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NOTES

ON THE PROBLEM OF SUITING THE MATING PARTS FOR A ONE-SHOT ASSEMBLY

By Y. R. RAU
SQC Division, Indian Statistical Institute

SUMMARY. This note attempts a statistical evaluation of some of the strategies in vogue concerning a one-shot assembly problem involving two mating parts. This has been discussed from the point of view of (i) the probability of acceptable fit during assembly, (ii) the interchangeability of parts.

1. INTRODUCTION

In the manufacture of complex, custom-made equipments, each equipment assembled may be treated, for all practical purposes, as a unique or one-shot item. In such operations, it is usual to identify in advance the parts for a particular assembly and machine and assemble them to achieve the desired fit. There is no scope for a random or even selective assembly, since only one number of each part is available. If the desired fit is not achieved during assembly, the parts may have to be discarded (and replaced) or repaired resulting in heavy losses and delays. Therefore, the parts have to be 'machined to suit' so that the probability of an unacceptable fit during assembly is small. The suiting problem reduces to one of selecting the best strategy for machining the parts which, while ensuring the desired probability of an acceptable fit during assembly, does not unduly come in the way of interchangeability of parts.

2. DISCUSSION OF THE PROBLEM

We shall confine the discussions to a one-shot assembly involving two mating parts—called hereafter as the bore and the shaft. The excess of the bore diameter over the shaft diameter is called the clearance. The specified nominal diameters for the bore and the shaft are a and $a-c$ units respectively. When the two are assembled, the fit is deemed acceptable if the clearance lies within $c \pm t_c$ units.

The shaft diameter, being an outside dimension, is usually capable of being machined within a small range of variation around its nominal value. But the machining of the bore diameter may be more difficult specially when an associated characteristic such as the surface finish is also critical. In such cases, it may become necessary to finish-machine the bore until the desired surface finish is attained and live with the resultant bore diameter. Thus, the variation in the bore diameter is expected to be quite large, which may jeopardise the chance of an acceptable fit during assembly. Hence the need for selecting the right strategy for machining the bore and the shaft diameters.

Some of the commonly used strategies are :

Strategy I : Machine the bore and the shaft diameters independent of one another to their respective nominal diameters of a and $a-c$ units.

Strategy II : Machine the bore ahead of the shaft. If the bore diameter is x, redesignate the nominal shaft diameter as $x-c$.

Strategy III : Machine the bore, ahead of the shaft. If the bore diameter is x, redesignate the nominal shaft diameter as

(i) $a-c$ if $|a-x| \leqslant t$

(ii) $x-c$ if $|a-x| > t$

where t is a suitable value.

The Strategy I may not cause any difficulty in assembly if the machining variations of the bore and the shaft diameters are relatively small. If σ_1 and σ_2 are the standard deviations representing the machining variation of the bore and shaft diameters respectively, the probability of an acceptable fit with Strategy I will be

$$\geqslant \cdot 99 \text{ if } 2{\cdot}58\sqrt{\sigma_1^2+\sigma_2^2} \leqslant t_c. \quad \text{If } t_c < 2{\cdot}58\sqrt{\sigma_1^2+\sigma_2^2},$$

there will be a corresponding reduction in the probability of an acceptable fit, resulting in assembly difficulties. With Strategy II, the probability of an acceptable fit is high since the entire tolerance of $\pm t_c$ is available for accommodating machining variation of the shaft diameter. However, since the nominal shaft diameter itself varies with the bore diameter, the standard deviation representing the overall (unconditional) variation in shaft diameter will be considerably larger than σ_2. This has an adverse effect on the interchangeability of the parts thereby causing difficulties in the manufacture of spares. Strategy III strikes a balance—it results in a larger probability of acceptable fit compared to Strategy I, but also results in a smaller overall variation of the shaft diameter, compared to Strategy II.

It is advantageous to reword Strategy III as follows

Strategy III : Machine the bore ahead of the shaft. If the bore diameter is x, designate the nominal shaft diameter as

(i) $a-c$ if $|a-x| \leqslant h\sigma_1$

(ii) $x-c$ if $|a-x| > h\sigma_1$,

where h is a suitable constant.

It may be easily observed that Strategy I is a particular case of Strategy III when the value of h is chosen as ∞. Similarly, Strategy II is a particular case of Strategy III when h is made equal to zero. It is therefore sufficient to evaluate Strategy III for different values of h.

3. Probability of acceptable fit during assembly

We shall now evaluate the probability of acceptable fit during assembly for Strategy III for various values of h. Let X and Y be the random variables representing the bore and the shaft diameters respectively. Let $f(x, y)$, $f_X(x)$ and $f_{Y/X}(y/x)$ respectively represent the joint probability density of X and Y, the marginal probability density of X and the conditional probability density of Y given $X = x$. We shall assume that the marginal probability density of X is Normal with mean a and standard deviation σ_1. We further assume that the conditional probability density of Y given $X = x$ is again Normal with standard deviation σ_2 and a mean which is either $a-c$ or $x-c$ depending upon the value of x.

