Answer Question No. 1 and any three from the remaining questions

1. Find all mixed strategy Nash equilibria of the game given below.

	L	M	R
B	(4, 2)	(0, 0)	(0, 1)
S	(0, 0)	(2, 4)	(1, 3)

[10]

2. Consider sale of an indivisible object by auction. There are three bidders, with their valuations satisfying: $v_1 > v_2 > v_3$. Players bid simultaneously for the object. The player who gives the highest bid wins the object. If there is more than one finalist, the player who has the lowest (player) index wins the object. Find a NE in pure strategy in the following cases, and if that NE is not unique, then find at least another NE. You must explain why these are Nash equilibria.

   (a) First price auction: the highest bidder wins the object by paying its bid.

   (b) Second price auction: the highest bidder wins by paying the second highest bid.

   (c) Third price auction: the highest bidder wins by paying the third highest bid.

[10]

3. (a) Consider the following static normal form game played by two players:

|	L	M	R
U	5, 5	0, 0	0, 6
M	0, 0	4, 4	0, 1
D	6, 0	1, 0	1, 1

where player 1 chooses a row and player 2 chooses a column *simultaneously*. Find the set of rationalizable strategies for each player. Then justify the choice of at least one such strategy of a player.

I

(b) Find the maximin mixed strategy solution of the game of matching pennies.

[5+5=10]

4. State the Kuhn's theorem in the context of the relation between behavior strategies and mixed strategies. Check whether the theorem holds for the following game.

[3+7=10]
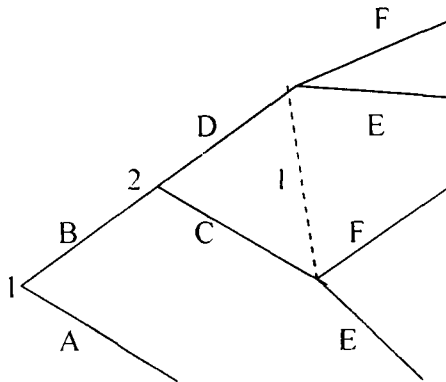
5. Player 1 and player 2 will play the following game.

	B	S
B	3, 1	0. 0
S	0. 0	1. 3

But player 1 is to first decide whether the game will be played in an AC room or in a non-AC room. Then they play the game. However, playing the game in an AC room costs player 1 Re 1.

(a) Portray the game in extensive form and predict the possible outcome.

(b) Also represent the game in strategic form.

[7+3=10]

Indian Statistical Institute
Economic Development I
MSQE I & II
Midterm Examination, 2012

Date: 12.9.12

Time: 2 hours

Answer as many questions as you can. However, the maximum you can score is 40. Marks allotted to a question are given in square brackets at the end of each question.

1. Consider an economy consisting of a constant returns sector and an increasing returns sector. Both sectors produce the same good using a single factor of production, labour. In the constant returns sector a unit of labour produces a unit of output independent of the scale of operation. In the increasing returns sector per unit labour requirement goes down as more labour is employed in this sector. Making suitable assumptions identify the stable long run equilibria and chalk out the dynamic path to equilibrium. Show how history and expectations can play important roles in determining the equilibrium path. [20]

2. Show that an economy with imperfect credit markets, lumpy investment cost of education and bequests and inheritance would converge to a bi-modal income distribution in the long run. How would the long run equilibrium change if
    (a) Credit markets were perfect?
    (b) Small increments in skill formation were possible by small incremental investments in education? [10+5+5]

    3. An economy produces a single good with a single factor of production, labour. Production of each unit of final good requires completing two tasks. For each task one unit of labour can be hired. Labour is available in two qualities: high and low. High quality labour can perform an assigned task with probability p while the low quality labour can perform with probability q, p > q. Output is produced if and only if both tasks are successfully completed. Production is undertaken by profit maximizing competitive firms. There are N units of high quality and M units of low quality labour in the economy.

a) Show that in equilibrium there will be skill clustering, that is high quality labour will work only with high quality labour and low quality labour only with low quality labour.

b) Determine equilibrium wages and expected output in terms of the parameters. [5+5]

Date:12.09.2012   Duration: 2 hours

Answer as many questions as you like. But you may at most score 50.

1. Write a C code that prints the line "MS(QE) 2012-13 batch is the best in ISI" without using a semicolon anywhere in the code. /*hint: make use of *decision control*/   5

2. What will the outputs be for the following codes: 3X3=9

   (i)   ```c
            #include<stdio.h>
            main( )
            {
            int a, b, sum ;
            for (a = 1 ; a <= 3 ; a++)
                {
                    for (b = 1 ; b<= 2 ; b++)
                    {
                        sum = a + b;
                        printf ("a = %d b = %d sum = %d\n", a, b, sum) ;
                    }
                }
            }
            ```

   (ii)     ```c
            #include<stdio.h>
            main()
            {
            int variable=11111;
            printf("%f",variable);
            }
            ```

   (iii)   ```c
            # include <stdio.h>
            main()
            {
            int i=200, j=300, k=100;
            int mohar;
            mohar=(i, k, j);
            printf("%d\n", mohar);
            }
            ```

3. Explain with suitable examples:                                          4X6=24

    a.   scope of an identifier and global variable

    b.   function call (by value) and (by reference)

    c.   (iteration and recursion procedure) OR (*while* and *for* loops) in loop control

    d.   positional number system and its base

4. (a) Is it possible to enter an input text line VEDIKA for the following code? What would be the output for such an attempt? What would the outputs be if input text line is (b) MALAYALAM (c) ARUNITA DAS? (d) What does any such output, mentioned above, indicate regarding the function VEDIKA?                                          2+2+2+3=9

```
# include <stdio.h>
void VEDIKA(void);
main()
{
        printf ("enter a text line\n");
        VEDIKA( );
        getchar( );
}
void VEDIKA (void)
{
        char c;
        if ((c=getchar ( )) !='\n')
        VEDIKA ( );
        putchar (c);
}
```

5.   (a) Convert from octal to binary and vice-versa: 2X(2+2)=8

    $(2673.54)_8$, $(11101110.000111)_2$

    (b) Convert from hexadecimal to binary and vice-versa:

    $(65D.6)_{16}$, $(1111001001.111)_2$

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2012-13

M. S. (Q.E.) I Year
Mathematical Methods I

Date: 14. 09. 12   **Full Marks: 60**   **Time: 3 hours**

Answer all Questions

1. (a) State the axioms of a Peano system $(\mathbb{N}, 0, s)$ and show that if for another set T, a bijection $f : \mathbb{N} \to T$ exists then a Peano system can be defined on T.

