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# A CLASS OF DUAL INTEGRAL EQUATIONS

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**Abstract.** In this paper we introduce three simple approximate procedures to solve a pair of dual integral equations with Bessel function of zero order as kernel both analytically and numerically after reducing it to a Fredholm integral equation of the second kind.

**1. Introduction.** Dual integral equations arise in a natural way in the course of solving a mixed boundary value problem. A typical boundary value problem of mixed type is one in which the boundary condition on a part of the boundary surface is given in one form while on the remaining part of this surface it is given in some other form. A fairly extensive account of dual integral equations with kernels as trigonometric functions, Bessel functions and Legendre functions etc., are available in the books by Sneddon (1966 and 1972), Sneddon and Lowengrub (1969) and Mandal and Mandal (1999). A large number of researchers contributed significantly to the methodology of solution as well as applications of dual integral equations with other special functions as kernel.

In the present paper, we study the pair of dual integral equations in the form of equations (2.1) and (2.2). A slight variation of these arises while studying the problem of heat conduction with mixed boundary conditions on the surface of an isotropic half-space (cf. Mandrik, 2001). This pair of dual integral equations is first reduced to a Fredholm integral equation of the second kind by a suitable choice of the unknown function. Next we study the solution of Fredholm integral equation by three methods, the first one is analytical while the second and third methods are numerical and the numerical results obtained by these three methods are compared for two special forms of the forcing function.

**2. Method of Solution.** We consider the pair of dual integral equations given by

$$\int_0^\infty A(x)(x^2+\lambda^2)^{\frac{1}{2}} J_0(xy)dx = f(y), \quad 0 < y < a, \tag{2.1}$$

$$\int_0^\infty A(x) J_0(xy)dx = 0, \quad a < y < \infty, \tag{2.2}$$

where $J_0(xy)$ is the Bessel function of first kind of order zero, $\lambda$ is a constant and $f(y)$ is the known forcing term. If we choose the unknown function $A(x)$ in the form

$$A(x) = \frac{x}{\sqrt{x^2+\lambda^2}} \int_0^a \phi(t)\sin(t\sqrt{x^2+\lambda^2})dt, \tag{2.3}$$

where $\phi(t)$ is an unknown analytic function, then noting the result (cf. Gradshteyn and Ryzhik, 1980)

$$\int_0^\infty \sin(t\sqrt{x^2+\lambda^2})\frac{xJ_0(xy)}{\sqrt{x^2+\lambda^2}}dx = \begin{cases} 0 & \text{for } y > t, \\ (t^2-y^2)^{-1/2}\cos(\lambda\sqrt{t^2-y^2}) & \text{for } y < t \end{cases} \tag{2.4}$$

we find that the equation (2.2) is automatically satisfied.

Substituting (2.3) into (2.1) and changing the order of integration we obtain

$$\int_0^a \phi(t)\left\{\int_0^\infty xJ_0(xy)\sin(t\sqrt{x^2+\lambda^2})dx\right\}dt = f(y), \quad 0 < y < a. \tag{2.5}$$

The relation (2.5) reduces to

$$\int_0^a \phi(t)\sin(\lambda t)dt + \int_0^a t\phi(t)\left\{\int_0^\infty \frac{xJ_0(xy)\cos(t\sqrt{x^2+\lambda^2})}{\sqrt{x^2+\lambda^2}}dx\right\}dt$$
$$= \int_0^y \rho f(\rho)d\rho, \quad 0 < y < a \tag{2.6}$$

after integration between 0 to $y(< a)$. Multiplying both sides of (2.6) by

$$\frac{2u\cos(\lambda\sqrt{y^2-u2})}{\sqrt{y^2-u^2}}$$

and integrating with respect to $u$ between $u = 0$ to $u = y$, and then differentiating both sides with respect to $y$, we find that $\phi(y)$ satisfies the Fredholm integral equation of the second kind given by

$$\phi(y) - \frac{1}{\pi}\int_0^a \phi(t)\left[\frac{\sin\{\lambda(y-t)\}}{(y-t)} - \frac{\sin\{\lambda(y+t)\}}{(y+t)}\right]dt$$
$$= \frac{2}{\pi}\int_0^y \rho f(\rho)\frac{\cos(\lambda\sqrt{y^2-\rho^2})}{\sqrt{y^2-\rho^2}}d\rho, \quad 0 < y < a. \tag{2.7}$$

An analytical method is employed by which $\phi(y)$ can be obtained in terms of a power series in $\lambda$, the coefficients being functions of $y$, which are obtained recursively. This series for $\phi$ will produce the unknown function $A(x)$. Also two collocation methods to solve (2.7) are used. The methods are described below :

METHOD I. The unknown function $\phi(y)$ satisfying the Fredholm integral equaion (2.7) can be represented in the form of the series (Mandrik, 2001)

$$\phi(y) = e^{-\lambda a}\sum_{n=0}^{\infty}\lambda^n\phi_n(y), \tag{2.8}$$

where $\phi_n(y)$ $(n = 0, 1, 2, \cdots)$ are unknown functions.

