ON APPROXIMATION OF 1940 RESISTANCE LAW BY SIMPLE FUNCTIONS AND ITS EFFECT ON TRAJECTORIES OF PROJECTILES

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SUMMARY. For computing the trajectory of a projectile the retardation coefficient is generally taken from Resistance Law Tables which have been constructed from a large number of experimental data. To calculate the trajectories on an electronic computer one has to store the tables in the memory of the computer. Here attempts have been made to replace 1940 Resistance Law Tables by one or more simple functions by the method of least squares. The effect of this approximation on the trajectories has also been studied. In the final form of the approximation in which the 1940 Resistance Law Tables are replaced by three simple functions valid in three different ranges it is found that the maximum error of approximation is 0.2% and that in most of the part it is much less. On computing a trajectory by using these functions instead of the tables one finds an error of 0.01 ft. in a range of 86503 ft. and of 0.84 ft. in a maximum altitude of 17165 ft.

1. Introduction

Though the general equations of motion of a symmetrical projectile are quite complicated (Mc Shane *et al*, 1953, p. 192), for computational work it is found that the solution of a relatively simple system of equations gives an excellent approximation to the actual motion of a projectile. The actual trajectory can be derived from this approximate trajectory by a theory of small correction or perturbation. For calculating this approximate trajectory one generally makes the following simplifying assumptions that the projectile moves with its axis tangential to its trajectory and the only forces acting on the projectile are the drag and the force of gravity. The density of air and velocity of sound are functions of height of the projectile above earth's surface and there is no wind.

The air resistance or drag D of the projectile is generally written in the form $D = \frac{1}{2}\rho A.\ V^2C_D$, where C_D is the coefficient of drag, ρ the density of air, A the cross-sectional area of the projectile and V the velocity of the projectile. The drag is also written in the from $D = m.\ C.\ V^2.\ H(y).\ G.$, where m is the mass of the projectile; $H(y) = \frac{\rho}{\rho_0}$, ρ_0 being the density of air at ground level and C is the reciprocal of the Ballistic coefficient and G is called retardation coefficient.

This retardation coefficient G is not independent of V but varies much more slowly than do D. For small yaws G is nearly constant. G is actually a function of Mach number and Reynolds number. The dependence on Reynolds number can usually be neglected but in precise work the coefficient should be regarded as a function of the Mach number rather than as a function of V in order to allow for the

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effects of variation of velocity of sound. The velocity of sound is proportional to the square root of absolute temperature. Since the absolute temperature varies with height, tables giving G as a function of V should be entered not at V, the velocity of the projectile but at

$$W = \left(\frac{T_0}{T}\right)^{1/2} \cdot \; V \qquad \ldots \quad (1.1)$$

in order to enter at the correct Mach number. Here T is the absolute temperature and T_0 the absolute temperature under standard ballistic conditions.

So for calculating the drag one should take into account the variation of density and temperature with height. While the variations of temperature and density with height can be approximated quite accurately by simple functions of the altitude, the retardation coefficient G as a function of W is generally given in the form of tables based on extensive experimental data. The tables again can vary from one group of projectiles to another depending on their shapes viz, Gâvres Tables, Siaccis Tables etc. (Davis *et al.*, 1958, p 48).

Consequently, in order to integrate numerically the differential equations governing the motion of a projectile on a digital computer one has to introduce a large table for the retardations coefficient. In the present work we have investigated the possibility of approximating the 1940 Resistance Law Tables by some simple function or functions and also have studied the effect of this approximation on the trajectories of projectiles.

It is found from experiments that the function G remains almost constant for Mach numbers well below one (i.e. speeds well below the speed of sound). In the neighbourhood of the velocity of sound the coefficient rises rapidly, reaches a maximum (in the case of the 1940 Resistance Law it is at about $W = 1200$) followed by a slow decline as the velocity increases.

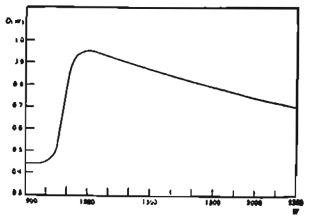

Fig 1. The curve of the 1940 Resistance Law.

APPROXIMATION OF RESISTANCE LAW BY SIMPLE FUNCTIONS

Since it is customary to approximate the coefficient of resistance by a function defined by a series of n-th power laws each good in a particular zone of velocities, a function of the form

$$K.W.\ (C_0+C_1W+C_2W^2)$$

where K, C_0, C_1, C_2 are constants, was first tried to approximate $G(W)$. On fitting a function of this form by the method of least squares it was found that the error was at some points of the table as large as 27%. When this function was used to calculate the trajectory of a projectile with initial velocity of 2700 ft/s with angle elevation of 30° the errors in calculation of range and maximum altitude were found to be 622 ft. in 86,503 ft. and 96 ft. in 17,165 ft. respectively.

The form of the graph of the retardation coefficient suggests the use of two different functions one to approximate the part between the constant value at subsonic region to the maximum of the curve which is in the sonic region i.e. for $1000 \leqslant W \leqslant 1210$ and the other to approximate the part of the curve starting from the maximum point to the supersonic region i.e. for $1210 < V \leqslant 3000$. For the first part we assumed a series of the form $a_0+\sum_1^3 a_n \cos nx$ where $X = \frac{(W-10^3)}{210}\pi$ so that the tangent to this curve is zero at $W = 10^3$ and $W = 1210$ and for the second part a quadratic function of the form $A+BW+CW^2$. The results of this approximation are definitely better. The maximum error was found to be 2.7% only. The trajectory, computed with the above founctions representing $G(W)$ and with the same initial conditions as in the previous case, was found to deviate only 5 ft. in range (correct value being 86503 ft.) and 0.8 ft. in maximum altitude (correct value being 17165 ft.).

To better the results even further it was noticed that the approximation in the supersonic range i.e. $1210 < V \leqslant 3000$ was much better than that in the subsonic range. Actually the maximum error in the supersonic range was about 0.75% while for the subsonic part it was about 2.7%. So it appeared that the approximation in the subsonic range needed improvement. Ultimately we broke up the range into three parts and approximated each part by different functions as follows.

For $100 \leqslant W \leqslant 1210$, $G(W) = a_0+\sum_{n=1}^{4} (a_n \cos nx+b_n \sin nx)$

For $1210 < W \leqslant 1270$, $G(W) = c+dW$

For $1270 < W \leqslant 3000$, $G(W) = A+BW+CW^2$.

For values of W less than 1000, $G(W)$ was all along taken to be a constant.

With these approximations the error was everywhere less than 0.2% and the errors in calculated range and maximum altitude came out to be 0.01 ft. in 86503 ft. and 0.8 ft. in 17165 ft. respectively. This is practically negligible.

2. The equations of motion of a projectile

With the simplifying assumptions mentioned earlier, the projectile is regarded as a material particle acted on by the force of gravity and by tangential retarding force due to the resistance of air. The acceleration due to gravity is considered to be constant in magnitude and direction which means that the earth is taken to be flat and that the height reached by the projectile is small compared to the radius of the earth.

