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A Novel Face Recognition System Using Hybrid Neural and Dual Eigenspaces Methods

David Zhang, Hui Peng, Jie Zhou, and Sankar K. Pal

Abstract—In this paper, we present an automated face recognition (AFR) system that contains two components: eye detection and face recognition. Based on invariant radial basis function (IRBF) networks and knowledge rules of facial topology, a hybrid neural method is proposed to localize human eyes and segment the face region from a scene. A dual eigenspaces method (DEM) is then developed to extract algebraic features of the face and perform the recognition task with a two-layer minimum distance classifier. Experimental results illustrate that the proposed system is effective and robust.

Index Terms—Dual eigenspaces method, eyes detection, face recognition, hybrid neural method.

I. INTRODUCTION

As one of the most challenging tasks for computer vision and pattern recognition, the problem of automated face recognition (AFR) has been a topic that has been studied for thirty years. Since the end of the 1980s, more and more researchers have devoted themselves to this subject, and many significant results have been achieved [1], [2]. A fully implemented AFR system usually involves two major tasks: (1) face detection and (2) face recognition. Locating face, or facial organs, in a scene is no doubt, the first essential step. The conventional approaches for face location include, image-based methods [3], model-based methods [4]–[6], and NN(Neural Networks)-based methods [7]–[9]. Based upon previous research, face recognition can be roughly divided into two categories [2], [10]–[12]: the connectionist approach, which is based upon the learning of neural networks, and the nonconnectionist method, which is based upon matching of face model. As an effective approach, the eigenfaces method [13]–[16] computes the principle components of an ensemble of facial images to serve as feature vectors. Intrinsically, these features seek to capture the holistic, or gestalt-like nature of facial images, and can be named as algebraic features.

Despite the research mentioned above, many drawbacks and difficulties still exist relating to the AFR system. What we first point out is the relationship between the human and machine in face recognition. Human beings have an innate and remarkable ability to handle face recognition on a daily basis. The mechanics of this have not been fully explored, however, so using a computational or analytical face model with enough flexibility and efficiency is not something that can yet be mimicked. From a cognitive view, two facts give us good enlightenment to propose a novel AFR system. The first is that human beings have a strong ability to learn from new samples and enrich patterns and knowledge stored in memory. This inspires us to develop an improved version of the NN-based method for eye detection in our AFR system.

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Although there are many different types of NN's that can be employed, what attracts our interest the most is radial basis function (RBF) networks, due to their rapid training, good generality, and simplicity in contrast to MLP [8]. The second, is the use of holistic features that are more important than those detailed features in the human recognition procedure. As a result, we are motivated to study algebraic features and propose a dual eigenspaces method (DEM) as an improvement of the conventional eigenface method.

Another point we must emphasize is that many difficult issue arise from different influences, such as, various scales, perspective angles, cluttered backgrounds, different illumination, and meaningful expressions. The AFR system we propose in this paper attempt to overcome these influences and achieve satisfied results. Our system will not only relax those constraints on the scale, posture, and countenance of subjects, but also tolerate many different nonrestricted image acquisition conditions.

II. EYES DETECTION USING HYBRID NEURAL METHOD

A. Facial Image Preprocessing

Our first step is resolution reduction using the neighborhood-averaging approach. A lower resolution image, e.g., 128×128 dimensions, can be generated so that the amount of image data fed into the neural networks is greatly reduced. The next step is gray-level normalization. The technique of histogram modification is performed to adjust the mean intensities and standard deviation of each image to the same value. This can partly reduce the sensibility to illumination strength.

B. Hybrid Neural Networks

Eyes play a critical role in face localization since their position is stable despite changes in facial expression. To detect eyes well, we present a hybrid neural method that combines an improved version of RBF networks with the hierarchical knowledge-based approach.

