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# MULTIDIMENSIONAL BARGAINING UNDER ASYMMETRIC INFORMATION*

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A buyer with private information regarding marginal valuation bargains with a seller to determine price and quantity of trade. Depending on parameter values, a high-valuation buyer wants either to reveal information to create value or to conceal it to capture value. In the first case, equilibrium trades are efficient. In the second case, the low-valuation buyer purchases less than her efficient quantity, and there can be a one-period delay in trade. The quantity distortion is the only inefficiency that persists when time between offers approaches zero. There exist equilibria that are independent of the seller's prior beliefs.

## 1. INTRODUCTION

Many real-world negotiations are multidimensional. Negotiations between a firm's management and its union involve deciding on the wage rate, work hours, and job security. Stakeholders and managers of a company have to negotiate over multiple issues, including methods of raising new capital and investment strategies. In negotiating procurement contracts, upstream and downstream firms need to agree on price, quantity, and quality. Furthermore, in many cases, the negotiators are asymmetrically informed about the costs and benefits of an agreement. In union-management negotiations, the management may have private information about future profitability, which can affect the gains to both parties from reaching an agreement on wages, layoff policy, etc. Similarly, in negotiations between vertically related firms, a retailer may have private information regarding the nature of demand, and this can influence the payoffs to both the manufacturer and the retailer from agreeing on a quality/quantity schedule.

Multidimensional negotiations under asymmetric information bring forth two important issues that warrant formal analysis. First, when two parties negotiate to reach an agreement over multiple issues, they try to achieve two objectives. One is to *create value*, i.e., to negotiate an agreement that maximizes the total surplus, and the

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other is to *capture value*, i.e., to capture as large a part of the surplus as possible. In the presence of information asymmetry, there can be a conflict in achieving these two goals simultaneously. Second, the process of information transmission in such negotiations can generate two kinds of inefficiency: costly delays in reaching an agreement and an agreement on a set of terms that does not maximize the total surplus. Union-management negotiations, for example, may lead to work stoppages (i.e., strikes) and/or an agreement under which too many or too few of the current workers are retrenched.[2]

This article explores these issues by studying a simple two-dimensional negotiation game. I consider a buyer/seller model where the buyer can buy multiple units of a good and has private information regarding her marginal valuation. The players negotiate to determine a one-time trade, which involves an agreement on price and quantity. The negotiation takes place via infinite-horizon alternating-offer bargaining, where (1) counteroffers can be made arbitrarily quickly, and (2) the uninformed seller can offer a "menu of trades" whenever he makes an offer.

This simple model of negotiation has three desirable features. First, it brings out the conflict between creating and capturing value. The "efficient" (i.e., surplus-maximizing) trade involves a larger quantity when the buyer's marginal valuation is high. However, only the buyer knows her valuation. For a high-valuation buyer, any action that reveals her type allows for value creation by enabling agreement on an efficient trade, but at the same time, such an action limits his or her ability to capture value, since the seller is able to extract a larger part of the surplus if he knows that the buyer's valuation is high. Second, the posited bargaining structure provides minimal precommitment ability to the players and makes their bargaining power (approximately) equal.[3] Third, the game allows for multiple channels of information transmission—the seller has the ability to "screen" the buyer's private information both within a period by offering a menu of trades and between periods by gradually improving the terms of trade over time, and the buyer can "signal" her information both by her rejection of offers and by her own offers. This is an appropriate extensive form to determine what kinds of inefficiency—delay in trade, trade at inefficient levels, or both—are generated by the negotiation process.

In the existing literature, two sets of articles—the incomplete-information bargaining models and the hidden-information principal-agent models—study negotiations under incomplete information. But they do so under specific assumptions. Almost all articles on incomplete-information bargaining study buyer/seller bargaining over the price of an indivisible good. These models posit that there is a fixed quantity to be traded. Thus the focus is on the attempt by the players to capture

[2]The literature on negotiations has recognized the potentially conflicting goals of creating and capturing value (see Sebenius, 1992). Kennan and Wilson (1993) note that while bargaining under incomplete information can lead to the two kinds of inefficiency, the extant bargaining literature has focused primarily on delay costs.

[3]This may indeed be the case in many situations—in union-management negotiations, as well as in negotiations between vertically related firms, bargaining power of the two parties can be close to being equal.

value, and the only inefficiency that can arise is delay in time to trade.[4] On the other hand, in the principal-agent models, the total surplus does depend on the agent's private information, and the negotiation is typically over price-quantity contracts. However, these models consider a one-shot game where the principal makes a take-it-or-leave-it offer and so has all the bargaining power. The analysis focuses on the principal's ability to create value (since he or she can capture all the surplus), and the only inefficiency that can exist is a quantity distortion.[5]

This article brings together these two strands of research in a model where the negotiators pursue the twin objectives of creating and capturing value and where both kinds of inefficiency—delay and quantity distortions—can exist in equilibrium. Further, the current model symmetrizes the players' bargaining power and their strategy spaces. This article characterizes the pure-strategy Perfect Bayesian equilibria (PBE) of the buyer/seller bargaining game that survive a modified version of the Perfect Sequential equilibrium (PSE) refinement of Grossman and Perry (1986a, 1986b). The analysis delineates how the parameters of the model determine the tradeoff between creating value and capturing value for a high-valuation buyer and how this tradeoff in turn determines the nature of equilibrium inefficiencies.

This article is organized as follows: The bargaining game is described in Section 2. In Section 3 I present a summary of the main results of the article and provide some intuition behind the results. The formal analysis is contained in Sections 4, 5, and 6. Section 4 describes the equilibrium refinement. Section 5 contains some preliminary results regarding continuation equilibrium paths in the bargaining game. The equilibrium-characterization results are presented in Section 6. Finally, in Section 7 I discuss the relationship between these results and those in the existing literature. Proofs of all results are presented in the Appendix.

## 2. THE BARGAINING GAME

A buyer and a seller bargain to determine a one-time trade. Agreement on a trade involves agreeing on a quantity $q$ and a price $p$. A feasible trade is denoted by $y=(q,p)\in[\mathscr{R}_+\times\mathscr{R}_+]$. The seller's cost of producing a quantity $q$ is $c(q)$, and the buyer's utility from consuming $q$ is $u(q,B)$. The parameter $B$ indexes the buyer's marginal valuation for the good, and it is her private information. It is common knowledge that the parameter $B$ takes one of two values—$H$(igh), or $L$(ow), with probabilities $e\in(0,1)$ and $(1-e)$, respectively.

Time is continuous, starting from $\tau=0$. The periods $t=0,1,2,\dots$ correspond to times $\tau=0,z,2z,\dots$, where $z>0$. In any period $t$, a predesignated player makes an

[4] The articles most closely related to this article are those which study alternating-offer bargaining under one-sided incomplete information. This set includes Rubinstein (1985), Grossman and Perry (1986b), Admati and Perry (1987), Gul and Sonnenschein (1988), and Ausubel and Deneckere (1992). A recent article by Wang (1998) does study incomplete-information bargaining over two issues. I discuss that article in Section 7.

[5] Baron and Myerson (1982), Sappington (1983), and Guesnerie and Laffont (1984) are three classic articles on the one-shot hidden-information principal-agent model. Dynamic agency models, where parties (re)negotiate long-term contracts, also have been studied in the literature. The relation between those studies and this article is discussed in Section 7.

offer, which his or her opponent either accepts or rejects and makes a counteroffer in period $t+1$. The seller's offer can be a menu of trades, in which case the buyer either accepts one particular trade from the menu or rejects the entire menu. A strategy for each player specifies, for every history of the game that ends with an opponent's offer, the player's acceptance behavior in that period and offer behavior in the next. In this article I restrict attention to pure strategies. Further, I impose the following restrictions on strategies: The seller's menu offer in any period can contain either a single trade or two distinct trades, and the buyer can only offer a single trade.[6]

The players have a common time discount rate $r > 0$. Given $z$, this corresponds to a perperiod discount factor $\delta \equiv e^{-rz} \in (0,1)$. If a trade $y = (q, p)$ is agreed on in period $t$, it is immediately implemented—the seller produces the quantity $q$ and sells it to the buyer at price $p$. Then the seller's payoff (evaluated at $t = 0$) is $\delta^t U_S(y)$, where $U_S(y) = p - c(q)$, and the payoff to the buyer with valuation parameter $B$ (who will henceforth be referred to as buyer $B$) is $\delta^t U_B(y)$, where $U_B(y) = u(q, B) - p$ for $B = L, H$. If there is no trade in finite time, each player gets zero payoff. I will focus on the case where the time between offers $z$ is close to zero.

The seller's cost function $c(\cdot)$ is twice continuously differentiable, increasing, and convex in $q$, with $c(0) = 0$. For buyer $B$, $u(q, B)$ is twice continuously differentiable, increasing, and concave in $q$. I define $R(q) \equiv u(q, H) - u(q, L)$ to be the difference in the utility of the two buyer types from consuming $q$ and assume that buyer $H$ has a higher marginal valuation than buyer $L$:

Assumption 1. $R(0) = 0$ *and* $R'(q) > 0$ *for all* $q \geq 0$.

Define $w(q, B) \equiv u(q, B) - c(q)$ to be the total surplus when the quantity $q$ is traded. I assume that there exists a unique maximizer $q_B^* \in (0, \infty)$ of $w(q, B)$, for $B = L, H$. The maximum surplus will be denoted by $w_B^* \equiv w(q_B^*, B)$. Assumption 1 implies that $q_H^* > q_L^*$ and $w_H^* > w_L^*$. I consider the case where the valuation parameters of the two buyer types are not too far apart. Specifically, I assume the following inequality, which will be sufficient to ensure positive trades for both buyer types in any equilibrium of the bargaining game:

Assumption 2. $w_L^* \geq w_H^*/2$.

[6]Given the focus on pure-strategy equilibria, these additional restrictions are reasonably innocuous. First, since the buyer can only be one of two types, there is no further loss of generality in positing that the seller's menu offer can contain at most two trades. Second, this is a "private values" model, where the buyer's private information does not directly affect the seller's payoff. This implies that in a game where the buyer is allowed to make menu offers, if the seller accepted from the buyer's offered menu after any history, he would accept that trade which is the "best" for him (i.e., that trade which gives the seller his highest payoff). Given this, one may impose the following restriction on the PBE: "Any two histories that are identical in terms of the seller's menu offers and the best offers by the buyer are *equivalent* in that seller is required to hold the same beliefs, and the players are required to play the same continuation PBE." It is easily established that under this restriction, every PBE outcome of the game where the buyer can make menu offers is a PBE outcome of the game where the buyer can make only a single offer.

As a benchmark, consider the alternating-offer bargaining game where the buyer's valuation parameter $B$ is commonly known. In this complete-information game, the following result can be established as a straightforward extension of the Rubinstein (1982) result.