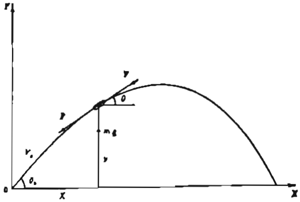

Fig 2. Forces acting on a projectile.

Taking x-axis horizontally in the direction of firing and y-axis vertically upwards through the point of projection, the system of differential equations expressing the motion of the centre of mass of an artillery shell in air can be put in the following form

$$\frac{dx}{dt} = u \qquad \ldots \quad (2.1)$$

$$\frac{dy}{dt} = up \qquad \ldots \quad (2.2)$$

$$\frac{dp}{dt} = -\frac{g}{u} \qquad \ldots \quad (2.3)$$

$$\frac{du}{dt} = -Cu^2\sqrt{1+p^2}\, H(y)\, G(W) \qquad \ldots \quad (2.4)$$

where

- u denotes the horizonal component of velocity of the centre of mass of the shell
- p the tangent of the angle between the velocity vector and the x-axis.
- g the acceleration due to gravity
- C the reciprocal of the ballistic coefficient
- $H(y)$ the relative change in density with height
- $G(W)$ the retardation coefficient where $W = V\sqrt{\frac{T_0}{T(y)}} = u\sqrt{1+p^2}\sqrt{\frac{T_0}{T(y)}}$ as mentioned in equation (1.1).

The functions $H(y)$ and $T(y)$ are available in Tables or can be approximated by suitable functions quite accurately. Generally they are taken in the form

$$H(y) = e^{-\lambda y} \qquad \ldots \quad (2.5)$$

where λ is a constant and

$$T(y) = T_0 - K_0 Y. \qquad \ldots \quad (2.6)$$

where K_0 is a constant, the temperature gradient.

The function $G(W)$ is generally given in the Resistance Law Tables. In the present work our main task is to replace the 1940 Resistance Law Tables by some suitable functions so that for numerical integration of the system of equations (2.1)... (2.4) we can use these functions for $G(W)$ instead of the tables.

3. Numerical integration of the system of differential equations

For numerical integration of the system of differential equations (2.1) ... (2.4), we resorted to Runge-Kutta method (Scarborough, 1958, p. 314). For the sake of computational advantages the following scalings were done

$\bar{x} = x.10^{-5}$; $\bar{y} = y.10^{-5}$; $\bar{p} = p.10^{-1}$; $\bar{u} = u.10^{-4}$ and $\bar{g} = g.10^{-4}$.

Then the above system reduces to

$$f_1 = \frac{d\bar{x}}{dt} = .1u \qquad \ldots \quad (3.1)$$

$$f_2 = \frac{d\bar{y}}{dt} = u\,\bar{p} \qquad \ldots \quad (3.2)$$

$$f_3 = \frac{d\bar{p}}{dt} = -.1 \cdot \frac{\bar{g}}{\bar{u}} \qquad \ldots \quad (3.3)$$

$$f_4 = \frac{d\bar{u}}{dt} = -10.C.\bar{u}^2\sqrt{.01+p^2}\;G(W)H(\bar{y}). \qquad \ldots \quad (3.4)$$

Given $\bar{x}_i$, $\bar{y}_i$, $\bar{p}_i$ and $\bar{u}_i$ i.e. the values of $\bar{x}$, $\bar{y}$, $\bar{p}$ and $\bar{u}$ at the i-th stage to get the values of $\bar{x}_{i+1}$, $\bar{y}_{i+1}$, p_{i+1} and $\bar{u}_{i+1}$ we first compute the following expressions

$$\left.\begin{aligned}
K_{1j} &= f_j(\bar{x}_i,\ \bar{y}_i,\ \bar{p}_i,\ \bar{u}_i)\\
K_{2j} &= f_j\left(\bar{x}_i+\frac{K_{11}}{2}h,\ \bar{y}_i+\frac{K_{12}}{2}h,\ \bar{p}_i+\frac{K_{13}}{2}h,\ \bar{u}_i+\frac{K_{14}}{2}h\right)\\
K_{3j} &= f_j\left(\bar{x}_i+\frac{K_{21}}{2}h,\ \bar{y}_i+\frac{K_{22}}{2}h,\ \bar{p}_i+\frac{K_{23}}{2}h,\ \bar{u}_i+\frac{K_{24}}{2}h\right)\\
K_{4j} &= f_j(\bar{x}_i+K_{31}h,\ \bar{y}_i+K_{32}h,\ \bar{p}_i+K_{33}h,\ u_i+K_{34}h)
\end{aligned}\right\} \qquad \ldots \quad (3.5)$$

where h is the interval of integration which we have taken constant and equal to 0.5 seconds.

Then the values of a_i's, $i = 1, 2, 3, 4$ are computed using the formula

$$a_i = (K_{1i}+2K_{2i}+2K_{3i}+K_{4i}) \cdot \frac{h}{6} \qquad \ldots \quad (3.6)$$

and finally $\bar{x}_{t+1}, \bar{y}_{t+1}, \bar{p}_{t+1}, \bar{u}_{t+1}$ are calculated from

$$\bar{x}_{t+1} = \bar{x}_t+a_1;\ \bar{y}_{t+1} = \bar{y}_t+a_2;\ \bar{p}_{t+1} = \bar{p}_t+a_3;\ \bar{u}_{t+1} = \bar{u}_t+a_4. \qquad ...(3.7)$$

This process is repeated till the value of $\bar{y}_n$ (for some n) becomes negative (i.e. till the projectile reaches the ground).

4. Approximating $G(W)$ by a function of the form $KW^{C_0+C_1W+C_2W^2}$

The retardation coefficient is often taken to be of the form KW^n where K is a constant and n takes up different integral values in different ranges of values of W. So it appears logical to assume n a function of W and a function of the form

$$G(W) = K\ .\ W^{C_0+C_1W+C_2W^2} \qquad ... \quad (4.1)$$

was chosen to approximate the 1940 Resistance Law Table.

The approximation was done by the method of least squares (Whittaker and Robinson, 1954, p.200). The normal equations are derived as follows.

