Sankhyā: The Indian Journal of Statistics
1995, Volume 57, Series A, Pt. 1, pp. 158-160

A NOTE ON THE L-CLASS OF LIFE DISTRIBUTIONS

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SUMMARY. It is shown that the L-class of life distributions is closed under weak convergence.

1. Introduction

Klefsjo (1983) has defined and extensively studied the L-class of life distributions. A distribution function F with support on $[0, \infty)$ and finite mean μ is said to belong to the L-class if for $s \geq 0$,

$$\int_0^\infty exp(-st)\overline{F}(t)dt \geq \mu/(1+s\mu), \qquad \ldots(1.1)$$

where $\overline{F} = 1 - F$. The right hand side of (1.1) may be seen to be the Laplace transform of an exponential distribution with mean μ. It is easy to show that (see Klefsjo (1983)) the L-class is strictly larger than the harmonically new better than used in expectation (HNBUE) class of life distributions. Hence the L-class also contains the smaller NBUE, NBU, IFRA and IFR classes of life distributions.

Basu and Simons (1983) considered the problems of weak convergence within the IFR family. Basu and Bhattacharjee (1984) have shown that the HNBUE family of life distribution is closed under weak convergence. We obtain the same result for the L-class of life distributions in this note.

2. The result

The result is stated in the following theorem.

Theorem. *Let $F_n, n = 1, 2, \ldots,$ be a sequence of L-life distributions. If F_n converges weakly to F, then F belongs to the L-class.*

Proof. Let $P(X_n \leq t) = F_n(t)$. Since,

$$\begin{aligned} \int_{[X_n>\alpha]} X_n dP &= \int I_{[X_n>\alpha]} X_n dP \\ &\leq (EX_n^2)/\alpha \\ &\leq 2\mu_n^2/\alpha \qquad \text{(see Chaudhuri (1993))} \end{aligned}$$

Paper received. May 1993; revised November 1993.
AMS (1980) *subject classification.* Primary 62N05; secondary 60F05.
Key words and phrases. Life distribution, L-class, weak convergence, L-order.

and

$$\mu_n = \int_0^\infty \overline{F}_n(t)dt \leq B < \infty,$$

for some real B, and for all n (see Basu and Bhattacharjee (1984)), X_n is uniformly integrable. Thus,

$$\lim_{n\to\infty} \int_0^\infty t dF_n(t) = \int_0^\infty t dF(t), \qquad \ldots(2.1)$$

(see Billingsley, 1991, p. 348).

(Note that this proof is much shorter and elegant than the one given in Basu and Bhattacharjee (1984)).

We have,

$$\begin{aligned} \int_0^\infty e^{-st}\overline{F}_n(t)dt &= \int_0^\infty e^{-st}(1 - F_n(t))dt \\ &= (1 - \int_0^\infty e^{-st} dF_n(t))/s. \end{aligned}$$

Thus, as $F_n \varepsilon L$,

$$\int_0^\infty e^{-st} dF_n(t) \leq (1 + s\mu_n)^{-1}, \quad s \geq 0. \qquad \ldots(2.2)$$

This implies

$$\int_0^\infty e^{-st} dF(t) \leq (1 + s\mu)^{-1}, \quad s \geq 0,$$

which follows from (2.1) and the Helly-Bray theorem. This completes the proof.

The above result can be extended in the following sense. Suppose $F_1, F_2, \ldots$ and $G_1, G_2, \ldots$ are two sequences of life distributions converging weakly to F and G, respectively. Further, let F_n and G_n have the same mean for each n and the second moments of G_n be uniformly bounded. Then an L-order between each F_n and G_n in the sense

$$\int_0^\infty e^{-st}\overline{F}_n(t)dt \geq \int_0^\infty e^{-st}\overline{G}_n(t)dt, \quad s \geq 0 \qquad \ldots(2.3)$$

implies an L-order between F and G. In the special case when G_n is exponential with mean μ_n, the second moment condition is automatically satisfied.

Acknowledgement. The author is grateful to the referee for helpful suggestions which have substantially improved the presentation of the paper. In fact, the extension mentioned in this note was indicated by the referee. This work was supported by a post-doctoral grant from CSIR, India.

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