Substituting (2.8) in (2.7) and after some calculations, we obtain

$$\sum_{n=0}^{\infty}\lambda^n\phi_n(y)-\sum_{n=0}^{\infty}\lambda^n\sum_{m=3}^{\infty}\frac{\sin\left(\frac{m\pi}{2}\right)}{m!}\frac{1}{\pi}\int_0^a\phi_{n-m}(t)\left\{(y-t)^{m-1}-(y+t)^{m-1}\right\}dt$$

$$=\sum_{n=0}^{\infty}\lambda^n\sum_{j=0}^{n}\frac{a^{n-j}}{(n-j)!}\frac{\cos\left(\frac{j\pi}{2}\right)}{j!}\frac{2}{\pi}\int_0^y\rho f(\rho)(\sqrt{y^2-\rho^2})^{j-1}d\rho. \tag{2.9}$$

Equating the coefficients of $\lambda^n$ in both sides of (2.9) we see that $\phi_n(y)$ satisfies the recurrence relation

$$\phi_n(y)=\frac{2}{\pi}\sum_{j=0}^{\infty}\frac{a^{n-j}}{(n-j)!}\frac{\cos\left(\frac{j\pi}{2}\right)}{j!}\int_0^y\rho f(\rho)(\sqrt{y^2-\rho^2})^{j-1}d\rho$$

$$+\frac{1}{\pi}\sum_{m=3}^{n}\frac{\sin\left(\frac{m\pi}{2}\right)}{m!}\int_0^a\phi_{n-m}(t)\left\{(y-t)^{m-1}-(y+t)^{m-1}\right\}dt$$

$$n=0,1,2,3,\cdots;\quad 0<y<a. \tag{2.10}$$

This can be written in the form

$$\phi_n(y)=\frac{2}{\pi}\sum_{j=0}^{\infty}\frac{a^{n-j}}{(n-j)!}\frac{\cos\left(\frac{j\pi}{2}\right)}{j!}\int_0^y\rho f(\rho)(\sqrt{y^2-\rho^2})^{j-1}d\rho$$

$$+\frac{1}{\pi}\sum_{m=3}^{n}\sum_{r=0}^{m-1}{}^{m-1}C_r\frac{\sin\left(\frac{m\pi}{2}\right)}{m!}\left[(-1)^r-1\right]y^{m-1-r}\int_0^a\phi_{n-m}(t)t^r dt$$

$$n=0,1,2,3,\cdots;\quad 0<y<a. \tag{2.11}$$

The recurrence relation (2.11) produces $\phi_n(y)$ $(n=0,1,2,\cdots)$ successively. In particular,

$$\phi_0(y)=\frac{2}{\pi}\int_0^y\frac{\rho f(\rho)}{(\sqrt{y^2-\rho^2})}d\rho,$$

$$\phi_1(y)=\frac{2a}{\pi}\int_0^y\frac{\rho f(\rho)}{(\sqrt{y^2-\rho^2})}d\rho,$$

$$\phi_2(y)=\frac{2}{\pi}\left[\frac{a^2}{2}\int_0^y\frac{\rho f(\rho)}{(\sqrt{y^2-\rho^2})}d\rho-\frac{1}{2}\int_0^y\rho f(\rho)\sqrt{y^2-\rho^2}d\rho\right],$$

$$\phi_3(y) = \frac{2}{\pi}\left[\frac{a^3}{6}\int_0^y \frac{\rho f(\rho)}{(\sqrt{y^2-\rho^2})}d\rho - \frac{a}{2}\int_0^y \rho f(\rho)\sqrt{y^2-\rho^2}d\rho\right] - \frac{2y}{3\pi}\int_0^a \phi_0(t)t\,dt,$$

etc. (2.12)

Thus all the $\phi_n$'s are determined analytically in principle. Substituting (2.8) in (2.3) and using (2.12) we get the unknown function $A(x)$ satisfying the pair of dual integral equations (2.1) and (2.2). Thus $A(x)$ is obtained analytically in principle.

