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Bertrand equilibria with entry: limit results

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Abstract

We study the limiting behaviour of Bertrand equilibria (where firms must supply the whole of the demand coming to them) for two different cases, first when entry is exogenous, and second when it is free. The limit equilibrium set is characterised for both cases for a large class of cost functions.

Keywords: Bertrand oligopoly; Free entry; Perfect competition; Folk theorem

JEL classification: D43; D41; L13

1. Introduction

This paper examines some limiting properties of a Bertrand–Chamberlin model of price competition. It is motivated by the 'folk theorem' for perfect competition which states that whenever firms are small relative to the market, the market outcome approximates a perfectly competitive equilibrium (i.e. the outcome where price equals marginal cost equals average cost). The theoretical research in this idea has concentrated mainly on quantity competition. The goal of this paper is to examine if the 'folk theorem' holds in case of price competition.

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In the quantity competition framework the idea can, in fact, be traced back to Cournot (1963). There are two possible approaches to the problem. Under the first approach, developed (among others) by Ruffin (1971) and Okuguchi (1973), one looks at the n-firm Cournot equilibrium taking demand and cost conditions as given. One then studies the limiting outcome as the number of active firms becomes very large. We call this the Ruffin–Okuguchi, or the *exogenous entry* approach. Under this approach the folk theorem holds only if average costs are (weakly) increasing. For U-shaped average cost curves the folk theorem invariably fails.

The alternative approach, pioneered by Novshek (1980), looks at the limiting outcome under free entry Cournot equilibrium, as firm size becomes progressively smaller compared to the market. We shall call this the Novshek or the *free entry* approach. Novshek (1980) demonstrates that under the second approach the folk theorem holds under very general conditions.

There are two ways the trading process following a price announcement can be modelled. The first approach, pioneered by Chamberlin (1933), is to assume that firms supply all demand.[1] The other approach, developed by Edgeworth (1922), is to assume that firms are free to supply less than the quantity demanded.

As argued by Dixon (1990), the difference between the two approaches can be traced to the cost of turning away customers. Under the Chamberlin (1933) approach such costs are assumed to be very high, so that customers are never turned away. Such costs, however, are assumed to be zero under the Bertrand–Edgeworth approach, and a firm supplies the minimum of the demand coming to it and its profit maximising output.

As examples of such costs we can mention reputational effects, as well as government regulations that force the firms to supply the whole of the demand. Vives (1999) argues that this may be the case in regulated industries like electricity or telephone in the U.S. Under the 'common carrier' regulation, for example, firms are required to supply all demand at the set prices. In case the supply of the commodity is exhausted, the consumers take a 'rain-check', a coupon to purchase the good at the posted price at a later date (see Spulber, 1989). Another case that would fit this assumption is that of certain sealed bid auctions, where firms make their bids under the assumption that if their bid wins, they would have to supply whatever demand that comes to them. In fact the existence of such costs is routinely adopted in fields like operations research (see Dixon, 1990, as well as Taha, 1982).

The reality of course would generally be somewhere in between, and, depending on the particular market in question, either one of the two approaches may be appropriate.

In this paper we adopt the Chamberlin (1933) approach to the problem, i.e.

[1] Bertrand implicitly uses this approach. It has also been adopted, among others, by authors like Vives (1990) and Bulow et al. (1985).

firms are assumed to supply the whole of the demand coming to them.[2] We study the properties of the *limit equilibrium set* both when entry is exogenous, as well as when it is free. Under the Ruffin–Okuguchi approach we solve for the n-firm Bertrand equilibrium where demand is given and all firms are active in equilibrium. The limit equilibrium set comprises all possible prices that can be obtained as the limit of a sequence of equilibrium prices as the number of active firms, n, is taken to infinity. Under the Novshek approach we consider an r-fold replication demand and then solve for free entry Bertrand equilibrium (i.e., an equilibrium with an endogenously determined number of active firms in which at least one firm chooses to remain inactive by setting a price that generates no sales). The limit equilibrium set comprises all possible prices that can be sustained as the limit of a sequence of free entry equilibrium prices (at which sales occur) as market size, r, is taken to infinity.

