ARROW AND HAHN ON GENERAL EQUILIBRIUM SOME REACTIONS

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Little did Walras realise, when he beheld his new born "Elements" (Walras, 1877) the impact it was destined to produce on generations of practitioners in economics. A lot has changed in the world of economic theory since its publication. Time has become predominantly "infinite" and linear functionals have come to "separate"; traders have formed a "continuum" and economies have had joy rides along "turnpikes", including "twisted" ones at times. However, in spite of the technological change at such an unprecedented rate, the "Elements" has defied obsolescence. The basic issues considered in it still keep scholars busy and as such, Arrow and Hahn's *General Competitive Analysis* (1971) is not unique in the class of publications on the subject. It has amongst others, rich predecessors like Koopmans' (1957) "Three Essays", and Debreu's (1959) "Theory of Value". Being the youngest in the line, it had, naturally, a lot to consolidate. However, when two people like Arrow and Hahn collaborate, the output ceases to be a mere consolidation of earlier work. As with Dorfman, Samuelson and Solow's *Linear Programming and Economic Analysis* (1958), almost everywhere in the book, "originality" keeps "breaking in". Arrow and Hahn's book has thus surpassed the narrow confines of a text-book and has emerged as a goldmine of new results and better proofs of old ones.

Of course, like many first editions, the book contains slips—minor, as well as not so minor ones. Before we make detailed comments on these, let us briefly indicate the contents of the book.

Chapter 1 begins with an interesting introduction to the history of General Equilibrium : of earlier attempts, of their failures and of later modifications and rigorous proofs given by earlier general equilibrium theorists. Chapter 2 studies the problem of the existence of equilibrium in the simplest possible framework, where there is a well-defined excess demand function at any price $p \epsilon S_n$: the unit simplex, and possesses the properties of (a) Walras Law (b) homogeneity of degree zero in the prices (c) continuity; and in this situation, the existence of equilibrium is exhibited. Such an exercise would be helpful in considering the more general cases, when some of the assumptions made above do not hold.

Chapter 3 considers the theory of production. Assumptions are provided under which, the set of feasible production allocations for the economy is bounded. A few of the standard propositions in the theory of the firm, relating to supply functions are established. Results similar to the Wong-Viner theorem are indicated. Chapter 4 deals with consumer behaviour theory. Under a suitable set of assumptions on the preferences, a continuous utility function is shown to exist over the consumption possibility set. The concept of Pareto optimality is replaced by a weaker notion of Pareto efficiency and the authors go on to demonstrate that every Pareto efficient utility allocation is supported by a *compensated equilibrium* price vector. The Slutsky equation and other related results are derived. Chapter 5 starts with rigorous definitions of the two kinds of equilibria—*compensated* and *competitive*. The relationship between the two is presented and conditions are provided under which a compensated equilibrium is a competitive equilibrium. The existence of a competitive equilibrium is next taken up in full generality, in the case of an economy consisting of finitely many agents. Some observations are made regarding problems which come up with the introduction of uncertainty in the model.

Chapter 6 goes on to use the tools developed in earlier chapters to prove the existence of equilibria in situations where the conditions of perfect competition may not hold. Thus, if (a) utilities vary with prices or (b) there are externalities of a particular type in production and consumption, then modifications of assumptions made in the previous chapter are shown to lead to the existence of an equilibrium. The method of earlier chapters is also used to study a two-period economy where commodities traded in currently, are the commodities of the present period and bonds. A unit bond is a promise to pay one unit of currency of account in the next period. By suitable modifications, an existence theorem, which claims the existence of equilibrium in current markets, is proved. Finally the case of monopolistic competition is considered; as is clear from the above description, this chapter branches off from the traditional general equilibrium theory into areas not usually considered in the literature.

The role of convexity is perhaps best understood, when one tries to prove existence theorems without convexity. Externalities, traditionally, are taken to introduce non-convexities in the economy. These matters are considered in Chapter 7. In this chapter, a general theorem is stated on the degree to which an economy not satisfying the convexity assumptions in production and consumption, can nevertheless possess an approximate competitive equilibrium i.e., a price vector, a consumption vector for each household that minimizes the cost of achieving a given utility level and satisfies a budget constraint at those prices, a production vector for each firm that maximizes profit at those prices, and the social excess demand for all commodities is bounded (bound determined by the amount of non-convexity present in the economy). Also this discrepancy is shown to go to zero as the size of the economy (e.g., the number of economic agents) increases.

