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Sediment-induced stratification in turbulent open-channel flow

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SUMMARY

A vertical gradient of suspended sediment concentration exists in a turbulent open channel flow, particularly near the bed where sediment erosion and deposition take place. This shows a remarkable effect on the flow dynamics. The density gradient of sediment-mixed fluid may become stably stratified, which results in damping of turbulence fluctuations. In this work, theoretical models for mean velocity and concentration distributions have been developed considering the effect of sediment-induced stratification and the modified mixing length due to high suspension together with viscous and turbulent shear stresses, which are the functions of concentration. The models are compared with comprehensive experimental data sets. The comparison reveals that (i) the calculated velocity and concentration profiles agree well with the observed data, (ii) the model constant due to stratification used for verification is consistent with the measurements in thermally stratified flows, and (iii) the higher the sediment suspension, the better the effect of density stratification and the less the impact of mixing length.

KEY WORDS: sediment suspension; stratification; viscosity; turbulent shear stress; Richardson number; velocity and concentration profiles

1. INTRODUCTION

In the hydraulic open channel flow of a clear fluid the vertical velocity profile is usually described by a log-law to the near wall region, and the deviation from log-law in the outer region can be expressed by adding Coles' wake function (1956), which depends on the Reynolds number. Researchers like Nezu and Rodi (1986), Cardoso *et al*. (1989) and Mazumder and Bandyopadhyaya (2001) have suggested that the standard log-law could be adjusted by choosing a suitable function known as the 'wake strength function'. Various investigations related to velocity and suspension concentration distributions have been undertaken in laboratory experiments by many authors, such as Vanoni (1946), Einstein and Chien (1955), Coleman (1981), Woo *et al*. (1988), Umeyama and Gerritsen (1992), Mazumder (1994) and others. Recently, Mazumder and Ghoshal (2002) developed theoretical models for velocity based on Prandtl's momentum transfer theory and sediment suspension concentration taking into account the viscous and turbulent shear stresses, which are the functions of volumetric concentration.

However, the above-mentioned models are not concerned with the damping effect of turbulence due to stratification of suspended sediments. When flow velocity suspends sediments, the vertical gradient of sediment concentration gives rise to a vertical gradient of density. This may affect the flow dynamics significantly to cause the near-bottom vertical velocity to deviate from its usual log-law. If the density decreases with height, the fluid is gravitationally stable and the work must be done on the fluid to mix it, in order to raise the potential energy. A vertical gradient of suspended concentration often exits in estuaries and lakes, particularly near the bottom, where the strong currents can cause erosion and deposition of bottom sediments generating a vertical stratification in suspended sediment, which can affect the flow. The problems of sediment-laden flows are also of direct interest to the field of marine and riverine sedimentation, coastal sediment transport and two-phase flow. Smith and McLean (1977) first introduced the role of turbulence damping due to the density stratification produced by the suspended sediment itself, which was assumed as an analogy with the atmospheric boundary layer, where density gradient is caused by vertical gradient of air temperature. Later, Gelfenbaum and Smith (1986) applied this technique to the velocity and sediment data of Vanoni (1946) and Einstein and Chien (1955). Soulsby and Wainwright (1987) summarized the structure of the stratification effects for a range of sediment sizes and the conditions of flow velocity; and provided a method for estimating the friction velocity u_* from the velocity data which takes into account the suspended sediment effects. Sheng and Villaret (1989) presented a simplified second-order turbulent closure model based on comprehensive Reynolds stress equations to examine the role of sediment-flow interaction in boundary layer dynamics for erosion and deposition. McLean (1991), Villaret and Trowbridge (1991) and Ghoshal and Mazumder (2003) studied the effects of density stratification on mean velocity and suspended sediment concentration in laboratory channels, in order to test the applicability of the stratified flow analogy to sediment-laden turbulent flow of water.

When the current carries a large amount of sediments in suspension, it affects the viscous and turbulent shear stresses due to the changes of viscosity and density of the fluid–sediment mixture. In order to develop theoretical models for mean velocity and suspension concentration, it is desirable to take into account the modified viscous and turbulent shear stresses in the formulation in addition to the effect of density stratification and the modified mixing length due to high suspension. The objectives of the present investigation are to determine the equations of mean velocity and suspension concentration for uniform open-channel flow taking into these physical effects, and to verify the predicted values with the experimental data. Comparison shows good agreement with the observed data.

2. VELOCITY DISTRIBUTION

We consider two-dimensional sediment-laden turbulent flow in an open channel with the origin of co-ordinates on the bed, where the x-axis is along the flow, the y-axis is perpendicular to the flow and h is the depth of water. The total shear stress τ for turbulent flow derived from the Reynolds stress equation is written as

$$\tau = \mu \frac{\mathrm{d}u}{\mathrm{d}y} + \rho \epsilon_m \frac{\mathrm{d}u}{\mathrm{d}y} \tag{1}$$

where u is the mean velocity parallel to the wall, μ is the coefficient of viscosity, ρ is the net density of the fluid–sediment mixture and ϵ_m is the momentum diffusion coefficient. The first term and second term of the right hand side of (1) represent, respectively, the viscous shear stress and Reynolds shear stress in the x-direction. When the turbulent flow carries high sediment in suspension over the erodible sand bed, the density of the mixture may be defined as $\rho = \rho_f + (\rho_s - \rho_f)C = \rho_f[1 + AC]$, where ρ_f is the density of clear fluid, ρ_s is the density of sediment, $A(= \frac{\rho_s}{\rho_f} - 1)$ is the relative density and C is the volume of sediment per volume of the water–sediment mixture. The coefficient of viscosity μ of the sediment–fluid mixture is a function volumetric concentration C and is given by Coleman (1981) as