PROPOSITION 1. *Define the following trades*:

$$a_B^* \equiv \left[ q_B^*, c(q_B^*) + (1/1 + \delta) w_B^* \right]$$

*and*

$$b_B^* \equiv \left[ q_B^*, c(q_B^*) + (\delta/1 + \delta) w_B^* \right]$$

*The complete-information bargaining game between the seller and buyer B has a unique subgame perfect equilibrium: After any history, the seller offers $a_B^*$, buyer B offers $b_B^*$, and player j (for $j = B, S$) accepts an offer y if and only if $U_j(y) \geq (\delta/1 + \delta) w_B^*$.*

In the equilibrium of the complete-information game, the trade offered by any player gives the "responder" $(\delta/1 + \delta)$ share of the total surplus, and the responder accepts an offer if and only if it gives him or her at least that share of the surplus. Given this, the crucial question with regard to the incomplete-information game is: Will one buyer type want to mimic the other type's complete-information strategy? It is easily proved that buyer $L$ never wants to mimic buyer $H$'s strategy: $U_L(b_L^*) > U_L(b_H^*) = (1/1 + \delta) w_H^* - R(q_H^*)$ and $U_L(a_L^*) > U_L(a_H^*) = (\delta/1 + \delta) w_H^* - R(q_H^*)$. However, things are different for buyer $H$. For her, the following inequalities hold[7]:

$$U_H(b_H^*) - \delta U_H(a_L^*) > U_H(b_H^*) - U_H(b_L^*) > U_H(a_H^*) - U_H(a_L^*)$$

where each of the three terms may or may not be positive.

Given the preceding inequalities, I define two parameter regimes of the model as follows:

The "no mimicking" (N-M) regime:

$$U_H(a_H^*) \geq U_H(a_L^*)$$

The "mimicking by $H$" (H-M) regime:

$$\delta U_H(a_L^*) > U_H(b_H^*)$$

In the N-M regime, all three terms in the preceding chain of inequalities are positive. In this regime, neither buyer type will want to mimic the other's complete-information strategy. In the H-M regime, on the other hand, all three terms in the chain of inequalities are negative. In this regime, buyer $H$ has a strong incentive to pose as buyer $L$—she would rather have the seller offer $a_L^*$ in the next period than have him accept $b_H^*$ in the current period. Subsequent analysis will show that when

[7]Note that $U_H(b_L^*) = R(q_L^*) + (1/1 + \delta) w_L^*$, and $U_H(a_L^*) = R(q_L^*) + (\delta/1 + \delta) w_L^*$. Using these expressions, the posited inequalities can be established easily. Observe that $U_H(a_L^*) > \delta U_H(b_L^*) > \delta U_H(a_L^*)$.

the time between offers is small, the parameters of the model belong in one of these two regimes (see Section 5).

## 3. OVERVIEW OF RESULTS

In this section I describe the main results of the article regarding the properties of the equilibrium outcomes of the bargaining game when the time between offers is small. I then provide some intuition behind the results by studying particular "screening" and "signaling" games.

I focus on those pure-strategy PBEs of the bargaining game that satisfy the Grossman and Perry PSE requirement that off-equilibrium beliefs be "self-fulfilling." However, instead of making their "support restriction" assumption, I use the Kohlberg and Mertens (1986) logic of "elimination of never weak best responses" (NWBR) to specify how the seller updates after reaching point beliefs. Further, the results in the H-M regime are obtained for "stationary equilibria," where the seller's offer strategy is required to be stationary in the sense that it is allowed to depend only on his current beliefs. (See Section 4 for the details.)

It is the specific parameter configurations of the model that determine the tradeoff between creating value and capturing value for a high-valuation buyer. In the N-M regime, buyer $H$ wants to reveal her type to create value. As a result, the unique equilibrium outcome is the *ex post* efficient complete-information equilibrium outcome described in Proposition 1. (See Proposition 4.) In the H-M regime, on the other hand, buyer $H$ wants to conceal her type to capture value. All stationary equilibrium outcomes in this regime are *ex post* inefficient. The inefficiency takes the following forms: There exists a quantity distortion in that buyer $L$ purchases less than her efficient quantity $q_L^*$ (while buyer $H$ buys $q_H^*$). In addition, there can be a one-period delay in trade. Since this delay is the time it takes to make a single counteroffer, it vanishes in the limit as time between offers approaches zero (i.e., as $z \to 0$). However, the quantity distortion persists. In all stationary equilibria, buyer $H$'s payoff is at least her "Rubinstein payoff" (i.e., her equilibrium payoff in the corresponding complete-information game), while buyer $L$'s payoff is strictly less than her Rubinstein payoff. In this sense, buyer $H$ gets nonnegative "information rents," while buyer $L$ gets negative information rents. This result also holds in the limit $z \to 0$. Finally, there exist stationary equilibria in the H-M regime whose outcomes are independent of the seller's prior beliefs. In particular, a belief-independent equilibrium outcome exists in the limit $z \to 0$. (See Propositions 5 and 6.)

To understand these results, recognize that the bargaining structure consists of alternate periods of screening by the uninformed seller and signaling by the informed buyer. In equilibrium, agreements are reached in one of three ways: Either the seller makes a menu offer and the two buyer types accept distinct trades—the "screening outcome"; or the two buyer types make separating offers and the seller accepts both offers—the "signalling outcome"; or the seller offers a single trade that buyer $H$ accepts, and buyer $L$ makes an offer in the following period that the seller accepts—the "hybrid screening-signaling outcome." The following discussion explains the properties of these different outcomes by looking at one-shot screening and signaling games.

Consider the following one-shot screening game: The seller makes a take-it-or-leave-it menu offer $\{(q_H, p_H), (q_L, p_L)\}$ to the buyer. If the buyer rejects the menu, she gets her reservation payoff $v_B$ if her type is $B$. Assume that $0 \leq v_H - v_L \leq R(q_H^*)$. The equilibrium outcome of this game is the solution to the problem of maximizing the seller's expected payoff subject to the buyer's "standard" incentive compatibility and individual rationality constraints. Let $q_L(e)$ maximize $[w(q,L) - (e/1-e)R(q)]$ subject to $q \geq 0$. Note that $q_L(e) < q_L^*$ for all $e \in (0,1)$ and that $q_L'(e) < 0$ for all $q_L(e) > 0$. The following proposition presents the solution to the screening problem. (Its proof follows from the arguments in the screening literature.)

PROPOSITION 2. *At the solution to the screening problem,* $q_H = q_H^*$, *and* $(q_L, p_L)$ *is such that* $U_L(q_L, p_L) = v_L$. *Further,* $(a)$ *if* $v_H - v_L \leq R(q_L(e))$, *then* $q_L = q_L(e)$, $(b)$ *if* $R(q_L(e)) < v_H - v_L < R(q_L^*)$, *then* $q_L = R^{-1}(v_H - v_L) < q_L^*$, *and* $(c)$ *if* $R(q_L^*) \leq v_H - v_L \leq R(q_H^*)$, *then* $q_L = q_L^*$. *Finally,* $p_H$ *is such that* $U_H(q_H, p_H) = v_L + R(q_L(e)) > v_H$ *in case* $(a)$ *and* $U_H(q_H, p_H) = v_H$ *in cases* $(b)$ *and* $(c)$.

To understand this result, start with the case where the buyer's reservation payoff is type-independent, i.e., where $v_H = v_L$. This is the case that is commonly considered in the hidden-information principal-agent models. In the current model, this case (with $v_H = v_L = 0$) corresponds to the scenario where the seller can make a one-off offer to the buyer. In this case, incentive constraints are downward-binding; i.e., they are binding for buyer $H$ and not for buyer $L$. This is why it is optimal for the seller to distort the quantity traded with buyer $L$ below her efficient quantity. By doing so, the seller can set a higher price for the amount $q_H^*$ and still ensure that buyer $H$ accepts that trade. This limits the information rent going to buyer $H$, $R(q_L(e))$ being the magnitude of this rent. The larger is the seller's prior belief that the buyer's type is $H$, the greater is her expected benefit from decreasing the quantity for buyer $L$. This is why the magnitude of the quantity distortion $[q_L^* - q_L(e)]$ is increasing in $e$.

When buyer $H$'s reservation payoff is greater than buyer $L$'s, one of three cases arise. If the difference in the payoffs is small [case $(a)$ in Proposition 2], the same solution as in the case where $v_H = v_L$ obtains. Alternatively, when the payoff difference is in an intermediate range [case $(b)$], the quantity for buyer $L$ need no longer be distorted all the way down to $q_L(e)$, since buyer $H$ now has to be given a high payoff to ensure that her individual rationality constraint is satisfied. In this case, the quantity distortion depends only on the difference in the reservation payoffs and not on the sellers' beliefs. Finally, when the payoff difference is sufficiently large [case $(c)$], buyer $H$'s incentive compatibility condition can be satisfied without having to distort the first-best quantity in buyer $L$'s trade.

In any "screening outcome" in the bargaining game, the reservation payoffs $v_H$ and $v_L$ correspond to the continuation payoffs for the two buyer types if they reject a seller offer. Every equilibrium has the property that after any history, buyer $H$'s continuation payoff is greater than buyer $L$'s. This is so because the total surplus is larger when the buyer's marginal valuation is high. It will be shown that the difference in continuation payoffs are such that in any screening outcome, case (c) obtains in the N-M regime and so the first-best quantities are traded, while case (b) obtains in the H-M regime and so there is a quantity distortion for buyer $L$.

Next, consider a one-shot signaling game. The informed buyer makes a single offer to the seller. If the seller rejects the offer, his reservation payoff depends on his posterior belief and is increasing in that belief. Denote this payoff by $v_S(\mu)$, where $\mu \in [0,1]$ is the updated belief that the buyer's type is $H$. The following proposition (whose proof follows from the analysis in the signaling literature) characterizes the Riley outcome—which is the efficient (i.e., least-cost) separating equilibrium outcome—in the signaling game.

PROPOSITION 3. *In the unique Riley outcome of the signaling game, buyer H offers the trade* $[q_H^*, c(q_H^*) + v_S(1)]$ *and buyer L offers the trade* $[q_L, c(q_L) + v_S(0)]$, *where* (*a*) *if* $v_S(1) - v_S(0) \leq w_H^* - w(q_L^*, H)$, *then* $q_L = q_L^*$, *and* (*b*) *if* $v_S(1) - v_S(0) > w_H^* - w(q_L^*, H)$, *then* $q_L < q_L^*$ *such that* $v_S(1) - v_S(0) = w_H^* - w(q_L, H)$.

Start with the case where the seller's reservation payoff does not increase in his or her posterior belief. This is the case that is commonly considered in standard signaling models. In the current model, this case (with $v_S(1) = v_S(0) = 0$) corresponds to the scenario where the buyer can make a one-off offer to the seller. In this case, since the buyer's type does not affect the seller's payoff from accepting a trade (this is the "private values" specification), the first-best quantities are traded. When the seller's reservation payoff is strictly increasing in his or her posterior belief, one of two cases arise. If this increase in payoff is small [case (*a*) in Proposition 3], buyer $H$ still prefers to make her first-best offer than to mimic buyer $L$, and so the first-best quantities are traded. (In no circumstances does buyer $L$ want to mimic buyer $H$.) Alternatively, if the increase in payoff is sufficiently large [case (*b*)], buyer $H$ wants to mimic buyer $L$. In this case, buyer $L$ "separates" from buyer $H$ by offering a trade that contains a quantity smaller than her first-best quantity.

