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ON THE NEAR OPTIMUM CONTINUOUS SAMPLING PLAN CSP-2 (WITH $k = i$) TO MINIMISE THE AMOUNT OF INSPECTION AND ITS PERFORMANCE AS COMPARED TO OPTIMUM CSP-1 PLAN

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SUMMARY. For both the continuous sampling plan CSP-1 introduced by Dodge and its later modification CSP-2 several combinations of (i, f) are possible that will ensure the same AOQL. Ghosh (1988) worked out the optimum CSP-1 plan and the unique combination of (i, f) that would minimise the amount of inspection when the incoming quality p is known to follow a one, two or three point Binomial distribution or any continuous distribution which can be approximated by discrete probabilities for some finite number of values of p. In the present paper we work out the near optimum CSP-2 plan with $k = i$ and the unique combination (i, f) that seeks to minimise the amount of inspection under similar assumptions of distribution for incoming quality p. The performances of the optimum CSP-1 and near optimum CSP-2 plan are also compared with respect to both average amount of inspection and average outgoing quality AOQ.

In the present paper we have also obtained an algebraic relation to find f for a given i which will ensure a desired AOQL for CSP-2 type of sampling inspection.

1. Introduction

Dodge (1943) introduced a random order continuous sampling plan CSP-1 and there after Dodge and Torrey (1949) offered additional continuous sampling plans CSP-2 and CSP-3. The CSP-1 plan provides for corrective inspection with a view to having a 'limiting average outgoing quality' AOQL which will not be exceeded no matter what quality is submitted. The plan visualised two phases of inspection. At the outset 100% of the units produced consecutively are inspected till i units in succession are found clear of defects. Then only a fraction f of the units, chosen at random, are inspected. If a sample unit is found to be defective immediately 100% inspection is resorted to until again i units in succession are found clear of defects.

Plan CSP-2 differs from plan CSP-1 in that once sampling inspection is started, 100% inspection is not invoked when a defect is found but is

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invoked only if a second defect occurs in the next k or less sample units. In other words if two defects observed during sampling are separated by k or less good inspected items, 100% inspection is reintroduced otherwise sampling inspection is continued.

The factor k may be theoretically assigned any value. However the authors studied elaborately only the CSP-2 plan with $k = i$ and for several combinations of (i, f) for a desired value of AOQL. By comparing the performances of CSP-1 and CSP-2 ($k = i$) the authors reported as follows :

"i is increased for given values of AOQL and f; for example for AOQL = 3% and f = 5%, i (= k) is 64 for plan CSP-2 whereas i = 50 for plan CSP-1. A larger proportion of total products is accepted on a sampling basis by CSP-2 plans when the incoming percentage defective is approximately less than 1.5 times AOQL. Likewise the average fraction of total product units inspected in the long run, including both sampling inspection and 100% inspection is somewhat less for CSP-2 plans than for corresponding CSP-1 plans when the incoming percent defective is less than some multiple of AOQL which varies with f; for example the multiple is approximately 2.0 for f = .20 and 1.5 for f = .05. For higher levels of p, plan CSP-2 requires more inspection than plan CSP-1".

In view of the design of the CSP-2 plan the p_t (%) is not a unique value as in the case of CSP-1. The authors recommended the comparison of maximum p_t (%) for CSP-2 with the unique p_t (%) for CSP-1. It is seen that p_t (%) for plan CSP-2 may be considerably higher than that for plan CSP-1 for the same value of f in both cases.

If the choice of one of the (either of two types of CSP plan or one out of many possible for either type) plans is not judiciously made one may have to undertake unnecessary extra inspection even at the risk of higher p_t (%). However no clearcut guidelines are available for such a choice. It is also not clear why 'f' of the CSP-1 and CSP-2 should be kept same. What would be the effect of following CSP-1 and CSP-2 plans having the same value of i ? Even after agreeing to same value of f for both CSP-1 and CSP-2 we are not sure which level of f is the most pragmatic choice. In other words we do not have any objective criterion for selecting a particular type and a unique (i, f).

Ghosh (1988) developed a procedure for selecting an optimum CSP-1 plan with a unique combination of (i, f) that would achieve the AOQL and also minimise the amount of inspection when the process average $\bar{p}$ is known. In fact the procedure can be employed even if the incoming quality p follows a one, two or three point Binomial or any continuous distribution which can be approximated by discrete probabilities for some finite number of values of p.

In the present paper we attempted to develop optimum CSP-2 plan under similar assumptions about the distribution of p. However, it is very difficult to get the optimum solution as the mathematical expressions become intractable. Nevertheless by making some approximations it is possible to obtain a near optimum plan relatively easily. The sensitivity of this approximate plan was found to be good enough for most practical situations. To achieve the same AOQL one can now choose one of the optimum CSP-1 or the near optimum CSP-2 plan after comparing the expected performances of the two plans.

Both CSP-1 and CSP-2 provide an expression for average outgoing quality AOQ in terms of i, f and p. In CSP-1 it is possible to find analytically f for a given choice of i for a required AOQL. However, in CSP-2, Dodge and Torrey obtained the (i, f) combination by a tedious trial and error procedure. However, in this paper we have obtained an analytical relation connecting i, f, and AOQL, so that for a given i and AOQL, f can be determined. The solution procedure is also iterative in this case but straightforward.

2. Notations

Throughout the paper we retain the symbols introduced by Dodge and Torrey and make use of the various results obtained by them. For the sake of easy reference they are listed below :

The symbol p_L denotes AOQL,

p_A average outgoing quality AOQ,

p_1 the quality level for which AOQL is reached

$\bar{p}$ the process average

p the incoming proportion defective,

and $q = 1-p$.

i denotes the number of defect free consecutive items which will direct a change from cent per cent inspection to sampling inspection. k denotes the minimum spacing between two defects in the sampling phase of inspection which will permit sampling inspection to continue. f denotes the sampling fraction. u stands for the expected number of items inspected on a 100% inspection basis. v stands for expected number of items that will be passed during sampling inspection. This includes the sampling units produced between successive sampled units.

In CSP type of plans, if there is a sudden deterioration of incoming quality p during sampling phase of inspection it will take some time to switch back to 100% inspection and consequently the items accepted by that time will contain more defectives than the usual case. Thus with each CSP plan having a particular (i,f) combination is associated a spotty quality $p_t(\%)$. Dodge has defined p_t (%) as the percent defective in a consecutive run of $N = 1000$ units for which the probability of acceptance under sampling phase is .10.

From the results of Dodge and Torrey we have

$$u = \frac{1-q^i}{pq^i} \qquad \dots \quad (1)$$

$$fv = \frac{1}{p} + \frac{q^k}{1-q^k}\left(\frac{1}{p}+k\right) + \frac{1}{p} - \frac{kq^k}{1-q^k}$$

$$= \frac{2-q^k}{p(1-q^k)} \qquad \dots \quad (2)$$

$$F(\text{amount of inspection}) = \frac{u+fv}{u+v} = \frac{\dfrac{1-q^i}{pq^i}+\dfrac{2-q^k}{p(1-q^k)}}{\dfrac{1-q^i}{pq^i}+\dfrac{2-q^k}{fp(1-q^k)}} \qquad \dots \quad (3)$$

and

$$p_A = p(1-F). \qquad \dots \quad (4)$$

3. THE (i, f) COMBINATION WHICH WILL ENSURE A DESIRED AOQL UNDER CSP-2 $(k = i)$ SCHEME OF INSPECTION

Exact relation : For $k = i$, it follows from (3) that

$$F = \frac{\dfrac{(1-q^i)^2+(2-q^i)q^i}{pq^i(1-q^i)}}{\dfrac{f(1-q^i)^2+(2-q^i)q}{fpq^i(1-q^i)}} \qquad \dots \quad (5)$$