1) *Invariant RBF Networks for Eyes Detection*: The structure of RBF networks is similar to a three-layer feedforward neural network [17], where the input layer is fully connected to the hidden layer, via unity weights, and the hidden layer, composed of a number of RBF nodes associated with two sorts of parameters, namely, the centers and widths. Each RBF node computes the Euclidean distance between its own center and the network input vector, which then transfers to a radial basis function. The most common radial basis function is the Gaussian function, in which case the activation h_j of hidden node j is calculated by

$$h_j = \exp \left\{ -\frac{1}{2\sigma_j^2} (X - C_j)^T (X - C_j) \right\}, j = 1, 2, \dots, J \quad (1)$$

where X is the input vector, J is the number of hidden nodes, C_j , and σ_j^2 are the centers and width of hidden node j , respectively. It is obvious that the radial basis function gives the highest output when the input is close to its center, decreasing monotonically as the distance from the center increases. The activation y_k of output node k is determined by

$$y_k = \sum_{j=1}^J \omega_{jk} h_j - \theta_k, k = 1, 2, \dots, K \quad (2)$$

where ω_{jk} is the weight from hidden node j to output node k , θ_k is the threshold of output node k , and K is the number of output nodes.

In general, the output nodes form a linear combination of the nonlinear basis functions, thus the overall network performs a nonlinear

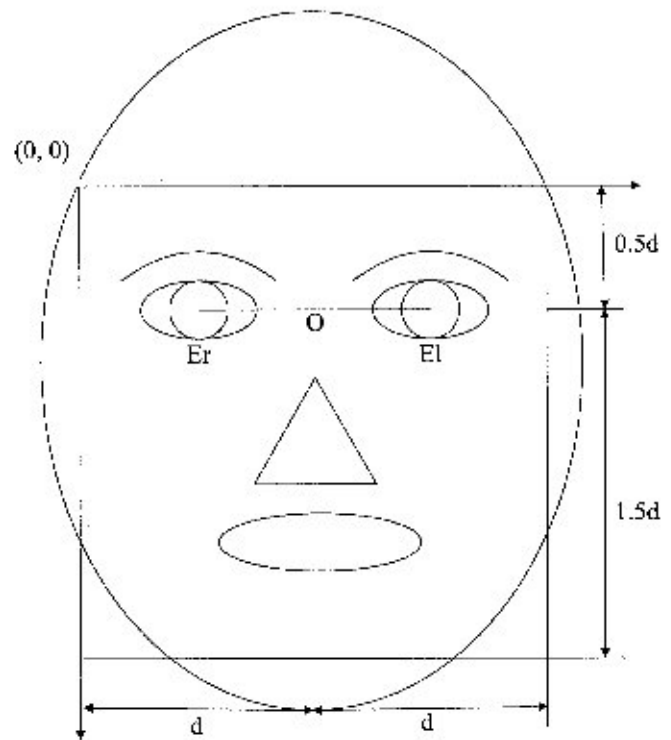


Fig. 1. Standard face alignment window.

transformation of the input. The response of the output node may be considered as a map $f: R^J \rightarrow R$, that is,

$$f(X) = \sum_{j=1}^J \omega_j h(\|X - C_j\|) \quad (3)$$

where $\|\cdot\|$ denotes the Euclidean norm. Since eye data forms itself into clusters in the original high-dimension space, the RBF networks can be used to identify eyes due to their partition in the input space.

In order to improve the invariance of RBF networks, a new algorithm is developed to determine the centers of hidden nodes in this paper. Instead of the typical classification of those training samples: eye and noneye data, more clusters may be created based on the following complex situations:

- 1) nuances between the left and right eye;
- 2) various eye sizes due to uncertain distances between the face and camera;
- 3) different eye orientations due to the unrestricted posture of the face.

According to the diversified factors combined above, eye training samples can be classified into more clusters. As to noneye samples that are extracted from facial images randomly, they can be also divided into several clusters. After selection of initial seeds of each cluster, c-means cluster algorithm is applied to determine the centers of each hidden node. Due to this improvement, our invariant radial basis function (IRBF) networks become more robust against variations of scale and orientation.