In any "signaling outcome" in the bargaining game, the payoff $v_S(\mu)$ corresponds to the seller's continuation payoff (based on his updated belief) when he rejects an offer. In any equilibrium, after any history, the seller's continuation payoff is increasing in this belief. It will be shown that in any signaling outcome, case (*a*) obtains in the N-M regime and case (*b*) obtains in the H-M regime. Further, a signaling outcome does not depend on the seller's prior belief for the same reason that the Riley outcome in the signaling game is belief-independent.

Finally, the "hybrid screening-signaling outcome" (which is a possible equilibrium outcome in the H-M regime) has the following features: The seller screens the two buyer types by offering a trade that is acceptable to buyer $H$ (and contains her first-best quantity $q_H^*$) but not to buyer $L$. In the next period, buyer $L$ offers a trade that is acceptable to the seller and which contains a quantity smaller than her first-best quantity $q_L^*$. Here, buyer $L$ uses both the quantity distortion and the one-period delay in trade to separate from buyer $H$. The logic behind the screening and the signaling outcomes also explains the properties of this hybrid outcome.

The preceding discussion provides some intuition for the equilibrium inefficiencies in the bargaining game. It also indicates the reason for the existence of belief-independent equilibria. Section 7 contains a further discussion of the properties of the equilibrium outcomes.

## 4. THE EQUILIBRIUM CONCEPT

In this section I formalize the specific equilibrium concept used in this article. I begin with some notation. Let $h^t$ denote a history of the bargaining game when the period $t$ offer is on the table. The seller's "belief function" is a mapping $\mu(\cdot)$ from the space of all possible histories in any period to $[0,1]$. For any history $h^t$, $\mu(h^t) \in [0,1]$ will denote the seller's belief that the buyer's type is $H$. Further, given any seller menu offer $\mathbf{y}$ in any period, I will let $y_B(\mathbf{y})$ denote the trade for which $U_B[y_B(\mathbf{y})] \geq U_B(y)$ for all $y \in \mathbf{y}$, for $B = L, H$. [For any $\mathbf{y}$, it can be that $y_H(\mathbf{y}) = y_L(\mathbf{y})$; this will of course be the case if $\mathbf{y}$ contains a single trade.]

A tuple $(\underline{\sigma}, \mu)$, where $\underline{\sigma} \equiv \{\sigma_S, \sigma_H, \sigma_L\}$ is the vector of the players' strategies and $\mu$ is the seller's belief function, will be a PBE if and only if the strategies are optimal given beliefs and the beliefs are Bayes-consistent with the strategies.[8] The PSE refinement of Grossman and Perry (1986a, 1986b) imposes a particular restriction on beliefs following an off-equilibrium history.[9] In order to formulate this restriction, one has to address the following issue: In any PBE, suppose that after a particular history the seller is specified to hold a point belief regarding the buyer's type. Then, under what subsequent histories should he be permitted to "switch away" from the belief? The PSE refinement incorporates a support restriction that posits that the uninformed player can *never* switch away from point beliefs. Instead of imposing such a strong restriction, I specify a weaker condition on when the seller can (and cannot) switch away from point beliefs.[10] The condition is motivated by the NWBR criterion of Kohlberg and Mertens (1986): An equilibrium outcome should survive elimination of strategies that are dominated in all equilibria consistent with that outcome. In what follows, $V_j^+$ will denote the supremum of the equilibrium continuation payoffs for player $j$ ($j = S, L, H$) over all histories.

ASSUMPTION 3. *Suppose that the seller's menu offer in any period contains a trade* $y$ *with* $U_B(y) \geq \delta V_B^+$, *and the menu is rejected. Then, and only then, the seller "prunes*

[8] Note that any $\mu$ that is a part of a PBE must satisfy the following restriction: If the buyer makes an offer in period $t$, and if the seller rejects that offer and makes a counteroffer in period $t+1$, then $\mu(h^{t+1}) = \mu(h^t)$.

[9] The PSE refinement has been criticized by Mailath et al. (1993). However, the alternative "forward induction" refinements put forward by these authors and others have been formalized only for finite games. It is partly for reasons of tractability that I follow the PSE refinement in the current infinite-horizon game.

[10] Many researchers have been critical of the support-restriction assumption—see Madrigal et al. (1987) and Noldeke and Van Damme (1990).

*type B," i.e., assigns probability zero to type B in the current period and in all future periods.*[11]

Assumption 3 states the precise conditions under which, having reached a point belief, the seller will not switch away from it. In particular, this restriction determines when buyer type $L$ should be pruned from the seller's belief set. Under this assumption, we have the following result. (Proofs of the lemmas in this section are presented in Section 1 of the Appendix.)

LEMMA 1. *In any PBE satisfying Assumption* 3, $V_S^+ \leq (1/1+\delta)w_H^*$, *and* $V_L^+ \leq (1/1+\delta)w_L^*$. *Consequently, after any history, the seller accepts a buyer offer* $y$ *if* $U_S(y) \geq (\delta/1+\delta)w_H^*$ *and rejects it if* $U_S(y) < (\delta/1+\delta)w_L^*$, *and given a seller offer* $\mathbf{y}$, *buyer L accepts* $y_L(\mathbf{y})$ *if* $U_L(y_L) \geq (\delta/1+\delta)w_L^*$, *and buyer H rejects* $y_H(\mathbf{y})$ *if* $U_H(y_H) < (\delta/1+\delta)w_H^*$.

Given Lemma 1, a buyer offer $y$ that gives the seller a payoff less than $(\delta/1+\delta)w_L^*$ will be referred to as *silence*, since it is going to be rejected in all continuation PBEs satisfying Assumption 3. Any other buyer offer will be termed a *serious offer*.

The following assumption incorporates the PSE notion of self-fulfilling beliefs. It specifies the buyer deviations for which the seller should update to believing that the buyer's type is $L$.[12]

ASSUMPTION 4. *Let* $(\underline{\sigma}, \mu)$ *be a PBE satisfying Assumption* 3. *Consider any history* $h^t$ *ending with a seller offer such that no buyer type has been pruned from the seller's belief set. Define* $\overline{V}_B(\underline{\sigma}, h^t)$ *to be buyer B's continuation payoff under* $\underline{\sigma}$ *from* $h^t$. *Suppose that the buyer makes an off-equilibrium offer* $y'$ *in period* $t+1$. *Then the seller is required to hold the point belief that the buyer's type is L if either* (*a*) $y'$ *is a serious offer*, $\delta U_L(y') > \overline{V}_L(\underline{\sigma}, h^t)$, *and* $\overline{V}_H(\underline{\sigma}, h^t) \geq \delta U_H(y')$, *or* (*b*) $y'$ *is silence*, $\delta^2 U_L(a_L^*) > \overline{V}_L(\underline{\sigma}, h^t)$, *and* $\overline{V}_H(\underline{\sigma}, h^t) \geq \delta^2 U_H(a_L^*)$.

[11] For ease of presentation, this assumption is stated in a stronger (and less rigorous) form than what is implied by NWBR. First, when $U_B(y) = \delta V_B^+$, rejection of the seller's menu is not necessarily a strictly dominated strategy for buyer $B$ (it is certainly weakly dominated). Thus, following the logic of NWBR, it should be stated that "when $U_B(y) = \delta V_B^+$, type $B$ is to be pruned if and only if the buyer rejects the seller's offer and then makes such a counteroffer that the continuation payoff for buyer $B$ after that history is strictly less than $V_B^+$." It can be checked that incorporating this qualification into Assumption 3 will not affect the subsequent analysis. Second, the last statement in Assumption 3 should be made subject to the following qualifier: "The assignment of zero probability to both buyer types concurrently is viewed as an empty restriction" because the pruning of all possible types of the informed player is inconsistent with *any* observed action. See Kohlberg (1990, Theorem 3′).

[12] While in principle this assumption can be stated more generally to include the set of buyer deviations for which the seller should update to believing that the buyer's type is $H$, I refrain from doing so because the assumption, as stated, is all that is needed for the subsequent analysis.

Assumption 4 states the following: Suppose that the buyer deviates after some history, and following her off-equilibrium offer, if the players play the equilibrium strategies in the complete-information game between the seller and buyer $L$, the deviation will generate a strictly greater payoff for buyer $L$ and not for buyer $H$ (relative to their respective equilibrium payoffs). Then, following the deviation, the seller should believe that the buyer's type is $L$.[13]

The next result is an immediate consequence of Assumption 4: If the seller (temporarily) believes that the buyer's type is $B$ (for $B = L, H$), then his equilibrium response is to play his equilibrium strategy in the complete-information game against buyer $B$.

LEMMA 2. *Consider any history ending with a buyer offer $\bar{y}$ that generates the point belief that the buyer's type is $B$. In any continuation equilibrium, if $U_S(\bar{y}) \geq (\delta/1 + \delta)w_B^*$, then the seller accepts $\bar{y}$, and if $U_S(\bar{y}) < (\delta/1 + \delta)w_B^*$, then he or she rejects $\bar{y}$ and offers $a_B^*$.*[14]

Henceforth, the term *equilibrium* will refer to a pure-strategy PBE that satisfies Assumptions 3 and 4. When studying equilibrium outcomes of the bargaining game in the H-M regime, I will focus on "stationary equilibria" as defined below.

DEFINITION. A *stationary equilibrium* is an equilibrium (i.e., a pure-strategy PBE satisfying Assumptions 3 and 4) that satisfies the following restriction: After any history, the seller's offer strategy depends only on the current beliefs that are generated by that history.[15]

## 5. PRELIMINARY RESULTS

I begin this section by showing that when the time between offers $z$ is smaller than a critical value $z^*$, the parameters of the model (generically) belong in one of the two regimes defined in Section 2. The subsequent analysis will only consider the case when $z < z^*$. (Proofs of the lemmas in this section are presented in Section 2 of the Appendix.)

[13] Condition (a) states that for a serious buyer offer, the seller should believe that the buyer's type is $L$ if acceptance makes only buyer $L$ strictly better off—this is Assumption B-1 in Rubinstein (1985). Condition (b) says that if the buyer remains silent, the seller should believe that the buyer's type is $L$ if the trade $a_L^*$ in the next period makes only buyer $L$ strictly better off—note that it is a dominant strategy for buyer $L$ to accept $a_L^*$ and that rejection of $a_L^*$ will cause the seller to prune type $L$ from his beliefs.

[14] In any PBE satisfying Assumptions 3 and 4, after any history that leads the seller to hold the point belief $\mu = 0$, the seller offers $a_L^*$. If this is rejected, he changes his point belief and sets $\mu = 1$ in all future periods. This indicates that the support-restriction assumption is contrary to the NWBR logic in this game.