The following notations are used

$$\phi(t) = (2\pi)^{-\frac{1}{2}} \exp(-t^2/2)$$

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)dt$$

$$b(x, y, \rho) = (2\pi\sqrt{1-\rho^2})^{-1} \exp\left[\frac{-(x^2-2\rho xy+y^2)}{2(1-\rho^2)}\right]$$

$$L(h, k, \rho) = \int_h^\infty \int_k^\infty b(x, y, \rho)dxdy$$

$$\sigma_1 = \alpha\sigma_2; \quad t_c = \beta\sigma_2$$

Then

$$f_X(x) = \frac{1}{\sigma_1}\phi\left(\frac{x-a}{\sigma_1}\right) \qquad \ldots \quad (3.1)$$

$$f_{Y/X}(y/x) = \begin{cases} \dfrac{1}{\sigma_2}\phi\left(\dfrac{y-a+c}{\sigma_2}\right) \text{ when } |a-x| \leqslant h\sigma_1 \\ \\ \dfrac{1}{\sigma_2}\phi\left(\dfrac{(y-x+c)}{\sigma_2}\right) \text{ when } |a-x| > h\sigma_1 \end{cases} \qquad \ldots \quad (3.2)$$

Probability of an acceptable fit during assembly (P)

$$= \iint_{|x-y-c| < t_c} f(x, y)dxdy$$

$$= \int_{-\infty}^{\infty} f_X(x)\left\{\int_{x-c-t_c}^{x-c+t_c} f_{Y/X}(y/x)dy\right\}dx$$

$$= \int_{|x-a|<h\sigma_1} f_X(x)\left\{\int_{x-c-t_c}^{x-c+t_c} f_{Y/X}(y/x)dy\right\}dx$$

$$+ \int_{|x-a|>h\sigma_1} f_X(x)\left\{\int_{x-c-t_c}^{x-c+t_c} f_{Y/X}(y/x)dy\right\}dx$$

$$= P_1+P_2 \text{ say.}$$

From result (1) of Appendix, we obtain

$$P = 2L(h, k, \rho)+2L(h, k, -\rho)+2\Phi(k)-4\Phi(-h)\Phi(-\beta)-1 \qquad \ldots \quad (3.3)$$

where $\rho = \alpha/\sqrt{1+\alpha^2}$ and $k = \beta/\sqrt{1+\alpha^2}$.

Values of P can be computed for any α, β and h using the Bivariate Normal Probability Tables [1 and 2]. It may be seen that for given α and β, the probability of acceptable fit P decreases as h increases in the range $(0, \infty)$.

4. Overall variation in the shaft diameter

In this section, we shall evaluate the overall variation in the shaft diameter for Strategy III for different values of h. Let $V(Y)$ and σ_Y respectively represent the overall variance and standard deviation of the shaft diameter. From result (2) of Appendix, we have

$$V(Y) = \sigma_2^2[1+2\alpha^2\{h\phi(h)+\Phi(-h)\}]$$

or

$$\frac{\sigma_Y}{\sigma_2} = \sqrt{1+2\alpha^2\{h\phi(h)+\Phi(-h)\}}$$

$h\phi(h)+\Phi(-h)$ is a decreasing function with h in the range $(0, \infty)$. σ_Y/σ_2 decreases from $\sqrt{1+\alpha^2}$ when $h = 0$ to 1 when $h = \infty$ and its value can be computed for any α, β and h using the univariate Normal Probability tables.

5. Uses of P and σ_Y/σ_2

Designating a value for h determines the machining strategy. In any given situation, values of α and β are known. Choice of a larger value for h decreases P while choice of a smaller h increases σ_Y. Thus a tradeoff is called for. If β is sufficiently large and α is sufficiently small, h can be made large say 3·0. However, if α is large and β is small, one may have to select a smaller value for h such as 2·0 or even 1·5 and put up with the inconvenience of larger σ_Y.

Appendix

1. $P_1 = \int_{-h}^{h} \phi(z)[\Phi(z\alpha+\beta)-\Phi(z\alpha-\beta)]\,dz$ where $z = \frac{x-a}{\sigma_1}$.

Using the well-known result

$$L(h, k, \rho) = \int_{h}^{\infty} \phi(z)\Phi\left(\frac{\rho z-k}{\sqrt{1-\rho^2}}\right) dz$$

and other properties of $L(h, k, \rho)$, it can be established that

$$P_1 = 2L(h, k, \rho)+2L(h, k, -\rho)+2\Phi(k)+2\Phi(h)-3$$

where $\rho = \alpha/\sqrt{1+\alpha^2}$ and $k = \beta/\sqrt{1+\alpha^2}$.

Similarly, it can be shown that

$$P_2 = 2-2\Phi(h)-4\Phi(-h)\Phi(-\beta).$$

It therefore follows that

$$P = 2L(h, k, \rho)+2L(h, k, -\rho)+2\Phi(k)-4\Phi(-h)\Phi(-\beta)-1.$$

2. We know

$$E(Y/X = x) = \begin{cases} a-c & \text{if } |x-a| \leqslant h\sigma_1 \\ x-c & \text{if } |x-a| > h\sigma_1 \end{cases}$$

$$V(Y/X = x) = \sigma_2^2.$$

Therefore.

$$E(Y^2/X = x) = \begin{cases} \sigma_2^2+(a-c)^2 & \text{if } |x-a| \leqslant h\sigma_1 \\ \sigma_2^2+(x-c)^2 & \text{if } |x-a| > h\sigma_1. \end{cases}$$

It can be established that $EY = a-c$ and

$$EY^2 = \sigma_2^2+(a-c)^2+2\sigma_1^2\{h\phi(h)+\Phi(-h)\}$$

$$\therefore \quad V(Y)/\sigma_2^2 = 1+2\alpha^2\{h\phi(h)+\Phi(-h)\}.$$

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