METHOD II. For simplicity we write the Fredholm integral equation (2.7) in the form

$$\phi(y) = g(y) + \mu\int_0^a \phi(t)K(y,t)dt, \quad 0 < y < a \tag{2.13}$$

where

$$K(y,t) = \frac{\sin\{\lambda(y-t)\}}{(y-t)} - \frac{\sin\{\lambda(y+t)\}}{(y+t)}, \tag{2.14}$$

$$g(y) = \frac{2}{\pi}\int_0^y \rho f(\rho)\frac{\cos(\lambda\sqrt{y^2-\rho^2})}{\sqrt{y^2-\rho^2}}d\rho \tag{2.15}$$

and

$$\mu = \frac{1}{\pi}.$$

Let us divide the interval $(0,a)$ into $k$ parts by the points

$$y_0 \equiv t_0 = 0, \quad y_j \equiv t_j = \sum_{n=1}^{j} h_n, \quad j = 1,2,\cdots,k,$$

where $h_n = y_n - y_{n-1}$, $n = 1,2,\cdots,k$. Using the approximate quadrature formula

$$\int_0^a K(y,t)\phi(t)dt \simeq \sum_{j=1}^{k} h_j K(y,t_j)\phi(t_j), \tag{2.16}$$

we see that the equation (2.13) takes the form

$$\phi(y) = g(y) + \mu\sum_{j=1}^{k} h_j K(y,t_j)\phi(t_j) \tag{2.17}$$

which must hold for all values of $y$ in the interval $(0,a)$. In particular, this equation is satisfied at the $k$ points $y_i$, $i = 1,2,\cdots,k$. This leads to the system of linear equations for $\phi(y_i)$ $(i = 1,2,\cdots,k)$

$$\phi(y_i) = g(y_i) + \mu\sum_{j=1}^{k} h_j K(y_i,t_j)\phi(t_j), \quad i = 1,2,\cdots,k. \tag{2.18}$$

Writing

$$g(y_i) = g_i, \quad \phi(t_j) \equiv \phi(y_j) = \phi_j, \quad K(y_i, t_j) = K_{ij} \tag{2.19}$$

the system of equations (2.18) is written in the compact form

$$\phi_i - \mu \sum_{j=1}^{k} h_j K_{ij} \phi_j = g_i, \quad i = 1, 2, \cdots, k. \tag{2.20}$$

The values of $\phi_i$ are now obtained by solving this linear system. These values of $\phi_i$ can be regarded as approximate solutions of the integral equation (2.13) at the points $y_1, y_2, \cdots, y_k$.

METHOD III. To solve the Fredholm integral equation (2.13) by Gauss's quadrature approximation method, we change the variable by the substitution $2t = a(u+1)$ and $2y = a(v+1)$, then the limits of integration of (2.13) becomes $-1$ to 1. The Fredholm integral equation (2.13) becomes

$$\psi(v) = h(v) + \frac{\mu}{2} \int_{-1}^{1} \psi(u) k(v, u) du, \quad -1 < v < 1 \tag{2.21}$$

where

$$\psi(v) = \phi\left(\frac{a(v+1)}{2}\right),$$

$$h(v) = g\left(\frac{a(v+1)}{2}\right),$$

$$k(v, u) = K\left(\frac{a(v+1)}{2}, \frac{a(u+1)}{2}\right).$$

Using the Gauss's quadrature formula, we obtain

$$\int_{-1}^{1} \psi(u) k(v, u) du \simeq \sum_{j=1}^{k} w_j \, \psi(u_j) k(v, u_j) \tag{2.22}$$

where $w_j$ are the weights of the Gauss's quadrature formula corresponding to the Gauss's quadrature nodes $u_j$. The equation (2.21) takes the form

$$\psi(v) = h(v) + \frac{\mu}{2} \sum_{j=1}^{k} w_j \, \psi(u_j) k(v, u_j) \tag{2.23}$$

which must hold for all values of $v$ in the interval $(-1, 1)$.

In particular, this equation is satisfied at the $k$ Gauss's quadrature nodes (cf. Scarborough, 1958), $v_i (= u_i)$, $i = 1, 2, \cdots, k$. Thus

$$\psi(v_i) = h(v_i) + \frac{\mu}{2} \sum_{j=1}^{k} w_j \, \psi(u_j) k(u_i, u_j), \quad i = 1, 2, \cdots, k. \tag{2.24}$$

Writing

$$h(v_i) = h_i, \quad \psi(v_i) = \psi(u_i) = \psi_i, \quad k(v_i, u_j) = k_{ij} \tag{2.25}$$

equation (2.24) is written compactly in the form

$$\psi_i - \frac{\mu}{2} \sum_{j=1}^{k} w_j \, k_{ij} \, \psi_j = h_i, \quad i = 1, 2, \cdots, k. \tag{2.26}$$

The values of $\psi_i$ and hence $\phi\left(a\left(\frac{u_i+1}{2}\right)\right)$ are now obtained by solving this linear system by any standard method.