The width σ_j^2 represents the variance of the data in Cluster j . It is commonly determined by the average distance between the cluster center C_j and the training samples in that cluster. For Cluster j

$$\sigma_j^2 = \frac{1}{M_j} \sum_{X \in \mathcal{U}_j} (X - C_j)^T (X - C_j) \quad (4)$$

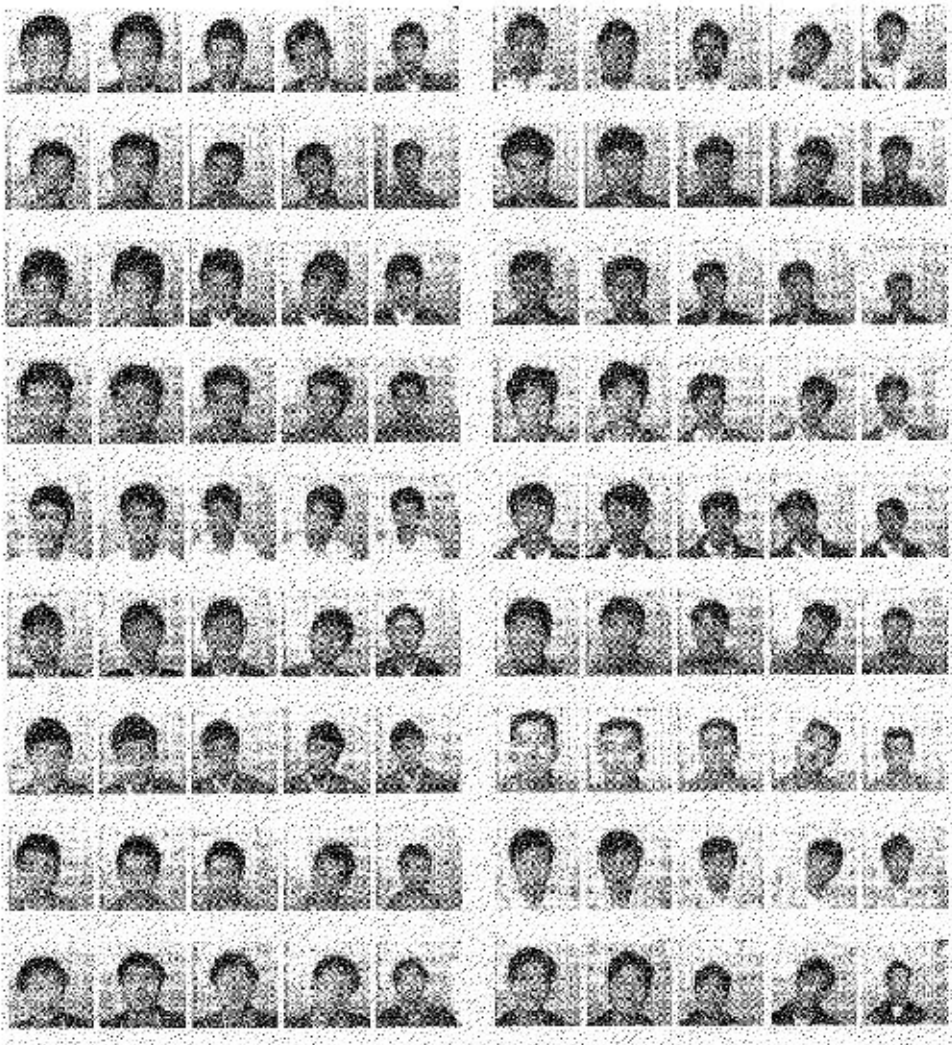


Fig. 2. Some typical facial images in our database.

where $X \in \Omega_j$ is a training vector in the cluster j , C_j is the cluster center, and M_j is the number of training samples in that cluster.