Denoting $\log G_i$ by F_i and $\log K$ by E where G_i denotes the i-th value of G in the Resistance Law Tables and if N be the total number of value of G in the Tables, we minimise the sum of squares of errors namely

$$S = \Sigma\{F_i-E-(C_0+C_1W_i+C_2W_i^2)\log W_i\}^2. \qquad ... \quad (4.2)$$

Differentiating S partially with respect to E, C_0, C_1 and C_2 and equating them to zero one gets four linear equations with four unknowns E, C_0, C_1, C_2.

$$\left.\begin{aligned}
\Sigma F_i &= NE+C_0\Sigma\log W_i+C_1\Sigma W_i\log W_i+C_2\Sigma W_i^2\log W_i\\
\Sigma F_i\log W_i &= E\Sigma\log W_i+C_0\Sigma(\log W_i)^2+C_1\Sigma W_i(\log W_i)^2+C_2\Sigma W_i^2(\log W_i)^2\\
\Sigma F_iW_i\log W_i &= E\Sigma W_i\log W_i+C_0\Sigma W_i(\log W_i)^2+C_1\Sigma W_i^2(\log W_i)^2+C_2\Sigma W_i^3(\log W_i)^2\\
\Sigma F_iW_i^2\log W_i &= E\Sigma W_i^2\log W_i+C_0\Sigma W_i^2(\log W_i)^2+C_1\Sigma W_i^3(\log W_i)^2+C_2\Sigma W_i^4(\log W_i)^2
\end{aligned}\right\} \qquad ... \quad (4.3)$$

On solving this system of equations the following values were obtained

$$\left.\begin{aligned}
K &= 0.99565969\times10^{30}\\
C_0 &= 19.269254\\
C_1 &= 100.95272\\
C_2 &= 90.709656
\end{aligned}\right\} \qquad ... \quad (4.4)$$

where in (4.1) W has been scaled down by a factor of 10^{-4} for convenience of computation. This function has been tabulated as approximation 1 (APP 1) in Table 1. The correct values of $G(W)$ taken from 1940 Resistance Law are given under column $G(W)$ and the percentage error is given in column ERR 1. As is evident from the tables the approximation is not satisfactory. The percentage error is all along quite high reaching values as large as 27.5%

In order to study how the errors in approximating the retardation coefficient $G(W)$ affect the trajectory of projectiles, a trajectory was computed with this approximation and compared with the trajectory computed with the correct values of $G(W)$ from the Resistance Law Table. Columns (x_c, y_c) of Table 2 give the exact trajectory and columns (x_1, y_1) of Table 2 the trajectory with this approximation. The initial velocity was taken to be 2700 ft/s, the angle of elevation 30° and ballistic coefficient 4.956 for this particular trajectory. The range is calculated from the last two values of x and y by assuming that part of the trajectory to be a straight line. If (x_{n+1}, y_{n+1}) be the point of the trajectory where y becomes negative for the first time then the range

$$R = x_n+(x_{n+1}-x_n)\cdot\frac{y_n}{y_n-y_{n+1}}. \qquad \dots \quad (4.5)$$

On calculating the range and the maximum altitude from the tables for both the trajectories one finds an error of 622 ft. in a range of 86503 ft. and 96 ft. in maximum altitude of 17165 ft. So the approximation of 1940 Law by a function of the form (4.1) is not satisfactory.

5. Approximating $G(W)$ by two functions for two ranges

The graph of $G(W)$ plotted from the 1940 Resistance Law Tables, Fig. 1, suggests the use of two different functions for approximating two parts of the curve. One sinusoidal curve for the part $10000 \leqslant W \leqslant 1210$ and a polynomial for the part $1210 < W \leqslant 3000$. For $W < 1000$ we are taking all along G as a constant and equal to 0.444. The range above 3000, we are not considering at all.

In consequence we assume

$$G(W) = a_0+\sum_1^3 a_n \cos nx \quad \text{for } 1000 \leqslant W \leqslant 1210, \qquad \dots \quad (5.1)$$

where

$$X = \frac{(W-10^3)}{210}\times\pi.$$

So that the tangent is zero at $W = 1000$ and $W = 1210$; and

$$G(W) = A+BW+CW^2 \quad \text{for } 1210 < W \leqslant 3000. \qquad \dots \quad (5.2)$$

The system of equations to be solved for fitting by the method of least squares the trigonometric series (5.1) for the first range is

$$\begin{bmatrix} N & \Sigma\cos x & \Sigma\cos 2x & \Sigma\cos 3x \\ \Sigma\cos x & \Sigma\cos^2 x & \Sigma\cos x\cos 2x & \Sigma\cos x\cos 3x \\ \Sigma\cos 2x & \Sigma\cos x\cos 2x & \Sigma\cos^2 2x & \Sigma\cos 2x\cos 3x \\ \Sigma\cos 3x & \Sigma\cos x\cos 3x & \Sigma\cos 2x\cos 3x & \Sigma\cos^2 3x \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \Sigma G \\ \Sigma G\cos x \\ \Sigma G\cos 2x \\ \Sigma G\cos 3x \end{bmatrix} \qquad \dots \quad (5.3)$$

The solutions are found to be

$$\left.\begin{aligned} a_0 &= 0.73085861 \\ a_1 &= -0.28289856 \\ a_2 &= -0.044988202 \\ a_3 &= 0.030620387 \end{aligned}\right\} . \qquad \dots \quad (5.4)$$

The normal equations for the quadratic curve are

$$\begin{bmatrix} N & \Sigma W_i & \Sigma W_i^2 \\ \Sigma W_i & \Sigma W_i^2 & \Sigma W_i^3 \\ \Sigma W_i^2 & \Sigma W_i^3 & \Sigma W_i^4 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} \Sigma G_i \\ \Sigma G_i W_i \\ \Sigma G_i W_i^2 \end{bmatrix} . \qquad \dots \quad (5.5)$$

The solutions are

$$\left.\begin{aligned} A &= 1.3916324 \\ B &= -0.41160082 \cdot 10^{-3} \\ C &= 0.04502329 \cdot 10^{-6} \end{aligned}\right\} . \qquad \dots \quad (5.6)$$

The result of this approximation is tabulated in Table 1 as APP-2 and the percentage error under the column ERR-2. As can be seen from the tables, this approximation of $G(W)$ by two functions (5.1) and (5.2) gives fairly good results. The maximum percentage error for the quadratic function is less than 0.2 for $W > 1260$ reaching 0.7 at $W = 1220$. The maximum error for the cosine series is nearly 2.7%.

The trajectory calculated with this approximation, with the same initial values as before, is gives in Table 2 under columns x_2 and y_2. One can compare this trajectory with the correct trajectory calculated from 1940 Resistance Law which is given under columns x_c, and y_c in Table 2. On calculating the range from the last two values of the table one finds an error of only 5 ft. in a range of 86503.2 ft. The error in maximum height comes out as 0.8 ft. in 17165 ft. This error in range is as small as 0.0058% and this can be neglected in most practical cases. One has to store now only 7 constants viz, a_0, a_1, a_2, a_3, A, B, C and a cosine subroutine in the memory of the computer. One of the main reasons for this good approximation seems to be the fact that the curves have been fitted by the method of least squares. The approximate values of $G(W)$ are sometimes higher than the correct values and sometimes lower so that the error in calculation of trajectory is continuously rectified during computation.

6. Approximation of $G(W)$ by three different functions

As is evident from Table 1, the approximation of the 1940 Resistance Law Tables in the supersonic range by a second degree polynomial is quite accurate in the range $1260 \leqslant W \leqslant 3000$, the error being less than 0.2%. The error gradually increases between $1260 > W \geqslant 1210$ reaching 0.7%, while the percentage error for the cosine series is comparatively higher the maximum being 2.7%. When we study the trajectory computed with these functions replacing the 1940 Resistance Law Tables, we find that the errors in horizontal distance and vertical height of the projectile begin to build up at a quicker pace only when the W comes in the subsonic range. So it appears that to get even better results the cosine series approximation must be improved.