$$= \frac{f}{f+(1-f)q^i(2-q^i)}$$

and

$$p_A = p\left[1-\frac{f}{f+(1-f)q^i(2-q^i)}\right] \qquad \dots \quad (6)$$

Hence

$$p_L = p_1\left[1-\frac{f}{f+(1-f)(1-p_1)^i(2-(1-p_1)^i)}\right]$$

where p_1 is the value of p for which AOQL is reached. To determine p_L we differentiate (6) with respect to p, equate it to zero and after substituting p_1 for p we get

$$1-\frac{A-fp_1(B+C)}{D}=0$$

where

$$A=f^2+f(1-f)(1-p_1)^i(2-(1-p_1)^i)$$
$$B=(1-f)\cdot-i\cdot(1-p_1)^{i-1}(2-(1-p_1)^i)$$
$$C=(1-f)(1-p_1)^i\cdot-i(1-p_1)^{i-1}\cdot-1$$
$$D=\{f+(1-f)(1-p_1)^i(2-(1-p_1)^i\}^2$$

or

$$1-\frac{f^2+f(1-f)q_1^i(2-q_1^i)+ip_1f(1-f)q_1^{i-1}(2-2q_1^i)}{\{f+(1-f)q_1^i(2-q_1^i)\}^2}=0$$

or

$$f^2+f(1-f)\,q_1^i(2-q_1^i)+ip_1f(1-f)\,q_1^{i-1}\,(2-2q_1^i)$$
$$=f^2+(1-f)^2q_1^{2i}\,(2-q_1^i)^2+2f(1-f)\,q_1^i\,(2-q_1^i)$$

which implies that

$$ip_1=\frac{q_1(2-q_1^i)}{2-2q_1^i}+\frac{1-f}{f}\cdot\frac{q_1^{i+1}(2-q_1^i)^2}{2-2q_1^i}. \qquad \dots \quad (7)$$

Writing $2-2q_1^i$ as s and $2-q_1^i=r$ we have

$$\frac{s}{r}\cdot\frac{ip_1}{1-p_1}=1+\frac{1-f}{f}\cdot(1-p_1)^i\,r \qquad \dots \quad (8)$$

or

$$\frac{1-p_1}{ip_1}\cdot\frac{r}{s}=\frac{f}{f+(1-f)(1-p_1)^i\cdot r}$$

or

$$1-\frac{1-p_1}{ip_1}\cdot\frac{r}{s}=1-\frac{f}{f+(1-f)(1-p_1)^i\,r}$$

It follows from (6) that

$$(is+r)\,p_1-r=isp_1\left[1-\frac{f}{f+(1-f)(1-p_1)^i\,r}\right]$$
$$=isp_L \qquad \dots \quad (9)$$

and hence

$$p_1=\frac{ip_L+\frac{r}{s}}{i+\frac{r}{s}} \qquad \dots \quad (10)$$

From (8) we have

$$\frac{s}{r}\cdot ip_1+p_1-1 = \frac{1-f}{f}(1-p_1)^{i+1}\cdot r$$

or

$$\frac{p_1(is+r)-r}{r} = \frac{1-f}{f}(1-p_1)^{i+1}\cdot r$$

or

$$\frac{isp_L}{r} = \frac{1-f}{f}(1-p_1)^{i+1}\cdot r \qquad \text{[from (9)]}$$

or

$$fip_L\cdot\frac{s}{r} = (1-f)(1-p_1)^{i+1}\cdot r$$

or

$$fip_L\frac{s}{r}+fq_1^{i+1}r = q_1^{i+1}\cdot r$$

and hence

$$f = \frac{q_1^{i+1}}{ip_L\frac{s}{r^2}+q_1^{i+1}}. \qquad \dots \quad (11)$$

Thus for a given i and p_L, p_1 can be obtained by solving equation (10) numerically. The iterative procedure can be initiated by taking $p_1 = \frac{ip_L+1}{i+1}$. Once p_1 is determined f can be obtained from (11). The (i, f) combination will meet the AOQL requirement no matter what quality p is submitted for inspection.

Approximate relation. Since it is intractable to get an expression for F and to study it for extremum value using these exact relations we make approximations p_1^* and f^* to p_1 and f respectively. Replacing $\frac{s}{r}$ which is less than but very nearly equal to 1 for moderately large values of i by 1 we have,

$$p_1^* = \frac{ip_L+1}{i+1} \qquad \dots \quad (12)$$

and

$$f^* = \frac{q_1^{*i+1}(2-q_1^{*i})}{ip_L+q_1^{*i+1}(2-q_1^{*i})} \qquad \dots \quad (13)$$

It can be easily shown that $p_1 > p_1^*$, $f > f^*$ and $F > F^*$. In order to study the error due to approximation the values of (f^*, f), $(AOQ^*(p_1))$, $AOQ(p_1))$ and $(F^*_{p_1}, F_{p_1})$ were obtained for some selected values of p_L covering the range of our usual working and for some selected values of i. The results are shown in Table 1.

TABLE 1. ERROR DUE TO APPROXIMATION

p_L	i	error in f	error in $AOQ(p_L)$	error in F
.01	5	.0107	.0023	.0115
	10	.0025	.0002	.0015
	15	.0026	.0001	.0016
.05	5	.0030	.0005	.0022
	10	.0018	.0002	.0017
	15	.0008	.0001	.0010
.10	5	.0002	.0004	.0017
	10	.0002	.0001	.0005
	15	.0001	.0000	.0002

It can be seen that for $i \geqslant 10$ actual and approximate values are in good agreement for all values of p_L. For higher values of p_L even for i as small as 5, the two figures match well. It will be seen later that for near optimal CSP-2 plans lower values of p_L requires larger values of i if $\bar{p}$ is small. When $\bar{p}$ is moderate, i is also moderate and the approximation holds good. Hence the near optimal plans developed here can be considered adequate for any practical situation.

4. On the nature of F^*

We now obtain an expression $F^*_p(i)$ for the amount of inspection for a given p and for (i, f^*) combination that will ensure a given p_L under the above approximations.

From (13),

$$\frac{1}{f^*} - 1 = \frac{ip_L}{q_1^{*i+1}(2-q_1^{*i})}$$

$$= \frac{ip_L}{1} \cdot \frac{1}{\dfrac{i^{i+1}q_L^{i+1}}{(i+1)^{i+1}}} \cdot \frac{1}{2-\dfrac{i^i q_L^i}{(i+1)^i}} \quad [\text{using (12)}]$$

$$= \frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{q_L} \cdot \frac{1}{q_L^i} \cdot \frac{1}{2-\left(\dfrac{i}{i+1}\right)^i q_L^i}$$

Hence using (5) we get,

$$F_p^*(i) = \frac{1}{1+\left(\frac{1}{f^*}-1\right)q^i(2-q^i)}$$

$$= \frac{1}{1+\frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{q_L}\left(\frac{q}{q_L}\right)^i \cdot \frac{2-q^i}{2-\left(\frac{i}{i+1}\right)^i q_L^i}} \cdot \quad \ldots \quad (14)$$

We note that the amount of inspection required to ensure a given AOQL does not involve p_1 and can be expressed in terms of i and p only. The present form may be compared with $F_p(i)$ for CSP-1 as given by equation (8) in Ghosh (1988).

For the sake of convenience in future we will write $F_p^*(i)$ as $F^*(x)$ where

$$F^*(x) = \frac{1}{1+cb^x \frac{(x+1)^{x+1}}{x^x} \cdot \frac{2-(ab)^x}{2-\left(\frac{ax}{x+1}\right)^x}} \quad \ldots \quad (15)$$

$a = 1-p_L$, $c = \frac{p_L}{1-p_L}$ and $b = \frac{1-p}{1-p_L}$ are constants for a given p and p_L. We note that $b < 1$ for $p > p_L$ and $a \leqslant 1$. For the purpose of comparison of CSP-2 with CSP-1 plan we will denote the amount of inspection in CSP-1 plan by $F(x)$.

