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Efficient Horizontal Mergers

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This paper considers the Debreu–Farrell–Färe measure of efficiency of merger to compare economic efficiencies of alternative merged entities in a homogeneous good industry. The comparability results rely on concentration curve dominance relation and identify the class of cost functions for which efficiency ranking of the entities becomes unambiguous. The results have been developed under alternative assumptions about the total output and number of firms of the merging subgroups. *Journal of Economic Literature* Classification Number: L41. © 1998 Academic Press

1. INTRODUCTION

Horizontal merger refers to an amalgamation of two or more firms producing the same good into one. An important contribution to the literature on horizontal merger is the pioneering article by Salant, Switzer, and Raynolds [20]. They employed a symmetric Cournot model with linear demand function and identical constant average cost for each firm to examine the incentive to merge. In the Salant–Switzer–Reynolds framework merger of two firms from a symmetric equilibrium with n firms should result in an equilibrium with $(n-2)$ old firms and a new firm that is larger than others. But since there is no technological asymmetry in the industry, the merged firm continues to have access to the same technology as others. Perry and Porter [17] explicitly specified a tangible asset that the merged firm acquires from its constituent firms. This structure addresses the industry asymmetries caused by the merger of subsets of firms. For characterizing the circumstances under which there is an incentive to merge, Perry and Porter [17] considered a dominant oligopoly model with a competitive fringe and also a second model in which there are large and small oligopolists. Farrel

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and Shapiro [8] analyzed mergers in Cournot oligopoly and developed a procedure for analyzing the effect of merger on rivals and consumers and thus provided sufficient conditions for profitable mergers to increase welfare. Werden [25] considered the concentration of the distribution of capital among firms to examine the welfare effects of merger on nonmerging firms and consumers.[1]

"Mergers may give rise to efficiency gains (for example, scale economies) that reduce the cost of production or distribution" (Perry and Porter [17, p. 219]). Long and Vousden [15], in the context of trade liberalization, have shown that merger between two firms will not be profitable unless it results in sufficiently large cost reduction.

The purpose of this article is to analyze horizontal merger from view point of cost efficiency. More precisely, suppose that we have two merged entities formed by combining firms in separate subgroups in a homogeneous good industry. The number of firms as well as the total outputs of the subgroups are not necessarily the same. Then we wish to identify the class of cost functions for which one merged entity becomes more efficient than another. The criterion we employ for this purpose is the output concentration curve dominance rule and its relation with the Debreu [6]–Farrell [10]–Färe [7] measure of efficiency of merger. We in fact show that if the concentration curve of one subgroup dominates that of another, then the merger of the latter subgroup of firms is more efficient than that of the former subgroup by the Debreu–Farrel–Färe measure for a certain class of cost functions. This procedure can be utilized to identify the most efficient merged entity.

The paper is organized as follows. The next section discusses the Debreu–Farrel–Färe measure of efficiency of merger. The main findings are presented in Section 3. Finally Section 4 concludes.

2. MEASURING EFFICIENCY OF MERGER

Consider a homogeneous good industry consisting of n firms. Let $x_i \geqslant 0$ be the output of firm i. The output vector $(x_1, x_2, ..., x_n)$ is denoted by x. The set of all output vectors in this n-firm industry is D^n, the nonnegative orthant of the Euclidean n-space R^n with the origin deleted. Thus, the industry may include some firms that exist but are currently producing no output. Further, by deleting origin from the output domain we ensure that at least one firm is producing positive output. For all $x \in D^n$; we write X for $\sum_{i=1}^{n} x_i$, the sum of output levels of the firms, and $b = (b_1, b_2, ..., b_n)$

[1] See Farrell and Shapiro [9] for a discussion on Werden's results. See also Hay and Werden [13]. for further analysis.

for x/X; the vector of output shares. Since the firms produce a homogeneous good, X and b are well defined. For any $x \in D^n$, we will write $\overset{0}{x}$ for that permutation of the output distribution such that $\overset{0}{x}_1 \geqslant \overset{0}{x}_2 \cdots \geqslant \overset{0}{x}_n$.

The Debreu [6]–Farrel [10] measure of technical efficiency for an arbitrary firm with output level m is

$$F(m, y) = \frac{1}{\max\{\lambda \in D^1 \mid y/\lambda \in L(m)\}}, \tag{1}$$

where L is the input requirement set; that is, $L(m)$ is the set of all input vectors that can produce at least the output level m. The denominator in (1) is the Malmquist [16]–Shepard [22] distance function that gives the maximum amount by which the input vector y can be scaled down such that the resultant input vector is still in the input requirement set $L(m)$.

