Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules*

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A large literature on incentive mechanisms represents incentive constraints by the requirement that truthful reporting be a Bayesian equilibrium. This paper identifies mechanism design problems for which there is no loss in replacing Bayesian incentive compatibility by the stronger requirement of dominant strategies. We identify contexts where it is possible to change the transfer payments of an optimal Bayesian mechanism so as to create dominant strategies and yet leave every participant's expected utility unchanged. We also address the issue of multiple equilibria and unique implementation. Contexts where these results apply include auctions, bilateral bargaining, procurement contracting, and intrafirm resource allocation. *Journal of Economic Literature Classification Numbers*: C72, C78, D82.

1. INTRODUCTION

Early work on incentives focused on the possibility of achieving a given performance standard (such as Pareto efficiency) while providing agents with dominant strategy incentives. For the most part, however, this literature arrived at impossibility results, e.g., Hurwicz [14], Gibbard [10], Satterthwaite [30], and Green and Laffont [11]. In contrast, d'Aspremont and Gerard-Varet [9] and Arrow [2] obtained positive results for environments with "quasi-linear" preferences by replacing dominant strategy equilibria with Bayesian equilibria. This undoubtedly initiated a shift to the analysis of Bayesian mechanisms. Following Myerson [27] and Myerson and Satterthwaite [28], a large literature has since explored the design of optimal Bayesian mechanisms for economic

settings where first-best outcomes are unattainable. These settings include bargaining, auctions, regulation, procurement contracting, and intrafirm resource allocation.

The corresponding problem of designing optimal dominant strategy mechanisms has been less systematically explored. However, for select mechanism design problems it has been observed that one can impose the requirement of dominant strategies rather than Bayesian equilibrium without changing the equilibrium payoffs of any of the parties involved. For example, it is well known that the second price auction, which generates dominant strategies, is optimal among all Bayesian incentive compatible mechanisms, for a private values auction with ex ante symmetric bidders.[1] Stated differently, the auctioneer can create dominant strategy incentives without reducing his expected revenue or any of the bidders' interim utilities. The purpose of this paper is to explore in some generality the class of Bayesian incentive compatible mechanisms for which equivalent dominant strategy mechanisms exist. This will help expose the common underlying logic of the disparate cases where this equivalence result is known to be true, as well as extend the argument to cases where it was hitherto unknown.

We see two major arguments for favoring dominant strategy over Bayesian mechanisms as long as the participants' utilities are unchanged. First, Bayesian mechanisms not only require the information structure to be common knowledge among all agents, they also assume that the designer knows the common prior beliefs. It has been argued (see Ledyard [18]) that this is too demanding in terms of informational requirements for the designer.[2] Further, there is no reason to believe that slight misspecifications of the prior will lead only to small deviations from optimality, since the equilibrium response of agents may shift discontinuously with small alterations in the prior.[3] The postulate that Bayesian players will necessarily play Bayesian equilibrium strategies also requires assumptions considerably more demanding than Bayesian rationality. For instance, the rationality of all players must also be common knowledge.[4]

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[1] The possibility of an equivalent dominant strategy mechanism has also been observed by Laffont and Tirole [17] and Kreps [15] (Chapter 18) in the context of procurement contracting.

[2] Also see Hagerty and Rogerson [12] for an argument that "good" mechanisms need to be "robust" with respect to alterations in prior beliefs. Specifically, they define a mechanism to be robust if it satisfies dominant strategy incentive compatibility and ex post individual rationality.

[3] While the Bayesian equilibrium correspondence is upper-semicontinuous for generic games, optimal mechanisms often judiciously employ indifference among alternative strategies for agents, with the consequence that the induced games are nongeneric.

[4] See Brandenburger and Dekel [4] for assumptions required to ensure that Bayes–Nash equilibrium strategies will be played by Bayesian players.

A second problem with Bayesian mechanisms is that they may possess multiple equilibria. As demonstrated by Demski and Sappington [8], Repullo [29], and others, some of the alternative equilibria may generate undesirable outcomes from the designer's point of view. The implementation literature has therefore focused attention on mechanisms with the property that all Bayesian equilibria yield desirable outcomes. Yet, it remains unspecified how agents coordinate their choices among alternative equilibria in those mechanisms.

While dominant strategy mechanisms still rely on the assumption of a common prior distribution, they ameliorate some of the problems of Bayesian mechanisms listed above. For instance, dominant strategy mechaisms are more robust in the sense that slight misspecification of the prior by the designer cannot lead to large losses, since dominant strategy equilibria are prior-independent. Losses arising from incorrect specification of priors are then comparable to those arising in Bayesian decision problems. One would also expect that dominant strategy mechanisms are not plagued by multiple equilibrium problems. While this is not always true, we characterize a class of Bayesian mechanisms that can be uniquely and equivalently implemented in dominant strategies.

We consider mechanism design problems where the organization has to choose a public decision that affects agents differently depending on their own environments (types). An allocation rule specifies for each environment a public decision as well as monetary transfers for the agents. Central to our analysis is the concept of an equivalent dominant strategy implementation of a Bayesian incentive compatible allocation rule. This concept requires that one can change the transfer functions so as to create dominant strategies and leave every agent's interim utility unchanged. In some cases the allocation rule is not given exogenously but rather emerges as the solution to a principal-agent problem. For those settings we identify characteristics of the agency problem (such as utility functions and beliefs) that ensure that the principal has nothing to lose by creating dominant strategy incentives.[5]

The paper is organized as follows. The basic model is set out in Section 2. Section 3 shows, in some generality, that an equivalent dominant strategy mechanism exists if and only if the decision rule (specifying the public decision) of the original Bayesian mechanism is dominant strategy implementable. Accordingly, we provide necessary and sufficient conditions

[5] Makowski and Ostroy [19] establish a related result. They focus on individually rational and Pareto-optimal mechanisms, and find that the class of environments for which such mechanisms exist does not change as the incentive compatibility requirement is weakened from dominant strategies to Bayesian incentive compatibility. A major difference between their analysis and ours is that we do not insist on balanced mechanisms, i.e., the sum of the transfer payments to the agents need not be zero.

for dominant strategy implementability. In Section 4, we show that these conditions can be simplified considerably, if additional structure is imposed on agents' preferences. In particular, we suppose that an agent's utility can be expressed by a one-dimensional condensation (or statistic) of the public decision. For a large class of principal-agent problems, this property (combined with additional regularity conditions) ensures that there is no loss from replacing Bayesian equilibrium constraints by dominant strategy requirements. Illustrations of our results are provided in the context of optimal auctions, team production, and bilateral trading. Section 5 considers environments where the one-dimensional condensation property is not satisfied. The results of that section are illustrated in the context of procurement contracting, and intrafirm resource allocation. Finally, we conclude in Section 6.