In order to study the nature of $F^*(x)$ in relation to $F(x)$ we make use of the following lemmas.

Lemma 1 : *$F^*(x) \gtreqless F(x)$ according as $x \lesseqgtr x_0$ where*

$$\frac{x_0}{x_0+1} = b.$$

Proof : It is easy to see that $\frac{2-(ab)^x}{2-\left(\frac{ax}{x+1}\right)^x} \lesseqgtr 1$ according as $\frac{x}{x+1} \lesseqgtr b$ for a given a.

and $F(x)$ has the form

$$\frac{1}{1+cb^x \frac{(x+1)^{x+1}}{x^x}} \ldots \text{ eqn. (9) of Ghosh (1988)}$$

Hence the result follows.

Lemma 2 : *$F^*(x)$ is decreasing at $x = x_0$ where* $\frac{x_0}{x_0+1} = b.$

Proof : Let us define $\psi(x) = cb^x \frac{(x+1)^{x+1}}{x^x} \cdot \frac{2-(ab)^x}{2-\left(\frac{ax}{x+1}\right)^x}$

and
$$\phi(x) = \log \psi(x)$$

Then
$$\phi(x) = \log c + x \log b + (x+1) \log (x+1) - x \log(x)$$
$$+ \log \{2-(ab)^x\} - \log \left\{2-\left(\frac{ax}{x+1}\right)^x\right\}$$

$$\phi'(x) = \log b + \log(x+1) - \log x - \frac{(ab)^x \log ab}{2-(ab)^x}$$
$$+ \frac{\left(\frac{ax}{x+1}\right)^x \cdot \left\{\log \frac{ax}{x+1} + \frac{1}{x+1}\right\}}{2-\left(\frac{ax}{x+1}\right)^x}$$

Since
$$F^*(x) = \frac{1}{e^{\phi(x)}+1} \text{ and } F^{*\prime}(x) = -\frac{e^{\phi(x)} \cdot \phi'(x)}{(e^{\phi(x)}+1)^2}$$

it follows that

$$F^{*\prime}(x) = \frac{e^{\phi(x)}}{(e^{\phi(x)}+1)^2}\left[\left(\log \frac{x}{x+1} - \log b\right) + \frac{(ab)^x \log ab}{2-(ab)^x}\right.$$
$$\left. - \frac{\left(\frac{ax}{x+1}\right)^x \left\{\log \frac{ax}{x+1} + \frac{1}{x+1}\right\}}{2-\left(\frac{ax}{x+1}\right)^x}\right] \qquad \ldots \quad (16)$$

At $x = x_0$

$$F^{*\prime}(x_0) = -\frac{e^{\phi(x_0)}}{(e^{\phi(x_0)}+1)^2} \cdot \frac{(ab)^{x_0} \cdot \frac{1}{x_0+1}}{2-(ab)^{x_0}}$$
$$< 0$$

and hence the result.

Lemma 3 : *If there exists a x for which $F^*(x)$ becomes minimum then $x > x_0$.*

Proof: For all $x < x_0$, $F^*(x) > F(x)$ and $F^*(x) = F(x)$ at x_0. Since $F(x)$ is minimum at x_0 (see Ghosh (1988) $F^*(x)$ at x_0 is smaller than $F^*(x)$ for all $x < x_0$. At x_0, $F^*(x)$ is decreasing and hence the result.

Lemma 4 : *Let x_0 be such that $b = \frac{x_0}{x_0+1}$. Then*

$$\log(ab) > \frac{\left(\frac{ax}{x+1}\right)^x}{(ab)^x} \cdot \log\frac{ax}{x+1} \text{ for all } x > x_0$$

for the usual range of a $(0 < a < 1)$.

Proof: We define $Z = \left\{\frac{x}{b(x+1)}\right\}^x \log\frac{ax}{x+1}$, we note that for x_0,

$Z = \log ab$.

Now

$$\frac{dZ}{dx} = \left\{\frac{x}{b(x+1)}\right\}^x \left\{\log\frac{x}{b(x+1)}\log\frac{ax}{x+1} + \frac{1}{x+1}\log\frac{ax}{x+1} + \frac{1}{x(x+1)}\right\}.$$

$$a_1 < a_2 \Rightarrow \frac{1}{x(x+1)} + \frac{1}{x+1}\log\frac{a_1x}{x+1} < \frac{1}{x(x+1)} + \frac{1}{(x+1)}\log\frac{a_2x}{x+1} \text{ for all } x$$

and since for $x > x_0$, $\frac{x}{b(x+1)} > 1$

$$\log\frac{x}{b(x+1)}\log\frac{a_1x}{x+1} < \log\frac{x}{b(x+1)}\log\frac{a_2x}{x+1} \text{ for all } x > x_0.$$

If we can show that

$$A = \log\frac{x}{b(x+1)}\log\frac{x}{x+1} + \frac{1}{x+1}\log\frac{x}{x+1} + \frac{1}{x(x+1)} < 0 \text{ for all } x > x_0$$

then the lemma will be true for all $a < 1$.

We note that if b is large and consequently x_0 is large, then for $x > x_0$.

(a) $\log\frac{x}{b(x+1)}\log\frac{x}{x+1} < 0$ as $\frac{x}{b(x+1)} > 1$

(b) $\frac{1}{x+1}\log\frac{x}{x+1} < 0$

and (c) $\frac{1}{x(x+1)} \to 0$.

So, we consider only small values of b, say $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ etc. and study the behaviour of $\frac{1}{x(x+1)}+\frac{1}{x+1}\log\frac{ax}{x+1}$ for some selected values of $a \leqslant 1$.

TABLE 2. VALUES OF $\frac{1}{x(x+1)}+\frac{1}{x+1}\log\frac{ax}{x+1}$

x \ a	.90	.95	.99	.995	1.0
1	0.1007	0.1278	0.1478	0.1509	0.1534
2	−0.0036	0.0144	0.0281	0.0298	0.0315
3	−0.0149	−0.0014	0.0069	0.0122	0.0114
4	−0.0167	−0.0048	0.0038	0.0044	0.0054
5	−0.0146	−0.0056	0.0013	0.0021	0.0029
6	−0.0133	−0.0055	0.0008	0.0011	0.0018
7	−0.0120	−0.0052	−0.0001	0.0005	0.0014
8	−0.0109	−0.0049	−0.0003	0.0002	0.0008
9	−0.0010	−0.0045	−0.0004	0.0001	0.0006
10	−0.0009	−0.0042	−0.0004	−0.0003	0.0004

The validity of the lemma is verified for

$$b \geqslant \frac{9}{10} = .90 \text{ for } a \leqslant .995$$

$$b \geqslant \frac{6}{7} = .86 \text{ for } a \leqslant .99$$

$$b \geqslant \frac{2}{3} = .67 \text{ for } a \leqslant .95$$

and for

$$b \geqslant \frac{1}{2} = .50 \text{ for } a \leqslant .90$$

Since a represent $1-p_L$ it is clear from the above that the result is true for $b \geqslant .9$ for $p_L \geqslant .005$.

We calculate for $a=1$ the values of $\frac{1}{x(x+1)}+\frac{1}{x+1}\log\frac{x}{x+1}+\log\frac{x}{b(x+1)}$ $\log\frac{x}{x+1}$ for different values of b such as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$ and $\frac{9}{10}$.