We now briefly describe our results.

First consider the Ruffin–Okuguchi approach. If average cost is increasing then, for the Cournot case, equilibrium price is near minimum average cost whenever the number of firms is large (Ruffin, 1971). Thus the folk theorem holds. For the Bertrand case, however, the limit equilibrium set is not a singleton, even though the limit set does include the perfectly competitive price. Thus the folk theorem fails for Bertrand–Chamberlin price competition, even though it holds under Cournot. If cost is continuous and average cost is U-shaped, then Ruffin (1971) shows that under Cournot competition, for a large number of firms, the equilibrium price must be near the limiting value of average cost as output approaches zero. Thus the folk theorem fails for Cournot competition. Under Bertrand competition we find that for some parameter values the limit equilibrium set is empty, whereas for other parameter values the limit set comprises an interval. Moreover, the lowest price in this interval is bounded away from the minimum average cost. Thus the folk theorem fails under price competition as well.

Now consider the free entry approach. Novshek (1980) examines the case when average cost is U-shaped and firms pursue Cournot competition. He finds that if market size is sufficiently large compared to firm size, then the equilibrium price is arbitrarily close to the competitive one. Under the Bertrand approach though, the limit equilibrium set comprises an interval. However, the competitive price does emerge as the lowest price of the limit equilibrium set.[3] Thus the folk theorem holds for the Cournot case, but fails under Bertrand competition.

[2] Dastidar (1995) has demonstrated the existence of pure strategy Bertrand equilibrium under the assumption that firms supply all demand. For the Bertrand–Edgeworth approach, one solution would be to look for equilibrium in mixed strategies, using the fixed point theorems developed by Dasgupta and Maskin (1986a,b). This approach is adopted, among others, by Allen and Hellwig (1986), Maskin (1986) and Vives (1986). Dixon (1987, 1990), however, demonstrates the existence of pure strategy equilibrium in Bertrand–Edgeworth models of price competition.

[3] For increasing average costs there is a non-existence problem under Cournot, as well as Bertrand competition. See Remark 1 later.

Thus our investigation casts serious doubts on the applicability of the folk theorem under Bertrand–Chamberlin competition.

Finally, let us relate our paper to the literature on the limiting properties of Bertrand–Edgeworth equilibrium where firms are free to supply less than the quantity demanded. In a model where costs of turning customers away are small, Dixon (1990) finds that for a large enough market all equilibrium prices would be close to the competitive one.[4] Allen and Hellwig (1986) and Vives (1986) both study capacity-constrained price games, though with different rationing rules. They both find that as the number of firms goes to infinity, the equilibrium price converges (in distribution) to the perfectly competitive outcome. Thus generally speaking the folk theorem holds under Bertrand–Edgeworth competition. In contrast we find that the folk theorem fails under a Bertrand–Chamberlin framework. This demonstrates that the limiting behaviour of price competitive models depends crucially on the costs of turning customers away.

The rest of the paper is organised as follows. Section 2 presents the basic model and assumptions. The results are collected together in Section 3. Section 4 concludes. Finally, Appendix A contains some preliminary results of a technical nature.

2. The model and assumptions

We consider a one stage game of price competition between n identical firms, all producing a single homogeneous good, where firms must meet all demand. The equilibrium concept is pure strategy Nash equilibrium in prices, i.e. Bertrand equilibrium.

We will deal with a fixed market demand given by $f(p)$, and a sequence of markets that grow by demand replication. For $r = 1, 2, 3, \ldots,$ the r-fold replication demand is $f(p{:}r) = rf(p)$. The demand function $f(p)$ satisfies the following assumption.

Assumption 1.

(a) $f{:}[0, \infty) \to [0, \infty)$.[5] Moreover, $f(p)$ is continuous.
(b) There exists a strictly positive number $\hat{p}$ such that $f(p) = 0$, $\forall p \geq \hat{p}$.
(c) $f(p)$ is strictly decreasing on $[0, \hat{p})$.

