing in a single project is often explained through credit constrained investors (Winton [98]). In his model, the cost of the project is higher than the resources with a single investor. In Zender[100] too, the investors are cash constrained. In this framework he shows that traditional return streams of fixed repayments and residual claims arise endogenously. Hence, standard debt contracts (Gale and Hellwig [44]) are signed by the firm with more than one lender. Therefore, in [98] and [100], multiple lending is exogenous and a pre requisite. In this thesis, multiple lending is endogenous.

The literature on priority rules among the creditors, is based upon the various categories of debt. The categories are either public versus private debt, or short term versus long term debts. The analysis is carried under the ex-ante efficiency and the ex-post efficiency criteria. The ex-ante efficiency criterion mitigates the underinvestment and overinvestment problems.<sup>3</sup> The ex-post criterion requires the optimal priority rules to be determined by socially and privately efficient liquidation/continuation decisions made by the lenders. Rajan [76], White [95] and Diamond [36] analyze the problem under the ex-ante efficiency framework. These models crucially depend on the information gathering abilities of the banks vis-a-vis the dispersed holders. For instance, while a large lender (say, the bank) can prevent the managers from continuing with a negative net present value (NPV) project, it may often demand liquidation even when continuing with the project would be efficient.

In Rajan [76], bank lending reduces managerial effort as well as agency costs as the bank may withdraw financing in the second period. The optimal capital structure may include both monitored loan as well as arm's-length debt. In this model, bank debt must have lower priority. White [95] and Diamond [36] assert that dispersed debt holders must have a higher priority over the concentrated debt holders. Bulow and Shoven [21] consider the optimal priority structure based on the difficulties faced by the firm in renegotiating

<sup>&</sup>lt;sup>3</sup>The basic underinvestment and overinvestment result is due to seminal works done in Myers [72], Myers and Majluf [73].

with the dispersed debt holders, as against the banks.

An alternate formulation of the hierarchy problem concerns the optimal mix of short term and long term debts. The short term and long term debts differ according to their different maturity structures. Berglöf and Thadden [12], Gertner and Scharfstein [45], Hart and Moore [54], Berkovitch and Kim [13] and Diamond [38], [39] derive the optimal priority structure in this framework. Under different setups, they find out that long-term debt should be junior to short-term debt [39]. More recently Park [26] confirms the above findings in a CSV framework. Guedes and Opler [46], provide a literature survey on various theoretical models that links optimal maturity to liquidity risk, credit quality, debt agency costs and taxes.

In addition to the above, often the priority structure offered by a firm is based on debt covenants. These covenants restrict the firm from issuing any future debts which at the very least have the same priority as the existing debts. Smith and Warner [90] consider a random sample of 87 public issues registered with Securities and Exchange Commissions (SEC) between January '74 and December '75, they find more than 90% of bonds contained restrictions on the issuing of additional debt with seniority.

As evident from the literature, most works justify the existence of seniority among different kinds of claims. These claims could be either differing across maturities or across its holding concentration. Our model is closer to the second kind. We investigate the priority rule among creditors who differ across their monitoring (information gathering) abilities. Therefore, creditors with lower monitoring costs could be interpreted as either banks or long term creditors. <sup>4</sup> In this light, Winton's[98] paper is closest to ours.

Winton [98] considers the issue of hierarchical claims where investors have monitoring costs of the type we are considering. In his model, the manager does not have any funds with which to start the project. She is, therefore, forced to approach credit constrained investors. Therefore, multiple lending by banks is a prerequisite. All investors lend fixed amounts. The number of lenders are determined by the division of total amount of investment necessary, by the identical amount each investor has to invest. Within this framework he shows that strict hierarchy of claims dominates over equal pri-

<sup>&</sup>lt;sup>4</sup>While analogy between banks and creditors with relatively lower monitoring costs is easy to appreciate, the analogy with long term debt is less obvious. However, one can interpret holders of long term claims as creditors who has to monitor the firm in more states than the short term debt holders. Our results in chapter 2 would establish the analogy even further.

ority of claims, and that the optimal contract is a standard debt contract with each of the investors. Since all lenders are identical, any of the lenders can be a senior claimant to another lender.

We predict that, in a firm with multiple investors, the senior most claimant is the investor with the highest auditing cost. Ascending order of seniority corresponds to an ascending order of monitoring costs of the lenders. These results conform with the universally observed seniority pattern(see Barclay and Smith [10], Harris and Raviv [50]). Across different firms, in order of descending seniority, they list secured creditors, taxes rents and wages, unsecured creditors and equity. In chapter 2, we show how our results support the above findings.

In chapter 4 we use the CSV framework to explain how in contrary to the literature, overinvestment is obtained with debt contracts.

In Myers [72] the agency cost of debt occurs from the fact that, when the firms are likely to go bankrupt in the future, equity holders may have no incentive to undertake value increasing projects. This is because the equity holders have to bear the entire cost of the new project while the gains may mostly accrue to the debt holders. In [73], the investors are less informed about the current firm insiders. There is a conflict between the existing shareholders and the new investors. They show that positive NPV projects are not taken by the manager, representing the interests of the existing shareholders, as gains from doing so mostly accrue to the new shareholders.

Papers that have similar framework to ours, but obtain underinvestment in a CSV framework include Gale and Hellwig [44], Mukherji and Nagarajan [71], Biais and Casamatta [15]. In these papers, CSV framework leads to underinvestment with the standard debt contract. The reason being, with costly monitoring to encounter the moral hazard problems, credit rationing occurs thereby reducing the aggregate investment in the project. The frameworks mentioned above differ slightly with each other. In Gale and Hellwig [44], the source of moral hazard is underreporting of entrepreneur's return. In Mukherji and Nagarajan [71] and Biais and Casamatta [15], managerial effort is unobservable. Lower managerial effort implies a higher bankruptcy probability.

In Gale and Hellwig, underinvestment is in the form of credit rationing. Credit rationing occurs in the sense of rationing the size of the loan. They find that the "... optimal (second-best)investment level never exceeds and typically falls short of the first best, [this is ]... the basic underinvestment result." Pg 648 [44]. However, in our model, in spite of having credit ra-

tioning similar to that in Gale and Hellwig [44], we obtain overinvestment. Our result depends upon the fact that investments made by the bank and the promoter are interdependent.

In Narayanan [75] overinvestment occurs when asymmetry of information is only regarding the new project to be undertaken. He shows that in this context, some projects with negative NPV are also undertaken. This is because, perfect screening of firms is impossible when the only observable signal is that projects are undertaken or not undertaken. Therefore, in equilibrium, firms are priced at an average value implying some projects with low NPV are also undertaken.

As mentioned earlier, our overinvestment result occurs owing to the fact that, the investment by the firm and the bank are interdependent. Therefore, though we have a CSV framework, we do not obtain underinvestment as in [44] and others. Our results are not identical to that of [75] either.

The model in chapter 4 also model explains the observed phenomenon of differences in capital structures across industries and countries. With the irrelevance of capital structure as proposed by Modigliani and Miller [68], considerable work has been done to establish the contrary. Theoretical articles by Jensen and Meckling [58], Myers [72], Jensen [57], Ross [83] etc have established the relevance of debt financing under different market conditions. However, no satisfactory answer exists till date. Hart [52] asserts that "[there is] ... yet a model to explain... the widespread .. variation in debt across industries, countries...[etc.] " (Hart [52] pp 150-151). We establish the link between bank financing and capital structure. Our model differs from the existing literature primarily based on the project description. Most of the theoretical models that derive the optimal leverage ratio have a fixed project initiation cost. Therefore, choice of debt level immediately determines the equity level and hence, the debt equity ratio. In contrast, in chapter 4, both debt and equity are individually determined involving optimization exercises.

Harris and Raviv [49] provides an overview of papers that try to explain various capital structures or try to derive optimal capital structures. In chapter 4, we provide an interesting insight into this problem. Our results help us in understanding the factors that may explain the differences in capital structure across various economies.

Harris and Raviv [48] as well as Ross [83] establish the existence of a positive correlation between leverage and default probability. However, Castanias [24] does not find any empirical support for the above. Our results help to resolve the ambiguity. In our model, a decrease in default probability

may lead to either an increase in leverage (Harris and Raviv [48] etc.) or a decrease in leverage ratios (Castanias [24]).

Borio [20] compares the capital structure of the G-7 countries and conclude that companies in Japan and Continental Europe are more highly levered than the Anglo-American companies. The reason behind this, he attributes, is the financial structure and systems prevailing in these countries. Similar findings are reported in Berglöf [11]. The reasons again are, the financial systems prevailing in these countries. Berglöf [11] asserts that while financing in countries like Japan, Germany, France and Italy are 'bank oriented', in USA, UK and Canada, financing is more 'market oriented'.

Rajan and Zingales [78] consider the OECD data for these G-7 countries. They consider three alternate definitions of leverage. Leverage in [78] is measured as either the debt to capital ratio, or the debt to net assets ratio or the debt to assets ratio.<sup>5</sup> Their findings are that the differences in leverage across the G-7 countries are not as large as previously thought. Only firms in the UK and Germany appear to be substantially less leveraged than firms in the other G-7 countries. From the observed pattern, Rajan and Zingales hypothesize that the factors that explain leverage ratios across countries are banking strength and size (as similar to those proposed in [11] and [20]), tax codes, bankruptcy laws, bond markets, ownership patterns etc.

Demigürc and Levine [29], Demigürc and Maskimovic [30] compare the debt equity ratios across various developing and developed nations. Demigürc and Levine [29] conclude that the more developed the stock markets, higher are the debt equity ratios. This, despite the fact that an improved stock market induces a firm to have more equity. They conclude, "...Firms in countries with underdeveloped stock markets first increase their debt equity ratios as their stock markets develop..." (pp234) [29] (see Demigürc and Levine [29] and Demigürc and Mascimovic [30] for details). Our results support the above observations.

Some of the results obtained in chapter 2 have interesting implications on voluntary information disclosure and group lending. We find a strong case in support of voluntary disclosure of information by the bank in chapter 2. According to Diamond [35], voluntary disclosure of privately observed information to the shareholders may be actually be optimal for the firm. This will take place if voluntary disclosures change the incentives for costly private

 $<sup>^5{</sup>m The}$  various definitions used in [78] takes care of the discrepancies that may arise out of various accounting practises.

information production. Admati and Pfleiderer [1] conclude that voluntary disclosure can be efficient if (i) firm values are correlated; (ii) disclosure is costly, and (iii) individuals use disclosure by one firm to value another. The intuition is that the firms, which do not disclose their private information, are not valued highly. In our model, voluntary disclosure of information leads to higher overall efficiency because this reduces the expected monitoring costs. In this light we also establish that group lending is efficient as it reduces the transaction costs in the economy. coordination among the banks as a group ensures that duplication of verification costs does not take place. More importantly, more efficient auditor audits. Efficient delegation of monitoring activities is recognized in the literature even in other contexts (see Diamond [34], Krassa and Villamil [62] etc). In this vein, it can be argued that loan syndication is a more efficient form of group lending than consortium arrangement. The main advantage of a syndicated loan system over a consortium arrangement is that the former reduces appraisal time and eliminates the duplication of transaction costs in the system. Therefore, it is not surprising that the syndicated arrangements are increasingly replacing the existing consortium arrangements, both in India and elsewhere (see Ravishanker [80] and Megginson et -al [64], for the emergence of syndicated loan system in India and in the international context respectively).

The importance of lead bank monitoring and its effect on project financing has been more recently dealt in Hansen and Terregrosa [47] and Jain and Kini [55]. In [47], lead bank monitoring is valuable as it reduces agency costs. However, they do not study whether this improves corporate performance. Jain and Kini, considers the demand for monitoring in the Initial Public Offer (IPO) market using the 1976-1990 Securities Data Corporation's New Data Base. They find that there is a demand for lead bank monitoring. Further, they report that monitoring by the lead bank leads to a better post issue performance. Our results emphasize the above observations. In particular, we obtain that, loan syndication under a lead bank monitoring increases both firm profitability and net surplus.

However, Rajan [76] and Diamond [37] mention some of the problems associated with bank lending. In Rajan [76], banks develop information monopolies over borrowers and can distort investment incentives by demanding a share of the rents from profitable projects as a condition for rolling over short term loans. Diamond [37] asserts that, when banks maximize the value of their short term claims, they ignore rents accruing to borrowers in later periods and thus liquidate the borrowers too often. We continue to con-

sider bank lending as an important factor that affect the capital structure and other related decisions, is in line with Rajan and Winton [77]. Rajan and Winton explores the extent to which bank lending together with debt covenants, affect the overall efficiency. They find that covenants and the ability to collateralize, make a loans effective priority contingent on monitoring by the lender. Therefore, both covenants and collateral can be motivated as contractual devices that increases a lender's incentive to monitor.

Some of the recent works that trace the importance of bank lending on capital structure are listed below. Johnson [59] and [60] find a positive relation between bank financing and leverage. For a large sample of COMPU-STAT firms, [60] finds that leverage is statistically and economically significantly higher for firms that use bank debts. Specifically, mean leverage is approximately 41% higher for banks that use bank debts than for those that not. Datta et-al [28], James [56] finds that bank loans increase firm values. Carey et-al [23] establishes that creditors with distinct information acquiring advantages, add significant firm values.

Chapter 5 models the role of financial intermediaries at information producers or credit rating agencies. For a discussion on the functioning of a rating agency, what it does and how it operates, see Rose [82]. The various instruments ranked/ rated by Moody's and Standard and Poor's, the two leading rating agencies in business, are explained in [82]. For the detailed guidelines regarding the operation of the Indian rating agencies, see the SEBI Manual [86], CRISILSCAN [27] etc.

The early papers which emphasize the role of intermediaries as information producers are Leland and Pyle [61] and Campbell and Kracaw [22]. These papers highlight the role of intermediaries that reduce the informational asymmetry between the firm and the investors. Both these papers highlight the advantages that financial intermediaries have in gathering information at a lower cost than 'atomistic' investors. Bhattacharya and Thakor [14] provides a literature overview of information producing intermediaries.

Two other papers that also emphasize the role of financial intermediaries in lowering signaling costs include Diamond [34], Ramakrishna and Thakor [79]. In Diamond [34], the asymmetric information arises out of moral hazard problems related to unobservable effort by the manager. The financial intermediary is of infinite size and, can impose non pecuniary penalty on the manager in the event of default. Ramakrishna and Thakor [79] study the formation of a financial intermediary as a collection of many independent information gatherers or screening agents. Similar to Diamond [34], the in-

termediary is infinitely large. Lower information gathering costs are achieved as the cost of monitoring the firm per screening agents in the intermediary falls as the number of agents in an intermediary increases.

In Millon and Thakor [67] a theory of information gathering agents (screening agents) is proposed. In their framework, the information gathering agents certify the true value of a firm to the outside investors. There are two sources of asymmetric information in their model. Firstly, the moral hazard problem due to unobservable managerial effort. Secondly, an industry specific shock that is unobservable to the investors, but the screening agents can learn about this shock after spending resources. The existence of screening agents are established in these framework.

Another role of information gathering intermediaries is that of investment bankers. Allen and Faulhaber [4] studies the role of underwriter in the IPO market, where the investment banker gathers private information about new issues and conveys part of this information through underpricing of these issues.

Chemmanur and Fulgheri [25] model reputation of investment bankers in producing such information. Their structure is closely related to ours. The similarity in framework arises from the fact that, the investment bank in [25] and the rating agency in our model announces the quality of projects that comes to them to the market. While [25] does not consider the inference technology explicitly, we do. The announcements are strategic in [25], while they are not so in our model. Both models have Bayesian investors who update their initial priors about the firm's prospects on hearing the intermediary's announcements.

Some of the other papers that attempt to model various functioning of rating agencies and its possible impact on the debt market include Altman and Kao [6] Altman [5], Altman and Saunders [7] and Machauer and Weber [65]. Altman and Kao [6] theoretically model the impact of rating migration over time. Specifically, it studies how the investment banker changes its ratings of new IPOs as well as the older ones over time. Altman [5] and Altman and Saunders [6], empirically test their theoretical findings using Moody's and Standard and Poor's bond ratings over 1976-1990. Bond ratings are usually first assigned by rating agencies to public debt at the time of issuance and are periodically reviewed by the rating companies. A change in rating reflects the agency's assessment that the company's credit quality has improved (upgrade) or deteriorated (downgrade). These announcements have significant impact on the prices of these issues portraying the informa-

tion content of rating migrations to the investors (Altman [5]). Machauer and Weber [65] determine how bank lending is usually followed in two steps. First, the banks assess the default risk of the borrower and makes this information public. The actual lending takes place in the next stage.

In the Indian context, almost identical ratings of debt instruments by various rating agencies are reported in Venkatesh and Gupta [93]. Ravishanker and Thakur [81], asserts that, over time, on an average there is a significant upgrading of debt instruments in India.

Apart from financial intermediation, the role of agencies or 'experts' as information producers have also gained prominence. Biglaiser [16], Albano and Lizzeri [3] model the role of experts who evaluate product qualities. These models have similar framework to ours but they study very different problems.

## Chapter 2

# Multiple Lending and Seniority in Claims

### 2.1 Introduction

In this chapter, we focus on two major aspects of debt financing. One, we study why multiple banks lend to the same project, even though they are risk neutral and not constrained by availability of funds. Second, we determine the optimal hierarchical arrangement of the banks.

We adopt the costly state verification framework. The lenders monitor only in bad states (Moore [70]). Here, monitoring by the lender is equivalent to a commitment to verify or audit the realization reported by the borrower (Moore [70], Townsend [92]). The lender audits the firm because it may default even when the actual fund available with it does not warrant such default. If the creditors have no mechanism to verify the reports made by the firms, the latter will be encouraged to understate project realizations.

In our model the possible creditors to the firm are called the lending institutions. The lending institutions usually encompass a wide range of project financiers. These lenders have different auditing and capital costs. The lending institutions in India can be divided into depository and non-depository institutions. The commercial banks are examples of depository institutions.

<sup>&</sup>lt;sup>1</sup>Yafeh and Yosha [99] provide empirical support for this hypothesis.

<sup>&</sup>lt;sup>2</sup>There can be another type of monitoring. In this type, the lenders monitor the borrowers to ensure investment of resources in such a manner as to minimize the risk of default. A substantial portion of finance literature deals with this type of auditing (Diamond [34], [39] and Dietrich and Wihlborg [97]).

The commercial banks raise most of their resources through deposits. The depositors are protected by deposit insurance schemes. Commercial banks have lower opportunity costs of capital in comparison to institutions which do not have government guarantees. To attract capital, the latter institutions have to commit to expend more resources towards auditing. Such commitments serve as signals to the small investors, who provide capital to these institutions, that their resources are in safe hands. The investment banks typify non-depository institutions.

In our model, we find that both multiple lending and endogenization of claim priority are possible, where the banks are not credit-constrained. In particular, we solve for (a) the situation where multiple lending will be optimal, (b) the optimal amounts lent by each bank and (c) the optimal hierarchy of claims. Our findings are as follows. The hierarchy among creditors is exclusively determined by their auditing costs. However, the optimal number of creditors is determined by the auditing cost as well as the opportunity cost of capital.

We also find support in favour of why voluntary disclosure of information by the firm may be efficient. In our context, voluntary disclosure would amount to the fact that the firm publicly announces if it defaults on any creditor. We also establish *group lending* to be an efficient outcome, as it reduces the expected auditing costs in the economy. We therefore justify the recommendations of the Narasimham Committee [74] which suggested that banks should lend collectively to a project. Finally, we answer why a system of *syndicated* lending is more efficient than a system of *consortium* lending.

We find that, syndicated lending allows information flows from the lead bank to other lenders in the syndicate. This reduces the total expected auditing cost as all banks do not need to audit. Any auditing cost is a deadweight loss to the system. Since banks make zero profits, anything that increases the net surplus (for instance, through lower total auditing costs). improves the return to the promoter. Hence the promoter prefers syndicated lending to one where banks lend as independent entities.

The rest of the chapter is organized as follows. In Section 2.2, we describe the basic model. In Section 2.3, we derive the main set of results. In particular, the optimal hierarchy of claims is derived. We also demonstrate the situations where borrowing from multiple lenders would be desirable. Section 2.4 allows the lenders to communicate with each other. Some important implications of our results are discussed in section 2.5. Section 2.6 concludes the chapter.

### 2.2 The Model

The economy consists of two competitive lending institutions or lenders,  $L_i$ , i = 1, 2 and a promoter (entrepreneur), P.<sup>3</sup> The promoter has a project with uncertain return, z. It requires a fixed investment X, part of which the promoter can raise as debt, D and the remainder, if any, as equity. X is common knowledge.

**A.1:** z has a density function f(z). Its distribution function is F(z), with support  $[0, \bar{z}], 0 < \bar{z} < \infty$  and F(0) = 0.

There are four stages. In the first stage, the promoter decides whether to borrow. If the promoter decides not to borrow, then the project will be fully financed through equity. If she decides to borrow, she determines the number of lending institutions to borrow from. If the promoter borrows from multiple lenders, she announces the hierarchy on the basis of which their claims will be met. In the second stage, the lenders announce their debt claims as well as their investment levels (the loan) to the promoter. The total investment provided by the lenders is denoted by I. If I < X, an amount K = X - I has to be raised through the capital market as equity. The promoter does this in the third stage, with the project being initiated with the announced investment levels. In the fourth stage, the actual realization takes place and z is distributed between the promoter and the lender(s), with debt having senior claim over equity. The complete sequencing is illustrated below.

 $<sup>^3</sup>$ We restrict our model to only two lenders, for the sake of simplicity. The model can easily be extended to include more lenders.

<sup>&</sup>lt;sup>4</sup>Park [26] has a model where our first and second stages are collapsed into one, and the promoter decides on the amount of the loan from the bank(s). As we will show, the debt claim of each bank is a function of the actual value of the monitoring cost. For the firm to decide on the bank debt claim, it will have to know this exact value. For deciding the seniority of the creditors' claims, the promoter need to know only the relative monitoring costs of the banks, not their exact values. With full information on the bank's costs, the two approaches will give identical results.

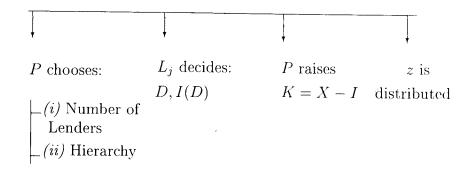


Figure 2.1

As in Moore [70], the lenders incur a fixed auditing cost,  $\theta$ , whenever the promoter defaults on her debt repayments. By incurring this cost, the lenders can observe the actual realization of the project. The auditing technology is perfect, i.e., once this cost is incurred, the lending institution knows the true realization accurately.<sup>5</sup> In the event that the promoter does not default, the lenders do not audit.

The only uncertainty in our model is about the project realization. However, in the fourth stage, the promoter knows the actual realization whereas  $L_i$  has to expend resources,  $\theta_i$ , to learn z.

We assume linear costs of raising funds for the promoter, q and the

<sup>&</sup>lt;sup>5</sup>Ex post it may not always be in the interest of the lenders to audit the promoter whenever she defaults. However, without any such commitment on the part of the lenders, the promoter may not have any incentive to report the true realization. Hence, she may want to default even when the project realization does not warrant such default. In other words, such a commitment rules out strategic defaults referred to in Bolton and Scharfstein [17] and Hart and Moore [53]. We are implicitly assuming that the promoter knows that she will be 'audited' with probability one, if she defaults. Consequently, she will be caught and then be required to pay heavy fines. More realistically, this audit probability may be less than one (Mookherjee and Png [69]) and yet, deter the promoter from lying provided the fines are sufficiently high. It will become evident that this does not affect the nature of our results as long as the expected cost of underreporting is such that the promoter does not indulge in strategic default.

2.2. THE MODEL 27

lenders, r. In general, q will be higher than the lender's cost of capital, r. If,  $q \le r_i$ , i = 1, 2, the promoter will never take a loan.

**A.2:** 
$$\theta_1 > \theta_2 > 0$$
 and  $0 < r_1 < r_2 < q < z^e/X$ , where  $E(z) = \int_0^{\bar{z}} z dF(z) = z^e$ .

Lenders specializing in auditing may not be the most efficient fund raisers. Deposit institutions like commercial banks, may raise funds easily, covered as they usually are with deposit insurance. With deposit insurance, they will be less inclined to develop sufficient expertise in monitoring. This could result in higher auditing costs for the commercial banks. Non-depository institutions, on the other hand, with better monitoring expertise will have lower auditing costs. The assumption  $q < z^e/X$  ensures that the project is viable, and will be funded by the promoter even if she decides not to borrow. The trade-off between the two costs ensures the functioning of both these kinds of lending institutions in the market.

Thus, given A.2, from now on, whenever we refer to lender  $L_1$  we will mean the lender with the higher auditing cost, and a lower cost of raising capital. Similarly,  $L_2$  would refer to the lender with a lower auditing cost and a higher capital cost.

The auditing cost is a cost in addition to the cost of capital. However, this cost is borne by the bank in the event of a default only. Of course, this affects the bank's payoffs in default states and, hence, the overall profitability of the loan to the bank. This, in turn, affects the bank's investment, given the debt claim. Since the equity market is more costly, lower bank investment hurts the overall profitability of the project. Thus, in principle, the promoter will prefer both capital and auditing costs to be low. In this paper we show that, when banks are differently ranked by their two cost components, and there is credit rationing (banks do not supply all the capital necessary for the project), there exists an optimal hierarchy that is endogenously determined.

We organize the model by dividing it into three parts. In the first part, we deal with a single lender. The next part deals with the case where there is strict hierarchy in claims of lenders. In the third part, we allow for multiple lenders with equal priority in claims.

### 2.2.1 Lender j is the *sole* claimant.

The expected return to  $L_i$  is denoted by  $R_i$ . Therefore,

$$R_{j} = \int_{0}^{\delta_{j}} z dF(z) + \int_{\delta_{j}}^{\bar{z}} \delta_{j} dF(z) - \int_{0}^{\delta_{j}} \theta_{j} dF(z)$$
$$= z^{e} - \int_{\delta_{j}}^{\bar{z}} (z - \delta_{j}) dF(z) - \theta_{j} F(\delta_{j})$$
(2.1)

where  $\delta_j$  is the debt claim of  $L_j$  in the project. In the first line of (2.1), the first term on the right hand side represents the expected return to  $L_j$  from the project when the promoter defaults (i.e,  $z < \delta_j$ ). The second term gives the expected return to the lender when her claim of  $\delta_j$  is satisfied (i.e.  $z \ge \delta_j$ ). The final term is the expected auditing cost to  $L_j$ . Recall that a lender audits the firm only when the firm defaults in paying her debt claim and not otherwise.

Let  $\delta_j^*$  maximize<sup>6</sup>  $R_j$  given in equation (2.1). Denote  $R_j^* \equiv R_j(\delta_j^*)$ . A necessary condition for  $0 < \delta_j^* < \bar{z}$  is that at  $\delta_j^*$ 

$$\frac{dR_j}{d\delta_j}|_{\delta_j = \delta_j^*} = 0 \Rightarrow [1 - F(\delta_j^*)][1 - \theta_j h(\delta_j^*)] = 0, \tag{2.2}$$

where,

$$h(.) \equiv \frac{f(.)}{[1 - F(.)]}$$

h(.) is the hazard rate. To ensure the second order condition for a solution, we assume the following about the hazard rate.

**A.3:** The hazard rate is increasing, i.e, h'(.) > 0.

<sup>&</sup>lt;sup>6</sup>Variables with asterisks denote optimal values.

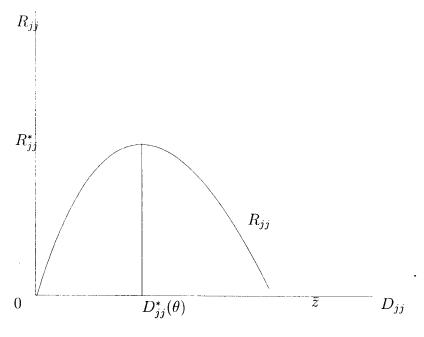


Figure 2.2

In figure 2.2, the net return function for  $L_j$  initially increases with a rise in debt claim and then decreases. A higher debt claim implies a higher return during the non-default states. It also increases the probability of default. As a result, the expected auditing cost of the lender increases. Therefore, there exists an optimal debt claim (here  $\delta_j^*$ ), such that the net return to  $L_j$  is maximized. It may well be the case that the optimal lending by the banks may not cover the amount required for investment. This is a form of credit rationing (Moore [70]). The underlying reason being, beyond  $\delta_j^*$ ,  $L_j$ 's expected return falls with a higher debt claim. This induces the lender to invest a lower amount in the project. A higher debt claim increases the lender's expected auditing cost as the probability of default increases.

Earlier, we stated that lending institutions operate in a competitive environment. This essentially means that they make zero profit. If  $\eta_j$  is the investment made by  $L_j$  in the project, then the zero profit condition is simply  $R_j - r_j \eta_j = 0$ , j = 1, 2. This solves for  $\eta_j$  being equal to  $R_j/r_j$ .

With a single lender, the investment of the promoter will be the difference between the total investment, X, and the investment made by the lender.