   (b) Define the natural partial order on $(\mathbb{N}, 0, s)$. Show that every nonempty subset of $\mathbb{N}$ has a smallest element.

   $$[10 + 10 = 20]$$

2 (a) Define countable sets. Prove that if A is a countable set then so is $A \times A$.

   (b) State the Least Upper Bound axiom for real numbers. Find the l.u.b and g.l.b of *any one* of the following sets (indicating if they do not exist) from the definition.
   (i) For $t > 0$, (given), $\{a + ta^{-1} : a$ is a positive rational number$\}$
   (ii) For $t > 0$, (given). $\{x \sin x^{-1} : 0 < x < t\}$.

   $$[15 + 5 = 20]$$

3. (a) Define the lim sup and lim inf of a bounded sequence $\{x_n : n \geq 1\}$ of real numbers.

   (b) Show that for a bounded sequence $\{x_n : n \geq 1\}$ of real numbers
   $$\limsup x_n = \liminf x_n,$$
   if and only if $\{x_n\}$ is a Cauchy sequence.

   $$[5 + 15 = 20]$$

Supplementary Examination

Mid-semestral Examination 2012-13

MS(QE) I & MSTAT II

Microeconomic Theory I

Date: 01.10.2012   **Maximum Marks:** 40   **Duration:** 2 hours

(1) (a) Show that if a preference relation R on X can be represented by a utility function then it is rational. **(4)**

   (b) Suppose that X is *finite* and R is a rational preference defined on X. Consider the function $u^* : X \to \Re$ such that $\forall\ x \in X$, $u^*(x) = |X| - |\{z \in X \mid zPx\}|$. Is the function $u^*(.)$ a valid utility representation of the preference relation R on X? Justify your answer. **(8)**

(2) Define the weak axiom of revealed preference. Show that if the Walrasian demand function $x(p,w)$ is homogeneous of degree zero and satisfies Walras' law, then the weak axiom of revealed preference holds if and only if it holds for all compensated price changes. **(1+13=14)**

(3) Define monotonicity, strong monotonicity, weak monotonicity and local non-satiation of a preference relation R on X. Show that if the preference relation R on X is complete, then strong monotonicity implies monotonicity. Also show that if R on $X = \Re_+^L$ is monotone, then it is locally non-satiated. Finally, show that if R on $X = \Re_+^L$ is locally non-satiated, transitive and weak monotone, then it is monotone. **(4+2+4+4=14)**

Indian Statistical Institute
Supplementary First Midsemester examination
MSQE I
Mathematical Methods I

Date: October 9, 2012   Maximum marks: 60   Duration: $1\frac{1}{2}$ hrs.

1. Let $\mathbb{N}$ denote the set of nonnegative integers. Show that the cartesian product $\mathbb{N} \times \mathbb{N}$ is countable.

   [20]

2. Let $\mathcal{F}$ be the set of functions $f : \mathbb{N} \to \{0,1\}$. Show that $\mathcal{F}$ is not countable.

   [20]

3. Give an example of a bounded function $f : \mathbb{R} \to \mathbb{R}$, which is continuous except at integers and satisfies

$$\liminf_{n \to \infty} f(x_n) = f(x),$$

   for any sequence $\{x_n\}$ convergent to x, for every $x \in \mathbb{R}$.

   [20]

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2012-13

Course Name: MSQE I and II

Subject Name: Economic Development

Date: 16.11.12   Maximum Marks: 60   Duration: 3 hours

Answer any three questions

1. Consider an economy consisting of one formal and one informal sector. There are two political parties A and B engaged in political competition. An informal sector worker has to join one of the two parties for protection and political favour. Only the ruling party can give a political favour. A worker lives for two periods and votes only when young. As an old party loyalist he gets a higher rent than when young, provided his party is in power. The formal sector voters vote according to the signal they get about the ruling party. If they get a good signal, they vote for the ruling party and if not they vote for the opposition. Let π be the fraction of formal voters getting a good signal and let $G(x) \equiv$ prob $(\pi \leq x)$ be the distribution of π. Total population of each generation is normalized to unity. The ruling party chooses the size of the formal sector by choosing how much to invest in infrastructure. *Assuming that G(x) is a uniform distribution,* determine the equilibrium winning probability for the ruling party and the size of the formal sector.   [20]

2. Setting up a suitable model, show how reciprocity can act as the basis of an informal insurance arrangement between two players with uncertain income streams which are (ex ante) identical, independent and infinite. What happens if the horizon is finite, that is, there is a period T after which both players die?   [15+5]

3. Show how sharecropping can emerge as the optimal contract when the effort level exerted by the tenant cannot be observed by the landlord. What role does the assumption of limited liability on the part of the tenant play in the analysis?   [15+5]

4. Consider an agricultural commodity whose output is seasonal and demand is continuous. A small number of oligopolistic traders control the market. Show that the degree of price rise varies inversely with the degree of oligopoly of the market. Find the competitive sales path and show that it is socially optimal.   [10+10]

Indian Statistical Institute

First Semestral Examination 2012-13

MS(QE) I & MSTAT II

Microeconomic Theory I

Date: $19/11/12$ **Maximum Marks: 60**   **Duration: 3 hours**

(1) Provide the two definitions of continuity of preference relation R defined on X. Show that if $u(.)$ is a continuous utility function representing the preference relation R on X, then R on X must be continuous. **(2+6=8)**

(2) Suppose that $u(.)$ is a continuous utility representation of a locally non-satiated preference relation R defined on $X = \Re_+^L$ and that the price vector is $p >> 0$. Show that if x^* is the optimal in the expenditure minimization problem when $u > u(\underline{0})$, then x^* is optimal in the utility maximization problem when the wealth level is $p.x^*$. Also show that the maximized utility level in this utility maximization problem is exactly u. **(10+2=12)**

(3) Suppose that $f(.)$ is the production function associated with a single-output technology, and let Y be the production set of this technology. Prove the following.
   (a) Y satisfies constant returns to scale if and only if $f(.)$ is homogeneous of degree one.
   (b) Y is convex if and only if the production function $f(.)$ is concave.
   (8+8=16)

(4) Let Y^+ be the additive closure of Y, that is, the smallest production set that is additive and contains Y. Show that if Y is convex, then $Y^+ = \cup_{n=1}^{\infty} nY$ where for any positive integer n, $nY = \{ny \in \Re^L : y \in Y\}$. **(9)**

(5) Define Pareto efficiency and weak Pareto efficiency. Prove that Pareto efficiency implies weak Pareto efficiency. Also show that if the preference relation is defined on $X_i = \Re_+^L$ for all $i = 1, \ldots, I$, and all consumers' preferences are continuous and strongly monotonic then weak Pareto efficiency implies Pareto efficiency. **(2+3+10=15)**

1

INDIAN STATISTICAL INSTITUTE

Semester Examination : Semester I (2012-2013)

M.S.Q.E. 1st Year

Statistics

Date: 2⊘. 11. 12   Maximum marks: 100   Time: 3 hours.