$$\mu = \mu_f(1 + 2.5C + 6.25C^2 + 15.62C^3) = \mu_f g(C) \tag{2}$$

where μ_f is the dynamic viscosity of clear water. Using (2) in (1), the total shear stress in the sediment-laden turbulent flow is written as

$$\tau = \mu_f g(C)\frac{\mathrm{d}u}{\mathrm{d}y} + \rho_f(1 + AC)\epsilon_m\frac{\mathrm{d}u}{\mathrm{d}y} \tag{3}$$

The vertical distribution of total shear stress τ can be written as a function of dimensionless depth $\xi = y/h$ and is given by

$$\tau = \rho_f u_*^2\left[1 - \xi + A\int_\xi^1 C\mathrm{d}\xi\right] \tag{4}$$

Combining (3) and (4), the expression for velocity gradient $\frac{\mathrm{d}u}{\mathrm{d}\xi}$ explicitly yields

$$\frac{[(1 + AC)\epsilon_m + \nu_f g(C)]}{hu_*^2[1 - \xi + A\int_\xi^1 Cd\xi]}\frac{\mathrm{d}u}{\mathrm{d}\xi} = 1 \tag{5}$$

In order to evaluate the applicability of the stratified flow analogy to the sediment mixed turbulent flow, we have used a theoretical framework in which the suspended particles stratify the flow and have an influence analogous to that of a downward heat flux in the stably stratified atmospheric surface layer. Following Smith and McLean (1977), a relationship between the constant stress layer and a gradient Richardson number R_i has been assumed in the stably stratified boundary layer; and it is extended to a sufficiently large value of y using the Coles' strength parameter Π as

$$(1 - \gamma\beta_* R_i)\frac{\mathrm{d}u}{\mathrm{d}\xi} = \frac{u_*}{l_*} + \frac{\Pi}{\kappa}\pi u_*\sin(\pi\xi) \tag{6}$$

where l_* is the mixing length, κ is the von-Karman constant, γ is the ratio of the sediment diffusion coefficient to the momentum diffusion coefficient of water, β_* is a constant found to be 4.7 ± 0.5 by Businger *et al*. (1971) from the data of the Kansas experiment, and R_i is the gradient Richardson number, defined as the ratio of buoyant production to shear production of turbulent kinetic energy, and is given by

$$R_i = \frac{-(\rho_s - \rho_f)gh}{\rho_f}\frac{\mathrm{d}C}{\mathrm{d}\xi}\left(\frac{\mathrm{d}u}{\mathrm{d}\xi}\right)^{-2} \tag{7}$$

When $R_i = 0$, the fluid is neutrally stratified, and for $R_i \geq 0.25$, the turbulence production in fluid is completely diminished (Tennekes and Lumley, 1980). According to Umeyama and Gerritsen (1992), the mixing length l_* is modified for the entire boundary layer thickness, which is a function of concentration, and it yields

$$l_* = \kappa\xi(1-\xi)^{\frac{1}{2}\left(1+\beta\frac{C}{C_{\xi_a}}\right)} \tag{8}$$

where C_{ξ_a} is the concentration at the reference level $\xi = \xi_a$, and the constant β is determined from the experimental data. Eliminating the velocity gradient $\frac{du}{d\xi}$ from (5) and (6), we obtain the momentum diffusion coefficient ϵ_m as

$$\epsilon_m = \frac{1}{1+AC}\left[\frac{\left(1-\xi+A\int_\xi^1 C\mathrm{d}\xi\right) h\, u_*\, l_*(1-\gamma\beta_* R_i)}{1+l_*\frac{\Pi}{\kappa}\pi\sin(\pi\xi)} - \nu_f g(C)\right] \tag{9}$$

Using (7) and (8) in (6), the velocity gradient can be rewritten as

$$\frac{\mathrm{d}u}{\mathrm{d}\xi} = \frac{u_*}{\kappa\xi(1-\xi)^{\frac{1}{2}\left(1+\beta\frac{C}{C_{\xi_a}}\right)}}\left\{1+\xi(1-\xi)^{\frac{1}{2}\left(1+\beta\frac{C}{C_{\xi_a}}\right)}\Pi\pi\sin(\pi\xi)\right\}\left[1+\gamma\beta_* Agh\frac{\mathrm{d}C}{\mathrm{d}\xi}\left(\frac{\mathrm{d}u}{\mathrm{d}\xi}\right)^{-2}\right]^{-1} \tag{10}$$

It is observed that the derived Equation (10) for the velocity gradient is inaccessible at the free surface $\xi = 1$. The validity of the equation at the free surface is not important, because the available observed data are limited to only $\xi < 1$, so we have not considered the region immediately adjacent to the free surface. In fact, the observed maximum flow velocity generally occurs somewhere in the flow below the free surface $\xi < 1$. Such a phenomenon may be attributed to secondary circulation or some other effect on the free surface.

