Mid-semester Examination: (2013-2014)
M. S. (Q. E.) I Year
Computer Prog. & Applications

Date:26.08.13 | Full Marks: 50 | Duration: 2 hours

Attempt all questions:

1. Perform the following decimal additions in binary number system, clearly showing the steps. Express the results in hexadecimal and octal number systems:

(i) 41.50 + 2.25
(ii) 11.75 + 9.25

5+5

2. What do you mean by programming language? How is it different from natural language? Explain any two of the following concepts: a) machine level language, b) high level language, c) hardware organization of computer 2+2+ (3+3)

3. Write a pseudocode and draw the flow chart for executing the program:

$S=|A| + |B|$ [where, $|X|$ =>modulus(X)]

3+3

4. What are the outputs of the following programs:

(i)
```
#include<stdio.h>
main()
{
  int var=5;
  printf("%f",var);
}
```

(ii)
```
# include <stdio.h>
main()
{
    int i=2, j=3, k=0;
    int p;
    p=(i, k, j);
    printf("%d\n", p);
}
```

(iii)
```
# include <stdio.h>
main()
{
    int var1=2,var2=12,var3=12;
    var1=var2= =var3;
    printf("%d",var1);
}
```

......contd on page 2

3+3+3

5. Identify the error/errors, if any, in the following programs along with their outputs:

(i)
```
# include <stdio.h>
main()
{
    char name;
    name='Hello';
    printf("%c", name);
}
```

(ii)
```
#include <stdio.h>
main()
{
    char x,y;
    int z;
    x=a;
    y=e;
    z=x+y;
    printf("%d",z);
}
```

3 + 3

6. Is the following code error free? If not, identify them. Otherwise say, how many times can it be executed and why?

```
#include<stdio.h>
.main( )
{
        char  another ;
        int  num ;
        do
        {
                printf ( "Enter a number " ) ;
                scanf ( "%d", &num ) ;
                printf ( "square of %d is %d", num, num * num) ;
                printf ( "\nWant to enter another number y/n " ) ;
                scanf ( " %c", &another ) ;

        } while (another = = 'y' ) ;
}
```

3

7. Write a program to obtain the factorial of a number. 6

INDIAN STATISTICAL INSTITUTE

MID-SEMESTER EXAMINATION 2013

Course name: MSQE I & M.STAT II
Subject name: Microeconomic Theory I
Date: 30.08.13
Maximum marks: 40
Duration: 2 hours
Please answer Sections I and II on separate answer books. Each part is worth 20 marks.

Section I

Q1. Let $\succeq$ be a *preference relation* over a non-empty set of alternatives X. Suppose also that $\succ$ and $\sim$ are the *strict preference relation* and the *indifference relation* respectively associating with $\succeq$.

(i) Show that $\succ$ is irreflexive. [2]

(ii) Define a utility function representing $\succeq$. Show that if $U : X \to \mathbb{R}$ is a function and $\succeq$ is rational, then "U is a utility function representing $\succeq$" is equivalent to "$x \succ y \Leftrightarrow U(x) > U(y)$ for any $x, y \in X$". [1+4]

(iii) Suppose $\succeq$ is rational and $U : X \to \mathbb{R}$ is a function. Show that if $U(x) = U(y) \Rightarrow x \sim y$ and $U(x) > U(y) \Rightarrow x \succ y$, then U is a utility function representing $\succeq$. [3]

Q2. Let $(\mathscr{B}, C(\cdot))$ be a choice structure. The *strict revealed preference relation* $\succ^*$ is defined by $x \succ^* y \Leftrightarrow \exists\ B \in \mathscr{B}$ such that $x, y \in B, x \in C(B)$ and $y \notin C(B)$.

(i) Define a *weak axiom of revealed preference* (or, simply $WARP$). Prove or, disprove: If $(\mathscr{B}, C(\cdot))$ satisfies $WARP$, then $\succ^*$ is transitive. [1+2]

(ii) Let $\mathscr{B} = \{\{x, y\}, \{x, y, z\}\}$ and $C(\{x, y\}) = \{x\}$. Show that if $(\mathscr{B}, C(\cdot))$ satisfies $WARP$, then $C(\{x, y, z\}) = \{x\}$ or, $= \{z\}$ or, $= \{x, z\}$. [2]

(iii) Answer any **one** question.