2. DEFINITIONS

We consider a standard adverse selection model involving a principal and n-agents, $N = \{1, ..., n\}$. The principal (designer) seeks to implement some decision y belonging to a feasible set Y. If the decision $y \in Y$ is implemented, Agent i bears a cost of $C_i(y, \theta_i)$, or, alternatively, obtains a benefit of $-C_i(y, \theta_i)$.[6] This cost is observable to Agent i only, since the parameter $\theta_i \in [\underline{\theta}_i, \bar{\theta}_i]$ is Agent i's private information. For notational convenience we shall write $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ and $\Theta = X_{i=1}^n \Theta_i$. We suppose that the actual environment $\theta = (\theta_1, ..., \theta_n)$ is drawn from a common prior distribution given by the cumulative distribution $F(\theta_1, ..., \theta_n)$. To compensate the agents for their costs, the principal can make monetary transfers denoted x_i for Agent i. Throughout this paper we maintain the assumption that all parties are risk-neutral. Hence, Agent i's utility payoff is $x_i - C_i(y, \theta_i)$.

In general, the principal will seek to implement different decisions for different environments. We shall refer to a function $y: \Theta \to Y$ as a *decision rule*. Similarly, there will be functions $x_i: \Theta \to \mathbb{R}$ referred to as *transfer rules*. An $(n+1)$ tuple of functions $(y, x_1, ..., x_n): \Theta \to Y \times \mathbb{R}^n$ will be called an *allocation rule*. Since our analysis will at first disregard issues of multiple equilibria, we can invoke the Revelation Principle and focus on mechanisms in which agents have an incentive to report truthfully. For a given allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$, let

$$\Pi_i(\theta_{-i}, \bar{\theta}_i | \theta_i) \equiv x_i(\theta_{-i}, \bar{\theta}_i) - C_i(y(\theta_{-i}, \bar{\theta}_i), \theta_i)$$

[6] We shall assume that $C_i(y, \theta_i)$ is continuously differentiable with respect to θ_i. Furthermore, we restrict attention to allocation rules such that the functions $x_i(\cdot)$, $C_i(y(\cdot), \cdot)$, $(\partial/\partial\theta_i)$ $C_i(y(\cdot), \cdot)$, $C_i(y(\cdot), \bar{\theta}_i)$, and $(\partial/\partial\theta_i) C_i(y(\cdot), \bar{\theta}_i)$ (for arbitrary $\bar{\theta}_i$) are bounded and measurable with respect to both, the probability measure induced by $F(\cdot)$ and the Lebesgue measure on Θ.

denote Agent i's utility payoff, if his environment is θ_i, he reports $\tilde{\theta}_i$ and the other agents report θ_{-i}.

DEFINITION 1. The allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ is dominant strategy incentive compatible (DSIC) if

$$\Pi_i(\theta_{-i}, \theta_i | \theta_i) - \Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i) \geqslant 0$$

for all $\theta_{-i} \in \Theta_{-i}$, $\tilde{\theta}_i \in \Theta_i$, $\theta_i \in \Theta_i$, and $i \in N$.

Let $E_{\theta_{-i}}[\Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i] \equiv \int_{\Theta_{-i}} \Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i) \, dF(\theta_{-i} | \theta_i)$ denote Agent i's expected utility if his type is θ_i, he reports $\tilde{\theta}_i$, and everybody else reports truthfully. (Here, $F(\theta_{-i} | \theta_i)$ denotes the conditional distribution upon observing θ_i).

DEFINITION 2. The allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ is Bayesian incentive compatible (BIC) if

$$E_{\theta_{-i}}[\Pi_i(\theta_{-i}, \theta_i | \theta_i) - \Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i)] \geqslant 0$$

for all $\theta_i \in \Theta_i$, $\tilde{\theta}_i \in \tilde{\Theta}_i$, and $i \in N$.

The following definition is central to our analysis.

DEFINITION 3. A BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ can be equivalently implemented in dominant strategies if

    (i)   there exist transfer rules $(\bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$ such that the allocation $(y(\cdot), \bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$ is DISC and

    (ii) $E_{\theta_{-i}}[x_i(\theta_i, \theta_{-i}) - \bar{x}_i(\theta_i, \theta_{-i}) | \theta_i] = 0,$ for all $\theta_i \in \Theta_i$, $i \in N$.

The requirement of equivalent implementation is that the public decision rule $y(\cdot)$ is unchanged, while the original transfers $x_i(\cdot)$ can be replaced by $\bar{x}_i(\cdot)$ so as to make truthtelling a dominant strategy while leaving every agent's interim utility unchanged. Note that the assumption of a common prior $F(\cdot)$ ensures that the expected transfer payments to the principal are also unchanged if $(x_1(\cdot), ..., x_n(\cdot))$ is replaced by $(\bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$. In the following sections, we assume that agents' type are drawn independently, i.e., $F(\theta) = X_{i=1}^n F_i(\theta_i)$. The concluding section discusses the extent to which our results extend to correlated environments.

3. CONDITIONS FOR EQUIVALENT DOMINANT STRATEGY IMPLEMENTATION

In this section we find conditions for an arbitrary BIC allocation rule to admit an equivalent dominant strategy implementation. In later sections we

impose more structure on the problem and focus on BIC allocation rules that are optimal for particular mechanism design problems.

DEFINITION 4. The public decision rule $y(\cdot)$ is implementable in dominant strategies if there exists a transfer rule $(x_1(\cdot), ..., x_n(\cdot))$ such that the allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ is DSIC.

The following result shows that an equivalent dominant strategy mechanism exists whenever the public decision rule is dominant strategy implementable. Hence, requirement (ii) of Definition 3 will be satisfied whenever requirement (i) is satisfied.

PROPOSITION 1. *The BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ can be equivalently implemented in dominant strategies if and only if the decision rule $y(\cdot)$ is implementable in dominant strategies.*

Proof. "Only if" is obvious. To prove the "if" part, let $(\tilde{x}_1(\cdot), ..., \tilde{x}_n(\cdot))$ be a set of transfer rules such that the decision rule $(y(\cdot), \tilde{x}_1(\cdot), ..., \tilde{x}_n(\cdot))$ is DSIC. Using only local incentive constraints, we know from Laffont and Maskin [16] that

$$\tilde{x}_i(\theta_{-i}, \theta_i) = C_i(y(\theta_{-i}, \theta_i), \theta_i) + \int_{\theta_i}^{\theta_i} \frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, t), t) \, dt + \tilde{\varepsilon}_i(\theta_{-i}) \qquad (1)$$

for some arbitrary function $\tilde{\varepsilon}_i(\theta_{-i})$. Since $(y(\cdot), x_1(\cdot), ... x_n(\cdot))$ is BIC, the local incentive constraints imply that

$$E_{\theta_{-i}}[x_i(\theta_{-i}, \theta_i)]$$

$$= E_{\theta_{-i}}\left[C_i(y(\theta_{-i}, \theta_i), \theta_i) + \int_{\theta_i}^{\theta_i} \frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, t), t) \, dt + \varepsilon_i(\theta_{-i}) \right].$$

Now define $\bar{x}_i(\theta) = \tilde{x}_i(\theta) - \tilde{\varepsilon}_i(\theta_{-i}) + \varepsilon_i(\theta_{-i})$ for all $i \in N$. It follows directly that $(y(\cdot), \bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$ is DSIC and that $E_{\theta_{-i}}[x_i(\theta_{-i}, \theta_i) - \bar{x}_i(\theta_{-i}, \theta_i)] = 0$. Hence, $(y(\cdot), \bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$ is an equivalent dominant strategy implementation of the BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$. ∎

In order to make use of Proposition 1 we characterize decision rules $y(\cdot)$ that are dominant strategy implementable. The following result provides a sufficient condition, which extends a condition developed by Mirrlees [26] for a single agent problem.