Note: Answer all questions. Maximum you can score is 100.

1. (a) Let Y be a gamma random variable with parameters (s, α). If the conditional distribution of X given $Y = y$ is Poisson with mean y, find the conditional distribution of Y given $X = x$.   [10]

   (b) Let X follow $U[0,1]$ distribution. Find the distribution of $Y = -logX$.   [5]

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from $U[0, \theta]$ distribution. Find two unbiased estimators of θ using Method of Moments and Maximum Likelihood Estimation procedures. Compare their variances.   [15]

3. Let $X_1, X_2, \ldots, X_n$ be i.i.d. observations from Poisson(λ) distribution. Find the MVUE of λ by showing that the estimator attains the Cramer-Rao Lower bound.   [15]

4. Suppose it is claimed that the proportion (p) of people having internet access in a large population is more than 50%. Based on a random sample of size 100 from this population, it was found that 55 people have internet access.

   (a) Make a test of hypothesis at 5% significance level to decide on the claim.   [10]

   (b) If the alternative hypothesis is $p = 0.55$, find the power of the test.   [8]

   (c) What will be the required sample size if the power in part(b) is desired to be 90%? [7]

5. Suppose a random sample of 100 boys revealed that the average height at age 6 was 46 inches with an s.d. of 1.5 inches, and the average height at age 18 was 70 inches with an s.d. of 2.5 inches. The correlation between the heights at age 6 and at age 18 was about 0.8.

   (a) Predict the average height of a boy at age 18 whose height at age 6 was 49 inches. [5]

   (b) Predict the average height of a boy at age 6 whose height at age 18 was 74 inches. [5]

   (c) Test whether the regression slope parameter in part(a) is zero or not.   [10]

INDIAN STATISTICAL INSTITUTE

Semester Examination : Semester I (2012-2013)

M.S.Q.E. 1st Year

Statistics

Date: 20. 11. 12   Maximum marks: 100   Time: 3 hours.

Note: Answer all questions. Maximum you can score is 100.

1. (a) Let Y be a gamma random variable with parameters (s, α). If the conditional distribution of X given $Y = y$ is Poisson with mean y, find the conditional distribution of Y given $X = x$.   [10]

   (b) Let X follow $U[0,1]$ distribution. Find the distribution of $Y = -logX$.   [5]

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   (b) Predict the average height of a boy at age 6 whose height at age 18 was 74 inches. [5]

   (c) Test whether the regression slope parameter in part(a) is zero or not.   [10]

6. Two methods A and B were used to determine the latent heat of fusion of ice. The investigators wished to find out whether the methods differed. Two methods were applied to 13 and 8 ice samples respectively, and the change in total heat from ice to water in calories per gram were calculated. The averages for methods A and B were 80.02 grams and 79.98 grams respectively with standard deviations 0.024 and 0.031 grams respectively.

   (a) Find a 95% confidence interval for the actual magnitude of the average difference between the two methods.   [10]

   (b) Test the hypothesis that there is no difference between the methods at the 5% significance level. Compare your decision with part (a).   [10]

End Semestral Examination: (2012-13)
MS (QE) I Year
Computer Programming and Applications

Date: 29·11·12   Maximum Marks: 100   Duration: 3 hours

Answer as many as you wish. But you may at most score 100.

1. Write brief notes on any three of the following:

(i)   Central processing unit (ii) Memory unit (iii) Positional number systems (iv) Bisection method   6x3=18

2. (i) Explain with suitable example, what do you mean by a flowchart? (ii) Draw a flowchart for generating prime numbers up to 200. (iii) Write also the corresponding C code.   5+6+7=18

3. In anticipation of a century by Sehwag batting on 90, two programs provided below are kept prepared for announcing breaking news in two different television channels. Sehwag got out on 96, which was therefore given as input for runs in each of the above codes. What will be the output for breaking news in these channels? Justify your answer.   3+3=6

Channel 1

```
#include<stdio.h>
main( )
{
    int runs;
    printf("Enter the runs: ");
    scanf("%d", &runs);
    if (runs=100)
    printf("Breaking News: Another
century by Sehwag");
    else
    printf("Breaking News: Sehwag
narrowly misses century");
}
```

Channel 2

```
#include<stdio.h>
main( )
{
    int runs;
    printf("Enter the runs: ");
    scanf("%d", &runs);
    if (runs==100);
    printf("Breaking News: Another
century by Sehwag");
}
```

4. (i) What do you mean by ASCII value of characters in C? (ii) What is the difference between *char num = 1* and *char num = '1'* ? (iii) What is a null character? (iv) What do you mean by *escape sequence*? (v) Define a string. (vi)Write a C program that prints its input one word per line. (vii) Write another C program that removes all the vowels from sentences that are not more than 100 characters long.

3+3+2+2+3+7+8=28

........page 2

1

5. a) Explain each of the following terms in brief:
   (i) malloc( ) and free( ) (ii) memory stack and heap (iii) iteration and recursion procedure
   b) Write a C program that iteratively generates the Fibonacci series (take the number of terms as input).
   c) Then write another C code to generate the same series recursively.   (6x3)+7+7=32

6. (i) What are array indices? (ii) Explain what do you mean by a pointer variable and what are the pointer operators? (iii) Explain how an array element corresponding to a particular index can be accessed with the help of pointer arithmetic? (iv) Write two C codes to explain how array elements can be passed to a function by value as well as by reference. (v) Predict and justify the output from the following program: 3+4+3+(7+7)+4=28

```c
#include<stdio.h>
   main( )
{
   int m=2012, *s, qe=100;
   s=&m;
   *s=2013;
   s=&qe;
   *s=m;
   printf("%d  %d  %d", m, *s, qe);
}
```

Indian Statistical Institute
Supplementary First Semestral examination
MSQE I
Mathematical Methods I

Date: 30.11.2012   Maximum marks: 100   Duration: 3 hrs.