Thus, the expected profit to the promoter is given by,

$$\pi_j = \int_{\delta_j}^{\bar{z}} (z - \delta_j) dF(z) - qK. \tag{2.3}$$

where  $K = X - \eta_j$ . While the first term in equation (2.3) represents the residual claim of the promoter on the project, the second term represents the cost of equity.

The net surplus is the sum of the returns accruing to the lender and the promoter. Therefore, the net surplus,  $s_j$ , is given as  $s_j = R_j - r_j \eta_j + \pi_j$ . The lenders earn zero profits in equilibrium. This implies that any contract that maximizes the promoter's profit automatically maximizes the net surplus. Therefore, any arrangement that is optimal for the promoter is also efficient. Using (2.1) and (2.3), we get

$$s_j = z^e - qX - \theta_j F(\delta_j) + (q - r_j)\eta_j$$

To arrive at the net surplus, we have to deduct three sets of costs: the two opportunity costs of investment - one for the lender and the other for the promoter and the expected deadweight loss of auditing.<sup>7</sup>

### 2.2.2 Lender j is the senior claimant

Strict hierarchy implies that, the promoter can pay the junior lender only if the senior lender has been paid in full. Henceforth, we will use the following convention: the subscript ij in variable  $x_{ij}$  will imply that i is senior. Thus,  $R_{ij}$  will be the return of lender j when i is senior. If, however, there is only one subscript, such as in  $I_j$ , it will denote the aggregate value of bank investment when j is senior. Then, the expected returns to the lenders are given by

$$R_{jj} = \int_{0}^{D_{jj}} z dF(z) + \int_{D_{jj}}^{\bar{z}} D_{jj} dF(z) - \theta_{j} F(D_{jj}),$$

$$= z^{e} - \int_{D_{jj}}^{\bar{z}} (z - D_{jj}) dF(z) - \theta_{j} F(D_{jj}). \tag{2.4}$$

$$R_{ji} = \int_{D_{jj}}^{D_{j}} (z - D_{jj}) dF(z) + \int_{D_{j}}^{\bar{z}} D_{ji} dF(z) - \theta_{i} F(D_{j}),$$

$$= \int_{D_{jj}}^{\bar{z}} (z - D_{jj}) dF(z) - \int_{D_{j}}^{\bar{z}} (z - D_{j}) dF(z) - \theta_{i} F(D_{j}). \tag{2.5}$$

<sup>&</sup>lt;sup>7</sup>Since, by assumption,  $q > r_j$ , the first best surplus is  $z^e - min(r_1, r_2)X$ . However, because of the auditing costs, we are in the second best.

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Note that being the senior claimant is equivalent to being the sole lender. Therefore,  $R_{ij}$  is similar to  $R_j$ .

The junior claimant gets paid only after the senior claimant is paid. Therefore, the junior lender does not get paid at all when  $z < D_{jj}$ . However, for  $D_{jj} \le z < D_j$ , the junior lender gets paid  $z - D_{jj}$ . Thus, the junior lender faces default whenever the realization of z is less than  $D_j$ . The expected audit cost to the junior lender is, therefore,  $\theta_i F(D_j)$ . The expected return to  $L_i$  when  $z \ge D_j$  is  $D_{ji}[1 - F(D_j)]$ .

As before, we use the zero profit condition for banks to get their levels of investment in the project. The investment by  $L_k$ , denoted by  $I_{jk}$ , is given by  $I_{jk} = R_{jk}/r_k \quad \forall j, k = 1, 2$ . The aggregate investment,  $I_j$  is given by  $I_j = I_{jj} + I_{ji}$ .

The expected profit to the promoter is

$$\Pi_j = \int_{D_j}^{\bar{z}} (z - D_j) dF(z) - q(X - I_j). \tag{2.6}$$

The surplus from this set up is given by

$$S_{i} = z^{e} - qX - \theta_{i}F(D_{i}) - \theta_{i}F(D_{i}) + (q - r_{i})I_{i} + (q - r_{i})I_{i}, \quad i \neq j.$$

### 2.2.3 Lenders have equal priority in claims.

In this case, each unit realization from the project is proportionally distributed among the lenders till their claims are met. It is not possible for any particular lender to have his claim fully satisfied without the claim of the other lender being fully satisfied. This is in contrast to that of strict hierarchy in claims. With equal priority, therefore, both creditors have to audit when z is less than the total claims of the creditors. Here, there is no senior claimant. We will denote this by using 0 in the subscript. Thus,  $D_{0j}$  will denote the debt claim of j under equal priority.

Let the share of  $L_i$  on the project be denoted as  $\alpha_i$ . Thus,

$$\alpha_i \equiv \frac{D_{0i}}{D_{01} + D_{02}} = \frac{D_{0i}}{D_0}, \quad \forall i = 1, 2$$

<sup>8</sup>Being junior, he has to audit in more states than the senior claimant, as  $D_j = D_{jj} + D_{ji}$ .

<sup>&</sup>lt;sup>9</sup>The sharing rule.  $\alpha_i$ , is obtained once the lenders decide upon  $D_{0i}$ , i = 1, 2.

The expected return from the project to  $L_i$  is

$$R_{0i} = \int_{0}^{D_{0}} \alpha_{i} z dF(z) + \int_{D_{0}}^{\bar{z}} D_{0i} dF(z) - \int_{0}^{D_{0}} \theta_{i} dF(z)$$

$$= \alpha_{i} \left\{ z^{e} - \int_{D_{0}}^{\bar{z}} (z - D_{0}) dF(z) \right\} - \theta_{i} F(D_{0})$$
(2.7)

Here, both the lenders have to audit simultaneously in all states where the firm defaults, i.e,  $z < D_0$ . Similar to the case of strict hierarchy, the aggregate investment in the project by the lenders is given by  $I_0$ , where  $I_0 = I_{0i} + I_{0j} = R_{0i}/r_i + R_{0j}/r_j$ .

The expected profit to the promoter,  $\Pi_0$ , is

$$\Pi_0 = \int_{D_0}^{\bar{z}} (z - D_0) dF(z) - q(X - I_0). \tag{2.8}$$

The total surplus in this case is,

$$S_0 = z^e - qX - \sum_{i=1}^2 \theta_i F(D_0) + \sum_{i=1}^2 (q - r_i) I_{0i}.$$

### 2.3 Results

The promoter has six different options of raising funds. Two of these involve borrowing from only  $L_1$  or only  $L_2$ , respectively. She has another three options, where she borrows from both the lenders — making  $L_1$  senior,  $L_2$  senior, and giving them equal priority. Finally, the project could be financed entirely through equity. Recall that the superscript '\*' denotes the optimal value. Equations (2.9) and (2.10)denote the optimal surpluses when  $L_1$  and  $L_2$  are senior, respectively. Equation (2.11) denotes the optimal surplus when both the lenders have equal priority in claims. Equations (2.12) and (2.13) denote the optimal surpluses when the promoter borrows from  $L_1$  and  $L_2$  respectively. If we denote the surplus under all equity financing by  $s^*$ , as in equation (2.14), we have the following expressions for the different surpluses in the six cases.

$$S_1^* = z^e - qX - \theta_1 F(D_{11}^*) + \theta_2 F(D_1^*) + \sum_{i=1}^2 (q - r_i) I_{1i}^*.$$
 (2.9)

$$S_2^* = z^e - qX - \theta_2 F(D_{22}^*) + \theta_1 F(D_2^*) + \sum_{i=1}^2 (q - r_i) I_{2i}^*.$$
 (2.10)

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$$S_0^* = z^e - qX - \sum_{i=1}^2 \theta_i F(D_0^*) + \sum_{i=1}^2 (q - r_i) I_{0i}^*.$$
 (2.11)

$$s_1^* = z^e - qX - \theta_1 F(\delta_1^*) + (q - r_1)\eta_1^*. \tag{2.12}$$

$$s_2^* = z^e - qX - \theta_2 F(\delta_2^*) + (q - r_2)\eta_2^*.$$
 (2.13)

$$s^* = z^e - qX. (2.14)$$

The promoter's optimal borrowing decision with the associated hierarchy is simply the surplus with the highest value among all these surpluses. This is sufficient, since we have already argued that the surplus and the promoter's profit are the same in each case.

Before we explore the results involving optimal hierarchy and multiple lending, we enquire as to whether or not the *standard debt contract* (SDC) is the optimal contract in this context. The standard debt contract (Gale and Hellwig, 1985; p.648) represents " ... a contract which requires a fixed payment when the firm is solvent, requires the firm to be declared bankrupt if this fixed payment cannot be met and allows the creditor to recoup as much of the debt as possible from the firm's assets."

### **Theorem 2.1** The optimal contract is the standard debt contract. <sup>10</sup>

The intuition underlying the above result is straightforward. As q > r, efficiency requires that the entire funding be undertaken by the financial institutions. The question is, how will they collect the returns on their investment. The assumption is that the promoter knows the value of z at no cost, but the institutions have to incur a cost  $\theta$  to know z. Therefore, if the institutions are shareholders, their payment will be a function of the proportion of the shareholding and, hence, they will have to audit to know how much they should get. This will hold for all values of z. The debt contract lowers this audit cost because the institutions incur the audit cost only if there is a default.

Wang and Williamson [94] derive that the SDC is optimal when screening by the lenders is costly and the borrowers self select. They obtain that the unique equilibrium separating contract for good borrowers is a debt contract.

<sup>&</sup>lt;sup>10</sup>The proofs of all the results are in the appendix.

Dowd[40] and Krassa - Villamil [62] establish the optimality of debt contract when the firm borrows from multiple borrowers.<sup>11</sup>

### 2.3.1 Optimal Hierarchy

We now characterize the optimal hierarchy offered by the promoter to her creditors.

**Proposition 2.1** Let A.1-A.3 hold. If both the lenders invest positive amounts to the project and there is a strict hierarchy of claims, then  $L_1$  is the senior claimant.

The junior claimant always audits in more states than the senior claimant. The senior claimant audits in states where  $0 \le z < D_{jj}^*$ , while the junior audits in states where  $0 \le z < D_j^*$ . Both banks lend positive amounts with  $D_{jj}^* < D_j^*$ . Since  $L_1$  has the higher auditing cost, making it the senior lender reduces the total expected auditing cost.

Proposition 2.2 Under A.1-A.3, equal priority in claims is never optimal.

Propositions 2.1 and 2.2 taken together imply that, in the event of multiple lending, the optimal debt claim structure involves strict hierarchy of claims and the lender with the higher auditing cost is the senior claimant. This result conforms to the observed pattern of seniority in claims (Harris and Raviv [50], Barclay and Smith [10]). They find that dispersed creditors will be senior to the bank, which in turn will be senior to the promoter. Translated to our model, dispersed creditors are expected to have higher monitoring costs than banks. In India, non-depository investment institutions and the commercial banks often lend long-term capital to the same

<sup>&</sup>lt;sup>11</sup>A possible scenario involving the non-optimality of debt contracts is the case where auditing is stochastic (e.g, Mookherjee and Png [69]). In this setup, they proceed to show that 'equity like' contracts are optimal. However, the model in [69] studies quite different economic problems.

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project. Our model suggests that when both these types of lenders are involved in a project, the depository institutions will be senior to the other type. In the Indian context, Anant, Gangopadhyay and Goswami [8], empirically corroborate our hypothesis.

**Proposition 2.3** Under A.1 and A.3, if the lenders have the same auditing costs, then the promoter will choose only one bank.

The proposition implies that, if multiple lenders invest in the same project, they will have different auditing costs. In Winton [98], multiple lending was obtained with identical auditing and capital costs as the lenders were assumed to be credit constrained. Lenders are not credit constrained in our model and multiple lending will be determined endogenously. We now find out situations where multiple lending is optimal.

### 2.3.2 Multiple Lending

In order to identify the conditions under which multiple lending is optimal, we first derive a feasibility range of the parameters in the model. For the rest of this section, we simplify the algebra by assuming a specific distribution function for z. We will indicate the results for more general distributions, wherever necessary.

**A.4:** z is uniformly distributed between 0 and 1.

Observe that A.4 satisfies both A.1 and A.3.

Assuming interior solutions for the maximization of the bank's expected return, and using the first order conditions derived from equations (2.1), (2.4) and (2.5), we have

$$\delta_1^* = D_{11}^* = 1 - \theta_1, \quad \delta_2^* = D_1^* = 1 - \theta_2, \quad D_{12}^* = \theta_1 - \theta_2, \quad \text{with} \quad 0 < \theta_2 < \theta_1 < 1.$$

The optimal investment levels by the bank(s) are:

$$\eta_1^* = \frac{(1-\theta_1)^2}{2r_1}, \qquad \eta_2^* = \frac{(1-\theta_2)^2}{2r_2}.$$

$$I_{11}^* = \frac{(1-\theta_1)^2}{2r_1}$$
  $I_{12}^* = \frac{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}{2r_2}.$ 

Therefore,

$$I_1^* = I_{11}^* + I_{12}^* = \frac{(1-\theta_1)^2}{2r_1} + \frac{\theta_1^2 - \theta_2^2 - 2\theta_2(1-\theta_2)}{2r_2}.$$

(Recall from Proposition 2.1 that the bank with the lower auditing cost is never the senior lender; hence, there are no values for  $I_{21}^*$  and  $I_{22}^*$ .) If  $I_{12}^* = 0$ , we are in the case where bank 1 is the only lender. On the other hand, observe that,  $I_{12}^* > 0 \Rightarrow \theta_1 > [\theta_2(2 - \theta_2)]^{(1/2)}$ .

Let  $\pi^*$  measure the expected profit to the promoter when the entire project is funded by equity. Given propositions 2.1-2.3, we need to consider the relative value of  $\Pi_1^*$  with those of  $\pi^*$ ,  $\pi_1^*$  and  $\pi_2^*$  to check when multiple lending is optimal. Using A.4,

$$\Pi_{1}^{*} = 0.5 - qX - \sum_{i=1}^{2} \theta_{i} (1 - \theta_{i}) 
+ \frac{q - r_{1}}{r_{1}} \left\{ \frac{(1 - \theta_{1})^{2}}{2} \right\} + \frac{q - r_{2}}{r_{2}} \left\{ \frac{\theta_{1}^{2} - \theta_{2}^{2} - 2\theta_{2} (1 - \theta_{2})}{2} \right\}, 
\pi_{1}^{*} = 0.5 - qX - \theta_{1} (1 - \theta_{1}) + \frac{q - r_{1}}{r_{1}} \left\{ \frac{(1 - \theta_{1})^{2}}{2} \right\}, 
\pi_{2}^{*} = 0.5 - qX - \theta_{2} (1 - \theta_{2}) + \frac{q - r_{2}}{r_{2}} \left\{ \frac{(1 - \theta_{2})^{2}}{2} \right\} 
\pi^{*} = 0.5 - qX.$$
(2.15)

**Proposition 2.4** Let A.2 and A.4 hold. The promoter borrows from lender i only if

$$r_i \leq \overline{r}_i \equiv \frac{q(1-\theta)}{1+\theta_i} \quad \forall i = 1, 2.$$

Proposition 2.4 gives us a necessary condition. If  $r_i \leq \overline{r}_i$ , then it is profitable for the promoter to at least borrow from  $L_i$  rather than finance the project entirely through equity. However, even if  $r_i \leq \overline{r}_i$ , it is possible that the promoter does not borrow from  $L_i$ . This could be the case if borrowing from  $L_j$  alone is more profitable.

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**Proposition 2.5** Let  $r_i \leq \overline{r}_i$ . Under assumptions A.2 and A.4, multiple lending is optimal if and only if

$$q\left\{1 - \frac{2\theta_2(1-\theta_2)}{\theta_1^2 - \theta_2^2}\right\} \ge r_2 \ge r_1\left\{\frac{1+\theta_1}{1-\theta_1}\right\}.$$

Observe that, given risk neutrality the returns to each agent can be added up to get the total surplus. Thus, each dollar going to the bank implies a dollar being lost by the promoter. The promoter has to give enough to the bank to cover its capital cost, plus the auditing cost in the event of default. Consider the senior lender, bank 1. The audit cost per unit of return is  $[\theta_1 F(D_{11}^*)]/(R_1^*)$ . The effective cost to the bank, per unit of loan, is, therefore.

$$r_1 \left[ 1 + \frac{\theta_1 F(D_{11}^*)}{R_1^*} \right]$$

With banks making zero profits, the payment to the bank must exactly cover this cost. If this payment is less than the equity cost, q, then the bank is an attractive investor to the promoter. Similarly, the effective cost per unit of return to bank 2 is

$$r_2 \left[ 1 + \frac{\theta_2 F(D_1^*)}{R_{12}^*} \right]$$

Putting in the explicit solutions for the uniform case, worked out above, we get the respective effective costs for banks 1 and 2 as

$$r_1 \frac{1+\theta_1}{1-\theta_1}, \quad r_2 \frac{{\theta_1}^2-{\theta_2}^2}{{\theta_1}^2-{\theta_2}^2-2{\theta_2}(1-{\theta_2})}$$

We can rewrite the condition in Proposition 2.5 in two parts:

$$q \geq \max \left\{ r_1 \frac{1+\theta_1}{1-\theta_1}, \quad r_2 \frac{{\theta_1}^2 - {\theta_2}^2}{{\theta_1}^2 - {\theta_2}^2 - 2{\theta_2}(1-\theta_2)} \right\}$$

$$r_1 \frac{1+\theta_1}{1-\theta_1} \leq r_2 \frac{{\theta_1}^2 - {\theta_2}^2}{{\theta_1}^2 - {\theta_2}^2 - 2{\theta_2}(1-\theta_2)}$$

The first part guarantees that lending by both banks is feasible; the second generates the required hierarchy.

Given below are a set of numerical values for the parameters to show that the condition in proposition 2.5 is satisfied.

**Example 1:** Let  $\theta_1 = 0.5$ ,  $\theta_2 = 0.125$ ,  $r_2 = \{\gamma q\}/15$  and  $r_1 = \{\phi r_2\}/3$ . where,  $\gamma, \phi \in (0, 1]$ . Further, let q < 0.5/X. The above parametric configurations satisfy A.4. Also,  $\theta_1, \theta_2, r_1$  and  $r_2$  satisfy the conditions in proposition 2.5. Note

$$\begin{split} \Pi_1^* - \pi_1^* &= \frac{(1 - \gamma)}{128\gamma} \ge 0 \quad \forall \gamma \in (0, 1], \\ \Pi_1^* - \pi_2^* &= \frac{45(1 - \phi)}{2\gamma\phi} \ge 0 \quad \forall \phi \in (0, 1]. \end{split}$$

The above parametric values imply that  $\Pi_1^* \geq Max\{\pi_1^*, \pi_2^*\}$ . Therefore, the promoter borrows from both the lenders making  $L_1$  senior.

For a more general distribution function, we cannot get explicit values for  $\bar{r}_i$ . However, for completeness, we state the following result:

#### Proposition 2.6 Let A.1-A.3 hold.

(a) The promoter borrows from lender i only if

$$r_i \le \overline{r}_i \equiv \frac{qR_i^*}{R_i^* + \theta_i F(\delta_i^*)} \quad \forall i = 1, 2.$$

(b) If the condition in (a) is satisfied, then multiple lending is optimal if and only if

$$q\frac{R_{12}^*}{R_{12}^* + \theta_2 F(D_1^*)} \ge r_2 \ge r_1 \frac{R_1^* + \theta_1 F(\delta_1^*)}{R_1^*}.$$

It is important at this stage to compare our findings with some of those in the literature regarding multiple lending. As discussed earlier, one common strand is the importance of the behavioral role played by different claimants in on-going projects that face the choice of liquidation or continuation. In Bolton and Scharfstein [17] and Dewatripont and Maskin [32], the firm has multiple investors because multiple creditors can extract more cash flows from the firm than a single creditor during bankruptcy. Therefore, *strategic defaults* are less likely with multiple investors as renegotiation is likely to break down.<sup>12</sup>

<sup>&</sup>lt;sup>12</sup>See also Hart and Moore [53], [54], Berglöf and Thadden [12], Diamond [37] and [38].

In Bolton and Scharfstein [17], the various creditors are secured by specific assets. In their model, the manager identifies complementary assets and then raises investment from the creditors making them secured in terms of these assets. The optimal number of creditors is then obtained by weighing the liquidation and continuation values of the firm. However, in this framework, as the creditors are secured by specific assets, they have equal priority. During default, the creditors liquidate the firm and collect their dues on the basis of the respective assets pledged to them. This gives rise to the equal priority structure. Therefore, in their paper, equal priority is not obtained as an exercise in deriving the optimal hierarchy structure. The main advantage of our model is that, both multiple lending as well as the strict hierarchy of claims is endogenous.

# 2.4 Communicating Banks

So far, we have implicitly assumed that the information obtained from auditing is available to the auditing lender alone. Now suppose that the information obtained by the auditor is also observable by the other lender. Thus, as before, the senior lender audits whenever its claim is not met. The junior lender observes this, and *learns* that the realization from the project is not sufficient to meet the senior claimant's debt obligation. Hence, the junior lender does not expect to get paid and, most importantly, does not audit in these states. This allows it to save on auditing costs.

Observe that, in a hierarchy, once default has occurred, there is no incentive for the senior lender to hide its audit information from the junior creditor. With equal priority in claims, there is an incentive for the auditing lender to hide the information regarding the actual realization of the state. This is because, with equal priority, each lender gets a fraction of the realized value in default states. If one lender does not know the true realization, the other can get a higher value.

In this section, variables with superscript 'C' indicate communication among lenders. The expected revenue of the senior claimant is the same as in the previous sections, even with communication. However, the junior claimant gains from communication. The junior lender will now have to audit in only those states where  $D_{11}^C \leq z < D_1^C$ . Therefore, the expected returns

to  $L_1$  and  $L_2$  will be:

$$R_{11}^{C} = z^{e} - \int_{D_{11}^{C}}^{\bar{z}} (z - D_{11}^{C}) dF(z) - \theta_{1} F(D_{11}^{C}).$$

$$R_{12}^{C} = \int_{D_{11}^{C}}^{\bar{z}} (z - D_{11}^{C}) dF(z) - \int_{D_{1}^{C}}^{\bar{z}} (z - D_{1}^{C}) dF(z) - \theta_{2} [F(D_{1}^{C}) - F(D_{11}^{C})].$$

With communication,  $R_{11}^C$  will remain the same, i.e.,  $R_{11}^C = R_{11}$ . Proposition 2.1 suggests that in the event of strict hierarchy, the lender with the higher auditing cost will be the senior claimant. Communication among the lenders will not affect this result. This can be seen as follows. Consider any lender. The lender increases its debt claims from zero that level where the marginal revenue from an additional unit of debt equals the marginal cost of auditing the additional default state. Note that, the marginal revenue for the junior claimant will always be lower than the senior claimant. In addition, if the junior claimant has higher auditing costs, then its marginal cost will also be higher than the senior claimant. Now, suppose,  $L_2$  is the senior lender. Denote the optimal debt claim by  $L_2$  as  $D_{22}^C$ . Note that for  $L_2$ , any additional debt claim over  $D_{22}^C$  would imply that the marginal revenue is lower than the marginal costs of doing so. For  $L_1$  who is junior, any positive debt claims would mean that it would get paid only if returns exceed  $D_{22}^{C}$ . Therefore, the marginal revenues to  $L_1$  will be lower than the marginal cost for any positive debt claims it has. This implies that if  $L_1$  is junior, it will not have any positive debt claims and hence will not invest at all. Note that this reasoning is independent of the communicating abilities of the banks. Therefore, if the promoter borrows from both the lenders, the optimal arrangement is to make  $L_1$  senior. The expected return to the junior lender  $L_2$ , will be  $R_{12}^C =$  $R_{12} + \theta_2 F(D_{11})$ . This implies that  $I_{12}^C > I_{12}$  and the expected auditing cost decreases with communication. The following results are readily obtained.

**Proposition 2.7** Let A.1-A.3 hold. With multiple lenders, the expected profit to the promoter is more when the lending institutions communicate with each other.

**Proposition 2.8** Under A.2, A.4 and communicating banks, multiple lending is optimal if and only if

$$q\left\{1 - \frac{2\theta_2(\theta_1 - \theta_2)}{\theta_1^2 - \theta_2^2}\right\} \ge r_2 \ge r_1\left\{\frac{1 + \theta_1}{1 - \theta_1}\right\}.$$

It is interesting to note that, the upper bound on  $r_2$  is greater in proposition 2.8 than in proposition 2.5. This implies that the probability of obtaining multiple lending increases with communication. Since communication reduces the (total) expected auditing costs, the aggregate net surplus is increased. The promoter, therefore, finds it optimal to borrow from multiple lenders for a wider range of parametric values. Alternatively, the overall efficiency improves with communication since the duplication of monitoring effort is avoided. Two facts emerge particularly interesting in this context—the 'disclosure rules' of the firm and group lending. A recent literature concerns the efficiency of voluntary disclosure of information by the firm (Diamond [35], Admati and Pfleiderer [1]). However, their results that relate voluntary disclosure to efficiency, study a very different model.

Our result provides another reason as to why voluntary disclosure of information can lead to higher overall efficiency. If the firm announces its default on the senior lender's claim, this would deter the junior claimant from monitoring when the firm defaults on the senior claimant. Thus, the expected monitoring costs would be lower.

In the next section we consider the implications of our results on group lending.

# 2.5 Implications

One of the recommendations of the Narasimham Committee [74] for Indian public banks was that, banks must lend collectively to any new project. The rational given was, group lending brings about significant economies of scales. When banks lend as a group, they jointly pool resources and share other responsibilities. Under group lending, the banks can either undertake consortium or syndicated lending. In a system of loan consortium, after independently assessing a firm's project and its credentials, a group of banks lend collectively to it. Under the syndicated loan system, a lead arranging bank determines the exposure level of bank finance for a particular borrower. The lead bank then syndicates responsibilities to the other banks at a later date. The lead arranging bank does the entire documentation. Therefore, unlike consortium lending where all banks have to appraise the client, here appraisal by the lead bank is sufficient for project financing.

<sup>&</sup>lt;sup>13</sup>For a discussion on the various aspects of group lending, see Shekhar and Shekhar [88].

### 2.5.1 Consortium Lending

In March 1977, the Reserve Bank of India advised the banks on the need to expedite the formation of consortium arrangements in cases involving multiple banking. Its guidelines suggested that wherever there is multiple banking arrangement, it is advisable to form a consortium. Also, where the aggregate credit limit sanctioned by banks to a single party amounts to Rs 50 million or more, formation of consortium should be considered obligatory. The activities of the consortium so formed were to be supervised by a Consortium Committee comprising of senior executives of the member banks. The consortium selects a 'lead bank' to deal with most of the transactions with the firm. The lead bank is usually the bank with the highest exposure.<sup>14</sup>

Within the consortium, the members (banks) decide upon the common rate of interest, lending terms and conditions, monitoring and other activities. The common rate of interest is arrived at by considering the simple weighted average of the rates of interest charged (suggested) by the member banks. The weights are their relative percentages of loan exposure to the total amount lent by the consortium. Monitoring and other activities are often decided on the basis of comparative advantages of the member banks.

A review of the consortium arrangements was undertaken in 1993. The changes were necessary as it was felt that, in general, the consortium arrangements were too inflexible. The main objections were that, loan or credit appraisal by a consortium were often too time consuming and involved unnecessary duplication of transaction costs. Loan appraisal by a consortium required approval by all its members. Therefore, duplication of transaction costs and unnecessary delay in obtaining clearance certificates were not unusual. The Narasimham Committee report[74] recommends a gradual shift from the consortium to the syndicated loan system.

### 2.5.2 Syndicated Loan System

Under the syndicated loan system, a lead bank determines what is to be the exposure level of bank finance for a particular borrower. The lead bank then takes on the entire loan into its book. This loan is syndicated to the other banks at a later date. The lead bank does the entire documentation.

<sup>&</sup>lt;sup>14</sup>In the cases where none of the member banks have a significant loan exposure in comparison to each other, the lead bank is decided by the committee. The appointment of the committee and the lead bank is usually done to reduce the delay in actual transactions.

Therefore, the main advantage of a syndicated loan system over a consortium arrangement is that the former reduces appraisal time and eliminates the duplication of transaction costs in the system. Therefore, it is not surprising that the syndicated arrangements are increasingly replacing the existing consortium arrangements, both in India and elsewhere (see Ravishanker [80] and Megginson et -al [64], for the emergence of syndicated loan system in India and in the international context, respectively).

Coordination among the banks in a group ensures that duplication of verification costs does not take place. More importantly, auditing is done by the more efficient auditor. Efficient delegation of monitoring activities is recognized in the literature even in other contexts (see Diamond [34], Krassa and Villamil [62] etc). In this light it can be argued that loan syndication is a more efficient form of group lending than consortium arrangement. More recently, Park [26] proposes that an optimal debt contract delegates monitoring the most efficient monitor.

The intuition for this again lies in the reduction of transaction costs associated with both these forms of arrangements. Proposition 2.7 presents us with a strong intuition as to why group lending would be more efficient. In our paper, syndicated lending allows information flows from the lead bank to other lenders in the syndicate. This reduces the total expected auditing cost as all banks do not need to audit. Any auditing cost is a deadweight loss to the system. Since banks make zero profits, anything that increases the net surplus (for instance, through lower total auditing costs), improves the return to the promoter. Hence the promoter prefers syndicated lending to one where banks lend as independent entities.