PROPOSITION 2. *A decision rule $y(\cdot)$ is dominant strategy implementable if*

$$\gamma_i(\theta_{-i}, t \mid \theta_i) \equiv \frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, t), \theta_i) \tag{2}$$

is decreasing in t for all $\theta_{-i} \in \Theta_{-i}, \theta_i \in \Theta_i, i \in N$.

Proof. We have to establish transfer rules $\tilde{x}_i(\cdot)$ that make the allocation rule $(y(\cdot), \tilde{x}_1(\cdot), ..., \tilde{x}_n(\cdot))$ DSIC. Using the transfers in (1), we find that $\Pi_i(\theta_{-i}, \theta_i \mid \theta_i) \geqslant \Pi_i(\theta_{-i}, \tilde{\theta}_i \mid \theta_i)$ if and only if

$$C_i(y(\theta_{-i}, \tilde{\theta}_i), \tilde{\theta}_i) - C_i(y(\theta_{-i}, \tilde{\theta}_i), \theta_i) \leqslant \int_{\theta_i}^{\tilde{\theta}_i} \frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, t), t) \, dt. \tag{3}$$

This inequality follows from the hypothesis that

$$\frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, t), t) \geqslant \frac{\partial}{\partial \theta_i} C_i(y(\theta_{-i}, \tilde{\theta}_i), t)$$

for $\tilde{\theta}_i > \theta_i$. In case $\tilde{\theta}_i < \theta_i$ the same conclusion is obtained by reversing the inequalities twice. ∎

The monotonicity condition (2) may appear to be necessary for dominant strategy implementability. However, at the current level of generality, this turns out not to be true. To see this, define $\phi_i(t, \theta_i) \equiv C_i(y(\theta_{-i}, t), \theta_i)$. If $y(\cdot)$ is to be dominant strategy implementable, the associated transfer must satisfy (1), and for any $\theta_i, \tilde{\theta}_i$:

$$\int_{\tilde{\theta}_i}^{\theta_i} \frac{\partial}{\partial \theta_i} [\phi_i(\tilde{\theta}_i, t) - \phi_i(t, t)] \, dt \geqslant 0. \tag{3a}$$

There exist functions $\phi_i(t, \theta_i)$ such that the partial derivative of $\phi_i(\cdot, \cdot)$ with respect to θ_i, $(\partial/\partial\theta_i) \phi_i(t, \theta_i)$, does not satisfy the monotonicity requirement in (2), i.e., there exist values of θ_i such that $(\partial/\partial\theta_i)\phi_i(t, \theta_i)$ is strictly increasing in t over some range. At the same time, the functions $\phi_i(\cdot, \cdot)$ may satisfy

$$\frac{\partial}{\partial \theta_i} \phi_i(\theta_i, t) \leqslant \frac{\partial}{\partial \theta_i} \phi_i(t, t) \leqslant \frac{\partial}{\partial \theta_i} \phi_i(\tilde{\theta}_i, t)$$

whenever $\theta_i > t > \tilde{\theta}_i$, which in turn implies (3a). However, the following restriction on agents' cost functions ensures that (2) is necessary and sufficient.

DEFINITION 5. The cost functions $C_i(\cdot, \cdot)$ satisfy the weak single crossing property provided the following holds. If for any two public

decisions y_1, $y_2 \in Y$ there exists $\bar{\theta}_i \in \Theta_i$ such that $(\partial/\partial\theta_i) C_i(y_1, \bar{\theta}_i) > (\partial/\partial\theta_i)$ $C_i(y_2, \bar{\theta}_i)$, then $(\partial/\partial\theta_i) C_i(y_1, \theta_i) > (\partial/\partial\theta_i) C_i(y_2, \theta_i)$ for all $\theta_i \in \Theta_i$.

PROPOSITION 3. *Suppose agents' cost functions satisfy the weak single crossing property. Then the BIC allocation rule* $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ *can be equivalently implemented in dominant strategies if and only if the monotonicity condition* (2) *is satisfied.*

Proof. In light of Proposition 2, we have to establish the "only if" part. Suppose (2) is violated, and there exists i, $\theta_{-i} \in \Theta_{-i}$ and $\theta_i, \hat{\theta}_i, \bar{\theta}_i \in \Theta_i$, with $\hat{\theta}_i > \bar{\theta}_i$ such that $(\partial/\partial\theta_i) C_i(y(\theta_{-i}, \hat{\theta}_i), \theta_i) > (\partial/\partial\theta_i) C_i(y(\theta_{-i}, \bar{\theta}_i), \theta_i)$. The weak single crossing property implies that

$$\int_{\bar{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} C_i(y(\theta_{-i}, \hat{\theta}_i), t)\, dt > \int_{\bar{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} C_i(y(\theta_{-i}, \bar{\theta}_i), t)\, dt.$$

Using inequality (3), the dominant strategy requirement implies

$$\int_{\bar{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} C_i(y(\theta_{-i}, \bar{\theta}_i), t)\, dt$$

$$\geq \int_{\bar{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} C_i(y(\theta_{-i}, t), t)\, dt \geq \int_{\bar{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} C_i(y(\theta_{-i}, \hat{\theta}_i), t)\, dt,$$

and, thus, we obtain a contradiction. ∎

4. ENVIRONMENTS EXHIBITING THE ONE-DIMENSIONAL CONDENSATION PROPERTY

In this section we restrict attention to cost functions $C_i(\cdot, \cdot)$ that depend on the public decision y only via a one-dimensional statistic.

DEFINITION 6. The cost function $C_i(y, \theta_i)$ satisfies the one-dimensional condensation property if there exist functions $h_i: Y \to \mathbb{R}$ and $D_i(\cdot, \theta_i)$: $\Theta_i \times \mathbb{R} \to \mathbb{R}$ such that

(i) $D_i(\cdot, \theta_i)$ is twice continuously differentiable, and

(ii) $C_i(y, \theta_i) = D_i(h_i(y), \theta_i)$.

In addition, $C_i(\cdot, \cdot)$ satisfies the single crossing condition if $(\partial^2 D_i/\partial h_i \partial\theta_i) > 0$.

Though the one-dimensional condensation property is restrictive when y is multidimensional, it nevertheless allows us to address a variety of familiar mechanism design problems, as shown later in this section.

PROPOSITION 4. *Suppose agents' cost functions satisfy the one-dimensional condensation property and the single-crossing condition. Then the BIC allocation rule* $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ *can be equivalently implemented in dominant strategies if and only if for all* $i \in N$, *all* $\theta_{-i} \in \Theta_{-i}$:

$$h_i(y(\theta_{-i}, t)) \text{ is decreasing in } t. \tag{4}$$

Proof. Because of Proposition 3, it suffices to show that the weak single-crossing property follows from the one-dimensional condensation property and the single-crossing condition. To see this, note that $(\partial/\partial\theta_i) D_i(h_i(y_1), \tilde{\theta}_i) > (\partial/\partial\theta_i) D_i(h_i(y_2), \tilde{\theta}_i)$ for some $y_1, y_2, \tilde{\theta}_i$ implies $h_i(y_1) > h_i(y_2)$. The single-crossing property further implies that $(\partial/\partial\theta_i) D_i(h_i(y_1), \theta_i) > (\partial/\partial\theta_i) D_i(h_i(y_2), \theta_i)$ for all $\theta_i \in \Theta_i$. ∎

The one-dimensional condensation property admits a simple characterization of the class of BIC allocation rules that can be equivalently implemented in dominant strategies. In fact, for this class of cost functions, we can go further and provide conditions for the existence of unique and equivalent dominant strategies. As argued in the Introduction, the possibility of multiple equilibria is one of the problems afflicting many Bayesian mechanism. However, examples by Dasgupta, Hammond and Maskin [7] and Repullo [30] show that dominant strategy mechanisms may suffer from multiplicity problems as well. For instance, consider an allocation rule in which two different reports by some agent give this agent the same utility, but generate different utility values for some other agent. In such cases there are multiple dominant strategy equilibria that are not utility-equivalent for all agents.