Answer all Questions

1 (a) Define a topological space $(X, \mathcal{T})$, with examples. Let A be a subset of X. Suppose that for each $x \in A$ there is an open set $U \in \mathcal{T}$ containing x such that $U \subset A$. Show that A is open in X.

[5 + 10 = 15]

(b) For $A \subset X$. define the interior and boundary of A in terms of the topology $\mathcal{T}$ on X. Show that if the set A is closed then the boundary of A is a subset of A.

[10]

2 (a) Define a metric space (X, d). Show that for the set $\mathbb{R}^n$,

$$d(x, y) = \max_{1 \leq i \leq n} |x_i - y_i|$$

defines a metric where $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ are two arbitrary points in $\mathbb{R}^n$.

[3 + 7 = 10]

(b) Find the minimum value of $d(x, 0)$ subject to the condition $\sum_{i=1}^{n} x_i^2 = 1$, where $0 = (0, 0, \ldots, 0)$ and $x \in \mathbb{R}^n$.

[5]

(c) Let (X, d) be a metric space. Define a compact subset of (X, d). Let $K \subseteq X$ be a compact subset and $f : K \to \mathbb{R}$ be a continuous function. Show that the image $f(K)$ is a bounded subset of $\mathbb{R}$.

[3+ 7 = 10]

3 (a) Suppose $\mathcal{V}$ is a finite dimensional vector space over $\mathbb{R}$. Define linear independence of a finite set of vectors in $\mathcal{V}$ and a basis of $\mathcal{V}$.

[5]

1

$\mathcal{P}.\mathcal{T}.0.$

(b) Let $U = \{u_1, u_2, \ldots, u_n\}$ be a basis of $\mathcal{V}$. For any $v \in \mathcal{V}$, let

$$v = \sum_{i=1}^{n} \lambda_i^U(v) \, u_i$$

denote the unique linear combination which represents the vector v with respect to the basis U. Show that $(\lambda_1^U, \lambda_2^U, \ldots, \lambda_n^U)$ is a set of linearly independent linear functionals defined on $\mathcal{V}$ satisfying $\lambda_i^U(u_j) = 1$ if $i = j$ and 0 otherwise.

[15]

(c) Let $W = \{w_1, w_2, \ldots, w_n\}$ be another basis of $\mathcal{V}$ defined by $w_i = a_i u_i$ for some positive real numbers $a_1, a_2, \ldots, a_n$. In view of (b) above, express $(\lambda_1^W(v), \lambda_2^W(v), \ldots, \lambda_n^W(v))$ as a linear combination of $(\lambda_1^U(v), \lambda_2^U(v), \ldots, \lambda_n^U(v))$ for any arbitrary $v \in \mathcal{V}$.

[10]

4. Define convex sets in $\mathbb{R}^n$. State and prove the separating hyperplane theorem for disjoint convex sets $K_1, K_2 \subset \mathbb{R}^n$. You need to state the assumption needed for the result precisely in the statement of the theorem.

[20]

Indian Statistical Institute
Semester Examination: 2012-2013 ·
MS(QE) I/ M.Stat.II: 2012-2013
Game Theory I

Date:26/12/2012   Maximum Marks: 100   Duration: 3 Hours

Answer ALL questions

1. (a) A seller throws his house for sale. It has a listed price of Rs. 1,50,000/- which is the buyer's maximum price, and its invoice price is Rs 1,20,000/- which is the seller's reservation price. Consider an alternating offer game in the Rubinstein framework. Assume that each player has the common discount factor, $\delta = 1/2$.

   (i)   If the buyer is the player to give the opening offer, what will be the price at which trade will occur if the game is played for <u>three</u> periods only?

   (ii) If the game has an infinite time horizon, what will be the optimal buyer's offer?

   (iii) If the seller, instead of the buyer, gives the opening offer, what will be the result?

   [10+8+7=25]

2. (a) What is the 'Nash reversion strategy' in an infinitely repeated Bertrand game? •Show that the Nash reversion strategy constitutes a subgame perfect Nash equilibrium of this game if and only if $\delta \geq 1/2$, where δ is the discount factor.

   (b) Consider playing a prisoners' dilemma game repeatedly finite number of times. What is the outcome of the game? Derive your result.

   [13+12=25]

3. (a) Find all mixed strategy Nash equilibria of the following two player game:

	L	M	R
B	(2, 2)	(0, 3)	(1, 2)
S	(3, 1)	(1, 0)	(0, 2)

(b) There are three one-rupee notes to be divided between two players. It's a two period game. In the first period player 1 proposes a division. If player 2 accepts it, the division is implemented, and if it is rejected, the game goes to the second period in which case player 2 gives a counter offer. Again, if it is accepted, this is implemented, but if it is rejected, no player gets anything (say, the money goes to a charity fund). Each player has time preference given by the (common) discount factor, δ; $0 < \delta < 1$. Find the subgame perfect Nash equilibrium of this game.

[10+15=25]

4. Two people have Rs. 10/- to divide between themselves. They use the following procedure. Each person names a number of rupees (a non-negative integer), at most equal to 10. If the sum of the amounts the people name is at most 10, then each person receives the amount she named (and the remainder, if any, is destroyed). If the sum of the amounts that the people name exceeds 10, and the amounts named are different, then the person who named the smaller amount receives the amount and the other person receives the remaining amount. If the sum of the amounts that the people name exceeds 10 and the amounts named are the same, then each person receives Rs 5/-.

(a) Construct the payoff matrix and solve the game by iterated elimination of weakly dominated strategies.

(b) Write down the payoff function of each player, determine their best response functions, plot them in a diagram and finally find the Nash equilibria of the game.

[12+13=25]

Back Paper

Indian Statistical Institute

First Semestral Examination 2012-13

MS(QE) I & MSTAT II

Microeconomic Theory I

Date:28.12.2012   **Maximum Marks: 100**   **Duration: 3 hours**

(1) Let R be a preference relation defined on X. Prove the following.

 (a) R is monotone if and only if the utility function $u(.)$ representing it is increasing.

 (b) R is convex if and only if the utility function $u(.)$ representing it is quasi-concave.

 (c) R is strictly convex if and only if the utility function $u(.)$ representing it is strictly quasi-concave.