All firms have identical cost functions denoted by $c(q)$ and average cost functions denoted by $AC(q)$. We allow for a wide variety of cost functions

[4] Dixon (1987) also studies the limiting properties of price equilibria in a Bertrand–Edgeworth model. He, however, examines epsilon-equilibria, rather than Nash equilibria.

[5] Note that this implies that $f(0)$ is finite.

including continuous cost with increasing marginal (and hence average) cost, continuous cost with U-shaped marginal and average cost, and cost functions with discontinuities at the origin due to avoidable (i.e. non-sunk) fixed costs and U-shaped average cost.

We make the following assumption on the cost function.

Assumption 2.

(a) $c:[0, \infty) \to [0, \infty)$. Moreover, $c(0) = 0$ and $c(q) > 0$, $\forall q > 0$.
(b) The cost function is continuous everywhere except possibly at the origin.[6]
(c) $AC:(0, \infty) \to (0, \infty)$. There exists a non-negative number q^* such that $AC(q)$ is strictly decreasing on $(0, q^*)$ and strictly increasing on (q^*, ∞).[7]

We then introduce some notation. Define

$$b = \lim_{q \to 0} AC(q).$$

Allowing infinity as a possible limit, from Assumption 2(c) it follows that b is well defined. Also define

$$c^* = \inf_q AC(q).$$

Consider the case when q^* is strictly positive. Clearly, $c^* = AC(q^*) < b$. We will use the term *U-shaped* average cost for this class of cost functions, even though average cost need not be strictly U-shaped. For example, $\lim_{q \to \infty} AC(q)$ could be finite even though $AC(q)$ may be increasing $\forall q > q^*$.

Next consider the case when $q^* = 0$. Clearly, the cost function must be continuous at zero and moreover $c^* = b$. We will use the term *increasing average cost* to denote this class of cost functions.

Assumption 3. A firm must supply the whole of the demand it faces.

Combining the demand and the production sectors we obtain a *market*. This notion is formally defined below.

Definition. The *market* $M(r, n)$ has the r-fold replication demand $f(p, r)$ and n identical firms, each with cost function $c(q)$.

If the n firms choose the prices $p_1, p_2, \ldots, p_n$, then the demand curve facing firm i is

[6] Note that $AC(q)$ is well defined and continuous on $(0, \infty)$.
[7] Note that q^* may be 0, in which case $AC(q)$ is increasing everywhere.

$$D_i(p_1,\ldots,p_i,\ldots,p_n)=\begin{cases}0, & \text{if } p_i>p_j, \text{ for some } j,\\ \dfrac{rf(p_i)}{m}, & \text{if } p_i\leq p_j,\ \ \forall j, \text{ and } \#(l\colon p_l=p_i)=m.\end{cases}$$

The corresponding profit of the ith firm is

$$E_i(p_1,\ldots,p_n)=\begin{cases}0, \text{ if } p_i>p_j, \text{ for some } j,\\ (p_i-AC(D_i(p_1,\ldots,p_n)))D_i(p_1,\ldots,p_n), & \text{if } p_i\leq p_j, \forall j.\end{cases}$$

In this paper we restrict attention to pure strategies.

Definition. A price vector $(p_1,\ldots,p_i,\ldots,p_n)$ is a *Bertrand equilibrium* for the market $M(r, n)$ if, $\forall i$ and $\forall p_i'$,

$$E_i(p_1,\ldots,p_i,\ldots,p_n)\geq E_i(p_1,\ldots,p_i',\ldots,p_n). \tag{1}$$

We then formalise the notion of the limit equilibrium set under both approaches. In case of Cournot competition Ruffin (1971) and Okuguchi (1973) examine the nature of equilibria when market demand is given (so that $r=1$) and all firms are active, where active means producing a positive output. We examine the same question, but in a Bertrand framework. In a Bertrand framework firms are said to be active only if their price is among the lowest. We are thus interested in the properties of the following set.