[15] Grossman and Perry (1986b) show that their PSE assumption implies stationarity in the one-dimensional bargaining game. In the current game, however, Assumptions 3 and 4 do not imply stationarity by themselves. Gul and Sonnenschein (1988) consider a related stationarity assumption.

LEMMA 3. *There exists $z^* > 0$ such that for all $z \in (0, z^*)$, the parameters are in the N-M regime if $(w_H^* + w_L^*) > 2w(q_L^*, H)$, and they are in the H-M regime if $(w_H^* + w_L^*) < 2w(q_L^*, H)$.*

I now identify continuation equilibrium paths in the bargaining game after specific histories. First, I look at the candidate continuation equilibria in the following situation: Consider any history such that the seller's current belief is his prior, and it is the buyer's turn to make an offer. What are the continuation equilibrium strategies in this case? To answer this question, I define the trade $x_L^0 \equiv (q_L^0, p_L^0)$ to be buyer $L$'s optimal serious offer such that buyer $H$ prefers the trade $b_H^*$ to $x_L^0$.

DEFINITION. The trade $x_L^0 \equiv (q_L^0, p_L^0)$ uniquely maximizes $U_L(y)$ subject to the constraints: $U_S(y) \geq (\delta/1 + \delta)w_L^*$ and $U_H(y) \leq U_H(b_H^*) \equiv (1/1 + \delta)w_H^*$.

The trade $x_L^0$ has the following properties: If $U_H(b_H^*) \geq U_H(b_L^*)$, then $x_L^0 = b_L^*$. If $U_H(b_H^*) < U_H(b_L^*)$, then $q_L^0$ is the smaller solution to $w(q, H) = (1/1 + \delta)(\delta w_L^* + w_H^*)$, and $p_L^0 = c(q_L^0) + (\delta/1 + \delta)w_L^*$. In the latter case, $q_L^0 \in (0, q_L^*)$ for all $z > 0$, and $\lim_{z \to 0} q_L^0 \in (0, q_L^*)$.

LEMMA 4. *Consider any period t such that $\mu(h^{t-1}) = e$, and it is the buyer's turn to offer. Then, in any continuation equilibrium, either both buyer types pool and remain silent, or buyer H offers $b_H^*$ and buyer L offers $x_L^0$ and the seller accepts these offers.*

Lemma 4 characterizes the signaling outcome in the bargaining game. It shows that any seller-acceptable buyer offer must be separating. The arguments behind this result are similar to those which prove that in a monotonic signaling game a pooling equilibrium does not survive any of the "standard" refinements (see Cho and Sobel, 1990). When the two buyer types make separating offers, they reveal their types, and so the seller's continuation payoff from rejecting is $(\delta/1 + \delta)w_B^*$ when the buyer's type is $B$ (for $B = L, H$). Given this, the form of the belief-independent separating offers follow from Proposition 3 and the definition of the trade $x_L^0$. Specifically, case $(a)$ of the proposition holds in the N-M regime (where $q_L^0 = q_L^*$), and case $(b)$ holds in the H-M regime (where $q_L^0 < q_L^*$). Subsequent analysis will show that buyer $L$'s equilibrium payoff is no less than $\delta U_L(x_L^0)$. Assumption 2 guarantees that $U_L(x_L^0) > 0$.[16]

Next, consider the following situation: Given the history in any period $t$, the seller's current belief is his prior, and it is his turn to make an offer. Let his menu offer be $\mathbf{y}$, and suppose that buyer $H$'s equilibrium response is to accept $y_H(\mathbf{y})$. Let $U_H(y_H) = v_H$. Then if buyer $L$ rejects $\mathbf{y}$, her optimal serious offer in period $t + 1$ is the trade $x_L(v_H)$ as defined below.

[16] Define $b_L^0$ as the trade that maximizes $U_L(y)$ subject to the constraint that $U_S(y) \geq (\delta/1 + \delta)w_H^*$. Since $w_L^* \geq w_H^*/2$, $U_L(b_L^0) > 0$ for all $\delta < 1$, thus ensuring that $U_L(x_L^0) > 0$.

DEFINITION. For any $v \geq (\delta/1+\delta)w_H^*$, the trade $x_L(v) \equiv [q_L(v), p_L(v)]$ uniquely maximizes $U_L(y)$ subject to the constraints: $U_S(y) \geq (\delta/1+\delta)w_L^*$ and $\delta U_H(y) \leq v$.

The trade $x_L(v)$ has the following properties: In the N-M regime, $x_L(v) = b_L^*$ for all $v \geq U_H(a_H^*)$. In the H-M regime, $x_L(v) = b_L^*$ for all $v \geq \delta U_H(b_L^*)$, and for all $v \in [U_H(a_H^*), \delta U_H(b_L^*)]$, $q_L(v)$ is the smaller solution to $w(q,H) = (\delta/1+\delta)w_L^* + v/\delta$, and $p_L(v) = u(q_L(v), H) - v/\delta$. In the latter regime, $q_L(v) \leq q_L^*$ for all $v \geq U_H(a_H^*)$, and $q_L(v)$ rises in $v$ from $q_L^0$ to $q_L^*$.

Recognize that when buyer $L$ rejects the seller's menu $\mathbf{y}$ in period $t$, it may be optimal for her to remain silent in period $t+1$ rather than to offer $x_L(v_H)$. When is this the case? If and only if the trade $a_L^*$ in period $t+2$ is more profitable for buyer $L$ and less profitable for buyer $H$—that is, when $\delta U_L(a_L^*) > U_L(x_L(v_H))$ and $v_H \geq \delta^2 U_H(a_L^*)$. This leads me to define the following continuation payoff, $V_L(v)$, for buyer $L$.

DEFINITION. For any $v \geq (\delta/1+\delta)w_H^*$,

$$V_L(v) = \begin{cases} \delta U_L(a_L^*) & \text{if } \delta U_L(a_L^*) > U_L(x_L(v)) \text{ and } v \geq \delta^2 U_H(a_L^*) \\ U_L(x_L(v)) & \text{otherwise} \end{cases}$$

LEMMA 5. *Consider any period $t$ such that $\mu(h^{t-1}) = e$, and it is the seller's turn to offer. Suppose that the seller offers a menu $\mathbf{y}$ and that buyer $H$'s equilibrium response is to accept $y_H(\mathbf{y})$. Let $U_H(y_H) = v_H$. Then $v_H \geq (\delta/1+\delta)w_H^*$, and in any continuation equilibrium, either $U_L(y_L) \geq \delta V_L(v_H)$, in which case buyer $L$ accepts $y_L(\mathbf{y})$, or $U_L(y_L) < \delta V_L(v_H)$, in which case buyer $L$ rejects $\mathbf{y}$ and offers $\tilde{y}$ in period $t+1$, where*

$$\tilde{y} = \begin{cases} \textit{silence} & \textit{if } \delta U_L(a_L^*) > U_L(x_L(v_H)) \textit{ and } v_H \geq \delta^2 U_H(a_L^*) \\ x_L(v_H) & \textit{otherwise} \end{cases}$$

Lemma 5 characterizes continuation equilibria when buyer $H$ is the first to accept a seller offer. It provides a partial characterization of the hybrid screening-signaling outcome of the bargaining game by showing how the seller can screen the two buyer types by making an offer that is acceptable only to buyer $H$. When this is the case, and if buyer $H$'s acceptance payoff is $v_H$, buyer $L$'s equilibrium continuation payoff (evaluated at $t$) is $\delta V_L(v_H)$. Further, in the H-M regime, buyer $L$'s serious offer in the following period contains a quantity distortion.

Finally, consider the following scenario in the H-M regime. As in the preceding case, consider any period $t$ of the bargaining game where, given the history, the seller's current belief is his prior, and it is his turn to make an offer. But now suppose that the seller is required to make incentive-compatible offers—an offer $(q_H, p_H)$ to buyer $H$ that gives her an exact payoff $v$ and an offer $(q_L, p_L)$ to buyer

$L$ that gives her at least $\delta V_L(v)$. What are such offers that maximize the seller's payoff? To determine this, we have to solve the following problem:

THE PA($v$) PROBLEM. Maximize $eU_S(q_H, p_H) + (1-e)U_S(q_L, p_L)$ subject to the incentive compatibility (IC) constraints, $U_H(q_H, p_H) \geq U_H(q_L, p_L)$ and $U_L(q_L, p_L) \geq U_L(q_H, p_H)$, and the individual rationality (IR) constraints, $U_H(q_H, p_H) = v$ and $U_L(q_L, p_L) \geq \delta V_L(v)$.

The next lemma derives the unique solution to the PA($v$) problem in the H-M regime.

LEMMA 6. *In the H-M regime, for* $v \in [U_H(a_H^*), U_H(a_L^*)]$, *the following pair of trades constitutes the unique solution to the PA(*$v$*) problem:* $(q_H^*, u(q_H^*, H) - v)$ *for buyer H and* $\{Q_L(v), u(Q_L(v), L) - \delta V_L(v)\}$ *for buyer L, where* $Q_L(v) = R^{-1}(v - \delta V_L(v)) \in (0, q_L^*)$.

For any $v \geq U_H(a_H^*)$ in the PA($v$) problem, buyer $H$'s reservation payoff is greater than buyer $L$'s: $v > \delta V_L(v)$. Given this, the solution to the problem is easily understood from Proposition 2. Specifically, in the H-M regime, for $v \in [U_H(a_H^*), U_H(a_L^*)]$, case ($b$) of the proposition obtains, and this explains the nature of the quantity distortion in the optimal trade for buyer $L$. The subsequent analysis will show that the screening outcome of the bargaining game in the H-M regime will be the solution to the PA($v$) problem for some particular value of $v$. Thus Lemma 6 provides a characterization of the screening outcome in that regime.

## 6. THE EQUILIBRIUM OUTCOMES

This section contains the main results of this article. I present a set of characterization results for the equilibrium outcomes of the bargaining game. (Proofs of all results in this section are in Sections 3 and 4 of the Appendix.)

6.1. *Equilibria in the N-M Regime.* In the N-M regime, the unique equilibrium outcome of the bargaining game is the complete-information equilibrium trade given the buyer's true type. Uniqueness and efficiency of the equilibrium outcome are a consequence of the following result: In the N-M regime, in any continuation equilibrium after any history, the two buyer types make separating offers that are their respective complete-information equilibrium offers. The next lemma establishes this result and its implications for the players' equilibrium continuation strategies.

LEMMA 7. *In any equilibrium in the N-M regime, after any history,* ($a$) *when the buyer makes an offer in any period t with* $\mu(h^{t-1}) = e$, *buyer H offers* $b_H^*$ *and buyer L offers* $b_L^*$, ($b$) *given any seller offer* $\mathbf{y}$, *buyer H accepts* $y_H(\mathbf{y})$ *if and only if* $U_H(y_H) \geq (\delta/1+\delta)w_H^*$, *and* ($c$) *when the seller makes an offer in any period t with* $\mu(h^{t-1}) = e$, *he offers the menu* $\{a_H^*, a_L^*\}$.