After several trials it was found that the approximation of 1940 Law by the following three functions valid in three ranges gives very good results.

For $$1000 \leqslant W \leqslant 1210,\; G(W) = a_0 + \sum_{1}^{4} (a_n \cos nx + b_n \sin nx) \qquad \ldots (6.1)$$

where $X = \left(\frac{W-10^3}{200}\right)\pi.$

For $$1210 < W \leqslant 1270,\; G(W) = d_0 + d_1 W. \qquad \ldots (6.2)$$

For $$1270 < W \leqslant 3000,\; G(W) = A + BW + CW^2. \qquad \ldots (6.3)$$

On fitting the curve (6.1) by the method of least squares and solving the normal equations, the constants come out to be

$$\begin{aligned} a_0 &= +0.769737 & b_1 &= -0.088167 \\ a_1 &= -0.274767 & b_2 &= -0.012182 \\ a_2 &= -0.090770 & b_3 &= +0.029389 \\ a_3 &= +0.021537 & b_4 &= +0.006091. \\ a_4 &= +0.018603 \end{aligned} \qquad \ldots (6.4)$$

The constants for the straight line approximation (6.2) are

$$\begin{aligned} d_0 &= 1.17736 \\ d_1 &= -0.186 \times 10^{-3}. \end{aligned} \qquad \ldots (6.5)$$

The quadratic curve (6.3) is the same as was used in the last approximation and so the constants are same as in (5.6).

For this approximation of the 1940 Resistance Law by three different functions the error never exceeds 0.2% and in most of the part it is much less.

Columns APP-3 and ERR-3 of Table 1 give the values of $G(W)$ by this approximation and the percentage errors at the points given in column W.

The trajectory calculated with the same initial values as before viz, $V_0 = 2700$ ft/s, $\theta_0 = 30°$, but using this new approximation of the 1940 Law is given in columns x_3 and y_3 of Table 2, so that one can compare with the exact values given in columns x_c and y_c.

The range calculated from the last two values by the formula (4.5) is found to be 86503.18 ft. where the correct range is 86503.19 ft. The error is practically negligible. So it seems one can use this approximation of 1940 Law quite confidently to calculate the trajectory of a projectile. This involves 14 constants and one subroutine for sine or cosine functions.

TABLE 1. THE 1940 RESISTANCE LAW, THE APPROXIMATIONS AND PERCENTAGE ERRORS

$G(W)$ = The 1940 Resistance Law. APP-1, APP-2, APP-3 = The First, Second and Third Approximations. ERR-1, ERR-2, ERR-3 = The Corresponding Percentage Errors

W	$G(W)$	APP-1	ERR-1	APP-2	ERR-2	APP-3	ERR-3
1000	0.44434	0.53271	−19.887	0.43359	2.419	0.444340	0.0
1002	0.44444	0.53574	−20.543	0.43368	2.422	0.444407	0.007
1004	0.44457	0.53877	−21.188	0.43393	2.394	0.444591	−0.005
1006	0.44474	0.54179	−21.821	0.43435	2.336	0.444867	−0.029
1008	0.44495	0.54480	−22.442	0.43495	2.248	0.445218	−0.060
1010	0.44521	0.54781	−23.046	0.43572	2.132	0.445631	−0.095
1012	0.44553	0.55082	−23.632	0.43667	1.988	0.446098	−0.128
1014	0.44592	0.55381	−24.196	0.43782	1.817	0.446618	−0.157
1016	0.44640	0.55681	−24.733	0.43915	1.623	0.447194	−0.178
1018	0.44699	0.55979	−25.235	0.44070	1.408	0.447835	−0.189
1020	0.44769	0.56277	−25.705	0.44245	1.171	0.448555	−0.193
1022	0.44853	0.56574	−26.132	0.44442	0.915	0.449373	−0.188
1024	0.44953	0.56870	−26.510	0.44663	0.645	0.450311	−0.174
1026	0.45070	0.57166	−26.838	0.44908	0.360	0.451395	−0.154
1028	0.45208	0.57461	−27.103	0.45178	0.067	0.452655	−0.127
1030	0.45369	0.57755	−27.300	0.45474	−0.231	0.454123	−0.096
1032	0.45556	0.58048	−27.422	0.45798	−0.530	0.455834	−0.060
1034	0.45771	0.58341	−27.463	0.46150	−0.827	0.457823	−0.025
1036	0.46018	0.58633	−27.413	0.46531	−1.115	0.460125	0.012
1038	0.46298	0.58924	−27.271	0.46943	−1.394	0.462777	0.044
1040	0.46615	0.59214	−27.028	0.47387	−1.656	0.465814	0.072
1042	0.46973	0.59503	−26.676	0.47862	−1.894	0.469269	0.098
1044	0.47373	0.59792	−26.215	0.48371	−2.107	0.473176	0.117
1046	0.47818	0.60080	−25.642	0.48914	−2.292	0.477562	0.129
1048	0.48311	0.60366	−24.954	0.49491	−2.442	0.482453	0.136
1050	0.48853	0.60652	−24.153	0.50103	2.558	0.487872	0.135
1052	0.49448	0.60937	−23.235	0.50749	−2.632	0.493837	0.130
1054	0.50095	0.61221	−22.211	0.51432	−2.668	0.500360	0.118
1056	0.50797	0.61505	−21.079	0.52149	−2.662	0.507450	0.102
1058	0.51554	0.61787	−19.849	0.52902	−2.615	0.515110	0.083
1060	0.52366	0.62068	−18.528	0.53690	−2.528	0.523337	0.062
1062	0.53233	0.62349	−17.124	0.54512	−2.402	0.532125	0.039
1064	0.54154	0.62628	−15.648	0.55368	−2.241	0.541459	0.015
1066	0.55128	0.62906	−14.110	0.56256	−2.047	0.551320	−0.007
1068	0.56152	0.63184	−12.523	0.57177	−1.825	0.561685	−0.029
1070	0.57225	0.63460	−10.896	0.58128	−1.579	0.572525	−0.048
1072	0.58343	0.63736	− 9.243	0.59109	−1.313	0.583806	−0.064
1074	0.59503	0.64010	− 7.575	0.60117	−1.032	0.595488	−0.077
1076	0.60701	0.64284	− 5.902	0.61151	−0.742	0.607531	−0.086
1078	0.61932	0.64556	− 4.236	0.62209	−0.448	0.619887	−0.092

TABLE 1 (*Contd.*). THE 1940 RESISTANCE LAW. THE APPROXIMATIONS AND PERCENTAGE ERRORS

$G(W)$ = The 1940 Resistance Law. APP-1, APP-2, APP-3 = The First, Second and Third Approximations. ERR-1, ERR-2, ERR-3 = The Corresponding Percentage Errors