On the other hand, a natural measure of economic efficiency E of a firm is the ratio of minimal to actual input costs for producing an arbitrarily given output level m. That is,

$$E(m, y, w) = \frac{C(m, w)}{w . y}, \tag{2}$$

where w is the vector of input prices and $C(m, w)$ is the cost function:

$$C(m, w) = \min_y \{w . y \mid y \in L(m)\}. \tag{3}$$

Loosely speaking, E determines a firm's success in choosing an optimal set of inputs for producing a given (but arbitrary) level of output. Evidently, for all positive output levels m, positive prices w, and $y \in L(m)$, $0 < E \leqslant 1$, a high value of E shows a higher degree of economic efficiency.

It has been noted (Debreu [6], Russell [19]) that the two efficiency measures are related as follows: For all (m, y, w),

$$E(m, y, w) \leqslant F(m, y). \tag{4}$$

We assume that the factor markets are perfectly competitive. Therefore, all the n firms in the industry face the same factor prices, which are taken as being given. Therefore, we can rewrite the cost function $C(m, w)$ as $f(m)$ for some real valued increasing function f of output m. Increasingness of f ensures that cost is increasing with output. Now, if all the n firms are merged into one firm, the industry output X is produced by a single firm. Färe [7] suggested the use of the quotient of the Debreu–Farrel measures corresponding to the reference technologies $L(X)$ and $\sum_{i=1}^n L(x_i)$ as the efficiency gain function from combining the producers. If the input vectors employed are cost minimizing, then the gain function can be written as

$$G_f(x) = \frac{f(X)}{\sum_{i=1}^{n} f(x_i)}, \tag{5}$$

where $x \in D^n$ is arbitrary. We note that the gain function in (5), which we refer to as the Debreu–Farrell–Färe (DFF) measure of efficiency of merger, implicitly assumes that when a group of firms merge, the merged entity produces the premerger combined output of its constituents.

Now, in terms of the DFF measure G_f efficiency will increase from combining the firms if $G_f < 1$, that is, if $\sum_{i=1}^{n} f(x_i) > f(X)$. A cost function that satisfies the condition for all output distributions is called subadditive. Subadditivity thus means that it costs less to produce various outputs together than to produce them separately. If a cost function has a decreasing marginal, then it is subadditive. Subadditivity also follows from economics of scale, a weaker requirement than declining marginal costs (Jacquemin [14, p. 20]). However, economies of scale is a sufficient but not a necessary condition for subadditivity (Sharkey [21]).

If the cost function is subadditive, then the measure G_f is bounded between zero and one. As G_f approaches its upper bound, the gain in efficiency from merger becomes very minor. On the other hand, gain becomes substantial as G_f tends to zero. A lower value of the measure indicates a higher efficiency gain from merger.

3. EFFICIENT MERGERS

To explain the role of the DFF measure in intra-industry merger analysis, we need to relate it to a concentration index. An index of concentration of an n-firm industry is a real valued function on D^n and it measures the extent to which economic activity is controlled by large firms. A highly concentrated index will take on a large value for the index and is expected to be closer to the monopoly end of the monopoly to competition spectrum than an industry with a low value for the index. An index of concentration I defined on D^n should satisfy certain properties. These are symmetry (I should remain invariant under permutations of its arguments), zero output independence (addition or deletion of a firm which produces no output does not alter I), the output transfers principle (I increases under a transfer of output from a small firm to a large firm), homogeneity (I is homogeneous of degree zero in its arguments), continuity, normalization (when all the n firms have equal market shares, I should be $1/n$), and the replication principle (I gets multiplied by the factor $1/k$ if the industry is replicated k items). Symmetry means that firms are not distinguished by anything other than their output levels. Zero output independence gives the main difference between inequality and concentration. It is generally agreed that adding a person with zero income to a population increases inequality. In contrast, adding a firm that produces no output should not affect the dominance of large firms, that is, concentration. The output transfers principle is clearly a desirable property of I. According to homogeneity, I depends on output shares only, not on absolute output levels. Continuity means that I will not be oversensitive to small changes in one or more of its arguments. Finally, normalization and the replication principle are concerned with cardinality properties of I. (See Hannah and Kay [11]; Blackorby, Donaldson, and Weymark [1]; Waterson [24]; Chakravarty and Weymark [5]; Chakravarty [3], and Hay and Morris [12] for further discussions.)