The conjectures of Edgeworth and the contribution of Debreu and Scarf (1963), which have now come to be known as the theory of the core, are considered in Chapter 8. All recent contributions in this area have been such as to demand quite a sophisticated level of mathematics from the readers. Arrow and Hahn have chosen well to exclude the measure theoretic treatments from this section. The competitive equilibrium lies in the core—but exactly how does the core approximate the competitive equilibrium? In this chapter, a substantial generalization of the Debreu-Scarf limit theorem is provided. As is indicated in the discussion relating to Theorem 2 (Arrow and Hahn, 1971, page 190) the theorem is proved for cases where households may have non-convex preferences. Consequently, the Debreu-Scarf result gets substantially weakened. Instead of the core shrinking to the competitive equilibrium, it is shown that an allocation in the core approximates a compensated equilibrium (in an appropriate sense).[1]

It is in Chapter 9 that Arrow and Hahn provide a lucid description of the conditions under which an economy has a unique equilibrium. The so-called uniqueness conditions which exist in the literature, namely those due to Gale and Nikaido (1958) are weakened in several ways. This might be indicated in the following manner. The Jacobian of the excess supplies, denoted by $J(p)$, is assumed to be such that all its principal minors have positive determinants for all values of p lying in S_n : this is the Gale-Nikaido condition, called GP in Arrow and Hahn (1971). The Arrow-Hahn contribution lies in weakening the above assumption to the one where $J(p)$ has GP only for $p \epsilon E = (p/S(p) \geqslant 0)$, the set of equilibrium price vectors.

Chapter 10 considers the problems involving the existence of comparative statics information. Usually, comparative statics results are derived via the calculus and as such are concerned with infinitesimal changes. In this chapter, following Morishima (1964), binary comparisons of price situations are made and hence, these lead to results concerned with 'large' shifts in prices. Chapters 11-13 are concerned with the study of the problem of stability of competitive equilibrium. There is no other collection of such results, except perhaps the survey by Negishi (1962), and in most cases, rigorous proofs are provided.

Chapter 14 deals with the Keynesian model. Previously a temporary equilibrium (Chapter 6) was shown to exist under an assumption that economic agents had no commitments left from the past, and also, there was no money in the economy. One of the main things studied in this chapter is whether, under the allowance of the two points noted above, a temporary equilibrium exists. It is not, as the authors are careful to point out, a detailed study of Keynes. Appendix A is on Positive Matrices and is a particularly neat exposition of the Perron-Frobenius theory. Appendix B is on Convex and related sets and consists of much of the background necessary to understand the material in Chapters 7 and 8. Appendix C contains a rigorous statement and proofs of Scarf's algorithm for approximating fixed points of mappings.

[1]Note that Arrow and Hahn consider a rather arbitrary expansion of the economy : it is not the duplication process considered by Debreu and Scarf (1963).

On the whole, one must say that the choice of topics covered in the appendices is excellent and the treatment rigorous—quite a refreshing change from the usual shoddy treatments that one finds in appendices.

This completes the description of the contents of the book. It is indeed quite an impressive coverage of general equilibrium theory. It is a pity, however, that problems of multisectoral growth are not discussed in this work. We proceed now to indicate the places where the formal arguments need tightening or where the proofs go astray.

In Chapter 3, the production set of the firm f, Y_f, is assumed to be closed, convex and to contain the point 0 (i.e., the origin). It is then claimed (see, Arrow and Hahn, 1971, Section 4, page 69) that if Y_f also happens to be bounded above, then "the function $p \cdot y_f$ attains a maximum on Y_f". This statement need not be true for all $p \epsilon S_n$: the n-dimensional unit simplex. For example, in the two dimensional case, let $p = (0, 1)$. If Y_f is as shown below, in Figure 1, then there does not exist any $y_f \epsilon Y_f$ at which $p \cdot y_f$ attains a maximum. It can be easily ascertained, however, that the assertion is true for all $p \epsilon S_n$ such that $p \gg 0$. Theorem 4 of the same section as above claims that $\pi_f(p) = \max\limits_{y_f \epsilon Y_f} p \cdot y_f$ is a strictly convex function of p when Y_f is bounded and strictly convex and admits free disposal. This statement, once again, seems to be false. $\pi_f(p)$ is certainly convex, but even the strict convexity of Y_f may not guarantee the strict convexity of $\pi_f(p)$. Consider, for example, the case described in Figure 2.

