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Indian Statistical Institute

Mid-Semester Examination (2011-2012)

M.Tech. (CS) First Year

Discrete Mathematics

Date: September 1, 2011 Maximum Marks: 60 Time: 2.5 hours

Answer as much as you can. This paper carries 75 marks. The maximum you can score is 60 marks. Marks alloted to each question are indicated within square brackets near the right margin.

1. Use a combinatorial argument to show that $\binom{n}{s} = \frac{n}{s}\binom{n-1}{s-1}$. [5]

2. Consider binary strings of length $(m+n)$ with exactly m 1's and n 0's. Count the number of these strings with exactly k runs, where a run is a maximal substring of consecutive 1's. For example, 1011100110 has 3 runs. [5]

3. Prove each of the following statements, explicitly mentioning the method of proof:
   (a) If k is odd, then 2^{n+2} divides $k^{2^n} - 1$ for all natural number n.
   (b) $6 - \sqrt{35} < 1/10$ (assume that a calculator is not available). [6+4=10]

4. Given any 9 integers whose prime factors lie in the set 3,7,11, prove that there must be two whose product is a square. [5]

5. Show that for any integer $a \geq 2$, the diagonal Ramsey number $R(a,a) < 4^{a-1}$. [5]

6. Let $P_m(n) = \Sigma_{k=1}^{n-1} k^m$ denote the sum of the m^{th} powers of the integers from 0 to $n-1$. Find the exponential generating function $P(x,n)$ for $P_m(n)$. Verify your answer for $m = 2$ and $m = 3$. [10]

7. Solve or give tight bounds for the following recurrences:
   (a) $a_n = 9a_{n-1} - 30a_{n-2} + 44a_{n-3} - 24a_{n-4}$, and $a_0 = 5$, $a_1 = 12$, $a_2 = 38$, $a_3 = 126$
   (b) $a_n = 7a_{n-1} - 12a_{n-2} + 3n4^n$, and $a_0 = 0$, $a_1 = 2$.
   (c) $T(n) = 2T(n/4) + \sqrt{n}$, and $T(1) = 1$ [8+7+5=20]

8. How many integers between 1 and 500 inclusive are divisible by none of 2, 3, and 5? [5]

9. What is the significance of $S(n,k)$, the Stirling's number of the second kind? Express it as a recurrence relation. [5]

10. Write the generating function for the number of partitions of a positive integer n in which each part is at most 5. [5]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2011-12 (First Semester)

Course Name: M.Tech. in Computer Science
Subject Name: Computer Organization

Date: 02.09.2011 *Maximum Marks: 40* *Duration: 2 hours*

Instructions: *Answer all questions. All parts of a question must be answered in the same place.*

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1. Multiple Choice Questions (More than one may be correct) 2 X 5 = 10

(a) In Static RAM
- No refreshing is needed when powered
- Bits are stored as on/off switches
- Bits are stored as charges in capacitors
- None of the above

(b) Which of the following is the correct order according to increasing speed of access?
- Registers < Cache < Main memory < Secondary Memory
- Cache < Main Memory < Registers < Secondary Memory
- Secondary Memory < Main Memory < Cache < Registers
- Registers < Cache < Secondary Memory < Main Memory

(c) In DRAM
- Bits are stored as on/off switches
- Bits are stored as charges in capacitors
- Refreshing is needed when powered
- None of the above

(d) The idea of cache memory is based on
- Locality of Reference
- Main memory access is slow
- Fact that references tend to cluster
- None of the above

(e) In Memory mapped I/O
- Devices and memory share an address space
- I/O looks just like memory read/write
- There are no special commands for I/O
- None of the above

2. a) Compare the approaches of programmed I/O and interrupt-driven I/O. [5]

b) When a device interrupt occurs, how does the processor determine which device issued the interrupt? [3]

c) In virtually all systems which include DMA modules, DMA access to main memory is given higher priority than CPU access to main memory. Why? [2]

3. a) Consider 4 GB, byte-addressable main memory. Each word is 32 bits wide. Assume that cache block size is 4 words and it contains 64 KB data. What is the total storage needed for the cache? [4]

b) Assume the performance of 1-word wide primary memory organization is

- 4 clock cycles to send the address
- 56 clock cycles for the access time per word
- 4 clock cycles to send a word of data

Given a cache block of 4 words, calculate the miss penalty. [2]

Re-compute the miss penalty if we have Main memory width of 2 words. [2]

c) Can a direct mapped cache sometimes have a higher hit rate than a fully associative cache with LRU replacement policy (on the same reference pattern and with the same cache size)? If so, give an example. If not, explain why not? [2]

4. a) State and explain Amdahl's law. [2]

b) It is found that in a typical execution of a circuit simulation program 80% of all instructions do floating point operations. If we add the fastest available floating point co-processor to the system, what is the upper bound on the achievable speed up? [4]

c) Consider a program execution profile where in 1000 memory references there are 40 misses in the first level cache and 20 misses in the second-level cache.

- What are the local and global miss rates? [2]
- Assume the miss penalty from L2 cache to Memory is 100 clock cycles, the hit time of L2 cache is 10 clock cycles, the Hit time of L1 is 1 clock cycles, and there are 1.5 memory references per instruction. What is the average memory access time per instruction? Ignore the impact of writes. [2]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination : Semester I (2011-2012)

M. Tech (CS) 1st Year

Probability & Stochastic Processes

Date: 5. 9. 11 Maximum marks: 45 Time: 2 hours.

Note: Answer all questions. Maximum you can score is 45.

1. Two people toss a fair coin n times each. Find the probability that they will see the same number of heads. [8 points]

2. Let $B_1, \ldots, B_n$ be a partition of the event B. For an arbitrary event A show that:

$$P(A \mid B) = P(A \mid B_1)P(B_1 \mid B) + \ldots + P(A \mid B_n)P(B_n \mid B).$$

[7 points]

3. An airport bus deposits 25 passengers at 7 stops. Assume that each passenger is as likely to get off at any stop as another and that they act independently. FInd the expected number of stops the bus will make. [8 points]

4. Suppose X and Y are independent exponential random variables with parameters λ and μ. Find $P(X < Y)$. [7 points]

5. A person takes four tests in succession. The probability of his passing the first test is p. Then, the probability of his passing each succeeding test is p or $p/2$ according as he passes or fails the preceding one. He qualifies provided he passes at least three tests. Given that he qualified, what is the probability that he passed the first test? [12 points]

6. Let $\phi(x) > 0$ for $x > 0$ be monotonically increasing and suppose that for some random variable X, $E(\phi(\mid X \mid)) = M$ exists. Prove that,

$$P(\mid X \mid \geq t) \leq \frac{M}{\phi(t)}.$$ [8 points]

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination (2011-2012)

M.Tech. (CS) – First year

Switching Circuit and Logic Design

Maximum marks: 80

Date:5.9.11 **Time : 2 hours 30 mins**

Answer all the questions

1.a). A multiple-output function is given below.

$X = A'C + AB'C + A'B$
$Y = BC'D + B'D' + A'B$
$Z = A'C + B'D' + C'D'$

i). Implement the functions by using ROMs.

ii). Implement the functions by using PLAs

iii). Implement the functions by using PALs

iv). Compare the size of the ROM implementation with that of PLA implementation.

[5+5+5+3]

2 a). What is the condition for two columns of a PLA to be compatible?

b). Applying that condition find the compatibility matrix for the input variables of the problem given in question 1.

c). From the matrix find the first companion pair.

d) what are the different types of faults possible in a PLA

[. 2+10+5+5]

3. Realize the function given below

$F(x_1, x_2, x_3, x_4) = \sum m(1, 4, 5, 6, 7, 12, 13, 15)$.

P.T.O.

i) with an 8-to-1 multiplexer

ii) with some 4-to-1 multiplexers [5+5]

4a). Define Boolean Difference of a function. How Boolean difference is used for finding all test sets for a line stuck-at-fault?

[5]

b) Apply Boolean difference to find test vectors for detecting line X_3 stuck- at-1 for the function,

$F=(X_1+ X_2) X_4' + X_3 X_4$ [10]

5a). What is a self dual function? How many distinct self dual n-variable functions can occur?

b).Determine if the function $F=\sum (0,3, 5,6,8.11,12,13)$ is self dual or not?

[10+5]

INDIAN STATISTICAL INSTITUTE

PERIODICAL EXAMINATION
M.TECH.(CS) I YEAR

ELEMENTS OF ALGEBRAIC STRUCTURES

Date: 07.09.2011 Maximum marks: 70 Duration: 2 hrs

The paper contains 85 marks. Answer as much as you can, the maximum you can score is 70.

1. (a) If A and B are countable sets, show that $A \cup B$ is countable.
   (b) Find the multiplicative inverse of 53 modulo 275.

$(5+5=10)$

2. Let G be the set of all formal products of the form $x^i y^j$ for $i = 0, 1$ and $j = 0, 1, \ldots, n-1$ satisfying the following relations:

   $x^i y^j = x^{i'} y^{j'}$ if and only if $i = i'$ and $j = j'$; $x^2 = y^n = e$ for some $n > 2$; and $xy = y^{-1}x$.

   (a) Find the form of the product $(x^i y^j)(x^k y^\ell)$ as $x^\alpha y^\beta$.
   (b) Show that G is a non-abelian group of order $2n$.

$(4+6=10)$

3. (a) If G is a group such that $(ab)^2 = a^2b^2$ for all $a, b \in G$, then show that G is abelian.
   (b) Express as a product of disjoint cycles: $(1,2,3)(4,6)(1,5,7,9,8)(5,6,3)$.

$(4+6=10)$

4. (a) Show that if G is a group having no non-trivial sub-groups then G must be finite and of prime order.
   (b) Let $U_n = \{i : 1 \leq i < n, \gcd(i,n) = 1\}$. Show that U_8 is not a cyclic group.

$(6+4=10)$

5. (a) If N and M are normal sub-groups of G, prove that NM is also a normal subgroup of G.

(b) Let G be a group and $g \in G$. Define a map $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Show that ϕ is an isomorphism.

$(5 + 5 = 10)$

6. Let G be a finite cyclic group of order r. Show that the automorphism group of G is ismorphic to U_r.

(10)

7. (a) Let G be a finite group and T an automorphism of G with the property that $T(x) = x$ if and only if $x = e$. Show that every $g \in G$ can be written as $g = x^{-1}T(x)$.

   (b) Suppose that in (a) we further have $T^2 = I$, where I is the identity map on G. Then show that G is abelian.

$(5 + 5 = 10)$

8. Let G be a finite group, X be a finite set and $\phi : G \times X \to X$ such that

   for $g, h \in G$, and $x \in X$, $\phi(gh, x) = \phi(g, \phi(h, x))$;
   $\phi(e, x) = x$ for all $x \in X$.

   Define a relation $\sim$ on X as follows: for $x, y \in X$, the relation $x \sim y$ holds if there is a $g \in G$ such that $\phi(g, x) = y$.

   (a) Show that $\sim$ is an equivalence relation.

   (b) For $x \in X$, define $G(x) = \{g \in G : \phi(g, x) = x\}$. Show that $G(x)$ is a sub-group of G.

   (c) Let $B(x)$ be the equivalence class of $\sim$ containing x. Show that the index of $G(x)$ in G is equal to the number of elements in $B(x)$.