The key assumption in our framework is that the lenders have different monitoring as well as capital costs. This is given in A.2. The strict hierarchy is obtained because the lenders have different monitoring costs. Further, we obtain multiple lending because the lender with the higher monitoring cost has the lower capital cost. If the lenders had either identical monitoring costs or capital costs, there would be no multiple lending. This is because, if the lenders were identical, multiple lending would imply unnecessary duplication of auditing efforts. Therefore, the promoter can borrow from any one of the lenders. If the lenders have one cost component identical and differ on the other, one of the lenders would be more efficient than the other. In this case, the promoter will borrow only from the efficient lender and multiple lending cannot occur. In reality, all financial institutions are not identical. While some financial institutions specialize in project financing, investment

banking, etc., there are others who enjoy an advantage in raising funds from the market. As discussed after A.2, the cost of raising capital is lower with depository institutions as compared to non-depository institutions. Similarly, lenders specializing in project financing are expected to be more efficient project monitors than those institutions that do not specialize in this activity.

### 2.6 Conclusion

We consider lenders auditing the promoter in the default state to prevent strategic default. The lenders choose the debt claims so as to reduce the occurrence of default states. This reduces the expected auditing costs. This formulation is similar to Moore [70]. The optimal hierarchy depends solely upon the differential auditing costs of the lenders. The optimal number of lenders chosen by the promoter depends upon the following tradeoff, between high debt claims and high bank investment, on the one hand and low debt claims and low bank investment, on the other.

The two crucial features distinguishing our model from that of Winton [98] are (a) differential auditing costs across lenders and (b) the calculation of optimal loan supply by the lenders. The differences in auditing costs form the basis for a hierarchical claim structure. The optimal loan supply by the lenders, along with a difference in auditing costs, determines the optimal number of lenders.

Our results are summarized as follows. For multiple lending to occur, it is necessary that the lenders have different auditing and capital costs. Our model predicts that the lender with the higher auditing cost is the senior claimant. Taken to its logical extreme, this result also explains why firms are the residual claimants. Our results do not support the case for an equal priority in claims.

We also establish that group lending is an efficient outcome, and that a system of *syndicated* lending is more efficient than a system of *consortium* lending.

## 2.7 Appendix to Chapter 2

The following lemmas will be used during the course of proving the results.

**Lemma 2.1** The optimal debt claim to  $L_1$  when it is the senior claimant is equal to the debt claim if  $L_1$  were lending alone. Further, the aggregate debt claim with  $L_1$  as the senior claimant is equal to the debt claim when  $L_2$  alone lends to the project. I.e.,  $D_{11}^* = \delta_1^*$ . and  $D_1^* = \delta_2^*$ .

**Proof:** From equations (2.1) and (2.4), we obtain that,

$$R_{11} = z^{e} - \int_{D_{11}}^{\bar{z}} (z - D_{11}) dF(z) - \theta_{1} F(D_{11})$$

$$R_{1} = z^{e} - \int_{\delta_{1}}^{\bar{z}} (z - \delta_{1}) dF(z) - \theta_{1} F(\delta_{1}).$$

Therefore, the conditions for the interior solutions for  $D_{11}^*$  and  $\delta_1^*$  are:

$$\theta_1 h(D_{11}^*) = 1 = \theta_1 h(\delta_1^*) \implies \delta_1^* = D_{11}^*.$$

Further, from equation (2.5), we obtain

$$R_{12} = \int_{D_{12}}^{D_1} (z - D_{11}) dF(z) + \int_{D_1}^{\bar{z}} D_{12} dF(z) - \theta_2 F(D_1).$$

The condition for the interior solution of  $D_1^*$  is  $\theta_2 h(D_1^*) = 1$ . Also,  $\theta_2 h(\delta_2^*) = 1$ . Thus,  $\delta_2^* = D_1^*$ .

**Lemma 2.2** The optimal debt claim with  $L_2$  as the sole claimant is more than the optimal debt claim when  $L_1$  is the sole claimant, i.e.,  $\delta_2^* > \delta_1^*$ .

**Proof:** From Lemma 2.1 we have  $h(\delta_1^*) = 1/\theta_1$  and  $h(\delta_2^*) = 1/\theta_2$ . As  $\theta_1 > \theta_2$  (from A.2) and h'(.) > 0 (from A.3), we have  $\delta_2^* > \delta_1^*$ .

#### Lemma 2.3

$$F(D_0^*) \ge \frac{1}{D_0^*} \int_0^{D_0^*} F(z) dz$$

Proof:

$$\begin{split} F(D_0^*) & \equiv & \frac{1}{D_0^*} F(D_0^*) D_0^* \\ & = & \frac{F(D_0^*)}{D_0^*} \int_0^{D_0^*} dz \ge \frac{1}{D_0^*} \int_0^{D_0^*} F(z) dz \end{split}$$

as F(.) is a non-decreasing function.

**Lemma 2.4** The aggregate debt claim with equal priority in claims is more than the optimal debt claim with any lender lending alone. I.e.,  $D_0^* \geq \delta_i^*$ , i = 1, 2.

**Proof:** The condition for an interior solution for  $D_{0i}^*$  is

$$[1 - \theta_i f(D_0^*) - F(D_0^*)] + \alpha_j [F(D_0^*) - \frac{1}{D_0^*} \int_0^{D_0^*} F(z) dz] = 0.$$

Lemma 2.3 implies  $[1 - \theta_i f(D_0^*) - F(D_0^*)] \le 0$ . Combining this with equation (2.2) and A.3, the result follows.

Corollary 2.1 Denote  $R_i \equiv R_i(\theta_i, \delta_i)$ , where

$$R_i = z^e - \int_{\delta_i}^{\bar{z}} (z - \delta_i) dF(z) - \theta_i F(\delta_i).$$

Therefore,  $R_1^* \equiv R_i(\theta_i, \delta_i^*) = R_{11}^*$  and  $R_2^* \equiv R^*(\theta_2, \delta_2^*) = R_{12}^* + R_1^* + \theta_1 F(D_{11}^*)$ .

**Proof:** The result is obtained by applying Lemma 2.1 in equations (2.1). (2.4) and (2.5).

### • Proof of Theorem 2.1.

The proof will be organized as follows. Denote  $\{\Delta\}$  as the SDC and  $\{\Phi\}$  as an alternative contract offered by the promoter to the lender(s). The result will be established by showing that for any contract  $\{\Phi\}$ .  $\Pi(\{\Delta\}) \geq \Pi(\{\Phi\})$ , where  $\Pi$  is the expected profit to the promoter. Note that the optimal contract - the one which maximizes the net surplus- is also the one which maximizes the promoter's profit.

We prove the result by considering the two possible cases - (i) the promoter borrows from  $L_j$  alone and (ii) the promoter borrows from both  $L_i$  and  $L_j$ .

Case I: The promoter signs with  $L_j$  alone.

Describe the alternate contract  $\{\Phi'\}$  as a combination of debt and equity. Given  $\delta_j$ , let  $\beta' \in [0,1]$  be the equity share of  $L_j$ . Therefore,  $\forall z \geq \delta_j$ ,  $\beta'(z-\delta_j)$  is the return from equity to  $L_j$ , while  $(1-\beta')(z-\delta_j)$  is the promoter's return.

From (2.1) we have

$$R_{j}(\{\Phi'\}) = \int_{0}^{\delta_{j}} z dF(z) + \int_{\delta_{j}}^{\bar{z}} \delta_{j} dF(z) + \int_{\delta_{j}}^{\bar{z}} \beta'(z - \delta_{j}) dF(z) - \int_{0}^{\bar{z}} \theta_{j} dF(z)$$
$$= \int_{0}^{\delta_{j}} z dF(z) + \int_{\delta_{j}}^{\bar{z}} \{\beta'z + (1 - \beta')\delta_{j}\} dF(z) - \theta_{j}.$$

Note that if  $\beta' > 0$ , then  $L_j$  has to incur  $\theta_j$  in all the states. The lender does not know the true realization of z without auditing. However, with only debt, once the lender's claim is paid, he does not audit. With equity, it has to audit even when  $z \geq \delta_j$ . Therefore, the optimal debt claim set by  $L_j$ . denoted by  $\delta_j^*$  is given by:

$$\frac{\partial R_j(\{\Phi'\})}{\partial \delta_j^*} = (1 - \beta')[1 - F(\delta_j^*)] = 0,$$

implying in equilibrium, either  $\beta'=1$  or  $\delta_j^*=\bar{z}$ . Note that with either  $\beta'=1$  or  $\delta_j^*=\bar{z}$ ,

$$R_{j}(\lbrace \Phi' \rbrace) = \int_{0}^{\delta_{j}} z dF(z) + \int_{\delta_{j}}^{\bar{z}} \lbrace \beta' z + (1 - \beta') \delta_{j} \rbrace dF(z) - \theta_{j}$$
$$= z^{e} - \theta_{j}.$$

With  $\beta' = 1$ , we have

$$\Pi(\lbrace \Phi' \rbrace) = -qX + \frac{q}{r_i} \left\{ z^e - \theta_j \right\}.$$

With the SDC,  $\{\Delta\}$ , we have

$$R_{j}(\{\Delta\}) = R_{j}^{*}$$
  
 $\Pi(\{\Delta\}) = \int_{\delta_{i}^{*}}^{\bar{z}} (z - \delta_{j}^{*}) dF(z) + \frac{q}{r_{j}} \{R_{j}^{*}\}.$ 

Therefore,

$$\begin{split} \Pi(\{\Delta\}) - \Pi^{max}(\{\Phi^*\}) &= \int_{\delta_j^*}^{\bar{z}} (z - \delta_j^*) dF(z) + \frac{q}{r_j} \left\{ R_j^* - z^e + \theta_j \right\} \\ &> \frac{q}{r_j} \left\{ R_j^* - z^e + \theta_j \right\} > 0. \end{split}$$

The last inequality follows from the fact that,  $R_j^* \equiv R_j(\delta_j^*) > R_j(\delta_j = \bar{z}) = z^e - \theta_j$ , and  $\delta_j^* = \arg \max_{\delta_j} = R_j(\delta_j)$ .

Case II: The promoter signs with both the lenders.

In this case, the alternative contracts, could be either  $\{\Phi_A\}$ , such that  $L_j$  is offered SDC while  $L_i$  is offered equity for  $i \neq j$  or  $\{\Phi_B\}$ , such that both  $L_i$  and  $L_j$  are offered equity.

The lender that is offered the equity contract, will have to audit in all the states. This is similar to Case I. As established earlier, the promoter will earn greater profit by offering that lender  $\{\Delta\}$ .

Thus, the SDC,  $\{\Delta\}$ , is the optimal contract.

#### • Proof of Proposition 2.1.

It will be sufficient for the proof to show that  $L_1$  cannot be the junior claimant. This will leave us with the only other possible option, that  $L_1$  is the senior claimant. Suppose,  $L_1$  is the junior claimant, since,  $D_{22}^*$  maximizes  $R_{22}$  and from h'(.) > 0, we have

$$1 - \theta_{2} f(D_{22}^{*}) - F(D_{22}^{*}) = 0$$

$$\{1 - F(D_{22}^{*})\}[1 - \theta_{2} h(D_{22}^{*})] = 0$$

$$\{1 - F(D_{22}^{*})\}[1 - \theta_{1} h(D_{22}^{*})] < 0 \quad \theta_{1} > \theta_{2}$$

$$1 - \theta_{1} f(z) - F(z) < 0, \quad \forall z \geq D_{22}^{*}.$$

$$\int_{D_{22}^{*}}^{D_{2}^{*}} [1 - \theta_{1} f(z) - F(z)] dz < 0 \quad \text{since } D_{2}^{*} > D_{22}^{*}.$$

From (2.5), we have  $\forall D_{21}^* > 0$ ,

$$\begin{split} R_{21}^{\star} &= \int_{D_{22}^{\star}}^{D_{2}^{\star}} (z - D_{22}^{\star}) dF(z) + \int_{D_{2}^{\star}}^{\bar{z}} D_{21}^{\star} dF(z) - \theta_{1} F(D_{2}^{\star}). \\ &< \int_{D_{22}^{\star}}^{D_{2}^{\star}} (z - D_{22}^{\star}) dF(z) + \int_{D_{2}^{\star}}^{\bar{z}} D_{21}^{\star} dF(z) - \theta_{1} \{ F(D_{2}^{\star}) - F(D_{22}^{\star}) \}. \end{split}$$

$$= \int_{D_{22}^{\star}}^{D_{2}^{\star}} [1 - \theta_{1} f(z) - F(z)] dz$$
  
< 0.

Thus  $L_1$  gets negative returns when it is junior. The zero profit condition ensures that  $L_1$  will not lend any positive amount to the project. This contradicts the fact that both the lenders are lending positive amounts.

#### • Proof of Proposition 2.2.

From equations (2.6) and (2.8), we obtain

$$\Pi_{1}^{*} - \Pi_{0}^{*} = (q - r_{1})(I_{11}^{*} - I_{01}^{*}) + (q - r_{2})(I_{12}^{*} - I_{02}^{*}) 
+ \theta_{1}[F(D_{0}^{*}) - F(D_{11}^{*})] + \theta_{2}[F(D_{0}^{*}) - F(D_{1}^{*})] 
= (q - r_{1})(I_{11}^{*} - I_{01}^{*}) + (q - r_{2})(I_{12}^{*} - I_{02}^{*}) 
+ \theta_{1}[F(D_{0}^{*}) - F(\delta_{1}^{*})] + \theta_{2}[F(D_{0}^{*}) - F(\delta_{2}^{*})] 
\geq (q - r_{1})(I_{11}^{*} - I_{01}^{*}) + (q - r_{2})(I_{12}^{*} - I_{02}^{*}).$$

The inequality follows from Lemma 2.4. Therefore,

$$\Pi_{1}^{*} - \Pi_{0}^{*} \geq (q - r_{1})(I_{11}^{*} - I_{01}^{*}) + (q - r_{2})(I_{12}^{*} - I_{02}^{*}) 
\geq \frac{q - r_{2}}{r_{2}} \left\{ R_{11}^{*} - R_{01}^{*} + R_{12}^{*} - R_{02}^{*} \right\}, 
= \frac{q - r_{2}}{r_{2}} \left\{ R_{2}^{*} - \left[ \int_{0}^{D_{0}^{*}} z dF(z) + \int_{D_{0}^{*}}^{\bar{z}} D_{0}^{*} dF(z) - \theta_{2} F(D_{0}^{*}) \right] \right\} 
+ \frac{q - r_{2}}{r_{2}} \theta_{1} [F(D_{0}^{*}) - F(D_{1}^{*})], 
\geq \frac{q - r_{2}}{r_{2}} \left\{ R_{2}^{*} - R(\theta_{2}, D_{0}^{*}) \right\} 
> 0$$

The second inequality follows from  $r_1 > r_2$ . The third inequality is obtained using  $D_0^* \ge D_1^*$ . The last inequality follows from the definition of  $R_2^*$ .

#### • Proof of Proposition 2.3.

From the Proof of proposition 2.1, it is clear that, if the lenders have the same auditing costs, i.e, if  $\theta_1 = \theta_2$ , then maintaining strict hierarchy is not possible. The only case that remains to be examined is the equal priority

case. The lenders  $L_i$  and  $L_j$ , differ only in their capital costs. Therefore, it is sufficient for the proof to show that with  $\theta_1 = \theta_2 = \theta$ ,  $\pi_1(\theta) \ge \Pi_0(\theta)$ . With  $\theta_1 = \theta_2 = \theta$ , we have,

$$\begin{split} \pi_1^*(\theta) - \Pi_0^*(\theta) &= \theta[2F(D_0^*) - F(D^*)] \\ &+ \frac{q - r_1}{r_1} (R^* - R_{01}^*) - \frac{q - r_2}{r_2} R_{02}^* \\ &\geq \frac{q - r_2}{r_2} \left\{ R^* - \left[ \int_0^{D_0^*} z dF(z) + \int_{D_0^*}^{\bar{z}} D_0^* dF(z) - 2\theta F(D_0^*) \right] \right\} \\ &\geq \frac{q - r_2}{r_2} \left\{ R^* - \left[ \int_0^{D_0^*} z dF(z) + \int_{D_0^*}^{\bar{z}} D_0^* dF(z) - \theta F(D_0^*) \right] \right\} \\ &= \frac{q - r_2}{r_2} \left\{ R^* - R(\theta, D_0^*) \right\} \\ &\geq 0, \end{split}$$

where,  $D^*$  is obtained from the fact that,  $\theta h(D^*) = 1$  and  $R^* = R(\theta, D^*)$ . Note that,  $\theta_1 = \theta_2 \Rightarrow R_{01}^* = R_{02}^*$ . The first inequality is obtained from A.2. while the last inequality follows from the definition of  $R^*$ .

### • Proof of Proposition 2.4.

From the set of equations in (2.15) we get.

$$\pi_i^* = 0.5 - qX - \theta_i(1 - \theta_i) + \frac{q - r_i}{r_i} \left\{ \frac{(1 - \theta_i)^2}{2} \right\},$$
  
$$\pi^* = 0.5 - qX.$$

Therefore,

$$\pi_i^* \ge \pi^* \Rightarrow r_i \le \overline{r}_i \equiv \frac{q(1-\theta_i)}{1+\theta_i}.$$

### • Proof of Proposition 2.5.

From the set of equations given in (2.15), we obtain

$$\Pi_1^* - \pi_1^* = (q - r_2) \frac{\theta_1^2 + \theta_2^2 - 2\theta_2}{r_2} - \theta_2 (1 - \theta_2)$$

$$= \frac{q}{2r_2} \{\theta_1^2 + \theta_2^2 - 2\theta_2\} - \frac{\theta_1^2 - \theta_2^2}{2}$$

$$\Pi_1^* - \pi_2^* = (q - r_1) \frac{(1 - \theta_1)^2}{2r_1} + (q - r_2) (I_{12}^* - \eta_2^*) - \theta_1 (1 - \theta_1)$$

$$= q(1 - \theta_1) \left\{ \frac{1 - \theta_1}{2r_1} - \frac{1 + \theta_1}{2r_2} \right\}.$$

Therefore,

$$\Pi_1^* \geq \pi_1^* \iff q \left\{ 1 - \frac{2\theta_2(1 - \theta_2)}{\theta_1^2 - \theta_2^2} \right\} \geq r_2, \text{ and } \Pi_1^* \geq \pi_2^* \iff r_2 \geq r_1 \left\{ \frac{1 + \theta_1}{1 - \theta_1} \right\}.$$

Further,

$$q\left\{1 - \frac{2\theta_2(1 - \theta_2)}{\theta_1^2 - \theta_2^2}\right\} \ge r_2 > 0 \Rightarrow R_{12} > 0.$$

### • Proof of Proposition 2.6.

The proof is similar to that of proposition 2.5. The conditions in (a) and (b) of the proposition is readily obtained by using the following inequalities:

$$\pi_i^* \ge \pi^*; \quad \forall j = 1, 2 \quad \text{and} \quad \Pi_1^* \ge \max\{\pi_1^*, \pi_2^*\}.$$

The final expressions in (a) and (b) are obtained by using the fact that the net surplus in the system is the same as the expected profits to the promoter. Therefore, the values of  $\Pi_1^*$ ,  $\pi_1^*$  and  $\pi_2^*$  are obtained from equations (9), (12) and (13) respectively. Finally, note that, with general distribution forms,  $\pi^* = z^e - qX$ .

### • Proof of proposition 2.8 is identical to that of proposition 2.5.

# 2.8 Notations used in Chapter 2

Table 2.1: Notations Under Seniority of Claims,  $(L_j \text{ is Senior})$ 

Agents	Relevant Variables	Description of the Variables
Lender $j$ $L_j$	$D_{jj} \ I_{jj}$	Debt claim of the senior claimant Investment by the senior claimant
	$R_{jj}^{jj}$	Expected return to the senior claimant
Lender i	$D_{ji}$	Debt claim of the junior claimant
$L_i$	$I_{ji}$	Investment by the junior claimant
	$R_{ji}$	Expected return to the junior claimant
Promoter	$D_j$	Aggregate debt claim
P	$I_j$	Aggregate investment by the lenders
	$\Pi_j$	Expected profit to the promoter

Table 2.2: Notations Under Equal Priority in Claims

Agents	Relevant Variables	Description of the Variables
	$D_{0i}$	Debt claim of $L_i$
$L_i$	$I_{0i}$	Investment by $L_i$
	$R_{0i}$	Expected return to $L_i$
Promoter	$D_0$	Aggregate debt claim
P	$I_0$	Aggregate investment by the lenders
	$\Pi_0$	Expected profit to the promoter

# Chapter 3

# Multiple Lending With Asymmetric Information

### 3.1 Introduction

In the previous chapter, we studied the conditions under which (i) multiple banks lend to the same project and (ii) the priority rule associated with such lending. However, there the only asymmetric information between the promoter and the creditors was regarding the ex post project realization. We now incorporate an additional source of asymmetric information. In particular, we assume that the promoter's true 'type' is known only to her. The lenders do not know the actual 'type' of the promoter. We identify the conditions under which multiple lending is optimal. The costly state verification framework (Moore [70], Townsend [92] and Seward [87]) is retained in this chapter to prevent the promoter from under reporting. We compare our findings in this chapter with those obtained in the previous chapter and with Winton [98].

We assume that the lenders are identical. However, in proposition 2.3 of the previous chapter, we established that, with identical lenders and no asymmetric information regarding the promoter's type, it is always more profitable for the firm to borrow from any one lender than from both of them. This is because, with perfect information regarding the promoter's type, borrowing from multiple investors would entail an unnecessary duplication of monitoring effort. This would lead to a lower net surplus to the promoter. Thus, we obtained that multiple lending is never optimal with

identical lenders.

In section 3.2, we present a simple model that establishes that multiple lending with identical lenders is possible if there is an asymmetric information regarding the lender's type. we show that, though the lenders do not know whether the promoter has a high or a low profitability project, nonetheless, the lenders can offer contracts to the promoter that separates the types. The contract offered, entails the promoter with a high profitability project to separate its type from the other, by borrowing from multiple lenders.

### 3.2 The Model

We extend the basic model in section 2.2 of chapter 2 to include asymmetric information among the agents.

The economy consists of two competitive lending institutions and a promoter. The promoter P, has a project with uncertain returns, z. We assume that z has a density function f(z) and a distribution function F(z), with support  $[\underline{z}, \overline{z}]$ ,  $0 < \underline{z} < \overline{z} < \infty$  and  $F(\underline{z}) = 0$ .

### **A.1:** The distribution function F(z) follows uniform distribution.

The project requires a fixed investment X which can be raised as debt and/or equity. The amount X is common knowledge to the promoter and the lenders.

There are two sources of asymmetric information. One, as mentioned in chapter 2, is the private knowledge to the promoter regarding the actual project realization. The lenders audit the promoter to tackle this uncertainty. Lender i will incur a fixed auditing cost,  $\theta_i$ , i = 1, 2, whenever the promoter defaults on her debt repayments. By incurring  $\theta_i$ , i observes the actual realization of the project. Similar to chapter 2, the auditing technology is perfect.

The second source of asymmetric information is regarding the promoter's type. The promoters' type,  $\tau$  is either G or B, i.e,  $\tau \in \{G, B\}$ . The true type is known only to the promoter herself. The lenders have a probability distribution regarding the promoter's type. The initial probability distribution of the promoter's type is given by  $\Pr{\{\tau = G\} = p \text{ and } \Pr{\{\tau = B\} = 1 - p.}}$ 

Type G has a project with returns z such that  $\underline{z} = t, \overline{z} = 1 + t$  t > 0. Therefore, for G, z is uniform over [t, 1 + t]. Type B has a project return z. such that  $\underline{z} = 0, \overline{z} = 1$ . Therefore, for B, z is uniform over [0, 1].

The sequencing of the model is as follows. In the first stage, the lenders offer two contracts to the promoter. The contracts specify the number of lenders who wish to lend, their respective debt claim, D and investment. I respectively. In the next stage, the promoter decides upon which contract to accept. If the total investment provided by the lender(s) is less than X, the remainder, X-I, is raised by the promoter as equity. This takes place in the third stage. In the final stage, the project realizations are distributed among the promoter and the lender(s) according to the contract specifications.

Apart from the monitoring costs, lender i also has a unit cost of raising funds,  $r_i$ , i = 1, 2. The cost of raising one unit of capital for the promoter is q. We assume that the lenders are identical.

A.2:

$$\theta_1 = \theta_2 = \theta; \quad q > r_1 = r_2 = r > 0.$$

We assume that q > r. The promoter, being small compared to the institutional lenders, will find it costlier to raise capital from the market as compared to the lenders. Besides, if  $q \le r_i$ , i = 1, 2, the promoter will never approach  $L_i$  for loan.

Denote  $D^M$  as the aggregate debt claims and  $I^M$  as the aggregate investment levels by the lenders, when they collectively lend. Similarly, denote  $D^S$  and  $I^S$  as the debt and investment level when only one of the lenders lend. The superscript M and S denote multiple and single lending, respectively. We assume that with multiple lending, strict hierarchy is followed. Denote  $D^M_{11}, I^M_{11}$  as the debt claim and investment of the senior lender, while  $D^M_{12}, I^M_{12}$  as the debt and investment of the junior lender. As the lenders are identical, without loss of generality, we assume that  $L_1$  is the senior claimant.

Let,  $\Gamma = \{D^S, I^S\}$  and  $\Omega = \{D^M, I^M\}$  be the two contracts offered by the lenders to the promoter. The contract  $\Gamma$  involves a debt claim of  $D^S$  with a corresponding investment level  $I^S$ . Contract  $\Omega$  involves a debt claim of  $D^M = D^M_{11} + D^M_{12}$  and investment of  $I^M = I^M_{11} + I^M_{12}$ . The promoter decides which contract to accept. If she accepts  $\Omega$ , she retains an amount  $Max\{z-D^M,0\}$  from the project for herself and finances  $X-I^M$  with equity. Similarly, if she accepts  $\Gamma$ , she retains  $Max\{z-D^S,0\}$  from the project and invests  $X-I^S$  as equity in the firm.

The expected return to any lender when it lends alone, is given by  $R^S$ .

Therefore,

$$R^{S} = \int_{\underline{z}}^{D^{S}} (z - \theta) dz + D^{S} \int_{D^{S}}^{\overline{z}} dz.$$
 (3.1)

Denote  $R_{11}^M$  as the expected return to the senior lender,  $L_1$  and  $R_{12}^M$  as the expected return to the junior lender,  $L_2$ , respectively. Then,

$$R_{11}^{M} = \int_{\underline{z}}^{D_{11}^{M}} (z - \theta) dz + D_{11}^{M} \int_{D_{11}^{M}}^{\overline{z}} dz$$
 (3.2)

$$R_{12}^{M} = \int_{D_{11}^{M}}^{D_{11}^{M}} (z - D_{11}^{M}) dz + (D_{1}^{M} - D_{11}^{M}) \int_{D_{1}^{M}}^{\overline{z}} dz - \theta F(D_{1}^{M}).$$
 (3.3)

The equilibrium we are interested in has, type 'G' offering contract  $\{\Omega\}$  involving multiple lending while type 'B' offers  $\{\Gamma\}$  involving borrowing from a single lender. Therefore, in equilibrium, whenever contract  $\{\Gamma\}$  is observed by the lenders they know that  $\underline{z}=0$  and  $\overline{z}=1$ . Similarly, whenever contract  $\{\Omega\}$  is observed by the lenders they know that  $\underline{z}=t$  and  $\overline{z}=1+t$ . In other words, we have

$$R^{S} = \int_{0}^{D^{S}} (z - \theta) dz + D^{S} \int_{D^{S}}^{1} dz$$

$$= D^{S} (1 - \theta) - 0.5 [D^{S}]^{2}. \qquad (3.4)$$

$$R_{11}^{M} = \int_{t}^{D_{11}^{M}} (z - \theta) dz + D_{11}^{M} \int_{D_{11}^{M}}^{1+t} dz$$

$$= D_{11}^{M} (1 + t - \theta) - 0.5 [D_{11}^{M}]^{2} + \theta t - 0.5 t^{2}. \qquad (3.5)$$

$$R_{12}^{M} = \int_{D_{11}^{M}}^{D_{11}^{M}} (z - D_{11}^{M}) dz + (D_{1}^{M} - D_{11}^{M}) \int_{D_{1}^{M}}^{1+t} dz - \theta (D_{1}^{M} - t)$$

$$= -0.5 ([D_{1}^{M}]^{2} - [D_{11}^{M}]^{2}) + (1 + t) (D_{1}^{M} - D_{11}^{M})$$

$$- \theta (D_{1}^{M} - t). \qquad (3.6)$$

Competition among the lenders ensure that funds are supplied to the firm, till no additional profits are to be made by doing so. Therefore, investment by the bank, with the contract  $\{\Gamma\}$ , is

$$I^{S} = \frac{1}{r} \left\{ D^{S} (1 - \theta) - 0.5 [D^{S}]^{2} \right\}. \tag{3.7}$$

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The aggregate investment by the banks with the contract  $\{\Omega\}$ , denoted  $I^M$ , is obtained from  $rI^M = R_{11}^M + R_{12}^M$ . Using equations (3.5) and (3.6), we obtain

$$I^{M} = \frac{1}{r} \left\{ D^{M} (1+t) + \theta [2t - D_{11}^{M} - D^{M}] - 0.5[D^{M}]^{2} - 0.5t^{2} \right\}.$$
 (3.8)

We denote  $\Pi^{\Gamma}(\tau)$  and  $\Pi^{\Omega}(\tau)$  as the expected profit to the promoter of type  $\tau$ , when she accepts contracts  $\{\Gamma\}$  and  $\{\Omega\}$  respectively. Therefore, we have

$$\Pi^{\Gamma}(B) = \int_{D^{S}}^{1} (z - D^{S}) dz - qX + qI^{S}.$$

$$\Pi^{\Omega}(B) = \int_{D^{M}}^{1} (z - D^{M}) dz - qX + qI^{M}.$$

$$\Pi^{\Gamma}(G) = \int_{D^{S}}^{1+t} (z - D^{S}) dz - qX + qI^{S}.$$

$$\Pi^{\Omega}(G) = \int_{D^{M}}^{1+t} (z - D^{M}) dz - qX + qI^{M}.$$

Solving the above we obtain,

$$\Pi^{\Gamma}(B) = 0.5(1 - D^{S})^{2} - qX + qI^{S}.$$

$$\Pi^{\Omega}(B) = 0.5(1 - D^{M})^{2} - qX + qI^{M}.$$

$$\Pi^{\Gamma}(G) = 0.5(1 + t - D^{S})^{2} - qX + qI^{S}.$$

$$\Pi^{\Omega}(G) = 0.5(1 + t - D^{M})^{2} - qX + qI^{M}.$$
(3.9)

The following parameters are defined for notational simplicity:

$$A \equiv D^{S} - D^{M}; \quad C \equiv 1 - 0.5(D^{S} + D^{M}); \quad V \equiv \frac{q}{r}\theta[D_{11}^{M} - (D^{S} - D^{M})].$$

Define  $\bar{t}$  and  $\underline{t}$  such that,

$$\bar{t} \equiv \{2\theta + D^M\} + \sqrt{(2\theta + D^M)^2 + 2\{AC[1 - r/q] + \theta D_{11}^M - (D^S - D^M)\}} 
\underline{t} \equiv \{2\theta + D^M - A\} + \sqrt{(2\theta + D^M - A)^2 + 2\{AC[1 - r/q]\}}.$$

The following result gives us a separating equilibrium, where G accepts  $\Omega$  and B accepts  $\Gamma$ .