The following theorem shows that the DFF efficiency measure G_f can be interpreted as a concentration index under certain minor assumptions about the cost function.

THEOREM 1 (Chakravarty [2, 3]). *Suppose that the cost function f is continuous, strictly concave in output levels, and $f(0)=0$. Then the DFF merger efficiency measure G_f can be regarded as a concentration index in the sense that it agrees with continuity, zero output independence, symmetry, and the output transfers principle.*

It may be important to note that although the DFF measure can be considered a concentration index, a concentration index may not be representable in the form given by (5). For instance, there does not exist a strictly concave cost function f using which we can express the Rosenbluth [18] index of concentration,

$$I_R(x)=\frac{X}{\sum_{i=1}^{n}(2i-1)\overset{0}{x}_i}, \tag{6}$$

in the form given by G_f. (The index I_R fulfills all the properties of a concentration index considered in Theorem 1.)

Now, two different merged entities may be ranked in different directions by the DFF measure for distinct cost functions. Therefore, it will be worthwhile to investigate whether one entity can be unambiguously regarded as more efficient than another by the efficiency measure in (5) for a certain class of cost functions. We know that the DFF measure can be interpreted as a concentration index and that the ordering of output distributions by concentration indices can be obtained through pairwise comparison of concentration curves of distributions (Hannah and Kay [11]; Chakravarty and Eichhorn [4]). Therefore, for ranking alternative merged entities in terms of efficiency, we consider the concentration curve, a plot of the cumulative output shares against the number of firms, with firms ranked from the largest to the smallest. Formally, for any $x \in D^n$ the concentration curve $K(x, i)$ is obtained by plotting $\sum_{j=1}^{i} \overset{0}{x}_j / X$ against i, where $i = 1, 2, ..., n$. For any two output distributions $x, z \in D^n$, the concentration curve of x dominates that of z (xKz, for short) if

$$\sum_{j=0}^{i} \frac{\overset{0}{x}_j}{X} \geqslant \sum_{j=1}^{i} \frac{\overset{0}{z}_i}{Z} \tag{7}$$

for all $i = 1, 2, ..., n$, with $>$ for at least one $i < n$. That is, xKz holds if the concentration curve of x lies nowhere below that of z and at some places (at least) strictly above the latter. Note that, since the concentration curve satisfies zero output independence and homogeneity of degree zero, we can consider the comparability criterion in (7) in the cases of variable number of firms and variable total output, in addition to equal total output and equal number of firms case.

Suppose two merged entities are formed within the same n-firm industry by combining n_1 and n_2 firms, respectively, where $n_1 + n_2 \leqslant n$. The entities produce the premerger combined outputs of the respective subgroups. We denote the premerger output vector of the n_i (merging) firms by x^i, $i = 1, 2$. Assume without loss of generality that $x^1 = (x_1, ..., x_{n_1})$, $x^2 = (x_{n_1+1}, ..., x_{n_2})$. X^1 and X^2 are the total outputs of the subgroups; that is, $X^1 = \sum_{i=1}^{n_1} x_i$ and $X^2 = \sum_{i=n_1+1}^{n_2} x_i$. We then wish to determine the class of cost functions that will enable us to rank the entities by the DFF measure under alternative assumptions about the number of merging firms and total outputs of the merging firms. The following theorem provides a sufficient condition along this line.

THEOREM 2. *Suppose* $n_1 = n_2$ *and* $X^1 = X^2$. *Then* $x^1 K x^2$ *implies that* $G_f(x^1) > G_f(x^2)$ *for all strictly concave cost functions* f.