$(4 + 3 + 8 = 15)$

INDIAN STATISTICAL INSTITUTE

MidSemestral Examination

M. Tech (CS) - I Year (Semester - I)

Data and File Structures

Date :9.09.11 Maximum Marks : 60 Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. Write an efficient algorithm that compute the height of a binary tree T. [8]

2. You are given an array of infinite length contining zeros followed by one. How fast can you locate the first one in the array. [8]

3. A linked list has exactly n nodes. The elements in these nodes are selected from the set $\{0, 1, ..., n\}$. There are no duplicates in the list. Design an $O(n)$ worst case time algorithm to find which one of the elements from the above set is missing in the given linked list using only a constant amount of additional storage. [10]

4 Let A be an array of size n, containing positive or negative integers, with $A[1] < A[2] < \ldots < A[n]$. Design an efficient algorithm (should be more efficient than $O(n)$) to find an i such that $A[i] = i$ provided such an i exists. What is the worst case computational complexity of your algorithm? [12]

5. Given a tree T, how efficiently can you verify whether T is a binary search tree. [10]

6. Consider a binary heap containing n numbers (the root stores the greatest number). You are given a positive integer $k < n$ and a number x. You have to determine whether the kth largest element of the heap is greater than x or not. Your algorithm must take $O(k)$ time. You may use $O(k)$ extra storage. [12]

7. An array $A[1..n]$ is unimodal if it consists of an increasing sequence followed by a decreasing sequence, or more precisely, if there is an index $m \in \{1, 2, \ldots, n\}$ such that $A[i] < A[i+1]$ for all $1 \leq i < m$, and $A[i] > A[i+1]$ for all $m \leq i < n$. In particular, $A[m]$ is the maximum element, and it is the unique *locally maximum* element whose neighboring elements $A[m-1]$ and $A[m+1]$ have values less than it. Give an algorithm to compute the maximum element of a unimodal input array $A[1..n]$ in $O(logn)$ time. [10]

8. Design an $O(n)$-time algorithm that, given a real number x and a sorted array S of n numbers, determines whether or not there exist two elements in S whose sum is exactly x. [10]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: 2010-11
M. Tech. (CS) I Year
Optimization Techniques

Date: ~~29.08.11~~ 12.09.11 Duration: 2 Hours

The paper contains questions of 70 marks. You may attempt all questions. Maximum marks you can score is 60.

1.(a) Describe (not just diagrams) two sets that are not convex. Justify your answer.

(b) Give example of two real-valued functions that are not convex. Justify your answer.

(c) Define a convex programming problem with an example.

(d) Prove that for a convex programming problem, a local optimum is also a global optimum.

[4+4+4+8 = 20]

2. For an LPP, define: basic solution and feasible solution. Give example of a basic solution which is not feasible and a feasible solution which is not basic.

[10]

3. State the optimality criteria for the Simplex Algorithm. Prove that when the algorithm stops satisfying the criteria, cost function has been optimized.

[3+12=15]

4. Solve the following LPP using the two-phase method.

maximize $x + y$

subject to

$$x - y \geq 1$$
$$x - y \leq 0$$
$$x, y \geq 0$$

[15]

P.T.O.

Consider the following transportation problem.

outlet / warehouse	1	2	3	4	
1	2	5	1	4	20
2	3	1	2	5	60
3	4	3	2	1	40
	40	10	30	40	

(a) Find a bfs.

(b) Find another bfs that has less cost than the previous one.

[5+5=10]

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Indian Statistical Institute
Mid-Semestral Examination: 2011-2012
(first year first semester)
Course Name: M. Tech in Computer Science
Subject Name: Programming Languages & Methodology

Date: 13-09-2011 Maximum Marks: 40 Duration: 2 hours

Instructions:

You **may** attempt **all** questions which carry a total of **45** marks. However, the maximum marks you can score is only **40**.

1. (a) Consider the following simple language. In this language a and b represent names of integer variables. Each statement may have a label as a prefix. Statements in the language include:

Statements	Meaning of the statements
$a = b$	*Assign a the value of b*
$a = a + 1$	*Add* 1 *to a*
$a = a - 1$	*Subtract* 1 *from a*
if $a = 0$ *then goto* L	*If* $a = 0$ *transfer control to statement* L
if $a > 0$ *then goto* L	*If* $a > 0$ *transfer control to statement* L
goto L	*Transfer control to statement* L
halt	*Stop execution*

Write the following program in this language using the combination of statements above:
Given a and b, compute $x = a * b$. [3]

(b) Give an example of an operation in C that has a side-effect and is self modifying too. [2]

(c) Explain the meaning of static (i.e. compile-time) and dynamic (i.e. run-time) type checking. Compare the advantages and disadvantages of these two approaches to type checking from the point of view of the language implementer, and the programmer. [2 + 2]

2. (a) Suppose the declaration of a vector V is given using an enumeration type as the subscript range. For example, the following is a typical declaration of a vector in Pascal.

```
Class=(Fresh, Soph, Junior, Senior, Graduate);
V: array[Class] of real;
```

Show the storage representation (including descriptor) that would be appropriate for V and give the accessing formula for computing the location of a component $V[i]$. [3 + 2]

(b) Briefly discuss the appropriate storage representation for an *unordered set* for the cases where the size of the underlying universe of values that may appear in the set data object is known to be large. [2]

(c) What is binding time? What is the trade-off involved with early versus late binding? [1 + 1]

3. (a) What is garbage collection? [2]

(b) Discuss the comparative advantages/disadvantages of *reference counts* and *mark-and-sweep* collection as a means of garbage collection. [4]

(c) If a language has garbage collection, must it necessarily be compiled or can it be interpreted? Explain briefly. [2]

(d) Give an example of an overloaded operator in a language you are familiar with. [1]

4. (a) What is a dangling reference? Are the following C statements creating any dangling reference? What about garbages? Explain briefly. [2 + 2 + 2]

```
int *p,*q;
p=malloc(sizeof(int));
q=p;
free(p);
```

(b) What are the dangers in allowing memory deallocation to be under the direct control of the programmer? [2]

(c) What does type equivalence mean? [1]

5. (a) What distinguishes declarative languages from imperative languages? [2]

(b) Differentiate records from variant records with suitable examples. [2]

(c) In order to remove the overhead of a function call, a programmer decides to replace all calls to a function f with the macro F, where f and F are defined as follows:

```
int f(int x)
{
return (x+x);
}
#define F(X) (X)+(X)
```

Give two valid C expressions involving f which produce different results when F is substituted for f. Justify your answer. [2]

(d) Draw the runtime stack, showing the activation records that would be pushed as a result of the call factorial(3) (using the recursive version of factorial as shown below). Just show the value of N in each activation record (don't worry about the return address). Also indicate the value that is returned as the result of each call. [3]

```
int factorial(int N)
{
if (N == 0)
  return (1);
else
  return (N*factorial(N-1));
}
```

INDIAN STATISTICAL INSTITUTE
First Semestral Examination: 2011-12 (First Semester)

Course Name: M.Tech. in Computer Science
Subject Name: Computer Organization

Date: 14. 11. 2011 *Maximum Marks: 80* *Duration: 3 hours*

Instructions:

*Answer **all** questions from Part A and any **four** from Part B.*

Part-A: *Answer all questions*

1. Each question below carries 2 marks. More than one option may be correct. You will get full credit only by choosing all the correct answers. Clearly write the answers chosen by you on the answer sheet. 2 X 5 = 10

   a) The number of cycles required to complete n tasks in a k stage pipeline is
      i. k + n -1
      ii. nk + 1
      iii. k
      iv. None of these

   b) Register Indirect Addressing mode is useful for
      i. Procedure Returns
      ii. Case / switch statements
      iii. Virtual functions
      iv. All of the above

   c) A 16 bit address bus implies an address space of
      i. 64K
      ii. 16K
      iii. 8K
      iv. None of the above

   d) Consider the following snippet of C code:

```
for (i = 0; i < 20; i++)
    for (j = 0; j < 10; j++)
        a[i] = a[i] * j;
```

   The code above exhibits:
      i. Spatial Locality
      ii. Temporal Locality
      iii. None of the above

   e) A byte addressable computer has memory capacity of 2^m KB and can perform 2^n operations. An instruction involving 3 operands and one operator needs a maximum of
      i. 3m bits
      ii. 3m + n bits
      iii. m + n bits
      iv. 3n + m bits

2. True / False questions — 2 X 5 = 10

For each of the following statements, state which are true and which are false. Provide a one-line explanation wherever needed.

a) Pentium and IBM 370 are both big endian machines.

b) In twos complement arithmetic, if the exclusive-OR of the carry bits into and out of the leftmost column is 1, then there is an overflow condition, otherwise, there is no overflow.

c) The wider the data bus, the greater the number of bits transferred at one time.

d) A micro-operation consists of one or more instructions.

e) An arithmetic left shift always produces the same result as logical left shift on an unsigned operand.

Part-B: *Answer any four questions:*

1. **[6 + 4 + 5]= 15**

A. Consider the pipelined execution of these instructions for the MIPS 5-stage pipeline.

```
DADD    R1, R2, R3
LD      R4, 0 (R1)
SD      R4, 12 (R1)
OR      R8, R1, R4
XOR     R10, R1, R11
```

Explain how the above execution may generate data hazards. Can all the data hazards be handled using data forwarding?

B. Explain the concept of structural hazards in a pipeline and the associated resolution mechanisms.

C. Compare zero, one, two and three address machine codes in terms of number of instructions for the computation below:

$$X = (A + B \times C) / (D - E + F)$$

for each of the four machines. The instructions available for use are as follows:

0 address	1 address	2 address	3 address
PUSH M POP M ADD MUL SUB DIV	LOAD M STORE M ADD M MUL M SUB M DIV M	MOVE (X ← Y) ADD (X ← X + Y) SUB (X ← X – Y) MUL (X ← X * Y) DIV (X ← X / Y)	MOVE (X ← Y) ADD (X ← X + Z) SUB (X ← X – Z) MUL (X ← Y * Z) DIV (X ← Y / Z)

2. **[6 + 5 + 2 + 2]=15**

A. For the following data structure, draw the big-endian and little-endian layouts. The value of each variable is written as a comment beside the variable. For example, the initial value of **a** is 0x**1112**. Note that there is no initial value for **c**.

```
struct {
    short a;    //0x1112
    int b;      //0x11121314
    int c;
    char *d;    //0x31323334
    int *e;     //0x41424344
    char f[7]  // 'A', 'B', 'C', 'D', 'E', 'F', 'G', 'H'
} s1;
```

B. An address field in an instruction contains decimal value 14. Where is the corresponding operand located for

- Immediate addressing
- Direct Addressing
- Indirect Addressing
- Register Addressing
- Register Indirect Addressing

C. Suppose a stack is to be used by a processor to manage procedure calls and returns. Can the program counter be eliminated by using the top of the stack as a program counter?

D. Is there any possible justification for an instruction with two opcodes?

3. **[6 + 5 + 4]=15**

A. Consider a library of books as a cache system with practically infinite levels of hierarchy. Smaller libraries can borrow books from larger libraries, which can buy them from stores. Similarly, smaller stores can buy books from larger stores. Assume that the time to borrow a book from the nearest library is one unit and the probability of finding a book there is **h**. Given that a larger library or store serves the demands of more users and at the same time have a bigger stock, we assume that the probability of finding a book in the local stock is **h** for all libraries and stores. We also assume that these libraries and stores process media electronically and the access time between any two consecutive levels remains one unit. What will be the average access time for retrieving a book from the nearest library?

B. A processor runs a variety of programs, some with heavily clustered data localities in the memory and others with highly fragmented data. Assuming that the cache size is fixed, suggest a miss-guided dynamic scheme for the cache that will attempt to keep the hit rate high.

C. How does SDRAM differ from ordinary DRAM?

4. **[6 + 9]=15**

A. Explain the concept of dedicated and multiplexed bus types. Contrast the centralized and distributed bus arbitration approaches.

B. Write the sequence of micro-operations required for a CPU with a standard internal bus structure to add a number to the accumulator AC when the number is:

- An immediate operand
- A Direct-address operand
- An Indirect-address operand

5. **[3 + 6 + 6]=15**

A. Represent the following decimal numbers in both binary sign-magnitude and twos complement form using 16 bits:

a. +401
b. -14

B. Use the Booth algorithm to multiply 25 (multiplicand) by -13 (multiplier), where each number is represented using 16 bits.