3. SUSPENDED SEDIMENT CONCENTRATION DISTRIBUTION

In a steady and uniform two-dimensional sediment-laden turbulent flow, where the concentration is constant in time and varies only with vertical co-ordinate y, the vertical distribution of suspended sediment concentration C with particle settling velocity W could be obtained (Hunt, 1954) as

$$\epsilon_s\frac{\partial C}{\partial y} + C\frac{\partial C}{\partial y}(\epsilon_m - \epsilon_s) + (1-C)CW = 0 \tag{11}$$

where ϵ_s and ϵ_m are the diffusion coefficients of sediment and water respectively. Equation (11) satisfies the continuity condition of sediment and water, which is derived for the volumetric concentration of the sediment. The solution of (11) is simplified if the diffusion coefficients of sediment and water are assumed to be equal. The equality of ϵ_s and ϵ_m is not strictly accurate but is a close approximation for low concentration. If the density of the fluid mixture is increased by sediments, buoyancy force is increased and hence the substantial reduction of particle settling velocity in suspension occurs. The effective settling velocity of the sediment W in sediment-laden flow varies with the volumetric concentration C as a result of hindered settling (Fredsoe and Deigaard, 1992), obtained as

$$W = w_0(1 - C)^{\alpha} \tag{12}$$

where w_0 is the fall velocity of a single grain in a clear fluid and α is the exponent of reduction of fall velocity, which depends on the particle Reynolds number R_g as: $\alpha = 4.35\ R_g^{-0.03}$ for $0.2 \le R_g \le 1.0; \alpha = 4.45\ R_g^{-0.01}$ for $1.0 \le R_g \le 500; \alpha = 2.39$ for $500 \le R_g$. The validity of (12) has been tested in modelling the reduction of particle fall velocity in sediment-laden flows by several investigators (Woo *et al.*, 1988; Ni and Wang, 1991; Mazumder, 1994). The momentum diffusion coefficient ϵ_m in a fluid–particle mixture is related to sediment diffusion coefficient, ϵ_s by

$$\epsilon_s = \gamma \epsilon_m \tag{13}$$

where the ratio γ has to be determined from the experimental conditions. Using (12) and (13) in (11), the expression for $\frac{\mathrm{d}C}{\mathrm{d}\xi}$ can be written as

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{-hC(1 - C)^{\alpha+1} w_0}{\epsilon_m[\gamma + (1 - \gamma)C]} \tag{14}$$

where the ϵ_m is given by (9). This is the desired expression for the concentration field which is coupled with the velocity field through the expression of eddy diffusivity ϵ_m, in which the effect of stratification due to suspended sediments is present.

In order to solve the coupled Equations (10) and (14) by the Runge-Kutta method for velocity and concentration simultaneously, it is necessary to provide the required boundary conditions for u and C at the reference level ξ_a as

$$u = u_{\xi_a} \text{ and } C = C_{\xi_a} \text{ at } \xi = \xi_a \tag{15}$$

The precise location of the reference level is arbitrary. To solve coupled differential Equations (10) and (14), we omit the term $A \int_{\xi}^{1} C\mathrm{d}\xi$, as inclusion of this term makes the problem too complicated. At the first step we assume $\beta_* = 0$ (without stratification) in both Equations (10) and (14) to obtain the first estimates of $\mathrm{d}u/\mathrm{d}\xi$ and $\mathrm{d}C/\mathrm{d}\xi$ respectively. Next, these two values are used in the right hand sides of (10) and (14) for the new estimates of velocity and concentration gradients. The free parameters β and γ are adjusted for a good agreement between theoretical and experimental results.

4. COMPARISON WITH EXPERIMENTAL DATA

The theoretical models for the mean velocity and suspension concentration for sediment-laden turbulent flows are compared with the most comprehensive experimental data of Vanoni (1946), Einstein and Chien (1955), Coleman (1981) and Lyn (1986). In this work, comparison with the data sets of Einstein and Chien (1955) is shown only; the detailed comparisons of other data sets are available in Ghoshal (2004). These data sets were collected under controlled conditions over plane sediments beds in laboratory channels, and these are most frequently used in the literature of sediment transport. In order to compute the velocity and concentration profiles from (10) and (14) based on the conditions (15), it is important to know the empirical constants $\kappa, \Pi, \beta_*, \beta$ and γ. Here we have used the von-Karman constant $\kappa = 0.4$ and the wake strength parameter $\Pi = 0.2$ under fully developed turbulent flow conditions (Nezu and Rodi, 1986), and for the stably stratified flow, the empirical constant $\beta_* = 4.7$ is taken from Businger *et al.* (1971) for our computational purpose. The specific gravity of sediments used for all the experiments is 2.65. The quantities β and γ are the free parameters estimated by adjusting to best fit with the observed velocity and concentration data.