TABLE 3. VALUES OF $\frac{1}{x(x+1)}+\frac{1}{x+1}\log\frac{ax}{x+1}+\log\frac{x}{b(x+1)}\log\frac{ax}{x+1}$ for $a=1$

x \ b	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$	$\frac{6}{7}$	$\frac{7}{8}$	$\frac{8}{9}$	$\frac{9}{10}$
1	.15								
2	−.09	.03							
3	−.10	−.02	.0114						
4	−.09	−.03	−.0090	.0054					
5	−.09	−.03	−.0163	−.0045	.0029				
6	−.08	−.03	−.0188	−.0088	−.1568	.0018			
7	−.0783	−.03	−.0191	−.0105	−.0050	−.0018	.0015		
8	−.0670	−.03	−.0192	−.0116	−.0068	−.0035	−.0011	.0008	
9	−.0613	−.0310	−.0186	−.0118	−.0075	−.0046	−.0023	−.0007	.0006
10	−.0565	−.0291	−.0179	−.0117	−.0079	−.0052	−.0032	−.0017	−.0005

It is clear from Table 3 that for $b \leqslant .9$ the expression A is negative for all $x > x_0$ and for all $a \leqslant 1$. Similar calculations for a large number of $b > .9$ and $.995 < a < 1$ also gave A negative as it was expected. This can be verified easily by extending Table 2 and Table 3.

Though no completely analytical proof could be given a large number of numerical examples showed that the lemma is true for all $b < 1$ and for all $a < 1$ and in particular for $a \leqslant .995$ i.e. for $p_L \geqslant .005$, which incidentally is the most commonly used range of p_L.

Lemma 5 : *For all* $x > x_0$, *log* $(ab) > \dfrac{\left(\frac{ax}{x+1}\right)^x}{(ab)^x}\;\dfrac{\log \frac{ax}{x+1}(2-(ab)^x)}{\left\{2-\left(\frac{ax}{x+1}\right)^x\right\}}$

Proof : Since for $x > x_0$, $\frac{x}{x+1} > b$

$$2-\left(\frac{ax}{x+1}\right)^x < 2-(ab)^x.$$

Thus, (i)
$$\frac{2-(ab)^x}{2-\left(\frac{ax}{x+1}\right)^x} > 1,$$

and (ii)
$$\frac{\left(\frac{ax}{x+1}\right)^x \log \frac{ax}{x+1}}{(ab)^x} < 0.$$

Now, use of Lemma 4 completes the proof.

Lemma 6 : $\frac{(ab)^x \log ab}{2-(ab)^x}$ *is greater than* $\frac{\left(\frac{ax}{x+1}\right)^x - \log (ax)}{2-\left(\frac{ax}{x+1}\right)^x}$ *for all* $x > x_0$ *and for all a and b less than* 1.

Proof : This follows immediately from the Lemmas 4 and 5.

Theorem 1 : *There exists a unique* $x(> x_0)$ *for which* $F^*(x)$ *attains its minimum value.*

Proof : From Lemmas 2, 3, 4, 5 and 6 we have for $x > x_0$

(i) $\log \frac{x}{x+1} - \log b > 0$

(ii) $$\frac{(ab)^x \log ab}{2-(ab)^x} - \frac{\left(\frac{ax}{x+1}\right)^x \log \frac{ax}{x+1}}{2-\left(\frac{ax}{x+1}\right)^x} > 0$$

and (iii) $$-\frac{\left(\frac{ax}{x+1}\right)^x \cdot \frac{1}{x+1}}{2-\left(\frac{ax}{x+1}\right)^x} > 0$$ but tends to zero for large x.

Hence it follows from equation (16) that there exists a $x > x_0$ for which $F^{*\prime}(x) = 0$ and afterwards $F^{*\prime}(x)$ becomes positive. Hence the result follows.

N.B. 1 : Our experience shows that x for for which $F^*(x)$ is minimum is usually one or two more than x_0 which minimises $F(x)$ of CSP−1.

5. THE UNIQUE COMBINATION (i^*, f^*) THAT MINIMISES F^*, THE AMOUNT OF INSPECTION FOR A GIVEN PROCESS AVERAGE $\bar{p}$ AND A DESIRED p_L

The algorithm may be stated as follows :

Step 1. Solve $\frac{i}{i+1} = \frac{1-\bar{p}}{1-p_L}$ and take $i_0 = [i]$.

Step 2. Compute $F^*_{\bar{p}}(i)$ for $i_0, i_0+1, \ldots,$ till an i is obtained for which $F^*_{\bar{p}}(i+1) > F^*_{\bar{p}}(i)$.

Step 3. Take this i as i^* and compute the corresponding f^* using equation (13).

N.B. 2 : This is an approximate plan but can be taken to be good enough for all practical purposes if $i^* \geqslant 10$. If $i^* < 10$ and the value of p_L is low then the approximate plan may be replaced by the exact one. For this we find the exact value of (i, f) by solving numerically eqn. (10) andt hen using eqn. (11). The optimum (i, f) is then obtained by evaluating $F_{\bar{p}}(i)$ in the vicinity of i^*.

N.B. 3 : If $\bar{p} < p_L$, there is no positive integer i_0 for which $\frac{i_0}{i_0+1} = \frac{1-\bar{p}}{1-p_L}$ and hence we can not find any optimum plan as $F^*_{\bar{p}}(i)$ is an ever decreasing function of i and the amount of inspection goes on decreasing as i increases.

The approximate optimum inspection plan for a wide range of AOQL and process average $\bar{p}$ is given in appendix 1. The plan gives $(i^*, f^*, F^*_{\bar{p}}(i))$. For $\bar{p} < p_L$, it is recommended that the plan for $\bar{p}$ which is just greater than p_L in the table should be used. For the sake of comparison, $(i, f, F_{\bar{p}}(i))$ for optimum CSP-1 plan is also incorporated in appendix 1.

6. THE OPTIMUM (i^*, f^*) WHEN THE INCOMING QUALITY p FOLLOWS A TWO POINT BINOMIAL DISTRIBUTION FOR A GIVEN p_L

It will be quite logical and practical too to assume that the incoming quality is controlled most of the time at $p_{(1)}$ and occasionally at $p_{(2)}$ rather than at a single process average $\bar{p}$.

We will workout the approximate optimum CSP-2 plan under the assumption that the incoming quality is $p_{(1)}$ with probability w_1 and is $p_{(2)}$ with probability w_2 so that $0 < p_L < p_{(1)} < p_{(2)} < 1$ and $w_1 + w_2 = 1$. Under the situation the average amount of inspection will be

$$F^*(i) = w_1 F^*_1(i) + w_2 F^*_2(i)$$

where

$$F^*_j(i) = \frac{1}{1+\frac{(i+1)^{i+1}}{i^i} \cdot \frac{p_L}{q_L}\left(\frac{q_{(j)}}{q_L}\right)^i \cdot \frac{2-q^i_{(j)}}{2-\left(\frac{i}{i+1}\right)^i q^i_L}}, \quad j=1, 2 \ldots \qquad (19)$$

The optimum i is obtained by solving the equation

$$\frac{dF^*(i)}{di} = 0.$$

The optimum plan is based on the following lemmas and theorem.

Lemma 7 : *$F_2^*(i)$ is always greater than $F_1^*(i)$ for all* $0 < p_{(1)} < p_{(2)} < 1$ *and for all positive i.*

Proof : $p_{(1)} < p_{(2)} \Rightarrow q_{(1)}^i > q_{(2)}^i$ and $q_{(j)}^i(2-q_{(j)}^i)$ is a decreasing function of p.

Hence the result.