C. Divide -119 by 11 in binary twos complement notation, using 12-bit words.

INDIAN STATISTICAL INSTITUTE

SEMESTRAL-I EXAMINATION (2011-12)
M.TECH.(CS) I YEAR

ELEMENTS OF ALGEBRAIC STRUCTURES

Date: 14.11.2011 Maximum marks: 100 Duration: 3 hours

The paper contains 120 marks. Answer as much as you can, the maximum you can score is 100.

1. (a) Let a and b be positive integers and d be their greatest common divisor. Show that there are integers x and y such that $d = ax + by$.

   (b) Let G be a group and define

   $$Z = \{z \in G : zx = xz \text{ for all } x \in G\}.$$

   i. Show that Z is a normal subgroup of G.

   ii. If T is any automorphism of G show that $T(Z) \subset Z$.

   $(8 + 5 + 7 = 20)$

2. (a) If D is an integral domain and if $na = 0$ for some $a \neq 0$ in D and some integer $n \neq 0$ prove that D is of finite characteristic.

   (b) Prove that any field is an integral domain.

   (c) Prove that any homomorphism of a field is either injective or takes each element to 0.

   (d) Let U, V be ideals of a ring R and UV be the set of all elements that can be written as finite sum of elements of the form uv where $u \in U$ and $v \in V$. Prove that UV is an ideal of R.

   (e) Let F and K be two fields with $F \subset K$ and suppose that $f(x), g(x) \in F[x]$ are co-prime in $F[x]$. Prove that they are co-prime in $K[x]$.

   (f) Prove that when F is a field, $F[x_1, x_2]$ is not a principal ideal ring.

   $(7 + 5 + 5 + 7 + 6 + 5 = 35)$

3. (a) Prove that the kernel of a vector space homomorphism is a subspace.

   (b) If A and B are subspaces of V, prove that $(A + B)/B$ is isomorphic to $A/(A \cap B)$.

   (c) If T is an injective homomorphism of a vector space V *onto* a vector space W, prove that T maps a basis of V onto a basis of W.

   (d) Let F be a field. Is $F[x]$ a vector space over F? If so, is it finite dimensional? Justify.

(e) Let V be a finite dimensional inner product space and $B = \{w_1, \ldots, w_m\}$ is an orthonormal set in V such that

$$\sum_{i=1}^{m} |\langle w_i, v \rangle|^2 = \|v\|^2$$

for every $v \in V$. Prove that B is a basis of V.

$(5 + 8 + 7 + 5 + 10 = 35)$

4. (a) Find a polynomial of degree 3 which is irreducible over $GF(5)$.

   (b) Suppose that F is a finite extension of $GF(p)$ which contains all the zeros of the polynomial $x^{p^m} - x$. Show that these zeros form a field.

$(5 + 5 = 10)$

5. Let $F = GF(2)$ and $\tau(x)$ be an irreducible polynomial of degree d in $F[x]$.

   (a) Let K be an extension of F such that there is an $\alpha \in K$ satisfying $\tau(\alpha) = 0$. Show that the zeros of $\tau(x)$ are

   $$\alpha, \alpha^2, \alpha^{2^2}, \ldots, \alpha^{2^{d-1}}.$$

   (b) Show that $\tau(x) = \tau(1 + x)$ if and only if $\tau(x) | (x^{2^i} + x + 1)$ for some i.

$(7 + 13 = 20)$

Indian Statistical Institute
Semestral Examination: 2011-2012
(first year first semester)
Course Name: M. Tech in Computer Science
Subject Name: Programming Languages & Methodology

Date: 16-11-2011 Maximum Marks: 80 Duration: 3 hours

Instructions:

You **may** attempt **all** questions which carry a total of **90** marks. However, the maximum marks you can score is only **80**.

1. (a) Discuss the implementation of *recursive* subprogram control. [5]

   (b) How many asterisks in terms of k will be printed by the following C function if it is called by **count**(2^k)? Justify your answer. [3 + 2]

```
void count(int n)
{
 printf("*");
    if(n > 1)
    {
      count(n/2);
      count(n/2);
    }
}
```

   (c) What will be the return value of the following C function if it is called by $f(0, 1, 0, n)$? Justify your answer. [3 + 2]

```
int f(int i, int j, int k, int m)
{
    if(k == m)
      return(j);
    else
      return(f(j, i+j, k+1, m));
}
```

2. (a) What are *coroutines*? In what way are the *coroutines* different from conventional subprograms? [2 + 2]

   (b) Suppose a language allows initial values to be specified for local variable declaration. Discuss the implementation strategies that such an initialization might take in the cases where (i) local variables are *retained* between calls and (ii) local variables are *deleted* between calls. [3 + 3]

   (c) What is *polymorphism*? What makes a polymorphic object different from other objects? Can the following two C++ functions having the same name be considered as *overloaded*?

```
int f(int a);
float f(int a);
```

   [2 + 2 + 1]

3. (a) What is *dynamic scope rule*? What will be the output of the following program under *dynamic scope rule*? [2 + 2]

```
Program dynamic_scope;
var n : char;
    procedure W();
    begin
      write(n);
    end;
    procedure D();
    var n : char;
    begin
      n := 'X';
      W();
    end;
begin
  n := 'Y';
  W();
  D();
end.
```

(b) What are the differences between the *eager evaluation* and *lazy evaluation* methods for evaluating an expression? Provide a *Boolean expression* that will produce different results under *lazy* and *eager* evaluation. [3 + 2]

(c) Define *abstract data type*. Discuss the relative advantages and disadvantages of *direct encapsulation* and *indirect encapsulation* models for abstract data object. [2 + 4]

4. (a) Suppose that the *call by name* method is used to pass parameters to the procedure *swap* below.

```
procedure swap(var x, y : integer);
    procedure g() : integer;
    var z : integer;
    begin
      z := x; x := y; return z;
    end;
begin
  y := g();
end.
```

Consider that an assignment $E_1 := E_2$ is implemented as follows:

compute the location of E_1;
compute the value of E_1;
place the value of E_2 into the location of E_1;

Will the call *swap*(i, $A[i]$) be able to exchange the values of i and $A[i]$? Justify your answer. [2 + 2]

(b) Define *exception*, *exception handler*, and "*raising an exception*". Give the names of two different places where the control can be transferred back after processing of an exception is finished. [3 + 2]

(c) What will be the output of the following program if *call by reference* method is used to pass parameters to the procedure f? Will there be any change in

the output if *call by value result* is used instead of *call by reference* to pass parameters to the procedure f? Justify your answer. [2 + 2 + 2]

```
Program test;
var i, j : integer;
    procedure f(var x, y : integer);
    begin
      i := y;
    end;
begin
 i := 2; j := 3;
 f(i, j);
 write(i, j);
end.
```

5. (a) Outline the key features that an "object-oriented" language must have. Discuss to what extent the programming language C++ has these features. [4 + 3]

   (b) What is a *virtual function*? What is the purpose of having a *null virtual function* in C++. [2 + 2]

   (c) What will be the output of the following C++ program? Justify your answer. [2 + 2]

```
class Base {
public:
    virtual char f() { return 'X';}
    char g() { return 'X';}
    char testF() { return f();}
    char testG() { return g();}
};
class Derived : public Base {
public:
    char f() { return 'Y';}
    char g() { return 'Y';}
};

int main()
{
  Base B;
  Derived D;
  printf("%c %c",B.testF(), B.testG());
  printf("%c %c",D.testF(), D.testG());
  return 0;
}
```

6. (a) Outline four key concepts that must be addressed in order to consider parallelism in a programming language. Suppose a programming language provides the following "**and** construct" for allowing parallel execution:

$$statement_1 \textbf{ and } statement_2 \textbf{ and } \cdots \textbf{ and } statement_n$$

The "**and** construct" has the semantics that each of the various $statement_i$ execute in parallel; the statement following the statement containing the "**and** construct" does not begin until all the parallel statements terminate. Discuss one possible implementation of such an "**and** construct". What are the possible values of y that may be printed on execution of the following program statements, depending on different order of execution of the statements in the "**and** construct"? [4 + 4 + 3]

```
x := 1;
x := 2 and y := x+x;
print(y);
```

(b) What does the following C function do if *unsigned int* uses 32-bit representation? [2]

```
int fun(unsigned int n)
{
    return (((n | (~n + 1)) >> 31) & 1);
}
```

(c) What will be the output of the following C program? [2]

```
int main()
{
    int i = 0, n = 0;
    while (i <= 6)
    {
        i++;
        if (i == 2)
            continue;
        n = n + i;
        printf("%d",n);
        if(n >= 12)
            break;
    }

return(0);
}
```

Indian Statistical Institute

First Semester Examination (2011-2012)

M.Tech. (CS) First Year

Discrete Mathematics

Date: November 18, 2011 Maximum Marks: 100 Time: 3.5 hours

Answer any five questions. Marks alloted to each question are indicated within square brackets near the right margin. You are allowed to carry with you TWO A4-sized reference sheets with your name and roll no. written on them. These are not to be shared and to be submitted with your answerscripts at the end of this exam.

1. (a) The king of Bombagarh had 4 sons, 10 of his male descendants had 3 sons each, 15 had 2 sons each, and all others died childless. How many male descendants did the king have?

   (b) Show that the block graph of any connected graph is a tree.

   (c) For a graph $G = (V, E)$ and a positive integer $k \geq 1$; $G^{(k)}$ is defined as the graph with vertex set $V(G)$ and two distinct vertices have an edge between them if and only if their distance in G is at most k. Prove that for any tree T, the graph $T^{(3)}$ has a Hamiltonian cycle.

   [5+7+8=20]

2. (a) Let G be a graph with minimum degree $\delta \geq 3$ and girth $g \geq 3$,

   (i) Prove that if g is odd, then G has at least $1 + \frac{\delta}{\delta-2}((\delta-1)^{(g-1)/2} - 1)$ vertices.

   (ii) Further, if the graph G above is also δ-regular, then what is its diameter?

   (b) For the wheel graph W_n, determine the number of fundamental cut-sets.

   [10+5+5=20]

3. (a) Let G be a graph with vertices a and b, and let $X \subset V - \{a, b\}$ be an a-b separator in G. Show that X is minimal as an a-b separator if and only if every vertex in X has a neighbour in the component C_a of $G - X$ containing a, and another in the component C_b of $G - X$ containing b.

   (b) Show that the edges of K_4 can be partitioned into exactly two edge-disjoint spanning trees. Generalize that the edges of K_n, where n is even, can be partitioned into exactly $n/2$ edge-disjoint spanning trees.

   [10+(4+6)=20]

(P.T.O.)

4. (a) Prove that every outerplanar graph is 3-colorable.

   (b) Derive the chromatic polynomial for the graph $G = K_3 \cup K_1$.

   (c) Let the vertices of the cycle graph C_{14} be labeled as $0, 1, 2, \ldots, 13$. Consider the graph H, called *Heawood graph*, obtained from C_{14} by adding the edges $(i, i+5)$ for $i = 0, 2, 4, 6, 8, 10, 12$. Prove that H is bipartite.

   [6+4+10=20]

5. (a) What should be the criterion for which a digraph has an Eulerian directed tour? Justify your answer.

   (b) Define (i) a vertex cover, and (ii) an edge cover of a graph. Can their cardinalities be same? Give an example or a counterexample.

   (c) Prove that if a digraph is a tournament, then $\Sigma_{v \in V}(d_{in}(v))^2 = \Sigma_{v \in V}(d_{out}(v))^2$.

   [6+(4+4)+6=20]

6. (a) A uniform rod of length $1m$, is to be decorated by dividing its surface into five $20cm$ bands and coloring each band red or blue. In how many distinct ways can this be done?