**Proposition 3.1** When  $\underline{t} < t < \overline{t}$ , type G borrows from multiple lenders accepting the contract  $\Omega$ , and type B borrows from a single lender accepting the contract  $\Gamma$ .

The above sufficiency condition requires that the differences in types across the promoters must neither be too far nor too close. In other words, the difference in the types, here t, must be bounded from above and below.

Note that, ceteris paribus, for any given level of aggregate debt, the total investment by two identical lenders is less than if any one of them were to be the sole lender. This is because multiple lending duplicates expected monitoring costs. The investments made by the lenders internalize this cost, reducing the aggregate investment. Therefore, the cost of signaling is the lower aggregate investment by multiple lenders, at a given level of debt claim. The benefit of signaling, on the other hand, is the higher residual claim of the promoter. If the superior project (G) is very close to the inferior project (B) (t) is low), then net effect of these is negligible and G will not find it profitable to signal. On the other hand if G is much superior to G G is very high), then G can offer a high enough debt claim to one lender that cannot be offered by G. It is only in an intermediate range, that G cannot offer high debt claims to one bank, but can offer more modest debt, but to two banks.

The key assumption here is the existence of asymmetric information regarding the promoter's type. The promoter knows her type while the lenders only know the probability distribution from which type is drawn. We work with this assumption because the focus of our study is to establish that multiple lending is obtainable even with identical lenders if there exists asymmetric information regarding the promoter's type. Winton [98] established multiple lending with identical lenders who were fund constrained. In our study, we assume throughout that, the lenders are not fund constrained. The reason why we have multiple lending with identical lenders is because of asymmetric information between the lenders and the promoter regarding her type. Recall that in Chapter 2, all information at the time of making the loan was symmetric, but lenders had different cost configurations. Here, lenders are identical, but do not have the same information as the borrower (though between them they have the same information). Taken together, we have established the rationale behind multiple lending by looking at two polar cases.

### 3.3 Conclusion

The above discussion suggests another scenario where multiple lending is optimal for the promoter. Under asymmetric information, the promoter will approach multiple lenders for project financing even though the lenders are identical. Asymmetric information among the lenders and the firm is crucial for multiple lending with identical lenders. This is distinct from the approach taken by Winton [98]. In [98], multiple lending occurs with identical lenders due to credit-constrained investors. However, as is evident from the above set up, if the lenders are not fund constrained, the promoter will borrow from multiple investors to signal her type. In our model the lenders are not credit constrained. Our result depends upon the asymmetric information between the promoter and the lenders. In this set-up, the promoter with a better project separates herself from the other by borrowing from multiple lenders.

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## 3.4 Appendix to Chapter 3

#### • Proof of Proposition 3.1.

**Proof:** We show that  $\underline{t} < t < \overline{t}$ 

It is sufficient to ensure that the promoter of type G makes more profit by borrowing from multiple lenders and so accepts  $\Omega$ , i.e,  $\Pi^{\Omega}(G) > \Pi^{\Gamma}(G)$ . Also, the promoter of type B earns more profit when she borrows from a single lender. Hence, she accepts  $\Gamma$ , i.e  $\Pi^{\Gamma}(B) \geq \Pi^{\Omega}(B)$ . From equation (3.9) we have

$$\Pi^{\Gamma}(B) > \Pi^{\Omega}(B) \Rightarrow q(I^{S} - I^{M}) > AC$$
  
 $\Pi^{\Omega}(G) \leq \Pi^{\Gamma}(G) \Rightarrow q(I^{S} - I^{M}) \leq AC + At$ 

From equations (3.7) and (3.8), we get

$$q[I^S - I^M] = \frac{q}{r} \{AC + [0.5t - 2\theta - D^M]t\} + V.$$

Therefore, the sufficient condition satisfying  $\Pi^{\Omega}(G) > \Pi^{\Gamma}(G)$  and  $\Pi^{\Omega}(B) \leq \Pi^{\Gamma}(B)$ , is

$$AC + At > \frac{q}{r} \{AC + [0.5t - 2\theta - D^{M}]t\} + V$$

$$0 > \frac{q}{r} \{AC + [0.5t - 2\theta - D^{M}]t\} - AC - At + V \quad (3.10)$$

$$AC \leq \frac{q}{r} \{AC + [0.5t - 2\theta - D^{M}]t\} + V$$

$$0 \leq \frac{q}{r} \{AC + [0.5t - 2\theta - D^{M}]t\} - AC + V \quad (3.11)$$

Note V is independent of t. Further, from the above two inequalities we have at t=0

$$0 > \left\{\frac{q}{r} - 1\right\} AC + V \quad \text{(from equation 3.10)}$$
$$0 \leq \left\{\frac{q}{r} - 1\right\} AC + V \quad \text{(from equation 3.11)}$$

The two inequalities cannot be satisfied implying that  $t \neq 0$ . This implies that the separating equilibrium requires a lower bound on t such that t > 0.

Also as t is the lowest possible project realization,  $D^M$ , the aggregate debt claim on the project must be as much as t, i.e.,  $D^M \geq t$ . Therefore,  $[0.5t - 2\theta - D^M]t$  decreases in t. Therefore, we have

$$Lim_{t\to\infty}[0.5t-2\theta-D^M]t\to-\infty.$$

This implies from (3.11) that

$$Lim_{t\to\infty}AC \leq -\infty$$
.

The above condition cannot be satisfied as both A and C are finite. This implies that t is bounded above. Therefore, the separating equilibrium requires an upper lower bound on t such that t > 0 and is finite.

Therefore, we can now claim that there exists  $\underline{t}, \overline{t} > 0$ , such that

$$\begin{split} AC\{\frac{q}{r}-1\} + \frac{q}{r}[0.5t - 2\theta - D^M]t + V & \geq 0 \ \forall t \leq \overline{t} \\ AC\{\frac{q}{r}-1\} + \frac{q}{r}[0.5t - 2\theta - D^M]t + V - Ai & < 0 \ \forall t \geq \underline{t}. \end{split}$$

where

$$\bar{t} = \{2\theta + D^M\} + \sqrt{(2\theta + D^M)^2 + 2\{AC[1 - r/q] + \theta D_{11}^M - (D^S - D^M)\}} 
\underline{t} = \{2\theta + D^M - A\} + \sqrt{(2\theta + D^M - A)^2 + 2\{AC[1 - r/q]\}}.$$

Therefore,  $\underline{t} < t < \overline{t}$  is a sufficient condition ensuring that G borrows from multiple lenders and accepts  $\Omega$ , while B borrows from a single lender and accepts  $\Gamma$ .

# 62 C HAPTER~3.~~MULTIPLE~LENDING~WITH~ASYMMETRIC~INFORMATION

# Chapter 4

# Monitoring and Optimal Investment

# 4.1 Introduction

The recent regulations governing the banking sector in India, require banks and other financial institutions to operate in a more competitive framework. These regulations were carried out broadly, in two phases. It started with the *Debt Recovery Tribunal*, set up in 1993. This was followed in the same year by the recommendations of the Narasimham Committee [74] for the Indian public banks. In accordance with the guidelines of the Debt Recovery Tribunal and the recommendations of the Narasimham Committee, the banks are being made increasingly accountable for their actions. In this chapter, we develop a model of bank financing incorporating some of these features.

We consider a simple model where the lenders (banks) monitor the borrower (firm) to prevent the latter from strategic under reporting of project realization. The additional feature in this chapter is the source of uncertainty regarding the project realizations. Apart from the inherent risk associated with the project, expected profitability also depends upon the aggregate investment in the project.

In this model, aggregate investment in the project can be raised by any combination of debt and equity. The total investment in the project can

<sup>&</sup>lt;sup>1</sup>For example, the banks are required to make provisions for Non Performing Assets (NPAs), writing off bad debts etc. Therefore, the onus rests on the banks to recover risky debts.

be viewed as the fund required to purchase machinery and technology, or improve human resources, etc. This means that, better the quality of these resources, acquired by more investments, higher is the probability that the project succeeds. In other words, we believe that, firms capable of raising more capital are less likely to fail. The profitability of the project is measured as the difference between the expected return from the project and the costs of investment.

Elsewhere in the literature, usage of the fund raised by the firm is either to repay the old investors (Hart and Moore[54]) or to cover the cost required to undertake a project (see Berkovitz and Kim [13], Rajan[76] among others). Dowd [41] considers a model where the investment raised by the firm affects the return from the project without affecting the probability of success.

In our setup, we find that a debt contract leads to overinvestment. This result contradicts some of the previous works. Our results crucially depend upon the fact that the investments made by the bank and the promoter, enhance each other. However, in the other papers in the literature, the investments made by the different agents do not affect the incentives for each other to invest.

Our model also offers a possible explanation for differing debt equity ratios across economies. This is explained by the differences in auditing costs across these economies. Our model establishes an inverse relation between the 'average' auditing cost and the debt equity ratio.

The remainder of this chapter is organized as follows. Section 4.2 presents the basic model. The equilibrium of the model is described and derived in section 4.3. Section 4.4 deal with the underinvestment and overinvestment results. In section 4.5, some of the implications of the model are highlighted. Section 4.6 concludes the chapter. The proofs of the results appear in appendix A while in Appendix B, we find the relationship between equity and underinvestment.

## 4.2 Description of the Model

The economy consists of only two sets of agents, an entrepreneur/ promoter P, and a bank B. The promoter has a project that converts input of X into output of z. The input is the aggregate investment in the project provided by the bank and the promoter. The amount, X is not fixed and is obtained as the aggregate investment by the bank,  $I_B$  and by the promoter,  $I_P$ . The

project realizations are uncertain because of two factors. First, because z is inherently risky, and second, the default probabilities are also related to aggregate investment in the project, where the aggregate investment is itself a variable. The project has two stages. In the first stage, X determines the probability of obtaining z.

Denote  $I_B$  and  $I_P$  as the investments by the bank and the promoter respectively with  $I_B, I_P \geq 0$ . Therefore,  $X = I_B + I_P$ . The costs of raising these funds are  $r(I_B)$  for the bank and  $q(I_P)$  for the promoter.

**A.1:** 
$$r(I_B) = r.I_B \text{ and } q(I_P) = q.I_P, \text{ with } q > r > 0.$$

The costs of raising funds for both the parties are assumed to be linear functions. Further, it is assumed that the bank can raise funds at a lower cost than the promoter. This assumption is not unreasonable, given that the bank has a superior network as compared to the promoter.

Let p(X) be the probability of success. Here 'success' is to be interpreted as the state where z is realized. Typically, with an aggregate investment of X, p(X) is the probability that the project will realize z. However, z itself is uncertain and hence, could be zero. Nonetheless we will call a state to be 'success' if the realization in that state is z. With a probability of 1 - p(X) the realization from the project is zero. We assume the following about the probability function.

**A.2:** 
$$p(X) = 1 - e^{-X}$$
. Therefore,  $p(0) = 0$  and  $\lim_{X \to \infty} p(X) \to 1$ . Further,  $p'(X) > 0$  and  $p''(X) < 0$ .

The inherent riskiness of the project is given by the distribution function F(z). Assumption A.3 characterizes F(z).

**A.3:** z has a density function f(z). Its distribution function is F(z), with support  $[0, \bar{z}], 0 < \bar{z} < \infty$  and F(0) = 0.

The project described by A.2 and A.3 implies first order stochastic dominance. This can be seen as follows. Let  $z_1 \in [0, \overline{z}]$  and  $X_1 > 0$  be any

<sup>&</sup>lt;sup>2</sup>The two stage project formulation is similar to Shleifer and Vishny [89]. In [89], the firm has a two stage project. In the first stage, the firm draws at random, a project. The first stage of uncertainty is involved in this lottery. The project drawn in the previous stage has random returns. Our formulation is similar to theirs. However, we study different problems from them.

aggregate investment level. Therefore, probability that the project realization is at least as much as  $z_1$ , is

$$1 - F_{X_1}(z_1) \equiv Prob_{X_1}\{z \ge z_1\} = p(X_1)[1 - F(z_1)].$$

In the above expression,  $F_{X_1}(z_1)$  measures the probability that the realizations are at most  $z_1$  when the aggregate investment in the project is  $X_1$ . Also,  $Prob_{X_1}\{z \geq z_1\}$  denotes the probability that  $z \geq z_1$  with an aggregate investment of  $X_1$ . Note that  $Prob_{X_1}\{z \geq z_1\}$  increases as  $X_1$  increases. Therefore, for any  $z_1$ ,

$$F_{X_1}(z_1) \le F_{X_2}(z_1) \quad \forall X_1 \ge X_2.$$

This is the first order stochastic dominance condition as in Rothschild and Stiglitz [84].

The entrepreneur can finance the project in any of the following three ways.<sup>3</sup> Firstly, she can finance the project entirely through equity. In which case she invests an amount  $X = I_P$  in the project and does not seek any funding from the bank. Given A.2, the project will realize z with a probability of  $1 - e^{I_P}$ . The entrepreneur retains the entire amount of z for herself whenever z occurs.

The second mode of financing is entirely by debt financing. The promoter can finance the project by borrowing an amount  $I_B$  from the bank without contributing any amount herself. The bank will lend  $X = I_B$  in return for a claim of D on the project. In this case, the project succeeds with a probability of  $1 - e^{I_B}$ . The entrepreneur being the residual claimant, earns  $Max\{z-D,0\}$ .

Finally, the entrepreneur may contribute  $I_P$  as her own equity participation while borrowing  $I_B$  from the bank. In this case, the project succeeds with a probability of  $1 - e^{I_B + I_P}$ .

There are three stages. In the first stage, the promoter decides whether or not to borrow and subsequently, her own equity contribution. Note that,

<sup>&</sup>lt;sup>3</sup>In the main text of the chapter, we deliberately rule out yet another possibility of financing - that of, the bank financing the project through equity. However, this case is dealt in appendix B. We omit the discussions in the main text as we want to focus on the role of debt contracts. Bank financing through equity is akin to the universal banking model (Wihlborg and Dietrich [97]). The crucial feature in our model is the role of the large investor monitoring. For the various effects of bank monitoring see Carey et al [23] and Datta et al [28].

if the promoter decides not to borrow, then the project will be fully financed through equity. In the second stage, if the promoter borrows from the bank, the latter announces the debt claim as well as the investment (the loan supply) to the promoter. In the final stage, the actual realization takes place and z is distributed among the promoter and the bank, with debt having senior claims over equity.

As in the previous chapters, the bank incurs a fixed auditing cost,  $\theta$ , whenever the promoter defaults on her debt repayments. By incurring this cost, the lenders can observe *noiselessly*, the actual realization of the project and hence can prevent the firm from strategic under reporting thereby preventing strategic default by the promoter (Bolton and Scharfstein [17] and Hart and Moore [53]).

An important feature of this model is that, in order to undertake the project, borrowing is not compulsory for the firm. This is because (i) we do not assume that the project requires a fixed initiation cost and, (ii) we also do not assume that the fund available with the firm is inadequate to initiate the project. Therefore, in this model it is possible for the firm to undertake the project without external borrowing. Additional funding increases the probability of success.

Denote D as the debt claim of the bank in the project and  $z^e \equiv \int_0^{\overline{z}} z dF(z)$  as the expected value of the project. Further denote  $R_B$  and  $R_P$  as the expected gross returns to the bank and the promoter respectively. The expressions for  $R_B$  and  $R_P$  are as follows.

$$R_{B} = \int_{0}^{D} z dF(z) + \int_{D}^{\overline{z}} D dF(z) - \int_{0}^{D} \theta dF(z)$$
$$= z^{e} - \int_{D}^{\overline{z}} (z - D) dF(z) - \theta F(D). \tag{4.1}$$

In the first line of (4.1), the first term on the right hand side represents the expected return to the bank from the project when the promoter defaults (i.e, z < D). The second term gives the expected return to the lender when her claim of D is satisfied (i.e,  $z \ge D$ ). The final term is the expected auditing cost to the bank. The above expressions imply that the lender audits the firm only when the firm defaults and not otherwise.

The gross returns to the promoter is given by.

$$R_P = \int_D^{\bar{z}} (z - D) dF(z) \tag{4.2}$$

We now denote  $\Pi_B$  and  $\Pi_P$  as the net returns (profits) to the bank and the promoter respectively.

$$\Pi_{B} = p(X)R_{B} - [1 - p(X)]\theta - rI_{B} 
= p(X)[R_{B} + \theta] - \theta - rI_{B}$$
(4.3)

$$\Pi_P = p(X)R_P - qI_P \tag{4.4}$$

The right hand side of equation (4.3) is explained as follows. With a probability of p(X) the project succeeds and the bank earns a gross return of  $R_B$ . With a probability of 1 - p(X) the project returns are zero. In order to prevent strategic default, the bank has to incur  $\theta$  whenever  $z \leq D$ . Therefore, the bank incurs  $\theta$  in the following events. One, when z is realized with probability p(X) but  $z \leq D$ . Therefore, the expected monitoring cost in this case is  $p(X)\theta F(D)$ . Two, when the project fails, with a probability of 1 - p(X). The expected monitoring cost in this case is  $[1 - p(X)]\theta$ . The total expected monitoring cost for the bank is

$$\theta\{1 - p(X)[1 - F(D)]\}.$$

Using equations (4.1) and (4.2), we have,

$$\Pi_{B} = p(X) \left\{ z^{e} - \int_{D}^{\overline{z}} (z - D) dF(z) \right\} 
- \theta \{ 1 - p(X)[1 - F(D)] \} - rI_{B}$$
(4.5)

$$\Pi_P = p(X) \int_D^{\bar{z}} (z - D) dF(z) - q.K \tag{4.6}$$

# 4.3 Solving for Equilibrium

In this section we present the equilibrium analysis of the model. The equilibrium conditions and the values are derived in section 4.3.1.

### 4.3.1 Equilibrium Analysis

The stages are solved backwards. Therefore, we solve for stage 2 first where the bank chooses  $D^*$  to maximize  $\Pi_B$  and then  $I_B^*$  is obtained from the zero profit condition for the bank. Note that, from equation (4.3), choosing  $D^*$  to maximize  $\Pi_B$  is equivalent to maximize  $R_B$ . Finally, stage 1 is solved where the promoter chooses  $I_P^*$  to maximize  $\Pi_P$  given  $\{D^*, I_B^*\}$ .

#### Solving Stage II:

Let  $D^*$  maximize  $R_B$ . Further, denote  $R_B^*$  as the maximum expected gross returns the bank can carn. Therefore,

$$R_B^* \equiv R_B(D^*, \theta) = z^e - \int_{D^*}^{\overline{z}} (z - D^*) dF(z) - \theta F(D^*).$$

Note that, from equation (4.3), higher  $R_B$  implies higher  $\Pi_B$ . Therefore, choosing  $D^*$  to maximize  $R_B$  is equivalent to maximizing  $\Pi_B$ .

A necessary condition for  $0 < D^* < \overline{z}$  is that at  $D^*$ 

$$\frac{dR_B}{dD} = 0 \Rightarrow [1 - F(D^*)][1 - \theta h(D^*)] = 0, \text{ where } h(.) \equiv \frac{f(.)}{[1 - F(.)]}. (4.7)$$

Note that h(.) is the hazard rate. The following assumption on the hazard rate ensures the second order condition for an interior solution.

### **A.4:** The hazard rate is increasing, i.e., h'(.) > 0.

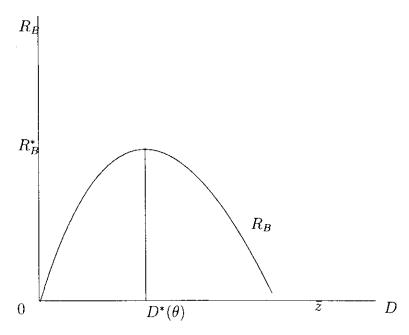


Figure 3.1

In Figure 3.1, the net return function for B is concave. It initially increases and then decreases with a rise in debt claim. This is because of the two opposing effects associated with an increase in the debt claims of the bank. To the bank, a higher debt claim increases the return during the non-default states. However, a higher D also increases the probability of default. The optimal debt claim is obtained as a tradeoff between these two opposing effects. Therefore, there exists an optimal debt claim,  $D^*(\theta)$ , such that the net return to B is maximized.

The concavity of the  $R_B$  function implies a form of credit rationing (Moore [70], Stiglitz and Weiss [91]). This is because, beyond  $D^*(\theta)$ , B's expected return falls with a higher debt claim. This induces the lender to invest a lower amount in the project.

The zero profit condition for the bank is obtained by setting  $\Pi_B = 0$  in equation (4.3). This implies

$$I_B^* = \frac{1}{r} \{ p(X)[R_B^* + \theta] - \theta \}. \tag{4.8}$$

#### Solving Stage I:

We now solve for  $I_P^*$ . Note that

$$I_P^* = arg \max_{I_P} \Pi_P = arg \max_{I_P} p(X)R_P^* - q.I_P,$$

where,

$$R_P^* \equiv R_P(D^*) = \int_0^{D^*} (z - D^*) dF(z).$$

Finally, we denote  $X_B$  as the aggregate investment, when the promoter borrows from the bank.

Denote  $A \equiv q(R_B^* + \theta) + rR_P^*$ . The explicit solutions of the key variables are: <sup>4</sup>

$$I_B^* = \frac{R_B^*}{r} - \frac{q(R_B^* + \theta)}{A}.$$
 (4.9)

$$X_B^* = ln\left(\frac{q(R_B^* + \theta) + rR_P^*}{qr}\right) \tag{4.10}$$

$$I_P^* = ln\left(\frac{q(R_B^* + \theta) + rR_P^*}{qr}\right) - \left\{\frac{R_B^*}{r} - \frac{q(R_B^* + \theta)}{A}\right\}.$$
 (4.11)

$$p(X_B^*) = 1 - \frac{qr}{A}. (4.12)$$

<sup>&</sup>lt;sup>4</sup>The derivations of explicit form solutions are given in the appendix.

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We now make two assumptions regarding the feasibility of the parameters.

**A.5:**  $\theta \leq \overline{\theta}$  where  $\overline{\theta}$  solves

$$R_B^*(\overline{\theta}) = \frac{qr[R_B^*(\overline{\theta}) + \overline{\theta}]}{q[R_B^*(\overline{\theta}) + \overline{\theta}] + rR_P^*(\overline{\theta})}.$$

The above assumption implies that  $I_B^* \geq 0$ . This follows from equation (4.9) and from lemma 4.7 which states that  $I_B^*$  is decreasing in  $\theta$ . Let  $\overline{D}$  solve  $\overline{\theta}h(\overline{D}) = 1$ . Therefore,

$$\begin{array}{lcl} R_B^{\star}(\overline{\theta}) & = & z^e - \int_{\overline{D}}^{\overline{z}} (z - \overline{D}) dF(z) - \overline{\theta} F(\overline{D}) \\ R_P^{\star}(\overline{\theta}) & = & \int_{\overline{D}}^{\overline{z}} (z - \overline{D}) dF(z). \end{array}$$

Note that, from (4.9),  $I_B^*(\overline{\theta}) = 0$ .

**A.6:** 
$$z^e > q > 0$$
.

Assumption A.6 implies that, the promoter can undertake the project without borrowing from the bank. The proof of this appears as lemma 4.2 in the appendix. Some of the results are given in the next section.

#### 4.3.2 Results

We begin by defining the following:

Investment Enhancing Property (IEP): A project will be said to satisfy IEP if, in equilibrium, an additional unit of investment by any one investor induces more investment by the others.

In our case, the project will satisfy IEP if an increase in  $I_P$  or  $I_B$  leads to an increase in  $I_B$  or  $I_P$  respectively.

**Proposition 4.1** Under A.1-A.4, the project described in our model satisfies *IEP*.

The above result is obtained owing to the fact that higher investment by either party increases the probability of success. A higher success probability in turn implies that the expected profitability of the project is higher. This induces the other party to invest more.

**Proposition 4.2** Ceteris paribus, the bank's investment,  $I_B^*$  increases if either-

- (a) the monitoring cost of the bank  $(\theta)$  decreases; or
- (b) the capital cost of the bank (r) decreases, or
- (c) the capital cost of the promoter (q) decreases.

The negative impact of  $\theta$  and r on  $I_B^*$  is expected. The more interesting aspect is the negative relationship between q and  $I_B^*$ . This is entirely attributed to the IEP. An increase in q would imply that the promoter would invest less amount in the project. This adversely affects the success probability of the project. In turn, the IEP implies that the bank invests less amount in the project.

**Proposition 4.3** The aggregate investment,  $X_B^*$  increases if either

- (a) the capital costs of the bank decreases, or
- (b) the capital cost of the promoter decreases. or
- (c) the monitoring cost of the bank increases.

Interestingly, an increase in the bank's cost components has differential impact on the aggregate investment level. While an increase in the capital cost reduces the aggregate investment, an increase in the monitoring costs increases the aggregate investment.

Part (c) of proposition 4.3 is particularly interesting. With a high  $\theta$ , the claim of the bank on the project is lower. This implies that the residual claim of the promoter in the project is very high. A high  $\theta$  curtails the bank's investment but increases the promoter's investment. The increase in investment by the promoter is more than the bank's reduction in investment. Note that, propositions 4.2 and 4.3 has obvious implications for the debt equity ratios which will be discussed later.

### 4.4 Optimal Investment Results

With information asymmetry between the borrower and the creditor, optimality of investment becomes a crucial issue. In this section we investigate the *underinvestment* and the *overinvestment* problem occurring in the framework.

With the bank and the promoter both financing the project, the equilibrium net surplus in the system, denoted by  $S^*$  is

$$S^{*} = \Pi_{P}^{*} + \Pi_{B}^{*}$$

$$= p(X_{B}^{*})R_{P}^{*} - qI_{P}^{*} + p(X_{B})[R_{B}^{*} + \theta] - \theta - rI_{B}^{*}.$$

$$= p(X_{B}^{*})\{R_{P}^{*} + R_{B}^{*} + \theta\} - \theta - qI_{P}^{*} - rI_{B}^{*}$$

$$= p(X_{B}^{*})\{z^{e} + \theta[1 - F(D^{*})]\} - \theta - qI_{P}^{*} - rI_{B}^{*}$$

$$= p(X_{B}^{*})z^{e} - \theta\{1 - p(X_{B}^{*})[1 - F(D^{*})]\}$$

$$-qI_{P}^{*} - rI_{B}^{*}.$$

$$(4.13)$$

The third equality is obtained by using Lemma 4.5 given in appendix A.

Note that the net surplus is maximum when the project is funded at the least cost combination. This entails that the expected monitoring cost is zero and that X is raised at a per unit cost of r. Let  $X_F^*$  denote the first best level of investment. Therefore,

$$X_F^* = \arg\max_X p(X)z^e - rX,$$

i.e.,

$$X_F^* = \ln\left(\frac{z^e}{r}\right). \tag{4.14}$$

The results concerning optimal investment levels are presented below.

Proposition 4.4 Under A.1 - A.4 there is overinvestment.