DEFINITION 7. A BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$, can be equivalently and uniquely implemented in dominant strategies if there exists an equivalent DSIC allocation rule $(y(\cdot), \bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$, with the property that for every dominant strategy equilibrium $\sigma(\theta) = (\sigma_1(\theta_1), ..., \sigma_n(\theta_n))$:[7]

(i) Transfers coincide:

$$\bar{x}_i(\theta) = \bar{x}_i(\sigma(\theta)), \quad \text{for all} \quad \theta \in \Theta, i \in N.$$

[7] We emphasize that the notion of uniqueness in Definition 4 refers only to agent's utilities. If one insisted on a unique dominant strategy equilibrium, focus on revelation mechanisms would no longer be warranted. For instance in cases involving "pooling," revelation mechanisms necessarily possess multiple equilibria, all of which generate identical outcomes. In these situations there exist "shrunk" revelation mechanisms, i.e., where redundant type messages are deleted, such irrelevant multiplicities can be avoided. It may be verified that the following discussion would be unaltered if we were to require that there exist some mechanism (possibly non-revelation) with a unique dominant strategy equilibrium, which generates outcomes equivalent to the original BIC allocation rule.

(ii) Public decisions coincide:

$$y(\theta) = y(\sigma(\theta)) \qquad \text{for all} \quad \theta \in \Theta, \, i \in N.$$

PROPOSITION 5. *Suppose agents' cost function satisfy the one-dimensional condensation property and the single-crossing condition, and that $D_i(\cdot, \cdot)$ is strictly increasing in $h_i(\cdot)$. Then any BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ can be equivalently and uniquely implemented in dominant strategies if and only if for all $i \in N$:*

(i) *$h_i(y(\theta_{-i}, t))$ is decreasing in t for all θ_{-i},*

(ii) *if for some $i \in N$, $h_i(y(\theta_{-i}, t))$ is constant in t over some interval (θ_i, θ_i^*) for all θ_{-i}, then $y(\theta_{-i}, t)$ is also constant in t over this interval for all θ_{-i}.*

Proof. See Appendix.

Proposition 5 shows that a mild strengthening of the monotonicity property (4) is required to rule out multiple dominant strategy equilibria. Note that if the public decision y is one-dimensional and all $h_i(\cdot)$ are strictly monotone in y, then the monotonicity property (4) is necessary and sufficient for equivalent and unique implementation.

Our analysis so far has considered arbitrary BIC allocation rules. We now focus on allocation rules that emerge as the solution to a principal-agent problem. Suppose a principal contracts with n agents and seeks to maximize the expected value of some gross benefit $B(y)$ less the transfers paid to the agents.

PROGRAM 1.

$$\max_{(y(\cdot), x_1(\cdot), ..., x_n(\cdot))} E_\theta \left[B(y(\theta)) - \sum_{i=1}^{n} x_i(\theta) \right]$$

subject to

(i) $E_{\theta_{-i}}[\Pi_i(\theta_{-i}, \theta_i | \theta_i)] \geqslant 0$ *for all* $\theta_i \in \Theta_i$,

(ii) $E_{\theta_{-i}}[\Pi_i(\theta_{-i}, \theta_i | \theta_i) - \Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i)] \geqslant 0$ *for all* $\theta_i \in \Theta_i$, $\tilde{\theta}_i \in \Theta_i$.

Requirement (i) in Program 1 represents an interim participation constraint, where each agent's reservation utility is normalized to zero. Requirement (ii) is the Bayesian incentive compatibility constraint. We define Program 2 as the optimization problem obtained from Program 1 by replacing the BIC constraint in (ii) with the stronger DSIC constraint

$$\Pi_i(\theta_{-i}, \theta_i | \theta_i) - \Pi_i(\theta_{-i}, \tilde{\theta}_i | \theta_i) \geqslant 0 \qquad \text{for all} \quad \theta_i, \tilde{\theta}_i \in \Theta_i, \, \theta_{-i} \in \Theta_{-i}.$$

Assuming solutions to these optimization problems exist, we shall denote their optimal values by Γ (Program 1) and $\bar{\Gamma}$ (Program 2) respectively.[8]

PROPOSITION 6. *Suppose agents' cost functions satisfy the one-dimensional condensation property and the single-crossing condition. Furthermore suppose*

(i) $F_i(\cdot)$ *has a (positive) density function* $f_i(\cdot)$ *with the property that the inverse hazard rate* $(F_i(\theta_i)/f_i(\theta_i))$ *is increasing in* θ_i, *and*

(ii) $(\partial^2 D_i/\partial\theta_i\partial h_i)$ *is increasing in* θ_i.

Then any allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ *that solves Program 1 can be implemented equivalently in dominant strategies, and therefore,* $\Gamma(Program\ 1) = \bar{\Gamma}(Program\ 2)$.

Proof. We show that for any $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ solving the Bayesian problem in Program 1 the decision rule $y(\cdot)$ satisfies the monotonicity condition (4). As shown in the proof of Proposition 1, (local) Bayesian incentive compatibility implies that

$$E_\theta[x_i(\theta_{-i}, \theta_i)]$$

$$= \int_{\underline{\theta}_i}^{\theta_i} E_{\theta_{-i}}\left[D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i) \right.$$

$$\left. + \int_{\underline{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} D_i(h_i(y(\theta_{-i}, t)), t)\, dt + \varepsilon_i(\theta_{-i}) \right] dF_i(\theta_i)$$

$$= E_\theta[D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i)]$$

$$+ E_{\theta_{-i}}\left[\int_{\underline{\theta}_i}^{\theta_i} \int_{\underline{\theta}_i}^{\theta_i} \frac{\partial}{\partial\theta_i} D_i(h_i(y(\theta_{-i}, t)), t)\, dt\, dF_i(\theta_i) + \varepsilon_i(\theta_{-i}) \right]. \quad (5)$$

We note first that in the Bayesian mechanism the individual rationality constraint will only be binding for the highest cost type $\bar{\theta}_i$, i.e., $E_{\theta_{-i}}[\Pi_i(\theta_{-i}, \bar{\theta}_i | \bar{\theta}_i)] = 0$. As a consequence, we have $E_{\theta_{-i}}[\varepsilon_i(\theta_{-i})] = 0$. Integration by parts of the second term on the right hand side of (5) yields

$$E_\theta[x_i(\theta_{-i}, \theta_i)] = E_\theta[V_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i)],$$

where $V_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i) \equiv D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i) + (F_i(\theta_i)/f_i(\theta_i))(\partial/\partial\theta_i) D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i)$ denotes Agent i's virtual cost (following the terminololgy of Myerson [27]).