Definition. $S=\{p$: there is a sequence $p(n)$ that converges to p such that for each sufficiently large n, all firms setting price $p(n)$ is an equilibrium for the market $M(1, n)\}$.

Thus S is the set of all prices p such that if the number of firms is large enough then there is some equilibrium where all firms are active and the equilibrium price is arbitrarily close to p.

In case of Cournot competition Novshek (1980) examines the nature of equilibrium when market size is large compared to firm size and there is free entry of firms. Hence there are always inactive firms in equilibrium. For Bertrand competition, this is equivalent to finding some n for which there is an n-firm equilibrium in which not all n firms set the lowest price. If m firms are active and $n-m$ (>0) firms are inactive, then for any $n'>n$, there is a corresponding equilibrium in which the only change is that the additional $n'-n$ firms are added to the inactive group.[8] Under the Bertrand free entry approach we are thus interested in the following set.

[8] Of course this does not mean that the set of lowest equilibrium prices is the same with n' as with n. It only means that if some firms are inactive with n firms, then the lowest price in that equilibrium will also be the lowest price in some equilibrium for every $n'>n$.

Definition. $T = \{p$: there is a sequence $p(r)$ that converges to p such that for each sufficiently large r, there is an integer n and an equilibrium for the market $M(r, n)$ in which $p(r)$ is the lowest price, but not all firms set the price $p(r)\}$.

Thus T is the set of all prices p such that if the demand sector is large enough then there is an equilibrium involving both active and inactive firms such that the equilibrium price is arbitrarily close to p.

We introduce a final set of notations before we begin the analysis. Let

$$d(r) = \min\{p\text{: } AC(rf(p)) = p\} \text{ and } d^* = \lim_{r\to\infty} d(r).$$

We then state some properties of $d(r)$ and d^* given assumptions 1 and 2.[9] These are required in Theorem 2 below.

1. $d(1)$ is well defined if and only if the graphs of $f(p)$ and $AC(q)$ intersect at least once in the $p-q$ space[10].
2. Assuming that $d(r)$ is well defined, $d(r) < \hat{p}$, and, for r large, $d(r)$ is strictly increasing in r.
3. d^* is well defined if and only if $c^* < \hat{p}$. Moreover, d^* could be either equal to $\hat{p}$[11], or strictly less than $\hat{p}$[12], but when it is well defined d^* is always strictly larger than c^*.

3. The results

In this section we characterise the limit equilibrium set under both approaches.

We first consider the Ruffin–Okuguchi approach. Theorem 1 below characterises the set S for various cost functions.

Theorem 1. *Assume that $f(p)$ and $AC(q)$ intersect at least once (so that $d(1)$ is well defined) and Assumptions 1, 2 and 3 hold.*

(a) *If average cost is increasing, then $S = [c^*, d(1)]$.*[13]
(b) *If average cost is U-shaped and $b \leq d(1)$, then $S = [b, d(1)]$.*

[9] The proofs have been relegated to Appendix A.
[10] Note that if $d(r)$ is not well defined, then even a monopolist facing the r-fold replication demand would have unique optimal output zero.
[11] When $AC(q) > \hat{p}$ for q large.
[12] When $\lim_{q\to\infty} AC(q) < \hat{p}$.
[13] Since average cost is not defined for $q = 0$, $f(p)$ and $AC(q)$ must intersect for some $q > 0$, and $c^* < d(1)$.

