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Voluntary Contribution to Public Goods : A Note on Consistent Conjectures and Other Equilibria

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The paper generalizes contemporary work to prove the existence of a *consistent conjectures* equilibrium *(CCE)* with public goods involving smaller contributions than a *Nash* equilibrium *(NE)*. It also points out the possibility of an identification problem associated with the difficulty of isolating a *CCE* from an *NE* and a *Stackelberg* equilibrium. All the cases under consideration involve *complete* or *full* free-riding by a set of n-1 agents with unrestricted preferences over the n-th one, for whom the preferences may or may not be restricted.

JEL classification : H41

1. Introduction

One of the most exacting concepts to be applied to the study of voluntary contribution to public goods is the notion of a *consistent conjectures equilibrium (CCE)*. Following the lead given by Bresnahan (1981) and Perry (1982) in the context of oligopoly theory, Cornes and Sandler (1986, p.154) describe the *CCE* as "...an equilibrium in whose neighborhood every agent's conjectural variation ...is identical to the optimal actual response of the other agents." In the industrial organization literature, the notion of a *CCE* was the outcome of a dissatisfaction with the nature of extreme ignorance characterizing firm behaviour under *Nash* conjectures. It was felt that when the number of firms is neither too large nor small, "...each...can anticipate that its price or quantity decisions may call forth a response from rivals."[1] A similar sentiment was expressed in the context of public goods theory by Cornes and Sandler (1985, p. 125) : "If, however, information is incomplete and players gradually learn about their economic environment as they repeatedly make allocation decisions and observe outcomes, then the assumption that each takes the behaviour of others as unresponsive to his own optimizing response is clearly a very special case."

Sugden (1985, p. 123) argued on the other hand that the idea of learning behaviour, when pushed to its logical limit, viz. to the *CCE* concept, produces "...the conclusion that no public good would ever be supplied in significant amounts by voluntary contributions." While Sugden was correct in his claim that a *CCE* would normally lead to under-provision when compared to a *Nash* equilibrium *(NE)*, Cornes and Sandler (1986, p. 155)) pointed out that "The general properties of such equilibria are problematic..." unless "...we are prepared to impose extra structure on the model."

Bordignon (1993) succeeded in deriving a few of these properties for a *CCE*. The most striking of his results is that the symmetric *CCE* commonly assumed for a *homogeneous* agents model is unlikely to exist under fairly reasonable assumptions. He proves, moreover, the existence of an asymmetric *CCE* for a world of two homogeneous agents displaying zero income effects with respect to the public good and one free-riding *completely* over the other.[2]

The present exercise begins with a generalization of the last mentioned of Bordignon's results to a *heterogeneous* n-agents world. In particular, it is shown that there exists a *CCE* involving complete free-riding by n-1 agents with unrestricted preferences over the n-th one for whom the public good is *non-normal*, though not necessarily inferior. The result establishes the robustness of Bordignon's claim to the extent that it requires only *one* agent (rather than *all*) to have retricted preferences and permits the cardinality of the set of agents to be *any* finite number. When restricted to a 2-agent economy, the *CCE* in question is seen to coincide with a *Stackelberg* equilibrium *(SE)* in which the agent for whom the public good is normal can act as a leader to *completely* free-ride over the one for whom it is not so.

It is also proved below that for the same model, the Sugden conclusion of under-provision is reversed, the total provision of the public good being strictly greater in the free-riding *CCE* described above as compared to an *NE*[3] in which at least one of the agents who free-rides in the *CCE* contributes a nonzero quantity and the public good is *strictly* inferior for the n-th agent.[4] It is common practice of course to suppose that a public good must be *normal* to hold any interest for an economist. Comment 5 below argues, however, that there are interesting cases where a public good may in fact be *inferior*.[5] Finally, the paper provides a sufficient condition under which a boundary *CCE* coincides allocationwise with an *NE* as well as an *SE*. The coincidence of the different types of equilibria in all the two agent settings discussed in this note give rise to obvious problems of identification.

2. *Complete* free-riding as opposed to an *incomplete* one refers to a boundary situation where the free-riding agents do not contribute *at all*. The ditinction has its origin in Cornes and Sandler (1984-b).