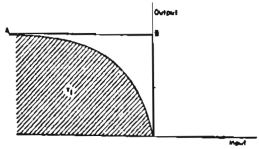

Fig. 1

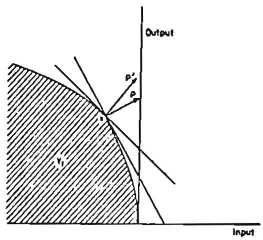

Fig. 2

Y_f is a strictly convex set with a non-differentiable boundary at the point x. Given the prices p and p', $\pi_f(p)$ and $\pi_f(p')$ are attained at the same point, viz., x. The proof in Arrow and Hahn (1971), proceeds as follows. Suppose $\pi_f(p)$ is attained at y_f and $\pi_f(p')$ is attained at y'_f. Then, consider, $p(\alpha) = \alpha p' + (1-\alpha)p$, where $0 < \alpha < 1$.

Let $y_f(p)$ correspond to $\pi_f(p(\alpha))$. Then, by the strict convexity of Y_f,

$$p{\cdot}y_f(\alpha) < p{\cdot}y_f \quad \text{and} \quad p'{\cdot}y_f(\alpha) < p'y_f'. \qquad \dots \quad (1)$$

Hence multiplying the first inequality by $(1-\alpha)$ and the second by α and adding,

$$(1-\alpha)p{\cdot}y_f(\alpha)+\alpha p'{\cdot}y_f(\alpha) = p(\alpha)\,y_f(\alpha) < (1-\alpha)p{\cdot}y_f+\alpha p'{\cdot}y_f'.$$

The proof goes wrong when we consider a point like x in Figure 2. It is obvious that given any convex combination of p and p', the profit maximizing point always remains x. Consequently, the strict inequalities in (1) have to be replaced by equalities and the remainder of the proof does not go through. For the theorem to be true, one needs stronger assumptions than the strict convexity of Y_f. In particular, one might like to assume that Y_f is strictly convex and there exists a unique tangent plane at each point on the boundary of Y_f.

In Chapter 4, a new proof is given for the existence of a continuous utility indicator over the consumption set of an individual. The proof is extremely elegant and very lucidly presented. The motivation of the proof is made clear at each stage. The extension of the utility function to the entire consumption possibility set from a subset of it is only one example of the ingenuity which the authors keep demonstrating throughout the book. There is only one minor comment that we would like to make. In Arrow and Hahn (1971, page 83), the following statement is made : "Since $\rho(x')$ is a continuous function bounded from below...and $C(x)$ is closed, $\rho(x')$ has a minimum as x' varies over $C(x)$'. It so happens that $\rho(x')$ does achieve a minimum over $C(x)$ in their case, but not for the reasons they give in the above mentioned statement, which is not true, in general. For example, consider the function $y = 1/x^2$ defined over the set $[0, \infty)$. The set is closed in the real line (as against the extended real line) and $1/x^2$ is bounded below on $[0, \infty)$. It is trivial to see that the minimum is not attained.

The last section of Chapter 4 derives some of the standard results in the pure theory of demand, including the Slutsky equation. The method is based on Mckenzie (1956-57). Mckenzie's main achievement in this paper consisted in giving up two of the major assumptions usually made in texts like Samuelson (1947) or Hicks (1939). Both Samuelson and Hicks assumed strict convexity of preferences and twice differentiability of the utility function. Mckenzie was able to give up the latter assumption altogether and replace the first one by a substantially weaker one and still retain the majority of results in demand theory. Arrow and Hahn fail to mention this at all and, as such, much of the flavour of the exercise is lost upon the reader.