Lemma 7 leads to the following result:

PROPOSITION 4. *In the N-M regime, equilibria exist in the "seller-initiated" game (i.e., where the seller makes the first offer) and in the "buyer-initiated" game. The equilibrium outcome is unique: If the buyer's type is $B$ (for $B = L, H$), the trade $a_B^*$ is executed at $t = 0$ in the seller-initiated game, and the trade $b_B^*$ is executed at $t = 0$ in the buyer-initiated game. In the limit $z \to 0$, there is a unique equilibrium outcome irrespective of who makes the first offer: At $t = 0$, buyer $B$ buys $q_B^*$ paying $[c(q_B^*) + w_B^*/2]$.*

In the N-M regime, the high-valuation buyer's dominant incentive is to create value by revealing her type. Here, buyer $H$ gets enough surplus from her complete-information trade so that it is not worth her while to pretend that her valuation is low by restricting the quantity demanded, even though the per-unit price would then be lower. This leads to *ex post* efficient trading as the unique equilibrium outcome.[17]

6.2. *Stationary Equilibria in the H-M Regime.* As indicated earlier, I focus on characterizing stationary equilibria in the H-M regime. In a stationary equilibrium, the seller's offer strategy after any history is a mapping from the belief space to the space of feasible trades. Let $\mathbf{Y}$ denote the seller's equilibrium offer after any history $h$ such that $\mu(h) = e$. Also, let $U_H(y_H(\mathbf{Y})) \equiv V_H$. The next lemma shows that (1) either $\mathbf{Y}$ contains the trade $a_H^*$ that is accepted by buyer $H$ while buyer $L$ rejects $\mathbf{Y}$ or (2) $\mathbf{Y}$ contains two distinct trades such that $y_B(\mathbf{Y})$ is accepted by buyer $B$ for $B = L, H$. In the latter case, $\mathbf{Y}$ solves the PA($v$) problem (described in Section 5) for $v = V_H$.

LEMMA 8. *(a) $V_H \in [U_H(a_H^*), U_H(a_L^*)]$ and buyer $H$ accepts $y_H(\mathbf{Y})$. (b) If $U_L[y_L(\mathbf{Y})] < \delta V_L(V_H)$, then $y_H(\mathbf{Y}) = a_H^*$, and buyer $L$ rejects $\mathbf{Y}$ and offers $x_L^0$ in the next period. (c) If $U_L(y_L) \geq \delta V_L(V_H)$, then $\mathbf{Y}$ solves PA$(v = V_H)$.*

This lemma completes the characterizations of the screening outcome and the hybrid screening-signaling outcome in any stationary equilibrium in the H-M regime.

PROPOSITION 5. *In the H-M regime, when the time between offers is close to zero, stationary equilibria exist in the "seller-initiated" game and in the "buyer-initiated" game. Every stationary equilibrium in the seller-initiated game generates one of the following paths:*

> *Path S1: The seller offers a menu $\{Y_H^0, Y_L^0\}$ at $t = 0$, and buyer $B$ ($B = L, H$) accepts $Y_B^0$. Here $\{Y_H^0, Y_L^0\}$ solves $PA(v = U_H(Y_H^0))$, and $U_H(Y_H^0) \in [U_H(a_H^*), U_H(a_L^*)]$.*

[17] This outcome is obviously belief-independent. Note that in this private values model, if only the informed buyer could make offers, the efficient trades would always occur in equilibrium. Proposition 4 shows that in the N-M regime, the efficiency result obtains even when the two parties alternate in making offers.

*Path $S2$: The seller offers $a_H^*$ at $t = 0$ that buyer $H$ accepts; buyer $L$ offers $x_L^0$ at $t = 1$ that the seller accepts.*

*Every stationary equilibrium in the buyer-initiated game generates one of the following paths:*

*Path $B1$: At $t = 0$, buyer $H$ offers $b_H^*$, buyer $L$ offers $x_L^0$, and the seller accepts both offers.*

*Path $B2$: The buyer is silent at $t = 0$; the seller offers a menu $\{Y_H^1, Y_L^1\}$ at $t = 1$, and buyer $B$ $(B = L, H)$ accepts $Y_B^1$. Here, $\{Y_H^1, Y_L^1\}$ solves $PA(v = U_H(Y_H^1))$, and $U_H(Y_H^1) \in [(1/\delta)U_H(b_H^*), U_H(a_L^*)]$.*

*In the limit $z \to 0$, there is no delay in trade irrespective of who makes the first offer.*

Proposition 5 delineates the properties of all stationary equilibrium outcomes in the H-M regime when $z$ is close to zero. Here, buyer $H$ wants to conceal her valuation to capture value. Specifically, if she accepts the complete-information trade for buyer $L$, her loss from buying a smaller quantity is more than compensated for by the lower per-unit price. Thus there is a quantity distortion in buyer $L$'s trade. Over and above this, she also can face a one-period delay in trade vis-à-vis buyer $H$. It is these inefficiencies in buyer $L$'s trade that makes buyer $H$ indifferent between revealing and concealing her valuation in equilibrium.

My next objective is to provide a more precise characterization of a subset of the set of stationary equilibria in the H-M regime. To this end, I look at "separating equilibria" as defined below.

DEFINITION. A *separating equilibrium* is an equilibrium that satisfies the following restriction: When the buyer makes an offer after any history $h$ such that $\mu(h) \in (0, 1)$, the two buyer types make distinct (i.e., separating) offers.

The following result characterizes the set of separating equilibrium outcomes in the H-M regime when the time between offers is close to zero and shows that this set is indeed a subset of the set of stationary equilibrium outcomes.

PROPOSITION 6. *In the H-M regime, separating equilibria exist for $z$ close to zero, and every separating equilibrium outcome can be supported as a stationary equilibrium. Depending on parameter values, a separating equilibrium in the seller-initiated game generates either Path $S1$ or Path $S2$ (as described in Proposition 5). Every separating equilibrium in the buyer-initiated game uniquely generates Path $B1$. In the limit $z \to 0$, the following is the unique separating equilibrium outcome irrespective of who makes the first offer: At $t = 0$, buyer $H$ buys $q_H^*$ paying $[c(q_H^*) + w_H^*/2]$, and buyer $L$ buys $\bar{q}_L^0 \equiv \lim_{z \to 0} q_L^0$ paying $[c(\bar{q}_L^0) + w_L^*/2]$.*

Propositions 5 and 6 establish the nature of equilibrium inefficiencies in the bargaining game in the H-M regime for any given $z$ close to zero, as well as in the limit $z \to 0$. Further, these results enable the comparison of the buyer's equilibrium payoffs to her Rubinstein payoffs. And finally, the results demonstrate the existence of stationary equilibria that do not depend on the seller's prior beliefs. The separating equilibrium outcome in the buyer-initiated game is belief-independent, as is the limit ($z \to 0$) separating equilibrium outcome.

This article has considered the case where the buyer's private information takes only one of two values. I conclude this section by commenting on the case where there are more than two types of buyers. While an extensive discussion of this case is outside the scope of this article, the following point is to be noted: In the current game, the "no mimicking" regime exists only when there is a "sufficient gap" between the two buyer valuations.[18] As one considers an increase in the number of possible buyer types, with adjoining types getting closer, the "no mimicking" regime may fail to exist. In the extreme, if the buyer's type is drawn from a continuum, every interior buyer type will want to mimic the type "just below" it. Thus, given the structure of payoffs and information asymmetry in the current article, all equilibrium outcomes are likely to be *ex post* inefficient in a continuum-of-types model. However, there may exist other multidimensional negotiation scenarios where an informed player's dominant incentive will be to create value even in a continuum-of-types model. Then the negotiation outcome may be *ex post* efficient for the same underlying reason that it is in the "no mimicking" regime in the current game. This remains an open issue for future research.

## 7. RELATIONSHIP TO THE LITERATURE

In this concluding section I discuss the relationship between the results in the current article and those in the existing bargaining and principal-agent literatures. Given the discussion regarding parameter regimes at the end of Section 6, I restrict attention to the case where the parameters belong in the H-M regime in the current bargaining game.

As compared with most articles on bargaining under incomplete information, the current model introduces two new features: (1) the quantity of trade is variable, with the efficient quantity depending on the buyer's type, and (2) the uninformed seller can make menu offers. Recognize that given the first feature, the second one is a natural specification.[19] However, it is not the seller's ability to make menu offers that is critical in generating the important properties of the bargaining equilibria. In a game where the seller can only make a single offer, there exist stationary equilibria whose outcomes have similar properties as the equilibrium outcomes described in Section 6. The seller's inability to make menu offers has the effect of delaying trade with the low-valuation buyer by one period without affecting the qualitative proper-

[18] The N-M regime exists for $z$ close to zero if and only if $(w_H^* + w_L^*) > 2w(q_L^*, H)$. When the parameters $L$ and $H$ are arbitrarily close in value, and as a result $q_L^*$ is arbitrarily close to $q_H^*$, the difference between $w(q_L^*, H)$ and $w_H^*$ $(\equiv w(q_H^*, H))$ is second-order small. In this case, the required inequality cannot be satisfied.

[19] In the existing literature, bargaining games are studied for the case where offers can be made arbitrarily quickly so as to minimize precommitment and exogenous delay built into the extensive form. The same logic dictates that in the current game the uninformed player should be allowed to make menu offers.

ties of the equilibrium trades. In particular, in the "single offer by the seller game," the limit ($z \to 0$) outcome described in Proposition 6 is a stationary equilibrium outcome in the H-M regime.

It is the first feature that generates the fundamental difference between the current model and the one-dimensional bargaining models. Each player's offer in the current model is a price-quantity pair. Since the indifference curves of the two buyer types exhibit single crossing, the buyer types can separate while making offers and reveal their private information. This is not possible when the players are bargaining just over price, given a fixed quantity. In the H-M regime, the quantity dimension is used by the uninformed seller in screening and by the informed buyer in signaling. The upshot of this is the existence of a quantity distortion in all stationary equilibria. This obviously has no analogue in the one-dimensional bargaining models. Further, given that information is transmitted by quantity choices, the maximum equilibrium delay is the time it takes to make a single counteroffer.

It is instructive to compare the properties of the "limit equilibria" (i.e., when $z$ limits to zero) in the current bargaining game with those in Grossman and Perry (1986b) and Gul and Sonnenschein (1988). Both articles consider alternating-offer buyer/seller bargaining over an indivisible good, where the buyer's private valuation is distributed over an interval $[\underline{b}, \bar{b}]$, and the seller's cost is zero.[20] The two articles impose different (but related) sets of refinement restrictions on the set of PBE. In the equilibria in these articles, the delay inefficiency vanishes as the time between offers approaches zero. This is congruent with the result in the current bargaining game. However, since delay can be the only inefficiency in the indivisible-good models, both articles predict that the limit equilibrium outcome is *ex post* efficient. This is not the case in the limit equilibria in the H-M regime of the current model, where the quantity distortion persists.