3. That is, *Nash* equilibrium under *Nash* conjectures.

4. This does not contradict Sugden's claim which was restricted to normal goods alone. It should be noted here that Cornes and Sandler (1984, 1985) were the first to make a case for improved contributions to public goods under non-*Nash* conjectures.

5. In the context of a voluntary contributions model, the theoretical implications of inferior public goods were discussed in the seminal paper by Chamberlin (1974). Sugden (1985) had also brought up the possiblility.

2. The Model

Individual i is assumed to maximize the utility function

$$U^i(G, x_i) = U^i(g_i + G_{-i}, x_i) \qquad i=1, 2, \ldots, n \tag{1}$$
$$U^i_j > 0, \qquad j=1, 2$$

subject to a budget (alternatively, technical) constraint

$$pg_i + x_i = w_i, \tag{2}$$

where G, G_{-i} and g_i represent respectively the total contribution, aggregate contribution by all agents except i and individual i's contribution to the public good, x_i represents i's consumption of the private good, w_i stands for his wealth and p refers to the relative price (opportunity cost) of the public good. In what follows, we shall assume that $p=1$. Substitution of (2) into (1) yields

$$U^i(g_i + G_{-i}, w_i - g_i), \qquad i=1, 2, \ldots n. \tag{1'}$$

Maximization of (1) subject to (2) is then equivalent to a simple maximization of (1'). Taking care of the possibility of boundary optima, i.e. of free-riding, the relevant first order condition *(FOC)* for i's equilibrium is

$$U^i_1(G, x_i)\left(1 + \frac{dG^e_{-i}}{dg_i}\right) - U^i_2(G, x_i) \leq 0 \tag{3}$$

where $\frac{dG^e_{-i}}{dg_i}$ denotes i's conjectural variation, i.e., i's expectation about the impact of a change in his contribution on that of the remaining agents. As opposed to this, their *optimal* response to a change in i's contribution may be written $\frac{dG^o_{-i}}{dg_i}$.

The vector $(g^*_1, x^*_1; g^*_2, x^*_2; \ldots; g^*_n, x^*_n)$ constitutes a *CCE* if

$$(g^*_i, x^*_i) \text{ maximizes } U^i(g_i + G^*_{-i}, w_i - g_i) \text{ given } \frac{dG^e_{-i}}{dg_i}, \; i=1, 2, \ldots, n \tag{i}$$

and

$$\frac{dG^e_{-i}}{dg_i} = \frac{dG^0_{-i}}{dg_i} \text{ in a small neighbourhood of } (g^*_1, x^*_1; g^*_2, x^*_2; \ldots; g^*_n, x^*_n). \tag{ii}$$

Comment 1. As is well-known, an *NE* typically violates condition (ii).

3. A *CCE* with Complete Free-riding

Comment 2. Suppose the public good is ***non-normal*** for agent n. Then the slope of his *Nash* reaction curve $G_n(G_{-n})$ will be bounded above by -1 for all values of G_{-n},[6] *including* $G_{-n}=0$, at which the slope is interpreted as the right hand derivative $G_n'(0)^+$. If the public good is strictly inferior for agent n, then $G_n'(G_{-n})$ is strictly smaller than -1 everywhere. The comment leads to the following results.

Proposition 1. Suppose the public good is ***non-normal*** for agent n. Then there exists a *CCE* with agents 1, 2, ..., n-1 free-riding *completely* over agent n.

Proof. Let $\bar{g}_n=G_n(0)$ be agent n's ***standalone*** contribution.[7] Then $\Omega=(0,\ w_1\,;\ 0,\ w_2\,;\ \ldots\,;\ 0,\ w_{n-1}\,;\ \bar{g}_n,\ w_n-\bar{g}_n)$ is a *CCE* supported by the following conjectural variations in a local neighbourhood of the point $(g_1,\ g_2,\ \ldots,\ g_n)=(0,\ 0,\ \ldots,\ \bar{g}_n)$:

Agent i $$\frac{dG^e_{-i}}{dg_i}=G_n'(0)^+,\quad i=1,2,\ldots,n\text{-}1\,;$$

Agent n $$\frac{dG^e_{-n}}{dg_n}=0\,.$$

Differentiation of $U^i\,(g_i+G_{-i},\ w_i-g_i)$, $i=1,2,\ \ldots,\ n-1$ yields

$$U^i_1\Big(\,(1+\frac{dG^e_{-i}}{dg_i})-\frac{U^i_2}{U^i_1}\Big),$$

which, under the assumed conjectural variations, equals

$$U^i_1\ (1+G_n'(0)^+)-\frac{U^i_2}{U^i_1} \tag{4}$$

at $G_{-i}=\bar{g}_n$. By virtue of Comment 2 and the assumption of positive marginal utility for both goods, the expression in (4) is clearly negative. On the other hand, by the very definition of a *Nash* reaction function, agent n maximizes his satisfaction at $(g_n,\ x_n)=(\bar{g}_n,\ w_n-\bar{g}_n)$ given $G_{-n}=0$ and his conjectural variation. Hence, condition (i) of a *CCE* is satisfied.

To verify condition (ii) of a *CCE*, it is easy to see that agent n moves along his *Nash* reaction function for small variations in G_{-n} given that he entertains *Nash* conjectures. Hence, the beliefs of agents $i=1,\ 2,\ \ldots,\ n$-1 are vindicated. Similarly,

6. See, for example Cornes and Sandler (1984-a), Sugden (1985) or Dasgupta and Itaya (1992).

7. This, as well as the terminology "*complete crowding out* contribution" used in Proposition 2 below, are due to Varian (1994).

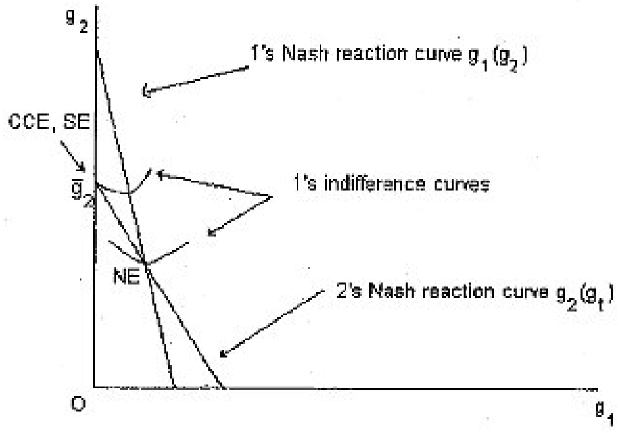

Figure 1

Proposition 1 : Public good non-normal for agent 2

given his conjectural variation, agent $i=1, 2, \ldots, n\text{-}1$ chooses $g_i=0$ in a small neighborhood of $G_{-i}=\bar{g}_{-n}$, since expression (4) continues to be negative. Hence, agent n's conjectures are consistent.□

Comment 3. Fig. 1 borrows a diagrammatic device from Cornes and Sandler (1986) to represent Proposition 1 for the case $n=2$. The diagram shows further that the equilibrium in question can also be viewed as an *SE* where agent *1*, acting as a leader, free-rides *completely* over agent *2*. The fact that the same equilibrium allocation may be looked upon as both a *CCE* and an *SE* implies the presence of an identification problem. Further, the public good being allowed to be *normal* for agent *1*, the result is comparable to that of Varian (1994), who showed that in an *SE*, an agent who likes the public good *most* might *completely* free-ride over an agent who likes it *least*. It is worth noting, however, that as opposed to the present framework, Varian's result required the public good to be *non-normal* for both agents.

Comment 4. Proposition 1 allows the preferences of agents $i=1, 2,\ldots, n\text{-}1$ to be perfectly arbitrary as far as their attitudes to the public good is concerned. Moreover, it is obvious that agent n's preferences too are less restrictive than what Proposition 1 might lead one to suspect, for the conclusion of the Proposition remains valid so long as

$$U_1^i\ ((1+G_n'(0)^+)-\frac{U_2^i}{U_1^i})<0$$

at $G_{-i}=\bar{g}_n$. Nonnormality of the public good for agent n is merely a sufficient condition for this inequality to be satisfied.

Corollary 1. Suppose that the public good is *strictly* inferior for agent n and the preferences of agents $i=1, 2, ..., n-1$ are such that at least one of them contributes a strictly positive quantity of the public good in an *NE*. Then the total contribution for this equilibrium is less than that for the *CCE* described above.