 (7+14+14=35)

(2) Suppose that $\pi(.)$ is the profit function of the production set Y and that $y(.)$ is the associated supply correspondence. Assume that Y is closed and satisfies the free disposal property. Then prove that

 (a) $\pi(.)$ is homogeneous of degree one.

 (b) $\pi(.)$ is convex.

 (c) $y(.)$ is homogeneous of degree zero.

 (d) If Y is convex then $y(p)$ is a convex set for all p.

 (e) If Y is strictly convex then $y(p)$ is single valued (if not empty).

 (8+8+8+8+8=40)

(3) Define Pareto efficiency and weak Pareto efficiency. Prove that Pareto efficiency implies weak Pareto efficiency. Construct an example to show that the converse is not necessarily true. **(5+8+12=25)**

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2012-13

M.S. (Q.E.) I & II

Environmental Economics

Date: 18 February, 2013   Maximum Marks: 65   Duration: 2 hrs.

1. (a). It is known that impacts of emissions by polluting firms on ambient pollution concentrations differ by location of firms. In that situation how should an efficient emission fee for each firm be set by a regulator following equimarginal principle?

    Two identical firms save money from polluting. Marginal savings from emitting an amount e_i by i^{th} firm are given by $14 - 7e_i$. Two firms differ in their impact on ambient pollution concentrations. One unit of emission results 2 units by firm 1 and 3 units by firm 2 of ambient pollution. Marginal damage is assumed to be same as total ambient pollution. Find the appropriate amount of emission at firm level and at ambient level.

    (b). Show how the decision of a regulator to choose an appropriate regulatory measure out of emission fee and quantity regulation is affected when pollution generated by individual firm is not observable to the regulator but total ambient pollution level is observable to him.

    [9+5+6=20]

2. Consider an example of three species : (i) grain (consumed by rodents); (ii) rodents (consumed by predators like owl, foxes) and (iii) owl and foxes. Individuals' resource allocation decisions are affected by grains (for bread making) and predators only. But rodents affect the allocation decision of individuals indirectly and create externality which is unknown to the individuals. Analytically show, through a general equilibrium model, how this misunderstood ecosystem externality reverberates through both economic and ecological systems. Also show it graphically.

    [20]

3. Consider a small country facing market failure for producing a good with adverse environmental impact leaving uncontrolled and prospect of trade liberalization, in which its own actions do not affect the rest of the world.

    (a). Show how does the welfare of small country change if the country shifts from autarchy to open trade.

    (b). Show the relative efficiency of trade and environmental policies in reducing the environmental degradation.

    [15+10=25]

INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2012-13
M. S. (Q.E.) .I Year
Modern Growth Theory

Date: / 9.ø ?./.3   **Maximum Marks: 40**   **Duration:** $2\frac{1}{2}$ **Hours**

1   Examine the validity of any two of the following statements $[12 \times 2]$

   a)   Hicks-neutral technical change is consistent with the existence of steady-state growth equilibrium in a neo-classical one sector growth model.

   b)   In Mankiw-Romer-Weil model, steady-state growth equilibrium is unique.

   c)   In the absence of technical progress and with constant returns to scale production technology, steady-state equilibrium is always a 'no-growth' equilibrium.

$[8 \times 2]$

2   a)   Consider a Solow growth model where 25% of national income is saved and labour force grows at 5% rate. If the production function is $Y = \sqrt{KL}$, find out the per-capita income in the steady-state growth equilibrium.

   b)   Consider a two sector dynamic model where capital stocks in two sectors accumulate over time satisfying following equations of motions.

$$\dot{K_1} = K_1 \, K_2 - 10$$

$$\dot{K_2} = K_1 - K_2$$

Examine the problem of existence, uniqueness and stability of the steady-state equilibrium using a phase diagram and evaluating properties of the Jacobian.

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INDIAN STATISTICAL INSTITUTE
Mid-Semester Examination: 2012-13
MS (QE) I YEAR
Econometric Methods I
Date: 26 February 2013   Maximum Marks: 100   Duration: 3 hours

Note: Answer question 1 and any three from the rest of the questions]

1. Data on three-variable linear regression problem $y = b_1+b_2x_2+b_3x_3+e$ yield the following results:

$$X'X = \begin{bmatrix} 33 & 0 & 0 \\ 0 & 40 & 20 \\ 0 & 20 & 60 \end{bmatrix}, X'y = \begin{bmatrix} 132 \\ 24 \\ 92 \end{bmatrix} \text{ and } \Sigma(y-\bar{y})^2 = 150.$$

(a) What is the sample size?
(b) Write down the normal equations and solve for the regression coefficients.
(c) Estimate the standard error of b_2 and test the hypothesis that b_2 is zero.
(d) Compute R^2 and interpret it. Also interpret the values of the regression coefficients.
(e) Predict the value of y given $x_2 = -4$ and $x_3 = 2$.
(f) Comment on the possibilities of any of the regressors being dummy variable.

$$[1+9+8+6+2+2=28]$$

2. Write a brief account of different types of data that one comes across in econometric analysis. Describe the problems encountered with these data. Also describe some methods of refining the data.   [10+8+6=24]

3. Suppose the students of first year in a college either come from Bengali medium schools or from English medium schools. How will you compare the two groups of students on the basis of marks obtained in a certain examination using a regression model with dummy variable giving a value '0' for students of English medium school and '1' for the other group? Derive the results clearly stating the assumptions you are making at each stage. What will happen if you take '1' and '3' instead of '0' and '1' respectively?   [24]

4. (i) Define coefficient of determination (R^2). (ii) Write down an alternative form of it and prove its equivalence. (iii) Interpret its value as a goodness of fit parameter keeping in views the alternative forms of R and/or R^2. (iv) Can you always use the value of R^2 to compare the goodness of fit of different forms of regression equations? Explain giving appropriate examples.   [2+10+4+8=24]

5. State the assumptions of Classical Linear Regression Model (CLRM) giving reasons why these assumptions are necessary in this model. Derive Least Squares estimate of the regression coefficient in this model. Prove that it is BLUE.   [8+6+10=24]

6. Write short notes on any three of the following:
    (a) Detection of Outlying Observations by Dummy Variables.
    (b) Prediction by Dummy Variables
    (c) Partial correlation coefficient.
    (d) Regression without intercept term.   [8×3=24]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700 108
MID-SEMESTRAL EXAMINATION 2012 - 13
M.S.(Q.E.) 1st Year
Time Series Analysis & Forecasting

Date: $01 \cdot 03 \cdot 13$ Time: **2 hours**

[This question paper carries a total of 60 marks. You can answer any part of any question; but the maximum than you can score is 50.]