**3. Numerical Illustration.** As numerical illustration, we first consider the forcing function $f(y) = c$ (constant) then the equation (2.11) reduces to

$$\phi_n(y) = \frac{2c}{\pi} \sum_{j=0}^{\infty} \frac{a^{n-j}}{(n-j)!} \frac{\cos\left(\frac{j\pi}{2}\right)}{(j+1)!} y^{j+1}$$

$$+ \frac{1}{\pi} \sum_{m=3}^{n} \sum_{r=0}^{m-1} {}^{m-1}C_r \frac{\sin\left(\frac{m\pi}{2}\right)}{m!} \{(-1)^r - 1\} y^{m-1-r} \int_0^a \phi_{n-m}(t) t^r \, dt$$

$$n = 0, 1, 2, 3, \cdots; \quad 0 < y < a. \tag{3.1}$$

Thus

$$\phi_0(y) = \frac{2cy}{\pi},$$

$$\phi_1(y) = \frac{2cay}{\pi},$$

$$\phi_2(y) = \frac{2c}{\pi}\left[\frac{a^2 y}{2} - \frac{y^3}{6}\right],$$

$$\phi_3(y) = \frac{2c}{\pi}\left[\frac{a^3 y}{6} - \frac{a y^3}{6}\right] + \frac{4ca^3 y}{9\pi^2},$$

$$\phi_4(y) = \frac{2c}{\pi}\left[\frac{a^4 y}{24} - \frac{a^2 y^3}{12} + \frac{y^5}{120}\right] + \frac{4ca^4 y}{9\pi^2}, \quad \text{etc.} \tag{3.2}$$

We choose the constant $c = 1$, and $\lambda = 0.1$, $a = 1$, $n = 20$. Then the unknown function $\phi(y)$ calculated at the nodes (Scarborough, 1958) $y_k$, $k = 1, \cdots, 9$ by using (2.8), (2.20) and (2.26) are presented in Table-1.

Table-1 : Value of $\phi(y_k)$ at $y_k$

| $y_k$ | Method-I | Method-II | Method-III |
|---|---|---|---|
| 0.015919 | 0.010135 | 0.010136 | 0.010135 |
| 0.081984 | 0.052193 | 0.052197 | 0.052196 |
| 0.193314 | 0.123062 | 0.123070 | 0.123069 |
| 0.337873 | 0.215063 | 0.215073 | 0.215071 |
| 0.500000 | 0.318194 | 0.318526 | 0.318200 |
| 0.662127 | 0.421245 | 0.421250 | 0.421245 |
| 0.806686 | 0.513039 | 0.513037 | 0.513032 |
| 0.918016 | 0.583663 | 0.583654 | 0.583648 |
| 0.984080 | 0.625539 | 0.625525 | 0.625518 |

Using the above values of $\phi$, computed by the three methods, we obtain the values of the unknown function $A(x)$ of the dual integral equations at some representative points given in Table-2. $A(x)$ is computed from (2.3) by employing Gauss quadrature.

Table-2 : Value of $A(x_k)$ at $x_k$

| $x_k$ | Method-I | Method-II | Method-III |
|---|---|---|---|
| 0.2 | 0.084381 | 0.084392 | 0.084381 |
| 0.6 | 0.245125 | 0.245158 | 0.245124 |
| 0.9 | 0.351236 | 0.351283 | 0.351233 |
| 1.4 | 0.484640 | 0.484710 | 0.484637 |
| 1.9 | 0.549322 | 0.549412 | 0.549320 |
| 2.4 | 0.539428 | 0.539533 | 0.539428 |
| 2.8 | 0.481870 | 0.481982 | 0.481871 |
| 3.1 | 0.415003 | 0.415117 | 0.415006 |
| 3.7 | 0.242005 | 0.242117 | 0.242012 |