Lemma 8 : *Let i_1 and i_2 be the optimum values of i in relation to $p_{(1)}$ and $p_{(2)}$ respectively for CSP-1 plan and i_1^* and i_2^* be the optimum values in relation to $p_{(1)}$ and $p_{(2)}$ under near optimum CSP-2 plan. Let d be maximum of i_1^* and i_2^*. Then d is greater than both i_1 and i_2.*

Proof : For CSP-1 we have

$$\frac{i_1}{i_1+1} = \frac{1-p_{(1)}}{1-p_L} \text{ and } \frac{i_2}{i_2+1} = \frac{1-p_{(2)}}{1-p_L} \text{ (see Ghosh (1988)}$$

and $p_{(1)} < p_{(2)}$. Hence $i_1 > i_2$. $i_1 > i_2 \Rightarrow i_1^* > i_2$. And i_2^* is always greater than i_2 in view of Lemma 3. Hence the result.

Theorem 2 : *There exists at least one i_0^* such that $i_2 \leqslant i_0^* \leqslant d$ for which the amount of inspection $F^*(i)$ is minimum provided* $0 < p_L < p_{(1)} < p_{(2)} < 1$.

Proof : We have $F^{*\prime}(i) = w_1 F_1^{*\prime}(i) + w_2 F_2^{*\prime}(i)$. It is known that

(i) For $i \leqslant i_2$, $F_1^*(i) > F_1^*(i_1^*)$ and $F_2^*(i) > F_2^*(i_2^*)$

(ii) For $i \geqslant d$, $F_1^*(i) > F_1^*(i_1^*)$ and $F_2^*(i) > F_2^*(i_2^*)$

(iii) For $i_2 < i < d$, for some successive values of i, $F_1^{*\prime}(i) < 0$ and $F_2^{*\prime}(i) < 0$; there after $F_1^{*\prime}(i) < 0$ and $F_2^*(i) > 0$. For later values of i both $F_1^{*\prime}(i) > 0$ and $F_2^{*\prime}(i) > 0$.

(iv) $w_1 > 0$, $w_2 > 0$.

Hence there is at least one $i_0^*(i_2 < i_0^* < d)$ for which $F^{*\prime}(i) = 0$ and $F^*(i)$ is minimum.

N.B. 4 : Unless $p_{(1)}$ and $p_{(2)}$ are very close i_1^* will be greater than i_2^* and hence $d = i_1^*$ and the search will remain confined between (i_2^*, i_1^*).

7. The optimum (i^*, f^*) when the incoming quality p follows a three point Binomial distribution or any continuous distribution which can be represented by discrete probabilities at some isolated points.

To settle the three point Binomial case we make use of the following theorem.

Theorerm 3 : *Let the underlying distribution for incoming quality p be $p_{(J)}$ with probability w_J such that* $0 < p_{(1)} < p_{(2)} < p_{(3)} < 1$ and $\sum_{i=1}^{3} w_J = 1$. *Then there exists at least one i_0^* for which $F^*(i) = \sum_{J=1}^{3} w_J F_J^*(i)$ is minimum provided each $p_{(J)}$ is greater than p_L. Also that i_0^* lies in the range $i_3 < i_0^* < d$ where d is maximum of i_1^*, i_2^* and i_3^* (the optimum choices for $p_{(1)}$, $p_{(2)}$ and $p_{(3)}$ for CSP-2) and i_3 is optimum for $p_{(3)}$ for CSP-1.*

Proof : The discussions in the proof of Theorem 2 can be extended easily to prove the above theorem.

Since $p_{(1)}$ and $p_{(3)}$ usually are not very close d is usually i_1^* and the search for optimum i can be made in the region (i_3^*, i_1^*).

A trivial extension of Theorem 3 provides the basis for finding i_0^* in the most general case where p may follow a continuous distribution. Let the distribution be approximated by probabilities w_J for $p_{(J)}$, $J = 1,2,\ldots n$ such that $\sum_{J=1}^{n} w_J = 1$ and that $0 < p_L < p_{(1)} < p_{(2)} \ldots < p_{(n)} < 1$. Then i_0^* lies between i_n and maximum of $(i_1^*, \ldots, i_n^*)$. Usually i_0^* will be obtained by evaluating $F_{(i)}^*$ in the range (i_n^*, i_1^*) and taking that value i for which $F_{(i)}^*$ is minimum.

Remarks : (1) If the incoming quality is lower than or equal to p in one or more cases in the distribution of p then the search for i_0^* is to be widened between i_J^* and ∞ as some i may tend to infinity. It may also happen that no optimum i exists as $F_{(i)}^*$ may turn out to be an ever decreasing function of i.

In such a situation the value of the relevant $p_{(J)}$ may be taken just greater than p_L (as considered in Appendix 1). Since all i_J^* $(J = 1, 2, \ldots, n)$ are now finite there will exist at least one finite i_0^* for which the amount of inspection for the modified situation is minimum.

(2) It was possible to prove in the case of optimum CSP-1 that the value of i which minimises the amount of inspection when incoming quality p follows a distribution is unique. Though this stronger result could not be proved for CSP-2 it has been found for the large number of cases studied that i_0^* is really a unique value.

(3) To study the effect of (i, f) in case of CSP-2 on the amount of inspection, $F_{(i)}^*$ under the stated approximation is shown graphically in Appendix 2 for some selected values of p_L for the cases where incoming quality p exhibits two or three point Binomial distribution.

8 Comparison of near optimum CSP–2 plan with optimum CSP–1 plan

In order to compare the performance we study the AOQ curve and the amount of inspection curve for different possible values of p in the range (0, 1) for two pairs of CSP-1 and CSP-2 plans which are optimal for $\bar{p} = 0.06$ and 0.09, each one ensuring the same AOQL of 0.05. This is shown in Appendix 3.

For a given $\bar{p}$ near optimal CSP-2 needs somewhat lesser inspection. Remembering that for exact optimum CSP-2 $F > F^*$, the observed difference with optimum CSP-1 is too small to be of practical importance.

It can be seen that for smaller values of p, CSP-1 needs lesser amount of inspection.

We find that in one case amount of inspection for CSP-2 is smaller than that of CSP-1 for higher values of p.

But we also encounter an example where amount of inspection for CSP-2 is higher than that of CSP-1 for all values of p. This is contrary to our expectation.

Dodge and Torrey expected that for same value of f and AOQL, inspection under CSP-2 will be less than that under CSP-1 if p is less than some multiples of AOQL and for higher levels of p inspection will be more for CSP-2. Even though this observation may be generally true, it has not much significance for comparisons are to be made between two optimum plans. We note that for optimum CSP-1 and 2 plans the values of i are more or less same while f may very widely. Thus at least one of the plans considered by Dodge and Torrey in their comparison was not optimal. Though the amount of inspection for optimum CSP-2 is not smaller than the corresponding optimum CSP-1 plan the AOQ curve is better for optimum CSP-2 plan. It is also expected from the relation $p_A = p(1-F)$.

Another important criterion will be to compare the p_t (%) for both type of plans to know the protection offered against a sudden deterioration in quality. The p_t (%) for CSP-1 is a fixed value and this is compared, as recommended by Dodge and Torrey, with the maximum p_t (%) that may result for CSP-2 type of inspection. The maximum p_t (%) in a continuous run of 1000 items that may result under CSP-2 can be directly read from Fig. 2 of Dodge and Torrey (1951). The relevant figures for two cases are shown in Table 4.

TABLE 4. COMPARISON OF p_t (%)

	$p_L = 0.05$			
index	$\bar{p} = .09$		$\bar{p} = .19$	
	CSP—1	CSP—2	CSP—1	CSP—2
i	23	24	9	10
f	.0836	.1360	.3169	.4136
p_t (%)	2.7	4.0	0.7	2.5

Since p_t (%) is also more for CSP-2 it appears that the near optimum and hence the optimum CSP-2 plan with $i = k$ has little more to offer than optimal CSP-1 plan and as such CSP-1 appears to be the only choice.