Coming down to the details, the idea of Mckenzie's proof is extremely simple. He notes, to start with, that the minimum cost function $C_h(p, u_h)$ (using the notation of Arrow and Hahn) is a concave function of p given any u_h. As such, the function $C_h(p, u_h)$ is twice differentiable in p almost everywhere (in the sense of Lebesgue measure). Moreover, for p strictly positive, it can be shown that $\frac{\partial C_h}{\partial p_j} = x_{hj}(p, u_h)$, where, once again in the notation used by Arrow and Hahn, $x_{hj}(p, u_h)$ is the compensated demand function for the j-th good. Hence, whenever the second derivatives exist, concavity of $C_h(p, u_h)$ implies that the matrix $\left(\frac{\partial^2 C_h}{\partial p_i \partial p_j}\right)$ is negative semi-definite. That is, the matrix of the pure substitution terms of traditional consumer behaviour theory is negative semi-definite. (See, for example Samuelson, 1947, page 113).

As was mentioned above, this derivation does not require convexity of preferences. Mckenzie merely assumes local non-satiation. In Arrow and Hahn, semi-strict convexity of preferences along with non-satiation yields local non-satiation. However, the conditions are by no means necessary. Also, a twice differentiable utility function is not required, since the method of proof depends on the twice differentiability of $C_h(p, u_h)$, which is guaranteed almost everywhere, irrespective of any assumptions regarding the differentiability of the utility function. There seems to be an error in the Arrow and Hahn statement of these results (Arrow and Hahn, 1971, page 104, Theorem 9). First of all, when preferences are strictly convex, it is claimed that $C_h(p, u_h)$ is *strictly* concave in p. For reasons similar to those discussed in the case of the theory of production, this statement is false. All that one can claim is concavity. Also the method of proof suggested depends on the existence of $x_h(p, u_h)$, the point in the consumption set at which $C_h(p, u_h)$ is attained. Once again, as in the theory of production, $C_h(p, u_h)$ may not be attained in the consumption possibility set. However, $C_h(p, u_h)$ can be shown to be concave in any case. The following proof taken from Mckenzie demonstrates. Let $p, p' \epsilon S_n$, $p(\alpha) = \alpha p' + (1-\alpha)p$, $0 \leqslant \alpha \leqslant 1$. Then from the definition of $C_h(p, u_h)$, for any $e > 0$, there is $z \epsilon X_h(u_h)$ such that

$$C_h(p(\alpha), u_h) \geqslant p(\alpha)\cdot z - e = \alpha p'z + (1-\alpha)pz - e$$
$$\geqslant \alpha C_h(p', u_h) + (1-\alpha)C_h(p, u_h) - e.$$

Since the above holds for any $e > 0$, we have

$$C_h(p(\alpha), u_h) \geqslant \alpha C_h(p', u_h) + (1-\alpha)C_h(p, u_h),$$

which is concavity. In the above demonstration, we have used the notations of Arrow and Hahn.

Chapter 5 deals with the existence of competitive equilibrium. The proof is undoubtedly, one of the best given so far in the literature. The proof is completed in two steps. First of all, under the assumptions made on the model, a compensated equilibrium is shown to exist. The Kakutani fixed point theorem is used for this purpose. A compensated equilibrium allocation turns out to be a competitive equilibrium if the "cheapter point" assumption[3] is satisfied for every consumer. This is

[3]The cheaper point assumption may be stated thus: let S denote the consumption possibility set; then for any price p and $x \epsilon S$ and U an arbitrary neighbourhood of x, there is $x' \epsilon U \cap S$ such that $p.x' < p.x$.

precisely what is guaranteed when the assumption of "indirect resource relatedness" of households is made. In the second stage of the proof, Arrow and Hahn make the latter assumption to show that the compensated equilibrium is a competitive equilibrium also. The assumption of resource relatedness may be slightly more satisfactory as compared to the cheaper point assumption, in that it can be given an easy economic interpretation. In their treatment of the existence problem, Arrow and Hahn are concerned solely with the case of finitely many firms. As such, the case of free entry of firms is left out. In the absence of externalities, free entry implies that the aggregate production possibility set (i.e. the sum of individual closed and convex production possibility sets) would approximate a closed convex cone. In such a case, as traditional theory suggests, while aggregate production is determinate in an equilibrium, individual production is not. Such a case has been studied by Mckenzie (1959) and a mention of it would not be out of place. In discussing the case of finitely many firms, Arrow and Hahn are concerned solely with the parametric role of prices in a competitive equilibrium. However, a text-book on competitive analysis should have some discussion regarding free entry also, since this has been one of the most important characterizations of a competitive market.