(c) *If average cost is U-shaped and* $b > d(1)$, *then* S *is the empty set.*

Proof. First recall that $c^* = b$ when average cost is increasing, so $S = [b, d(1)]$ in both (a) and (b). To show that no price less than b or greater than $d(1)$ can be in the limit set, suppose $p(n)$ converges to p as n increases and for each sufficiently large n, all n firms setting a price $p(n)$ is an equilibrium for $M(1, n)$. Then $p(n)$ is never less than c^*, so that output per active firm is no more than $f(c^*)/n$, which converges to zero as n becomes large. Thus if $p < b$, then for sufficiently large n, $p(n) < AC(f(p(n))/n)$, and $p(n)$ cannot be an equilibrium price. If $p > d(1)$, then for sufficiently large n, profit per active firm, $[p(n) - AC(f(p(n))/n)]\,f(p(n))/n$, is less than $(\hat{p} - c^*)f(d(1))/n$, which converges to zero. Undercutting to the price $(p + d(1))/2$ yields a strictly positive profit that does not converge to zero as n increases. Thus, for n large, undercutting is strictly profitable.

To show that every price in the interval $[b, d(1)]$ is in the limit set, note that if $p > c^*$, then, for any sufficiently large n, if n firms set such a price and share demand, each firm will produce an output at which p exceeds average cost, and thus obtains a positive profit. Undercutting is unprofitable since $d(1)$ is the lowest price at which a monopolist avoids a loss. The remaining case is $p = c^* = b$ with increasing average cost. This p can be obtained as the limit of an appropriate sequence of equilibrium prices, $p(n)$, just discussed. □

The intuition is very simple. For some candidate equilibrium price p, if $p < b$, then for n large the average cost is close to b and the firms make losses. Whereas for $p > d(1)$, undercutting is profitable. Finally consider p such that $b < p < d(1)$. Since $p > b$, for n large, firms make positive profits. Moreover, since $p < d(1)$, undercutting is not profitable. Thus for n large all such prices can be sustained as equilibrium.

Ruffin (1971) shows that when average cost is increasing the Cournot analogue of the set S is $\{c^*\}$. So with a large number of active firms, the equilibrium price must be near the minimum average cost. For the Bertrand case, however, the equilibrium limit set is not a singleton, even though the perfectly competitive outcome does belong to the limit set. Thus the folk theorem fails under price competition, even though it holds for Cournot competition.

If, however, cost is continuous, and average cost is U-shaped, then Ruffin (1971) shows that the Cournot analogue of the set S is $\{b\}$. Thus with a large number of active firms the equilibrium price must be near $\{b\}$, and not near minimum average costs. Thus the folk theorem fails. Under Bertrand competition we find that for some parameter values the limit equilibrium set must be empty, whereas for other parameter values the limit set comprises an interval. Moreover, the lowest price in this interval is bounded away from the minimum average cost. Hence for U-shaped average costs the folk theorem fails under the Ruffin–Okuguchi approach for both quantity and price competition.

We then consider the free entry approach. In Theorem 2 below we characterise the set T for U-shaped average cost functions.

Theorem 2. *Assume that $c^* < \hat{p}$ (so that d^* is well defined) and Assumptions 1, 2 and 3 hold. If average cost is U-shaped, then $T = [c^*, \min\{b, d^*\}]$.*

Proof. To show that no price outside $[c^*, \min\{b, d^*\}]$ can be in the limit set, suppose $p(r)$ converges to p as r increases, and for each sufficiently large r, there is an n and an equilibrium for the market $M(r, n)$ in which $p(r)$ is the lowest price, but not all firms set price $p(r)$.

Clearly, $p(r) \geqslant c^*$, $\forall r$, so $p \geqslant c^*$. We then argue that any equilibrium price $p(r) < d^*$. There are two cases to consider.

Case 1. $d^* = \hat{p}$. Define $p'(r)$ as satisfying $rf(p) = q^*$. Clearly, $p'(r) < \hat{p}$. [Suppose to the contrary that $p'(r) \geqslant \hat{p}$. Then $q^* = rf(p'(r)) \leqslant rf(\hat{p}) = 0$. Since $q^* > 0$, this is a contradiction.] Also note that $\lim_{r\to\infty} p'(r) = \lim_{r\to\infty} f^{-1}(q^*/r) = f^{-1}(0) = \hat{p}$. Since $\hat{p} > c^*$, for r sufficiently large, $\hat{p} > p'(r) > c^*$. Now suppose to the contrary that $p(r) \geqslant d^* = \hat{p}$. Then $p(r) \geqslant \hat{p} > p'(r) > c^*$. But then an inactive firm could undercut by charging $p'(r)$, and make a strictly positive profit.

