CONCEPT OF IMPOSED GRADIENT ON EMBRYONIC DEVELOPMENT

A.K. Saha

Embryology Unit, Indian Statistical Institute, 203, Barrackpore Trunk Road, Calcutta 700 035, India

ABSTRACT

It is observed in Nature that the embryonic developments of different kinds of species require different types of environments. It is assumed in this paper that there may exist some kind of imposed gradient or pre-assigned gradient, generated as a resultant effect of the environment, acting on the developing embryo. This system has been studied by constructing a suitable mathematical model of epigenetic mechanism. It has been observed that the introduction of this imposed gradient can easily change any exiting pattern of the embryo. Model analysis has been performed in the line of Turing approach.

Key words : Epigenetic mechanism, Hill coefficient, Bifurcation, imposed gradient.

INTRODUCTION

The aim of this work is to show how an existing pattern within an embryo generated by a reaction diffusion system can be deformed by imposing a gradient. It is assumed that the gradient is generated in the extra cellular space of the embryo via some unidirectional signals. As an application we have considered here a simple mathematical model proposed by Tapaswi and Saha (1986) on epigenetic mechanism which is essentially the case in many different embryos.

MATHEMATICAL MODEL

The mathematical model, considered here, involves mRNA(X), regulator (Y) (which itself is a protein, perhaps an enzyme) and morphogen (Z) taking into account the self and cross-diffusion of morphogen (Z).

$$\frac{\delta\vec{\Phi}}{\delta t} = F(\vec{\Phi}) + \gamma_3 H(\vec{\Phi}) + (D + \eta\theta D^1)\Delta^2\vec{\Phi} \qquad (1)$$

with zero-flux boundary condition. We shall consider a cylindrically shaped embryo as a model. Therefore Δ^2 denotes the Laplace operator in the cylindrical coordinates. Here ρ and $\gamma_i > 0$ $(i = 1,2,3)$ are the system parameters and ρ is the Hill coefficient, η is a real parameter and θ is the imposed gradient.

$$\vec{\Phi} = (x,y,z)^T, F(\vec{\Phi}) = \left(\frac{1}{1+y^p} - \gamma_1 x, x - \gamma_2 y, y\right)^T H(\vec{\Phi}) = (0,0-z)^T,$$

$D = (a_{ij})$; $i,j = 1,2,3$; where all a_{ij} are zero except $a_{23} = D_{23} \neq 0$ and $a_{33} = D_{33} \neq 0$. $D^1 = (b_{ij})$ $i,j = 1,2,3$; where all b_{ij} are zero except $b_{33} = D_{33} \neq 0$.

INHOMOGENEOUS CONCENTRATION PATTERNS IN CYLINDRICAL COORDINATES

When the imposed gradient $\theta = 0$, (1) becomes

$$\frac{\delta\vec{\Phi}}{\delta t} = F(\vec{\Phi}) + \gamma_3 H(\vec{\Phi}) + D\Delta^2\vec{\Phi} \qquad (2)$$

with zero-flux boundary condition. The inhomogeneous concentration patterns emerge as a solution of

$$\delta F(\omega_0)U + \gamma_3 \delta H(\omega_0)U + D\Delta^2 U = 0 \qquad (3)$$

Here $\omega_0 = (x_0,y_0,z_0)$ is the steady-state. (3) has a non-trivial solution $U(r) \neq 0$ when

$$\det[\delta F(\omega_0) + \gamma_3 \delta H(\omega_0) - k_{nkj}^2 D_{33}] = 0 \qquad (4)$$

which implies

$$\gamma_3 = \gamma_3^c = K_{nkj}^2 D_{33}\left[\frac{\sigma}{\gamma_2(1+\gamma_1\gamma_2\rho y_0^{\rho+1})} - 1\right] \qquad (5)$$

where $D_{23} = -D_{23}^1$ as $D_{23}^1 > 0$ and

$$\sigma = \frac{D_{23}^1}{D_{33}}$$

(5) represents the line of bifurcation below which the system is stable and above unstable . Thus Turing (Turing, 1952) structure is possible when $\gamma_3 \geq \gamma_3^C$. Solutions of (3) is given by

$$U_L = \nu_{nkj}\delta^T \cos(k\pi/h) z J_n(x_{nj} r/R)\cos n\phi \qquad (7)$$

ν_{nkj} and δ^T can be found from $< U_L, U_L > = 1$ and $[\delta F(\omega_0) + \gamma_3 \gamma\delta H(\omega_0) - k_{nkj}^2 D]\,\delta = 0$. Here h and R are the height and radius of the embryo. For $n = 0, k = 0$ and $j = 3$, we have a real pattern of 3-germ layers ectoderm, mesoderm and endoderm as is known to actually occur in gastrula stage of embryogenesis of large number of species (see figure 1).