4.1. Einstein and Chien's data (1955)

Experiments were conducted in a steel recirculating flume which was 35.7 cm deep, 30.7 cm wide and 120 m long. The sizes of the mean sediments used in the experiments were the fine sands ($D_{50} = 0.274$ mm) for runs S-11 to S-16, medium sands ($D_{50} = 0.94$ mm) for runs S-6 to S-10 and coarse sands ($D_{50} = 1.30$ mm) for runs S-1 to S-5. The sediment concentration near the bed was in the range 30–600 gm/lit. The velocity distribution was measured at 25–31 vertical points between one-third and one-half of the depth to the flume bed. The comparison of computed velocity and suspended sediment concentration profiles, respectively, with six observed data (runs S-11 to S-16) of Einstein and Chien (1955) for fine sands ($D_{50} = 0.274$ mm) has been depicted in Figures 1(a,b) using the reference velocity u_{ξ_a} and concentration C_{ξ_a} at the lowest elevation ξ_a from the bed. Figures 2(a,b) show the velocity and suspension concentration profiles, respectively, for five observed data (runs S-6 and S-10) for medium sands ($D_{50} = 0.94$ mm). Similar comparison for velocity and concentration with observed data (runs S-1 to S-3) for coarse sands ($D_{50} = 1.30$ mm) has been made in Figures 3(a,b). The fitted values of β and γ for these runs are summarized in Table 1. It is observed from Figures 1(a,b) that favorable agreement between the theoretical and observed values occurs using stratification due to suspended sediment, but in Figures 2(a) and 3(a) the fit to the velocity is perfect using stratification effect except near the bed for all the runs. The fit to the concentration profiles is more or less improved by adding stratification for medium and coarse sands (Figures 2b and 3b). However, overall, quite good agreement is observed between the computed and observed data for all sand sizes (fine, medium and coarse) including the stratification effect. Moreover, it is assessed from the table that the estimated value of β is 10 for the runs of fine grained sediments (S-12 to S-16) and in the range 15–18 for the runs (S-7 to S-10) of medium sands except for the runs S-11 and S-6, where the value of β is 1. The value of β is 8 for coarse grained sediments (S-1 to S-3). Furthermore, the value of γ increases consistently with increase of grain sizes—fine to coarse grained sediments (Kaushal *et al*., 2002).

The calculated velocity and concentration profiles with and without stratification effect are plotted against the measured data of Vanoni (1946), Einstein and Chien (1955), Coleman (1981) and Lyn (1986) in Figures 4(a,b). The fitted values of β and γ for other experiments are also given in Table 1. In comparing the measured and computed velocity in Figure 4(a), it shows that the inclusion of stratification due to suspended sediment in the formulation yields a significant effect. From Figure 4(b), it is also noted that, although the data are scattered for both stratification and non-stratification cases, the results due to stratification in concentration profiles show better matching between the measured and computed data. The flux Richardson number R_i against height ξ above the bottom is plotted in Figure 5 for runs S-11 to S-16 of Einstein and Chien (1955), where the stratification effect is important. It is observed from the figures that R_i increases with height and tends to zero near the free surface. The value of R_i reaches to a maximum in the region $0 \leq \xi \leq 0.2$. Figures 6(a,b) exhibit similar profiles of R_i with height ξ for various runs from Vanoni (1946) and Coleman (1981).

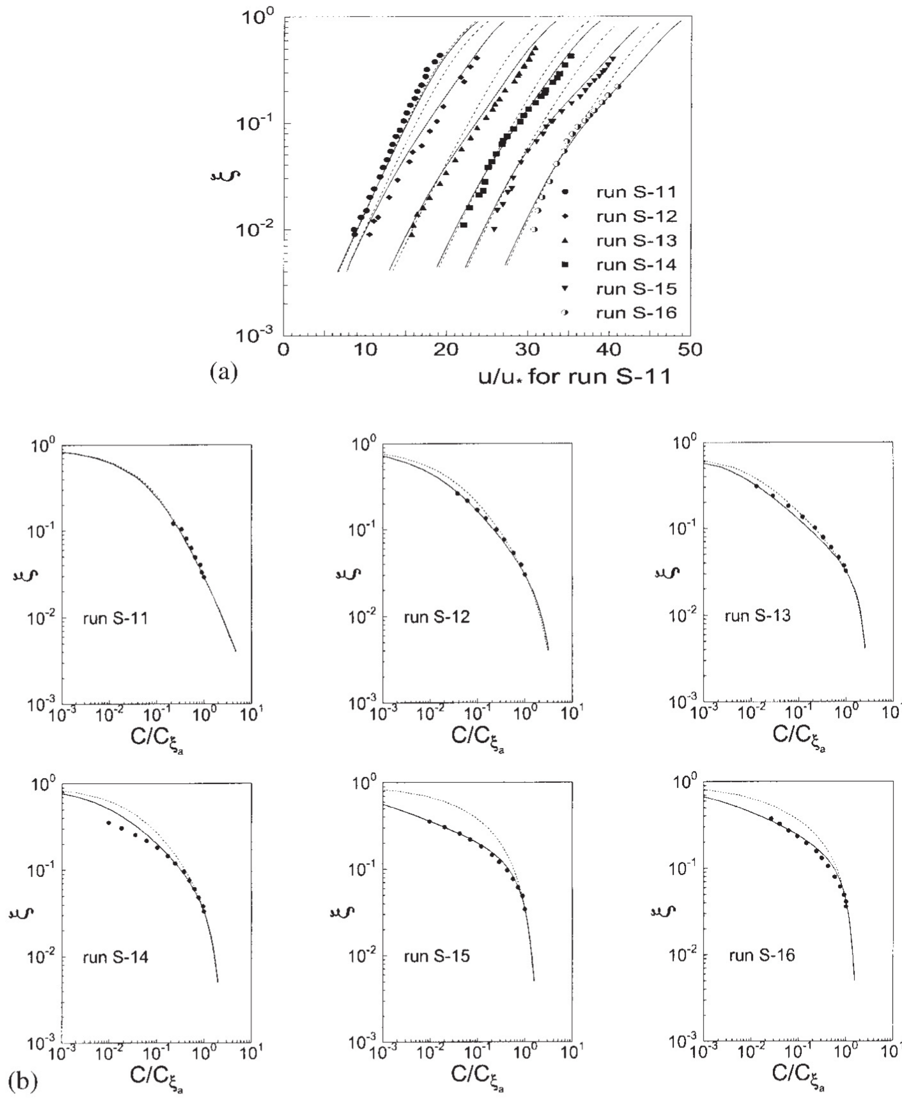

Figure 1. Comparison of calculated and measured velocity and concentration for the runs S-11 to S-16 of Einstein and Chien (1955) for fine sands ($D_{50} = 0.274$ mm); the continuous line includes stratification effect, the dashed line ignores the stratification effect and the symbols represent the experimental data