(a) Suppose that X is finite and $\succeq$ is a rational preference relation over a non-empty set of alternatives X. Show that $C^*(B; \succeq) = \{x \in B : x \succeq y \text{ for all } y \in B\}$ is non-empty for all non-empty $B \subseteq X$. [5]

(b) Let $(\mathscr{B}, C(\cdot))$ be a choice structure. If a rational preference relation $\succeq$ *rationalizes* $C(\cdot)$ relative to $\mathscr{B}$, then show that $C(B_1 \cup B_2) = C(C(B_1) \cup C(B_2))$ for every $B_1, B_2 \in \mathscr{B}$ such that $B_1 \cup B_2 \in \mathscr{B}$ and $C(B_1) \cup C(B_2) \in \mathscr{B}$. [5]

Section II

Q1. Suppose there is one firm and one consumer. There are two goods, good l, and one numeraire. The initial endowment of good l is 0, and that for the numeraire is $w_m > 0$. Let the consumer's utility function be $m + \phi(x)$, where $\phi(x) = \alpha + \beta \ln x$, for some $(\alpha, \beta) \gg 0$. Also, let the firm's cost function be $c(q) = \sigma q$, for some scalar $\sigma > 0$. Assume that both the firm and the consumer act as price-takers, and that the consumer receives all the profits of the firm. The price of the numeraire is normalised to 1, and the price of good l is denoted by p.

(a) Derive the competitive equilibrium price and output of good l. [2]

(b) How do these vary with α, β and σ? [3]

Q2. A tax is to be levied on a commodity traded in a competitive market.

(a) Suppose a specific tax is levied, so that an amount t is collected for every unit traded. Show that the ultimate cost to consumers and amount traded are independent of whether the consumers or the producers pay the tax. [10]

(b) Suppose an ad valorem tax is levied, so that an amount τp is collected for every unit traded, if market price is p. Are the ultimate cost to consumers and amount traded independent of whether the consumers or the producers pay the tax? [5]

Indian Statistical Institute

M.S.Q.E. 1st Year : 2013–2014
Mid-Semester Examination
Subject: Mathematical Methods

Date: 02/09/2013 Time: 3 hours Marks : 80

Answer Group-A and Group-B on separate answer scripts.

Group-A

1. Show that the determinant of the *Vandermonde matrix*

$$A_n = \begin{bmatrix} a_1^{n-1} & a_1^{n-2} & \cdots & a_1 & 1 \\ a_2^{n-1} & a_2^{n-2} & \cdots & a_2 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_n^{n-1} & a_n^{n-2} & \cdots & a_n & 1 \end{bmatrix}$$

is $\prod_{1\leq i<j\leq n}(a_i - a_j)$. [10]

2. Prove that a set of vectors $\{x_1, x_2, \ldots, x_m\}$ in a vector space V is linearly dependent if and only if at least one of the vectors in the set can be written as the linear combination of the others. [5]

3. Let A, B, C are arbitrary matrices for which matrix multiplication is defined. Then show that $A \cdot (B \cdot C) = (A \cdot B) \cdot C$. [5]

4. If an $n \times n$ square matrix A has a left inverse B and a right inverse C, then $B = C$. [5]

5. Construct two matrices A and B, such that $AB \neq BA$. [5]

6. Construct a singular matrix A, such that $A^2 = A$. [5]

7. Prove or disprove : If A is not the zero matrix and $AB = AC$, then $B = C$. [5]

Group-B

1. Give examples of functions :

   (a) **One-to-one** but not **onto**.

   (b) **Onto** but not **One-to-one**. [3 + 3]

2. Prove that $|a_1 + a_2 + \ldots + a_n| \leq |a_1| + |a_2| + \ldots + |a_n|$, for all $n \geq 1$. [6]

3. Using only postulates, prove that if a and b are real numbers, then $(-a) * (-b) = a * b$. [5]

4. Show that the set of all integers that are multiples of 2 or 3 is countable. [8]

5. Show that $\sqrt{2}$ is not a rational number, however it is a real number. [5 + 10]

Indian Statistical Institute

Mid-semester Examination: 2013-2014

MS(QE) I / M.Stat.II: 2013-2014

Game Theory I

Date: 04/09/2013 Maximum Marks: 40 Duration: 3 Hours

Answer TWO questions from each group

Group A

1. (a) The members of a group of people are affected by a policy, modelled as a number. Each person i has a favourite policy, denoted by x_i^*. She prefers the policy y to policy z if and only if y is closer to x_i^* than is z. The number N of people is odd. The following mechanism is used to choose a policy: each person names a policy, and the policy chosen is the median of those named (i.e. the policies named are put in order and the one in the middle is chosen). Formulate this as a strategic game and state whether choosing x_i^* is a weakly dominant strategy. Explain your answer.

3+7

2. (a) Suppose that two players choose locations a_1 and a_2 in the unit interval (where a_1 and a_2 are both measured from the left). Each player wishes to be as close as possible to the other. The payoff of each player is given by $-|a_1 - a_2|$. Show that every action of each player is rationalisable, while the set of Nash equilibria is $\{(a_1, a_2) : a_1 = a_2\}$.