Proof. Let $g_n=g_n^*$ and $G_{-n}=G_{-n}^*$ in the *NE*. By assumption, $G_{-n}^* > 0$. Further, by definition of an *NE*, $g_n^*=G_n(G_{-n}^*)$. By Comment 2 however, $G_n'(\cdot) < -1$ when the public good is inferior for agent n. Hence, $\bar{g}_n=G_n(0)>g_n^*+G_{-n}^*$. □

Comment 5. Corollary 2 shows that there are circumstances where non-*Nash*, as opposed to *Nash*, behaviour leads to an improvement in total contributions.[8] It imposes the condition, however, that G is an *inferior* good for at least one agent in the model.[9] The restriction by itself is mild insofar as it is only *one* out of n agents that is required to have such preferences. Besides, examples of inferior public goods are not hard to find, as the following discussion indicates.

If both G and x are to be looked upon as macro aggregates of heterogeneous collections of public and private goods, it might be natural to conclude that each is *non-inferior*. A less aggregative approach would suggest a different viewpoint however, especially where side by side with a public good, a more expensive and superior variety of the commodity is available in a private form and the agent is concerned with an optimal allocation of his budget between the two. The obvious examples are public and private schools, public and private health care, communally hired security services and private security, etc..

The utility derived from G, viz., the public form of the commodity, is partly attributable to the fact that an agent gains by consuming it directly (by sending one's child to a public school, having one's family treated at a public hospital or property guarded by security personnel hired for the community as a whole, etc.). Yet another part, however, is in the nature of an external effect and derived indirectly

8. For $n=2$, the result is true whether one views the non-*Nash* equilibrium as a *CCE* or an *SE*. This can be verified from Fig. 1 by comparing the respective horizontal intercepts of the 45^0 lines through the *CCE* (cum *SE*) and *NE* equilibria, since they represent total contributions under the two regimes.

9. The preferences of the remaining agents are of course not restricted in any significant manner.

10. There is room for conflict here inasmuch as a communally hired watchman cannot attend to the security needs of an individual at the same time that he protects a neighbour's property. In fact, it is not uncommon for agents to make side payments, thereby attempting to privatize the *public* good! The idea extends *mutatis mutandis* to doctors working for the public health scheme or teachers employed by a public school.

from the fact that its consumption by *other* agents leads to an overall improvement in the quality of life (better educated or more healthy neighbours, safer precincts etc.).[10] With a rise in the level of private wealth, an agent is able to afford more of the private form of the commodity, viz. x, so that his demand for the latter goes up (greater dependence on the private physician, sending a second child to private school, etc.). This in turn makes redundant for him a part of G that had earlier been satisfying his direct needs. As a result, a utility maximizing agent would not only be demanding more of x but also *less* of G. In other words, G would qualify as an inferior public good.

It is instructive to consider the manner in which the inferiority of public goods manifests itself in the voluntary contributions context. An agent experiences an effective decrease in the price of the public good if his contribution to G and purchase of x remain unchanged in the face of a decision by others to increase their contributions. The reallocation caused by the improved liquidity leads him to demand a lesser amount of G than before if the latter is indeed inferior as argued above. Moreover, the greater the contribution by others, the greater the liquidity, and hence, the greater the negative income effect, quite in the spirit of the *Giffen* paradox. A reversal of the sequence of arguments provides now the intuition underlying Corollary 1.[11]

Corollary 2. Suppose the agents are identical and that their preferences display a zero income effect with respect to the public good. Then, the allocation Ω of Proposition 1 constitutes a *CCE*.

Proof. Since the preferences of agents 1, 2, ..., $n-1$ were unrestricted in Proposition 1, the result follows trivially.□

Comment 6. For $n=2$, Corollary 3 was proved by Bordignon (1993) who offered it as an example of an *asymmetric CCE* in a *homogeneous* agents model.

4. Coincidence of *CCE*, *NE* and *SE* : A Boundary Optimum

Comment 7. This section removes the restrictions imposed in the previous one on the nature of the public good and concentrates on the case where $n=2$. This is necessary to facilitate comparisons with an *SE*.