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[8] We note that under the assumptions of Proposition 6, a solution will exist provided the set Y is compact and the benefit function $B(\cdot)$ is continuous.

Acknowledging only the local incentive compatibility constraints, Program 1 thus amounts to pointwise maximization of

$$B(y) - \sum_{i=1}^{n} V_i(h_i(y), \theta_i). \tag{6}$$

By assumptions (i) and (ii) the function $(\partial/\partial h_i) V_i(h_i(y), \theta_i)$ is increasing in θ_i. A "revealed preference" argument then shows that any decision rule $y(\cdot)$ maximizing (6) has to be such that $h_i(y(\theta_{-i}, t))$ is decreasing in t.[9] Further, if the transfers are given by (5), the corresponding allocation rule will be globally Bayesian incentive compatible. We conclude that there exists an optimal solution to Program 1 which is also feasible (and optimal) for Program 2. ∎

We note that under the conditions of Proposition 6, it would have been possible to construct an equivalent dominant strategy implementation that satisfies the participation constraints ex-post. Setting $\varepsilon_i(\theta_{-i}) \equiv 0$ implies

$$\Pi_i(\theta_{-i}, \theta_i | \theta_i) \geqslant 0 \qquad \text{for all} \quad \theta_i \in \Theta_i, \theta_{-i} \in \Theta_{-i}.$$

The proof of Proposition 6 reveals the basic argument underlying our result. The curvature and monotonicity assumptions on $D_i(\cdot, \cdot)$ ensure that the decision rule $y(\cdot)$ of the optimal Bayesian mechanism is such that $h_i(y(\theta_{-i}, t))$ is decreasing in t. As a consequence, $y(\cdot)$ is dominant strategy implementable. By Proposition 1 this implies that the BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ can be implemented equivalently in dominant strategies.

The natural question then is whether the two concepts of incentive compatibility could ever cause the corresponding optimal decision rules to differ. To illustrate that this may indeed be the case, suppose that cost functions do not satisfy the assumptions of Proposition 6. Conceivably, there could be a unique decision rule $y(\cdot)$ maximizing (6) pointwise such that $h_i(y(\theta_{-i}, t))$ is strictly increasing in t, on some interval, for some θ_{-i}.

[9] For given θ_{-i}, let y and $\hat{y}$ denote optimal production rules when agent i's type is θ_i and $\hat{\theta}_i$ respectively. Then

$$B(y) - \sum_{j \neq i} V_j(h_j(y), \theta_j) - V_i(h_i(y), \theta_i)$$

$$\geq B(\hat{y}) - \sum_{j \neq i} V_j(h_j(\hat{y}), \theta_j) - V_i(h_i(\hat{y}), \theta_i)$$

$$B(\hat{y}) - \sum_{j \neq i} V_j(h_j(\hat{y}), \theta_j) - V_i(h_i(\hat{y}), \hat{\theta}_i)$$

$$\geq B(y) - \sum_{j \neq i} V_j(h_j(y), \theta_j) - V_i(h_i(y), \hat{\theta}_i).$$

Adding these two inequalities, one obtains that $h_i(y(\theta_{-i}, t))$ is decreasing in t.

As a consequence, $(\partial/\partial\theta_i) D_i(h_i(y(\theta_{-i}, t)), \theta_i)$ will be increasing in t on that interval for those θ_{-i}, yet

$$E_{\theta_{-i}}\left[\frac{\partial}{\partial\theta_i} D_i(h_i(y(\theta_{-i}, t)), \theta_i)\right]$$

is decreasing in t. An argument analogous to that in Proposition 2, then shows that this decision rule, combined with the transfers obtained from the local BIC constraints, is an optimal Bayesian mechanism. Yet, by Proposition 4, this allocation rule cannot be equivalently implemented, and the principal would be strictly worse off by insisting on dominant strategies.

Our analysis applies to a variety of economic settings that have been considered in the mechanism design literature.

(i) Auctions

In the auction design problems considered by Myerson [27], Bulow and Roberts [5], and others, the decision space Y is the simplex in $\mathbb{R}^n$ with y_i representing the probability that the ith bidder receives an indivisible object sold by the principal. If the principal's valuation of the object is some number B^*, the benefit function in Program 1 becomes $B(y) = B^* \cdot (1 - \sum_{i=1}^n y_i)$. The ith bidder's valuation of the object is $\eta_i \in (\underline{\eta}_i, \bar{\eta}_i)$. With risk-neutrality an agent's expected utility becomes $\eta_i \cdot y_i - p_i$ where p_i denotes the expected price paid for the object. Defining $x_i = -p_i$ and $\theta_i \equiv -\eta_i$, the conditions of Proposition 6 will be satisfied provided the inverse hazard rate $(F_i(\theta_i)/f_i(\theta_i))$ is monotone increasing in θ_i.[10] Our result thus generalizes the well known fact that with symmetric bidders the second price auction (which provides dominant strategies) is an optimal Bayesian mechanism. With asymmetric bidders, the following dominant strategy mechanism is optimal:

$$y_i = 1 \quad \text{if and only if} \quad \alpha_i(\theta_i) \leqslant \min_{j \neq 1} \alpha_j(\theta_j), \tag{7}$$

where $\alpha_j(\theta_j) \equiv \theta_j + (F_j(\theta_j)/f_j(\theta_j))$. In the event of winning the object the ith bidder pays $p_i \equiv \alpha_i^{-1}(\beta_i(\theta_{-i})) + T_i$, where

$$\beta_i(\theta_{-i}) \equiv \begin{cases} \alpha_i(\underline{\theta}_i) & \text{if} \quad \min_{j \neq i} \alpha_j(\theta_j) < \alpha_i(\underline{\theta}_i) \\ \min_{j \neq i} \alpha_j(\theta_j) & \text{if} \quad \min_{j \neq i} \alpha_j(\theta_j) \in [\alpha_i(\underline{\theta}_i), \alpha_i(\bar{\theta}_i)] \\ \alpha_i(\bar{\theta}_i) & \text{if} \quad \min_{j \neq i} \alpha_j(\theta_j) > \alpha_i(\bar{\theta}_i). \end{cases}$$

The constant K_i is chosen such that the lowest valuation type $\bar{\theta}_i$ of Agent i is indifferent between participating and not participating. Since the decision

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[10] If $G_i(\eta_i)$ denotes the prior distribution for η_i, and g_i the associated density, the ratio $(F_i(\theta_i)/f_i(\theta_i))$ will be increasing in θ_i if and only if $(1 - G_i(\eta_i))/g_i(\eta_i)$ is decreasing in η_i.

rule in (7) also satisfies the hypothesis of Proposition 5, we obtain an equivalent and unique dominant strategy implementation of the optimal Bayesian auction mechanism.