1) (a) Discuss what is meant by seasonality of a time series.

   (b) Suppose that a time series contains trend, seasonality and noise components. Describe a procedure for obtaining trend and seasonal components present in this time series.

   [4+10 = 14]

2) (a) Show how the three conditions for weak stationarity of a time series can be derived from the condition for strong stationarity.

   (b) Let $\{ x_t \}$ be a normal white noise process with mean μ and variance σ^2. Consider the time series $y_t = x_t \, x_{t-2}$. Determine the mean, variance and autocovariance function of $\{ y_t \}$, and check if it is weakly stationary. Is $\{ y_t \}$ also strongly stationary? Justify your answer.

   [5+10 = 15]

3) (a) What is unit root in the context of time series? Explain.
   (b) Derive the conditions for stationarity (in terms of parameters) of an AR (2) process.
   (c) Discuss the nature of ACF (without any detailed derivations) of an ARMA (1,1) process.

   [4+7+5= 16]

4) (a) Suppose that $\{ x_t \}$ follows the following process:
      $$(1 + 0.6B) \, x_t = (1 - 0.3B) \, a_t, \qquad a_t \sim WN \, (0, 1).$$

      Find the coefficients Π_j's in the representation $x_t = \sum_{j=0}^{\infty} \Pi_j a_{t-j}$,

   (b) Suppose that a time series $\{ x_t \}$ follows an AR(1) process with coefficient α, and another time series $\{y_t\}$ follows an AR (2) process whose roots of the underlying characteristic equation are $1/\alpha$ and $1/\beta$, $\beta \neq \alpha$. Also, x_t and y_t are independent. Obtain the process to be followed by $\{z_t\}$ where $z_t = x_t + y_t$.
      [*No standard result/theorem can be used.*]

   [5+10=15]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: (2012-2013)

MS(QE) I

Microeconomic Theory II

Date: 04.03.2013   Maximum Marks: 40   Duration: $2\frac{1}{2}$ hours.

Note: Answer Group A and Group B in separate answer scripts.

Group-A

Note: Answer both questions.

(1) Define budget correspondence and demand correspondence. Making suitable assumptions prove that the budget correspondence is continuous in price and endowment. Also prove that if the cheaper point assumption holds at price-endowment (p_0, w_0), then the demand correspondence is upper semi-continuous at (p_0, w_0). (2+6+5=13)

(2) Consider an economy with 2 individuals 1, 2 and two goods x, y. Individual 1 has 1 unit of x and 0 unit of y. Individual 2 has 0 unit of x and 1 unit of y. Individual 1 has a utility function $U_1 = x_1^{\frac{1}{2}} y_1^{\frac{1}{2}}$ and individual 2 has a utility function $U_2 = x_2^{\frac{3}{4}} y_2^{\frac{1}{4}}$ where x_i, y_i are consumptions of the two goods of individual i, $i = 1, 2$.

(a) Find the set of Pareto optimal allocations.

(b) Find the competitive allocation. (3+4=7)

Group-B

Note: Answer both questions.

(1) Suppose that the rational preference relation $\mathcal{R}$ defined on the set of all simple lotteries $\mathcal{L}$ satisfies the continuity and the independence axioms. Show that there exists degenerate lotteries $\underline{L}$ and $\overline{L}$ such that $\overline{L} \mathcal{R} L \mathcal{R} \underline{L}$ for all $L \in \mathcal{L}$. (14)

(2) Given a (twice differentiable) Bernoulli utility function $u(.)$ for money, define the Arrow-Pratt coefficient of absolute risk aversion and the coefficient of relative risk aversion. Show that nonincreasing relative risk aversion implies decreasing absolute risk aversion but the converse is not necessarily true. (2+4=6)

INDIAN STATISTICAL INSTITUTE
Second Semester Examination: 2012-13
MS (QE) I YEAR
Econometric Methods I
Date: 13 April 2013   Maximum Marks: 100   Duration: 3 hours

Note: Answer question 1 and any **three** from the rest of the questions]

1. An investigator estimates a linear relation and associated standard errors by applying the OLS to the data:

X	2	3	1	5	9
Y	4	7	3	9	17

   He is subsequently informed that the variance matrix for the disturbances underlying the data is

   $$Var(e) = \sigma^2.diag\{0.10, 0.05, 0.20, 0.30, 0.15\}.$$

   Use this information to calculate the correct standard error for the OLS estimate of the regression coefficient and compare with that obtained from the conventional formula. Also find the GLS estimate of the regression coefficient and its standard error.   [16+12=28]

2. Suppose in the following regression model
   $$y = x_1\beta_1 + x_2\beta_2 + \ldots + x_K\beta_K + e,$$

   the variables $x_1, x_2, \ldots, x_K$ are multiplied by $c_1, c_2, \ldots, c_K$ respectively, where β_1 is the intercept. Compare the changes that will occur in the LS estimates of the regression coefficients and their variances and covariances. Also discuss what will happen to the prediction of y. In particular discuss the special case where $c_1 = 1$ and $c_i = 1/(sd\ of\ x_i)$, for $i = 2, 3, \ldots, K$.   [16+6+2=24]

3. Define Autoregressive and Distributed-Lag Models giving interpretations of the associated parameters. What are the problems of estimation of Distributed-Lag Models? How did Koyck transform the Distributed-Lag Model into an Autoregressive Model? How did he propose to estimate it and why? Give justifications for your answers. Derive the mean lag and the median lag of Koyck Model.   [4+2+5+5+8=24]

4. What do you mean by the problem of multicollinearity in the data? How will you detect the existence of multicollinearity? Write some possible solutions to this problem. Do you agree with the statement that "if all the simple correlations are small then the problem of multicollinearity will not arise"? Give explanations for your answer.   [4+6+12+2=24]

5. Examine the validity of the assumptions of CLRM under the presence errors-in-variables in the regression set up. Discuss the identification problem in this model. Describe two methods of estimation in errors-in-variables model.   [6+6+12=24]

6. Discuss the problems of over-, under-, and exact identification using structural and reduced forms in the context of simultaneous equations model by taking suitable examples. Also verify the rank and order conditions for identification of the equations in these examples.   [16+8=24]

7. Write short notes on any **three** of the following:
   (a)   Adjusted R^2.
   (b)   Two Stage Least Squares Estimation.
   (c)   Granger Causality.
   (d)   IV estimation.   [8×3=24]

INDIAN STATISTICAL INSTITUTE
Second Semester Examination: (2012-2013)
MS(QE) I & MSTAT II
Microeconomics II

Date: 30.04.2013   Maximum Marks: 60   Duration: 3 hrs.