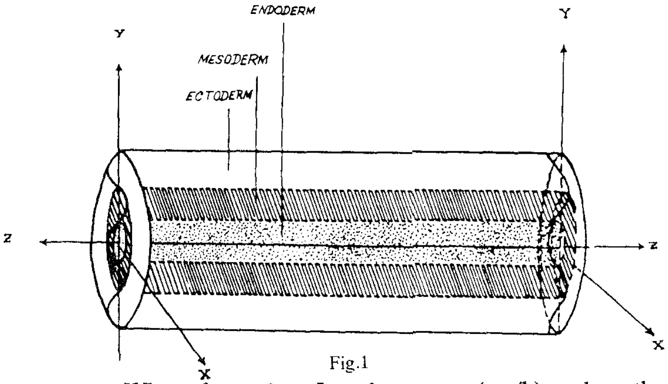

Fig.1

When $\theta \neq 0$. Let $\theta = \cos(\alpha\pi/h)z$, then the inhomogeneous concentration pattern emerges as a solution of

$$\delta F(\omega_0)U_L + \gamma_3^c \delta H(\omega_0)U_L + (D+\eta\theta D^1)\Delta^2 U_L = 0 \tag{8}$$

The solution of (8) is

$$\begin{aligned} U_L = {} & v_{nkj}\,\delta^T \cos(k\pi/h)z\, J_n(x_{nj}r/R)\cos n\phi \\ & + \eta a^T \cos((k-\alpha)\pi/h)z\, J_n(x_{nj}r/R)\cos n\phi \\ & + \eta b^T \cos((k+\alpha)\pi/h)z\, J_n(x_{nj}r/R)\cos n\phi \end{aligned} \tag{9}$$

The above solution is obtained by using the method developed by Hunding and Brons (1990). Now for $\nu = 0, k = 0$, $j = 3$ and $\alpha = 1$ we have the following pattern (see figure 2). Due to the imposed gradient, the 3-germ layers ectoderm mesoderm and endoderm tend to vanish at or nearer the posterior end.

For $n = 0, k = 0, j = 3$ and $\alpha = 2$ we have the following pattern (see figure 3). Due to

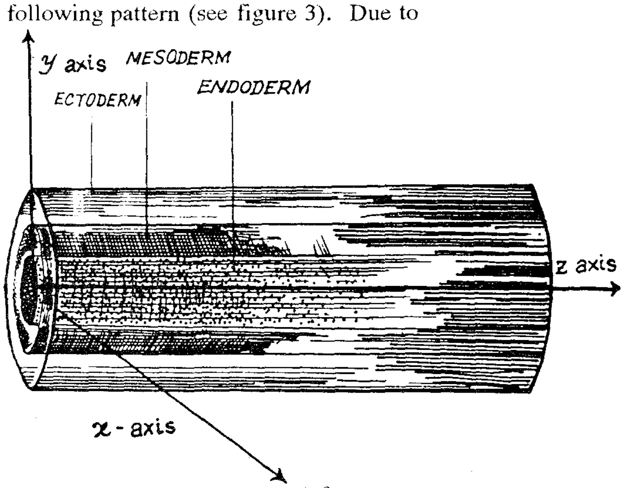

Fig.2

imposed gradient the triple structure ectoderm, mesoderm and endoderm vanishes just at the middle of the embryo.

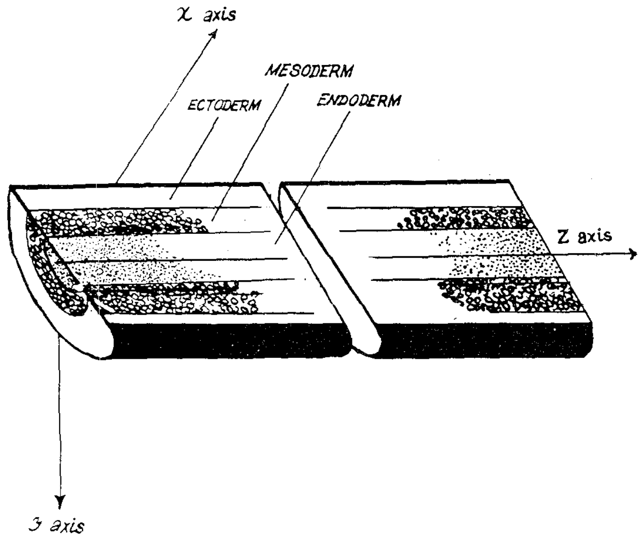

Fig.3

DISCUSSIONS

Main points are now summarised :

(a) There exists two gradient systems, namely, an imposed gradient θ in the form of signal and the local concentration gradient of the morphogen.

(b) It has been observed that due to the effect of imposed gradient θ a repetition of a pattern may occur along the anterior-posterior axis of the cylindrically shaped embryo.

REFERENCES

[1] Hundign, A and Brons, M. (1990) : "Bifurcation In A Spherical Reaction diffusion Systems with Imposed Gradient "Physica D, 44, 285-302.

[2] Tapaswi, PK and Saha, AK. (1986): "Pattern Formation and Morphogenesis A Reaction Diffusion Model" Bull. Math. Biol., 48, 213-228.

[3] Turing, AM (1952) : "The Chemical Basis of Morphogenesis" Phil. Trans. R. Soc., B 237, 37-72.