(ii) Team Production

For another application of Proposition 6, consider the team production models of McAfee and McMillan [20] and Melumad, Mookherjee and Reichelstein [23]. There, n agents jointly produce an output; y_i may be interpreted as the contribution of Agent i, $B(y_1, ..., y_n)$ as the team output, and $D_i(y_i, \theta_i)$ as Agent i's unobservable production cost. With additional mild assumptions, the optimal production assignments $y(\theta)$ (which maximize (6)) will also satisfy the conditions of Proposition 5, thus ensuring uniqueness. The above conclusion generalizes to some contexts involving cost externalities. Suppose each agent's utility function is of the form $x_i - K_i(e_i)$, where $K_i(e_i)$ is a level of disutility from effort e_i. The amount of effort needed to produce contribution y_i, given the contributions of other agents y_{-i}, is given by $e_i(h_i(y_i, y_{-i}), \theta_i)$. Suppose the function $e_i(\cdot, \cdot)$ is increasing in both arguments, that h_i is increasing in y_i, and $(\partial^2 e_i / \partial h_i \partial \theta_i) > 0$. Proposition 6 then applies, if we define $D_i(h_i(y_i, y_{-i}), \theta_i) \equiv K_i(e_i(h_i(y_i, y_{-i}), \theta_i))$. Equation (1) with $\tilde{z}_i(\theta_{-i}) \equiv 0$ shows the transfers that make the optimal decision rule dominant strategy incentive compatible.

It is important to note that the preceding formulation assumes that the principal monitors individual contributions y_i. A central point of McAfee and McMillan [20] is that the principal does not lose anything by monitoring only the group output $B(y_1, ..., y_n)$, provided $B(\cdot)$ satisfies certain curvature conditions. In other words, there is no value to monitoring individual contributions, a conclusion diametrically opposite to that of Alchian and Demsetz [1]. This result relies critically on the Bayesian formulation of the problem. McAfee and McMillan consider incentive schemes for each agent that are linear in the group output. It is possible to choose the individual output shares so that truthful reporting and subsequent delivery of the assigned contribution is a Bayesian equilibrium. Typically, though, this scheme will not admit a dominant strategy implementation: Agent i will not choose his assigned y_i if he expects other agents to deviate. Hence the value of monitoring individual contributions may be viewed as obtaining dominant strategies and avoiding multiple equilibrium problems.

(iii) Efficient Bilateral Trading Mechanisms

Myerson and Satterthwaite [28] consider the problem of designing efficient mechanisms for trading an indivisible object between a buyer and

a seller. The seller's valuation of the object is v_1, while the buyer's is v_2. If p is the probability of trade, and x the price paid, the parties' utilities are $x - v_1 p$ and $v_2 p - x$, respectively. Myerson and Satterthwaite consider the problem of finding a mechanism $\{p(v_1\ v_2), x(v_1, v_2)\}$, that maximizes the expected gains from trade $E[(v_2 - v_1)\ p(v_1, v_2)]$, subject to Bayesian incentive compatibility and interim participation constraints. They show that there exists an efficient trading mechanism in which $p(v_1, v_2)$ is monotone decreasing in v_1, and monotone increasing in v_2.

Our results imply that this mechanism can be equivalently and uniquely implemented in dominant strategies. To see this, define $y \equiv p$, $\theta_1 \equiv v_1$, $\theta_2 \equiv -v_2$, $x_1 \equiv x$, $x_2 \equiv -x$. The "cost" functions then satisfy the conditions identified in Propositions 4 and 5. However, the "transfers" between the buyer and seller in the equivalent dominant strategy mechanism will be balanced only in expectation, and not necessarily ex post. In other words, the presence of a risk-neutral third party (such as a broker) allows efficient trading to be implemented in dominant strategies by a "robust" mechanism.[11] Myerson and Satterthwaite also characterize trading mechanisms that maximize the expected profit of the broker, subject to Bayesian incentive compatibility and individual rationality constraints. Proposition 6 shows that the broker has nothing to lose by adopting a dominant strategy mechanism that guarantees ex post individual rationality.

5. GENERAL ENVIRONMENTS

Our analysis so far has relied substantially on the one-dimensional condensation property. In more generality, we seek conditions on agents' cost functions that ensure satisfaction of the monotonicity requirement in (4). The following result is a variant of Proposition 4.

PROPOSITION 4'. *Suppose agents' cost functions display the single-crossing property in each component, i.e.,*

$$\frac{\partial^2}{\partial \theta_i \partial y_j} C_i(y, \theta_i) \geq 0 \qquad \textit{for all} \quad 1 \leq j \leq k.$$

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[11] Hagerty and Rogerson [12] define a trading mechanism to be robust if it satisfies dominant strategy incentive compatibility, as well as ex post individual rationality. Confining attention to ex post balanced mechanisms, they show in the Myerson–Satterthwaite model, the only robust mechanisms are posted price mechanisms, where a given price is posted by a planner, and trade occurs at this price if and only if both parties are willing. Our result implies that, in contrast, there is no loss in restricting attention to "robust" mechanisms, provided transfers are only required to balance ex ante.

Then, the BIC allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ *can be equivalently implemented in dominant strategies provided* $y_j(\theta_{-i}, t)$ *is decreasing in* t *for all* $\theta_{-i} \in \Theta_{-i}$ *and all* $1 \leqslant j \leqslant k$.

The proof of this result is omitted, since it is fairly straightforward. We consider two applications of Proposition 4'.

(iv) *Auctioning Procurement Contracts*

Laffont and Tirole [17] study the problem where n firms compete for a procurement contract from the government. The observed cost of firm i, if awarded the contract, is $c_i = \theta_i - e_i$, where θ_i is privately known to firm i, and e_i is the firm's unobservable effort. The firm's profit becomes $x_i - \psi(e_i)$, where x_i is the net transfer to i from the government, over and above actual cost. The utilitarian government's objective is to maximize the expected value of $S - (1 + \lambda)(x_i + c_i) + x_i - \psi(e_i)$, where S is the benefit from the good delivered and $(1 + \lambda)$ is the social cost of transferring one unit of money to firm i.

In this model, the public decision y can be identified with the vector $(p_1, ..., p_n, c)$, where p_i denotes the probability of awarding the contract to firm i, and c is the level of cost mandated for the chosen firm.[12] The cost function of the ith firm then becomes $C_i(y, \theta_i) = p_i \psi(\theta_i - c)$, which does not satisfy the one-dimensional condensation property. Nevertheless, it satisfies the single-crossing property in each component.[13] Given the curvature assumptions $\psi'(\cdot) > 0, \psi''(\cdot) > 0, \psi'''(\cdot) > 0$, Laffont and Tirole show that the optimal Bayesian mechanism involves a public decision rule which is monotone in each argument.[14] Hence Proposition 4' implies that the optimal mechanism can be equivalently implemented in dominant strategies. This point is noted by Laffont and Tirole for the case of ex ante symmetric firms.

(v) *Intrafirm Recource Allocation*

In Harris, Kriebel and Raviv [13] a multidivisional firm has to decide on the allocation of an intermediate input. To produce some exogenously given output z_i, each division i ($1 \leqslant i \leqslant n$) can use an intermediate input y_i and its own (unobservable) effort. The intermediate input is provided by division 0 which in turn uses a primary input y_0 and its own effort in the

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[12] In the presence of forecasting or auditing errors, this is replaced by award probabilities for different firms, and an incentive contract for the chosen firm which relates the transfer t^i to observed cost c^i.

[13] We need to identify $-c$ rather than c as a component of the public decision.