5. ESTIMATION OF NEAR-BED VELOCITY AND CONCENTRATION

In solving the coupled differential Equations (10) and (14), we assumed in the earlier section the known velocity $u(\xi_a)$ and the concentration $C(\xi_a)$ at an available reference level $\xi = \xi_a$. The determination of u and C at an arbitrary reference level $\xi = \xi_a$ as a function of flow characteristics is a difficult problem. However, besides using the known velocity and concentration at an arbitrary

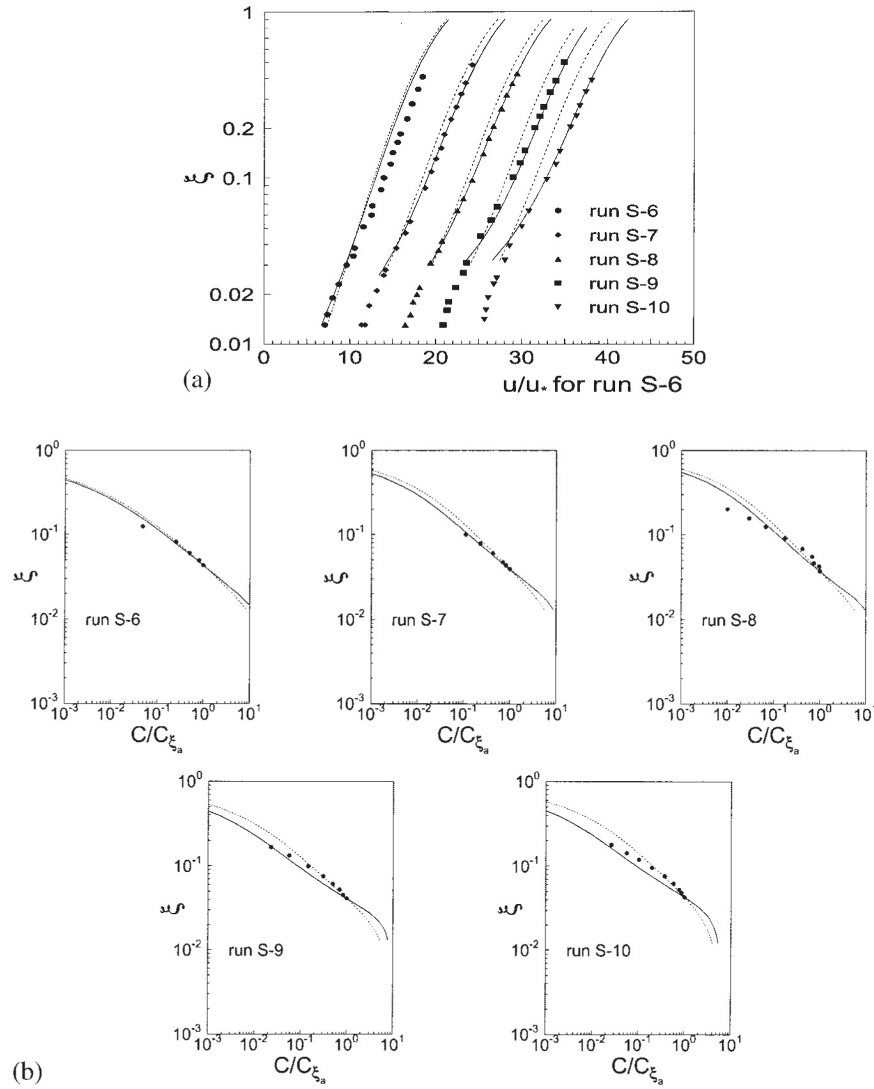

Figure 2. Comparison of calculated and measured velocity and concentration for the runs S-6 and S-10 of Einstein and Chien (1955) for medium sands ($D_{50} = 0.94$ mm); lines and symbols are as in Figure 1

location, we have made an attempt to estimate the velocity and concentration at a bed layer level. This idea is an alternative way of estimating the bottom velocity and concentration at the bed layer level. This seems to be more reasonable from the physical point of view than defining the reference velocity and concentration at an arbitrary location far above the bottom, as already a few diameters away from the bed the sediment particles are kept in suspension by the turbulence of the fluid and should therefore be regarded as sediment in suspension.