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11. The following excerpt from the editorial (dated September 28, 1994) of The Times of India, a daily with one of the largest circulations in the country, confirms the point: "Over the last decade and a half, the private health care sector has grown rapidly while the public, nonprofit sector has languished due to paucity of funds. In normal times and for nonemergency cases, private hospitals are being increasingly preferred because patients believe they can secure better care ..."

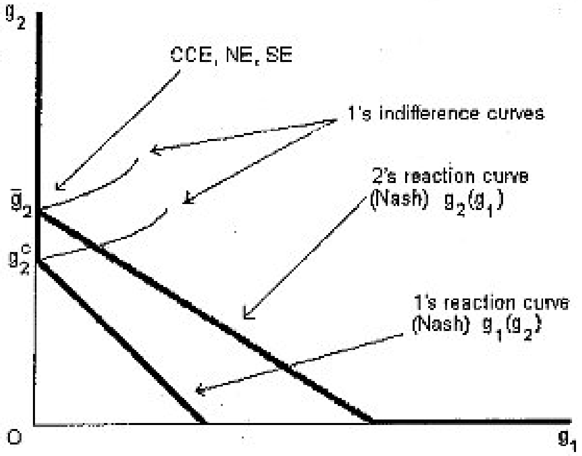

Figure 2

Proposition 2 : Public good not restricted to be non-normal for any agent

Let g_2^c stand for the *complete crowding out* contribution by agent *2*, i.e., it represents the minimum contribution of agent *2* that drives down agent *1*'s contribution to zero under *Nash* conjectures. So long as the private good is normal (which is assumed), $G_1(g_2)=0$ for all $g_2 \geq g_2^c$. A second propossition follows.

Proposition 2. Suppose $\bar{g}_2 > g_2^c$. Then, $\Omega' = (0, w_1 ; \bar{g}_2, w_2 - \bar{g}_2)$ is simultaneously an *NE*, an *SE* and a *CCE*.

Proof. Under the assumed condition, the point $(0, \bar{g}_2)$ represents the intersection of two *Nash* reaction curves in the $g_1 - g_2$ plane (See Figure 2). Consequently, Ω' is an *NE*. Hence, by Fact 3 of Varian (1994), Ω' is an *SE* also.

To see that Ω' is a *CCE*, note first of all that the slope of an indifference curve for agent *1* in the $g_1 - g_2$ plane is[12]

$$\frac{dg_2}{dg_1} = \frac{U_2^1}{U_1^1} - 1 \ .$$

This is true in particular at the boundary point $(0, g_2^c)$, where the slope is interpreted as the right hand derivative $(\frac{dg_2}{dg_1})^+$. By definition of g_2^c, however, $(\frac{dg_2}{dg_1})^+$ is no less than 0 for all $(0, g_2)$ such that $g_2 \geq g_2^c$. This follows since the vertical section above g_2^c forms the degenerate part of agent *1*'s *Nash* reaction curve (i.e., the one that involves a zero contribution by the agent). Hence,

12. See Cornes and Sandler (1986), p.72.

$$U_1^1((1+G_2'(0)^+)-\frac{U_2^1}{U_1^1}<0 \qquad (5)$$

at $g_2=\bar{g}_2$. The arguments of Proposition 1 may now be applied *mutatis mutandis* to see that the conjectural variations assumed there would also support Ω' as a *CCE*. □

Comment 8. Proposition 2, while guaranteeing the allocationwise coincidence of a *CCE* and an *NE*, does not imply a coincidence of the supporting conjectures. The latter can be shown to be trivially true when the utility functions involve zero income effects with respect to the private good.

Comment 9. Proposition 2 establishes the possibility of an identification problem regarding the nature of the voluntary contributions equilibrium even in the absence of restrictions on the utility functions.

5. Conclusion

The note obtains somewhat dramatic characteristics of voluntary contributions equilibria. These involve (i) a reversal of the well-known result that a *CCE* involves smaller contributions than an *NE* and (ii) an identification problem associated with the difficulty of isolating a *CCE* from other types of allocations. As the quote from Cornes and Sandler (1986) in the Introduction indicates, relatively little is known about the general properties of a *CCE* with public goods. The cases studied here might provide some insights into these.

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