Weight adjustment between the hidden layer and the output layer deals with the least mean square (LMS) rule. Consider the squared error of the networks as follows:

$$J(\omega) = \frac{1}{2} [Y(n) - O(n)]^2 \quad (5)$$

where $Y(n)$ is the activation of output layer and $O(n)$ is the desired output. LMS rule is to minimize the error by adjusting the weight vector, i.e.

$$\Delta\omega = -\eta \frac{\partial J(\omega)}{\partial \omega} \Big|_{\omega=\omega(n)} \quad (6)$$

where η is the learning rate. This yields

$$\Delta\omega = \eta [O(n) - Y(n)] h'(X(n)) \quad (7)$$

Our proposed IRBF networks serve as a filter between an input facial image and a mapping image where the peaks of intensities are referred to all possible eye candidates. It receives a vector that is constructed from the scanning window, and generates an output value ranging from 1 to 0, signifying the presence, or absence of an eye, respectively. After scanning the whole input image, the mapping image is completely obtained. Those pixels with value 1 construct some candidates of eyes regions; on the contrary, those pixels with value 0 form noneyes regions.

2) *Knowledge-Based Approach for Validation:* The candidates of eye regions obtained by IRBF networks might contain some false regions, thus a knowledge-based approach is proposed to remove them. Only the final two candidates survived from this stage will be regarded as the regions of both eyes. After edge smoothing and noise reducing, each eye candidate region is labeled, with its corresponding parameters, including gravity center, area, length, and slope of the longest line.

Based on a prior knowledge of the geometrical relationship between two eyes, a database of rules is constructed to validate these candidates. First, we evaluate each candidate one by one. A candidate will be removed if it satisfies one of the following conditions:

- The area of this region is smaller than threshold, such as 8 pixels;
- The length of the longest line is shorter than threshold, such as 4 pixels;
- The slope of the longest line is higher than threshold, such as 1.

Then, the remaining candidates are dealt with using the following knowledge-based rules:

- If the number of candidates is less than 2, it means our system can not find eyes correctly, so a "reject" result will be given;
- If the number of candidates is exactly equal to 2, then the two regions will be selected as the target regions;
- If the number of candidates is greater than 2, we will evaluate all possible combinations of any two candidates. For each combination, we draw a line to link the gravity centers of two regions,