   (b) Two switching functions f and g of three Boolean variables are considered equivalent if $f(x_1, x_2, x_3) = g(\pi(x_1, x_2, x_3))$, where the permutation π on a bit string of length 3 complements a string by interchanging 0 and 1. For example, $\pi(x_1, x_2, x_3) = (\bar{x_1}, \bar{x_2}, \bar{x_3})$.

   (i) Derive the cycle index of the group consisting of identity and permutation π above.

   (ii) Find the number of distinct three-variable switching functions that have exactly four 1s in the range.

   [8+(6+6)=20]

7. (a) Write any one of the following symbolically as a formulae:

   (i) The handshaking lemma for undirected graphs;

   (ii) The Ramsey number $R(3, 3) = 6$.

   (b) Express the following argument as a propositional formulae and establish its validity by the tableau method:

   "If it has snowed, it will be poor driving. If it is poor driving, I will be late unless I start early. Indeed, it has snowed. Therefore, I must start early to avoid being late."

   (c) Briefly state the implications of soundness, completeness and compactness of a proof procedure.

   (d) Let $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ and $\mathcal{N}$ be the model with universe $\mathbb{N}$ in which the predicate symbols $=, \leq, <$ and the expression $x + y = z$, all have their usual meanings. Establish whether the following are true:

   (a) $\exists y \forall x (y < x)$

   (b) $\forall x \exists y (x < y \rightarrow \exists y 3 < y)$.

   [5+6+3+(3+3)=20]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech (CS) - I Year (Semester - I)

Optimization Technique

Date 24.11.11 [redacted]

Maximum Marks : 100 Duration : 4 Hours

Note : You may answer any part of any question, but the maximum you can score is 100.

Answer GROUP A and GROUP B in separate sheets

GROUP A

1. Consider the following LPP.

$$\begin{aligned} \text{Minimize} \quad & x_1 + x_2 + x_3 + x_4 + x_5 \\ \text{subject to} \quad & 3x_1 + 2x_2 + x_3 = 1 \\ & 5x_1 + x_2 + x_3 + x_4 = 3 \\ & 2x_1 + 5x_2 + x_3 + x_5 = 4 \\ & x_i \geq 0, i = 1, 2, \ldots, 5 \end{aligned}$$

   a) Solve the above problem using simplex algorithm.

   b) Write down the dual of the above problem.

   c) Using complementary slackness conditions, solve the dual problem.

[8+4+8=20]

2. Prove that the North-West corner rule gives a basic feasible solution to any transportation problem. [15]

GROUP B

3. Consider the problem:

$$\begin{aligned} \text{Maximize} \quad & x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 8 \\ & -x_1 + x_2 \leq 2 \\ & x_1 - x_2 \leq 4 \\ & x_2 \geq 0 \text{ and integer, } x_1 = 0, 1, 4, \text{ or } 6 \end{aligned}$$

   Reformulate the problem as an equivalent integer linear program.

[15]

[P.T.O.]

4. Consider the following integer programming problem:

$$\begin{aligned} \text{Maximize} \quad & x_1 + 5x_2 \\ \text{subject to} \quad & -4x_1 + 3x_2 \leq 6 \\ & 3x_1 + 2x_2 \leq 18 \\ & x_1, x_2 \geq 0 \text{ and integer.} \end{aligned}$$

   a) Derive the LP relaxation of the above problem.

   b) Generate the optimal tableau for the relaxed LP.

   c) Solve the above integer programming problem using the branch-and-bound technique. You may graphically solve each linear programming problem that is encountered.

   d) Derive cuts from each of the rows in the optimal linear programming tableau.

[4+8+12+6=30]

5. Consider a flow network represented by a directed graph $G = (V, E)$ where each edge e is associated with a capacity c_e, and the vertices s and t represent source and sink nodes respectively. If all capacities in the flow network G are integers, prove that the *Ford-Fulkerson Algorithm* can be implemented to run in $O(mC)$ time, where m represents the cardinality of set E and C is the sum of capacities of all the edges coming out from s.

[10]

6. Prove that a matching M in a graph G is maximum if and only if there is no augmenting path in G with respect to M. [15]

7. Define the undirected chinese postman problem. Present a high-level description of an algorithm generating the optimum path for the postman. [4+8=12]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech (CS) - I Year (Semester - I)

Data and File Structures

Date 23.11.11 Maximum Marks : 100 Duration : 3 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1. Recall that a multiway tree T can be represented as a binary tree B by using the *firstChild* and *nextSibling* pointers. Here *firstChild* pointer of a node v in B points to the first child of node representing v in tree T and *nextSibling* pointer of node v in B points to a sibling of node representing v in tree T. For any node in B, the left link represents firstChild and the right link represents nextSibling. Inorder traversal of tree T can be defined as inorder traversal of subtree pointed by leftmost link of root, followed by the visit of the root, followed by inorder traversal of subtrees pointed by remaining links of root node of T in left to right order. Similarly, we can define preorder and postorder traversal analogus to the preorder and postorder traversal of binary tree.

   Justify the statement "*an inorder traversal of B is equivalent to preorder or postorder or inorder traversal of T*" with brief explanation or present a counter example. [20]

2. Suppose that the keys $\{1, 2, \ldots, n\}$ are inserted into an empty AVL tree in the sequence $1, 2, \ldots, n$. Find the key values where the rotation is not required while inserting them. Justify your answer. [20]

3. Recall that in B-tree of minimum degree t, each non-leaf node other than root has at least t children and at most $2t$ children. Suppose that the keys $\{1, 2, \ldots, n\}$ are inserted into an empty B-tree with minimum degree 2 in the sequence $1, 2, \ldots, n$. How many nodes does the final B-tree has? Justify your answer. [20]

4. What is the largest possible number of internal nodes in a *red black* tree with *black height* k? What is the smallest possible number? [10+5]

5. Prove that for any α, $1/3 < \alpha \leq 1/2$, $WB[\alpha] = WB[1/2]$, where $WB[\alpha]$ is the set of binary trees with weight balance α. [15]

[P.T.O]

6. Design a data structure and a search algorithm to solve the following problem. It should be able to answer the query in $O(\log^2 n)$ time. A high-level description of your data structure, the search algorithm and its running time are sufficient.

You have to store a set of n vertical segments, where each segment extends from a point $p_i = (x_i, y_i)$ down to the x-axis. You may assume that all coordinates are positive. The query consists of a single point $q = (q_x, q_y)$. Imagine that you shoot a bullet horizontally to the left from q. Return the *first* vertical line segment in the data structure that is hit by this bullet path. If no segment is hit, then return null.

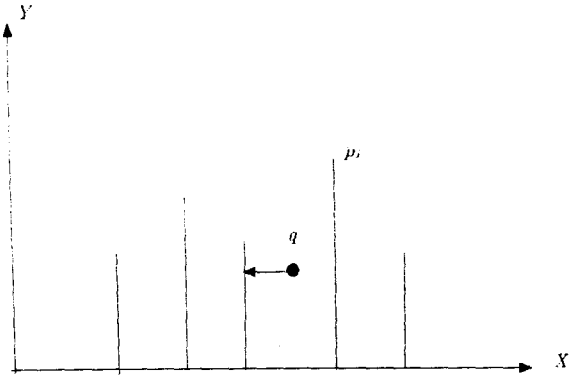


Figure 1: xxx

[20]

INDIAN STATISTICAL INSTITUTE
Semestral Examination (2011-2012)

M.Tech. (CS) – First year

Switching Circuit and Logic Design

Maximum marks: 100

Date: 25.11.11 **Time : 3 hours**

Answer any four questions

1a).A logic circuit has five inputs : $X_1\ X_2\ X_3\ X_4\ X_5$. Output Z_0 is to be 1 when the majority of the inputs are 1. Output Z_1 is to be 1 when fewer than four of the inputs are 1,provided at least one of the inputs is 1. Output Z_2 is to be 1 when two, three or four of the inputs are 1. Design a minimal sum-of- products realization of the described circuit.

b)Let $F=\sum(5,6,13)$ and $G=\sum(0,1,2,3,5,6,8,9,10,11,13)$;
Find H such that $F = G \cdot H'$. [15 + 10]

2. The state-table for a sequential machine is given below.

Present state	Next state,Output X=0	x=1
A	A, 1	B, 0
B	C, 0	A, 0
C	B, 0	C, 1

a)Find a homing sequence for the above sequential machine.

b)Check if this machine has a distinguishing sequence with the help of a successor tree.

c) The circuit is initially provided with an input sequence **01** to which it responds by producing an output sequence **10**. It is next provided with the input sequence **101010101001.**. Find the correct output sequence for this input sequence.

[5 + 10 + 10]

3.a)Why internal variables assignment for fundamental-mode sequential circuits is difficult? What are critical races and cycles? [10]

b) Determine all the races in the following state table. Indicate which of them are critical and which are not. Find a race free assignment for the given state table.

[5 + 2 + 8]

P.T.O.

	Next State			
	Input Combinations			
Present State	00	01	11	10
00	00	01	11	11
01	01	01	11	10
10	00	10	11	10
11	11	10	11	11

4. For an incompletely specified machine, whose state table is shown below, determine a minimum state reduced machine for the given state table. Draw the merger table. State if this minimization is unique. [12 + 8 + 5]

Present state	Next state,Output X=0	x=1
A	E,1	F,-
B	D,0	-,-
C	A,-	E,1
D	F,1	E,0
E	C,-	A,1
F	B,0	D,-

5. i) Determine a minimum-row state table for a clock mode sequence detector with one input and one output line, which produces an output **1** for every fourth **1** input (not necessarily consecutive).

ii) Design the above circuit using **D** flip-flop. [15 + 10]

INDIAN STATISTICAL INSTITUTE

First Semestral Examination : (2011-2012)

M. Tech (CS) 1st Year

Probability & Stochastic Processes

Date: 25. 11. 11 Maximum marks: 100 Time: 3 hours.

Note: Answer all questions. Maximum you can score is 100.
You may use any result proved in class after stating the results clearly in the proper place.
Notations are as used in class.

1. Customers arrive at a computer store in a given day following a Poisson distribution with paramter λ, and purchases a computer with probability p independently of other customers. Find the probability

   (a) that the store sells no computer in a given day.

   (b) that the store sells exactly k computers in a given day. [10+ 10]

2. Suppose X and Y are independent exp(λ) random variables, $W = \min(X, Y)$, and $Z = \max(X, Y)$.

   (a) Find $E(W)$ and $V(W)$. [5]

   (b) Find $E(Z)$ and $V(Z)$. [10]

3. Consider a discrete time Markov Chain with state space $\mathcal{S}$ and transition matrix P.

   Prove that $P_{ii}^{m+n} \leq 1 + P_{ii}^{m} * P_{ii}^{n} - max(P_{ii}^{m}, P_{ii}^{n})$. [10]

4. Consider a discrete time Markov Chain with state space $\mathcal{S} = \{0, 1, 2, 3, 4, 5\}$, and the transition matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{5} & \frac{2}{5} & \frac{2}{5} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

   (a) Determine which states are recurrent and which states are transient. [5]

   (b) Find the stationary distribution concentrated on each of the irreducible closed sets. [10]

5. Suppose in a branching chain, the probability that an individual produces k offsprings is given by the probability mass function $f(k)$, for $k \geq 0$.