Proof. Given $x^1 K x^2$, along with $n_1 = n_2$ and $X^1 = X^2$, we can say that x^1 is regarded as more concentrated than x^2 by all concentration indices that satisfy symmetry and the output transfers rule (Hannah and Kay [11]). Since the DFF efficiency measure is symmetric and strict concavity of the cost function guarantees that it meets the output transfers rule (Chakravarty [2, 3]), we must have $G_f(x^1) > G_f(x^2)$ for all strictly concave cost functions f. ∎

What Theorem 2 says is the following: Given two subgroups of firms with the same total output and the same number of firms, if the concentration curve of the premerger output distribution of subgroup 1 firms dominates that of subgroup 2 firms, then merger of subgroup 2 firms is more efficient than that of subgroup 1 firms for all strictly concave cost functions. Thus, in this case we do not need to compute the values of the DFF efficiency index for determining efficiency rankings of the subgroups. The intuitive reasoning behind this result is as follows. By concentration curve dominance large firms in subgroup 1 possess higher output shares than large firms in subgroup 2. Decreasingness of marginal cost then ensures that the same total output can be produced at a lower cost by firms in subgroup 1 than by firms in subgroup 2. Since the total outputs of the two merged entities are the same, their total costs are also the same. Consequently, merger of subgroup 2 firms gives rise to a higher amount of cost reduction than merger of subgroup 1 firms. Therefore, a policy maker who views higher cost reduction as better will prefer the merger of subgroup 2 firms over that of subgroup 1 firms. Since the concentration curve dominance relation is transitive, we can extend our analysis to any number of subgroups and identify the most efficient merged entity under the conditions stated in Theorem 2. However, the concentration curve dominance relation is a quasiordering—it is transitive, but not complete. Therefore, if the concentration curves of x^1 and x^2 cross, we can get two strictly concave cost functions f_1 and f_2 such that $G_{f_1}(x^1) > G_{f_1}(x^2)$ but $G_{f_2}(x^1) < G_{f_2}(x^2)$; that is, the two merged entities are ranked in different directions by the DFF measure. Thus, in such a case the policy maker has to withhold judgments on comparability of cost efficiencies of merged subgroups. Since concentration is a many faceted phenomenon, this quasiordering of merger efficiencies should not be taken as a serious shortcoming.

Theorem 2 has been demonstrated under the suppositions that the number of firms as well as the total outputs of the two merged subgroups are the same. The assumptions are quite restrictive. We relax these conditions in the next theorem.

THEOREM 3. *Suppose that* $x^1 K x^2$ *holds under the general conditions* $n_1 \neq n_2$ *and* $X^1 \neq X^2$. *Assume that the DFF efficiency measure* G_f *is differentiable. Then the only cost function for which we have* $G_f(x^1) > G_f(x^2)$ *is*

$$f(x_i) = Ax_i^{\alpha}, \tag{8}$$

where $A > 0$ *and* $0 < \alpha < 1$ *are constants.*

Proof. Given $x^1 K x^2$ along with $X_1 \neq X_2$ and $n_1 \neq n_2$, we can say that x^1 is should be more concentrated than x^2 by all concentration indices that satisfy zero output independence, symmetry, the output transfers principle, and homogeneity (Chakravarty and Eichhorn [4]; Chakravarty [3]). This in particular implies that $G_f(x^1) > G_f(x^2)$ for all strictly concave cost functions f that satisfy the condition $f(0) = 0$ and that make the DFF efficiency measure G_f homogeneous of degree zero.

To determine the form(s) of the cost function(s) for which the DFF measure is homogeneous of degree zero, suppose that $n = 2$. Then the DFF measure can be written as

$$G_f(x_1, x_2) = \frac{f(x_1 + x_2)}{f(x_1) + f(x_2)} = h(f), \qquad \text{say}, \tag{9}$$

where h is an operator on f. Since (x_1, x_2) belongs to D^2, $f(0) = 0$, and the cost function is increasing in output levels, both $f(x_1 + x_2)$ and $f(x_1) + f(x_2)$ are positive. In fact, G_f becomes identically one if one of the two output levels is zero. So, let us suppose that both x_1 and x_2 are positive. Now by Euler's theorem for homogeneous functions, from (9), we have

$$x_1 \frac{f'(x_1 + x_2)(f(x_1) + f(x_2)) - f'(x_1)(f(x_1 + x_2))}{(f(x_1) + f(x_2))^2} + x_2 \frac{f'(x_1 + x_2)(f(x_1) + f(x_2)) - f'(x_2)(f(x_1 + x_2))}{(f(x_1) + f(x_2))^2} = 0, \tag{10}$$

where f' is the derivative of f. Multiplying both sides of (10) by $(f(x_1) + f(x_2))^2$ and rearranging the resulting expression we get

$$\frac{f(x_1 + x_2)}{f(x_1) + f(x_2)} = \frac{(x_1 + x_2)\, f'(x_1 + x_2)}{x_1 f'(x_1) + x_2 f'(x_2)}. \tag{11}$$