In Chapter 8, there is perhaps a small point to be noted. In proving Theorem 2 of this chapter, use is made of Lemma 1(a) of Chapter 4 (see page 102 in Arrow and Hahn, 1971). The said Lemma makes use of semi-strict convexity of preferences. As is shown in Lemma 1(b) of Chapter 4, semi-strict convexity of preferences along with continuity leads to convex indifference maps. Hence, a somewhat better way of characterizing Arrow and Hanh's weakening of the Debreu-Scarf assumptions (see Arrow and Hahn, 1971, page 190) would be to say that the assumption of continuity of preferences is given up, rather than the convexity of indifference maps.

Moving on to the chapter on uniqueness, (Chapter 9), the Gale-Nikaido (1965) conditions are considerably weakened, as we indicated above. Along with other assumptions, Arrow and Hahn employ the numeraire assumption (Arrow and Hahn, 1971, page 208, Assumption 2(N)). This reads :

In every equilibrium of the system, there is a good, give it the label n, for which $\sum_i s_i(p) = -\infty$ whenever $p_n = 0$. This good is called the numeraire.

A better statement of this assumption would have been : Given any no. $N > 0$, there is $\beta > 0$, such that in every equilibrium of the system, there is a good, give it the level n, for which $\sum_i s_i(p) < -N$ whenever $p_n = 1$, $\|p\| > \beta$. This good is is called the numeraire.

The latter statement is consistent with the numeraire being a good whose price is fixed at 1 (once we allow this, it is difficult to explain how $p_n = 0$ is possible).

In Chapters 11-13, there are several places where the treatment is faulty. We would like to begin by indicating that (Arrow and Hahn, 1971, page 273, paragraph beginning "Now by A.4, the path...) the role of the boundedness of the solution path has not been made clear. From reading the paragraph, one would imagine that boundedness of the path led to $V(\ )$: the Liapunov function, being bounded. Now, boundedness of the solution path leads to the existence of limit points and hopefully, one would prove convergence to such a limit point, which would turn out to be an equilibrium of the economy. The boundedness of the function $V(\ )$ does not enter into the argument except trivially. $V(\ )$ is chosen to be positive at all prices and the crux of the argument is to show that $\dot{V} < 0$ along the solution path. If we have such a $V(\ )$, then, $0 \leqslant V(p(t)) \leqslant V(p(0))$ and boundedness is immediate quite irrespective of whether the solution path is bounded or not.

On page 288 (Theorem 4) and Page 295 (Theorem 9), the proofs provided are erroneous. It should be pointed out that the Liapunov functions to choose, in case of the former is $V(p) = \underset{i}{\text{Max}}\ |p_i - 1|$ and in case of the latter, $H(p) = \text{Max}\ |F_i/p_i|$. As the choices of $V(\ )$ and $H(\ )$ stand i.e., without the absolute value signs, the proofs do not go through.

On page 293, the assumption $Z_n(p(n)) > 0$ where $p(n)$ is defined to be the price vector with $p_n = 0$ is certainly odd and should be restated as we indicated in the case of the numeraire assumption above. A new result in this connection is the one which connects dominant diagonal with global stability (Arrow and Hahn, 1971, page 292). A special form of the definition of diagonal dominance is used. It would have been clearer to have indicated, that the speciality lay in requiring that it is the prices which yield the dominant diagonal. A partial demonstration of the conjecture made by the authors (Arrow and Hahn, 1971, pages 293-294) regarding dominant diagonal and stability may be found in Mukherji (1973); the demonstration there is not quite the result that Arrow and Hahn have in mind—it shows that at least in one case, if the weights $h(p)$ which yield dominant diagonal are allowed to vary with prices, we need not have any information regarding how $h_j(p)$ changes with p. This is the case when $h_j(p)$ also happens to be the speed of adjustment in the j-th market.