(b) Find out the set of rationalisable strategies and the pure strategy Nash equilibria for the following game:

	2	b_1	b_2	b_3	b_4
1	a_1	2, 9	9, 2	4, 7	2, 3
	a_2	9, 2	2, 9	4, 7	2, 3
	a_3	7, 4	7, 4	5, 5	2, 3
	a_4	2, 2	2, 2	2, 0	12, 1

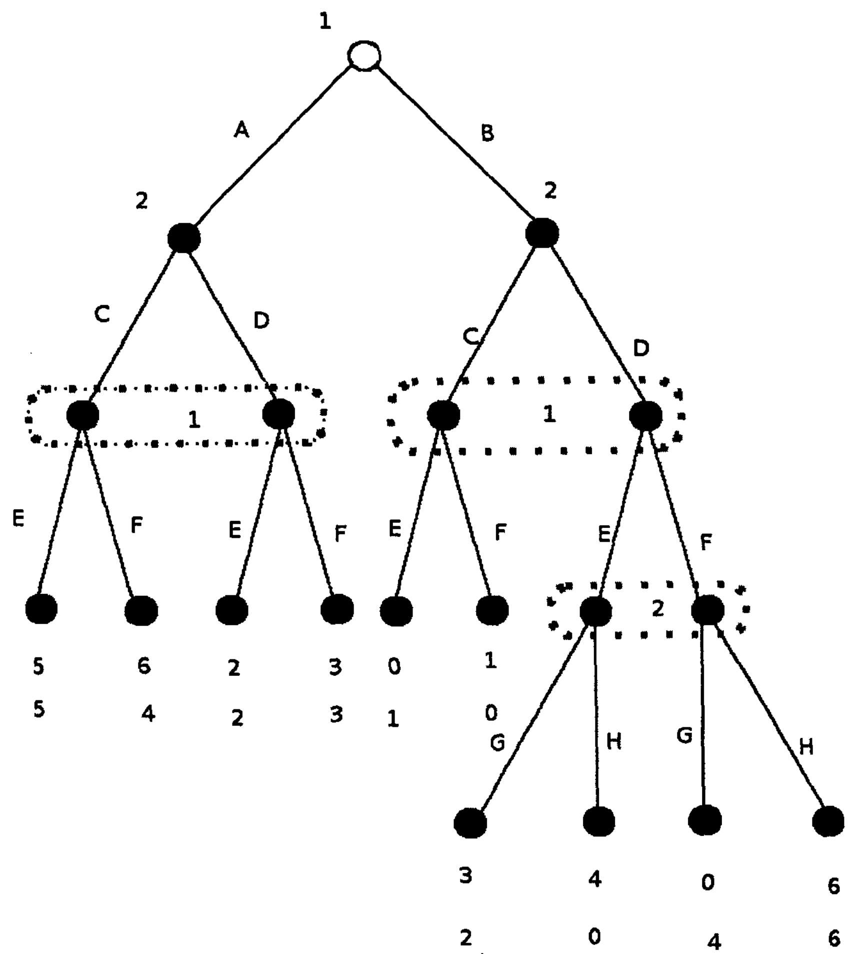
5+5

3. (a) Consider a game in which the following simultaneous move game is played twice:

	2	b_1	b_2	b_3
1	a_1	10, 10	2, 12	0, 13
	a_2	12, 2	5, 5	0, 0
	a_3	13, 0	0, 0	1, 1

The players observe the actions chosen in the first play of the game prior to the second play. What are the pure strategy subgame perfect Nash equilibria of this game?

(b) Provide the strategic form representation of the following extensive form game. How many subgames are there in this game?



6+(3+1)

Group B

4. (a) Using the properties of mixed strategy equilibrium check whether the mixed strategy profile $(\sigma_1, \sigma_2) = \left(\left(\frac{3}{4}, 0, \frac{1}{4}\right), \left(0, \frac{1}{3}, \frac{2}{3}\right)\right)$ is a mixed strategy Nash equilibrium for the following game:

2		L	C	R
1	T	2, 2	3, 3	1, 1
	M	5, 3	0, 7	2, 3
	B	3, 4	5, 1	0, 7

(b) For the following game, find all Nash equilibria (including mixed strategy Nash equilibria):