It is, therefore, necessary to explore the properties of CSP-2 plan with $i \neq k$ so that we can identify the combination (i, f, k), if it exists, for which the optimum CSP-2 plan will have lesser amount of inspection and/or greater protection against spotty quality than the optimal CSP-1 plan.

Acknowledgement. The author is grateful to Dr. A. C. Mukhopadhyay, Head, Computer Science Unit for the discussions he had with him..

Appendix—1

VALUES OF i, f (i^*, f^*) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGE $100p$

Incoming process average	AOQL (%) 1.0		2.0		3.0		4.0		5.0	
	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP
1.0	98 .1213 27.20	101 .1952 29.02								
2.0	98 .1213 50.00	101 .1952 49.96	97 .0253 15.60	99 .0456 15.93						
3.0	48 .3167 66.67	53 .4159 66.52	97 .0253 33.33	99 .0456 33.33	97 .0063 10.37	97 .0124 11.04				
4.0	32 .4462 75.00	36 .5473 74.81	48 .1235 50.00	51 .1891 49.94	97 .0063 25.00	97 .0124 25.00	95 .0019 8.40	95 .0038 8.51		
5.0	24 .5387 80.00	27 .6338 79.81	32 .2262 60.00	35 .3125 59.89	46 .0538 40.00	49 .0936 39.98	95 .0019 20.00	95 .0038 20.00	94 .0006 6.90	94 .0012 6.90
6.0	19 .6068 83.33	22 .6879 83.14	24 .3118 66.70	26 .4171 66.51	31 .1281 50.00	33 .1949 49.94	47 .0266 33.33	48 .0476 33.38	94 .0006 16.70	94 .0012 16.67
7.0	16 .6527 85.72	18 .7345 85.53	19 .3864 71.40	21 .4903 71.25	23 .2008 57.10	25 .2844 57.04	31 .0733 42.90	32 .1226 42.83	46 .0140 28.60	47 .0253 28.57

N.B. In the absence of an optimum plan for $\bar{p} \leqslant p_L$, the plan for $\bar{p}$, which is just greater than p_L, is recommended.

Appendix—1 (*contd.*)

VALUES OF i, f (i^*, f^*) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGE $100p$

Incoming process average	AOQL (%) 1.0 CSP1	1.0 CSP2	2.0 CSP1	2.0 CSP2	3.0 CSP1	3.0 CSP2	4.0 CSP1	4.0 CSP2	5.0 CSP1	5.0 CSP2
8.0	13 .7030 87.50	16 .7591 87.33	15 .4621 75.00	18 .5406 14.81	18 .2709 62.60	20 .3616 62.37	23 .1281 50.00	25 .1888 49.94	31 .0433 37.50	32 .0748 37.48
9.0	11 .7393 86.90	14 .7846 88.73	13 .5067 77.80	15 .5963 77.57	15 .3271 66.70	17 .4182 66.51	18 .1863 55.60	20 .2587 55.46	23 .0835 44.45	24 .1361 44.40
10.0	10 .7583 90.00	12 .8110 89.84	11 .5666 80.00	13 .6367 79.80	13 .3723 70.00	15 .4610 69.84	15 .2360 60.00	16 .2842 59.69	18 .1305 50.00	19 .2008 49.93
11.0	9 .7780 90.90	11 .7898 90.76	10 .5839 81.80	12 .6580 81.63	11 .4253 72.70	13 .5083 72.54	13 .2779 63.60	14 .3304 63.49	15 .1728 54.55	16 .2547 54.45
12.0	8 .7982 91.67	10 .8382 91.58	9 .6128 83.30	10 .7028 83.14	10 .4552 75.00	11 .5608 74.80	11 .3259 66.70	12 .4332 66.51	13 .2100 58.35	14 .2990 58.22
13.0	7 .8192 92.30	9 .8522 92.18	8 .6435 84.60	9 .7263 84.43	9 .4878 76.90	10 .5892 76.72	10 .3587 69.20	11 .4624 69.05	11 .2570 61.54	12 .3514 61.40
14.0	7 .8192 92.90	8 .8666 92.73	7 .6762 85.70	8 .7507 85.54	8 .5232 78.60	9 .6189 78.36	9 .3917 71.50	10 .4997 71.24	10 .2851 64.31	11 .3812 64.14

N.B. In the absence of an optimum plan for $\bar{p} < p_L$, the plan for $\bar{p}$, which is just greater than p_L, is recommended.

Appendix—1 (*contd.*)

VALUES OF i, f (i^*, f^*) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGE $100p$

Incoming process average	AOQL (%) 1.0		2.0		3.0		4.0		5.0	
	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2
15.0	6 .8408 93.30	7 .8810 93.20	7 .6762 86.70	8 .7507 86.50	7 .5619 80.00	8 .6503 79.60	8 .4285 73.30	9 .5272 73.14	9 .3169 66.70	10 .4136 66.52
16.0	6 .8408 93.80	7 .8810 93.68	6 .7109 87.50	7 .7760 87.31	6 .6041 81.20	8 .6503 81.07	7 .4696 75.00	8 .5631 74.79	8 .3531 68.77	9 .4489 68.58
17.0	5 .8631 94.10	6 .8957 94.01	6 .7109 88.30	7 .7760 80.07	6 .6041 82.30	7 .6833 82.14	6 .5156 76.50	7 .6014 76.29	7 .3944 70.59	8 .4873 70.40
18.0	5 .8631 94.40	6 .8957 94.34	5 .7479 88.90	6 .8021 88.72	5 .6503 83.40	6 .7181 83.17	6 .5156 77.70	7 .6014 77.57	6 .4417 72.24	7 .5292 72.03
19.0	5 .8631 94.80	6 .8957 94.66	5 .7479 89.50	6 .8021 89.31	5 .6503 84.20	6 .7181 84.00	5 .5672 79.00	6 .6426 78.75	6 .4417 73.69	7 .5292 73.50
20.0	4 .8862 95.00	5 .9107 94.90	4 .7873 90.00	5 .8292 89.86	5 .6503 85.00	6 .7181 84.83	5 .5672 80.00	6 .6426 79.78	5 .4962 75.00	6 .5743 74.79

N.B. In the absence of an optimum plan for $\bar{P} < p_L$, the plan for p, which is just greater than p_L, is recommended.

Appendix—1 (*contd.*)

VALUES OF i, f (i^*, f^*) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGE 100p

Incoming process average	AOQL (%) 6.0		7.0		8.0		9.0		10.0	
	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2
1.0										
2.0										
3.0										
4.0										
5.0										
6.0	93 .0002 5.80	93 .0004 5.81								
7.0	93 .0002 14.30	93 .0004 14.30	93 61×10^{-6} 4.97	93 122×10^{-6} 4.97						

N.B. In the absence of an optimum plan $\bar{p} < p_L$, the plan for $\bar{p}$, which is just greaer than p_L, is recommended.