We now come to the uniqueness result which is demonstrated in terms of an adjustment process (Arrow and Hahn, 1971, page 304). The part which seems to be difficult to follow is the part relating to the boundedness of the solution path. The argument runs as follows : if $\|P(t)\| \to \infty$ then $Z_1(P) \to \infty$. Then by Walras law and for $\|P\|$ large enough, $\widetilde{Z}(P)'\widetilde{Z}(P)$ must be increasing if $\|P\|$ increases further ($\widetilde{Z}(P)$ denotes the excess demand vector without the numeraire excess demand).

This is true if there are only two goods (including the numeraire). Certainly, $\widetilde{P}'\widetilde{Z}(P) < 0$ for $\|P\|$ large enough; but why should that imply $\widetilde{Z}(P)'\widetilde{Z}(P)$ to be increasing ? We, at least, could not find the proof of this statement.

There is an example of global instability (Arrow and Hahn, 1971, page 300). However, it is to be noted that this is really the case where a proper choice of the numeraire, yields a stable system for all speeds of adjustment. The example is built on there being a single good which exhibits Giffen's paradox. Choose this good to be the numeraire, and stability independent of the speed of adjustment follows.

However, for the constant returns economy, with a single primary factor, it is possible to formulate an adjustment to indicate that the unique equilibrium is globally stable under this adjustment process. A formulation of this process is provided on page 316. Unfortunately, the proof of global stability in this connection is erroneous. The mistake occurs at the stage where the authors define $k(p) = \max p_i/p_i^*$ for all i; the proper definition would be $k(p) = \max_{i \neq 0} p_i/p_i^*$, where 0 stands for the single primary factor. Latter parts of the proof, consequently, have to be changed.

We have, so far, been discussing processes where trading at dis-equilibrium prices is not allowed, i.e., tatonnement processes. One of the major contributions in the area of non-tatonnement processes is due to Uzawa (1962). However, the behavior of prices in the contribution (Uzawa, 1962) is quite unclear. The trouble with the Uzawa paper lies in the fact that the price adjustment equations are not specified. That this specification would be enough to show that prices converge to an equilibrium, has been shown elsewhere (Mukherji, 1974). The difficulty with the Arrow-Hahn treatment lies first, in their weaker assumption of quasi-concave utility functions for individuals. On page 331, they conclude from the relation (4) on that page, that $\sum_i \bar{U}_{hi}\lambda Z_{hi} > 0$. However, because of the approximation involved, they should be able to deduce only the weak sign $\geqslant$. Also, the inequality at the bottom of page 331 should be weak, for the same reasons. Thus, the proof of Theorem 3 and hence, the proof of Theorem 4 does not go through. Secondly, even if we allow for concavity of the utility functions, Arrow and Hahn do not prove what they state in their Theorem 4 (1971, page 334). What they succeed in proving is, that if one had as initial conditions $p(0) = \bar{p}$, $Y(0) = Y^*$: a Pareto optimal distribution, then the solution path would converge to an equilibrium (p^*, Y^*), with no trades taking place. Further, if the process starts with arbitrary initial conditions, then there are limit points such as $(\bar{p}, Y^*)$, where $\bar{p}$ may or may not support the Pareto optimal distribution Y^*. This, of course, is not enough to imply the convergence of prices, in general. However, their assertion in Theorem 4 (Arrow and Hahn, 1971, page 334), is correct, as we indicated above.

In the above discussion, we have perhaps dwelt at length on points which may be of minor interest. A review can hardly ever do justice to such an encyclopaedic work. Having indicated the parts which we felt could be improved upon, it would be incorrect not to point out what we liked in the book. The discussions at each stage, of every assumption made and the indication of exactly what purpose is served by these assumptions, is highly praiseworthy. For each topic covered, whether it be the problem of existence or uniqueness or the stability of competitive equilibrium, Arrow and Hahn have collected a huge body of results. The well-known ones are all included; there are, in addition, several new results as we have indicated above. The student is saved from a lot of bother in looking up the literature from the pages of the different journals; also, the new results bring one to the very frontiers of the subject. The notes at the end of each chapter serve as an excellent source of information, should someone be interested in looking up the original contributions. Indeed, it would be not incorrect to say that the study of general equilibrium theory is simpler, now that graduate students have Arrow and Hahn to turn to.

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