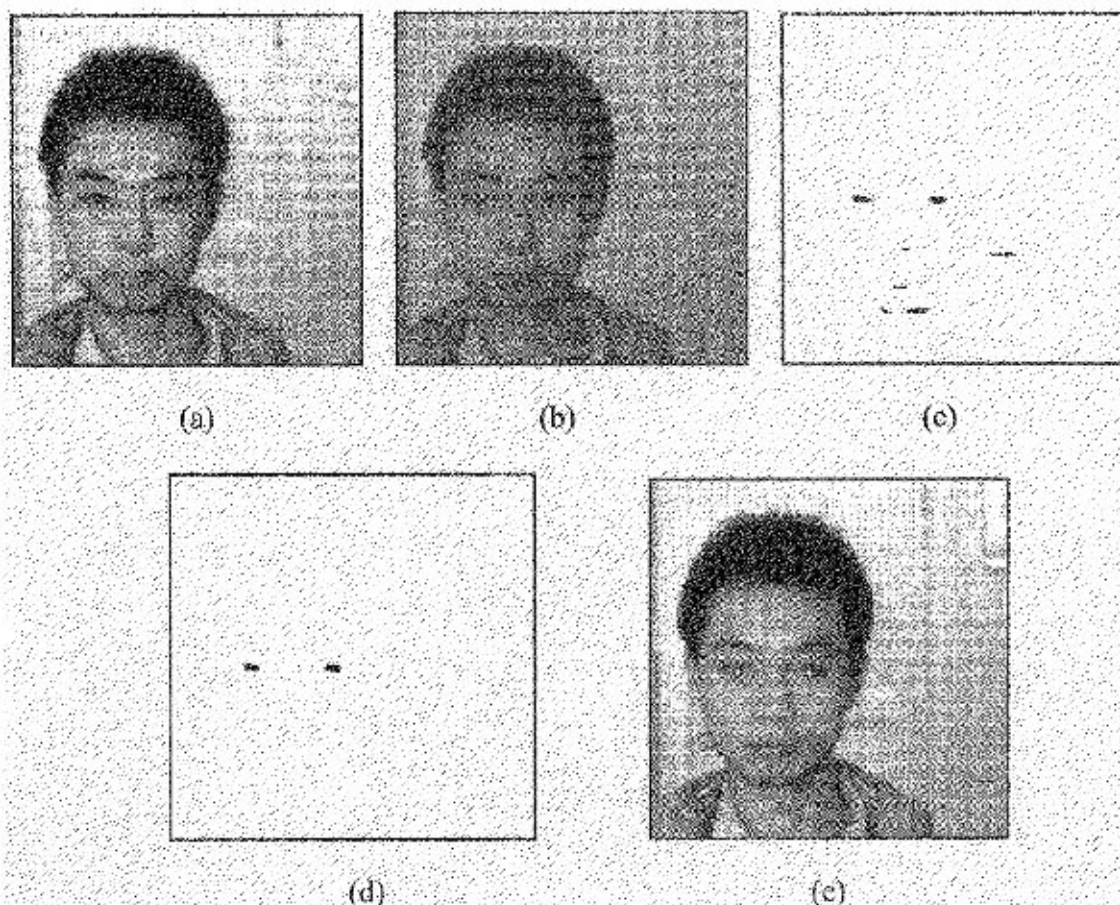


Fig. 3. Processing results at each detection stage. (a) Original image. (b) Normalized image. (c) Candidates of eyes. (d) Target regions. (e) Final result.

and then calculate the angles between this line and the longest line of each region, respectively. The combination that has the smallest average value of two angles will be served as the target regions.

Finally, the gravity centers of each target region will be marked as the central points of the left and right eyes.

C. Face Segmentation and Alignment

It is important to segment and normalize facial image to geometrically align it with the standard sample before face recognition. Regarding both eyes as steady landmarks of face, they can be served as anchors for image alignment (See Fig. 1). In order to explain easily, we represent the central point of right (left) eye as E_r (E_l); the line from E_r to E_l as $\overline{E_r E_l}$; the middle point of $\overline{E_r E_l}$ as O ; and the length of $\overline{E_r E_l}$ as d . The alignment procedure is described as follows:

- 1) Rotation invariance: Move the image so that $\overline{E_r E_l}$ can keep horizontal.
- 2) Shift invariance: Translate the image to arrange the point O at the relative fixed position $(0.5d, d)$.
- 3) Scale invariance: Crop the image with a standard window as shown in Fig. 1, then scale it from $2d \times 2d$ to 128×128 pixels, so that the distance between both eyes can keep constant, *i.e.*, 64 pixels.

After such an alignment procedure, the face is extracted and fixed at the same position with the same size and orientation approximately. In other words, we achieve a kind of geometrical invariant representation of face in the image plane. At the same time, the influences of background and hairstyle are eliminated because the standard window is enclosing only the face.

III. FACE RECOGNITION BY DUAL EIGENSPACES METHOD

Face recognition is the core module of our AFR system. As stated above, Eigenface method appears as a fast, simple, and practical method. Pentland *et al.* [14] extended their early work on eigenface to eigenfeatures corresponding to face components, such as eyes, nose, and mouth. This method achieved a satisfying recognition rate on the FERET database. In [15], Pentland *et al.* proposed another improved Eigenface method, which decomposes the original space into a principal subspace and its orthogonal components.

Different with the eigenface methods mentioned above, a new scheme called "dual eigenspaces method" (DEM) is described in this section, which is designated to combine K-L expansion technique with the coarse-to-fine matching strategy. In DEM, besides of the unitary eigenspace, we introduce another kind of eigenspace for each individual to characterize the variations among each person's face. The coarse classification is first performed in unitary eigenspace, then a few candidates are chosen and further analyzed in each candidate's individual eigenspace for the finer classification.