   (a) If $f(1) < 1$, show that every state other than 0 is transient. [5]

   (b) If $f(k) = p(1-p)^k$, for $k \geq 1$ and $0 < p < 1$, then show that the extinction probability $\rho = 1$ if $p \geq \frac{1}{2}$, and $\rho = \frac{p}{1-p}$ if $p < \frac{1}{2}$. [10]

[P.T.O]

6. Customers arrive at a bank following a Poisson Process with rate λ.

   (a) If three customers arrive by the first hour, find the probability that

      i. all of them arrived within the first half an hour

      ii. at least one of them arrived within the first 15 minutes. [5 + 7]

   (b) If $N(t)$ denotes the number of customers arriving within time t, for $s < t$, show that the distribution of $(N(s)|N(t) = n)$ is Binomial$(n, \frac{s}{t})$. [8]

7. Suppose in a queueing process, customers arrive at a Possion rate λ, and are served at a Possion rate μ.

   (a) Given that a single server is currently serving a customer, find the probability that during his service period, at least two more customers will arrive. [5]

   (b) Suppose there are 5 servers who are all currently busy and no customer is waiting in the queue. Find the probability that the next customer upon arrival finds exactly 2 servers free. [10]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL-I BACK PAPER EXAMINATION (2011-12)
M.TECH.(CS) I YEAR

ELEMENTS OF ALGEBRAIC STRUCTURES

Date: 30.12.2011 Maximum marks: 100 Duration: 3 hours

The paper contains 100 marks. Each question carries 10 marks. Answer all questions.

1. Let m and n be positive integers such that $\gcd(m, n) = 1$. Show that given any two integers a and b, there exists an integer x such that $x \equiv a \bmod m$ and $x \equiv b \bmod n$.

2. Let S_3 be the group of all permutations of the set $\{a, b, c\}$. In S_3 show that there are two elements x and y such that $(x \cdot y)^2 \neq x^2 \cdot y^2$.

3. Let G be any group and g a fixed element of G. Define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that g is an isomorphism of G onto G.

4. Given an example of an integral domain which is of positive characteristic, but, has an infinite number of elements.

5. If R is a ring with identity 1 and ϕ is a homomorphism of R *onto* R', prove that $\phi(1)$ is the identity of R'.

6. Find the greatest common divisor of the following polynomials over the field of rational numbers.
$$x^2 + 1 \text{ and } x^6 + x^3 + x + 1.$$

7. Let F be the field of real numbers and let V be the set of all sequences $(a_1, a_2, \ldots, a_n, \ldots)$, $a_i \in F$, where equality, addition and scalar multiplication are defined componentwise. Prove that V is a vector space over F.

8. If F is the field of real numbers, prove that the vectors $(1, 1, 0, 0)$, $(0, 1, -1, 0)$ and $(0.0, 0.3)$ in F^4 are linearly independent over F.

9. Let V be an inner product space and $\{w_1, \ldots, w_m\}$ is an orthonormal set in V. Show that
$$\sum_{i=1}^{m} |\langle w_i, v \rangle| \leq \|v\|^2 \qquad \text{for any } v \in V.$$

10. Show that the polynomial $\tau(x) = x^3 + x + 1$ is irreducible over F_2, the field of two elements. Write down the elements of $F_2[x]/(\tau(x))$.

Indian Statistical Institute
Mid-semestral Examination: 2011-2012
(first year second semester)
Course Name: M. Tech in Computer Science
Subject Name: Computer Network

Date: 20-02-2012 Maximum Marks: 30 Duration: 2 hours

Instructions:

You **may** attempt **all** questions which carry a total of **36** marks. However, the maximum marks you can score is only **30**.

1. (a) Consider a multi-channel communication network where packets are arriving for channel i according to a Poisson distribution with a mean arrival rate of λ_i packets/sec. Let the probability density function for packet size in bits be $\mu e^{-\mu x}$ with a mean of $1/\mu$ bits/packet. Assume that the capacity of channel i is C_i bits/sec. Derive an expression for T_i in terms of μ, C_i and λ_i where T_i is the mean delay including both queueing and transmission time for channel i. [5]

   (b) Consider a link of rate $R = 1$ Mbps. Suppose packets of size $L = 1250$ bytes arrive according to Poisson distribution to the link at an average rate of 1 packet/second. The total delay is the queuing delay plus the transmission delay. Find the average queuing delay and average total delay. [2]

   (c) Two networks each provide reliable connection-oriented service. One of them offers a reliable byte stream and the other offers a reliable message stream. Are these identical? If so, why is the distinction made? If not, give an example of how they differ. [2]

   (d) What is E_b/N_o? Express E_b/N_o in terms of SNR. [1+2]

2. (a) What is the difference between circuit switching and packet switching? [2]

   (b) Consider sending a packet of F bits over a path of Q links. Each link transmits at R bps. Assume that propagation and queuing delays are negligible.

      i. Suppose the network is a packet-switched datagram network and a connection-oriented service is used. Suppose each packet has $h \times F$ bits of header where $0 < h < 1$. Assuming t_s setup time, how long does it take to send the packet? [3]

      ii. Suppose that the network is a circuit-switched network. Furthermore, suppose that the transmission rate of the circuit between source and destination is $R/24$ bps. Assuming t_s setup time and no bits of header appended to the packet, how long does it take to send the packet?[3]

   (c) Both Shannon and Nyquist theorems place an upper limit on the bit rate of a channel based on two different approaches. How are the two related? [2]

   (d) What are the advantages and disadvantages of a ring topology? [2]

3. (a) Briefly explain the principle of pulse code modulation (PCM). Compare the performance of PCM with delta modulation (DM) for voice signals. [2 + 2].

   (b) A signal is quantized using 8-bit PCM. The bit rate of the system is equal to 50×10^6 bits/sec. What is the maximum bandwidth for which the system operates satisfactorily? [3]

   (c) For the bit stream 00110100010, sketch the waveforms using BFSK and BPSK. [3]

   (d) Derive an expression for baud rate as a function of bit rate for multilevel PSK. [2]

Indian Statistical Institute

Semester-II 2011-2012

M.Tech.(CS) - First Year

Mid-term Examination (23 February, 2012)

Subject: Operating Systems

Total: 70 marks Maximum marks: 60 Duration: 2.5 hrs.

Please keep your answers brief and to the point.

1. (a) List and briefly describe (in 2-3 sentences each) the 7 major constituents of a process' context. Which of these items need to be *copied* when saving a process' context? Justify your answer.

   (b) Draw a clearly labelled diagram that shows how proc structures are linked to each other to form three data structures. (The purpose of the data structures and the relationship between the proc structures should be clear from your diagram.)

   (c) When a process P calls `exit()`, how are the parent and children of P affected?

   [(14+2)+10+6=32]

2. (a) Explain whether starvation can happen under the following scheduling strategies: (i) FCFS; (ii) non-preemptive priority-based scheduling. Clearly state any assumptions that you make.

   (b) What is the difference between turnaround time and response time?

   (c) Let P_c, P_i be two conventional processes that are computation-intensive and interactive, respectively. Using appropriate excerpts from the code, explain how the Linux 2.4 scheduler tries to give more importance to P_i than P_c.

   (d) With reference to the Linux 2.6 scheduler, what is the *base time quantum*? How is it calculated? What are the minimum and maximum values that the base time quantum can take for conventional processes?

   [8+2+5+(2+4+4)=25]

3. Consider a stack implemented as a linked list. The pop function is given below.

```
NODE *pop(STACK *s)
{ NODE *tmp;
  if (s->top == NULL) return NULL;
  tmp = s->top;
  s->top = tmp->next;
  tmp->next = NULL;
  return tmp;
}
```

   (a) Suppose two processes P_1 and P_2 share a stack. Show how inconsistent results may be obtained if the two processes execute the pop function concurrently on the shared stack.

(You should show the initial state of the stack, the intended result of the two **pop** operations, the interleaved execution of the two processes, and the actual results obtained.)

(b) Show how you would re-design **pop** to prevent such a possibility if the hardware provided support for an atomic **TestAndSet** operation.

[9+4=13]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination
M. Tech(CS) I year: 2011–2012
Software Engineering

Date: 24. 02. 2012 Marks : 50 Time : 2 Hours

Answer any part of any questions. The maximum marks you can get is 50. The question is of 55 marks. Please try to write all the part answers of a question at the same place.

1. In a university, students belonging to different courses can participate in two types of courses: a fixed number of credit courses and any number of audit courses. The marks obtained in the credit courses contribute to the overall grade of the student. The student must also score a minimum marks in each of the audit courses. The overall marks in a particular subject consists of the marks obtained in the Assignments, Mid-Term examination and Semester examination. The weightage of these three parts may vary depending on the subject.

   After the completion of the examinations, the concerned teacher submits the marks before a given deadline. The students can view their marks after each of the examinations. The final score is calculated by the system, given the weightage of the assignments, mid-term and semester examinations. The system then calculates the final marks and decides if the student has to take any supplementary examination. The system should be so designed that the students are allowed to view their marks by entering his/her name and a secret password.

   (a) Draw a DFD corresponding to the above problem.

   (b) Prepare an SRS for the same.

   (c) Provide a brief description of what kind of software, hardware and networking platforms are required to implement this project.

   [10 + 10 + 10 = 30]

2. (a) What is FILE in C?

   (b) An ASCII text file "aa.txt" contains "1 2" (there is a blank space between 1 and 2). What will be the output of the following program? Explain.

```
#include <stdio.h>
main(){
        int i;
        FILE *fp;
        fp = fopen("aa.txt", "r");
        while (!feof(fp)){
                fscanf(fp, "%d", &i);
                printf("%d\n", i);
        }
        fclose(fp);
}
```

   (c) Explain the "fread" and "fwrite" functions with examples.

   (5 + 10 + 10 = 25)

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2011-12(Second Semester)
Course Name: M.Tech. in Computer Science
Subject Name: Automata Languages and Computation

Date: 27. 02. 2012 *Maximum Marks: 60* *Duration: 2 hours 30 mins*

Instructions:

Answer all questions. All parts of a question must be answered in the same place. The maximum marks you can score is 60. The question paper consists of questions carrying total marks of 70.

---

1 Answer with short justifications (example or counterexample as the case maybe), whether the following **claims** are **true** or **false**. No credit will be given for answers without justification. Assume that the alphabet is $\Sigma = \{0, 1\}$ unless specified otherwise. [6 * 4 = 24]

(a) $A \subseteq \Sigma^*$ is said to *co-finite* if $\Sigma^* - A$ is *finite*.

Claim: The size of $2^{\Sigma^*}_{\text{co-finite}} = \{A \subseteq \Sigma^* : A \text{ is co-finite}\}$ is uncountably infinite.

(b) Let $L_{c1} \subseteq L_{c2} \subseteq \Sigma^*$.

Claim: If L_{c1} is not a regular set, then L_{c2} can never be a regular set.

(c) Let $L_d = \{1^n : n \leq 1000 \text{ and } n \text{ is a prime}\}$

Claim: A DFA accepting L_d may have less than 900 states.

(d) **Claim**: If we restrict the production rules of a CFG to the form $A \rightarrow \sigma B$ or $A \rightarrow B\sigma$ or $A \rightarrow \sigma$, where $\sigma \in \Sigma$ and A,B are non-terminals, then the generated language of the grammar is regular.

(e) **Claim**: The following grammar (**S** is the start symbol, **S** and **A** are non-terminals, while **a** and **b** are terminals) is not ambiguous

S -> SAS | a
A -> ASA | b

(f) **Claim**: If M is a NFA that recognizes a regular language G, then swapping the accept and non-accept states in M yields a new NFA that recognizes the complement of G.

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination, Semester II, 2011-12
M.Tech. (Computer Science)
Computer Architecture

Full Marks : 100 Date : 29. 02. 2012
Time : 3 Hours

Note : Answer all questions. Marks on each part of a question are indicated in the right margin within parentheses.

1. Consider a user program which consists of 10 Giga of machine language instructions. If this program is executed on a computer which has an average cycle per instruction (CPI) of 1.2 and a 1 GHz clock, then calculate the time needed to execute this program.