The overinvestment result is in sharp contrast to the literature involving optimal investment in a CSV framework. In Gale and Hellwig [44], Mukherji and Nagarajan [71], Biais and Casamatta [15], CSV framework leads to underinvestment with the standard debt contract (SDC). <sup>5</sup> The reason being.

<sup>&</sup>lt;sup>5</sup>The standard debt contract (SDC) represents " ... a contract which requires a fixed payment when the firm is solvent, requires the firm to be declared bankrupt if this fixed payment cannot be met and allows the creditor to recoup as much of the debt as possible from the firm's assets." (page 648, Gale and Hellwig [44]).

with costly monitoring to counter the moral hazard problems, credit rationing occurs thereby reducing the aggregate investment in the project.

In Gale and Hellwig, credit rationing occurs in the sense of rationing the size of the loan.<sup>6</sup> They find that the "... optimal (second-best) investment level never exceeds and typically falls short of the first best...[this is] the basic underinvestment result." Pg 648 [44]. Our findings are the opposite. Credit rationing, similar to Gale and Hellwig takes place in our framework. This follows from the concavity of the gross return function of the bank. The bank invests lower amount in the project if it is offered higher debt claims than  $D^*$ . However, in spite of credit rationing, we obtain overinvestment as compared to the first best level.

Why do our result and those of Gale and Hellwig [44] differ? This is mainly because of the IEP in our model. In simple terms, IEP means that the incentive to invest by any one agent increases with the investments made by the other agents. As the aggregate investment increases, it lowers the default probability. This acts as a sufficient incentive for the parties to invest more. The investments made by the bank and the promoter enhances each other.

We now investigate the effects of the parameters in the model on overinvestment. For this, define  $\Delta X \equiv X_B^* - X_F^*$  as the extent of overinvestment. Therefore,

$$\Delta X = \ln\left(\frac{R_B^* + \theta}{r} + \frac{R_P^*}{q}\right) - \ln\left(\frac{z^e}{r}\right).$$

Proposition 4.5 Ceteris paribus, the extent of overinvestment is higher if

- (i) the bank has higher monitoring cost, or
- (ii) the bank has higher capital cost, or
- (iii) the promoter has lower capital cost.

With higher monitoring cost, the gross return to the bank and hence its investment, is lower. However, a high  $\theta$  also implies that the residual claim of the promoter is higher and, so is her investment. The reduction in investment by the bank is offset by the increase in investment by the promoter. Therefore, the aggregate investment increases. As the first best level depends only upon r, the extent of overinvestment increases with an

<sup>&</sup>lt;sup>6</sup>Other forms of credit rationing involves denying credit to some firms entirely as in Stiglitz and Weiss [91].

increase in  $\theta$ . As r increases, the first best investment level decreases. The aggregate investment also reduces because of a rise in r. However, the fall in aggregate investment is lower than the fall in the first best investment level. Further, as q increases, the promoter invests less in the project thereby reducing the aggregate investment.

The role of IEP investment leading to overinvestment becomes more prominent in the next result.

**Proposition 4.6** For low monitoring costs, the investment made by the bank is more than the first best investment level. I.e.,  $\exists \theta^C$  where  $\theta^C < \overline{\theta}$  such that

$$\begin{array}{lcl} \forall \theta & \leq & \theta^C, & I_B^*(\theta) \geq X_F^* \\ \forall \theta & > & \theta^C, & I_B^*(\theta) < X_F^*. \end{array}$$

The first best level of investment involves- (i) no monitoring; and (ii) the entire funds raised should be at the least cost, r. The above conditions will be satisfied when the financier of the project is a single entity (this rules out strategic defaults) and has the least capital cost, r. In other words, this is as if, the bank owns the project. When the bank invests along with the promoter, then the bank has to incur  $\theta > 0$ , to prevent strategic default. One would expect the bank to invest lower amounts as compared to the fact where it lends alone. However, paradoxically, even with  $\theta > 0$ , the bank may invest more than before i.e.,  $I_B^* > X_F^*$  if  $\theta$  is low enough. This occurs because, as the promoter invests along with the bank, it increases the overall success probability. Thus, the expected return to the bank is higher than before as the default probability is lower. This is only possible if investments by the bank and the promoter enhance each other.

We conclude this section with some implications of proposition 4.5. Let us consider two banks, indexed k = 1, 2. Let  $(\theta_k, r_k)$  denote the monitoring cost and the capital cost for bank k. Denote  $R_k^*$  as the gross return to bank k. Also denote,  $R_{Pk}^*$  as the gross returns to the promoter when bank k lends. Therefore,

$$R_k^* \equiv R_B^*(D_k^*, \theta_k) = z^e - \int_{D_k^*}^{\overline{z}} (z - D_k^*) dF(z) - \theta_k F(D_k^*)$$

$$R_{Pk}^* \equiv R_P^*(D_k^*, \theta_k) = \int_{D_k^*}^{\overline{z}} (z - D_k^*) dF(z). \quad \forall k = 1, 2.$$

**Proposition 4.7** The extent of overinvestment is higher with bank k than bank j iff

$$q(R_k^* + \theta_k) + r_k R_{Pk}^* \ge q(R_j^* + \theta_j) + r_j R_{Pj}^*$$

Further if,  $\theta_1 = \theta_2 = \theta$ , i.e., the banks have identical monitoring costs. Then the extent of overinvestment is higher with bank k than j iff  $r_k > r_j$ .

If  $r_k = r_j = r$ , i.e., if the banks have identical capital costs, then the extent of overinvestment is higher with bank k iff  $\theta_k > \theta_j$ .

In the following section, we consider some of the implications of our findings.

### 4.5 Implications on Optimal Leverage Ratio

Based on the model developed so far, we now characterize the optimal debt equity ratio. As the investment by the bank and the promoter can be solved in terms of the parameters of the model, the debt equity ratio can be obtained in terms of the parameters alone. Define

$$\beta \equiv \frac{I_B^*}{I_B^*}$$

as the optimal leverage ratio. In particular,  $\beta$  measures the ratio of bank financed investment to the aggregate investment.

Proposition 4.8 The debt equity ratio decreases as

- (i) the auditing cost of the bank increases, or
- (ii) the capital cost of the bank increases.

The intuition behind the above result follows from the discussion following proposition 4.5. We showed that higher monitoring costs will induce lower investments by the bank (proposition 4.2 part (a)). Moreover, with higher  $\theta$  the aggregate investment in the project goes up (proposition 4.3 part (c)). Part (ii) of proposition 4.8 is obtained due to the following reasons. As the capital cost of the bank increases, the bank as well as the promoter curtails their respective investment. However, the curtailment by the bank outweighs that of the promoter. The above result conforms Harris and Raviv's [48] conjecture that the leverage ratio is negatively related to investigation cost.

**Proposition 4.9**  $\exists \hat{\theta} \leq \overline{\theta}$  such that, the debt equity ratio increases as the firm's capital cost increases for  $\theta \leq \hat{\theta}$ . However, the debt equity ratio decreases as the firm's capital cost increases for  $\theta > \hat{\theta}$ .

Proposition 4.9 suggests that with a rise in the promoter's capital costs, the overall debt equity ratio will increase if the bank has a low auditing cost and decrease if it has high auditing cost. As q increases, the investment by the bank as well as the promoter, decreases. However, if the expected monitoring costs are not too high, the banks' reduction in investment takes place at a lower rate than the reduction in investment by the promoter. IEP drives the result again. Suppose the auditing cost is 'high'. This would imply that the net expected returns for the bank is low. Contrast this with the case where the monitoring cost is low. A unit reduction in investment by the promoter will reduce the expected net returns to the bank by larger amount if the monitoring cost is high. Therefore, the bank with the higher monitoring cost will reduce its investment by greater amount than the bank with the lower monitoring cost.

The above results have important implications in the context of observed leverage ratios across various countries.

Some of the observed debt equity ratios across countries are as follows. Australia (1.248), Hong Kong (1.322), Singapore (1.232), Malaysia (0.935), UK (1.480), US (1.791), Canada (1.600), India (2.700), Japan and Korea (3.6), FGR (2.732), France (3.613), Finland (4.932), Sweden (5.552) and Norway (5.375). The pattern, emerging from the above numbers, suggests that the debt equity ratios reduce with the development of financial markets. The ratios are particularly high for those markets which are still 'emerging financial markets', like the Scandinavian nations, India, Pakistan and Korea. in comparison to the already developed markets like UK, US, Canada. Japan and FGR have rather high debt- equity ratios than most. The high debt equity ratios in these two countries is often attributed to their existing bank structure. Often the financial system in Japan and Germany are described as 'bank' driven financial structures (see Borio [20], Rajan and Zingales [78] etc).

Borio [20] compares the capital structure of the G-7 countries and conclude that companies in Japan and Continental Europe are more highly levered than the Anglo-American companies. The reason behind this, he attributes, is the financial structure and systems prevailing in these countries.

Similar findings are reported in Berglöf [11]. The reasons again are, the financial system prevailing in these countries. [11] asserts that while financing in countries like Japan, Germany, France and Italy are 'bank oriented', in USA, UK and Canada, financing is more 'market oriented'.

The above findings are consistent with our results. As we predict, a closer bank - firm relationship would be reflected in higher leverage ratios. This is because, with banks having greater control over the firm, the auditing costs of the former will be lower inducing it to invest relatively more.

Another recent study by Biais and Casamatta [15] suggests that the optimal leverage ratio decreases with a worsening of the moral hazard problem. Moral hazard problem in their context simply means the risk taking behavior of the managers. In our model, a severe moral hazard problem due to strategic under reporting, will be tackled by the bank by intensifying its monitoring activities. This in turn would mean a higher monitoring cost. We predict that the optimal leverage ratio is lower in such a case.

Harris and Raviv [48] as well as Ross [83] establish the existence of a positive correlation between leverage and default probability. However, Castanias [24] does not find any empirical support for the above. Our results help to resolve the ambiguity. The default probability,  $Prob\{default\}$ , in our case is given by

Prob{
$$default$$
} =  $p(X_B^*)$ Prob{ $z \le D^*$ } +  $(1 - p(X_B^*))$   
 =  $1 - p(X_B^*)[1 - F(D^*)].$ 

Ceteris paribus, higher default probability would imply a low aggregate investment,  $X_B^*$ . A high aggregate investment can be brought about in two ways - either through higher equity levels  $I_P^*$ , or through high debt levels  $I_B^*$ . If  $I_P^*$  is higher while  $I_B^*$  does not change, this will lead to a lower leverage ratio. However, if  $I_B^*$  increases without an accompanying increment in  $I_P^*$ , then the leverage is higher. In our model, a decrease in default probability may lead to either an increase in leverage (Harris and Raviv [48] etc.) or a decrease in leverage ratios (Castanias [24].)

#### 4.6 Conclusion

This chapter presents a model of optimal investment when the banks lend in a CSV framework. In order to prevent strategic default, auditing by the bank is done only when the promoter defaults. The project available with the promoter is characterized by first order stochastic dominance. In particular, the probability of the project realizing positive cash flows depend upon the aggregate investment in the project. In equilibrium, the investment by the promoter and the bank perfectly complement each other in the following sense. An additional unit of investment by the promoter, induces more investment by the bank. We find that a standard debt contract leads to overinvestment.

The crucial assumption that drives our results is A.2, where the probability of success increases with aggregate investments. In simple terms, A.2 implies that, the incentive to invest by any one agent increases with the investments made by the other agent. Thus, ceteris paribus, if one agent invests more, it increases the probability of positive returns to the other by lowering the overall default probability — investments made by the bank and the promoter enhance each other. In the literature, one has underinvestment if the probability of success is independent of the scale of investment, as in Gale and Hellwig [44]. Here, we have overinvestment with A.2. What is interesting is that there is investment overshooting for even a small positive dependence of the success probability to total investment.

# 4.7 Appendices to Chapter 4

#### 4.7.1 Appendix A: Proofs

Lemma 4.1

$$\frac{dX_B}{dI_P} = \frac{r}{r - p'(X_B)[R_B^* + \theta]}.$$

**Proof:** From equation (4.8) we obtain,

$$r \frac{dI_{B}^{*}}{dI_{P}} = p'(X_{B})[R_{B}^{*} + \theta] \left\{ \frac{dI_{B}^{*}}{dI_{P}} + 1 \right\}$$

$$\Rightarrow \frac{dI_{B}^{*}}{dI_{P}} = \frac{p'(X_{B})[R_{B}^{*} + \theta]}{r - p'(X_{B})[R_{B}^{*} + \theta]}$$

$$\Rightarrow \frac{dX_{B}^{*}}{dI_{P}} = 1 + \frac{dI_{B}^{*}}{dI_{P}} = \frac{r}{r - p'(X_{B})[R_{B}^{*} + \theta]} \cdot \cdot$$

## • The explicit form solutions obtained in Section 4.3.1

As discussed in the text,  $K^*$  is obtained from

$$I_P^* = arg \max_{I_P} \Pi_P = p(X)R_P^* - q.I_P,$$

where,

$$R_P^* = R_P(D^*) = \int_0^{D^*} (z - D^*) dF(z).$$

Thus,  $I_P^*$  solves,

$$p'(X_B^*)R_P^*\frac{dX_B}{dI_P} - q = 0.$$

Using Lemma 4.1 we obtain

$$p'(X_B^*)R_P^*\left\{\frac{r}{r-p'(X_B^*)[R_B^*+\theta]}\right\}=q.$$

Thus,

$$p'(X_B^*) = \frac{qr}{rR_P^* + q[R_B^* + \theta]}.$$

From A.2 we obtain that p' = 1 - p. Therefore,

$$p(X_B^*) = 1 - \frac{qr}{q(R_B^* + \theta) + rR_P^*}$$

$$X_B^* = ln\left(\frac{q(R_B^* + \theta) + rR_P^*}{qr}\right).$$

Further, from the expression of  $I_B^*$  as given below,

$$I_B^* = \frac{1}{r} \{ p(X)[R_B^* + \theta] - \theta \},$$

by substituting the value of  $p(X_B^*)$ , one obtains

$$I_B^* = \frac{R_B^*}{r} - \frac{q(R_B^* + \theta)}{q(R_B^* + \theta) + rR_P^*}.$$

Finally,  $I_P^*$  is obtained from  $I_P^* = X_B^* - I_B^*$  as

$$I_P^* = \ln \left( \frac{q(R_B^* + \theta) + rR_P^*}{qr} \right) - \left\{ \frac{R_B^*}{r} - \frac{q(R_B^* + \theta)}{q(R_B^* + \theta) + rR_P^*} \right\}.$$

**Lemma 4.2** Under A.1 - A.4 and A.6, the promoter can undertake the project even without borrowing from the bank.

Proof: If the promoter finances the project alone then

$$R_P = \int_0^{\overline{z}} z dF(z) = z^e.$$

Therefore,

$$\Pi_P^0 = p(X_P)z^e - qX_P.$$

Here  $X_P$  is the aggregate investment in the project if the promoter finances the project alone. Thus,

$$\begin{array}{rcl} X_P^* & = & \arg\max_{X_P} \Pi_P^0 \\ & = & \ln\left(\frac{z^e}{q}\right). \end{array}$$

The equilibrium profit to the promoter is given by

$$\Pi_P^0 = z^e - q - q \ln\left(\frac{z^e}{q}\right).$$

Note that,  $\Pi_P^0 > 0 \iff z^e > q$ .

#### • Proof of proposition 4.1:

**Proof:** It is sufficient to show that, in equilibrium

$$\frac{dI_B^*}{dI_B^*} > 0.$$

Note that from the proof of lemma 4.1,

$$\frac{dI_B^*}{dI_P^*} = \frac{p'(X_B^*)[R_B^* + \theta]}{r - p'(X_B^*)[R_B^* + \theta]}.$$

Therefore,

$$\frac{dI_B^*}{dI_B^*} > 0 \Longleftrightarrow r > p'(X_B^*)[R_B^* + \theta],$$

as  $p'(X_B^*) > 0$  from A.2.

By substituting

$$p'(X_B^*) = \frac{qr}{q[R_B^* + \theta] + rR_P^*}$$

in the above expression we obtain,

$$r - \frac{qr}{q[R_B^* + \theta] + rR_P^*}(R_B^* + \theta) > 0.$$

Thus,

$$\frac{dI_B^*}{dI_P^*} > 0.$$

Lemma 4.3  $R_B^*$  decreases as  $\theta$  increases.

**Proof:** From the definition of  $R_B^*$  and using the *Envelope Theorem* we have

$$\frac{dR_B^*}{d\theta} = \frac{\partial R_B^*}{\partial D^*} \frac{dD^*}{d\theta} + \frac{\partial R_B^*}{\partial \theta}$$
$$= 0 - F(D^*)$$
$$< 0.$$

The second equality follows from the fact that  $D^* = arg \max_D R_B$ .

Lemma 4.4  $R_B^* + \theta$  increases as  $\theta$  increases.

Proof: Using the previous Lemma we obtain

$$\frac{d(R_B^* + \theta)}{d\theta} = 1 + \frac{dR_B^*}{dD^*}$$
$$= 1 - F(D^*)$$
$$> 0.$$

**Lemma 4.5**  $R_P^* + R_B^* = z^e - \theta F(D^*)$ . The aggregate gross returns from the project equals the expected returns of the project minus the expected verification costs.

**Proof:** From equations (4.1) and (4.2) we obtain

$$\begin{array}{rcl} R_B^* + R_P^* & = & z^e - \int_{D^*}^{\overline{z}} (z - D^*) dF(z) - \theta F(D^*) + \int_{D^*}^{\overline{z}} (z - D^*) dF(z) \\ & = & z^e - \theta F(D^*). \end{array}$$

Lemma 4.6  $R_P^*$  increases as  $\theta$  increases.

**Proof:** From Lemma 4.5 we have  $R_P^* = z^e - R_B^* - \theta F(D^*)$ . Therefore using Lemma 4.3 we obtain  $dR_D^*$ 

$$\frac{dR_P^*}{d\theta} = -\theta f(D^*) \frac{dD^*}{d\theta}.$$

From  $\theta h(D^*) = 1$  we obtain

$$\frac{dD^*}{d\theta} = -\frac{h(D^*)}{\theta h'(D^*)}.$$

Assumption A.4 implies that h'(.) > 0. Thus,

$$\frac{dR_P^*}{d\theta} > 0.$$

Lemma 4.7  $I_B^*$  decreases while  $X_B^*$  increases as  $\theta$  increases.

**Proof:** Note that

$$X_B^* = \ln\left(\frac{R_B^* + \theta}{r} + \frac{R_P^*}{q}\right).$$

The proof follows from Lemmas 4.5 and 4.6.

**Lemma 4.8** Both  $I_B^*$  and  $X_B^*$  decreases as r increases.

**Proof:** 

$$\frac{dI_{B}^{*}}{dr} = -\frac{R_{B}^{*}}{r^{2}} + \frac{q(R_{B}^{*} + \theta)R_{P}^{*}}{A^{2}} 
= -\frac{1}{r} \left\{ \frac{R_{B}^{*}}{r} - \frac{q(R_{B}^{*} + \theta)}{A} \frac{rR_{P}^{*}}{A} \right\} 
< -\frac{1}{r} \left\{ \frac{R_{B}^{*}}{r} - \frac{q(R_{B}^{*} + \theta)}{A} \right\} 
= -\frac{I_{B}^{*}}{r} 
< 0.$$

The first inequality follows from the fact that  $rR_P^* < A \equiv rR_P^* + q(R_B^* + \theta)$ .  $X_B^*$  decreasing in r follows straightforward from observation that

$$X_B^* = ln\left(\frac{q(R_B^* + \theta) + rR_P^*}{qr}\right).$$

**Lemma 4.9** Both  $I_B^*$  and  $X_B^*$  decreases as q increases.

**Proof:** The proof follows straight forward after re writing  $I_B^*$  and  $X_B^*$  as

$$I_{B}^{*} = \frac{R_{B}^{*}}{r} - \frac{R_{B}^{*} + \theta}{R_{B}^{*} + \theta + \frac{R_{P}^{*}}{q}}; \qquad X_{B}^{*} = \ln\left(\frac{(R_{B}^{*} + \theta)}{r} + \frac{R_{P}^{*}}{q}\right).$$

**Lemma 4.10** The probability of success,  $p(X_B^*)$  increases as  $\theta, q, r$  increases.

**Proof:** The proof follows from Lemmas 4.7 - 4.9 and p'(X) > 0.

#### • Proof of proposition 4.2

**Proof:** The proof follows straight from Lemmas 4.7. 4.8 and 4.9.

#### • Proof of proposition 4.3

**Proof:** The proof follows straight from Lemmas 4.7, 4.8 and 4.9.

#### • Proof of proposition 4.4

**Proof:** From equations (4.10) and (4.14) we have

$$X_B^* - X_F^* = \ln\left(\frac{R_B^* + \theta}{r} + \frac{R_P^*}{q}\right) - \ln\left(\frac{z^e}{r}\right).$$

From Lemma 4.3 we know that  $R_B^* + \theta$  is increasing in  $\theta$ . This implies

$$R_B^* + \theta \equiv R_B(D^*, \theta) + \theta \ge R_B(D_0, 0) = \int_0^{D_0} z dF(z) + \int_{D_0}^{\overline{z}} D_0 dF(z),$$

where  $D_0$  is the optimal debt claim announced by the bank when it has zero monitoring costs. Following equation (4.7), with  $\theta = 0$ ,  $D_0$  solves  $[1 - F(D_0)] = 0$ , implying  $D_0 = \overline{z}$ . Thus,

$$R_B^* + \theta \ge R_B(D_0, 0) = z^e$$
.

Therefore,

$$X_B^* = \ln\left(\frac{R_B^* + \theta}{r} + \frac{R_P^*}{q}\right) \ge \ln\left(\frac{z^e}{r} + \frac{R_P^*}{q}\right) > \ln\left(\frac{z^e}{r}\right) = X_F^*.$$

The last inequality follows from the fact that q > r > 0.

#### • Proof of proposition 4.5

**Proof:** 

$$\begin{split} \Delta X &= ln \left( \frac{R_B^* + \theta}{r} + \frac{R_P^*}{q} \right) - ln \left( \frac{z^e}{r} \right) \\ \frac{d\Delta X}{dr} &= -\frac{q(R_B^* + \theta)}{rA} - \frac{1}{r} \\ &= \frac{1}{r} \left\{ 1 - \frac{q(R_B^* + \theta)}{A} \right\} \\ &> 0. \\ \frac{d\Delta X}{dq} &= -\frac{rR_P^*}{A} \\ &< 0. \end{split}$$

#### • Proof of proposition 4.6

**Proof:** Let  $\theta^C$  be such that  $I_B^*(\theta^C) = X_F^*$ . Note that,  $X_F^*$  is independent of  $\theta$ . Therefore,

$$I_B^*(\theta^C) = ln\left(\frac{z^e}{r}\right) \ge 0 = I_B^*(\overline{\theta}).$$

Thus,  $\theta^C$  is feasible. As  $I_B^*(\theta)$  is decreasing in  $\theta$ , we have

$$X_F^* = I_B^*(\theta^C) \leq I_B^*(\theta) \quad \forall \theta \leq \theta^C$$
$$= I_B^*(\theta^C) > I_B^*(\theta) \quad \forall \theta > \theta^C.$$

#### • Proof of proposition 4.7

**Proof:** Define

$$\Delta X_k \equiv \ln \left( \frac{R_k^* + \theta_k}{r_k} + \frac{R_{Pk}^*}{q} \right) - \ln \left( \frac{z^e}{r_k} \right) \quad \forall k = 1, 2$$

as the extent of overinvestment with bank k. Therefore,

$$\Delta X_{k} - \Delta X_{j} = ln \left( \frac{(R_{k}^{*} + \theta_{k})}{r_{k}} + \frac{R_{Pk}^{*}}{q} \right) - ln \left( \frac{z^{e}}{r_{k}} \right)$$

$$- ln \left( \frac{(R_{j}^{*} + \theta_{j})}{r_{j}} + \frac{R_{Pj}^{*}}{q} \right) + ln \left( \frac{z^{e}}{r_{j}} \right)$$

$$= \frac{\frac{(R_{k}^{*} + \theta_{k})}{r_{k}} + \frac{R_{Pk}^{*}}{q}}{\frac{z^{e}}{r_{k}}} - \frac{\frac{(R_{j}^{*} + \theta_{j})}{r_{j}} + \frac{R_{Pj}^{*}}{q}}{\frac{z^{e}}{r_{j}}}$$

$$= \frac{1}{qz^{e}} \left\{ q(R_{k}^{*} + \theta_{k}) + r_{k}R_{Pk}^{*} - q(R_{j}^{*} + \theta_{j}) - r_{j}R_{Pj}^{*} \right\}.$$

Further, let  $\theta_k = \theta_j = \theta$ . Therefore, note that

$$R_k^* + \theta_k = R_i^* + \theta_i$$
 and  $R_{Pk}^* = R_{Pi}^*$ .

This follows from the fact that the gross returns to the promoter and the bank only depends upon the monitoring costs. Therefore, if the banks have the same monitoring costs, then the gross returns to the bank and the promoter respectively, are the same.

Thus, using the condition in proposition 4.7 we obtain

$$\Delta X_k - \Delta X_j = (r_k - r_j)R_P^* \Rightarrow \Delta X_k > \Delta X_j \Longleftrightarrow r_k > r_j.$$

Similarly, if  $r_k = r_j = r$ , then using the condition in proposition 4.7 we obtain

$$\Delta X_k - \Delta X_j = q\{R_k^* + \theta_k - R_j^* - \theta_j\} + r\{R_{Pk}^* - R_{Pj}^*\}.$$

As  $R_B^* + \theta$  and  $R_P^*$  are increasing in  $\theta$  (this follows from lemmas 4.4 and 4.6). we have  $\Delta X_k > \Delta X_i \iff \theta_k > \theta_i$ .

#### • Proof of proposition 4.8

**Proof:** The proof of (i) follows from the fact that  $I_B^*$  decreases while  $I_P^*$  increases as  $\theta$  increases (see Lemma 4.7). Proof of (ii). Note that, as  $X_B^* \equiv I_B^* + I_P^*$ ,  $\beta \equiv I_B^* / I_P^*$  will increase whenever  $I_B^* / X_B^*$  increases and vice versa. We therefore will look at the  $(I_B^* / X_B^*)$  expression for simplicity. Now,

$$sign\left(\frac{d\beta}{dr}\right) = sign\left(\frac{1}{(X_B^*)^2} \left\{ X_B^* \frac{dI_B^*}{dr} - I_B^* \frac{dX_B^*}{dr} \right\} \right).$$

Therefore,

$$\operatorname{Sign}\left(\frac{d\beta}{dr}\right) = \operatorname{Sign}\left\{X_B^* \frac{dI_B^*}{dr} - I_B^* \frac{dX_B^*}{dr}\right\}.$$

Now using the values for  $(dI_B^*/dr)$  and  $(dX_B^*/dr)$  we obtain.

$$\left\{ X_B^* \frac{dI_B^*}{dr} - I_B^* \frac{dX_B^*}{dr} \right\} = X_B^* \left\{ \frac{qR_P^*(R_B^* + \theta)}{A^2} - \frac{R_B^*}{r} \right\} + I_B^* \frac{q(R_B^* + \theta)}{Ar}.$$

At  $\theta = \overline{\theta}$ ,  $I_B^* = 0$ , implying that  $(d\beta/dr) < 0$ . The last equation is decreasing in  $\theta$ . Therefore, it remains to be shown that at  $\theta = 0$ ,  $(d\beta/dr) < 0$ .

Note that  $\theta = 0$  implies  $R_B^*(D_0, 0) = z^e$  and  $R_P^*(D_0, 0) = 0$ . This follows from the following observations.

$$R_B(D_0, 0) = \int_0^{D_0} z dF(z) + \int_{D_0}^{\overline{z}} D_0 dF(z),$$

where  $D_0$  is the optimal debt claim with  $\theta = 0$ . Following equation (4.7), with  $\theta = 0$ ,  $D_0$  solves  $[1 + F(D_0)] = 0 \Rightarrow D_0 = \overline{z}$ .

Thus, with  $\theta = 0$  we have

$$X_B^* = ln\left(\frac{z^e}{r}\right), I_B^* = \frac{z^e}{r} - 1, \quad R_P^* = 0, \quad R_B^* = R_B^* + \theta = z^e.$$

and further,

$$A = qz^e, \quad \frac{q(R_B^* + \theta)}{A} = 1.$$

We can now write,

$$\operatorname{Sign}\left(\frac{d\beta}{dr}\right)_{\theta=0} = \frac{1}{r} \left\{ \frac{z^e}{r} - 1 - \ln\left(\frac{z^e}{r}\right) \frac{z^e}{r} \right\}.$$

Let

$$x \equiv \frac{z^e}{r}$$
.