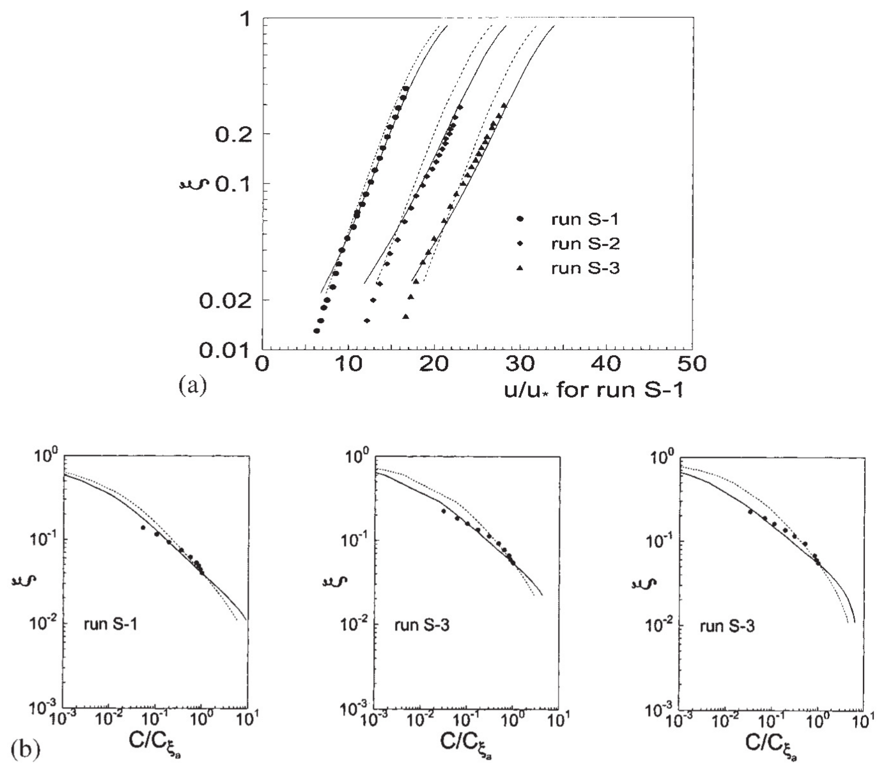

Figure 3. Comparison of calculated and measured velocity and concentration for the runs S-1 to S-3 of Einstein and Chien (1955) for coarse sands ($D_{50} = 1.3$ mm); lines and symbols are as in Figure 1

The velocity of sediment particles in the bed layer is a function of shear velocity u_* and critical shear velocity u_{*c} corresponding to the condition of the Shields grain movement for different grain sizes. The empirical relation for the velocity of a sediment particle in the bed layer $2D$ is given by Engelund and Fredsoe (1976) as

$$u = \delta u_* \left[1 - 0.7\sqrt{\left(\frac{u_{*c}}{u_*}\right)} \right] \tag{16}$$

where the value of δ is 9.0 for sand. Here it is assumed that the migration velocity of sediment size at the top of the bed layer is approximately equal to the fluid velocity at that layer (Mazumder, 1994). In our case we have estimated the value of $\delta = 3.65$ from (16) to match the computed velocity u with the extrapolated observed velocity of Einstein and Chien (1955) near the bed.

Zyserman and Fredsoe (1994) proposed a formula for determining the bed concentration at a reference level $\xi_a = 2D$ ($D =$ grain diameter in cm) as

$$C_{\xi_a} = \frac{0.331(\theta' - 0.045)^{1.75}}{1 + \frac{0.331}{0.46}(\theta' - 0.045)^{1.75}} \tag{17}$$

Table 1. Parameters used for computations

Data	Sand	D_{50} (mm)	Run no.	ξ_a (observed)	C_{ξ_a} (observed)	β (estimated)	γ (estimated)
Vanoni (1946)	Fine	0.10	20	0.067	0.00145	1.0	1.70
			21	0.077	0.00106	1.0	1.70
			S-11	0.029	0.01185	1.0	0.70
			S-12	0.030	0.07721	10.0	0.60
	Fine	0.274	S-13	0.032	0.13283	10.0	0.50
			S-14	0.033	0.14566	10.0	0.50
			S-15	0.034	0.22679	10.0	0.50
			S-16	0.036	0.23321	10.0	0.45
Einstein and			S-6	0.043	0.01057	1.0	1.10
Chien (1955)			S-7	0.039	0.03313	15.0	1.50
	Medium	0.940	S-8	0.037	0.03147	19.0	1.80
			S-9	0.041	0.05736	18.0	1.50
			S-10	0.043	0.08151	18.0	1.50
			S-1	0.040	0.02189	8.0	1.80
	Coarse	1.300	S-2	0.054	0.04566	8.0	1.80
			S-3	0.055	0.05679	7.0	1.70
Coleman (1981)	Fine	0.105	2	0.035	0.00085	1.0	1.80
			5	0.035	0.00400	1.0	1.50
Lyn (1986)	Fine	0.15	1565EQ	0.093	0.00214	1.0	1.00
	Fine	0.19	1965EQ	0.072	0.00235	1.0	1.20

where θ' is the Shields parameter related to skin friction. But use of this formula gives a highly overestimated concentration in Einstein and Chien's data at the $2D$ level, which is noted by comparing the computed concentration with the extrapolated value of Einstein and Chien's data. So estimation of concentration at the $2D$ level by (17) leads to an erroneous result except in the case of run S-15 and S-16, where the amount of material in suspension is very high.