A. Algebraic Features Extraction by K-L Transform

In traditional eigenface method, the eigenvectors of K-L generating matrix span a subspace where any facial image can be represented in terms of a weight vector. Intuitively, each eigenvector can be displayed as a sort of ghostly facial image in appearance. The weight vector is regarded as a sort of algebraic features for its characterization of human face. The generating matrix of K-L transform is the total scatter matrix

$$S_t = \frac{1}{M} \sum_{i=1}^{2N} (x_i - m)(x_i - m)^T \quad (8)$$

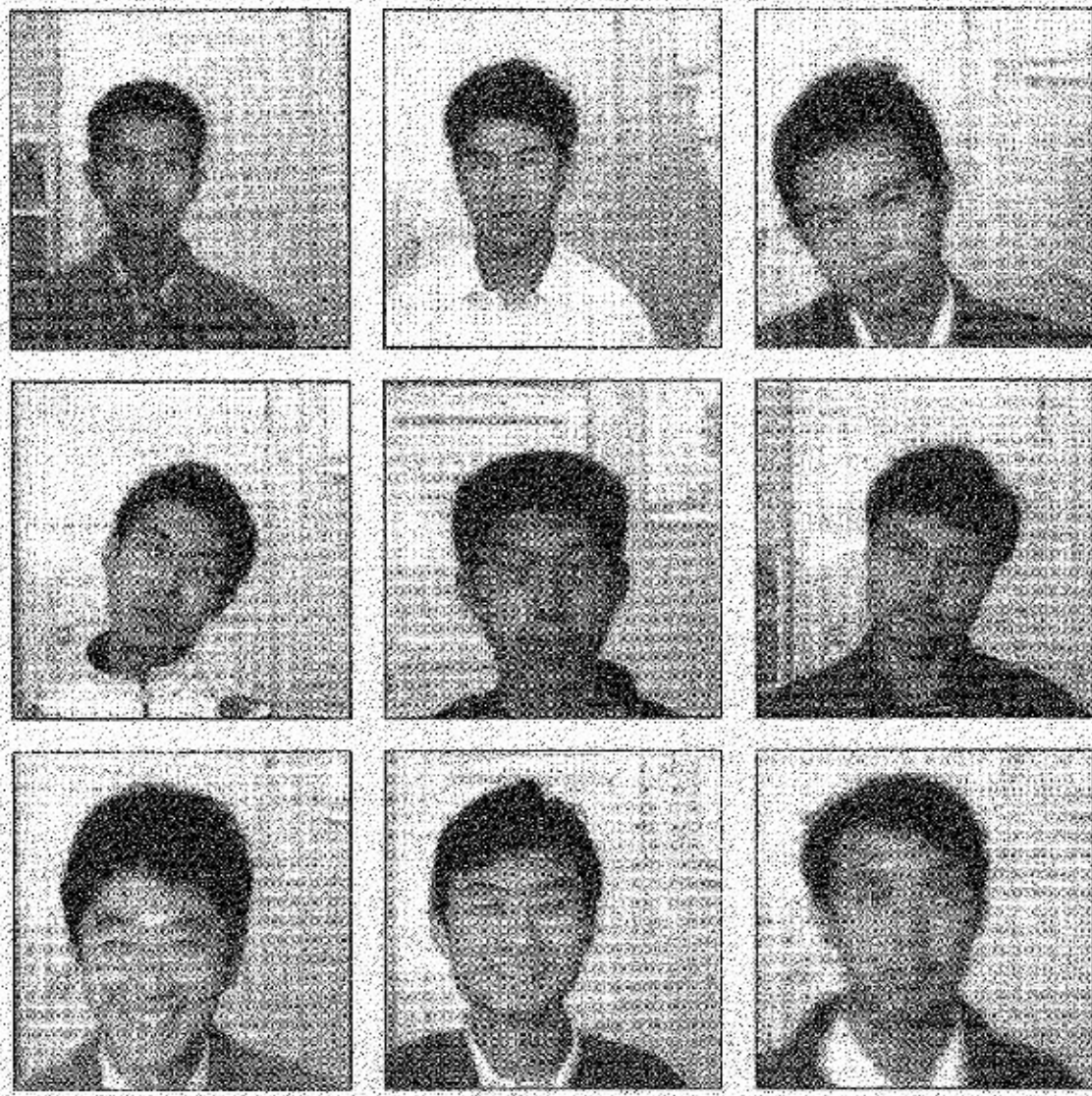


Fig. 4. Some detection results under different conditions.

where x_i denotes a vector of length N^2 ($N \times N$ facial image), m is the average image of all the training samples, and M is the number of images in the training set.