W	$G(W)$	APP-1	ERR-1	APP-2	ERR-2	APP-3	ERR-3
1080	0.63191	0.64827	− 2.589	0.63289	−0.156	0.632509	−0.095
1082	0.64475	0.65007	− 0.965	0.64389	0.133	0.645344	−0.092
1084	0.65777	0.65366	0.624	0.65506	0.412	0.658340	−0.087
1086	0.67091	0.65634	2.171	0.66638	0.675	0.671441	−0.079
1088	0.68413	0.65901	3.672	0.67782	0.923	0.684503	−0.068
1090	0.69735	0.66167	5.117	0.68935	1.147	0.697741	−0.056
1092	0.71054	0.66432	6.506	0.70095	1.350	0.710830	−0.041
1094	0.72303	0.66695	7.833	0.71258	1.527	0.723806	−0.024
1096	0.73656	0.66957	9.094	0.72422	1.676	0.736617	−0.008
1098	0.74928	0.67219	10.289	0.73583	1.705	0.749215	0.009
1100	0.76174	0.67479	11.415	0.74739	1.884	0.761553	0.025
1102	0.77389	0.67738	12.471	0.75886	1.942	0.773587	0.039
1104	0.78570	0.67995	13.459	0.77022	1.970	0.785277	0.054
1106	0.79711	0.68252	14.376	0.78143	1.967	0.796588	0.065
1108	0.80800	0.68507	15.223	0.79247	1.933	0.807489	0.074
1110	0.81862	0.68761	16.004	0.80330	1.872	0.817951	0.082
1112	0.82866	0.69014	16.716	0.81390	1.782	0.827954	0.085
1114	0.83819	0.69266	17.363	0.82423	1.665	0.837478	0.085
1116	0.84919	0.69516	18.138	0.83429	1.755	0.846512	0.080
1118	0.85565	0.69766	18.465	0.84403	1.358	0.855047	0.071
1120	0.86357	0.70013	18.926	0.85343	1.174	0.863078	0.057
1122	0.87093	0.70260	19.327	0.86249	0.970	0.870607	0.037
1124	0.87777	0.70506	19.676	0.87116	0.753	0.877638	0.015
1126	0.88412	0.70750	19.977	0.87945	0.528	0.884180	−0.007
1128	0.89000	0.70993	20.233	0.88732	0.301	0.890245	−0.028
1130	0.89544	0.71234	20.448	0.89478	0.074	0.895849	−0.046
1132	0.90046	0.71475	20.624	0.90180	−0.149	0.901008	−0.061
1134	0.90510	0.71714	20.767	0.90838	−0.363	0.905744	−0.071
1136	0.90938	0.71952	20.878	0.91452	−0.565	0.910079	−0.077
1138	0.91332	0.72188	20.961	0.92020	−0.753	0.914037	−0.079
1140	0.91694	0.72423	21.017	0.92542	−0.925	0.917643	−0.077
1142	0.92027	0.72657	21.048	0.93019	−1.078	0.920922	−0.071
1144	0.92333	0.72889	21.058	0.93452	−1.212	0.923899	−0.062
1146	0.92613	0.73120	21.047	0.93839	−1.324	0.926600	−0.051
1148	0.92870	0.73350	21.018	0.94183	−1.414	0.929050	−0.038
1150	0.93105	0.73579	20.972	0.94484	−1.481	0.931273	−0.024
1152	0.93320	0.73806	20.911	0.94744	−1.526	0.933292	−0.010
1154	0.93516	0.74031	20.836	0.94963	−1.547	0.935127	0.004
1156	0.93695	0.74256	20.747	0.94144	−1.546	0.936798	0.016
1158	0.93857	0.74479	20.647	0.95288	−1.524	0.938323	0.026

TABLE 1 (*Contd.*). THE 1940 RESISTANCE LAW, THE APPROXIMATIONS AND PERCENTAGE ERRORS

$G(W)$ = The 1940 Resistance Law. APP-1, APP-2, APP-3 = The First, Second and Third Approximations. ERR-1, ERR-2, ERR-3 = The Corresponding Percentage Errors

W	$G(W)$	APP-1	ERR-1	APP-2	ERR-2	APP-3	ERR-3
1160	0.94005	0.74700	20.536	0.95397	−1.481	0.939718	0.035
1162	0.94139	0.74921	20.415	0.95473	−1.417	0.940906	0.042
1164	0.94260	0.75139	20.285	0.95519	−1.336	0.942169	0.046
1166	0.94370	0.75357	20.147	0.95537	−1.236	0.943247	0.048
1168	0.94460	0.75573	20.002	0.95529	−1.122	0.944239	0.048
1170	0.94557	0.75788	19.850	0.95498	−0.995	0.945149	0.045
1172	0.94637	0.76001	19.692	0.95446	−0.855	0.945983	0.041
1174	0.94708	0.76213	19.529	0.95377	−0.706	0.946744	0.035
1176	0.94771	0.76423	19.360	0.95292	−0.550	0.947435	0.020
1178	0.94820	0.76632	19.187	0.95195	−0.389	0.948067	0.021
1180	0.94875	0.76840	19.009	0.95088	−0.224	0.948610	0.015
1182	0.94917	0.77046	18.828	0.94973	−0.059	0.949096	0.008
1184	0.94954	0.77251	18.644	0.94854	0.105	0.949516	0.003
1186	0.94985	0.77454	18.457	0.94733	0.265	0.949870	−0.002
1188	0.95011	0.77656	18.266	0.94612	0.420	0.950160	−0.005
1190	0.95032	0.77856	18.073	0.94493	0.568	0.950389	−0.007
1192	0.95049	0.78055	17.879	0.94378	0.706	0.950560	−0.007
1194	0.95062	0.78253	17.682	0.94270	0.833	0.950680	−0.006
1196	0.95071	0.78449	17.484	0.94170	0.947	0.950754	−0.005
1198	0.95077	0.78644	17.284	0.94081	1.048	0.950790	−0.002
1200	0.95080	0.78837	17.083	0.94002	1.134	0.950800	0.0
1210	0.95040	0.79782	16.063	0.93815	1.289	0.950920	−0.045
1220	0.94962	0.80689	15.030	0.95648	0.723	0.950440	−0.086
1230	0.94834	0.81560	13.997	0.95347	−0.541	0.948580	−0.025
1240	0.94672	0.82305	12.908	0.95046	−0.395	0.946720	0.0
1250	0.94486	0.83192	11.953	0.94747	−0.276	0.944860	0.0
1260	0.94278	0.83953	10.952	0.94448	−0.181	0.943000	−0.023
1270	0.94054	0.84677	9.970	0.94151	−0.103	0.941140	−0.064
1280	0.93817	0.85364	9.010	0.93854	−0.039	0.938538	−0.039
1290	0.93568	0.86015	8.073	0.93558	0.011	0.935579	0.011
1300	0.93310	0.86629	7.160	0.93263	0.050	0.932629	0.050
1320	0.92774	0.87751	5.414	0.92676	0.106	0.926756	0.106
1340	0.92218	0.88732	3.780	0.92092	0.137	0.920919	0.137
1360	0.91651	0.89577	2.263	0.91512	0.152	0.915118	0.152
1380	0.91078	0.90288	0.867	0.90935	0.157	0.909353	0.157
1400	0.90503	0.90872	− 0.408	0.90362	0.156	0.903625	0.155
1430	0.89640	0.91510	− 2.086	0.89510	0.145	0.895099	0.145
1460	0.88781	0.91907	− 3.521	0.88665	0.130	0.886654	0.130
1490	0.87928	0.92057	− 4.696	0.87829	0.113	0.878290	0.113
1520	0.87084	0.91989	− 5.632	0.87001	0.096	0.870008	0.096