Noting that the right-hand side of (11) is $h(x_1 f')$, it follows from (11) that

$$h(f) = h(x_1 f'). \tag{12}$$

Now, let f_1 and f_2 be two distinct differentiable cost functions such that $f_1(0) = f_2(0) = 0$. Suppose that

$$h(f_1) = h(f_2); \tag{13}$$

that is,

$$\frac{f_1(x_1 + x_2)}{f_1(x_1) + f_1(x_2)} = \frac{f_2(x_1 + x_2)}{f_2(x_1) + f_2(x_2)}. \tag{14}$$

Taking $x_1 = x_2$ in (14) we have

$$\frac{f_1(2x_1)}{f_1(x_1)} = \frac{f_2(2x_1)}{f_2(x_1)}, \tag{15}$$

from which by componendo it follows that

$$\frac{f_1(x_1)+f_1(2x_1)}{f_1(x_1)}=\frac{f_2(x_1)+f_2(2x_1)}{f_2(x_1)}. \tag{16}$$

Clearly, from (15) and (16) we can deduce that

$$\frac{f_1(x_1)}{f_2(x_1)}=\frac{f_1(2x_1)}{f_2(2x_1)}=\frac{f_1(x_1)+f_1(2x_1)}{f_2(x_1)+f_2(2x_1)}. \tag{17}$$

Next, take $x_2=2x_1$ in (14) to get

$$\frac{f_1(3x_1)}{f_1(x_1)+f_1(2x_1)}=\frac{f_2(3x_1)}{f_2(x_1)+f_2(2x_1)}, \tag{18}$$

or

$$\frac{f_1(3x_1)}{f_2(3x_1)}=\frac{f_1(x_1)+f_1(2x_1)}{f_2(x_1)+f_2(2x_1)}. \tag{19}$$

Therefore, from (17) and (19) we have

$$\frac{f_1(x_1)}{f_2(x_1)}=\frac{f_1(3x_1)}{f_2(3x_1)}. \tag{20}$$

Repeating the above procedure we can show that for any arbitrary positive integer r,

$$\frac{f_1(x_1)}{f_2(x_1)}=\frac{f_1(rx_1)}{f_2(rx_1)}. \tag{21}$$

For $x_1=v/s$, where s is a positive integer, (21) becomes

$$\frac{f_1(v/s)}{f_2(v/s)}=\frac{f_1(r(v/s))}{f_2(r(v/s))}. \tag{22}$$

Since $v=(v/s)\,s$, using (21) it can be established that

$$\frac{f_1(v)}{f_2(v)}=\frac{f_1(v/s)}{f_2(v/s)}, \tag{23}$$

which in view of (22) gives

$$\frac{f_1(r(v/s))}{f_2(r(v/s))}=\frac{f_1(v)}{f_2(v)}. \tag{24}$$

Purring $v=1$ in (24), we have

$$\frac{f_1(r/s)}{f_2(r/s)}=\frac{f_1(1)}{f_2(1)}=k, \tag{25}$$

where $k>0$ is a constant. Since r and s are arbitrary positive integers, we can say that

$$\frac{f_1(q)}{f_2(q)}=k \tag{26}$$

for all positive rationals q.

We will now show that (26) holds for all positive reals q. To see this, define the function $g(q)=(f_1(q)/f_2(q))-k$, where $q>0$ is any real. Given continuity of g, which follows from continuity of f_1 and f_2, and $g(q)=0$ for all positive rationals q, we need to show that $g(q)=0$ for all $q>0$. Suppose that for some irrational $p>0$, $g(p)=c\neq 0$. Choose $\varepsilon=|c|/2$. Since g is continuous, there exists a $\delta>0$ such that $|q-p|<\delta$ implies $|g(p)-g(q)|<\varepsilon=|c|/2$. Now, there are rational numbers in every open interval. Therefore, there exists a rational number t such that $t\in\{q\colon |q-p|<\delta\}$. This in turn implies that $|g(t)-g(p)|=|g(p)|=|c|<\varepsilon=|c|/2$, which is a contradiction. Thus, g is identically zero for all positive reals; that is,

$$\frac{f_1(q)}{f_2(q)}=k \tag{27}$$

for all $q>0$. Therefore, $f_1(q)=kf_2(q)$. This shows that $h(f_1)=h(f_2)$ implies $f_1(q)=kf_2(q)$. Therefore, $h(f)=h(x_1f')$ implies

$$f(x_1)=kx_1f'(x_1), \tag{28}$$

from which we have

$$f(x_1)=Ax_1^{\alpha}, \tag{29}$$

where $\alpha=1/k$ and $A=e^T>0$, with T being the constant of integration. Increasingness and strict concavity of f require that $0<\alpha<1$. Since the cost function is identical for all firms, we have $f(x_i)=Ax_i^{\alpha}$ for all i. This establishes the necessity part. The sufficiency can be verified by checking that f given by (8) makes the DFF efficiency measure homogeneous of degree zero. ▮