In our scheme, for higher computational simplicity without loss of accuracy, the between-class scatter matrix is adopted as the generating matrix

$$S_b = \frac{1}{P} \sum_{i=1}^P (m_i - m)(m_i - m)^T = \frac{1}{P} X X^T \quad (9)$$

where $X = [(m_1 - m), \dots, (m_P - m)]$, m_i is the average image of the i^{th} person's training samples, and P is the number of people in the training set. However, directly calculating the eigenvectors of the matrix, $S_b \in \mathbb{R}^{N^2 \times N^2}$, is an intractable task. Fortunately, this can be solved by using SVD theorem [18]. Firstly, a lower dimensional matrix is formalized as follows:

$$R = \frac{1}{P} X^T X \in \mathbb{R}^{P \times P} \quad (10)$$

Obviously, it is much easier to calculate its eigenvalues, $A = \text{diag}[\lambda_1, \dots, \lambda_{P-1}]$, and orthonormal eigenvectors, $V = [v_1, \dots, v_{P-1}]$. Then, the eigenvectors of S_b , i.e., eigenfaces, can be derived by SVD theorem

$$U = X V A^{-1/2} \quad (11)$$

where $U = [u_1, \dots, u_{P-1}]$ denotes the basis vectors which span an algebraic subspace called unitary eigenspace of the training set. Finally, the following result is obtained

$$C = U^T X = A^{1/2} V^T \quad (12)$$

where $C = [c_1, \dots, c_P]$ is referred to as the standard feature vectors of each person.

In traditional eigenface method, face recognition is performed only in the unitary eigenspace mentioned above. However, some eigenvectors might primarily act as "noise" for identification because they mainly capture unwanted variations due to illumination or facial expressions [13]. This makes the reduction in recognition rate when head pose, lighting condition or facial expression is varied. In order to further characterize the variations among each person's face and analyze their different distributions of the weight vectors in the unitary eigenspace, we construct new eigenspaces for each person by carrying out another K-L transform. For the i^{th} person, its generating matrix is selected as the within-class scatter matrix of all the weight vectors of its training samples

$$W_i = \frac{1}{M_i} \sum_{j=1}^{M_i} (y_i^{(j)} - c_i)(y_i^{(j)} - c_i)^T, \quad i = 1, \dots, P \quad (13)$$

where $y_i^{(j)} = U^T(x_i^{(j)} - \mu_i)$ is defined as the weight vector of the i^{th} person's training sample $x_i^{(j)}$ and M_i is the number of i^{th} person's images in the training set. Note that the eigenvector of each W_i is easily obtained. Here those minor components (MC's) are chosen to span each person's individual eigenspace denoted by \tilde{U}_i ($i = 1, \dots, P$). In cooperation with the unitary eigenspace, the construction of our dual eigenspaces has been completed.

B. Face Recognition by Two-Layer Classifier

In the recognition phase, a two-layer classifier is built up. In the top layer, a common minimum distance classifier is used in the unitary eigenspace. For a given input facial image, f , its weight vector can be derived with a simple inner product operation

$$y = U^T(f - \mu). \quad (14)$$

In this way, the coarse classification can be performed by the distance between y and each person's standard feature vector, c_i ($i = 1, \dots, P$). Then a few candidates who have the minimum distance are chosen for the finer classification. In the bottom layer, the weight vector, y , is separately mapped onto each candidate's individual eigenspace to yield coordinate vectors

$$\tilde{y}_i = \tilde{U}_i^T(y - c_i). \quad (15)$$

If $d_j = \min\{d_i : d_i = \|\tilde{y}_i\|\}$, the input image, f , can be recognized as the j^{th} person.