Appendix—1 (*contd.*)

VALUES OF $\hat{i}$, f ($*i$, $f*$) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAGE 100p

Incoming process average	AOQL (%) 6.0		7.0		8.0		9.0		10.0	
	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2	CSP1	CSP2
8.0	46 .0071 25.00	46 .0140 25.00	93 61×10^{-6} 12.50	93 122×10^{-6} 12.50	91 23×10^{-6} 4.42	91 47×10^{-6} 4.42				
9.0	30 .0287 33.33	31 .0496 33.32	46 .0037 22.20	46 .0074 22.20	91 23×10^{-6} 11.11	91 47×10^{-6} 11.11	90 8×10^{-6} 9.95	90 17×10^{-6} 8.95		
10.0	23 .0558 40.00	28 .1014 39.98	30 .0178 30.0	31 .0314 30.00	45 .0022 20.0	45 .0043 20.00	90 8×10^{-6} 10.06	90 17×10^{-6} 10.00	89 3×10^{-6} 3.6	89 6×10^{-6} 3.57
11.0	18 .0928 45.46	19 .1466 45.41	22 .0422 36.36	23 .0702 36.35	30 .0112 27.27	30 .0219 27.27	45 .0012 18.20	45 .0023 19.18	89 3×10^{-6} 9.10	89 6×10^{-6} 9.09
12.0	15 .1282 50.00	16 .1944 49.94	18 .0668 41.67	18 .1195 41.65	22 .0292 33.33	23 .0489 33.33	29 .0081 25.00	30 .0140 25.00	44 72×10^{-6} 16.70	44 143×10^{-6} 16.70
13.0	12 .1800 53.86	13 .2590 53.77	15 .0960 46.17	15 .1658 46.11	17 .0558 38.57	18 .0889 38.44	22 .0203 30.77	22 .0389 30.76	29 .0053 23.08	29 .0104 23.08
14.0	11 .2024 57.15	12 .2852 57.03	12 .1407 50.00	12 .2069 49.93	14 .0833 42.86	15 .1287 42.82	17 .0410 35.72	18 .0663 35.70	21 .0165 28.58	22 .0276 28.57

N.B. In the absence of an optimum plan for $\bar{p} \leq p_L$, the plan for p, which is just greater tha , is recommended.

Appendix—1 (*contd.*)

VALUES OF i, f (i^*, f^*) AND AMOUNT OF INSPECTION FOR CSP 1 (CSP 2 $k=i$) PLANS FOR GIVEN VALUES OF AOQL (%) AND PROCESS AVERAOE $100p$

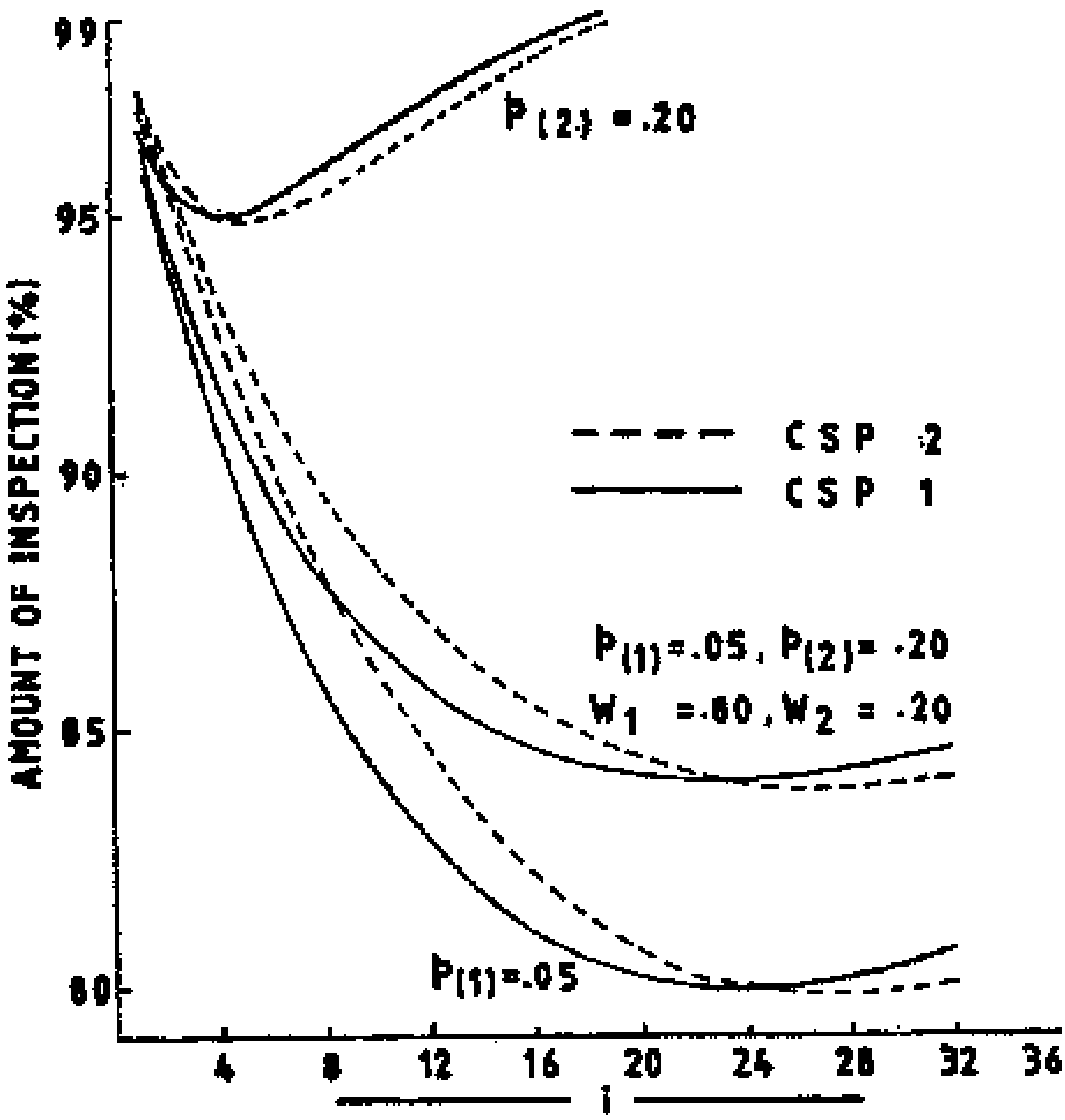
incoming process average	AOQL (%)									
	6.0		7.0		8.0		9.0		10.0	
	CSP1	CSP2	CSPI	CSP2	CSP1	CSP2	CSP1	CSP2	CSPI	CSP2
15.0	9 .2580 60.02	10 .3464 59.89	11 .1606 53.35	12 .2314 53.27	12 .1107 46.67	13 .1655 46.61	14 .0641 40.00	15 .1001 39.98	17 .0306 33.33	18 .0496 33.33
16.0	8 .2926 62.52	9 .3821 62.37	9 .2118 56.26	10 .2901 56.15	11 .1282 50.02	11 .2135 49.94	12 .0875 43.75	12 .1526 43.75	14 .0496 37.50	15 .0779 37.49
17.0	8 .2926 64.74	8 .4216 64.58	8 .2435 58.84	9 .3251 58.70	9 .1738 52.95	10 .2429 52.85	10 .1212 47.07	11 .1760 47.00	12 .0696 41.18	12 .1240 41.16
18.0	7 .3328 66.70	8 .4216 66.51	7 .2818 61.16	8 .3645 60.98	8 .2036 55.56	9 .2765 55.45	9 .1435 50.00	10 .2033 49.93	10 .0990 44.45	11 .1451 44.30
19.0	6 .3798 68.43	7 .4654 68.20	7 .2818 63.17	8 .3645 63.05	7 .2395 57.92	8 .3150 57.77	8 .1707 52.63	9 .2351 52.55	9 .1190 47.37	10 .1702 47.33
20.0	6 .3798 70.00	7 .4654 69.86	6 .3275 65.00	7 .4090 64.88	7 .2395 60.00	8 .3151 59.94	7 .2041 55.00	8 .2722 54.89	8 .1487 50.00	9 .1999 49.95