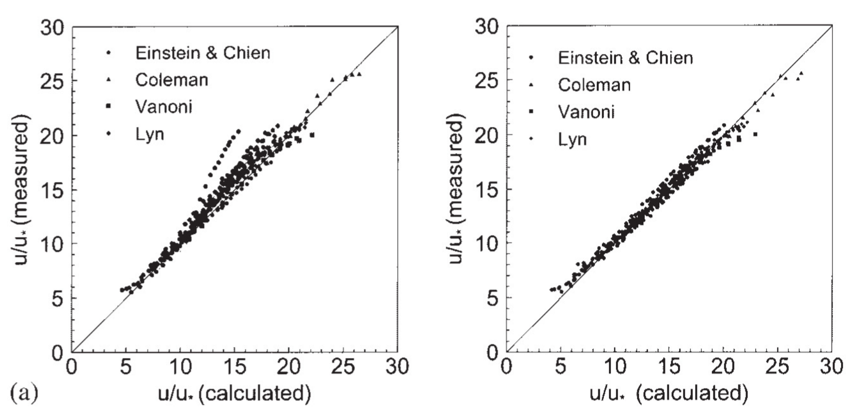

Figure 4. Comparison of calculated and measured velocity and concentration profiles without stratification (left) and with stratification (right)

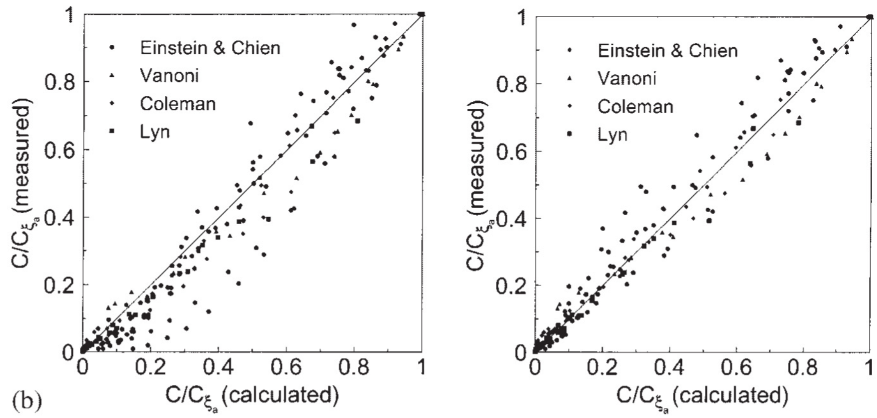

Figure 4. Continued

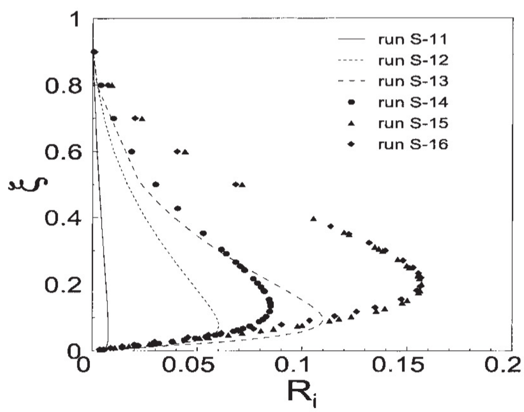

Figure 5. The variation of Richardson number with depth for runs S-11 to S-16 of Einstein and Chien (1955)

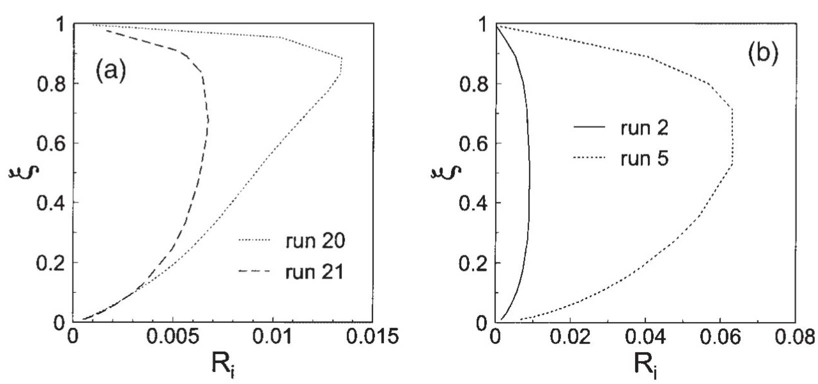

Figure 6. The variation of Richardson number with depth for (a) Vanoni (1946) and (b) Coleman (1981)