Further, there are important differences in the "limit payoffs" to the buyer. In Grossman and Perry (1986b), the equilibrium outcome (when it exists) converges to immediate trade at the price $\underline{b}/2$, which is the "Rubinstein price" for the lowest-valuation buyer. Thus the payoff to any buyer type $b > \underline{b}$ is strictly greater than his or her Rubinstein payoff. In Gul and Sonnenschein (1988), on the other hand, there can be multiple-limit equilibria, and the authors provide an example where the limit-equilibrium outcome converges to immediate trade at price $\underline{b}$. In this case, the equilibrium payoffs to some low-valuation buyers are smaller than their Rubinstein payoffs. In the limit-separating equilibrium in the H-M regime of the current bargaining game, buyer $H$'s payoff is her Rubinstein payoff, while buyer $L$'s payoff is strictly less than her Rubinstein payoff. In fact, the way that buyer $L$ separates from buyer $H$ is by agreeing on an inefficient trade and by then giving the seller *his* Rubinstein payoff with respect to buyer $L$.

The limit equilibrium in Grossman and Perry (1986b) is belief-independent. This is also the case for some limit equilibria in the current model. However, it is

[20] These articles study the "gap case" where $\underline{b} > 0$. Ausubel and Deneckere (1992) study a model that is similar to that in Gul and Sonnenschein (1988), except that they consider the case of "no gap" where $\underline{b} = 0$.

important to note the following point: In the limit equilibrium of the former model, the seller (effectively) makes a single offer of price $\underline{b}/2$, and all buyer types pool in accepting this offer. In contrast, there exist belief-independent limit equilibria in the current model where there is separation along the equilibrium path. Specifically in the H-M regime, whenever the stationary equilibrium outcome is a "signaling outcome," it is belief-independent. The limit-separating equilibrium outcome in the H-M regime is belief-independent because the signaling outcome, the screening outcome, and the hybrid outcome all coincide in the limit.

The preceding discussion has compared the current analysis to the one-dimensional bargaining models, which constitute the majority of the bargaining articles in the literature. Wang (1998) is one article that studies two-dimensional bargaining under asymmetric information, where an informed worker and an uninformed firm negotiate quality of output and wage. In contrast to the symmetric bargaining structure of this article, Wang considers a model where only the uninformed party can make offers. In this bargaining game, the author establishes the following result: The equilibrium agreement is the same as that in the one-shot principal-agent (i.e., screening) game, and the agreement is reached without any delay. This result, which is in sharp contrast to the results in this article, is a consequence of the posited bargaining structure. In Wang's model, the uninformed party "effectively" has all the bargaining power, and there is no tension between creating and capturing value. Given this, the discussion in the preceding sections of this article provide the intuition behind Wang's result.

How does the outcome of the principal-agent game compare with that in the current bargaining game? At the optimal trades of the one-shot screening game, there is a quantity distortion for the low-valuation buyer (see Proposition 2). Thus the equilibrium quantity distortion in the H-M regime of the bargaining game is qualitatively similar to that in the principal-agent formulation. However, the determinants of the magnitude of quantity distortion are very different in the two formulations. As explained in Section 3, the quantity distortion in the principal-agent model depends on the seller's prior belief. In contrast, the quantity distortion in the H-M regime in the bargaining game depends on the equilibrium-continuation payoffs of the players and is belief-independent conditional on the time of trade. In this context, the important point to note is the following: In any negotiation game under asymmetric information, equilibrium trades are determined by the interaction of "incentive compatibility" and "continuation individual rationality" constraints. It is precisely because the latter depend on the negotiation process that different bargaining structures predict different outcomes.

Finally, a limitation of the current study needs to be noted. The bargaining game in this article models the negotiation of a one-time trade and follows the rules of the standard Rubinstein/Stahl extensive form in which a player's acceptance irrevocably concludes the game. It is this feature that generates the result that there always exists a quantity distortion in the H-M regime. Note that in any stationary equilibrium path in this regime, at the time when the seller and the low-valuation buyer agree on an inefficient trade, the seller can infer the buyer's type, and thus both parties recognize that there exist additional gains from trade that they are foregoing. However, they cannot do anything about it because acceptance ends the game.

Such a specification may not be appropriate for studying long-term contractual relationships, where the acceptance of a long-term contract leaves open the possibility of subsequent renegotiation. Laffont and Tirole (1990) and Beaudry and Poitevin (1993) have studied agency relationships where it is possible for the players to renegotiate initially accepted contracts (also see Hart and Tirole, 1988). The first article considers a two-period model where the uninformed party makes all offers, and the second article considers infinite-horizon renegotiation in a "common values" model where the informed party makes all offers. These articles thus specify asymmetric bargaining positions of the two parties, as well as limited channels of information transmission. The equilibria in both analyses exhibit persistent quantity distortions. In contrast, preliminary analysis in Ausubel and Sen (1995) suggests that in an alternating-offers model of contract (re)negotiation, such quantity distortions may not persist indefinitely. These findings underscore the need for further research on long-term contracts by embedding the process of contract negotiation and renegotiation within a symmetric bargaining structure.

## APPENDIX

*Section* 1. This section presents the proofs of Lemmas 1 and 2.

PROOF OF LEMMA 1. Suppose that in some period $t$ in any continuation game, the buyer accepts a seller offer $y \equiv (q, p)$ such that $U_S(y) = V_S^+ - \epsilon$, for some $\epsilon$ arbitrarily close to zero. (The proof when the seller accepts a buyer offer like above is similar.) Then buyer $B$'s payoff (for $B = L, H$) at $t$ is $[w(q, B) - V_S^+ + \epsilon]$. Note that if the buyer rejects $y$ and offers $y' \equiv (q, p')$ in period $t + 1$, where $U_S(y') = \delta V_S^+ + \eta$ for some $\eta > 0$, the seller will accept this offer, and the buyer's payoff evaluated at $t$ will be $\delta[w(q, B) - \delta V_S^+ - \eta]$. Observe that if $V_S^+ > (1/1 + \delta)w_H^*$, then for $\epsilon$ close to zero, $\delta[w(q, B) - \delta V_S^+] > [w(q, B) - V_S^+ + \epsilon]$ for $B = L, H$. Then there exists an $\eta > 0$ such that the above-described deviation will be profitable for the buyer irrespective of his or her type. This proves that $V_S^+ \leq (1/1 + \delta)w_H^*$. Given this, it is easy to argue that in any continuation PBE, after any history, the seller will accept an offer if $U_S(y) \geq (\delta/1 + \delta)w_H^*$, and buyer $H$ will reject an offer $y_H(\mathbf{y})$ if $U_H(y_H) < (\delta/1 + \delta)w_H^*$.

Next, suppose that in some period $t$ in any continuation game, the seller accepts an offer $y \equiv (q, p)$ such that $U_L(y) = V_L^+ - \epsilon$, for some $\epsilon$ arbitrarily close to zero. Then the seller's payoff at $t$ is $[w(q, L) - V_L^+ + \epsilon]$. Define the trade $y'' \equiv (q, p'')$, where $U_L(y'') = \delta V_L^+ + \eta$ for some $\eta > 0$. First, suppose that $U_H(y'') < (\delta/1 + \delta)w_H^*$. In this case, note that if the seller rejects $y$ and offers the menu $\{a_H^*, y''\}$ in period $t + 1$, buyer $L$ will accept $y''$, and buyer $H$ will accept $a_H^*$, since rejection of the menu will cause the seller to prune buyer $L$ from his belief set, by Assumption 3. Then the seller's expected payoff (at $t$) will be $\delta\{\xi[(1/1 + \delta)w_H^*] + (1 - \xi)[w_L^* - \delta V_L^+ - \eta]\}$, where $\xi \in [0, 1]$ is his belief after observing $y$. Second, suppose that $U_H(y'') \geq (\delta/1 + \delta)w_H^*$. In this case, if the seller rejects $y$ and offers the single trade $y''$ in period $t + 1$, both buyer types will accept it, and the seller's payoff (at $t$) will be $\delta[w(q, L) - \delta V_L^+ - \eta]$. It is easy to show that if $V_L^+ > (1/1 + \delta)w_L^*$, then for $\epsilon$ close to zero, there exists an $\eta > 0$ such that both the "deviating" payoffs described earlier are greater than $[w(q,L) - V_L^+ + \epsilon]$ for all $\xi \in [0,1]$ and for all $q$. This proves that $V_L^+ \leq (1/1+\delta)w_L^*$. This implies that in any continuation PBE, buyer $L$ will accept any seller offer $y_L(\mathbf{y})$ if $U_L(y_L) \geq (\delta/1+\delta)w_L^*$. Finally, note that instead of accepting an offer $y$ such that $U_S(y) < (\delta/1+\delta)w_L^*$ after any history, the seller will be better off rejecting and offering the menu $\{a_H^*, a_L^*\}$ (buyer $L$ will accept $a_L^*$, and so buyer $H$ will either accept $a_H^*$ or $a_L^*$). Thus, in any continuation PBE, the seller will reject an offer $y$ if $U_S(y) < (\delta/1+\delta)w_L^*$. ∎

PROOF OF LEMMA 2. This lemma is proved by showing that under Assumption 4, the seller's supremum payoff in any seller-initiated continuation game starting with belief $\mu = 0$ ($\mu = 1$), denoted by $V_S^+(0)$ $[V_S^+(1)]$, is $(1/1+\delta)w_L^*$ $[(1/1+\delta)w_H^*]$. This can be shown using arguments that are very similar to the proof of Lemma 2.2 in Admati and Perry (1987). For instance, suppose that $V_S^+(0) > (1/1+\delta)w_L^*$. Then there exists a continuation equilibrium whose outcome, if the buyer's type is $L$, is agreement on a trade $y'$ in period $t$, where $U_S(y')$ is arbitrarily close to $V_S^{S+}(0)$. Suppose that the buyer makes this offer in period $t$. Note that $y'$ must contain a positive quantity, since otherwise $U_L(y') < 0$. Then, since $R'(q) > 0$, and since $U_S(y') > (1/1+\delta)w_L^*$, there exists a trade $y''$ such that $U_L(y'') > U_L(y')$, $U_H(y'') < U_H(y')$, and $U_S(y'') > \delta V_S^+(0)$. Thus, if buyer $L$ deviates in period $t$ and offers $y''$, the seller will accept $y''$ (by Assumption 4), and buyer $L$ will be better off under the deviation. ∎

*Section* 2. This section presents the proofs of Lemmas 3, 4, 5, and 6.