Note: Answer Group A and Group B in separate answerscripts.

Group A

Note: Answer all questions.

(1) Define the core of an exchange economy. Prove that as the number of agents becomes arbitrarily large, the core shrinks to the set of competitive equilibrium. (1+9=10)

(2) Consider a two period (period 0 and 1) exchange economy with financial securities. There is a single commodity which is received as endowment by the consumers and consumed at period 1. At period zero, securities can be traded. There are two consumers and two possible states of nature. Let ω_s^h denote consumer h's endowment of the consumption good in state s. We have

$$(\omega_1^1, \omega_2^1) = (3,1)$$

$$(\omega_1^2, \omega_2^2) = (1,2).$$

It is further given that state 1 occurs with probability 1/3 and state 2 with probability 2/3. The probabilities are common knowledge. The utility function of the two consumers are given by

$$U^1(x) = \frac{1}{1-\gamma}x^{1-\gamma}, \ \gamma = 3$$

$$U^2(x) = 2x,$$

where x is consumption. Moreover, there are two securities A and B. Security A has a return structure $r^A = (2,1)$. Security B is a call option whose primary security is security A with a strike price of 1. Answer the following questions:

(a) Specify each consumer's optimization problem at date 0.

(b) Specify security B's return structure.

(c) Derive the optimum consumption plans of the consumers (you need not consider corner solutions).

(d) Derive the equilibrium portfolios of securities of the consumers.

(e) Derive the risk neutral probabilities of the states.

(f) Derive the prices of the securities using the risk neutral probabilities. **(2+2+4+4+4+4=20)**

Group B

Note: Answer all the questions.

(1) Show that in any sub-game perfect Nash equilibrium of the screening game with unknown worker types the following results are true.

(a) In any equilibrium both firms earn zero profits.

(b) No pooling equilibrium exists.

(6+4=10)

(2) Consider the labour market model where the effort level of the tenant is not observable and not verifiable. Derive the first best and the second best contracts. **(20)**

Date: 4'05'13   Maximum Marks: 100   Time: 3 hours

Answer any five questions. Marks allotted to each question are given within parentheses.

1. (a) Discuss how correlogram analysis may be used to determine the order(s) of a given stationary time series.

   (b) Let $\{X_t\}$ be a time series given by $X_t = a_t - 3a_{t-1}, a_t \sim WN(0, \sigma^2)$. Also, $\{X_t\}$ has a representation through another time series $\{Y_t\}$ as $Y_t = (1 - 0.6B)^{-1}(1 - 3B)a_t$.

   Check if $\{X_t\}$ is stationary and invertible in both the representations. Find also the ACF of $\{Y_t\}$

   [10+10=20]

2. (a) Distinguish between deterministic trend and stochastic trend.

   (b) Define the Wiener process and indicate why this is relevant for unit root tests.

   (c) Discuss why the usual t-test for an appropriate null hypothesis involving the coefficient of an AR (1) model is not valid for testing the presence of unit roots in a time series.

   [5+7+8 = 20]

3. (a) Describe how efficient out-of-sample forecasts can be obtained for a time series. Also suggest, with justifications, two criteria for evaluating the out-of-sample performance of an estimated time series model.

   (b) State a simple state-space model along with all its assumptions. Show how updating equation(s) for prediction can be obtained by using the Kalman filter.

   [10 + 10 = 20]

4. (a) Explain what you mean by structural break(s) in a time series.

   (b) What is the most important limitation, from statistical consideration, of the ADF unit root test if it is applied without due consideration to structural breaks in a given time series? Give explanations in support of your answer.

(c) Describe an appropriate test for testing the presence of unit roots in case there is a known structural break in the deterministic trend of the time series.

[5+7+8 = 20]

5. (a) State one of the standard spectral representations of a stationary time series along with all the assumptions, and then show that this representation indeed satisfies all conditions of (weakly) stationary time series.

(b) Given a finite realization $\{x_1, x_2, \ldots, x_n\}$ of a stationary time series, obtain the periodogram of $\{x_1, x_2, \ldots, x_n\}$ and then show that this can be regarded as a sample analogue of $2\Pi f(\lambda)$ where $f(\lambda)$ is the spectral density function.

[10+10 = 20]

6. Gould and Nelson investigated the stochastic structure of the 'velocity of money', y_t, using the yearly observations from 1869 through 1960, by considering the following two models.

$$y_t - y_1 = \underset{(.0094)}{1.0044} \, (y_{t-1} - y_1) + e_t,$$

and

$$y_t = \underset{(0.0176)}{0.0141} + \underset{(0.0199)}{0.9702} \, y_{t-1} + e_t^*.$$

(i) Test for the presence of unit roots in $\{ y_t \}$ based on both the models, and check if the conclusions on unit roots are the same. In either case, give explanations for your conclusion.

(ii) Suppose that the observed values of 'velocity of money' for the first (i.e., 1869) and the last (i.e., 1960) years in the given time series are 0.98 and 3.82, respectively. Forecast the value for the year 1962 by both the models. Explain the difference in the forecast values by the two models, if any.

[10 + 10 = 20]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2012-13
M. S. (Q.E.) .I Year
Modern Growth Theory

Date: 07.05.2013   Maximum Marks: 60   Duration: 3 Hours

Answer any three

1 (a)   Derive the rate of growth of consumption in the Ramsey-Solow model.

   (b)   Show that the steady-state equilibrium satisfies saddle point stability in this model.

[12+8]

2 (a)   Derive the optimum tax rate in the Barro model of endogenous growth.

   (b)   Explain why transitional dynamic properties do not exist in this model.

[12+8]

3   'Economic inequality always produces a negative effect on economic growth' – Explain the validity of the statement in the light of the model developed by Alesina and Rodrick.

[20]

4   'In a planned economy, endogenous rate of human capital accumulation varies positively with the intensity of positive external effect of human capital' – Do you agree with this view? Explain your answer in the light of the Lucas model of endogenous growth.

[20]

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Date: /0· 05· /3   Maximum Marks 60   Duration 3 hours

Answer Group A and Group B in separate scripts

Group A
Answer **any two** questions

1. Consider an economy with the following AD and AS equations. (All variables are in logarithms and notations are standard). Further, all agents have *rational* expectations.