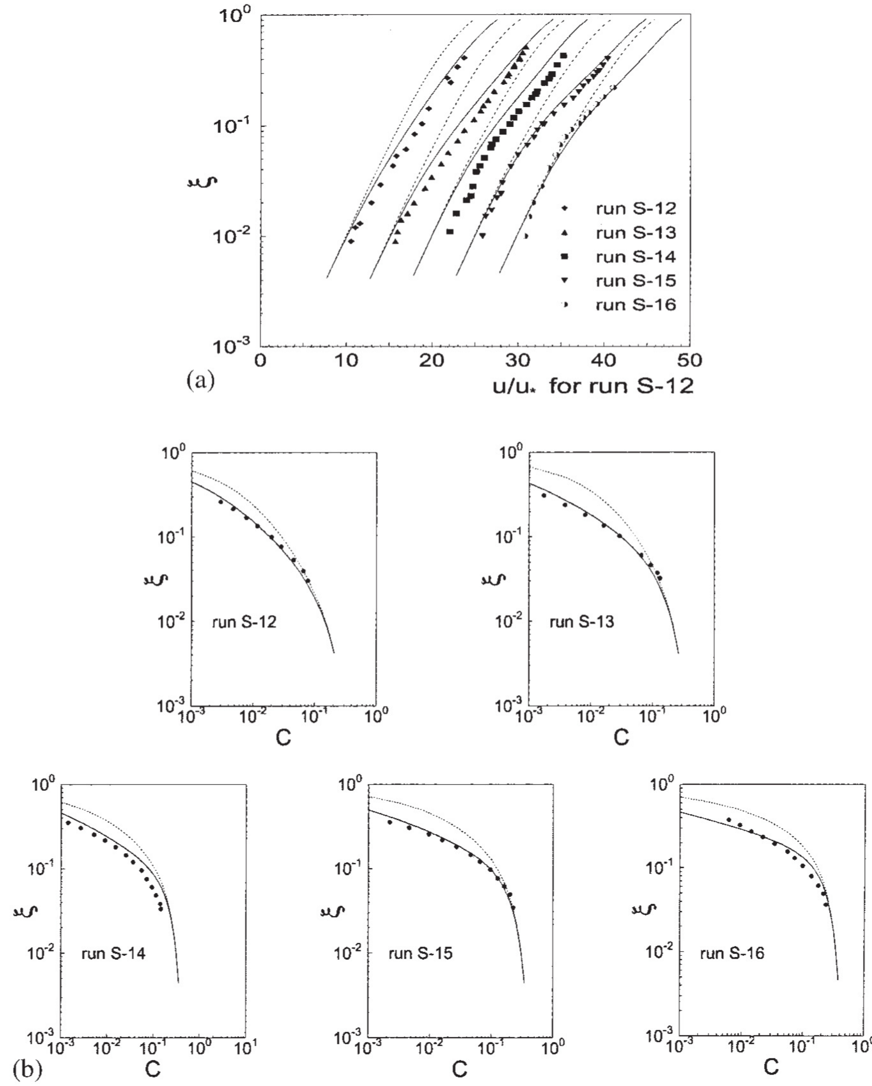

Figure 7. Comparison of calculated and measured velocity and concentration for the runs S-12 to S-16 of Einstein and Chien (1955) for fine sands ($D_{50} = 0.274$ mm) using the reference velocity from (16) and concentration from (20) at $2D$ level; lines and symbols are as in Figure 1

A similar result is observed when we estimate the concentration by van Rijn (1984) at a bed layer level $2D$. The concentration at the bed layer given by van Rijn (1984) is

$$C_{2D} = 0.18 C_b \frac{S_0}{D_*} \tag{18}$$

where C_b is the maximum theoretical value of bed concentration for firmly packed grains, S_0 is the normalized excess shear stress and D_* is the particle diameter, defined as

$$D_* = D\left(\frac{gA}{\nu_f^2}\right)^{1/3} \tag{19}$$

where D is the median grain size of sediment, A is the relative density of sediment and ν_f the kinematic viscosity of water. The computed bed layer concentration was overestimated and even more than that of Zyserman and Fredsoe (1994).

Finally, we made use of the expression of reference concentration proposed by Smith and McLean (1977) because it was supported by field measurements (Smith and McLean, 1977; Dyer, 1980). According to Smith and McLean (1977), the bed layer concentration C_{ξ_a} at the reference level ξ_a is given by

$$C_{\xi_a} = \gamma_0 \frac{u_*^2 - u_{*c}^2}{u_{*c}^2} \tag{20}$$

The value of the constant γ_0 was taken to be 0.004 by Smith and McLean (1977). We have estimated the value of γ_0 as 0.0055 as the computation of (20) with this value shows approximate agreement between the extrapolated value of the observed concentration of Einstein and Chien (1955) and the computed value of concentration by (20) at the level $2D$ except for the run S-11, where the amount of material in suspension is very low. Using the bed layer concentration from (20) at $2D$ level, we have computed the vertical velocity and concentration distribution of Einstein and Chien (1955) for S-12 to S-16, shown in Figures 7(a,b). The estimated value of β is 10 and the value of γ is less than 1. It is observed from Figures 1–3 in Section 4 (calculated from the arbitrary reference levels) that the density stratification due to suspended sediments has a remarkable effect only in runs S-12 to S-16, in which amount of sediment in suspension is high. Therefore, in this section we have considered only those runs of fine sediments for *direct* computation of vertical velocity and sediment concentration, considering the reference velocity and concentration at $2D$ level.

6. CONCLUSION

Theoretical models for mean velocity and sediment concentration at large suspension of sands have been developed based on the concept of sediment-induced stratification and modified mixing length together with viscous and turbulent shear stresses, which are the functions of concentration. Coupled differential equations arising from equations of Reynolds shear stress and mass have been solved with a view to compute the mean velocity and suspension concentration at any height above the bed, utilizing the following reference conditions: (i) the observed velocity and concentration at any arbitrary location and (ii) the computed fluid velocity and concentration at the bed layer level. Comparisons of theoretical models with the experimental data collected in laboratory channels have been made in order to test the applicability of the stratified flow analogy to sediment-laden turbulent flow. For both the reference conditions, analysis shows that the effect of stratification by suspended sediment is remarkable in the Einstein and Chien data, where the amount of sediment in suspension is high. But the real merit of estimation of velocity and concentration directly from the bed layer level is that it does not require an arbitrary choice of reference level. The derived velocity and concentration equations agree well with the experimental data, and thus the model constant used for sediment-induced stratification is consistent with the measurements of thermally stratified flows.

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