PROOF OF LEMMA 3. Define $\alpha \equiv U_H(a_H^*) - U_H(a_L^*) = (\delta/1+\delta)(w_H^* - w_L^*) - R(q_L^*)$, and $\beta \equiv U_H(b_H^*) - \delta U_H(a_L^*) = (1/1+\delta)(w_H^* - \delta^2 w_L^*) - \delta R(q_L^*)$. Observe that $d\alpha/d\delta > 0$, $d\beta/d\delta < 0$, and $\lim_{\delta\to 1}\alpha = \lim_{\delta \to 1}\beta = (w_H^* - w_L^*)/2 - R(q_L^*)$. Thus, if $w_H^* - w_L^* > 2R(q_L^*)$, there exists $\delta_\alpha < 1$ such that $\alpha \geq 0$ for all $\delta \geq \delta_\alpha$, and if $w_H^* - w_L^* < 2R(q_L^*)$, there exists $\delta_\beta < 1$ such that $\beta < 0$ for all $\delta \geq \delta_\beta$. Set $\delta^* = \max[\delta_\alpha, \delta_\beta]$, and define $z^*$ such that $\delta^* = e^{-rz^*}$. ∎

PROOF OF LEMMA 4. First, suppose that the two buyer types make a serious pooling offer $y^P$. If $U_S(y^P) = (\delta/1+\delta)w_L^*$, the seller's best response is to reject $y^P$ for the following reason: If he rejects $y^P$ and offers $\{y'_H, a_L^*\}$ in the following period, where

$$y'_H = \begin{cases} a_H^* & \text{if } U_H(a_H^*) \geq U_H(a_L^*) \\ [q_H^*, u(q_H^*, H) - U_H(a_L^*)] & \text{otherwise} \end{cases}$$

then buyer $L$ will accept $a_L^*$ and buyer $H$ will accept $y'_H$ (by Assumptions 3 and 4), and the seller's present value payoff will be greater than $(\delta/1+\delta)w_L^*$. Now suppose that $U_S(y^P) > (\delta/1+\delta)w_L^*$ and that the seller's equilibrium response is to accept $y^P$. Then, by Lemma 1, it must be that $U_H(y^P) \geq (\delta/1+\delta)w_H^*$. But then there exists a serious offer $y''$ satisfying $U_L(y'') > U_L(y^P)$ and $U_H(y'') < U_H(y^P)$. Thus, offering $y''$ is a profitable deviation for buyer $L$ (by Assumption 4). This proves that any pooling buyer offer must be silence.

Now suppose that the two buyer types make separating offers, $x_H$ and $x_L$. First, since buyer $H$ reveals her type, her supremum continuation payoff is $(1/1+\delta)w_H^*$. She can ensure this payoff by offering $b_H^*$. Thus $x_H = b_H^*$. Second, Assumption 4 and Lemma 2 imply that

$$x_L = \begin{cases} \text{silence} & \text{if } \delta U_L(a_L^*) > U_L(x_L^0) \text{ and } U_H(b_H^*) \geq \delta U_H(a_L^*) \\ x_L^0 & \text{otherwise} \end{cases}$$

Note that $U_L(x_L^0) > \delta U_L(a_L^*)$ in the N-M regime and $\delta U_H(a_L^*) > U_H(b_H^*)$ in the H-M regime. Thus $x_L = x_L^0$. By Lemma 2, the seller accepts $b_H^*$ and $x_L^0$. ■

PROOF OF LEMMA 5. Lemma 1 implies that $v_H \geq U_H(a_H^*)$. Further, Lemmas 1 and 2, Assumption 4, and the definition of the trade $x_L(v)$ imply that $\tilde{y}$ is buyer $L$'s equilibrium offer in period $t+1$ when she rejects the seller's period $t$ offer. This implies (by Lemma 2) that her continuation payoff evaluated at time $t$ is $\delta V_L(v_H)$. ■

PROOF OF LEMMA 6. In the H-M regime, $U_H(a_L^*) > \delta U_H(b_L^*) > U_H(a_H^*)$ and $\delta V_L(v) < v$ for any $v \geq U_H(a_H^*)$. Further, for $v \in [U_H(a_H^*), U_H(a_L^*)]$, $V_L(v) \geq U_L(x_L(v))] = v/\delta - R(q_L(v))$. Thus, for $v \in [U_H(a_H^*), U_H(a_L^*)]$, $v - \delta V_L(v) \in (0, R(q_L^*))$. Define $z_H^*(v) \equiv [q_H^*, u(q_H^*, H) - v]$, and $z_L^*(v) \equiv [q_L^*, u(q_L^*, L) - \delta V_L(v)]$. Note that $U_H(z_L^*(v)) = R(q_L^*) + \delta V_L(v)$, and $U_L(z_H^*(v)) = v - R(q_H^*)$. Also note that in the H-M regime, for all $v \in [U_H(a_H^*), U_H(a_L^*)]$, $\delta V_L(v) - U_L(z_H^*(v)) \geq \delta U_L(x_L(v)) - U_L(z_H^*(v)) = R(q_H^*) - \delta R(q_L(v)) > 0$, and $U_H(z_L^*(v)) - v = R(q_L^*) - [v - \delta V_L(v)] > 0$. These inequalities imply that for all $v \in [U_H(a_H^*), U_H(a_L^*)]$, buyer $H$'s IC constraint and buyer $L$'s IR constraint bind at the optimum of PA($v$), while buyer $L$'s IC constraint does not. Thus the solution to the following problem is the solution to PA($v$): Maximize $eU_S(q_H, p_H) + (1-e)U_S(q_L, p_L)$ subject to $U_H(q_H, p_H) = U_H(q_L, p_L) = v$ and $U_L(q_L, p_L) = \delta V_L(v)$. Straightforward optimization shows that the unique solution to this problem is as described in Lemma 6. In subsequent proofs, this solution will be denoted by $\{Z_H(v), Z_L(v)\}$. Note that $U_S(Z_H(v)) = w_H^* - v$, and $U_S(Z_L(v)) = w[Q_L(v), H] - v$. ■

*Section* 3. Here I present the proofs of Lemmas 7 and 8 and Propositions 4 and 5.

PROOF OF LEMMA 7. Consider buyer $H$'s continuation payoff starting from any $t$ when it is the buyer's turn to offer. First, if the buyer types make separating offers at $t$, then buyer $H$'s payoff is $(1/1+\delta)w_H^*$ (by Lemma 4). Second, if the buyer types remain silent at $t$, then either no agreement is ever reached, or an agreement is reached in the future in one of the following ways: (a) the two buyer types making separating offers, (b) buyer $L$ accepts a seller offer before buyer $H$ does, or (3) buyer $H$ accepts a seller offer no later than buyer $L$ does. In all these outcomes in the N-M regime, buyer $H$'s present value payoff is less than $(1/1+\delta)w_H^*$. Thus, in the N-M regime, buyer $H$'s supremum continuation payoff is $(1/1+\delta)w_H^*$. This result, in conjunction with Lemma 4, establishes parts (*a*) and (*b*) of the lemma. Then part (*c*) follows from Lemma 5, since $U_H(a_H^*) \geq U_H(a_L^*)$ and $V_L(v) = U_L(b_L^*)$ for all $v \geq U_H(a_H^*)$ in the N-M regime—specifically, if the seller's equilibrium menu offer following belief $\mu = e$ was different from the mean $\{a_H^*, a_L^*\}$, offering $\{a_H^*, a_L^*\}$ would be a profitable deviation for him. ■

Proof of Proposition 4. The previous lemmas prove that no other outcomes can be equilibrium in the N-M regime. An equilibrium generating the specified outcome is easily constructed (along similar lines as the construction in Section 4 of this Appendix) and is available from the author. The limit result is immediate. ■

Proof of Lemma 8. (*a*) If the equilibrium specifies that buyer $H$ rejects $y_H(\mathbf{Y})$ and buyer $L$ accepts $y_L(\mathbf{Y})$, then it also must specify that buyer $H$ offers $b_H^*$ in the next period. In this case, the seller can gainfully deviate and offer $\{a_H^*, y_L(\mathbf{Y})\}$ after any history when his or her belief is $e$ and have buyer $H$ accept $a_H^*$ and buyer $L$ accept $y_L(\mathbf{Y})$. If the equilibrium specifies that both buyer types reject $\mathbf{Y}$, then it also must specify that they make separating offers in the next period (by stationarity). In this case, the seller can gainfully deviate and offer the single trade $\{a_H^*\}$ and have buyer $H$ accept $a_H^*$. This proves that buyer $H$ accepts $y_H(\mathbf{Y})$ in a stationary equilibrium. Then $V_H \geq U_H(a_H^*)$ by Lemma 1. Further, in the H-M regime, $V_L(v) = U_L(b_L^*)$ for all $v \geq U_H(a_L^*)$. Thus, if $V_H > U_H(a_L^*)$, the seller can profitably deviate and offer $\{y''', a_L^*\}$, where $y'''$ has the same quantity as in $y_H(\mathbf{y})$ and satisfies $U_H(y''') = U_H(a_L^*)$.

(*b*) If $U_L(y_L(\mathbf{Y})) < \delta V_L(V_H)$, then, by Lemma 5, buyer $L$ rejects, and her continuation payoff is $\delta V_L(V_H)$. Recall that for all $v \in [U_H(a_H^*), U_H(a_L^*)]$ in the H-M regime, $\delta V_L(v) > U_L(z_H^*(v))$. Then stationarity implies that $y_H(\mathbf{Y}) = z_H^*(V_H)$. Further, if $V_H > U_H(a_H^*)$ when $U_L(y_L(\mathbf{Y})) < \delta V_L(V_H)$, then there exists an $\epsilon > 0$ such that if the seller offered the single trade $z_H^*(V_H - \epsilon)$, buyer $H$ would accept (by stationarity), and the deviation would be profitable. Thus $y_H(\mathbf{Y}) = a_H^*$. Then, since $\delta^2 U_H(a_L^*) > U_H(a_H^*) = \delta U_H(b_H^*)$ in the H-M regime, buyer $L$'s offer in the next period is $x_L(v = U_H(a_H^*)) = x_L^0$ (by Lemma 5).