Case 2. $d^* < \hat{p}$. Since $d^* < \hat{p}$, it follows that $f(d^*) > f(\hat{p}) = 0$. Thus for r sufficiently large, $rf(d^*) > q^*$. Moreover, since $AC(q)$ is strictly increasing for $q > q^*$, it follows that $AC(rf(d^*)) < d^*$. Now suppose to the contrary that $p(r) \geqslant d^*$. Then $p(r) \geqslant d^* > AC(rf(d^*))$. But then an inactive firm could deviate to price d^*, selling $rf(d^*)$ and earning a strictly positive profit.

Finally, if $b < p \leqslant d^*$, then an inactive firm could match the lowest price. With one more firm to share sales, each firm would sell less but the average cost must be less than p (since $AC(q) < b$ for $q \leqslant q^*$ and $AC(q)$ is increasing for $q > q^*$). Thus this deviation leads to a strictly positive profit.

Consider any p such that $c^* < p < \min\{b, d^*\}$. We argue that any such p must be in the limit set. Let $q(p)$ be the unique q with $0 < q < q^*$ such that $AC(q(p)) = p$. Let $N(r)$ be the largest integer such that $N(r) < rf(p)/q(p)$. For r sufficiently large, $AC(rf(p)/N(r)) < p \leqslant AC(rf(p)/N(r) + 1)$. Let $N(r)$ firms each set the price p in the r-market and share demand equally, and let one firm set a higher price. Then the active firms each earn a positive profit. If one of them or the inactive firm undercuts the price, then that firm must produce to meet a demand that exceeds $rf(p)$. But as r gets large, $AC(rf(p))$ either approaches $d^* > p$, or exceeds $\hat{p} = d^* > p$. Also, if an inactive firm matches the lowest price, by the properties of $N(r)$ the firm at best has a profit of zero. Thus for each sufficiently large r, p is an equilibrium price for r, and thus is in the limit set.

Finally, $p = c^*$ and $p = \min\{b, d^*\}$ can be obtained as limits of appropriate sequences of the r-market equilibrium prices, $p(r)$, just discussed. □

Novshek (1980) shows that when average cost is U-shaped the Cournot analogue of the set T is $\{c^*\}$, so that whenever market size is large relative to argmin $AC(q)$, then in any free-entry Cournot equilibrium the price must be near minimum average cost. Under the Bertrand approach we find that the limit equilibrium set comprises an interval though the competitive price does emerge as the lowest price of this set. Thus under Bertrand competition the folk theorem fails under the free entry approach, even though it holds for Cournot competition.

A few remarks are in order.

Remark 1. Note that in Theorem 2 above we did not consider the case of increasing average cost. This is because in both the Bertrand and the Cournot versions, the set T is empty. The equilibrium price with n active firms can never be as low as c^*, or the active firms would make losses. If it exceeds c^*, no firm would remain inactive since under Bertrand competition a firm could match the lowest price and earn a positive profit, while under Cournot competition a firm could, by producing a sufficiently small but positive output, earn a positive profit.

Remark 2. We did not consider cost functions for which average cost is either constant, or decreasing everywhere. The constant average cost case is well known. Here $b = c^* = d^*$, and $S = T = \{c^*\}$. If average cost is everywhere decreasing, then S and T are both the empty set. Given r, for any $n > 1$, any price above $d(r)$ would be undercut. If more than one firm sets the price $d(r)$, the firms make a loss, but if only one firm charges $d(r)$ then it could raise its price to just under the next lowest price. Ray Chaudhuri (1996), however, considers a model where prices are allowed to vary over a grid and average cost is decreasing. He shows that in the limit, as grid size approaches zero, the contestable outcome obtains.