(AD) $y = m - p$

(AS) $y = y^* + \beta\,[p - E(p \mid I)]$,   {For this economy, take $\beta = 1$, $y^* = 9$ units.}

where money supply (m) is a policy variable (to be specified in detail later).

(a) Explain the term $E(p \mid I)$. Does y^* have any particular interpretation?

To start with, suppose m is non-stochastic and the authority fixes m at 15 units and this is known to all. Find the *rationally expected* price level as well as the *actual* (i.e. market clearing) price level for the current period. Are they identical? Explain.

Find also the value of *actual y* for this period.

(b) Consider an alternative scenario where m is random and is given by the rule:

$m = 17 + u$ (u is a white noise with zero mean and a given standard deviation). Suppose, in the current period, the observed value of u turns out to be -2 so that the observed money supply is again 15. Find, once again for the current period, the *rationally expected* price level as well as the *actual* (i.e. market clearing) price level. Are they identical? Explain. Find also the value of *actual y*. Is it the same as the one in (a) above?

[8+ 7] = [15]

2. Consider an economy having a **fixed** exchange rate and **imperfect** capital mobility. The government of this economy, facing external payments deficits, unemployment, and budget deficits, adopts the advice of a committee recommending the imposition of tariff on imports.

Would this policy help to stimulate the economy, improve its trade balance and provide revenues to the government? Explain.

[*Hint*: A tariff at the rate θ raises the price of the foreign good from eP^* to $e(1 + \theta) P^*$.]

[15]

3. An open economy with involuntary unemployment and without any external capital transaction has to balance its external transactions through *only trade* in goods and services. Its trade balance relation is specified below.

$$TB = \overline{NX} + \alpha e \frac{P^*}{P} - mY \quad (\alpha > 0; \ m > 0),$$

where $\overline{NX}$ is *autonomous* amount of *net* export, Y is domestic output, e is nominal exchange rate and P and P^* are, respectively, domestic and foreign price level (both taken to be given). Let the set-up be an *IS-LM* one and i be the domestic nominal interest rate. Suppose, for some reason, the value of $\overline{NX}$ rises. Determine the new equilibrium values of e, Y, i and TB in each of the following two alternative cases:

(a) the economy has a *flexible* exchange rate; (b) the economy has a *fixed* exchange rate.

[7 + 8] = [15]

Group B

Answer all

1. Show how unemployment is sustained in equilibrium in the Shapiro- Stiglitz model.

What do you think would happen to equilibrium unemployment if the firms were to experience a technological progress (by this I simply mean that firms could produce more, say twice, the output they produced earlier, for any given level of input).

[12+3]

2 a) Show that in the OLG model an increased contribution to a "Pay as you go" pension fund will lower the steady state capital labour ratio.

In this context what can you say about the new steady state welfare of the economy compared to the earlier steady state.

b) Assume that the representative household's lifetime utility function is given by

$$V = V(C_1, C_2),$$ where C_i is consumption in period i, $V_i \equiv \frac{\partial V}{\partial C_i} > 0$ and $V_{ii} \equiv \frac{\partial^2 V}{\partial C_i^2} < 0.$ No restriction is put on V_{12}.

Households' intertemporal budget constraint is given by $C_1 + \frac{C_2}{1+r_1} = (1 + r_0)A_0 + \left[Y_1 + \frac{Y_2}{1+r_1}\right] \equiv \Omega$, where Y_i is the exogenous income in period 'i', A_0 is the initial financial wealth and r_i is the interest rate in period 'i'.

What can you say about the changes in period consumption C_i's due to a ceteris paribus change in wealth Ω?

[10+ 5=15]

INDIAN STATISTICAL INSTITUTE
203, B.T. ROAD, KOLKATA – 700108
Second Semester Back Paper Examination, 2012 – 13
M.S. (Q.E.) I Year
Time Series Analysis & Forecasting

Date: 02 . 07 . 13   Maximum Marks: 100   Time: 3 hours

Answer ALL questions. Marks allotted to each question are given within parentheses.

1. (a) Find the ACF of the following ARMA (2, 1) process:

$$x_t = 1.5x_{t-1} - 0.3x_{t-2} + a_t - a_{t-1}, \quad a_t \sim WN(0,\sigma^2).$$

   (b) Show that while the AR (2) process $x_t = x_{t-1} + cx_{t-2} + a_t$, where
   $a_t \sim WN(0,\sigma^2)$, is stationary for all values of c lying between – 1 and 0
   (*i.e.*, -1 < c <0), the AR (3) process given by $x_t = x_{t-1} + cx_{t-2} - cx_{t-3} + a_t$ is
   non-stationary for all values of c.

   (c) Find the 3-step ahead minimum MSE forecast at origin n of the following
   Time series

$$x_t = 2x_{t-1} - x_{t-2} + a_t - 0.4a_{t-1} + 0.3a_{t-2}, \quad a_t \sim WN(0,\sigma^2).$$

   [6+8+6 = 20]

2. (a) What are SACF and SPACF? Discuss briefly how these are used
   in correlogram analysis for deciding which of the standard stationary time
   series models should be used for a given time series.

   (b) Find the coefficients Φ_j, $j = 0,1,2,....$ in the time series representation

$$x_t = \sum_{j=0}^{\infty} \Phi_j a_{t-j}$$ of the ARMA (2, 1) process given by

$$(1\text{-}0.5B + 0.04B^2) \, x_t = (1 + 0.25B) \, a_t, \quad a_t \sim WN(0, \sigma^2).$$

   [12+8 = 20]

3. (a) Suggest an appropriate procedure for obtaining seasonal indices in a monthly time series from which trend and cyclical components have been removed.

(b) Describe the HEGY test for testing the presence of seasonal and nonseasonal unit roots in a quarterly time series.

[8 + 12 = 20]

4. (a) Describe the ADF test for unit roots in a time series and comment on its power.

(b) Describe the Quandt-Andrews test for detecting the presence of a structural break in a time series.

[10+10 = 20]

5. (a) Define the spectral density function, $f(\lambda)$, of a stationary process and then show that it is nonnegative for all $\lambda \in [-\pi, \pi]$. Also, find $f(\lambda)$ for an MA (1) process.

(b) State and prove the theorem on finding the spectral density function of a linear combination of stationary time series.

[10 + 10 = 20]