Therefore,

$$H(x) \equiv x - 1 - x \ln x$$
  
$$H'(x) = -\ln x.$$

By assumption  $z^e > q > r$ . This implies that H'(x) < 0 as x > 1. Therefore, H(x) attains the maximum value when x is minimum, i.e., when

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 $x \to 1$ . Note that H(1) = 0. This implies that the maximum value  $H(x) \le 0$ . This implies

$$\left\{ \frac{z^e}{r} - 1 - \ln\left(\frac{z^e}{r}\right) \frac{z^e}{r} \right\} \le 0$$

Therefore,

$$\left(\frac{d\beta}{dr}\right)_{\theta=0} \leq 0 \Rightarrow \left(\frac{d\beta}{dr}\right) < 0 \forall \theta > 0.$$

#### • Proof of proposition 4.9

**Proof:** Similar to the proof of proposition 4.8 we will look at the  $(I_B^*/X_B^*)$  expression for simplicity. Note that if  $(I_B^*/X_B^*)$  increases, then  $\beta$  increases. Now,

$$\frac{d\beta}{dq} = \frac{rR_P^*}{(X_B^*)^2 qA} \left\{ I_B^* - X_B^* \frac{q(R_B^* + \theta)}{A} \right\}.$$

Define  $\hat{\theta}$  such that

$$I_B^* = X_B^*(\hat{\theta}) \frac{q[R_B^*(\hat{\theta}) + \hat{\theta}]}{A(\hat{\theta})}.$$

Note that for  $X_B^*(\hat{\theta}) \geq 0$ ,  $I_B^*(\hat{\theta}) \geq I_B^*(\overline{\theta}) = 0 \Rightarrow \hat{\theta} \leq \overline{\theta}$ . Thus,  $\theta \leq \hat{\theta}$  is feasible. Therefore,

$$\theta > \hat{\theta} \implies I_B^* < X_B^*(\hat{\theta}) \frac{q[R_B^*(\hat{\theta}) + \hat{\theta}}{A(\hat{\theta})} \Rightarrow \frac{d\beta}{dq} < 0$$

$$\theta \le \hat{\theta} \implies I_B^* \ge X_B^*(\hat{\theta}) \frac{q[R_B^*(\hat{\theta}) + \hat{\theta}}{A(\hat{\theta})} \Rightarrow \frac{d\beta}{dq} \ge 0.$$

#### 4.7.2 Appendix B: Equity and Underinvestment

The basic overinvestment result obtained with debt in proposition 4.4 depends upon the fact that the project satisfies IEP. That is, the project has the feature that more investment by any one party induces higher investment by the other. We now show that, if the bank holds equity in the firm and in addition, monitors the borrower to prevent strategic default, we have underinvestment instead of overinvestment.

**Proposition 4.10** Under A.1-A.4, if the bank holds equity in the project. then there is underinvestment.

**Proof:** Let the equity contract be such that it specifies a share,  $0 < \alpha < 1$  on z to the bank. This implies that, out of each realization of z, the bank gets  $\alpha.z$  while the promoter retains the remainder,  $(1-\alpha)z$  for herself. Therefore.

$$R_B^* = \alpha \int_0^{\overline{z}} z dF(z) - \theta \int_0^{\overline{z}} dF(z)$$

$$= \alpha z^e - \theta$$

$$R_P^* = (1 - \alpha) \int_0^{\overline{z}} z dF(z)$$

$$= (1 - \alpha) z^e \quad \forall \alpha \in (0, 1).$$

Note that with the equity contract, the bank has to monitor in all possible realizations of z in order to prevent the promoter from making strategic defaults. Thus,

$$\Pi_B^* = p(X^e)\alpha z^e - \theta - rI_B^e$$
  

$$\Pi_P^* = p(X^e)(1-\alpha)z^e - qI_P^e.$$

In the above expression,  $I_B^e$ ,  $I_P^e$  and  $X^e$  denote the equilibrium levels of investment by the bank, the promoter and the aggregate investment respectively, when the bank holds equity.

Substituting the values of  $\Pi_B^e$  and  $\Pi_P^e$  in place of  $\Pi_B^*$  and  $\Pi_P^*$  in equation (4.10), we obtain

$$X^e = ln\left(\frac{\alpha z^e}{r} + \frac{(1-\alpha)z^e}{q}\right).$$

Finally using q > r > 0 we obtain,

$$X^{e} = ln\left(\frac{\alpha z^{e}}{r} + \frac{(1-\alpha)z^{e}}{q}\right) < ln\left(\frac{\alpha z^{e}}{r} + \frac{(1-\alpha)z^{e}}{r}\right)$$
$$X^{e} < ln\left(\frac{z^{e}}{r}\right) = X_{F}^{*} \quad \forall \alpha \in (0,1)$$

Proposition 4.10 states that if the outside investors only hold equity in the firm, then there is under investment. The transition from overinvestment to underinvestment comes as we move from a debt contract to an equity contract. With an equity contract, the states in which the bank has to monitor increases. This increases the expected monitoring costs of the bank. The aggregate returns from the project with equity contract is  $z^e - \theta$  which is less than that with debt  $(z^e - \theta F(D^*))$ . Therefore, the aggregate investment is much lower with equity than with debt.

# Chapter 5

# Rating Agencies- Efficiency and Regulation

#### 5.1 Introduction

In this chapter we study the role of a credit rating agency who provides brokerage services. As in Bhattacharya & Thakor [14], brokerage services among others, include screening and certification, underwriting/issuance etc. Here, we focus on the screening activities of such an intermediary. We provide a theoretical perspective to study the role of such agencies in achieving efficiency in the debt market.

Credit rating agencies evaluate and rate debt instruments. Bulk of the ratings done by the two leading rating agencies in India, CRISIL (Credit Rating Information Services of India Limited) and ICRA (Investment Information and Credit Rating Agency of India Limited), are fixed deposits and commercial papers. In other countries, apart from rating simple debt instruments, the rating agencies also provide the investors with information and ratings of countries, states, municipalities etc. The ratings of Moody's Investors Services are designed exclusively for the purpose of grading bonds according to their investment qualities. The Standard & Poor's corporate or municipal debt ratings is an assessment of the credit worthiness of a borrower with respect to its borrowing instruments.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>See Rose [82] for a discussion on the functioning of a rating agency. For the detailed guidelines regarding the operation of the Indian rating agencies, see the SEBI Manual [86], CRISILSCAN [27] etc.

The motivation for the current study lies mainly in the resultant debate and the suggested regulatory policies to govern credit rating agencies in India (see Venkatesh and Gupta [93], Ravishanker and Thakur [81]). These regulatory policies can be categorized into two distinct parts. One, regulating through an appropriate fee charged by the rating agencies, two, regulating through compulsory ratings or multiple ratings. In this chapter we develop a theoretical model highlighting the information generating role of such an agency. We then discuss some of the regulatory issues.

Most of the analysis is done with a monopolist rating agency. Competition is introduced as an extension to the current framework later. The benchmark model describing the functioning of a rating agency is presented in sections 5.2 and 5.2.1. We consider a simple environment where a firm has a project with binary payoffs. The firm has no wealth to initiate the project. Therefore, she borrows from the investor via debt. We begin by assuming that this debt level is exogenously given. This face value of the debt is assumed so low, that in the absence of a rating agency, the investor evaluates the project to be too risky to invest, given the information he has. Therefore, he refrains from investing in the firm. Thus the firm cannot raise this necessary funds to undertake the particular project. Therefore, even if the firm has a low default probability, she fails to raise the necessary investment. The rating agency acts as a third party information producer facilitating investment in the project.

In our model the investor is rational and has Bayesian beliefs. The investor updates his priors regarding the firm's type after each available information. Information production by a rating agency depends upon the screening technology it has and the evaluation standard it sets. The technology is able to distinguish between the types with a precision level that depends upon the evaluation standard.<sup>2</sup>

Our results indicate that only a portion of the types, come to the rating agency to get its instruments rated. Projects with high default probability do not come to the rating agency, and hence are not funded. Among the ones that come to the rating agency, only those projects which are announced

<sup>&</sup>lt;sup>2</sup>Alternatively, an investment banker can screen projects as certifying intermediaries. The certification fees paid by the firms coupled with the requirement that the investment banker bears some of the liabilities, leads to situations where only profitable projects are certified 'investment worthy' by the agency (Booth [19]). In contrast to a certifying intermediary, a rating agency also distinguishes across different projects via their announcements.

'good' by the rating agency, gets funded. The investor's decision to invest crucially depends upon the ability of the rating agency to individually identify each type and convey this information meaningfully to the market. In section 5.4, we find that the agency unambiguously improves efficiency. We then introduce a regulator who regulates the fee charged by the rating agency. We find that such regulation will not be able to increase the existing net surplus.

In section 5.5 we extend our model by making the face value of debt endogenous. Here, all the projects are funded even in the absence of a rating agency. Presence of a rating agency ensures that projects with very high default probabilities are not funded at all while all other projects are funded. However, we find that, the rating agency may actually end up reducing net surplus. Interestingly, higher are the returns from the project, more inefficient is the outcome in the presence of a rating agency. In this light we also investigate the possibility of regulator maximizing the net surplus by an appropriate choice of the fee charged by the rating agency. This is done in section 5.6.2. In section 5.6.3, we investigate other regulatory issues. We make two interesting observations. One, with appropriate regulation, the regulator can increase the net surplus in the system. Two, non price regulatory policies like compulsory ratings will in general be inefficient. We also find that the regulated fee will in generally be lower than the unregulated fee.

In section 5.7, we extend our analysis by allowing competition among the rating agencies. We construct a simple example to show that, competition may actually lead to inefficiency vis-a-vis monopoly.

Section 5.8 concludes the chapter.

# 5.2 Benchmark Model of Information Production

We consider a simple model where the economy consists of three sets of risk neutral agents - an investor, a firm and a credit rating agency (CRA).

Investor: The investor is endowed with capital. He can either invest a part of this in the risk free asset or can invest this capital with the firm. We assume that the risk free rate is zero. Therefore, investing one unit of capital in the risk free asset yields one unit of capital with certainty.

**Firm:** The firm/entrepreneur has a project with stochastic returns. The project requires one unit of capital as input to produce output tomorrow. For simplicity, we assume that the firm has no capital to initiate the project but the investor has enough funds. The firm has to raise the required amount from the investor if the project has to be started. Let z denote the (uncertain) returns from the project. We assume for simplicity that  $z \in \{0, \overline{z}\}$  where  $\infty > \overline{z} > 0$ . The outcome  $z = \overline{z}$  is to be interpreted as "success" with probability p where  $0 \le p \le 1$ . The probability of success is exogenous and type specific to the firm. The firm is of type p if its project succeeds with probability p.

To raise the required amount from the investor, the firm has to offer him a return of D per unit of amount borrowed,  $D \leq \overline{z}$ . As the project has only two possible realizations 0 and  $\overline{z}$ , a claim of D is only paid when  $\overline{z}$  is realized. Protected by limited liability, the firm pays nothing if the project fails. Thus. D is the face value of debt per unit borrowed by the firm. We implicitly assume that there is no *strategic default*, i.e, the firm does not default on meeting its debt obligations if the realization is  $\overline{z}$ .

**Information:** The true type of the firm is known only to the firm. This is the only source of asymmetric information. All other parameters in the economy are common knowledge. In other words, though p is not known to the others, the distribution function from which p is drawn is common knowledge. We denote this distribution function as F(p) with corresponding density function f(p).

**A.1:** The support of F(p) is [0,1] with density function f(p).

If the project gets funded, the expected profit to the firm of type p is:

$$\Pi_p = p\{\overline{z} - D\} + (1 - p)0.$$
 (5.1)

If the project succeeds, the firm earns  $\overline{z}$  from the project and has to pay D out of it to the investor. Therefore, with probability p it earns  $\{\overline{z} - D\}$ . If the project fails, the firm earns zero.

Aware of D, the investor's decision to invest will depend upon his expected returns from investing in the project vis-a-vis the outside option of putting the money in the risk free asset.

 $<sup>^3\</sup>mathrm{We}$  restrict ourselves to debt financing as they are the most commonly rated instruments by the CRA.

In the absence of any additional information being revealed, the investor calculates the expected probability of success of the firm that he is facing. This probability is E(p), where E(p) denotes the expectation operator on p. Therefore, by investing one unit of capital in the project the investor's expected returns are DE(p). The investor invests a unit of capital in the project if and only if  $DE(p) \geq 1$ . Denote  $\overline{p}$  such that  $\overline{p}.D = 1$ . Therefore, any type with  $p \geq \overline{p}$  will be termed 'good' whereas, any type such that  $p < \overline{p}$ , will be termed 'bad'. This notational convenience implies, the expected returns from investing in a 'good' firm is at least as much as investing in the risk free asset. Similarly, the expected returns from investing in a 'bad' firm is less than investing in the risk free asset.

#### 5.2.1 Functioning of the Rating Agency

The CRA conveys additional information about the firm's actual type to the investors.<sup>4</sup> The CRA acts as an information producer in three stages.

In the first stage, it sets an evaluation fee of W which must be paid by any firm that wishes to get itself rated by the CRA. This fee is charged ex ante and is the same for all types. Charging W ex post may not be incentive compatible. As we show later, the firm can take up the project only if it is announced 'good'. In order to get W ex post from the firm, it is necessary that the firm is able to undertake the project. This is because, the only source of payment by the firm to the CRA is from the project's cash flow. Therefore, the firm must be able to initiate the project in order to pay W. This will lead to a potential source of bias by the CRA towards giving better ratings to the firm than it actually deserves. The fee charged is same for all types because, at the time when the fee is paid, the rating agencies have no information regarding the firm's type and hence cannot make W depend upon p. However, W can depend on D and the other parameters in the model.<sup>5</sup>

In the second stage, the CRA learns about the firm with the help of the technology it has and the *evaluation standard* it sets. The accuracy with which it can distinguish p, depends upon the evaluation standard, e, it sets.

<sup>&</sup>lt;sup>4</sup>In actual practice, apart from the role of information production, the CRA may often act as an adviser to firms as it possesses additional macroeconomic information. However, in this model we consider only the information producing role of the CRA.

<sup>&</sup>lt;sup>5</sup>Later, when we consider competition, we consider the case when W is not chosen by the CRA.

For simplicity we assume that  $e \in [0, 1]$ .

Finally, the CRA announces its findings about the firm to the investor. The CRA announces the firm to be 'good' if it concludes that the firm's type is at least as much as  $\bar{p}$  and 'bad' if it concludes the firm's type to be less than  $\bar{p}$ . We assume away any *strategic announcements* by the CRA. That is the CRA always announces according to its findings.

The investor has Bayesian beliefs, and updates his priors regarding the firm's type, conditional on the announcements made about the firm. The decision whether or not to invest in the firm, depends upon these announcements. The investors invest if and only if the announcement is 'good'. The following notations will be used throughout this chapter. The announcements made by the CRA will be denoted by 'a'. An announcement 'good' will be denoted by 'g' and 'bad' will be denoted by 'b'. <sup>6</sup>

We assume that  $\{W,e\}$  are observable and verifiable by all the agents. In reality, W is observable. We make e observable to do away with the moral hazard problems between the CRA and the other agents in the economy. The possibility of moral hazard problem occurs because setting high evaluation standard is costly for the CRA.

We now briefly describe the role of the screening technology available with the CRA.

**Technology:** The technology available with the CRA is analogous to a type determining "black box". All the information available about the firm to the CRA are supplied as inputs. The technology then generates output in the form of reports that classify a firm having p either greater than or less than  $\bar{p}$ . The accuracy with which a type is inferred correctly depends upon e. A stricter evaluation standard (higher e) allows it to infer the actual type of a firm more accurately. In particular, with e=1 the technology is perfect. That is, the CRA knows the exact p. On the other extreme, with e=0, the technology does not convey any additional information regarding the firm's type other than the fact that p is drawn from F(p) and lies between 0 and 1. Different values of e can be attributed to the differences in actual

<sup>&</sup>lt;sup>6</sup>In reality the announcements are typically multi-dimensional, though we restrict ourselves to the case where the CRA just announces 'good' or 'bad'. In practice, the CRA classifies firms into categories starting from a 'high risk - speculative grade' to 'low risk - investment grade.'

<sup>&</sup>lt;sup>7</sup>One possible way to tackle such moral hazard problem is to consider reputation of the CRAs. See Chemmanur and Fulghieri [25] for a discussion on issues related to reputation of information producing intermediaries.

parameters CRAs may wish to investigate. In our model the evaluation standard is uni-dimensional. We make e uni dimensional as it considerably simplifies the algebra. One can interpret e having weighted components of all the parameters different agencies wish to investigate. Therefore, the agencies can be ranked according to e. Ceteris paribus, agencies with higher e can be thought to have less noisy announcements than agencies with lower e.

Corresponding to e, the technology generates a report, r for each type where  $0 \le r \le 1$ . The CRA observes r. The technology and the evaluation standard simply narrows down the possible values a type can take. This range is denoted by [l(p;e), u(p;e)] such that  $0 \le l(p;e) \le r \le u(p;e) \le 1$ . The technology and e determines l(p;e) and u(p;e). If  $r \ge \overline{p}$ , then the CRA concludes it to be good. Similarly, if  $r < \overline{p}$  it concludes the firm to be bad.

We assume the following about l(p; e) and u(p; e).

**A.2:** 
$$l(p;e) = pe$$
 and  $u(p;e) = 1 - e(1-p)$ .

A.2 implies that both u(.) and l(.) are differentiable in p and e. Further,  $u_p, l_p \ge 0$  and  $u_e \le 0 \le l_e$ . Also, l(0; e) = 0 and  $u(1; e) = 1 \ \forall e \in [0, 1]$ . Finally, l(p; 0) = 0, and u(p; 0) = 1 and, l(p; 1) = u(p; 1) = p.

The specifications  $u_e(.) \leq 0$  and  $l_e(.) \geq 0$  imply that with a higher (stricter) evaluation standard, the range of values any possible type can take is lower. At the extreme, e=1 corresponds to the "perfect" inference case. Note with e=1, l(p;1)=u(p;1)=p. Therefore, we have r=p. However, if e=0 then any type can generate a report between [0,1] with equal probability.

We now formally describe the announcements made by the CRA.

Announcement: Let  $\alpha \in [0, 1]$  denote the probability that any particular type will lead to an announcement 'good'. The probability with which any type is announced good is therefore simply the probability that  $r \geq \bar{p}$ .

<sup>&</sup>lt;sup>8</sup>In India, the two leading rating agencies CRISIL and ICRA, adopt different evaluation standards. In evaluating a firm, the former stresses the *legal position* of the firm, while the latter emphasizes its *managerial efficiency*.

<sup>&</sup>lt;sup>9</sup>Other possible *screening functions* that can qualify for our analysis include one proposed in Sah and Stiglitz [85]. The screening function in [85] assigns probabilities that a type is concluded accurately. This function depends upon deviations around the mean of the original distribution. However, the screening function proposed there does not depend upon any effort level or evaluation standard.

<sup>&</sup>lt;sup>10</sup>The subscripts p and e correspond to the first order partial derivatives with respect to p and e respectively. Therefore, an expression  $x_e$  denotes  $\partial x/\partial e$  etc.

For any p drawn from the distribution function F(p),  $\alpha(p,e,D)$  is given by

$$\alpha(p, 1, D) = \begin{cases} 1 & \text{if } p \geq \overline{p}; \\ 0, & \text{if } p < \overline{p} \end{cases}$$
and 
$$\forall e \in [0, 1),$$

$$\alpha(p, e, D) = \begin{cases} 0 & \text{if } u(p; e) < \overline{p}; \\ \frac{F(u(p; e)) - F(\overline{p})}{F(u(p; e)) - F((l(p; e))} & \text{if } u(p; e) \geq \overline{p} > l(p; e) \end{cases} . (5.2)$$

Note that  $\alpha(.)$  given in (5.2), satisfies the following conditions for  $e \in [0, 1)$ .

(i)  $\alpha(p, e, D)$  is continuous, differentiable in p and e;

(ii)  $\alpha(.)$  is non decreasing in p; and  $\alpha_e(.) \geq 0 \forall p \geq \overline{p}$  with  $\alpha_e(.) < 0 \forall p < \overline{p}$ .

The first condition describes the smoothness of the technology. The second condition is more crucial. First, it states that given any D, higher types are more likely to be announced 'good'. Second, with an improved evaluation standard, the probability with which a type is correctly inferred, increases.

**Example 1:** If  $F(p) \sim U[0,1]$  with u(p;e) = p + (1-e)(1-p) and l(p;e) = p - (1-e)p, then  $\forall e \in [0,1)$ 

$$\alpha(p, e, D) = \begin{cases} 0 & \text{if } p < 1 - \frac{(1-\overline{p})}{e}; \\ \frac{1-\overline{p}-e(1-p)}{(1-e)} & \text{if } \frac{\overline{p}}{e} \ge p \ge 1 - \frac{(1-\overline{p})}{e} \\ 1 & \text{if } p > \frac{\overline{p}}{e} \end{cases}.$$

If e = 1, then

$$\alpha(p, 1, D) = \begin{cases} 1 & \text{if } p \ge \overline{p}; \\ 0, & \text{if } p < \overline{p} \end{cases}.$$

# 5.3 Role of a Rating Agency when D is Exogenous

In this section we assume that D is exogenously specified.<sup>11</sup> In this setup, we will examine the role of a CRA. We shall then investigate the impact of the CRA on market efficiency.

Later, in section 5.5, we allow for endogenous D.

The starting point of our analysis is the case where D is low enough such that, in the absence of the CRA the investor is discouraged to invest in the firm. The following assumption rules out the possibility of trading between the firm and the investor in the absence of a CRA.

$$D\int_0^p pdF(p) \equiv DE(p) < 1.$$

A.3 implies that in the absence of any additional information that distinguishes the types, the investor's return is lower by investing in the firm than by investing in the risk free asset. Hence the investor does not invest in the project.

However, such a situation is inefficient (ex ante) as all types with  $p \ge 1/\overline{z}$  should be funded. We implicitly assume that  $\overline{z} > 1$ . This guarantees that  $\exists p \in (0,1)$  such that  $p\overline{z} \ge 1$ . Therefore, A.3 implies that some of the positive NPV projects are not funded.

The sequencing of the model is as follows.

In the first stage the CRA sets W and e. In the second stage, the firm has to make two decisions. The firm first decides whether or not to go to the CRA. If it decides to go, it borrows W from the investor to pay the CRA. We shall implicitly assume that once a firm has borrowed W, it has to go to the CRA.<sup>13</sup> Alternatively, the firm may decide not to go to the CRA and instead approach the investor directly for investment. In the third stage, the CRA rates the firm. The investor decides to invest additionally in the firm based on the announcement made by the CRA. Depending upon the investor's decision, the project is either taken up or not taken up by the firm. If the project succeeds the firm pays D(1+W) to the investor retaining the residual,  $\overline{z} - D(1+W)$  for itself. If the project fails, the returns to the firm and the investor is zero.

The equilibrium we are interested in involves the investor invests whenever the announcement is good and not otherwise. We assume that the investor believes that any type that does not go to the CRA has a p less than  $\bar{p}$ . Therefore, in equilibrium the investors do not invest if either the

<sup>&</sup>lt;sup>12</sup>If  $\overline{z} < 1$  then, the investor will never fund the project.

 $<sup>^{13}</sup>$ The investor can observe and verify whether a firm has gone to the CRA. One can think of legal enforcements which ensures that the firm has to go to the CRA if it borrows W.

announcement is 'bad' or if the firm does not go to the CRA. This 'equilibrium belief' is very similar to Milgrom [66]. However, the problem studied in Milgrom [66] is different. He studies the effect of fresh news arrival on a firms' value. In that model the beliefs of the investors are such that, in the event any information is withheld, it is automatically concluded as "bad news". In such cases, the investors attach lower value to the firm. Translated to our model, this would mean that, whenever the firm does not go to the CRA it is automatically inferred as 'bad' by the investor.

Investor's Decision: The investment decision of the investor depends upon the announcements made by the CRA. Denote,  $p_m$  as the minimum type that will come to the rating agency. The expected probability of success corresponding to any announcement is given by

$$E(p|\mathbf{a} = \mathbf{g}) = \frac{\int_{p_m}^1 p.\alpha(p;e)dF(p)}{\int_{p_m}^1 \alpha(p;e)dF(p)}.$$
 (5.3)

$$E(p|a = b) = \frac{\int_{p_m} p[1 - \alpha(p;e)]dF(p)}{\int_{p_m} [1 - \alpha(p;e)]dF(p)}$$
 (5.4)

The denominator in (5.3) and (5.4) indicates the total probability with which any type coming to the CRA is announced 'good' and bad' respectively. Recall  $\bar{p}$  is the type such that  $\bar{p}.D = 1$  and hence, an announcement 'good' corresponds to  $p \geq \bar{p}$ . Given any announcement 'a', the investor will invest if and only if  $E(p|\mathbf{a}) \geq \bar{p}$ . Therefore,

$$E(p|\mathbf{a}=\mathbf{g}) \ge \overline{p} \Rightarrow \int_{p_m}^1 (p-\overline{p})\alpha(p;e)dF(p) \ge 0.$$
 (5.5)

**Firm's Decision :** If the firm of type p goes to the CRA, the expected profit it earns is,

$$\Pi_{p} = \alpha(p; e) p\{\overline{z} - D(1+W)\} + [1 - \alpha(p; e)]0. \tag{5.6}$$

The firm can initiate the project if it is announced 'good' by the CRA. This occurs with a probability  $\alpha$ . With a total debt commitment of D(1+W), the net returns to the firm is  $\overline{z} - D(1+W)$ . However, the firm earns this with a probability p. Therefore, the expected net returns to the firm when it is announced 'good' is  $\alpha\{\overline{z} - D(1+W)\}$ . With probability  $(1-\alpha)$ , the firm

will be announced 'bad' and hence, cannot undertake the project. Therefore, the firm earns zero net returns when the announcement is 'bad'.

Note that, without any additional specification  $p_m = 0$ . As W is not incurred by the firm, the firm's cost of going to the CRA is zero. By going to the CRA, with a probability  $\alpha$  it can actually earn non negative expected net returns. Therefore, it will be in the interest of the firm to always go to the CRA irrespective of its type. To rule out such a case, we will assume that only types with  $\alpha(p;e) > 0$  goes to the CRA. Note that, once e is observed, all types can calculate  $\alpha(p,e)$  and hence would decide whether or not to go to the CRA. In other words, from equation (5.2), the decision of the firm whether or not to go, depends upon whether or not u(p,e) is greater than  $\overline{p}$ .

The expected profit of the CRA, denoted by  $\Pi_C$  is:

$$\Pi_C = W[1 - F(p_m)] - \overline{c}. \tag{5.7}$$

In the above equation,  $p_m$  depends upon e and hence,  $\Pi_C$  depends upon e as well. The expected gross returns to the CRA is the revenue it earns from all possible types which pay W. This is given by  $W[1-F(p_m)]$ . Finally,  $\bar{e}$  is the fixed set up cost of the CRA. A possible justification of assuming this cost to be independent of e is the fact that e is the evaluation standard. Therefore, the CRA chooses e and W such that  $\Pi_C$  is maximized.

Taking into account the actions of the various agents in the economy, the above exercise can be formally written as,

$$\max_{\{W,e\}} \Pi_C = W[1 - F(p_m)] \quad \text{s.t.}$$
 (5.8)

$$\alpha(p_m, e, D) \{ p_m[\overline{z} - D(1 + W)] \} \ge 0$$
 (5.9)

$$\int_{p_m}^1 (p - \overline{p})\alpha(p, e, D)dF(p) \ge 0.$$
 (5.10)

<sup>&</sup>lt;sup>14</sup>This can be supported by assuming that the firm incurs a small cost by going to the CRA.

<sup>&</sup>lt;sup>15</sup>However, if e is analogous to "effort level", then, a more realistic scenario would involve c = c(e) with c'(.) > 0 and c(0) = 0. Interpretation of e as the effort level may create additional complications. Observability of e becomes a crucial issue and hence, one has to look at the possibility of *moral hazard* between the CRA and the other agents. Later, we argue that most of the results obtained is robust to this assumption of constant set up cost.

Equation (5.9) denotes the Individual Rationality Condition (IRC) of the firm, while equation (5.10) denotes the Informative Condition (IC). In other words, (IC) indicates that the announcements of the CRA are informative.

The equilibrium described by  $\{W^*, e^*, p_m^*\}$  and the investor's decision, solves (5.8), (5.9) and (5.10).

**Proposition 5.1** In equilibrium  $\{W^*, p_m^*, e^*\}$  is such that:

- (a)  $W^* = \overline{z}/D 1$ ,
- (b)  $p_m^* = Max\{u^{-1}(\overline{p}, e^*), 0\}$
- (c)  $e^* > 0$  such that

$$\int_{p_m^*}^1 (p - \overline{p}) \alpha(p, e^*, D) dF(p) = 0.$$

(d) The investor invests whenever the announcement is good and does not invest otherwise. 16

The above result signifies the importance of the technology available with the CRA. The crucial requirement is that the technology is able to distinguish across types and thereby different types are announced 'good' with different probabilities. It is evident that if  $\alpha(.)$  was same across all types, i.e., types are indistinguishable after the announcements, then the investors do not invest. Therefore, if  $\alpha(p,.) = \theta$  (say), then

$$\forall p_m^* < \overline{p}, \quad \int_{p_m^*}^1 \theta(p - \overline{p}) dF(p) < 0.$$

Proposition 5.1 has interesting implications which are stated below as corollaries 5.1 and 5.2.

Corollary 5.1 In equilibrium, some 'bad' types get investment while some 'good' types do not.