Remark 3. Next consider technologies with capacity constraints. Our analysis could easily be modified to deal with this case. If average cost exceeds $\hat{p}$ for outputs near the limiting capacity level, then this case is already covered. If, however, average cost is less than and bounded away from $\hat{p}$ for outputs near the limit, then $d(r)$ should be defined as if the average cost function intersects the demand function at the capacity limit.

Remark 4. Suppose average cost is literally U-shaped and also suppose that there is an avoidable fixed cost, so that $d^* = \hat{p}$. We can use the same ideas as in the proof of Theorem 2 to show that for every sufficiently large r, there is a free entry equilibrium in which the lowest price is $d(r)$, but there is no free entry equilibrium with the lowest price exceeding $d(r)$. Since $d(r)$ is increasing in r, this means that the highest equilibrium outcome in terms of the price at which sales occur, gets worse as demand expands.[14]

Remark 5. Another strand of the literature considers markets in which both the firm and the consumer sector are replicated at the same rate, and requires that all firms be active in equilibrium.[15] Thus the market in this case is denoted by $M(n, n)$. As in Theorem 1 we assume that $d(1)$ is well defined and Assumptions 1, 2 and 3 hold. If all firms set price p, then each obtains a profit of $[p - AC(f(p))]\, f(p)$, so only prices at which a monopolist in $M(1, 1)$ at least breaks even could possibly be Bertrand equilibrium prices. However, any price above d^* could, for sufficiently large n, be profitably undercut. Thus the set of limit Bertrand equilibrium prices is the closure of the set

$$\{p: 0 < p < d^* \text{ and } AC(f(p)) \leqslant p\}.$$

The set is empty if $d(1) \geqslant d^*$. Note that this is possible only if $f(d(1)) < q^*$. Under the assumptions used in the Cournot strand of this literature, the set of limit Cournot equilibrium prices consists of the competitive equilibrium price for $M(1, 1)$ if it exists (i.e. if $f(d(1)) \geqslant q^*$) or is empty if $M(1, 1)$ has no competitive equilibrium (i.e. if $f(d(1)) < q^*$, so that the firm has a loss at the output level where the demand curve and the marginal cost curve intersect).

As a final comment on this case, suppose $f(d(1)) < q^*$ and we allow firms to be inactive in equilibrium. Then the limit sets are $[c^*, d^*]$ for Bertrand and $\{c^*\}$ for Cournot, as in the free entry case when $b \geqslant d^*$. Here firms are too large technologically relative to their share of the market to justify all firms being active in equilibrium.

4. Conclusion

In this paper we examine if the folk theorem of perfect competition holds under Bertrand–Chamberlin price competition where firms must supply all demand. We find that whenever the folk theorem fails under Cournot competition, then it fails under Bertrand competition as well. Whereas if the folk theorem holds under Cournot competition, then, under Bertrand competition, the limit equilibrium set

[14] The least of the equilibrium prices is not necessarily monotonic in r because of a possible integer problem. If $f(c^*) = q^*$, then for every $r > 1$ there is an equilibrium in which r firms set price equal to c^* and some inactive firms set a higher price. If $f(c^*) = q^*/2$, then for every even $r > 3$, there is an equilibrium in which $r/2$ firms set price equal to c^* and some inactive firms set a higher price, but for r odd, c^* is not a lowest equilibrium price.

[15] See, for example, Gabszewicz and Vial (1972) and Ushio (1985). Gabszewicz and Vial (1972) replicates both demand and firms in a general equilibrium framework. On the other hand, Ushio (1985) examines a very general replication procedure in a partial equilibrium framework. Clearly, a special case is where both the sectors are replicated at the same rate.

either does not exist, or constitutes an interval. In the last case the competitive price turns out to be the lowest price in this interval. Thus the folk theorem fails in this case as well.

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Appendix

In this section we derive the properties of the $d(r)$ function stated in Section 2.