TABLE 1 (*Contd.*). THE 1940 RESISTANCE LAW. THE APPROXIMATIONS AND PERCENTAGE ERRORS

$G(W)$ = The 1940 Resistance Law. APP-1, APP-2, APP-3 = The First, Second and Third Approximations. ERR-1, ERR-2, ERR-3 = The corresponding Percentage Errors

W	$G(W)$	APP-1	ERR-1	APP-2	ERR-2	APP-3	ERR-3
1550	0.86249	0.91722	− 6.346	0.86181	0.079	0.861806	0.079
1580	0.85423	0.91278	− 6.854	0.85369	0.064	0.853685	0.064
1610	0.84606	0.90676	− 7.174	0.84565	0.049	0.845646	0.049
1640	0.83798	0.89936	− 7.325	0.83769	0.035	0.837687	0.035
1670	0.82999	0.89078	− 7.322	0.82981	0.022	0.829810	0.022
1700	0.82210	0.88114	− 7.181	0.82201	0.011	0.822013	0.011
1730	0.81430	0.87060	− 6.922	0.81430	0.0	0.814298	0.0
1760	0.80658	0.85949	− 6.560	0.80666	−0.010	0.806664	−0.010
1790	0.79896	0.84776	− 6.107	0.79911	−0.019	0.799110	−0.019
1820	0.79143	0.83560	− 5.581	0.79164	−0.026	0.791638	−0.026
1850	0.78399	0.82315	− 4.995	0.78425	−0.033	0.784247	−0.033
1880	0.77664	0.81050	− 4.360	0.77694	−0.038	0.776937	−0.038
1910	0.76938	0.79777	− 3.690	0.76971	−0.043	0.769708	−0.043
1940	0.76221	0.78504	− 2.996	0.76256	−0.046	0.762560	−0.046
1970	0.75512	0.77240	− 2.288	0.75549	−0.049	0.755492	−0.049
2000	0.74813	0.75991	− 1.575	0.74851	−0.050	0.748506	−0.050
2030	0.74122	0.74765	− 0.867	0.74160	−0.051	0.741602	−0.051
2060	0.73440	0.73566	− 0.172	0.73478	−0.051	0.734778	−0.051
2090	0.72766	0.72401	0.501	0.72803	−0.051	0.728035	−0.051
2120	0.72102	0.71274	1.148	0.72137	−0.049	0.721273	−0.049
2150	0.71445	0.70188	1.759	0.71479	−0.048	0.714792	−0.048
2180	0.70798	0.69147	2.331	0.70829	−0.044	0.708292	−0.044
2210	0.70158	0.68155	2.855	0.70187	−0.042	0.701874	−0.042
2240	0.69528	0.67213	3.329	0.69554	−0.037	0.695536	−0.037
2270	0.68905	0.66325	3.744	0.68928	−0.033	0.689279	−0.033
2300	0.68291	0.65492	4.099	0.68310	−0.028	0.683104	−0.028
2330	0.67685	0.64716	4.387	0.67701	−0.024	0.677009	−0.024
2360	0.67088	0.63998	4.605	0.67100	−0.017	0.670996	−0.017
2390	0.66498	0.63341	4.747	0.66506	−0.013	0.665063	−0.012
2420	0.65917	0.62746	4.810	0.65921	−0.006	0.659212	−0.006
2450	0.65344	0.62214	4.790	0.65344	0.0	0.653441	0.0
2480	0.64778	0.61747	4.679	0.64775	0.004	0.647752	0.004
2510	0.64221	0.61345	4.478	0.64214	0.010	0.642144	0.010
2540	0.63671	0.61011	4.178	0.63662	0.015	0.636617	0.015
2570	0.63129	0.60745	3.777	0.63117	0.019	0.631170	0.019
2600	0.62595	0.60549	3.268	0.62581	0.023	0.625805	0.023
2630	0.62068	0.60425	2.647	0.62052	0.026	0.620521	0.026
2660	0.61549	0.60375	1.907	0.61532	0.028	0.615318	0.028
2690	0.61038	0.60400	1.045	0.61020	0.030	0.610196	0.030
2720	0.60533	0.60503	0.049	0.60516	0.029	0.605155	0.029
2750	0.60036	0.60687	− 1.084	0.60020	0.027	0.600195	0.027
2780	0.59547	0.60953	− 2.361	0.59532	0.026	0.595316	0.026
2810	0.59064	0.61305	− 3.794	0.59052	0.021	0.590518	0.020
2840	0.58588	0.61746	− 5.391	0.58580	0.013	0.585801	0.013
2870	0.58120	0.62281	− 7.159	0.58117	0.006	0.581166	0.006
2900	0.57658	0.62912	− 9.113	0.57661	−0.005	0.576611	−0.005
2930	0.57203	0.63645	−11.262	0.57214	−0.019	0.572137	−0.019
2960	0.56755	0.64485	−13.621	0.56774	−0.034	0.567745	−0.034
2990	0.56314	0.65438	−16.202	0.56343	−0.052	0.563433	−0.052

TABLE 2. THE TRAJECTORIES CALCULATED WITH 1940 RESISTANCE LAW AND THE THREE APPROXIMATIONS

T = Time, (XC, YC) = Trajectory with 1940 Resistance Law,
($X1$, $Y1$), ($X2$, $Y2$) ($X3$, $Y3$) = The Trajectories with the Three Approximate Formulas