The above result follows from the fact that  $p_m^* < \overline{p}$  and  $e^* < 1$ . This implies,  $\alpha(p.e^*) \le 1$  for all  $p \ge p_m^*$  with  $\alpha(p,e^*) < 1$  for some  $p \ge p_m^*$ . Why is  $p_m^* < \overline{p}$ ? Note that, all types that have  $\alpha(p,e^*) > 0$  will go to the CRA. From equation (5.2) we know that  $\alpha > 0$  if u(p,e) > 0. Therefore, the minimum

<sup>&</sup>lt;sup>16</sup>If u(p,e) = 1 - (1-p)e, then  $p_m^* = \text{Max}\{1 - \frac{1-\bar{p}}{e^*}, 0.\}$ .

type,  $p_m^*$  that will go to the CRA will have  $\alpha(p_m^*, e^*) \Rightarrow 0$ . Therefore, in equilibrium we must have,  $u(p_m^*, e^*) = \overline{p}$ . If  $p_m^* \geq \underline{p}$ , then  $u(p_m^*, e^*) > \overline{p}$ . Therefore, for  $u(p, e^*) = \overline{p}$  we must have  $p_m^* < \overline{p}$ . The above argument also depends upon the fact that  $e^* < 1$ . If  $e^* = 1$  then  $u(p_m^*, 1) = \overline{p}$ .

Denote  $\underline{p}$  such that  $DE(p|p \geq \underline{p}) = 1$ . In other words,  $\underline{p}$  denotes the minimum cut off type. The interpretation of  $\underline{p}$  is the following. If the investor knew that all the types have  $p \geq \underline{p}$ , she would have invested without any additional information. This is because, the average (expected) type of all those with  $p \geq \underline{p}$  is  $\overline{p}$ . However, if the minimum type faced by the investor has a success probability lower than  $\underline{p}$ , then additional information is needed for the investor to invest. The CRA provides the investor with this additional information. The information producing role of the CRA is highlighted in corollary 5.2.

Corollary 5.2 In equilibrium,  $p_m^* < \underline{p}$ . That is, the minimum type that comes to the CRA has a lower probability of success than the minimum cut off type.

Given e and D, all types with a positive probability of being announced 'good' will find it optimal to go to the CRA. However, types with very low p, i.e.,  $p < p_m^*$  do not go to the CRA. This screening alone is not sufficient. Corollary 5.2 states that the minimum type that will go to the CRA in equilibrium actually has a lower success probability than the minimum cutoff type, p. Therefore, the investor will not invest into any firm that has gone to the CRA. Her investment decision will depend upon the ability of the CRA to distinguish across the types that comes to it. This highlights the role of the CRA as an information producing agency as against a certifying agency. A certifying agency only issues operating licences to firms. It does not distinguish between any two types that are issued certificates. However, a rating agency distinguishes among types that come to it.

Proposition 5.1 indicates that the investor invests if and only if the announcements are good and not otherwise. The CRA's choice of e reflects this and it chooses the minimum e that guarantees investment for the good firm. In expected terms, the investor breaks even from investing in the project if the announcement is 'good'. That is, the expected net returns to the investor is zero if she invests in the firm which is rated 'good' by the CRA. The expected earnings to the investor when the announcement is 'bad' is

less than one and therefore, they do not invest if the announcement is 'bad'. Further, the investor knows that if the firm has not gone to the CRA, its type is strictly less than p (Corollary 5.2).

The above results explains 'honest mistakes' made by the CRA as a result of noisy inference. This imperfection arises because in equilibrium, the CRA sets a less than perfect level of evaluation standard. With  $e^* = 1$ ,  $p_m^*(1) = \overline{p}$ . This is because, when  $e^* = 1$ , then  $\alpha(p,1) = 0 \forall p < \overline{p}$ . Thus, no types with  $p < \overline{p}$  will go to the CRA. The CRA being a profit maximizer, would encourage maximum firms to come to it. Therefore,  $e^* = 1$  is never an equilibrium as the CRA can strictly do better by lowering e slightly and ensure that the investors invest when the announcement is 'good.' The CRA being a profit maximizer chooses  $e^* \in (0,1)$ .

In the next section we address issues related to efficiency of such arrangements.

# 5.4 Efficiency and Regulation

In this section we consider the impact of a rating agency on net surplus. We define net surplus as the surplus in the economy when the project is funded minus the surplus when it is not funded. We then consider the possibility that a regulator designs an appropriate fee structure to maximize the net surplus in the system.<sup>17</sup> We first analyze the role of the CRA on market efficiency. For completeness we assume that  $\bar{c} \leq W^*[1-F(p_m^*)]$ . This implies that the CRA operates profitably in the market.

Let  $S_C$  denote the net surplus when trading takes place in the presence of a CRA. Therefore,  $S_C$  is the sum of the net surplus for the three different agents - the investors, the firm and the CRA. Presence of a CRA improves efficiency if and only if  $S_C > 0$ .  $S_C$  is the second best net surplus level.

The first best level of net surplus involves only projects with non negative expected profitability be funded. This implies that only projects with  $p \geq 1/\overline{z}$  should be funded.

Thus,  $S_C$  is defined as

 $S_C$  = Expected profits to the firm + Expected profits

 $<sup>^{17}</sup>$ We only consider the cases where the regulator chooses W as the choice of e is CRA specific. Regulating 'evaluation standards' are not obvious and hence we do not consider such regulations.

to the CRA + Expected returns to the investors

$$= \left\{ \int_{p_{m}^{*}}^{1} p\alpha(p, e^{*}, D)[\overline{z} - D(1+W)]dF(p) \right\} + \left\{ W \int_{p_{m}^{*}}^{1} dF(p) - \overline{c} \right\}$$

$$+ \left\{ D(1+W^{*}) \int_{p_{m}^{*}}^{1} p\alpha(p, e^{*}, D)dF(p) - \int_{p_{m}^{*}}^{1} [W^{*} + \alpha(p, e^{*}, D)]dF(p) \right\}$$

$$= \int_{p_{m}^{*}}^{1} \alpha(p, e^{*}, D)\{p\overline{z} - 1\}dF(p) - \overline{c}.$$
(5.11)

In the first equation, the expressions in the curly brackets denote the expected profits to the firm, to the CRA and to the investor respectively. As the net surplus terms for the firm and the CRA is already discussed in details before, we will only discuss the net surplus term for the investor.

The net surplus for the investor is

$$\left\{D(1+W^*)\int_{p_m^*}^1 p\alpha(p,e^*,D)dF(p) - \int_{p_m^*}^1 [W^* + \alpha(p,e^*,D)]dF(p)\right\}.$$

Note that all types with  $p \geq p_m^*$  will borrow W and go to the CRA. In addition, any type with  $p \geq p_m^*$  will be announced 'good' with probability  $\alpha$ . Therefore, the investor will lend W to all types with  $p \geq p_m^*$  and lend one more unit of capital to all those types which are announced 'good'. Therefore, the opportunity cost to the investor of what he lends to the firm is  $\{W + \alpha(p, e^*, D)\}[1 - F(p_m^*)]$ . Note that, without the CRA, the surplus for the investor was 1. In the presence of the CRA, the investor invests  $\{W + \alpha(p, e^*, D)\}[1 - F(p_m^*)]$ . The investor continues to invest the remaining of her funds,  $1 - \{W + \alpha(p, e^*, D)\}[1 - F(p_m^*)]$  in the risk free asset. To arrive at the net surplus expression for the investor one has to subtract this opportunity cost from the expected returns he earns from investing the aforesaid amount in the firm.

In order to make any payments to the investor the firm must be announced 'good'. Once the firm is able to undertake the project, it will repay the investor D per unit borrowed with a probability p. Thus the investor gets back an expected returns of  $pD(1+W^*)$  from all types that are announced 'good'. Therefore, the expected returns to the investor is

$$D(1+W^*)\int_{p_m^*}^1 \alpha(p,e^*,D)pdF(p).$$

In the final expression of (5.11), the first term is the expected profitability of the projects funded by the CRA. The term  $\{p\overline{z}-1\}$  is the expected

profitability of each such projects which get financed with probability  $\alpha$ . We deduct the evaluation cost,  $\overline{c}$  from the expected profitability to arrive at the net surplus expression.

**Proposition 5.2** Under A.3, the presence of a CRA always increases net surplus.

In the absence of a rating agency, none of the projects were getting funded. The rating agency facilitates trading by ensuring that projects with p higher than  $p_m^*$  get funded with some positive probability  $\alpha$ . Therefore, some of the viable projects (the ones with  $p \geq 1/\overline{z}$ ) as well as some non viable projects (ones with  $p < 1/\overline{z}$ ) are now getting funded with positive probabilities. However, the aggregate surplus in the system increases as the surplus obtained from financing of the viable projects outweigh the loss in surplus arising due to financing of the non viable ones. From the resultant increase in surplus one has to deduct  $\overline{c}$ . Now, for the CRA to operate profitably it must be the case that  $W^*\{1-F(p_m^*)\} \geq \overline{c}$ , where  $W^*$  and  $p_m^*$  are obtained from proposition 5.1. This restriction (upper bound on  $\overline{c}$ ) ensures that the net surplus in the economy is higher with the CRA than without it.

So far the economy was not subjected to regulations. A natural question to ask is: Can regulation increase net surplus? We consider a regulator who wishes to maximize the net surplus by choosing  $W.^{18}$  All other agents in the economy move in the same sequence as described earlier. We denote the equilibrium variables under regulation with subscript 'R'.

In formal terms, the problem of regulator can be expressed as:

$$\max_{\{W_R\}} S_C = \int_{p_m^*}^1 \alpha(p, e^*, D) \{p\overline{z} - 1\} dF(p) - \overline{c} \qquad \text{s.t.}$$

$$\Pi_C = W_R[1 - F(p_m^*)] - \overline{c} \geq 0.$$

$$p_m \alpha(p, e^*, D) [\overline{z} - D(1 + W_R)] \geq 0.$$

$$\int_{p_m^*}^1 (p - \overline{p}) \alpha(p, e^*, D) dF(p) \geq 0.$$

In the result below we investigate the effectiveness of a policy that regulates the fee charged by the CRA.

<sup>&</sup>lt;sup>18</sup>Other form of regulatory policies will be dealt with later.

**Proposition 5.3** Under A.3, a regulated fee cannot increase the net surplus as compared to an unregulated fee.

Note that from (5.11),  $S_C$  is independent of W. This is because,  $p_m^*$  and  $e^*$  are independent of W. In other words, the fee charged by the CRA is merely a transfer from the investor to the CRA via the firm. This fee does not affect either the evaluation standard or the minimum type that goes to the CRA. Therefore, the regulator cannot set a fee  $W_R$  that will increase the net surplus.

The ineffectiveness of regulation here stems from the fact that D is exogenously given. With D held constant, the fee charged only determines the expected payoff to the firm. It neither affects the evaluation standard, nor does it affect the minimum type that comes to the CRA. Therefore, regulating fees will not change the net surplus. However, if the firm is allowed to choose D, any change in W may get reflected in the choice of D as well as determine the minimum type that will get its project funded. Therefore regulated fee may have more prominent impact there. In the next section we investigate this possibility by allowing D to be chosen by the firm. This is a more realistic scenario for two reasons. One, in reality firms do choose the face value of debt, and two, often the choice of D also acts as a signalling device for the firm (see Ross[83]).

# 5.5 Endogenous Choice of Debt Claims

In the framework developed so far, we now allow the firm to choose D. In the absence of a CRA, the firm belonging to a type p will set  $D_p$  such that it maximizes its net returns. As the expected net returns to the firm decreases in D, this implies that the firm will choose the minimum D that guarantees them investment. In equilibrium,  $\forall p$ ,  $D_p = D = 1/E(p)$ , and the investor invests. Thus, the equilibrium, in the absence of the CRA is a pooling equilibrium where all types set the face value of debt equal to 1/E(p) and the investor finances the project. However, implicit in this is the assumption that, such a debt claim is 'feasible'. 19

**A.4:** The firm's type p, follows uniform distribution.

<sup>&</sup>lt;sup>19</sup>Feasibility requires that  $\overline{z} > 1/E(p)$ .

Assumption A.4 results in considerable analytical and expositional simplicity without affecting the nature of the analysis substantially. We shall indicate wherever possible, the results with more general functional forms.

In contrast to the previous sections, we are now looking at the case where the firm can choose D. Therefore, the firm can set D such that D.E(p) = 1 and get investment. In the absence of a CRA, under A.4, the firm sets D = 2 and gets her project financed.

We now describe the role of the CRA in determining the market outcome. The agents in the economy move in the following sequence. In the first stage, the CRA sets W and e. In the next stage, the firm has two decisions to make. First, it has to decide whether or not to go to the CRA. Second. the firm decides D, irrespective of its decision to go or otherwise. If the firm decides to go to the CRA, the latter evaluates the firm and announces 'good' or 'bad'.

The investor's beliefs and her actions are as follows. Upon observing that the firm has gone to the CRA, she invests if and only if the announcement is 'good'. Alternately, the firm can directly approach the investor for funds. In this case, she invests depending upon her updated priors regarding the default probability of the firm. While updating, she takes into account whether the fee charged by the CRA is too high or not. If the expected probability of the types which do not go to the CRA is such that it is 'investment worthy', she invests in them. As distinct from the previous sections, we do not impose the following 'belief' that the investors never invest in a firm which has not sought rating from the CRA. This is because, the rating fee may be exceptionally high so as to deter some of the better types to seek ratings.

If the CRA faces a firm with a debt offer of  $D_p$ , it will announce 'good' if  $r \geq 1/D_p$  and announce 'bad' if otherwise. Here, the face value of debt may differ across types and hence, there does not exist a unique cut off type.

Given  $\{e, W\}$ , a firm of type p which decides to go to the CRA, will set  $D_p^*$  such that,

$$D_p^* = \arg\max_{D} \quad p.\alpha(p,e,D)\{\overline{z} - D(1+W)\} \quad \text{subject to} \quad \frac{\overline{z}}{1+W} \geq D_p^* \geq \frac{1}{p}.$$

The constraints guarantee (i) the feasibility and (ii) the minimum bound of  $D_p^*$ . The feasibility constraint implies that, the investor will not invest in any firm that offers  $D > \overline{z}/(1+W)$ . This is because, the investor will know that such debt obligations will never be satisfied by the firm. With  $\{e, W\}$ 

observable to the investor, the investor on observing  $D_p^*$ , can invert it to determine the type, p, that has set  $D_p^*$ . If  $D_p^* < 1/p$ , the investor will not invest, irrespective of the announcements.

With A.4 and  $\alpha(p, e, D)$  as given in example 1, we have

$$D_p^* = \arg \max_{D_p} p \frac{1 - 1/D_p - e(1 - p)}{1 - e} \{ \overline{z} - D_p(1 + W) \} \quad \text{if} \quad e \in [0, 1).$$

However, if  $e^* = 1$ , then,  $D_p^* = 1/p$ .

$$D_p^* = \begin{cases} \sqrt{\frac{\overline{z}}{(1+W)[1-e(1-p)]}} & \text{if } e \in [0,1); \\ 1/p & \text{if } e = 1 \end{cases}$$
 (5.12)

In addition,  $D_p^* \geq 1/p$ .

Denote  $p_m^e$  as the minimum type that goes to the CRA given  $\{e,W\}$ . Therefore, any type that does not go to the CRA has a lower probability of success  $(p < p_m^e)$ . A type that does not go to the CRA can attract investment if it sets  $D \geq 1/E(p|p < p_m^e)$ . However, this debt claim is only feasible if  $D \leq \overline{z}$ .

The expected profit of the firm that does not go to the CRA, denoted  $\Pi^N$  is:

$$\Pi^{N} = \begin{cases} p \left\{ \overline{z} - \frac{1}{E[p|p < p_{m}^{e}]} \right\} & \text{if } 1 < \overline{z}E[p|p < p_{m}^{e}]; \\ 0 & \text{if otherwise} \end{cases}$$

The equilibrium values of the various variables are described below in proposition 5.4.

**Proposition 5.4** With endogenous choice of debt claims and under A.1, A.2 and A.4, in equilibrium,

- (a) The rating agency sets  $e^* = 1$ ,
- (b)  $D_p^* = 1/p$ ,  $\forall p \ge p_m^e$ ,

(c)

$$W^* = \begin{cases} 0.5(\overline{z} - 1) & \text{if } 2 \le \overline{z} \le 3; \\ 1, & \text{if } \overline{z} > 3 \end{cases}.$$

(d)

$$p_m^e = \begin{cases} 0.5 + 0.5/\overline{z} & \text{if } 2 \le \overline{z} \le 3; \\ 2/\overline{z}, & \text{if } \overline{z} > 3 \end{cases}.$$

(e) Any type that does not go to the CRA is unable to undertake the project.

Proposition 5.4 establishes a separating equilibrium. Types with  $p \geq p_m^c$  go to the CRA, choose the face value of debt equal to 1/p. However, types with  $p < p_m^e$  does not go to the CRA and exit the market. <sup>20</sup>

The crucial question now is whether such a separating equilibrium is *ex* ante efficient as compared to the pooling equilibrium obtained in the absence of a rating agency? The next section provides us with the answer.

# 5.6 Efficiency and Regulatory Issues

## 5.6.1 Efficiency

With D chosen by the firm, the investor finances the project even in the absence of a CRA. Thus, both types of projects get funded- the ones those are profitable as well as the ones those are not. However, in the presence of a CRA, low profitability projects do not get funded. The net impact on efficiency would depend upon whether the CRA discourages economically viable projects as well.

In the absence of a CRA all projects get funded with certainty. The expected profitability from funding all the projects are  $z^e - 1$ , where  $z^e$  is the expected value of z. <sup>21</sup> Therefore, the net surplus, denoted by  $\overline{S}$  is given by.

$$\overline{S} = z^e - 1 = 0.5\overline{z} - 1.$$

The net surplus in the system when the CRA operates is given by:

$$\begin{split} S_C^1 &= \int_{p_m^e}^1 [p\overline{z} - 1] dF(p) - \overline{c} \\ &= \int_{Min\{0.5 + \frac{0.5}{2}, \frac{2}{5}\}}^1 [p\overline{z} - 1] dF(p) - \overline{c}. \end{split}$$

(i) 
$$e^* = 1$$
 and  $D_p^* = 1/p$ ,  $\forall p \ge p_m^e$ ,

(ii) 
$$W^* = \frac{p_m^e}{E(p|p < p_m^e)} - 1.$$

(iii) 
$$p_m^e = arg \max_{p_m} \quad \left\{ \frac{p_m}{E(p|p \leq p_m} - 1 \right\} \left[ 1 - F(p_m) \right].$$

(iv) Any type that does not go to the CRA is unable to undertake the project.

21 Note that the investor earns zero expected net returns as  $D_p = 1/E(p)$ .

<sup>&</sup>lt;sup>20</sup>Without A.4, i.e., with general distribution function, F(p), the result corresponding to proposition 5.4 is:

Note that, in the presence of the CRA, all types that come to it, i.e.,  $p \geq p_m^e$ , will get funded with probability one. The net return from such projects are  $pz^e - 1$ . The last equality follows from proposition 5.4.

Denote,

$$G(\overline{z}) \equiv \frac{\overline{z} + 1}{8\overline{z}} [3 - \overline{z}].$$

The following proposition indicates whether trading in the presence of the CRA improves efficiency. We do so by comparing  $S_C^1$  with  $\overline{S}$ .

**Proposition 5.5** If  $2 < \overline{z} \leq 3$ , then trading in the presence of a CRA is efficient if  $G(\overline{z}) > \overline{c}$ . Trading in the presence of a CRA is always inefficient if  $\overline{z} > 3$ .

Unlike in proposition 5.2, trading in the presence of a CRA does not necessarily improve net surplus. Note that, if  $\overline{z} \geq 3$ , then it does not. However, if  $2 < \overline{z} < 3$ , CRA might increase net surplus if  $\overline{c}$  is sufficiently small. That is, the net surplus will increase with the CRA if (i)  $\overline{c}$  is not too high, and (ii)  $\overline{z} < 3$ , the latter being a necessary condition for the CRA to improve upon efficiency. The interesting issue is that the source of inefficiency is not necessarily a high  $\overline{c}$ . Of course, a high  $\overline{c}$  such that  $\overline{c} > G(\overline{z})$  will always reduce the net surplus.

The observation that irrespective of  $\overline{c}$ , trading in the presence of a CRA is inefficient for  $\overline{z}>3$  is interesting. Higher  $\overline{z}$  implies that higher proportion of projects are profitable. In the limit, if  $\overline{z}\to\infty$ , almost all the projects are profitable. Without the CRA, all projects are financed. This implies that the inefficiency arising due to funding of unproductive projects are less. However, with the CRA, projects with  $p< p_m^e$  are not financed. Hence, the gap between all projects that ought to be financed (ones with  $p\geq 1/\overline{z}$ ) and all projects which are actually financed (ones with  $p\geq p_m^e$ ), reduces as  $\overline{z}$  increases. In the presence of the CRA,  $p_m^e>0.5$  implying that irrespective of how high  $\overline{z}$  is, only projects with p>0.5 are financed. Higher  $\overline{z}$  widens this gap even further. This is the source of inefficiency when the CRA operates as compared to the case where the investor and the firm directly trade with each other. To summarize the above, the CRA discourages proportionately more profitable projects than the non profitable ones.

The above result has a very interesting implication. It indicates that projects that are more and more viable, should not be subjected to ratings

from the CRA. We discuss this in details when we consider other regulatory issues.

## 5.6.2 Regulating Fees

We now investigate the possibility of a regulator increasing the net surplus in the system by an appropriate choice of W. It would then be interesting to ask whether by an appropriate choice of W, the regulator can ensure efficiency even in those cases where an unregulated CRA was inefficient. Therefore, we shall assume that  $\overline{z} > 3$ .

Let  $W_R$  denote the regulated fee. The regulator chooses  $W_R$  to maximize  $S_C^1$ . The following result is now obtained.

Proposition 5.6 Let  $\overline{z} > 3$ . If

$$\overline{c} < (\overline{z} - 2) \left\{ \sqrt{1 + \frac{1}{\overline{z}(\overline{z} - 2)}} - 1 \right\},$$

then there exists  $W_R$  where

$$\overline{W} \equiv \sqrt{1 - 2\overline{c}\overline{z}} > W_R \ge 0.5 \left\{ (\overline{z} - 1) - \sqrt{(\overline{z} - 1)^2 - 4\overline{c}\overline{z}} \right\} \equiv \underline{W}$$

such that trading in the presence of a rating agency is efficient. Further,  $W_R < W_U$ , where  $W_U$  is the unregulated fee.

Note that, for  $\overline{c} \to 0$ ,  $\overline{W} > \underline{W}$ . Therefore, a sufficiently low  $\overline{c}$  will guarantee the existence of a  $W_R$  as required in the above proposition.

With a low  $\bar{c}$ , the rating agency earns non negative profit, while a low  $W_R$  implies that more types go to the rating agency. As  $W_R \to 0$ ,  $p_m^* \to 1/\bar{z}$ . This is because,  $p_m^* = Min\{1, (1+W^*)/\bar{z}\}$  (proposition 5.4). Therefore, with low  $W_R$ , the types that get their project funded are the ones which are profitable. The ones that can not raise investment are mostly the projects with negative profitability. Thus, with lower  $W_R$ , proportionately fewer productive projects are discouraged. This is the source of increased efficiency with a regulated fee. This also explains the other important observation that, the regulated fee will in general be lower than the unregulated fee.

#### 5.6.3 Other Regulatory Issues

Apart from regulating fees, other form of regulatory policies include, (i) compulsory rating by at least one of the rating agencies; (ii) ratings from more than one agencies. etc.

We first discuss the issue of compulsory ratings in our framework.

Given  $\{e, W\}$ , a firm of type p which decides to go to the CRA, sets  $D_p^*$  identical to that obtained in equation (5.12). Compulsory ratings in our framework is reflected in the fact that as the investor does not invest in a firm that has not gone to the CRA. We have

$$\Pi^g = p\alpha(p, e, D_p^*)\{\overline{z} - D_p^*(1+W)\}$$

$$\Pi^N = 0.$$

In equilibrium, the minimum type,  $\hat{p}_m$ , that comes to the CRA is given by,  $\Pi_{\hat{p}_m}^g = 0$  along with the condition that  $D_{\hat{p}_m}^* \geq 1/\hat{p}_m$ .

**Proposition 5.7** Given any  $\{e, W\}$ , the minimum type that comes to the CRA in equilibrium, denoted,  $\hat{p}_m$ , is

$$\hat{p}_m = \frac{e(1+W)}{2\overline{z}} \left\{ 1 + \sqrt{1 + \frac{4\overline{z}(1-e)}{e^2(1+W)}} \right\}.$$

Further,  $\hat{p}_m$  is decreasing in e.

The observation that  $\hat{p}_m$  decreases as e increases, does not contradict corollaries 5.1 and 5.2. One of the requirements of the technology was that, given any D,  $\alpha$  was non decreasing in e for types with  $p \geq 1/D$  and strictly decreasing in e for types with p < 1/D. With D chosen by the firm, any type that goes to the CRA sets  $D_p \geq 1/p$ . This implies,  $\hat{p}_m$  is decreasing in e.

The equilibrium is described below.

**Proposition 5.8** Under A.1, A.2 and A.4, with compulsory rating system, in equilibrium,

$$e^* = 1$$
,  $D_p^* = 1/p$ ,  $W^* = 0.5(\overline{z} - 1)$ , and  $\hat{p}_m = 0.5 + \frac{0.5}{\overline{z}}$ .

Further, compulsory rating regulation, is inefficient as compared to no such regulations if  $\overline{z} > 3$ . However, for  $2 < \overline{z} \le 3$ , such regulations have no effect on net surplus.

The intuition is straightforward. Note that,  $\hat{p}_m = p_m^e$  for  $2 < \overline{z} \leq 3$ . However, for  $\overline{z} > 3$ ,  $p_m^e > \hat{p}_m$ .

Therefore, for  $2 < \overline{z} \le 3$ , the equilibrium values do not change, whether rating is compulsory or not. This can be seen by comparing propositions 5.4 and 5.8. However, with  $\overline{z} > 3$ , we obtain that  $\hat{p}_m < p_m^e$ . That is, more viable projects are discouraged with compulsory ratings.

Other forms regulatory policies may involve 'ratings from at least two of the leading rating agencies.' In the light of the above discussions, it is obvious that with such policies in place, more types, with  $p \ge 1/\overline{z}$ , quit the market. Therefore, such a policy would tend to be less efficient.

Based on the discussion so far, we can conclude with the following policy guidelines. (i) Rating agencies should only be regulated with fees. (ii) Policies like compulsory ratings or multiple ratings would be inefficient. (iii) The regulated fees should be lower than the unregulated fees.

# 5.7 Competitive Rating Agencies

The framework presented so far had the simplistic assumption of a monopolist rating agency. We now extend our model to include competitive rating agencies. We construct a simple example to show that, competitive rating agencies are not necessarily efficient vis-a-vis a monopolist rating agency.

We consider two *identical* rating agencies- A and B. The CRAs have identical set up costs,  $\overline{c}$ . They both have the same technology for a given evaluation standard, but can choose different evaluation standards. Let the debt claim be fixed at  $\overline{D}$  with  $\overline{D}E(p) < 1$  (A.3). We assume that the rating fees is the same for both agencies and is denoted by  $\overline{W}$ . Let  $e_i$ , i = A. B be the evaluation standard set by CRA i.

The sequencing is as follows:

- Stage I: The two rating agencies A and B, simultaneously set  $e_A$  and  $e_B$ .
- Stage II: The firm decides (i) whether or not to go to the CRA and (ii) which CRA to go to. If the firm decides to go, it borrows  $\overline{W}$  from the investor to pay the fees.
- Stage III: The CRA rates the firm if it comes to it and announces 'good' or 'bad'.
- Stage IV: The investor decides whether to invest or not to invest based on the announcements of the rating agency. The investor will lend an additional dollar to the firm if the announcement is good.

**Example:** Suppose  $\overline{z} = 2.5$ ;  $\overline{D} = 1.25$ ;  $\overline{W} = 0.8$ ;  $\overline{c} = 0.05$  and p is uniform over [0, 1].

Recall that  $\overline{p}$  solves  $p\overline{D}=1$ , i.e., if the investor had information about the firm's probability of success, then they would have invested in all firms who had  $p \geq \overline{p}$ . In this example,  $\overline{p}=0.8$ . However, we are in a situation where the investor does not know the firm types.

Suppose A is a monopolist. Let  $p_m$  denote the minimum cut off type that comes to A for ratings. Note that  $p_m$  depends upon e. But from footnote 16 we have  $p_m = Max\{1 - \frac{0.2}{e}, 0\}$ . From proposition 5.1 (c),  $p_m$  is given by,

$$\frac{\int_{p_m}^1 [0.2 - e(1-p)] p dp}{\int_{p_m}^1 [0.2 - e(1-p)] dp} = \overline{p} = 0.8.$$

Solving for  $p_m$  and e, we get  $p_m = 0.4$  and  $e = \frac{1}{3}$ . The expected profit to a firm with  $p \ge p_m$  is:

$$\Pi_p = p.\frac{3}{2}[0.2 - \frac{1}{3}(1-p)]\{2.5 - 1.25(1+0.8)\} \ge 0.$$

The expected profit to the CRA is:  $\Pi_A = \overline{W}[1 - F(p_m)] - \overline{c} = 0.43$ . Thus, the firm and the CRA are profitable.