   Consider a user program which is compiled on a *Computer A*, generating an object code having 10 Giga instructions with a mix of 40% ALU instructions, 30% load-store instructions and 30% branch instructions. The same user program, when compiled on another *Computer B* (with a different architecture), was translated to an object code in which the number of ALU instructions was reduced by a factor of two, while the number of all other instructions remained the same as that in case of *Computer A* . Assume that on both these computers, the CPI values for ALU instructions, load-store instructions and branch instructions are 1, 3 and 2, respectively. Now compute the following :

   i) MIPS value for *Computer A*, ii) MIPS value for *Computer B*, iii) execution time of the program on *Computer A* and iv) execution time of the program on *Computer B*.

   What conclusion can you draw from these computed values? (4 + 3 + 4 + 3 + 4 + 2 = 20)

2. Consider a 5-stage pipelined execution of instructions on a given computer. Assume that this pipeline structure is static and linear. Output of Stage i (1 <= i <= 4) of this pipeline is fed to the input of Stage (i + 1) of the pipeline. If the execution time of the Stages 1, 2, 3, 4 and 5 are 10 ns, 5 ns, 7 ns, 16 ns and 4 ns, respectively, then calculate the maximum possible speed-up by this pipeline structure over the unpipelined execution of the instructions.

   If the above pipeline structure is made non-linear by providing two units of Stage 4 (which takes 16 ns to complete its execution) and suitably using them in the pipeline, then will there be any improvement in the speed-up factor? If yes, then by how much ? (4 + 6 = 10)

3. Consider an instruction pipeline with five phases as *instruction fetch, instruction decode/resister fetch, execute/address calculation, memory access* and *write back*, respectively. Referring to this pipeline structure , explain the terms 'Structural hazards' and 'Control hazards', if any, in executing the instructions on this architecture. (6+4 = 10)

   Is it possible to eliminate structural hazards altogether in the above pipeline structure? If so, then how? If not, then what are those cases. (6)

If both branch address computation and branch condition evaluation are performed in the *execute/address calculation* phase of the above pipeline structure, then how many stall cycles are needed after every branch instruction? Can it be improved by any means? If so, then how?

(2+2 = 4)

Show how control hazards can be eliminated by rescheduling of instructions from different parts of the user program. What are the limitations of this approach? (6 + 4 = 10)

4. Consider the execution of the following machine language instructions on the above pipelined architecture as mentioned in Q.3 :

ADD	R1, R2, R3 ;	R1 ← R2 + R3
SUB	R4, R1, R5 ;	R4 ← R1 – R5
MUL	R6, R7, R1 ;	R6 ← R7 * R1
OR	R8, R1, R9 ;	R8 ← R1 OR R9
DIV	R10, R11, R1;	R10 ← R11 / R1

Indicate the possible data hazards that may be encountered in executing the above code section. Briefly mention any possible means to eliminate such data hazards. (6 + 6 = 12)

5. Represent the numbers below in floating-point form, following IEEE 754-1985 single precision standard :

i) 250 ii) 0 iii) $(-3)^{1/2}$ iv) 18.75×2^{-135}

(4 x 3 = 12)

6. Describe how the product of two floating point normalized numbers can be generated in the rounded form , rounded to the nearest value. Can there be any situation that this product becomes a denormal? If so, then how that can be tackled ? (12 + 4 = 16)

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination

M. Tech (CS) - I Year (Semester - II)

Design and Analysis of Algorithms

Date : March 2, 2012 Maximum Marks : 60 Duration : 3:00 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. You are given two sorted lists of size m and n stored in two arrays. Give an $O(\log m + \log n)$ time algorithm for computing the kth smallest element in the union of the two lists. **15**

2. You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \ldots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the nal hotel (at distance a_n), which is your destination.

You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty.that is, the sum, over all travel days, of the daily penalties.
Give an efcient algorithm that determines the optimal sequence of hotels at which to stop.

15

3. A server has n customers waiting to be served. The service time required by each customer is known in advance: it is ti minutes for customer i. So if, for example, the customers are served in order of increasing i, then the ith customer has to wait $\sum_{j=1}^{i} t_j$ minutes.

We wish to minimize the total waiting time $T = \sum_{i=1}^{n}$ *(time spent waiting by customer i)*:

Give an efcient algorithm for computing the optimal order in which to process the customers.

15

4. An array $A[1 \ldots n]$ is said to have a *majority element* if more than half of its entries are the same. Given an array, the task is to design an efcient algorithm to tell whether the array has a majority element, and, if so, find that element. The elements of the array are not necessarily from some ordered domain like the integers, and so there can be no comparisons of the form *"is* $A[i] > A[j]$*?"*. (Think of the array elements as GIF files, say.) However you *can* answer questions of the form: "*is* $A[i] = A[j]$*?*" in constant time.

15

5. For two problems *pr1* and *pr2*, $pr1 <<<_{f(n)} pr2$ means that every instance of *pr1* of size n can be solved using a constant number of instances of *pr2*, plus $O(f(n))$ additional time.

Show that, given two sorted arrays X and Y containing n elements each, finding sorted sequence of the elements of $X+Y = \{x+y \mid x \varepsilon X, y \varepsilon Y\}$ is 3SUM hard. **15**

INDIAN STATISTICAL INSTITUTE
Semestral Examination
M.Tech.(Computer Science)
First Year Semester II, 2011-12
Computer Architechture

Date: 23.04.2012
Time: 3 hours **Maximum Marks-100**

Note: Answer any five questions. Marks on different parts of a question are shown in the right margin within parentheses. Precise answers will fetch more credit.

1. a) Consider the following assemly language code for a certain instruction-pipelined computer system where LW stands for 'load word', SW stands for 'store word', ADD stands for 'addition' and SUB stands for 'subtraction' operations.

```
LW    R1,0(R2)     /*load word from memory location given by R2 in register R1*/
LW    R3, 0(R4)
ADD   R5,R1,R3     /* R5<----R1 + R3 */
SW    R5,0(R6)     /* store R5 in the memory location given by R6 */
LW    R7, 0(R8)
LW    R9, 0(R10)
SUB   R11,R7,R9
SW    R11,0(R12)
```

Considering data hazards, how many clock cycles will be needed to execute the above code? Can it ever be possible to reduce the number of cycles for completing the execution of the above assembly language code by rescheduling the instructions? If so, then how, and what will be the resulting number of cycles needed to execute the rescheduled code segment? **(2+6+2=10)**

b) Consider the following program segment in some high level language:

```
for (j = 0; j<101; j++) {
        a[j]= b[j] + c[j];          /*S1*/
        b[j+1]= a[j] + b[j] + d[j]; /*S2*/
        a[j+1]= b[j] + e[j];        /*S3*/
  }
```

Are there any loop carried dependencies in the above program segment? If so, find those out. Rewrite the above program segment to extract some kind of parallelism, if it exists, in executing the instructions of the program. Assume that you will be provided with multiple ALUs for such a situation. **(5+5=10)**

2. Consider the following assembly language program segment for a loop executed on an instruction-pipelined computer, where LD stands for load double word (8 bytes) instruction, ADD stands for addition of two double word floating point numbers, SD stands for a double word store instruction, SUBI stands for subtract an immediate operand from a register and BNEZ stands for branch on not equal to zero. Assume that each data element is stored in the memory as a double word.

```
Loop:   LD      F0, 0(R1)
        ADD     F4, F0, F2
        SD      0(R1),F4
        SUBI    R1,R1,#8
        BNEZ    R1,Loop
```

Assume that there is a stall of one cycle needed after the LD instruction and a stall of two cycles following the ADD instruction (as the data computed is required in the immediate next instruction). Also, a stall of one cycle is needed after the SUBI instruction as the BNEZ instruction computes the branch address and the condition for branching in the ID phase.

How many cylces per data element are needed for the above program segment and how can it be reduced by rescheduling the instructions? Give at laest one specific example for such a rescheduleed code segment with the corresponding number of cycles needed per data element.

Can it ever be possible to reduce the number of cylces needed per data element to i) exactly 3, ii) a value less than 4? Justify your answer with proper illustrative examples and appropriate reasoning in each case. **(3+5 +12=20)**

3. a) Explain how the branch penalty can be reduced by increasing the accuracy of branch prediction in loops of a program. How many prediction bits could be used for such prediction and why?

b) Consider the following high level program segment:

```
if(d==0)
      d=1;
if(d==1) {
...
};
```

The typical assembly language code segment corresponding to the above program segment may look as follows, assuming that register R1 stores the initial value of *d* :

```
        BNEZ    R1,L1       ; branch to L1 (d !=0)
        ADDI    R1,R0,#1    ; d==0, so d=1

L1:     SUBI    R3,R1,#1
        BNEZ    R3,L2       ; branch to L2 (d !=1)

...

L2:
```

Assume that *d* repeatedly assumes the values 0 and 2 alternately , i.e., *d* assumes the values 0, 2, 0, 2, 0, 2, . . .

Show that one prediction bit per loop does not work satisfactorily for accurate prediction of branches in this situation. Hence, explain how one correlation bit and one prediction bit can improve the accuracy of branch prediction for this specific example situation.

(8+12=20)

4. a) Explain the disadvantages of static re-scheduling of codes by compilers in order to avoid pipeline stall cycles. Can these disadvantages be removed by dynamic scheduling? Illustratre your answer by a suitable example.

b) Describe the basic features of Scoreboarding technique for dynamic scheduling with specific discussions on the following items :
i) dealing with structural hazards, ii) dealing with RAW hazards, WAR hazards and WAW hazards, iii) pipeline stages and their broad functions, iv) different information to be kept for controlling the total operation, and v) the required control logic. **(4+16=20)**

5. a) Consider a program of multiplying two $n \times n$ matrices Y and Z. Assuming that we have a cache memory just large enough to hold only $(n^2 + n)$ elements of the matrices, compute the total number of cache misses encountered thereof.

Now, assume that the cache size is further reduced, just large enough to hold B^2+2B matrix elements $(B<n)$. Under this scenario, rewrite your code for matrix multiplication (using any high level language) and then re-compute the total number of cache misses involved in executing your code. **(3+8+4=15)**

b). Consider a computer having 16KB instruction cache with a miss rate of 0.60% and another 16KB data cache with a miss rate of 6.40%. Assuming that about 75% of the memory accesses are for fetching the instructions, calculate the effective cache miss rate for this system.

If, instead of such a split cache, one unified-cache of size 32KB were considered for both instructions and data , then the cache miss rate were observed to be only 2.00%. Assuming that one cycle is spent on resolving the structural hazards on this unified cache (with instruction being fetched in first cycle and data being accessed in the second cycle), calculate the overall effective memory access time for both the split-cache system and the unified-cache system.
(1+4=5)

6. a) What are the basic pipeline stages of a GPU? Explain briefly the functionality of each stage in the pipeline. **(3+10=13)**

b) Indicate the different kinds of registers used in GPU and write down their functionality in the pipeline. How many cores are there in the Tesla Fermi GPU? **(2+4+1=7)**

INDIAN STATISTICAL INSTITUTE
Semestral Examination
M. Tech(CS) I year: 2011–2012
Software Engineering

Date: 27. 04. 2012 Marks: 100 Time: 3 Hours

Answer any 4 questions. Please try to write all the part answers of a question at the same place.

1. Consider a scenario where Indian Statistical Institute wishes to completely revamp the institute web site and replace it with a fully featured portal.

   (a) List the different tools that may be used during this project mentioning the purpose of using the tools and the project task in which the tool would be used.

   (b) What would be the deliverables of this project?

   [15+10=25]

2. Consider a software that can be downloaded to a mobile phone from a website. The software will be able to transfer all the contents of the address book of the mobile phone to another web server.

   (a) Provide a brief outline of the development process for this software.

   (b) What are the resources required for this software?

   (c) What are the security issues to be considered in this project?