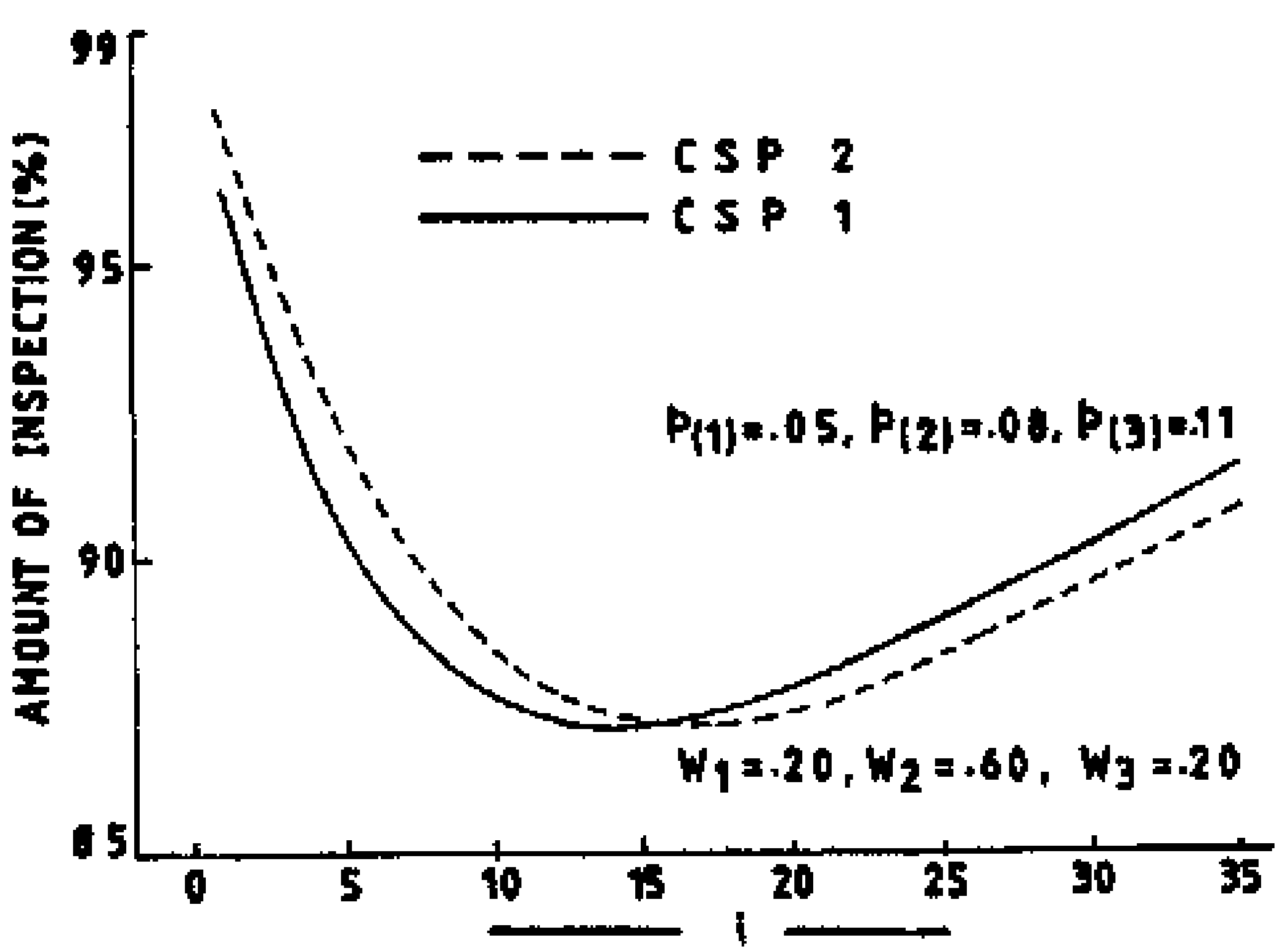
N.B. In the absence of an optimum plan for $\bar{p} \leqslant \bar{p}_L$, the plan for $\bar{p}$, which is just greater than $\bar{p}_L$, is recommended.

APPENDIX = 2

AMOUNT OF INSPECTION (%) FOR A GIVEN p_L FOR DIFFERENT VALUES OF i WHEN INCOMING QUALITY FOLLOWS A DISTRIBUTION

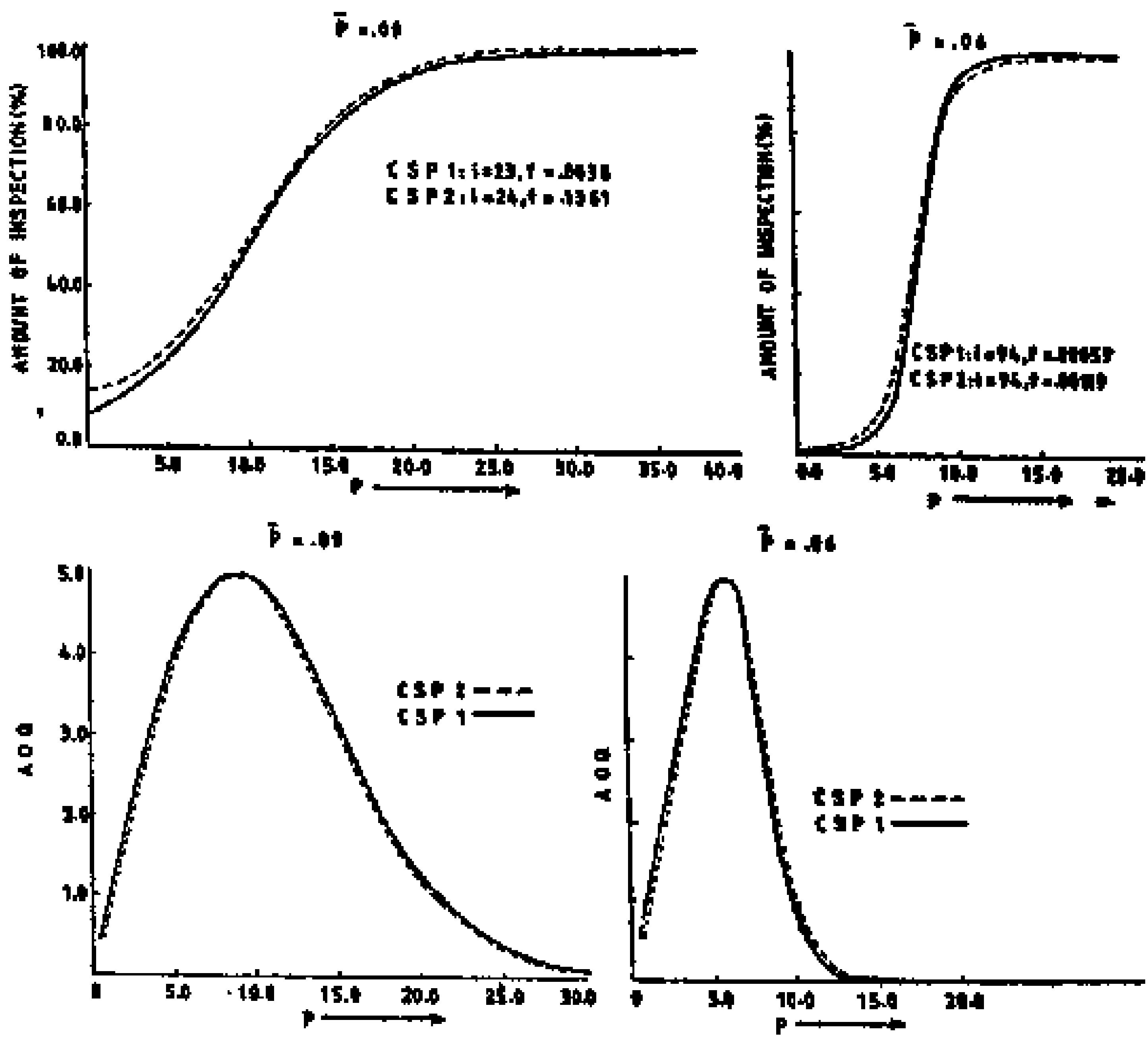
p_L =.01





Appendix—III

COMPARISON OF AMOUNT OF INSPECTION AND AOQ FOR OPTIMUM CSP PLANS
pL = .05



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