[14] If the bidding firms are ex ante symmetric, then it involes $p_i(\theta_1, ..., \theta_n) = 1$ if $\theta_i < \min_{j \neq i} \theta_j$ and 0 otherwise. The function $c(\theta_i)$ is increasing, since firms with higher cost will be asked to exert less effort.

production process. The primary input is supplied by headquarters. Division i's cost amounts to whatever effort it supplies. Since Harris, Kriebel and Raviv assume linear production functions, these costs become

$$C_i(y, \eta_i) = z_i - \eta_i \cdot y_i \qquad 1 \leqslant i \leqslant n$$

and

$$C_0(y, \eta_0) = \sum_{i=1}^{n} y_i - \eta_0 \cdot y_0.$$

The set of feasible decisions is

$$Y = \left\{ (y_0, y_1, ..., y_n) \middle| z_i - \eta_i \cdot y_i \geqslant 0, \qquad \sum_{i=1}^{n} y_i - \eta_0 \cdot y_0 \geqslant 0 \right\},$$

reflecting that divisions cannot use negative levels of effort.

Headquarters' total cost equals $y_0 + \sum_{i=0}^{n} x_i$. Following the same approach as in Proposition 6, the minimization of expected total cost requires pointwise minimization of total virtual cost, i.e.,

$$\min_{y \in Y} \left\{ y_0 + \sum_{i=0}^{n} V_i(y, \eta_i) \right\}, \tag{8}$$

where $V_i(y, \eta_i) \equiv C_i(y, \eta_i) - (1 - G_i(\eta_i)/g_i(\eta_i))(\partial/\partial\eta_i) C_i(y, \eta_i)$ for $0 \leqslant i \leqslant n$. Provided the ratios $(1 - G_i(\eta_i)/g_i(\eta_i))$ are decreasing we find that $(\partial^2/\partial\eta_i \partial y_j) V_i(y, \eta_i) \leqslant 0$. It then follows directly that there exists a solution $y^*(\eta)$ to the problem in (8) such that all components y_j are increasing in η_i.[15] If one again makes the linear transformation $\theta_i = -\eta_i$ (as described in connection with the auction problem above), then the conditions of Proposition 4′ are met: each $y_j^*(\theta_{-i}, t)$ is decreasing in t and the single crossing property, i.e., $(\partial^2/\partial y_i \partial \theta_i) C_i(y, \theta_i) \geqslant 0$, holds in all components for all i, $0 \leqslant i \leqslant n$.

We finish this section with a generalization of Proposition 6. Our result requires that the cost and benefit functions have suitable complementarity properties. A function $\phi(y_1, ..., y_k)$ is said to exhibit *complementarity*, if $(\partial^2\phi/\partial y_j \partial y_l) \geqslant 0$ for all $j \neq l$, $1 \leqslant j$, $l \leqslant k$.

PROPOSITION 6′. *Suppose*

(i) Y *is a compact cube in* $\mathbb{R}^k$.

(ii) *Agents' types are drawn independently and each* $F_i(\cdot)$ *has*

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[15] Harris, Kriebel and Raviv [13] confine attention to uniform distributions for which the inverse hazard rates are monotone decreasing.

a density function $f_i(\cdot)$ *with the property that the inverse hazard rate* $(F_i(\theta_i)/f_i(\theta_i))$ *is increasing in* θ_i.

(iii) $(\partial^2 C_i/\partial\theta_i\partial y_j)$ *is increasing in* θ_i, *and* $(\partial^2 C_i/\partial\theta_i\partial y_j) \geq 0$ *for all* $1 \leq j \leq k$.

(iv) *The functions:* $-C_i(y, \theta_i)$ *and* $-\partial/\partial\theta_i C_i(y, \theta_i)$ *exhibit complementarity, with respect to* $(y_1, ..., y_k)$ *for all* θ_i.

(v) $B(y)$ *is twice differentiable and exhibits complementarity.*

Then there exists an allocation rule $(y(\cdot), x_1(\cdot), ..., x_n(\cdot))$ *solving Program* 1 *that can be implemented equivalently in dominant strategies, and therefore,* $\Gamma(\text{Program } 1) = \Gamma(\text{Program } 2)$.

Proof. See Appendix.

The assumptions of Proposition 6′ ensure that the principal's objective function is supermodular in y and θ (see Milgrom and Roberts [25] and the references contained therein for an analysis of supermodular functions). As a consequence, there exists a solution to Program 1 such that all components of y change in a "synchronized" way, i.e., as some θ_i increases all $y_j(\cdot)$ decrease.[16]

6. CONCLUDING REMARKS

Throughout this paper we have assumed that the prior beliefs held by agents are independently distributed. With non-independent beliefs, equivalent dominant strategy implementation of Bayesian mechanisms is still possible if beliefs happen to satisfy a "spanning" condition. This condition, which has appeared in the work of Demski and Sappington [8], Cremer and McLean [6], Melumad and Reichelstein [23], and McAfee and Reny [22], requires conditional beliefs to have a full rank property. It implies the ability of the designer to extract all informational rents of agents by "screening by beliefs," i.e., construction of lotteries for each agent whose payoffs are conditioned on the announcements of other agents.

If each agent has a finite number of possible types, it can be shown that spanning implies that any public decision rule satisfying the monotonicity property (4) can be implemented in dominant strategies by a set of transfers that generate any desired interim utility levels for every agent. Therefore, spanning ensures the dominant strategy implementability of first-best

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[16] We note that the intrafirm resource allocation problem of Harris, Kriebel and Raviv [13] cannot be viewed as a direct application of Proposition 6′ since in their model the set Y is not a cube.

allocation rules in principal-agent environments of the kind represented in Program 1. In the absence of spanning, Bayesian incentive compatible allocation rules may not be equivalently implementable in dominant strategies; this has been established in an example by Cremer and McLean [6].

Finally, we have restricted attention to contexts where each agent's private information is represented by a single dimensional parameter. The approach of McAfee and McMillan [21] to analyzing multi-dimensional private information problems may be useful in extending our results in this direction.

Appendix

Proof of Proposition 5. We first establish sufficiency. Since $(y(\cdot),$ $x_1((\cdot), ..., x_n(\cdot))$ is BIC, it follows that

$$E_{\theta_{-i}}[x_i(\theta_{-i}, \theta_i)] = E_{\theta_{-i}}\left[D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i) \right.$$

$$\left. + \int_{\theta_i}^{\theta_i} \frac{\partial}{\partial \theta_i} D_i(h_i(y(\theta_{-i}, t)), t) \, dt \right] + \varepsilon_i.$$

Choose the transfer rules $\bar{x}_i(\cdot)$ according to:

$$\bar{x}_i(\theta_{-i}, \theta_i) = D_i(h_i(y(\theta_{-i}, \theta_i)), \theta_i)$$

$$+ \int_{\theta_i}^{\theta_i} \frac{\partial}{\partial \theta_i} D_i(h_i(y(\theta_{-i}, t)), t) \, dt + \varepsilon_i. \tag{9}$$