   [15+5+5=25]

3. In a large industrial complex, 10 lifts serve 50 floors. There are a few constraints.

   (a) More than 7 lifts cannot move in the same direction. A lift can change its direction only when there is no more floor to serve moving in that direction.

   (b) At any point of time a lift can accommodate a maximum of 10 people.

   To control the operations of these lifts, explain each of the phases of the software development process. Make any reasonable assumption that you may need and state the same with justification. [25]

4. Consider a Ticket Booking System for soccer matches where the spectators can reserve the tickets using Internet facility. The system should provide the particulars of the matches on entering the name of the team. The number of available seats for different class of tickets (position in the gallery, cost etc.) should be displayed. The user then enters the number of spectators with their details and seat preferences after which the system calculates the total cost and displays the particulars of the tickets. The user can buy the ticket online using credit cards. For this a card number is asked for. The system checks the validity of the credit card through a

third party banking system. The user can print the tickets on any printer and those printed tickets, along with the passports of the spectators, will be considered to be a valid document. The system also prepares a list of spectators for a particular match with relevant details. It should be noted that the spectators of two competing teams cannot be accommodated in the same gallery.

Draw a DFD and prepare an SRS for this system. [10 + 15 = 25]

5. Suppose you are the Product Manager of a software company that operates in the Education domain. Assume a product that can take care of the complete examination process.

   (a) Briefly describe the features and capabilities of this product.

   (b) Explain the issues related to the product in terms of (i) licensing model, (ii) support plan, and (iii) installation mechanism.

   [10+(5+5+5) = 25]

6. (a) Describe the differences between White box and Black box testing.

   (b) Briefly explain (i) Unit Testing, and (ii) Integration Testing.

   (c) Explain the roles of (i) verification, and (ii) validation in software testing.

   [10+(4+4)+(4+3) = 25]

7. What would be the differences in approach for verification and validation in two different situations:

   - for a mass marketed software product, and
   - for very large custom software development project done for a single customer.

   [15+10=25]

INDIAN STATISTICAL INSTITUTE
End-Semester Examination: 2011-12(Second Semester)
Course Name: M.Tech. In Computer Science I Yr'
Subject Name: Automata Languages and Computation

Date: 30. 04. 2012 *Maximum Marks: 80* *Duration: 3 hours*

Answer question 1 and any 4 questions from questions 2 to 7.

1.

Answer with short justifications whether the following **claims** are **true** or **false**. No credit will be given for answers without justification. Assume that the alphabet is $\Sigma = \{0, 1\}$ unless specified otherwise. [4 * 6 = 24]

a) L is a CFL, $x \in L$ and a proper prefix of x is also in L.
Claim: L cannot be accepted by a DPDA in empty stack.

b) **Claim**: If $L = \{1^p : p \text{ is a prime}\}$, then there is no context-sensitive language L' so that LL' is a regular language.

c) L is the language consisting of encodings of all Turing machines that accept nothing
Claim: Both L and its complement are recursively enumerable

d) **Claim**: Even if P≠ NP, every NP-complete problem can be solved using polynomial space.

e) L is the language consisting of encodings of all PDAs that accept everything
Claim: Both L and its complement are recursively enumerable.

f) Turing machines do not support recursion. Therefore, Turing machines are not as powerful as ordinary programming languages.

Claim: Church-Turing thesis is false.

2. [8 + 6 = 14 marks]

(a) Let α be a string (over some alphabet Σ). By odd(α), we refer to the string obtained by deleting symbols at all even positions of α. For example, if $\alpha = a_1a_2a_3\ldots a_n$, then $odd(\alpha) = a_1a_3a_5\ldots a_{n'}$ where n' is n or n – 1 according as whether n is odd or even. For a language $L \subseteq \Sigma^*$, define odd(L) = {odd(α) | α ∈ L}. Prove if L is regular, then odd(L) is regular too.

(b) Determine whether Regular sets are closed under each of the operations below. Provide a proof or a counterexample (as the case may be) for each of them.

i. Even(L) is the set of all strings x in L such that |x| is even.
ii. Triple(L) = {x | x=uvw, such that u, v, w are in L, and |u| = |v| = |w|}.

3. [8 + 6 = 14 marks]

(a) Prove that the following language over $\Sigma = \{a,b,c\}$ is not context-free.

$$L_3 = \{a^i b^j c^j \mid i,j \geq 0 \text{ and } i \geq j\}.$$

(b) A 2-way PDA is a machine that is just like a deterministic PDA except that it can move either left or right on seeing a particular symbol in a particular state. It accepts if it moves off the right end of the input in a final state.

Show that the set $\{0^n1^n0^n \mid n > 0\}$ is accepted by a 2-way PDA. Assume that there is a # symbol at the end of the string and a $ symbol at the beginning to mark the end points. To draw a 2-way PDA, just add R (right) or L (left) to each transition (so each arrow now will have an input symbol, a stack symbol, a push/pop command and an L or an R).

4. [(5 + 6) + 3 = 14 marks]

(a) For each of the following languages over the alphabet {0,1}, construct a context-free grammar **and** a pushdown automaton. Give a brief explanation for your construction. Assume x, y and w given below are strings over {0, 1}.

i. $L_1 := \{wyw^R \mid$ the length of y is even $\}$
ii. $L_2 := \{xy \mid |x| = |y|$ and $x \neq y\}$

(b) Let G be a CFG in Chomsky normal form that contains b variables. Show that, if G generates some string using a derivation with at least 2^b steps, then L(G) is infinite.

5. [(4 + 6) + 4 = 14 marks]

(a) For each of the following languages, provide a high level description of a Turing machine.

i. $L_1 := \{a^n\#a^n\#a^n: n \geq 0\}$, $\Sigma = \{a, \#\}$.

ii. $L_2 := \{a\#b\#c: a,b,c \in \{0,1\}^*$ and $a + b = c$, where a, b and c are interpreted as positive binary numbers$\}$, $\Sigma = \{0,1, \#\}$,

(b) Prove that the halting problem is decidable for LBA.

6. [4 + 10 = 14 marks]

(a) In the *Silly Post Correspondence Problem, SPCP,* in each pair the top string has the same length as the bottom string. Show that the SPCP problem is decidable.

(b) There are five languages (or equivalently, problems) A, B, C, D, and E. All we know about them is the following:

- A is in P.
- B is in NP.
- C is NP-complete.
- D is Recursive.
- E is Recursively Enumerable but not Recursive.

Consider the five statements below.

i. There is a reduction from E to D.
ii. There is a polytime reduction from C to B.
iii. There is a polytime reduction from A to B.
iv. There is a polytime reduction from B to the complement of C.
v. There is a reduction from D to C.

For each statement, comment with brief justification whether it is:

- CERTAIN to be true, regardless of what problems A through E are and regardless of the resolution of unknown relationships among complexity classes, of which "is P = NP?" is one example.
- MAYBE true, depending on what languages A through E are, and/or depending on the resolution of unknown relationships such as P = NP?
- NEVER true, regardless of what A through E are and regardless of the resolution of unknown relationships such as P = NP?

7. [8 + 6 = 14 marks]

(a) Suppose A and B are Turing recognizable languages. Which of these are guaranteed to be Turing recognizable? Briefly justify your answer in each case.

i. Union of A and B
ii. Complement of A
iii. Concatenation of A and B
iv. Intersection of A and B

(b) Consider the problem of determining whether a two-tape Turing machine ever writes a non-blank symbol on its second tape when it is run on input w. Formulate this problem as a language and show that it is undecidable.

Indian Statistical Institute

Second Semestral Examination: 2011-2012

Subject Name: M. Tech in Computer Science

Subject Name: Computer Networks

Date: 02-05-2012 Maximum Marks: 50 Duration: 3 hours

Instructions:

You **may** attempt **all** questions which carry a total of **55** marks. However, the maximum marks you can score is only **50**.

1. (a) Let $g(x) = x^3 + x^2 + 1$. Consider the information bits 110110. Find the codeword corresponding to these information bits if $g(x)$ is used as the generating polynomial. Can $g(x)$ detect double errors? If yes, explain why. If not, give an example of an error pattern that cannot be detected. [2+3]

   (b) Suppose that two check bits are added to a group of $2n$ information bits. The first check bit is the parity check of the first n bits, and the second check bit is the parity check of the second n bits. Characterize the error patterns that can be detected by this code. [3]

   (c) Describe in brief the distance vector method of updating routing table information. In particular, explain using an example how information about a node failure propagates using this algorithm. [3]

2. (a) Suppose that the pure ALOHA protocol is used to share a 56 Kbps satellite channel. Suppose that frames are 1000 bits long. Find the maximum throughput of the system in frames/second. [3]

   (b) Let G be the total rate at which frames are transmitted in a slotted ALOHA system. What proportion of slots goes empty when the system is operating at its maximum throughput? [3]

   (c) Briefly describe the basic bit-map collision-free protocol. How long does a station have to wait in the worst case before it can start transmitting its frame over a LAN that uses the basic bit-map protocol? [3+2]

3. (a) Compare the protocols Stop-and-Wait ARQ and Go-Back-N ARQ. Explain how the protocols react to the loss of an information frame and to the loss of an acknowledgment frame. [3 + 3]

   (b) Consider an error-free 64-Kbps satellite channel used to send 512-byte data frames in one direction, with very short acknowledgements coming back the other way. What is the maximum throughput for window sizes of 1, 7 and 15? The earth-satellite propagation time is 270 msec. [3]

(c) Why does UDP exist? Will it not be enough to let user-processes send IP packets? [2]

4. (a) Sixteen stations, numbered 1 through 16, are contending for the use of a shared channel by using the adaptive tree walk protocol. If all the stations whose addresses are prime numbers suddenly become ready at once, how many bit slots are needed to resolve the contention? [3]

(b) Computer A has 19.5 MBytes to send on a network and transmits the data in bursts of 6 Mbps. The maximum transmission rate across routers in the network is 4 Mbps. If Computer A's transmission uses a *leaky bucket*, how much capacity must the queue in the bucket hold not to discard any data? [3]

(c) Datagram fragmentation and reassembly are handled by IP and are invisible to TCP. Does this mean that TCP does not have to worry about data arriving in the wrong order? [2]

(d) Explain the basics of the MACA protocol. [3]

5. (a) What is CIDR? How does CIDR reduce the number of entries in a routing table? [1+3]

(b) A large number of consecutive IP addresses are available starting at 198.17.0.0. Suppose that two organizations, A and B, request 4000 and 2000 addresses, respectively, and in order. For each of these, write the first IP address assigned, the last IP address assigned, and the mask in the $w.x.y.z/s$ notation. [3]

(c) What is Network Address Translation (NAT)? How does NAT work? [1+3]

Indian Statistical Institute

Semester-II 2011-2012

M.Tech.(CS) - First Year

Semestral Examination (25 April, 2012)

Subject: Operating Systems

Total: 110 marks Maximum marks: 100 Duration: 4 hrs.

Please keep your answers brief and to the point.

(a) Given a resource allocation graph for a set of processes and resources, what possible conclusions can be drawn about the processes with respect to deadlock?

(b) Briefly describe and justify a deadlock prevention approach which ensures that the *circular wait* condition is never fulfilled in a system.

[6+8=14]

(a) Consider the following modified version of a standard scheme for solving the 2-process critical section problem (CSP).

```
shared char want[2] = {0,0};
shared int turn = 0;
1. P_i()
2. { while (1) {
3.      turn = j;
4.      want[i] = 1;
5.      while (want[j] && turn!=i);
6.      critical_section();
7.      want[i] = 0;
8.      remainder_section();
9.    }
10.}
```

Show that the above scheme does not guarantee mutual exclusion by constructing an appropriate interleaved sequence of instructions executed by two processes P_0 and P_1.