IV. EXPERIMENTAL RESULTS

The proposed AFR system has been implemented on a SUN Sparc20 workstation. In order to carry out our experiments, an image database of more than 600 facial images is firstly built up, which are acquired in the laboratory environment with a simple background. For each person, at least 13 facial images are taken under several different kinds of conditions (See Fig. 2).

- 1) The image scene is illuminated by a single light source from different direction;
- 2) The distance between the face and the camera can be roughly classified as three categories: near (< 1 m), medium (1 m ~ 3 m) and far (> 3 m);
- 3) There are variations in facial expressions, including smiling, surprise, anger and so on;
- 4) The subject is in an upright frontal position with tolerance for some tilting and rotation up to about 25 degrees.

In the detection phase, 30 faces are randomly selected for the training of IRBF network and the others are left for the test. For a given image, all candidates of eyes regions are first indicated after preprocessing. Since some false regions may be induced, the true regions of both eyes are finally located using the knowledge-based approach (See each stage in Fig. 3). In our experiment, 97.41% of test samples achieve correct detection results with a deviation limitation, 4. Fig. 4 demonstrates some successful examples under different changes.

All the detected results are transferred to the recognition phase. The face region is firstly extracted and aligned with the standard model. After that, its algebraic features achieved in dual eigenspaces are used by the two-layer classifier to obtain the final results. In our experiments, the number of each person's training samples varies from 2 to 12, while the remaining images constitute the test set. For the top layer, the top 18 eigenvectors are used. And for the lower layer, the number of eigenvectors equals to that of training samples.

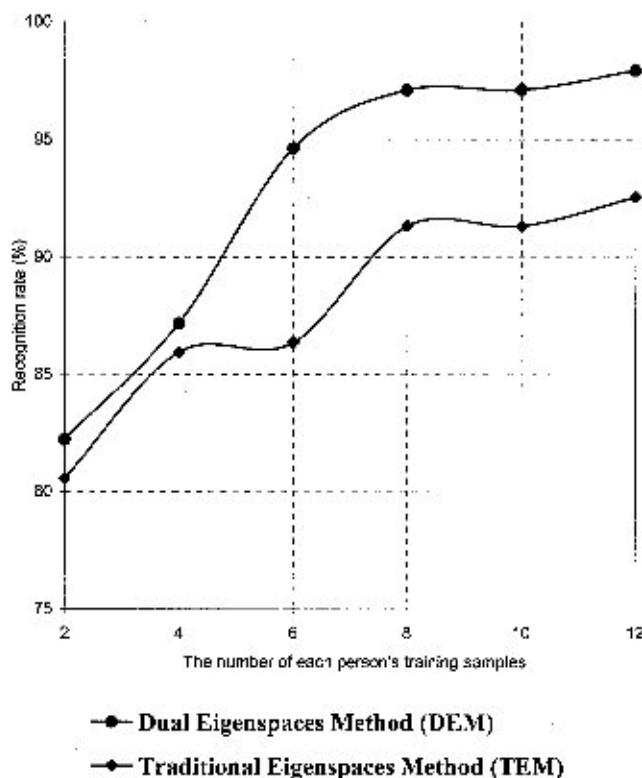


Fig. 5. Comparison of performances between the system using DEM and traditional eigenface method.

The recognition rates are depicted in Fig. 5, which indicate that DEM is obviously better than traditional eigenface method (in which the top 36 eigenvectors are used). For example, when six face images of each person are selected as training samples, there is a dramatic improvement in the recognition rate from 86.36% (traditional eigenface method) to 94.63% (DEM). Considering the characteristics of the test images that contain the changes of head posture, facial expressions and illumination directions, it is obvious that our system is effective to these ambiguous images.

V. CONCLUSIONS

In this paper, we have presented a novel AFR system which contains two modules: (1) eye detection based upon a hybrid neural method and (2) face recognition using the dual eigenspaces method. The simulation results demonstrate that the proposed system not only has considerable performance to detection and recognition of faces, but also is insensitive to different conditions such as, scale, posture, illumination, and facial expressions.

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