Next we consider the forcing function $f(y) = y^2$, then the equation (2.11) reduces to

$$\phi_n(y) = \frac{4}{\pi}\sum_{j=0}^{\infty}\frac{a^{n-j}}{(n-j)!}\frac{\cos\left(\frac{j\pi}{2}\right)}{(j+1)!}\frac{y^{j+3}}{(j+3)}$$

$$+\frac{1}{\pi}\sum_{m=3}^{n}\sum_{r=0}^{m-1}{}^{m-1}c_r\frac{\sin\left(\frac{m\pi}{2}\right)}{m!}\{(-1)^r-1\}y^{m-1-r}\int_0^a\phi_{n-m}(t)t^r dt$$

$$n = 0,1,2,3,\cdots;\quad 0 < y < a. \tag{3.3}$$

Thus

$$\phi_0(y) = \frac{4y^3}{3\pi},$$

$$\phi_1(y) = \frac{4ay^3}{3\pi}$$

$$\phi_2(y) = \frac{4}{\pi}\left[\frac{a^2y^3}{6}-\frac{y^5}{30}\right]$$

$$\phi_3(y) = \frac{4}{\pi}\left[\frac{a^3y^3}{18}-\frac{ay^5}{30}\right]+\frac{8a^5y}{45\pi^2},$$

$$\phi_4(y) = \frac{4}{\pi}\left[\frac{a^4y^3}{72}-\frac{a^2y^5}{60}+\frac{y^7}{840}\right]+\frac{8a^6y}{45\pi^2},$$

etc. (3.4)

Then $\phi(y)$ calculated at the same nodes by using the three methods are presented in Table-3.

**Table-3** : Value of $\phi(y_k)$ at $y_k$

| $y_k$ | Method-I | Method-II | Method-III |
|---|---|---|---|
| 0.015919 | 0.000002 | 0.000002 | 0.000002 |
| 0.081984 | 0.000234 | 0.000236 | 0.000235 |
| 0.193314 | 0.003066 | 0.003070 | 0.003069 |
| 0.337873 | 0.016369 | 0.016376 | 0.016374 |
| 0.500000 | 0.053041 | 0.053258 | 0.053047 |
| 0.662127 | 0.123155 | 0.123161 | 0.123158 |
| 0.806686 | 0.222668 | 0.222666 | 0.222663 |
| 0.918016 | 0.328107 | 0.328096 | 0.328091 |
| 0.984080 | 0.404115 | 0.404094 | 0.404090 |

Using again the above values of $\phi$, computed by the three methods, we obtain the values of the unknown function $A(x)$ of the dual integral equations at some representative points given in Table-4. $A(x)$ is computed from (2.3) again by employing Gauss quadrature.

**Table-4** : Value of $A(x_k)$ of $x_k$

| $x_k$ | Method-I | Method-II | Method-III |
|---|---|---|---|
| 0.2 | 0.033730 | 0.033737 | 0.033729 |
| 0.6 | 0.097376 | 0.097396 | 0.097374 |
| 0.9 | 0.138256 | 0.138286 | 0.138254 |
| 1.4 | 0.185929 | 0.185974 | 0.185926 |
| 1.9 | 0.201675 | 0.201733 | 0.201673 |
| 2.4 | 0.183661 | 0.183729 | 0.183661 |
| 2.8 | 0.147719 | 0.147792 | 0.147720 |
| 3.1 | 0.111172 | 0.111247 | 0.111175 |
| 3.7 | 0.024932 | 0.025006 | 0.024938 |

**Conclusion.** A pair of dual integral equations is solved here by reducing it to a Fredholm integral equation of second kind. An approximate analytical procedure is employed to solve the Fredholm integral equation in terms of a series whose terms are obtained recursively. This Fredholm integral equation is also solved numerically by employing two quadrature rules, one is based on the mid-point rule and the other is the Gauss's rule. As numerical illustrations, two forms of the forcing function is chosen. The unknown function satisfying the Fredholm integral equation and also the function satisfying the dual integral equations are computed at a number of point by following the three procedures, and displayed in a number of tables. These three methods produce almost the same numerical results for the unknown function $A(x)$.

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*References*

Gradshteyn, I.N. and Ryzhik, I.M. (1980) : Tables of Integrals, Series and Products, (Boston: Academic Press).

Mandal, B.N. and Mandal, N. (1999) : Advances in Dual Integral Equations, (Boca Raton: CRC Press).

Mandrik, P.A. (2001) Solution of the heat equation with mixed boundary conditions on the surface of an isotropic half-space, *Differential Equation*, **37**, 257.

Scarborough, J.B. (1958) : Numerical Mathematical Analysis, (Oxford; Oxford University Press).

**Sneddon, I.N.** (1966) : Mixed Boundary Value Problems in Potential Theory, (Amsterdam: North Holland Publ.).

**Sneddon, I.N. and Lowengrub, M.** (1969) : Crack Problems in the Mathematical Theory of Elasticity, (New York: John Wiley & Sons.)

**Sneddon, I.N.** (1972) : The Use of Integral Transforms, (New York: Mc Graw-Hill).

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