(b) Consider the following solution to the standard Dining Philosophers problem.

```
semaphore chopstick[5] = {1,1,1,1,1};

while (1) {
  think();
  wait(chopstick[i]);  wait(chopstick[(i+1) % 5]);
  eat();
  signal(chopstick[i]);  signal(chopstick[(i+1) % 5]);
}
```

Recall that the above solution is not deadlock-free. One way to avoid deadlock is to have odd philosphers pick up their left chopstick first, while even philosophers pick up their right chopstick first (or vice versa). Suggest an alternate deadlock-free, semaphore-based solution to this synchronisation problem. (HINT: Use 1b.)

(c) Now consider the following variant of the problem. One of the philosophers is highly respected by everyone else. When he announces that he is hungry, his neighbours give him

their forks. Can deadlock happen in the modified scenario? If yes, give an example of a deadlocked situation. If not, explain which of the deadlock conditions is/are violated.

[10+6+4=20]

3. (a) What is the disadvantage of having a large page / frame size in a page based memory management system? What is the disadvantage of having a small page / frame size?

   (b) Consider an operating system that uses paging. Process address space sizes are uniformly distributed between 1 byte and 64 KB. However, the size of the page table is the same for all processes. Also, each page table entry occupies 4 bytes. What is the optimum page size for this system?

   (c) With reference to page replacement, what is a *stack algorithm*?

   (d) *Using the definition above* and the following reference string, show that the FIFO page replacement strategy is not a stack algorithm.

   1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

   (e) How can you determine the minimum number of frames to be allocated to a process in a demand-paging system?

   (f) What is the difference between global and local page replacement strategies? How does the choice of this strategy affect the design of the *proc* structure and *u area*?

   [3+10+2+12+3+4=34]

4. (a) Describe the format of logical and linear addresses generated by i386 processors. (Include the bit-wise breakup of the address field in your answer.)

   (b) Clearly explain what hardware support is provided in this architecture to speed up the translation of 2-dimensional logical addresses to 32-bit linear addresses.

   [8+6=14]

5. Suppose you have to create an SVR2-like filesystem on a 512MB disk with 2KB block size. You estimate that the average file size is likely to be about 16KB.

   (a) Assuming that it takes 128 bytes to store each inode, how many blocks would you use to store the inode list for the file system? How many blocks would be available for use as data blocks? (You may make reasonable approximations.)

   (b) What problem would arise if your estimate in question 5(a) is (i) an overestimate (i.e., most files are shorter than 16KB) (ii) an underestimate? In your opinion, which of these problems is the more serious?

   (c) A process P has opened in `O_RDWR` mode a file `abc.txt` of size 300 bytes stored in the above filesystem. Calculate the total number of disk blocks read and written when P executes the following sequence of actions on this file. Ignore caching effects.

      (i) P seeks to byte offset 1000, and writes 10 bytes of data.

      (ii) P seeks to byte offset 22000 and writes 10 bytes of data.

      (iii) P seeks to byte offset 9000 and reads 10 bytes of data.

      Explain your answer.

   [8+8+12=28]

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech. - I Year (Semester - II)

Design and Analysis of Algorithms

Date : 04.05.12 Maximum Marks : 100 Duration : 3:30 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1. Given an array A having n elements, you are required to fill in an $n \times n$ matrix B so that whenever $i < j$, $B(i,j) = \sum_{l=i}^{j}(-1)^l.A[l]$. You dont have to worry about elements $B(i.j)$ with $i \geq j$.

   a) Give an efficient algorithm to do this.

   b) Prove the correctness of your proposed algorithm.

   c) Argue that your algorithm has the best possible time bound.

   [12+6+7=25]

2. Let $G = (V, E)$ be an undirected graph and let s, t be two vertices in G. Give an efficient algorithm which computes the number of shortest paths from s to t in G. [13]

3. You are given a complete binary tree T in which each node v is labelled with a real number x_v. A node v is said to be a local minimum if its value x_v is less than the values of all nodes to which it is joined by an edge. Give an algorithm to find a local minimum of T using atmost $O(logn)$ probes to the nodes of T. [13]

4. A directed graph is *semi-connected* if for any two vertices u, v, either u is connected to v or v is connected to u. Give an efficient algorithm to determine if a directed graph is semi-connected.

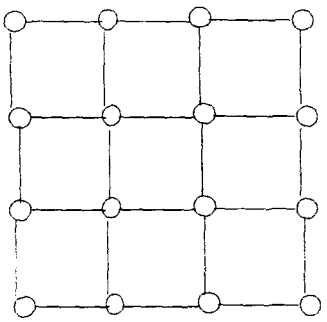
   [14]

5. Consider the following problem, called the zero-weight cycle (ZWC). You are given a directed graph $G = (V, E)$ with weighted edges where edge weights may be positive, negative or zero. The question is whether there exists a simple cycle of total weight 0? To prevent trivial solutions, we require that any solution consist of at least one edge. Prove the ZWC is NP-complete. (Hint: Reduction from directed Hamiltonian cycle.) [15]

6. Suppose you are given an $n \times n$ grid graph $G = (V, E)$, as shown in the following figure. Each node is associated with a nonnegative integer weight. You may assume that all weights are distinct. Your objective is to compute an independent set $V' \subseteq V$ so that the sum of weights of the vertices of V' is as large as possible. Consider a simple greedy

algorithm that works by selecting the vertex u of maximum weight from G and then deleting u and all its neighbors. This process is repeated until there are no more vertices left.

Prove that this algorithm produces an independent set for G whose total weight is at least $1/4$ of the total weight of the optimal solution.



[20]

7. Compute the prefix function π for the pattern *ababbabbabbababbabb* when the alphabet is $\Sigma = \{a, b\}$. [10]

8. Prove that, if $P \neq NP$, then for any $\epsilon > 0$, there is no polynomial time approximation algorithm with approximation ratio $3/2 - \epsilon$ for the bin packing problem. [15]

Indian Statistical Institute

Semester Examination: 2012

(first year second semester)

Course Name: M. Tech in Computer Science

Subject Name: Computer Networks (back paper)

Date: ~~27.07.2012~~ 31.07.2012 Maximum Marks: 100 Duration: 3 hours

Instructions:

You **may** attempt **all** questions which carry a total of **120** marks. However, the maximum marks you can score is only **100**.

1. (a) What are the main features of sliding window protocol? What is piggybacking and pipelining? [4+4+4]

   (b) Consider the operation of *go back-n protocol* with 3-bit sequence number. Why sender is restricted to transmit up to seven frames before being required to wait for acknowledgement? [4]

   (c) Explain in brief *flag bytes with byte stuffing* method of framing. [4]

2. (a) Explain 1-persistence, non-persistence and p-persistence CSMA protocol. [3–3+3]

   (b) What are hidden and exposed terminal problems in wireless communication? [4]

   (c) A channel has a bit rate of 4 kbps and a propagation delay of 20 msec. For what range of frame sizes does stop-and-wait give an efficiency of at least 50 percent? [4]

   (d) Sixteen-bit messages are transmitted using Hamming code. How many check bits are needed to ensure that the receiver can detect and correct single bit errors? [3]

3. (a) What is congestion? How is it different from flow control? [2+4]

   (b) What is a choke packet? How is it used for congestion control? [2+4]

   (c) A computer on a 6-Mbps network is regulated by a token bucket. The token bucket is filled at a rate of 1 Mbps. It is initially filled to capacity with 8 megabits. How long can the computer transmit at the full 6 Mbps? [6]

   (d) Give an argument why the leaky bucket algorithm should allow just one packet per tick independent of how large the packet is. [2]

4. (a) What is optimality principle in routing algorithm? [4]

   (b) Describe the main features of link state routing protocol? Write the major differences between distance vector routing and link-state routing. [8+4]

   (c) Discuss a way to restrict the number of duplicate packets generated by the flooding routing algorithm. [4]

5. (a) A network on the Internet has a subnet mask of 255.255.240.0. What is the maximum number of hosts the network can handle? [4]

(b) Convert the IP address whose hexadecimal representation is $C22F1582$ to dotted decimal notation. [4]

(c) Write the purpose of the following two fields of $IPv4$ header:

i. Time to live [3]

ii. Protocol [3]

(d) What are the key differences between *datagram* and *virtual-circuit* subnets? [6]

6. (a) Write short notes on the following:

i. Jitter Control [6]

ii. TCP Timers [6]

iii. UDP [8]

Indian Statistical Institute

Semester-II 2011-2012

M.Tech.(CS) - First Year

Backpaper Examination

Subject: Operating Systems

Total Marks: 65

Maximum marks: 60 Duration: 3.5 hrs.

02.08.12

Please keep your answers brief and to the point.

1. (a) List the types of events that cause a process to switch from user mode to kernel mode. For each type of event, explain in 1-2 lines
   - when the event occurs;
   - whether the event is synchronous / asynchronous;
   - whether the event is handled by the kernel in user context or system context.

   (b) Write a C program that does the following:
   - the original process creates a child process;
   - the parent process prints the child's process ID and exits;
   - the child executes `/bin/date` and exits.

   [6+4=10]

2. (a) In the context of process scheduling, what is *starvation*?

   (b) Can starvation occur under the following scheduling strategies? (i) FCFS; (ii) non-preemptive shortest job first. If so, construct a scenario where starvation occurs. If not, argue / prove that starvation cannot occur. You may assume that all CPU bursts are of finite duration.

   (c) Briefly describe a method by which starvation can be prevented in a priority-based scheduling strategy.

   [1+4+2=7]

3. (a) Recall that the integer value of a binary semaphore can only be 0 or 1. Write pseudo-code to show the functioning of the blocking version of the $wait_b$ and $signal_b$ operations on **binary** semaphores. (Do not write code for ordinary counting semaphores.)

   (b) Show how you would implement the *wait* and *signal* functions for ordinary (counting) semaphores using $wait_b$ and $signal_b$.

   [4+6=10]

4. A 32-bit machine has a page/frame size of 1 KB. Each page table entry occupies 4 bytes.

   (a) What is the size of the complete first-level (primary) page table for a process?

   (b) If page tables are also stored using paging, how many levels of paging will be required?

   (c) If a memory access takes 10ns, and the machine uses a TLB that provides a 70% hit-ratio (on average), what is the effective memory access time using multi-level paging? Neglect the time required for a TLB lookup.

   [2+2+2=6]

5. (a) Consider the following page reference string:

1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3

If a process is allocated 4 physical frames, how many page faults would occur if page replacements are done using the LRU algorithm.

(b) Explain how page-fault frequency monitoring can be used to prevent thrashing.

(c) What is the difference between global and local page replacement strategies? State one advantage of each strategy over the other.

(d) Draw a schematic diagram showing how address translation is done on i386 processors.

[5+3+3+6=17]

6. (a) Describe the structure of a directory's data block in an SVR2 filesystem.

(b) A text file contains only the 3 character string `abc`. Process A opens the file and forks a child process B. Draw a picture showing the relationship between the process file descriptor tables, global file table, and inode table. Each process (A and B) then reads characters one by one from the file and prints the characters on the screen. What is the output produced by each process? Justify your answer.

[3+5=8]

7. (a) Define a *safe sequence* for a resource allocation state (RAS).

(b) Consider the following RAS involving 5 processes and 4 resources. $Max[i,j]$ specifies the maximum number of instances that process i may request of resource j. $Alloc[i,j]$ gives the number of instances of resource j currently allocated to process i. $Avail[i]$ specifies how many instances of resource i are currently available.

$$Max = \begin{bmatrix} 4 & 2 & 3 & 1 \\ 6 & 2 & 3 & 4 \\ 3 & 2 & 1 & 3 \\ 1 & 2 & 1 & 3 \\ 3 & 0 & 3 & 2 \end{bmatrix} \quad Alloc = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad Avail = [\ 4\ 4\ 1\ 2\ ]$$

Calculate whether the system is in a safe state.

[2+5=7]