(*c*) If $U_L(y_L(\mathbf{Y})) \geq \delta V_L(V_H)$, then $V_H \in [U_H(a_H^*), U_H(a_L^*)]$, and buyer $H$ accepts $y_H(\mathbf{Y})$ and buyer $L$ accepts $y_L(\mathbf{Y})$ (by Lemma 5). If $\mathbf{Y}$ does not solve PA($V_H$), the seller can profitably deviate and offer the mean $\{Z_H(V_H), Z_L(V_H)\}$ after any history when his belief is $e$. In any stationary equilibrium, buyer $H$ will accept $Z_H(V_H)$, and so buyer $L$ will accept $Z_L(V_H)$. ■

Proof of Proposition 5. The characterization result for the seller-initiated game follows from Lemmas 5 and 8. In any stationary equilibrium in the buyer-initiated game, either both buyer types pool to remain silent at $t = 0$ or they separate. If they separate, the equilibrium outcome path is Path B1 by Lemma 4. Note that buyer $H$ will remain silent at $t = 0$ only if $\delta U_H(Y_H^1) > U_H(b_H^*)$. This and Lemmas 5 and 8 explain Path B2. The limit result is immediate. Finally, the statement of Proposition 6 (which is proved in the next section) establishes that stationary equilibria exist in the H-M regime for $z$ close to zero. ■

*Section* 4. This section presents the proof of Proposition 6. In what follows, it is maintained that the parameters belong in the H-M regime. Define the following payoffs:

$$V_S^0 \equiv eU_S(a_H^*) + (1-e)\delta U_S(b_L^*) \equiv e(1/1+\delta)w_H^* + (1-e)(\delta^2/1+\delta)w_L^*$$

$$\overline{V}_S \equiv \max\{eU_S(Z_H(v)) + (1-e)U_S(Z_L(v))|v \in [\delta U_H(b_L^*), U_H(a_L^*)]\}$$

and $\overline{V}_S(v) \equiv eU_S[Z_H(v)] + (1-e)U_S[Z_L(v)]$ for $v \in [U_H(a_H^*), \delta U_H(b_L^*)]$

Note that since $V_L(v) = U_L(b_L^*)$ for all $v \in [\delta U_H(b_L^*), U_H(a_L^*)]$, $\overline{V}_S$ is well defined.

*Claim* 1. *There exists* $z_0 > 0$ *and* $\Gamma_0 > 0$ *such that for all* $z < z_0$: (*a*) $\overline{V}_S < V_S^0$, (*b*) *for all* $v \in [(\delta/1+\delta)w_H^*, w_H^*/2 + \Gamma_0]$, $\overline{V}_S(v)$ *is continuous in* $v$, *and* $\lim_{z\to 0}\{\overline{V}_S(v)\} = e(w_H^* - v) + (1-e)w_L^*/2$, *and* (*c*) *for all* $v > w_H^*/2 + \Gamma_0$, $\overline{V}_S(v) < V_S^0$.

Proof. For all $v \in [\delta U_H(b_L^*), U_H(a_L^*)]$, $U_S(Z_H(v)) \le (1/1+\delta)w_H^*$, and $U_S(Z_L(v)) < (1/1+\delta)w_L^*$ for all $\delta < 1$. This implies that $\overline{V}_S < V_S^0$ for $z$ sufficiently close to zero. Next, since $\lim_{\delta\to 1}\delta^2 U_H(a_L^*) > w_H^*/2$, and since $V_L(v) = U_L(x_L(v))$ for $v < \delta^2 U_H(a_L^*)$, there exists $z_1 > 0$ and $\Gamma_0 > 0$ such that for all $z \le z_1$ and for all $v \in [U_H(a_H^*), w_H^*/2 + \Gamma_0]$, $V_L(v) = U_L(x_L(v))$, which is a continuous function of $v$. Thus, in this case, $\overline{V}_S(v) = e(w_H^* - v) + (1-e)\{w(Q_L(v), H) - v\}$ is a continuous function of $v$. Further, note (from the proof of Lemma 6) that when $V_L(v) = U_L(x_L(v))$, $R(Q_L(v)) = \delta R(q_L(v))$, and so, $\lim_{\delta\to 1}Q_L(v) = \lim_{\delta\to 1}q_L(v)$. Then, using the definition of $q_L(v)$, $\lim_{z\to 0}U_S(Z_L(v)) = w_L^*/2$, and we have the limiting result. Finally, consider any $v > w_H^*/2 + \Gamma_0$ and any $z$ sufficiently close to zero. If for this $v$ and $z$ it is the case that $V_L(v) = U_L(x_L(v))$, then the preceding result implies that $\overline{V}_S(v) < V_S^0$. Alternatively, if it is the case that $V_L(v) = \delta U_L(a_L^*)$, even then $\overline{V}_S(v) < V_S^0$ because $\lim_{z\to 0}U_S(Z_H(v)) < w_H^*/2$ [since $\lim_{\delta\to 1}\delta^2 U_H(a_L^*) = R(q_L^*) + w_L^*/2 > w_H^*/2$], and $\lim_{z\to 0}U_S(Z_L(v)) \le w_L^*/2$ [since $\lim_{\delta\to 1}\delta U_L(a_L^*) = w_L^*/2$]. From these arguments it is clear that there exists $z_0 > 0$ and $\Gamma_0 > 0$ such that for all $z < z_0$, the three parts of the claim hold. ■

For $z < z_0$, define $V^* \equiv \operatorname{argmax}\{\overline{V}_S(v)|v \in [(\delta/1+\delta)w_H^*, w_H^*/2 + \Gamma_0]\}$, which is nonempty by Claim 1. Define $\overline{V}_S^* \equiv \overline{V}_S(v^*)$, where $v^* \in V^*$. Existence of stationary equilibria in the H-M regime is proved by constructing stationary separating equilibria for $z < z_0$.

*Claim* 2. *In a separating equilibrium, after any history with* $\mu = e$: (*a*) *given a seller offer* **y**, *buyer H accepts* $y_H(\mathbf{y})$ *if and only if* $U_H(y_H) \ge U_H(a_H^*)$, *and* (*b*) *if* $\overline{V}_S^* > V_S^0$, *the seller offers* $\{Z_H(v^*), Z_L(v^*)\}$ *for some* $v^* \in V^*$, *and if* $V_S^0 > \overline{V}_S^*$, *the seller offers* $a_H^*$.

Proof. Claim 2 is proved by similar arguments as in the proof of Lemma 8 and is available from the author. ■

For $z < z_0$, Claims 1 and 2 imply the following results: The candidate separating equilibrium outcome in the buyer-initiated game is unique and belief-independent:

At $t = 0$, buyer $H$ offers $b_H^*$, buyer $L$ offers $x_L^0$, and the seller accepts these offers. In the seller-initiated game, if $\bar{V}_S^* > V_S^0$, every candidate separating equilibrium outcomes takes the following form: The seller offers $\{Z_H(v^*), Z_L(v^*)\}$ for some $v^* \in V^*$ at $t = 0$, and buyer $B$ $(B = L, H)$ accepts $Z_B(v^*)$, and if $V_S^0 > \bar{V}_S^*$, the unique candidate separating equilibrium outcome is as follows: The seller offers $a_H^*$ at $t = 0$ that buyer $H$ accepts, buyer $L$ rejects and offers $x_L^0$ at $t = 1$ that the seller accepts. In the seller-initiated game, the quantity distortion for buyer $L$ is belief-independent conditional on the time of trade.

I now construct the following separating equilibrium in the seller-initiated game in the H-M regime, for $z < z_0$, and when $\bar{V}_S^* > V_S^0$:

*The Seller's Beliefs*. $\mu(h^{-1}) = \mu(h^0) = e$, and the beliefs satisfy Assumption 3. Consider any history $h^t$ ending with a buyer offer $y^t$ such that no buyer type has been (or is required to be) pruned from the seller's belief set. Let $\mathbf{y}$ denote the seller's most recent offer. For $U_H(y_H(\mathbf{y})) \geq U_H(a_H^*)$, if $y^t$ is serious, then $\mu(h^t) = 0$ if $U_L(y_L(\mathbf{y})) < \delta U_L(y^t)$ and $U_H(y_H(\mathbf{y})) \geq \delta U_H(y^t)$, and $\mu(h^t) = 1$ otherwise; if $y^t$ is silence, then $\mu(h^t) = 0$ if $U_L(y_L(\mathbf{y})) < \delta^2 U_L(a_L^*)$ and $U_H(y_H(\mathbf{y})) \geq \delta^2 U_H(a_L^*)$, and $\mu(h^t) = 1$ otherwise. Alternatively, for $U_H(y_H(\mathbf{y})) < U_H(a_H^*)$, if $y^t$ is serious, then $\mu(h^t) = 0$ if $U_L(y_L(\mathbf{y})) < \delta U_L(y^t)$ and $U_H(b_H^*) \geq U_H(y^t)$, and $\mu(h^t) = 1$ otherwise; if $y^t$ is silence, then $\mu(h^t) = 1$.

*The Seller's Strategies*. When making the initial offer, offer $\{Z_H(v^*), Z_L(v^*)\}$, where $v^* \in V^*$. After any $h^t$ $(t \geq 1)$ ending with a buyer offer $y^t$, if $h^t$ is such that $\mu(h^t) = 0$ (1), then accept $y^t$ if and only if $U_S(y^t) \geq (\delta/1 + \delta) w_L^*$ $(U_S(y^t) \geq (\delta/1 + \delta) w_H^*)$ and offer $a_L^*$ $(a_H^*)$; if $h^t$ is such that $\mu(h^t) = e$, then accept $y^t$ if and only if $U_S(y^t) \geq \delta \bar{V}_S^*$ and offer $\{Z_H(v^*), Z_L(v^*)\}$.

*Buyer H's Strategies*. After any history $h^t$ (such that no buyer type has been pruned) ending with a seller offer $\mathbf{y}$, accept $y_H(\mathbf{y})$ if and only if $U_H(y_H) \geq (\delta/1 + \delta) w_H^*$ and offer $b_H^*$.

*Buyer L's Strategies*. After any history $h^t$ (such that no buyer type has been pruned) ending with a seller offer $\mathbf{y}$ with $v_H \equiv U_H(y_H(\mathbf{y}))$, if $v_H \geq U_H(a_H^*)$, then accept $y_L(\mathbf{y})$ if and only if $U_L(y_L) \geq \delta V_L(v_H)$ and offer $\tilde{y}$, where $\tilde{y}$ is as described in Lemma 5, and if $v_H < U_H(a_H^*)$, then accept $y_L(\mathbf{y})$ if and only if $U_L(y_L) \geq \delta U_L(x_L^0)$ and offer $x_L^0$. [After a history such that buyer $B$ has been pruned, the buyer (irrespective of his or her type) best responds to the seller's complete-information strategy against buyer type $B'$.]

Note that the preceding separating equilibrium is also a stationary equilibrium. For $\bar{V}_S^* < V_S^0$, a separating stationary equilibrium in the seller-initiated game can be constructed analogously, as can be a separating stationary equilibrium in the buyer-initiated game.

Finally, Claim 1 implies the following results: $\lim_{z \to 0} \bar{V}_S(v) < \lim_{z \to 0} V_S^0$ for all $v > w_H^*/2$, and $\lim_{z \to 0} \bar{V}_S(w_H^*/2) = \lim_{z \to 0} V_S^0$. Further, $\lim_{z \to 0} Z_H(w_H^*/2) = \lim_{z \to 0} \{a_H^*\} = [q_H^*, C(q_H^*) + w_H^*/2]$, and $\lim_{z \to 0} Z_L(w_H^*/2) = \lim_{z \to 0} \{x_L^0\} = [\bar{q}_L^0, C(\bar{q}_L^0$

$+ w_L^*/2]$. These results and Claim 2 establish that the outcome specified in Proposition 6 is the only candidate separating equilibrium outcome in the H-M regime in the limit $z \to 0$, irrespective of who makes the first offer.

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