The following result drops out as an interesting corollary to Theorem 3.

COROLLARY 4. *Suppose that the DFF efficiency measure G_f is differentiable in firm output levels. Then the only cost function for which G_f satisfies the output transfers principle, zero output independence, and homogeneity is $f(x_i) = Ax_i^{\alpha}$, where $A > 0$ and $0 < \alpha < 1$ are constants. Further, the resulting index satisfies continuity and symmetry also.*[2, 3]

Theorem 3 shows that if the number of firms and total outputs of the two merging subgroups are unequal, then only one cost function can rank the subgroups in terms of the DFF measure, given that the concentration curves of the two subgroups are nonintersecting. The result, however, relies on differentiability assumption of the efficiency measure. Since almost all cost functions are differentiable in output levels, this assumption, which is made for technical convenience only, does not appear to be quite restrictive.

It may be noted that in Theorem 3 we employ zero output independence to make the number of firms in the two subgroups equal. Homogeneity becomes helpful in making the total outputs the same. Therefore, the intuitive reasoning that we provided for Theorem 2 remains valid here also.

Clearly, we can prove a result that parallels Theorems 2 and 3 for the case where the number of firms of the merging subgroups are variable but their total outputs are the same. In this case $x^1 K x^2$ implies that $G_f(x^1) > G_f(x^2)$ holds for all strictly concave cost functions f which satisfy $f(0) = 0$. Thus, in addition to the form given by (8) the result also holds for a large class of cost functions. An example of this latter type of cost function is $f(x_i) = Bx_i/(1 + x_i)$, where $B > 0$ is a constant. Since the form given by (8) is necessary and sufficient for homogeneity of G_f, this example shows the importance of the homogeneity property for identification of (8). Finally, if the number of firms is fixed and total output is a variable, then $x^1 K x^2$ implies that $G_f(x^1) > G_f(x^2)$ for all strictly concave cost functions that make G homogeneous of degree zero. It should be clear that in this case also the only cost function that makes G_f homogeneous of degree zero is given by (8).

[2] An attempt has been made in Chakravarty [2] to prove a variant of Corollary 4. However, that proof was very sketchy and sloppy. The proof provided here is complete and quite rigorous. Further, no attempt was made in Chakravarty [2] to use the DFF measure for intra-industry merger analysis. Helpful suggestion from Partha Sarathi Chakraborty about the proof of Corollary 4 is acknowledged with thanks.

[3] The explicit form of the resulting concentration index is given by $M(x) = (\sum_{i=1}^{n} b_i^{\alpha})^{-1}$. This index is related to the Hannah–Kay [11] index $H(x) = (\sum_{i=1}^{n} b_i^{\alpha})^{1/(\alpha-1)}$ via the increasing transform $H(x) = (M(x))^{1/(1-\alpha)}$, where $0 < \alpha < 1$. Strictly speaking, $H(x)$ is defined for all $\alpha > 0$. As $\alpha \to 1$, $H(x)$ becomes one variant of the Theil [23] entropy index of concentration. On the other hand, for $\alpha = 2$, $H(x)$ becomes the well-known Herfindahl–Hirschman index of concentration.

4. CONCLUSIONS

Very few attempts have been made to analyze efficiency aspects of merger. Färe [7] employed the Debreu [6]–Farrel [10] measure of efficiency to demonstrate how under subadditivity of the cost function, merger of a group of firms can lead to efficiency gain. In this paper we show that the ranking of alternative horizontally merged entities in terms of the Debreu–Farrel–Färe measure of merger efficiency drops out as an implication of concentration curve dominance rule. We thus try to isolate the class of cost functions for which merger of one subgroup of firms turns out to be more efficient than another. In demonstrating our results we assume that the number of firms and total outputs of two merging subgroups of firms may or may not be the same.

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