Consider the first property. If $AC(q)$ and $f(p)$ intersect at least once, then, for all such intersection prices, $AC(f(p)) = p$, and $d(1)$ is well defined. Whereas if $AC(q)$ and $f(p)$ never intersect, then $AC(q)$ lies uniformly above $f(p)$,[16] and $d(1)$ is not defined.

Next consider the existence of d^*. For increasing average cost $d(r)$ is well defined if and only if $c^* < \hat{p}$. Recall that property 2 (to be proved below) states that whenever $d(r)$ is well defined, $d(r) < \hat{p}$, and, for r large, $d(r)$ is strictly increasing in r. Hence d^* is well defined if and only if $c^* < \hat{p}$. With U-shaped average cost if $c^* \geqslant \hat{p}$, then $d(r)$ is not defined for any r. Whereas if $c^* < \hat{p}$, then $d(r)$ is well defined for r large. Again the result follows from property 2.

We then establish properties 2 and 3. Consider two cases.

Case 1. $AC(q)$ is increasing in q.

The function $AC(rf(p))$ is decreasing in p, $\forall p \in [0, \hat{p})$. Moreover, whenever $r'' > r'$, $AC(r'' f(p)) > AC(r' f(p))$. Hence $d(r'') > d(r')$. Moreover, $\forall r$, $d(r) < \hat{p}$. We then consider the limit properties of $d(r)$ as r goes to infinity.

(1a). Suppose $\lim_{q\to\infty} AC(q) \geqslant \hat{p}$. Since $d(r)$ is increasing in r and $d(r) < \hat{p}$, $\forall r$, $d^* = \lim_{r\to\infty} d(r)$ is well defined. We claim that $d^* = \hat{p}$. Suppose not, i.e. let $d^* < \hat{p}$. Define $\tilde{q}$ such that $AC(q) = d^*$.[17]

Next define $\tilde{p}(r)$ as satisfying $rf(p) = \tilde{q}$.[18] Clearly, $\lim_{r\to\infty} \tilde{p}(r) = \hat{p} > d^*$. Hence

[16] Since $f(0)$ is finite $AC(q)$ cannot lie uniformly below $f(p)$).

[17] Since $\lim_{q\to\infty} AC(q) \geqslant \hat{p} > d^*$, this is well defined whenever $b \leqslant d^*$. If $b > d^*$, then $AC(f(p))$ is bounded away from d^*, $\forall p$. Contradiction.

[18] Allowing for non-integer values of r, for r sufficiently large $\tilde{p}(r)$ is well defined.

there exists $\tilde{r}$ such that $\forall r \geqslant \tilde{r}$, $\tilde{p}(r) > d^*$. Clearly, $AC(\tilde{r}\tilde{p}(\tilde{r})) > d^*$. But this is a contradiction.

(1b) Suppose $\lim_{q\to\infty} AC(q) = \tilde{d} < \hat{p}$. Then we claim that $d^* = \tilde{d}$. Since $AC(rf(p))$ is decreasing in p it follows that $d^* \leqslant \tilde{d}$. We can mimic the argument in case (1a) above to rule out the possibility that $d^* < \tilde{d}$. Thus $d^* = \tilde{d}$.

Case 2. $AC(q)$ is U-shaped.

Consider r large enough such that $f^{-1}(q^*/r) > c^*$. (Since $f(c^*) > 0$, such an r always exists.) Clearly, for any such r, $\forall p \leqslant f^{-1}(q^*/r)$, $AC(rf(p))$ is decreasing in p. Moreover, whenever $r'' > r'$, $\forall p \leqslant f^{-1}(q^*/r')$, $AC(r'' f(p)) > AC(r' f(p))$. Hence for all r large enough, $d(r)$ is increasing in r. Moreover, $\forall r$, $d(r) < \hat{p}$.

We can argue as in case 1 above that d^* is well defined. Moreover, if $\lim_{q\to\infty} AC(q) \geqslant \hat{p}$ then $d^* = \tilde{p}$, and if $\lim_{q\to\infty} AC(q) = \tilde{d} < \hat{p}$, then $d^* = \tilde{d}$.

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