By Proposition 4, the monotonicity requirement (i) of Definition 7 implies the existence of an equivalent DSIC mechanism. Suppose there exists an untruthful dominant strategy equilibrium $\sigma(\theta)$ in this mechanism, where type θ_i of Agent i reports θ_i^* instead. Since θ_i^* and θ_i are both dominant strategies for type θ_i, it follows that

$$\bar{\Pi}_i(\theta_{-i}, \theta_i^* | \theta_i) = \bar{\Pi}_i(\theta_{-i}, \theta_i | \theta_i), \quad \text{for all} \quad \theta_{-i}, \tag{10}$$

where $\bar{\Pi}_i$ denotes the ex post utility of Agent i in the DSIC mechanism $(y(\cdot), \bar{x}_1(\cdot), ..., \bar{x}_n(\cdot))$. Without loss of generality, suppose $\theta_i^* > \theta_i$. We claim that $h_i(y(\theta_{-i}, t))$ is constant over $t \in (\theta_i, \theta_i^*)$. If this were not the case, there would exist $\bar{\theta}_i \in (\theta_i, \theta_i^*)$ such that $h_i(y(\theta_{-i}, t)) > h_i(y(\theta_{-i}, \theta_i^*))$ for all $t \in (\bar{\theta}_i, \theta_i^*)$, for some θ_{-i}. We then find that

$$\bar{\Pi}_i(\theta_{-i}, \theta_i | \theta_i) - \bar{\Pi}_i(\theta_{-i}, \theta_i^* | \theta_i^*) = \int_{\theta_i}^{\theta_i^*} \frac{\partial}{\partial \theta_i} D_i(h_i(y(\theta_{-i}, t)), t) \, dt,$$

$$> \int_{\theta_i}^{\theta_i^*} \frac{\partial}{\partial \theta_i} D_i(h_i(y(\theta_{-i}, \theta_i^*)), t) \, dt$$

$$= D_i(h_i(y(\theta_{-i}, \theta_i^*)), \theta_i^*)$$

$$- D_i(h_i(y(\theta_{-i}, \theta_i^*)), \theta_i). \qquad (11)$$

The inequality step makes use of the assumption that the single crossing property holds strictly. Finally, the right hand side of (11) equals $\bar{\Pi}_i(\theta_{-i}, \theta_i^* | \theta_i) - \bar{\Pi}_i(\theta_{-i}, \theta_i^* | \theta_i^*)$, leading to a contradiction with (10).

It therefore follows that $h_i(y(\theta_{-i}, t))$ is constant over $t \in (\theta_i, \theta_i^*)$, for all θ_{-i}. Condition (ii) in Proposition 5 then implies that $y(\theta_{-i}, t)$ is constant in t for all θ_{-i}. Since this argument can be made separately for all $i \in N$, it follows by induction that $y(\sigma(\theta)) = y(\theta)$. A further implication is that $y(\sigma_{-i}(\theta_{-i}), \bar{\theta}_i) = y(\theta_{-i}, \bar{\theta}_i)$ for all $\bar{\theta}_i \in [\underline{\theta}_i, \theta_i]$. Therefore, it follows from (9) that $\bar{x}_i(\sigma(\theta)) = \bar{x}_i(\theta)$.

' To establish necessity, suppose $(y(\cdot), x_1(\cdot), \ldots, x_n(\cdot))$ can be equivalently and uniquely implemented. Requirement (i) in Proposition 5 follows from Proposition 4. To establish (ii) suppose $h_i(y(\theta_{-i}, t))$ is constant in t on the interval (θ_i, θ_i^*) for all θ_{-i}. The dominant strategy requirement implies that $x_i(\theta_{-i}, t)$ is constant in t for $t \in (\theta_i, \theta_i^*)$. Hence any reporting strategy

$$\sigma_i(\bar{\theta}_i) = \begin{cases} \bar{\theta}_i & \text{if} \quad \bar{\theta}_i \notin (\theta_i, \theta_i^*) \\ \omega_i(\bar{\theta}_i) & \text{if} \quad \bar{\theta}_i \in (\theta_i, \theta_i^*), \end{cases}$$

where $\omega_i(\cdot)$ is an arbitrary map from (θ_i, θ_i^*) to itself is also a dominant strategy for Agent i. By definition of uniqueness it follows that $y(\theta_{-i}, t)$ is constant in t implying requirement (ii) in Proposition 5. ∎

Proof of Proposition 6'. The first part of the proof is identical to that of Proposition 6. Using the local incentive constraints the optimal decision rule of the Bayesian problem requires maximization of

$$B(y) - \sum_{i=1}^{n} \hat{V}_i(y, \theta_i)$$

with $\hat{V}_i(y, \theta_i) = C_i(y, \theta_i) + (F_i(\theta_i)/f_i(\theta_i)) \cdot (\partial/\partial\theta_i) C_i(y, \theta_i)$. By Proposition 4 it is sufficient to show that $y_j(\theta_{-i}, t)$ is decreasing in t for all $1 \leq j \leq k$. Setting $\theta_i = -\eta_i$, let $V_i(y, \eta_i) \equiv \hat{V}_i(y, -\eta_i)$ and define

$$A(y, \eta) = B(y) - \sum_{i=1}^{n} V_i(y, \eta_i).$$

By Theorem 2 in Milgrom and Roberts [25] a function $\phi(x_1, ..., x_r)$ is supermodular if and only if $(\partial^2/\partial x_i \partial x_j) \phi(x_1, ..., x_r) \geq 0$ for all $1 \leq i, j \leq r$.[17] Hence, our complementarity assumptions on $B(\cdot)$ and $-C_i(y, \theta_i)$ combined with the single-crossing condition imply that the function $A(y, \eta)$ is supermodular.

For any given η define

$$Y^*(\eta) = \operatorname*{argmax}_{y \in Y} A(y, \eta).$$

Since Y is compact and $A(\cdot)$ is continuous, the set $Y^*(\eta)$ is non-empty and compact. By Theorem 4 in Milgrom and Roberts [25], the set $Y^*(\eta)$ is a sublattice, i.e., if $y, \tilde{y} \in Y^*(\eta)$ then $y \wedge \tilde{y} \in Y^*(\eta)$ and $y \vee \tilde{y} \in Y^*(\eta)$. It follows that there is a greatest element $y^*(\eta) \in Y^*(\eta)$, that is $y_j^*(\eta) \geq y_j$ for all $1 \leq j \leq k$, $y \in Y^*(\eta)$.

Finally, all components of the function $y^*(\eta_{-i}, t)$ are increasing in t. This is a consequence of Theorem 5 in Milgrom and Roberts [25] asserting that $y \in Y^*(\eta)$ and $\tilde{y} \in Y^*(\eta_{-i}, t)$ imply $y \vee \tilde{y} \in Y^*(\eta_{-i}, t)$ and $y \wedge \tilde{y} \in Y^*(\eta)$. Since $V_i(y, \eta_i) = V_i(y, -\eta_i)$ and $\theta_i = -\eta_i$, we conclude that the maximization problem

$$B(y) - \sum_{i=1}^{n} \hat{V}_i(y, \theta_i)$$

has a solution $y^*(\theta)$ with the property that all components $y_j^*(\theta_{-i}, t)$ are decreasing in t. ∎

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[17] A function $\phi: \mathbb{R}^r \to \mathbb{R}$ is supermodular if for all

$$x, y: \phi(x) + \phi(y) \leq \phi(x \wedge y) + \phi(x \vee y)$$

where $(x \wedge y)_i = \min(x_i, y_i)$ and $(x \vee y)_i = \max(x_i, y_i)$; see Milgrom and Roberts [25] and the references contained therein.

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