Finally, the aggregate surplus when there is a monopolist CRA is given by:

$$S_A = \int_{p_m}^{1} [p\overline{z} - 1]dp - \overline{c} = 0.4.$$
 (5.13)

Now suppose that both A and B operate. We show that the only equilibrium is  $(e_A, e_B) = (1, 1)$ .

Step I: Given the technology in (5.2), we know that if e = 1, then the CRA will announce the firm to be good (a = g) only for those types with  $p \ge \overline{p}$ ; the others will be announced bad (a = b). This means the firm will find it profitable, and hence approach either A or B only if  $p \ge 0.8$ . Let us assume that the firm goes randomly to any one CRA, so that the measure of types going to any rating agency is 0.1 (given that p is uniform over [0,1] and  $p \ge 0.8$ ). Denote  $\Pi_j^c$ , j = A, B as the expected profit to the rating agency j when the rating agencies are competing. Thus,

$$\Pi_A^c = \Pi_B^c = 0.8 \frac{1}{2} [1 - 0.8] - 0.05 = .03.$$

Thus, with  $(e_A, e_B) = (1, 1)$ , both CRAs make positive profits.

Step II: We now show that (1,1) is an equilibrium. Recall that, given  $e_A, e_B \in [0,1]$ , the only way a rating agency i, i = A, B can deviate from  $e_i = 1$ , is by setting an evaluation standard less than 1. Without loss of generality, consider  $e_A = 1$  and  $e_B < 1$ . Given our technology, the firm will be announced good with probability 1 by agency A if  $p \geq \overline{p} = 0.8$ . However, since  $e_B < 1$ , the same firm will be announced good with probability less than one by agency B. Thus, this type will not want to be rated by B. Type p < 0.8 will, on the other hand, be announced bad, with probability less than 1 by B and with probability 1 by A. So, types  $p < \overline{p}$  will want to go to B (see footnote 15). However, the investor knows this and, hence, will infer that a firm that has been announced good by B has a probability of success less than 0.8 and is not worth investing in. Thus, such a firm will not go to B to begin with. Hence B will make a negative profit (=-0.05):

#### Step III: $e_A \neq e_B < 1$ is not an equilibrium.

Without loss of generality, let  $e_B < e_A < 1$ . So, let  $e_B < e_A = e_B + \epsilon < 1$ . Observe, given our assumption on technology, all types with  $p \geq \overline{p} = 0.8$ , will be announced good with higher probability by A than by B. Therefore, they will go to A.

Now consider p slightly below  $\overline{p}$ . It may be announced good by either A or B. However, since no p, that is above  $\overline{p}$ , will want to go to B, the investor correctly infers that a firm that has a rating by B will have to have  $p < \overline{p}$  and is not worthy of investment. The firm knows this and will, therefore, not go to B. Thus B will make a negative profit. Therefore,  $e_B < e_A$  is not an equilibrium.

## Step IV: $e_A = e_B < 1$ is not an equilibrium.

From Step III, we know that no agency will set a standard below that of the other. We now argue that it will have an incentive to set a higher standard (if possible) than the other. At  $e_A = e_B < 1$ ,

$$\Pi_A(e_A = e_B, e_B) = \Pi_B(e_A = e_B, e_B) = \frac{1}{2}0.8[1 - \frac{0.2}{e_B}].$$

Suppose, A deviates by setting  $e_A = e_B + \epsilon < 1$ . Then, any type with  $p \ge 0.8$  will go to A as it will be announced good with a higher probability by A than by B. The investor correctly infers that no  $p \ge 0.8$  would go to B and

hence, does not invest in the firm if it goes to B. Therefore, while the firm does not go to B, it will go to A if it has  $p \ge \frac{1-\overline{p}}{e_B+\epsilon} = \frac{0.2}{e_B+\epsilon}$ .

$$\Pi_A(e_A = e_B + \epsilon, e_B) = 0.8[1 - \frac{0.2}{e_B + \epsilon}] > \Pi_B(e_A = e_B, e_B) = \frac{1}{2}0.8[1 - \frac{0.2}{e_B}]$$

The inequality in the above expression holds for all positive and small  $\epsilon$ . Thus,  $e_A = e_B < 1$  is not an equilibrium.

**Step V:** Combining steps I to IV, we have  $(e_A, e_B) = (1, 1)$  as the unique equilibrium.

With (e, e) = (1, 1), the minimum type that comes for rating is 0.8. Therefore, the aggregate surplus with competition, denoted  $S_{A+B}$  is,

$$S_{A+B} = \int_{\overline{p}}^{1} [p\overline{z} - 1]dF(p) - 2.\overline{c} = 0.15 < 0.4 = S_A$$

in equation (5.13). Thus,  $S_A > S_{A+B}$ .

This then is an example where a monopoly CRA is better than allowing competition among CRAs.

The interesting observation that emerges from the above example is that, a monopolist rating agency may actually be better than competitive rating agencies. There are two reasons for this. The first reason is that, with competition,  $\bar{c}$  is duplicated. The second reason is as follows. Note that, with  $\bar{z}=2.5$ , the cut off level of profitable projects is  $p=1/\bar{z}=0.4$ . The minimum type that gets funded with a monopoly is 0.4 while that with competition is 0.8. Therefore, in this example, while the monopolist rating agency funds all the profitable projects (and only the profitable ones), competition does not fund some of the profitable projects (ones with  $0.4 \le p < 0.8$ ).

Observe, that with competition there will always be some profitable projects which are not funded. This is because, with competition, the projects that get funded has  $p \geq 1/\overline{D} > 1/\overline{z}$ , where  $1/\overline{z}$  is the cut off level of profitable projects. However, with monopoly, the projects that get funded, may include profitable as well as non profitable projects. To see this, suppose,  $\overline{D} = 1.5$ , i.e.,  $\overline{p} = 2/3$ , then with monopoly, all types will get funded. This means, non profitable projects, i.e., p < 0.4, are also funded along with profitable ones. However, with competition, only types with  $p \geq 2/3$  gets funded. Therefore, while some profitable projects are not funded with competition, some non profitable projects may get funded with monopoly. Eventually, whether

competition is more efficient vis-a-vis monopoly, will depend upon whether the proportion of profitable projects ruled out with competition is more than the proportion of non profitable projects accepted with monopoly.

The key assumption in this chapter is the fact that the evaluation standard set by the rating agency is observable to both the firm and the investor and, the marginal cost of a higher standard is negligible (the cost to the rating agency is  $\bar{c}$ , independent of the standard e). In other words, the rating agency has no incentive to employ a lower standard than it has "committed to", implying no moral hazard on its part. The evaluation standard is a representation of the various parameters a rating agency might consider important in its rating process. The score models used by a rating agency take into account various parameters it considers to be important. For example, in India, while the rating agency ICRA emphasizes managerial efficiency, the other leading rating agency CRISIL, stresses on the legal position of the firm. The interpretation of e in this context is simply the possible parameters the rating agencies may wish to incorporate in their score models. Higher e implies that more parameters are being considered. The set of parameters used by the rating agencies is common knowledge (though not necessarily, how exactly they are used). Therefore, the evaluation standard is (at least partially) always observable. While allowing for moral hazard is an interesting issue, the purpose here is to concentrate on the modeling of rating agencies and their impact on efficiency.

# 5.8 Conclusion

This chapter presents a model of information producing rating agency. We use this framework to investigate the effects of regulation through price as well as non price measures.

Initially, we consider the simplistic case involving a monopolist rating agency. This is done through sections 5.3 - 5.6. Later, we extend our analysis in section 5.7 to a competitive rating agency.

We first consider a situation where in the absence of a rating agency, no trading takes place between the investor and the firm. For this, we assume that the debt claims are set exogenously. The equilibrium obtained in this case emphasizes the role of the rating agency as an information producer. We find that the CRA always improves efficiency. However, we find that a

regulator who regulates the rating fees, can not improve the net surplus in the system.

We find that, the firm can attract sufficient investment even in the absence of the rating agency. We then introduce the rating agency. The role of the rating agency then, is to reduce the signaling costs of the firm. We find a separating equilibrium where, better projects distinguish themselves by setting different debt levels. However, we show that the rating agency does not necessarily increase net surplus. This is because, in the absence of any rating agency, some projects were already getting financed. The efficiency will increase if and only if a rating agency either discards more negative NPV projects or accepts more positive NPV projects as compared to the situation involving no rating agencies. We then analyze the role of a regulator. We obtain that a regulator can increase the net surplus by a choice of the rating fees. Interestingly, we point out that, even in those cases where the rating agency was inefficient, regulating fees makes it efficient.

The important policy recommendations that emerges out of this study are: (i) regulating through fees charged by the rating agency, is more efficient than non price regulatory schemes. (ii) The regulated fees will in general be lower than unregulated fees to enhance efficiency.

We extend our model to incorporate competition among rating agencies. We construct a simple example to show that a competitive rating industry reduces welfare as compared to a monopolistic rating industry.

# 5.9 Appendix to Chapter 5

Define

$$H(p,e,D) \equiv \int_{p_m}^1 (p-\overline{p})\alpha(p,e,D)dF(p).$$

Note that H(.) is the left hand side of (IC) as given in equation (5.10). The following lemma will be useful in proving some of the results.

Lemma 5.1  $H_e \ge 0$ .

#### **Proof:**

$$H(p,e) = \int_{p_m}^{\overline{p}} (p-\overline{p}).\alpha(p,e,D)dF(p) + \int_{\overline{p}}^{1} (p-\overline{p})\alpha(p,e,D)dF(p)$$

$$H_e = \int_{p_m}^{\overline{p}} (p-\overline{p}).\alpha_e(p,e,D)dF(p) + \int_{\overline{p}}^{1} (p-\overline{p})\alpha_e(p,e,D)dF(p).$$

Given equation (5.2), it is immediate that  $H_e \geq 0$ .

#### • Proof of Proposition 5.1.

#### Proof:

(a) 
$$W^* = \overline{z}/D - 1$$
.

As  $\Pi_C$  is increasing in W, the CRA will set the maximum  $W^*$  such that the firm just breaks even. Therefore,  $\Pi_p = 0 \Rightarrow W^* = \overline{z}/D - 1$ .

(b) 
$$p_m^* = \text{Max}\{u^{-1}(\overline{p}, e^*), 0\}.$$

Note we have specified that  $\alpha(p,e) > 0 \quad \forall p \geq p_m^*$ . Therefore, from equation 5.2 we have  $\alpha(p_m^*,e^*) > 0 \Rightarrow u(p_m^*,e^*) > \overline{p}$ . This implies that  $p_m^* \geq \max\{u^{-1}(\overline{p},e^*),0\}$ . In equilibrium this will satisfy  $p_m^* = \max\{u^{-1}(\overline{p},e^*),0\}$ .

(c)  $e^* > 0$  such that  $H(p, e^*) = 0$  is the equilibrium evaluation standard set by the CRA.

The proof involve two steps. The first step asserts the existence of a  $e^* \in (0,1]$  such that  $H(p,e^*) = 0$ . In the second step, we will argue that  $e^*$  maximizes  $\Pi_C$ .

## Step I:

Suppose  $e^* = 0$ . Then, from (5.2) we have.  $p_m^* = 0$ . Therefore,

$$\int_{p_m^*}^1 (p-\overline{p})\alpha(p,0,D)dF(p) = [1-F(\overline{p})] \int_0^1 (p-\overline{p})dF(p) < 0.$$

This follows from our assumption that

$$DE(p) < 1 \Rightarrow \int_0^1 (p - \overline{p}) dF(p) < 0.$$

Therefore, in equilibrium,  $e^* > 0$ .

Claim:  $\exists e^* > 0$  such that  $H(p, e^*) = 0$ . If  $e^* = 1$ , then from equation (5.2) we have

$$\alpha(p, 1, D) = 1 \quad \forall p \ge \overline{p}$$
  
= 0  $\forall p < \overline{p}$ .

Note,  $e^* = 1 \Rightarrow p_m^* = \overline{p}$ . Therefore, we have

As H(.) is continuous and increasing in e (from Lemma 5.1),  $\exists e^*s.t.H(p,e^*) = 0$ .

Step II:  $e^*$  as obtained in step I maximizes the expected profit of the CRA.

Note that  $\forall e < e^*$ , H(p, e) < 0 and therefore,  $\Pi_C = 0$ . Now let  $e' > e^*$ . Then H(p, e') > 0, implying that the IC is satisfied.

However, from the expression of  $p_m^*$  obtained in (i) and the condition that  $u_e \leq 0$  we have  $p_m^*(e') \geq p_m^*(e^*)$ . This implies that

$$\Pi_C(e^*) = W[1 - F(p_m^*(e^*))] > W[1 - F(p_m(e'))] = \Pi_C(e').$$

Thus  $e^*$  is optimal.

(d)  $H(p, e^*) = 0$  implies that the investors just break even by investing in the firm whenever the announcement is good. Also, given that  $H(p, e^*) = 0$  and that  $\alpha(p, e, D)$  is increasing in p, it follows immediately that

$$\int_{p_m^*}^1 (p-\overline{p})[1-\alpha(p;e^*)]dF(p)<0,$$

implying that  $E[p|a=g]=\overline{p}$  and  $E[p|a=b]<\overline{p}$ .

Therefore, in equilibrium, the investor invests whenever the announcement is good and does not invest otherwise.

#### • Proof of Corollary 5.1.

**Proof:** The result is a straight forward interpretation of  $H(p_m^*, e^*) = 0$  as obtained from proposition 5.1. If  $p_m^* \geq \overline{p}$ , then  $H(p_m^*, e^*) > 0$ . This will contradict that  $e^*$  is an equilibrium.

#### • Proof of Corollary 5.2.

Proof:

$$H(\underline{p},e) \equiv \int_{p}^{1} (p-\overline{p})\alpha(p,e)dF(p).$$

Therefore,

$$H(\underline{p},0) \equiv \int_{\underline{p}}^{1} (p-\overline{p})\alpha(\underline{p},0)dF(p)$$
$$= \alpha(\underline{p},0) \int_{\underline{p}}^{1} (p-\overline{p})dF(p)$$
$$= 0.$$

This follows from the definition of p.

From lemma 5.1, we know that  $H_e \ge 0$ . Therefore,  $\forall e > 0 \quad H(\underline{p}, e) > 0$ . From the proof of proposition 5.1 we know that  $H(p_m^*, e^*) = 0$ . Therefore, it is immediate that  $p_m^* < \underline{p}$ .

### • Proof of Proposition 5.2.

**Proof:** 

$$\begin{split} S_{C} &= \int_{p_{m}^{*}}^{1} \alpha(p, e^{*}, D) \{ p\overline{z} - 1 \} dF(p) - \overline{c} \\ &\geq \int_{p_{m}^{*}}^{1} \{ \alpha(p, e^{*}, D) \{ p\overline{z} - 1 \} - W^{*} \} dF(p) \\ &= \int_{p_{m}^{*}}^{1} \{ \alpha(p, e^{*}, D) \{ p\overline{z} - 1 \} - \overline{p}\overline{z} + 1 \} dF(p) \\ &= \overline{z} \int_{p_{m}^{*}}^{1} \alpha(p, e^{*}, D) (p - \overline{p}) dF(p) \\ &+ (\overline{p}\overline{z} - 1) \int_{p_{m}^{*}}^{1} (1 - \alpha(p, e^{*}, D)) dF(p) \\ &= \overline{z} H(p_{m}^{*}, e^{*}) + (\overline{p}\overline{z} - 1) \int_{p_{m}^{*}}^{1} (1 - \alpha(p, e^{*}, D)) dF(p) \end{split}$$

$$= (\overline{p}\overline{z} - 1) \int_{p_m^*}^1 (1 - \alpha(p, e^*, D)) dF(p)$$
  
> 0.

The first inequality follows from  $W^*[1-F(p_m^*)] \geq \overline{c}$ . The third line obtained by substituting the value of  $W^*$  from proposition 5.1. The last equality is obtained from  $H(p_m^*, e^*) = 0$ .

#### • Proof of Proposition 5.3.

**Proof:** From (5.11),  $S_C$  is independent of W. This is because,  $p_m^*$  and  $e^*$ are independent of W obtained from proposition 5.1. The fee charged by the CRA is merely a transfer from the investor to the CRA. It does not affect either the evaluation standard or the minimum type that goes to the CRA. Therefore, the regulator cannot set a fee  $W_R$  that increases the net surplus.

#### • Proof of Proposition 5.4.

**Proof:** Proof of (a) and (b)

Let  $e^* < 1$ . Then two possibilities arise- either, If  $D_p^* \equiv D_p^*(e^*) < 1/p$ , or  $D_{p}^{*} \geq 1/p$ .

If  $D_p^* < 1/p$  then, the investors do not invest implying that  $p_m^e = 1$ . If  $D_p^* \ge 1/p$ . Then for all types that go to the CRA earns,  $p\{\overline{z} - D_p^*[1 + z]\}$ W].

If the CRA sets  $e^* = 1$ , then,  $D_p^*(e^*) = 1/p$ . This follows from the fact that  $\alpha(p,1) = 1 \forall p$  such that  $D_p^* \geq 1$  and  $\alpha(p,1) = 0$  otherwise. Therefore. we have  $D_p^* \ge 1/p$  if  $e^* < 1$  and  $D_p^* = 1/p$  if  $e^* = 1$ . As

$$\{\overline{z}-D_p^*(1+W)\}<\left\{\overline{z}-\frac{(1+W)}{p}\right\},$$

with  $e^* < 1$ , fewer types will go to the CRA as compared to  $e^* = 1$ . Thus, the CRA sets  $e^* = 1$  as  $\Pi_C = W[1 - F(p_m)] - \overline{c}$ .

With  $e^* = 1$ ,  $D_p^* = 1/p$ ;  $\forall p \ge p_m^e$ .

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## Proof of (c) and (d)

Denote  $\Pi_p^g$  as the expected profit to a firm of type p if it goes to the CRA. Therefore, as  $e^* = 1$ , we have

$$\Pi_p^g = p\{\overline{z} - \frac{(1+W^*)}{p}\}$$

$$\Pi_p^N = p\{\overline{z} - \frac{2}{p_m^*}\}.$$

Thus,  $p_m^e$  solves  $\Pi_p^g = \Pi_p^N$ . Therefore,

$$p_m^e = \begin{cases} 1 & \text{if } W^{**} > 1; \\ (1 + W^{*})/\overline{z} & \text{if } W^{**} \le 1 \end{cases}.$$

This is because, if  $W^* > 1$ , any type would earn higher expected returns by taking part in the pooling equilibrium where all types set  $D^* = 2$ . Therefore, if  $W^* > 1$ , then no types are willing to go to the CRA. However, as  $W^* \leq 1$ , the marginal type benefits from going to the CRA as long as  $\overline{z} \geq (1+W^*)D_p^*$ .

Therefore, with  $e^* = 1$ ,

$$\Pi_C = W \left\{ 1 - \left( \frac{1+W}{\overline{z}} \right) \right\} - \overline{c} \quad \text{if} \quad W \le 1$$
$$= -\overline{c} \quad \text{if} \quad W > 1.$$

Now,  $W^* = arg \max_W \Pi_C = W \left\{ 1 - \left( \frac{1+W}{\overline{z}} \right) \right\} - \overline{c}$ , implies that  $W^* = 0.5(\overline{z} - 1)$ . Note, for  $2 \le \overline{z} \le 3$ ,  $W^* \le 1$  while for  $\overline{z} > 3$ ,  $W^* > 1$ .

However, if  $W^* > 1$  then,  $p_m^e = 1 \Rightarrow \Pi_C = 0$ . Therefore, the CRA being profit maximizer sets  $W^* = min\{1, 0.5(\overline{z} - 1)\}$ . Therefore we have.

$$W^* = \begin{cases} 0.5(\overline{z} - 1) & \text{if } 2 \le \overline{z} \le 3; \\ 1, & \text{if } \overline{z} > 3 \end{cases}.$$

The equilibrium values for  $W^*$  imply

$$p_m^e = \begin{cases} 0.5 + \frac{0.5}{\overline{z}} & \text{if } 2 \le \overline{z} \le 3; \\ 2/\overline{z}, & \text{if } \overline{z} > 3 \end{cases}$$

#### $\underline{\text{Proof}}$ of (e)

We now show that, for  $\forall p < p_m^e$ , there does not exist D such that  $\Pi^N > \Pi^g > 0$ . That is, given  $\{e^*, W^*\}$ , any type that does not so to the CRA can not earn a strictly greater and positive profit.

Consider  $p_m^e$ .

$$p_m^e = \begin{cases} 0.5 + \frac{0.5}{\overline{z}} & \text{if } 2 \le \overline{z} \le 3; \\ 2/\overline{z}, & \text{if } \overline{z} > 3 \end{cases}.$$

Let  $2 < \underline{z} \le 3$ .

Note that,  $\Pi_{p_m^e}^g$  denotes the expected profits to type  $p_m^e$  when it goes to the CRA. Thus,

$$\Pi_{p_m^e}^g = p_m^e \{ \overline{z} - (1 + W^*) \frac{1}{p_m^e} \} = 0.$$

If the type  $p_m^e$  did not go to the CRA, the minimum debt level that it could have set to attract investment is  $2/p_m^e$ . Therefore,

$$\Pi^N_{p^e_m} \le p^e_m \{\overline{z} - \frac{2}{p^e_m}\} = p^e_m \{\overline{z} - \frac{2\overline{z}}{0.5\overline{z} + 0.5}\} \le 0 \le \Pi^g_{p^e_m}.$$

Let  $\overline{z} > 3$ .

Note that  $p_m^e = 2/\overline{z}$ . Therefore,

$$\Pi_{p_m^e}^g = p_m^e \{ \overline{z} - \frac{2}{p_m^e} \} = 0.$$

However,

$$\Pi_{p_m^e}^N \le p_m^e \{ \overline{z} - \frac{2}{p_m^e} \} = 0.$$

Therefore, given  $\{e^*, W^*\}$ , any type that does not go to the CRA can not set D such that  $\Pi_N > 0$ .

## Proof of proposition 5.5.

#### Proof:

Note that,

$$\begin{split} S_C^1 - \overline{S} &= \int_{\frac{1+W^*}{\overline{z}}}^1 [p\overline{z} - 1] dF(p) - \overline{c} - 0.5\overline{z} + 1 \\ &= \int_{\frac{1+\overline{z}}{2\overline{z}}}^1 [p\overline{z} - 1] dF(p) - \overline{c} - 0.5\overline{z} + 1 \quad \text{if} \quad \overline{z} \in [2, 3] \\ &= \int_{\frac{2}{\overline{z}}}^1 [p\overline{z} - 1] dF(p) - \overline{c} - 0.5\overline{z} + 1 \quad \text{if} \quad \overline{z} > 3 \\ &= \frac{\overline{z} + 1}{8\overline{z}} [3 - \overline{z}] - \overline{c} \\ &= G(\overline{z}) - \overline{c}. \end{split}$$

Therefore,  $G(\overline{z}) > 0$  if  $2 \le \overline{z} < 3$ . Thus, for sufficiently small  $\overline{c}$ ,  $S_C^1 - \overline{S} > 0$ . If,  $\overline{z} > 3$ , then  $S_C^1 - \overline{S} < -\overline{c}$ .

#### Proof of Proposition 5.6.

**Proof:** The regulated fee,  $W_R$ , must ensure,  $\Pi_C \geq 0$  and  $S_C^1 > \overline{S}_1$ .  $\Pi_C \geq 0$  implies

$$W_R \ge 0.5 \left\{ (\overline{z} - 1) - \sqrt{(\overline{z} - 1)^2 - 4\overline{c}\overline{z}} \right\}$$

Similarly,  $S_C^1 \geq \overline{S}_1$  implies

$$\sqrt{1-2\overline{c}z} > W_R$$
.

Therefore,  $\forall W_R$  such that

$$\sqrt{1-2\overline{c}\overline{z}} > W_R \ge 0.5 \left\{ (\overline{z}-1) - \sqrt{(\overline{z}-1)^2 - 4\overline{c}\overline{z}} \right\}.$$

trading in the presence of a rating agency is efficient.

It remains to be shown that, for  $W_R$  to be feasible,

$$\sqrt{1-2\overline{c}z} > 0.5 \left\{ (\overline{z}-1) - \sqrt{(\overline{z}-1)^2 - 4\overline{c}z} \right\}.$$

The above condition is satisfied if,

$$\overline{c} < (\overline{z} - 2) \left\{ \sqrt{1 + \frac{1}{\overline{z}(\overline{z} - 2)}} - 1 \right\}.$$

Proof of  $W_R < W_U$ .

Note that

$$2\overline{cz} < 2\overline{z}(\overline{z} - 2) \left\{ \sqrt{1 + \frac{1}{\overline{z}(\overline{z} - 2)}} - 1 \right\}.$$

Claim:

$$2\overline{z}(\overline{z}-2)\left\{\sqrt{1+\frac{1}{\overline{z}(\overline{z}-2)}}-1\right\}<1.$$

Proof by contradiction.

Suppose not, i.e,

$$2\overline{z}(\overline{z}-2)\left\{\sqrt{1+\frac{1}{\overline{z}(\overline{z}-2)}}-1\right\} \geq 1$$

$$\Rightarrow \sqrt{[\overline{z}(\overline{z}-2)]^2+\overline{z}(\overline{z}-2)}-\overline{z}(\overline{z}-2) \geq 1/2$$

$$\Rightarrow [\overline{z}(\overline{z}-2)]^2+\overline{z}(\overline{z}-2)-[\overline{z}(\overline{z}-2)]^2+\overline{z}(\overline{z}-2) \geq 1/4$$

$$0 \geq 1/4.$$

This contradicts that

$$2\overline{z}(\overline{z}-2)\left\{\sqrt{1+rac{1}{\overline{z}(\overline{z}-2)}}-1\right\}\geq 1.$$

Therefore, for  $\overline{z} > 3$ ,

$$W_R < \sqrt{1 - 2\overline{cz}} < 1 = W_U$$
.

#### • Proof of Proposition 5.7.

**Proof:** Claim:  $\hat{p}_m$  solves  $D^*_{\hat{p}_m} = 1/\hat{p}_m$ .

For all p such that  $D_p^* < 1/p$ , the investor does not invest irrespective of whether p goes to the CRA or not. As going to the CRA involves a small non pecuniary cost, all types with p such that  $D_p^* < 1/p$  stay out.

All types with  $D_p^* \ge 1/p$  go to the CRA, as the expected profit from going to the CRA is greater than not going to the CRA at all.

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 $\Pi_p$  being decreasing in  $D_p$  implies that,  $D_{\hat{p}_m}^* = 1/\hat{p}_m$ . Solving which gives.

$$\hat{p}_m = rac{e(1+W)}{2\overline{z}} \left\{ 1 \pm \sqrt{1 + rac{4\overline{z}(1-e)}{e^2(1+W)}} 
ight\}$$

or,

$$\hat{p}_m = Max \left\{ 0, \frac{e(1+W)}{2\overline{z}} \left\{ 1 \pm \sqrt{1 + \frac{4\overline{z}(1-e)}{e^2(1+W)}} \right\} \right\}.$$

If  $\hat{p}_m = 0$ , then  $D_{\hat{p}_m} \to \infty$  implying that in equilibrium

$$\hat{p}_m = \frac{e(1+W)}{2\overline{z}} \left\{ 1 + \sqrt{1 + \frac{4\overline{z}(1-e)}{e^2(1+W)}} \right\}.$$

Proof of  $\hat{p}_m$  decreasing in e.

$$\hat{p}_{m} = \frac{\left\{e(1+W) + \sqrt{[e(1+W)]^{2} + 4\overline{z}(1-e)(1+W)}\right\}}{2\overline{z}} \\
\frac{\partial \hat{p}_{m}}{\partial e} = \frac{1+W}{2\overline{z}} \left\{1 + \frac{e(1+W) - 2\overline{z}}{\sqrt{[e(1+W)]^{2} + 4\overline{z}(1-e)(1+W)}}\right\}$$

Claim:

$$1 + \frac{e(1+W) - 2\overline{z}}{\sqrt{[e(1+W)]^2 + 4\overline{z}(1-e)(1+W)}} < 0.$$

Proof by contradiction.

Suppose not, i.e.

$$1 + \frac{e(1+W) - 2\overline{z}}{\sqrt{[e(1+W)]^2 + 4\overline{z}(1-e)(1+W)}} \ge 0$$

$$\Rightarrow \sqrt{[e(1+W)]^2 + 4\overline{z}(1-e)(1+W)} \ge 2\overline{z} - e(1+W)$$

$$\Rightarrow (1+W) > \overline{z}.$$

This contradicts that  $\Pi_p \geq 0$ . Therefore,

$$\frac{\partial \hat{p}_m}{\partial e} < 0.$$

## • Proof of Proposition 5.8.

**Proof:** Note  $\Pi_C = W[1 - \hat{p}_m] - \overline{c}$ . As  $\hat{p}_m$  decreases in e, in equilibrium the CRA will set  $e^* = 1$ . It now follows straight forward that

$$D_p^* = 1/p$$
,  $W^* = 0.5(\overline{z} - 1)$ , and  $\hat{p}_m = 0.5 + \frac{0.5}{\overline{z}}$ .

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