T second	XC FT	YC FT	$X1$ FT	$Y1$ FT	$X2$ FT	$Y2$ FT	$X3$ FT	$Y3$ FT
0.5	1159.588	665.488	1159.704	665.555	1159.591	665.490	1159.585	665.487
1.0	2300.618	1312.347	2301.230	1312.697	2300.620	1312.353	2300.616	1312.347
1.5	3423.866	1941.144	3425.512	1942.081	3423.887	1941.157	3423.869	1941.147
2.0	4530.057	2552.411	4533.373	2554.287	4530.094	2552.432	4530.070	2552.420
2.5	5619.875	3146.647	5625.530	3149.833	5619.928	3146.677	5619.898	3146.661
3.0	6693.959	3724.318	6702.657	3729.186	6694.029	3724.357	6693.994	3724.339
3.5	7752.911	4285.863	7765.311	4292.771	7752.997	4285.912	7752.956	4285.891
4.0	8797.297	4831.697	8814.030	4840.074	8797.307	4831.753	8797.351	4831.729
4.5	9827.649	5362.207	9840.295	5374.152	9827.760	5362.270	9827.709	5362.243
5.0	10844.471	5877.760	10871.547	5892.635	10844.601	5877.828	10844.534	5877.798
5.5	11848.236	6378.701	11881.102	6396.731	11848.360	6378.772	11848.298	6378.740
6.0	12839.393	6865.357	12878.007	6886.726	12839.517	6865.429	12839.450	6865.394
6.5	13818.366	7338.037	13864.142	7362.890	13818.485	7338.107	13818.413	7338.070
7.0	14785.555	7797.033	14838.124	7825.476	14785.665	7797.009	14785.588	7797.059
7.5	15741.341	8242.621	15800.861	8274.725	15741.436	8242.681	15741.354	8242.638
8.0	16686.084	8675.064	16752.643	8710.864	16686.159	8675.116	16686.072	8675.070
8.5	17620.125	9094.611	17693.743	9134.110	17620.175	9094.652	17620.084	9094.604
9.0	18543.780	9501.409	18624.420	9544.670	18543.809	9501.526	18543.714	9501.476
9.5	19457.384	9895.951	19544.923	9942.743	19457.370	9895.963	19457.270	9895.911
10.0	20361.203	10278.183	20455.487	10328.618	20361.180	10278.178	20361.046	10278.123
10.5	21255.525	10648.397	21356.339	10702.178	21255.429	10648.373	21255.320	10648.316
11.0	22140.615	11006.788	22247.697	11063.000	22140.472	11006.743	22140.358	11006.684
11.5	23016.725	11353.540	23129.771	11413.853	23016.532	11353.474	23016.414	11353.412
12.0	23884.007	11688.831	24002.763	11752.201	23883.850	11688.742	23883.728	11688.677
12.5	24742.959	12012.829	24866.860	12079.103	24742.657	12012.716	24742.530	12012.649
13.0	25593.531	12325.695	25722.279	12394.712	25593.171	12325.557	25593.040	12325.488
13.5	26436.022	12627.584	26569.178	12699.178	26435.602	12627.420	26435.467	12627.349
14.0	27270.632	12918.643	27407.745	12992.645	27270.150	12918.453	27270.011	12918.380
14.5	28097.551	13199.014	28238.156	13275.254	28097.007	13198.798	28096.863	13198.723
15.0	28916.962	13468.832	29060.582	13547.141	28916.364	13468.590	28916.206	13468.513
15.5	29729.038	13728.228	29875.190	13808.439	29728.367	13727.960	29728.215	13727.881
16.0	30533.046	13977.327	30682.144	14059.276	30533.213	13977.033	30533.056	13976.952
16.5	31331.846	14216.249	31481.603	14299.778	31331.051	14215.929	31330.890	14215.846
17.0	32122.890	14445.109	32273.724	14530.066	32122.034	14444.764	32121.869	14444.679
17.5	32907.224	14664.019	33058.660	14750.258	32906.309	14663.649	32906.140	14663.562
18.0	33684.068	14873.086	33836.561	14960.470	33684.016	14872.692	33683.843	14872.603
18.5	34456.316	15072.413	34607.574	15160.813	34455.280	15071.996	34455.112	15071.905
19.0	35221.336	15262.100	35371.844	15351.308	35220.257	15261.660	35220.076	15261.567
19.5	35980.172	15442.243	36129.512	15532.324	35979.044	15441.781	35978.859	15441.686
20.0	36732.041	15612.035	36880.716	15703.700	36731.767	15612.452	36731.578	15612.355
20.5	37470.756	15771.265	37625.591	15865.623	37478.540	15773.762	37478.347	15773.663
21.0	38220.725	15920.321	38361.270	16018.189	38219.472	15925.799	38219.275	15925.698
21.5	38955.052	16069.187	39096.883	16161.492	38954.666	16068.647	38954.460	16068.544
22.0	39685.537	16202.044	39823.556	16295.623	39684.223	16202.387	39684.019	16202.282
22.5	40409.575	16327.072	40544.413	16420.670	40408.238	16327.099	40408.031	16326.992
23.0	41128.158	16443.447	41250.575	16536.717	41126.804	16442.859	41126.593	16442.750
23.5	41841.374	16550.345	41969.160	16643.848	41840.008	16549.743	41839.793	16549.632
24.0	42549.306	16648.437	42673.282	16742.142	42547.935	16647.822	42547.716	16647.709

TABLE 2 (*contd*). THE TRAJECTORIES CALCULATED WITH 1940 RESISTANCE LAW AND THE THREE APPROXIMATIONS

T = Time, (XC, YC) = Trajectory with 1940 Resistance Law.
($X1$, $Y1$), ($X2$, $Y2$), ($X3$, $Y3$) = The Trajectories with the Three Approximate Formulas

T second	XC FT	YC FT	X1 FT	Y1 FT	X2 FT	Y2 FT	X3 FT	Y3 FT
24.5	43252.036	16737.705	43372.053	16831.676	43250.665	16737.168	43250.442	16737.053
25.0	43949.641	16818.488	44065.582	16912.526	43948.276	16817.840	43948.050	16817.732
25.5	44642.195	16890.583	44753.974	16981.763	44640.843	16889.933	44640.613	16889.814
26.0	45329.770	16954.146	45437.332	17048.459	45328.437	16953.485	45328.204	16953.364
26.5	46012.433	17009.239	46116.755	17103.681	46011.126	17008.570	46010.890	17008.447
27.0	46690.250	17055.927	46789.340	17150.494	46688.976	17055.250	46688.737	17055.125
27.5	47363.284	17094.271	47458.180	17188.963	47362.050	17093.586	47361.807	17093.459
28.0	48031.505	17124.331	48122.365	17219.149	48030.407	17123.639	48030.161	17123.510
28.5	48695.240	17146.166	48781.982	17241.112	48694.105	17146.407	48693.856	17145.336
29.0	49354.275	17159.835	49437.115	17254.910	49353.190	17159.129	49352.947	17158.996
29.5	50008.752	17165.394	50087.846	17260.599	50007.742	17164.681	50007.486	17164.546
30.0	50658.722	17162.890	50731.251	17258.231	50657.784	17162.179	50657.524	17162.043
30.5	51304.233	17152.406	51376.406	17247.868	51303.372	17151.679	51303.109	17151.541
31.0	51945.331	17133.969	52014.382	17229.552	51944.553	17133.235	51944.286	17133.096
31.5	52582.061	17107.612	52648.249	17203.337	52581.370	17106.901	52581.100	17106.760
32.0	53214.465	17073.478	53278.072	17169.272	53213.865	17072.730	53213.592	17072.587
32.5	53842.584	17031.530	53903.914	17127.405	53842.077	17030.775	53841.801	17030.630
33.0	54466.456	16981.850	54525.835	17077.782	54466.045	16981.087	54465.765	16980.941
33.5	55086.118	16924.489	55143.892	17020.450	55085.804	16923.719	55085.521	16923.571
34.0	55701.606	16859.499	55758.140	16955.453	55701.388	16858.721	55701.102	16858.572
34.5	56312.953	16786.930	56368.630	16882.835	56312.820	16786.145	56312.540	16785.994
35.0	56920.192	16706.833	56975.412	16802.639	56920.158	16706.041	56919.866	16705.888
35.5	57523.353	16619.258	57578.631	16714.909	57523.404	16618.460	57523.110	16618.305
36.0	58122.465	16524.255	58178.032	16619.686	58122.594	16523.451	58122.302	16523.294
36.5	58717.556	16421.873	58773.955	16517.012	58717.754	16421.066	58717.471	16420.905
37.0	59308.652	16312.163	59366.340	16406.929	59308.903	16311.354	59308.613	16311.188
37.5	59895.778	16195.174	59955.222	16289.477	59896.079	16194.365	59895.844	16194.192
38.0	60478.958	16070.955	60510.63	16164.698	60479.288	16070.150	60479.098	16069.966
38.5	61058.214	15939.550	61122.613	16032.632	61058.555	15938.760	61058.426	15938.561
39.0	61633.568	15801.028	61701.182	15893.320	61633.898	15800.242	61633.849	15800.026
39.5	62205.039	15655.415	62276.370	15746.803	62205.335	15654.650	62205.387	15654.409
40.0	62772.646	15502.773	62848.202	15593.121	62772.904	15502.027	62773.056	15501.763
40.5	63336.407	15343.150	63416.701	15432.315	63336.671	15342.411	63336.874	15342.137
41.0	63896.330	15176.595	63981.887	15264.427	63896.655	15176.850	63896.855	15175.582
41.5	64452.467	15003.159	64543.778	15089.408	64452.873	15002.393	64453.017	15002.147
42.0	65004.776	14822.892	65102.392	14907.570	65005.337	14822.091	65005.378	14821.882
42.5	65553.308	14636.846	65657.743	14718.685	65554.056	14634.905	65553.952	14634.838
43.0	66098.067	14442.071	66209.843	14522.886	66099.037	14441.159	66098.750	14441.065
43.5	66639.064	14241.019	66758.708	14320.216	66640.283	14240.035	66639.764	14240.616
44.0	67176.311	14034.542	67304.335	14110.720	67177.797	14033.479	67177.065	14033.543
44.5	67709.817	13820.892	67846.744	13804.442	67711.581	13819.746	67710.602	13819.898
45.0	68239.590	13600.722	68385.936	13671.428	68241.634	13599.492	68240.404	13599.734
45.5	68765.637	13374.086	68921.915	13441.723	68767.966	13372.774	68766.479	13373.104
46.0	69287.964	13141.038	69454.684	13205.375	69290.545	13139.649	69288.833	12140.062
46.5	69806.578	12901.632	69984.244	12962.432	69809.401	12900.175	69807.473	12900.663
47.0	70321.486	12656.923	70510.595	12712.943	70324.521	12654.412	70322.405	12654.962
47.5	70832.693	12403.960	71033.735	12456.067	70835.904	12402.418	70833.632	12403.014
48.0	71340.202	12145.818	71553.661	12194.520	71343.548	12144.253	71341.158	12144.876

TABLE 2 (*contd.*). THE TRAJECTORIES CALCULATED WITH 1940 RESISTANCE LAW AND THE THREE APPROXIMATIONS

T = Time, (XC, YC) = Trajectory with 1940 Resistance Law

($X1$, $Y1$), ($X2$, $Y2$), ($X3$, $Y3$) = The trajectories with the three approximate formulas

T second	XC FT	YC FT	X1 FT	Y1 FT	X2 FT	Y2 FT	X3 FT	Y3 FT
48.5	71844.015	11881.536	72070.369	11925.701	71847.452	11879.977	71844.985	11880.606
49.0	72344.134	11611.179	72583.853	11650.536	72347.613	11609.661	72346.115	11610.262
49.5	92840.558	11334.806	73094.105	11369.085	72844.029	11333.336	72841.548	11333.903
50.0	73333.287	11052.477	73601.118	11081.403	73336.608	11051.094	73334.283	11051.590
50.5	73822.318	10764.254	74104.883	10787.547	73825.618	10762.988	73823.320	10763.383
51.0	74307.648	10470.200	74605.389	10487.575	74310.785	10469.081	74308.656	10469.345
51.5	74780.274	10170.378	75102.625	10181.547	74792.197	10160.437	74790.288	10169.538
52.0	75267.192	9861.852	75596.578	9869.522	75269.850	9864.121	75268.213	9864.027
52.5	75741.400	9553.686	76087.235	9551.564	75743.730	9553.200	75742.426	9552.878
53.0	76211.894	9236.947	76574.582	9227.734	76213.861	9236.740	76212.920	9236.156
53.5	76678.668	8914.700	77058.605	8898.009	76680.211	8914.808	76679.691	8913.931
54.0	77141.717	8587.014	77539.287	8562.723	77142.783	8587.474	77142.731	8586.270
54.5	77601.034	8253.950	78016.612	8221.675	77601.571	8254.808	77602.033	8253.245
55.0	78056.612	7916.004	78490.563	7875.024	78056.569	7916.880	78057.588	7914.926
55.5	78508.443	7572.024	78961.121	7522.839	78507.770	7573.763	78509.388	7571.386
56.0	78956.517	7223.290	79428.269	7165.194	78955.187	7225.530	78957.423	7222.699
56.5	79400.820	6869.478	79891.988	6802.160	79398.752	6872.258	79401.684	6868.940
57.0	79841.350	6510.665	80352.258	6433.814	79838.516	6514.016	79842.160	6510.187
57.5	80278.105	6146.927	80809.059	6060.231	80274.452	6150.887	80278.841	6146.517
58.0	80711.054	5778.343	81262.371	5681.489	80706.550	5782.947	80711.716	5778.008
58.5	81140.194	5404.995	81712.174	5297.667	81134.801	5410.276	81140.773	5404.743
59.0	81565.514	5026.963	82158.447	4908.846	81559.196	5032.954	81566.001	5026.801
59.5	81987.002	4644.331	82601.169	4515.109	81979.726	4651.063	81987.389	4644.266
60.0	82404.647	4257.182	83040.319	4116.538	82396.380	4264.695	82404.925	4257.223
60.5	82818.436	3866.601	83476.876	3713.218	82809.150	3873.004	82818.696	3865.756
61.0	83228.357	3469.676	83907.816	3305.237	83218.026	3478.804	83228.391	3469.951
61.5	83634.308	3069.403	84336.121	2892.680	83622.098	3079.473	83634.299	3069.897
62.0	84036.546	2665.143	84760.709	2475.639	84024.058	2675.095	84036.307	2665.682
62.5	84434.790	2256.714	85181.739	2054.202	84421.196	2268.459	84434.405	2257.395
63.0	84829.119	1844.297	85599.000	1628.461	84814.405	1856.933	84828.581	1845.127
63.5	85219.521	1427.985	86012.550	1198.609	85203.676	1441.565	85218.825	1428.970
64.0	85605.984	1007.871	86422.308	764.439	85589.001	1022.386	85605.126	1009.015
64.5	85988.498	584.048	86828.416	326.348	85970.374	599.504	85987.474	585.355
65.0	86367.053	156.611	87230.083	−115.669	86347.789	173.010	86365.860	158.064
65.5	86741.638	−274.344			86721.240	−257.004	86740.276	−272.703

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