2		L	C	R
1	T	4, 2	0, 0	0, 1
	B	0, 0	2, 4	1, 3

4+6

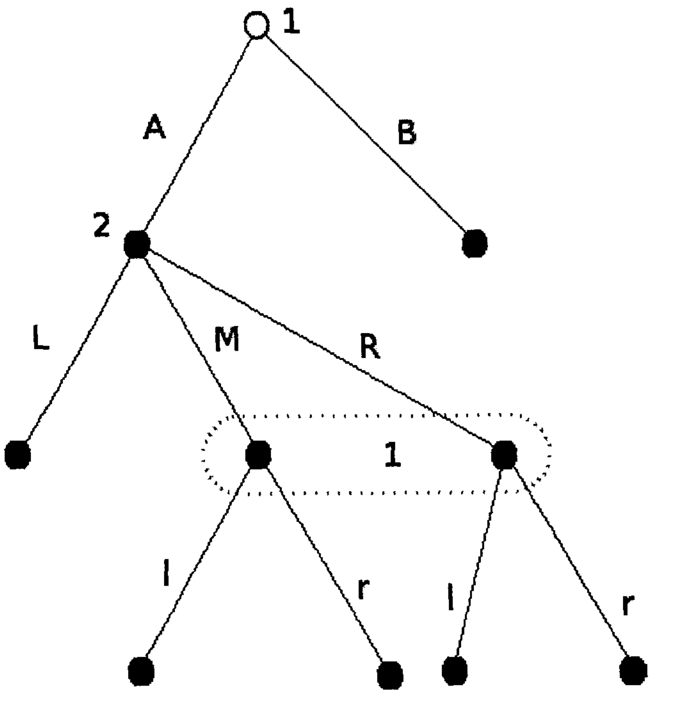
5. (a) Derive the least number of conditions for which the following game has no pure strategy Nash equilibrium.

2		L	R
1	T	a, b	c, d
	B	e, f	g, h

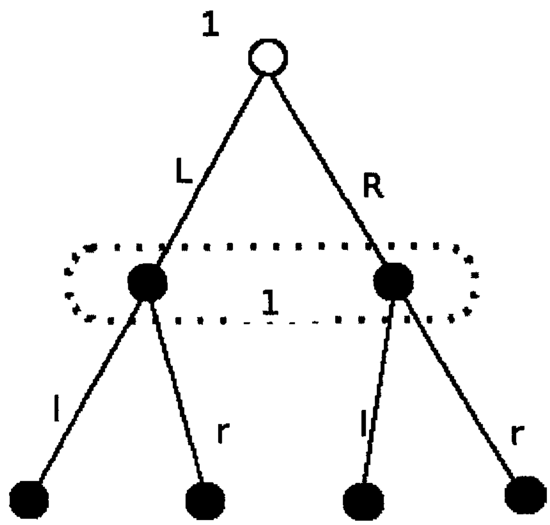
(b) For the game in 5 (a), show that if neither player has a strictly dominant strategy and the game has an equilibrium, then the equilibrium must be in mixed startegies.

5+5

6. (a) For the fllowing game player 1's pure strategy set is $[Al, Ar, Bl, Br]$. Let the mixed strategies over this set be (0.5, 0, 0.1, 0.4). Derive the behaviour strategy of player 1 that is equivalent to the given mixed strategy.



(b) Let the probabilities under mixed strategies associated with each of the pure strategies Ll and Rr be 0.5 for the game given below. Can you derive the equivalent behaviour strategy of the player for this game? Explain.



5+5

End Semestral Examination: (2013-14)

MS (QE) I Year

Computer Programming and Applications

Date: 11.11.2013 Maximum Marks: 100 Duration: 3 hours

Answer as many questions as you wish. But you may score at most 100.

1. Write brief notes on any three of the following: (i) Programming language (ii) Pseudocode and Flowchart (iii) Central Processing Unit (iv) Control Structures and their combinations 3X5=15
2. Explain with example the utility of the 'switch' and 'break' statements in C. Write a C program to check whether an integer is prime or not. 5+7=12
3. Explain with examples, what are the different categories of functions in C. Write a function in C language to calculate the highest common factor among two given integers. 4+8=12
4. What is a string? What do you mean by null character? Differentiate between the following pairs of functions using proper examples: (i) strcpy() and strncpy() (ii) strcat () and strncat () (2+2)+2X6=16
5. (a) Write a C code to approximate (for a predefined tolerance value) the square root of a number using an iterative method that starts from an initial guess. (b) Write another C code that prints out each digit from any given integer. 8+7=15
6. (a) Give examples to demonstrate the different processes to initialize a two-dimensional array. (b) Write a C code to print a given string in reverse order. (b) Write another C code to count the vowels that are present in the given string. 5+8+7=20
7. (a) Explain the preprocessing that is required to perform binary search in an array containing a set of integers. (b) Write the C code for this preprocessing. (c) Also write a function that takes as input an integer *x* and gives as output the position of *x* in the array if it is at all present. 4+6+8=18
8. Explain with example, what are local and global variables in C. Write a C program for printing the following figure by writing a suitable C program, where the number of lines in this pyramidal structure is taken as the input.

```
   *
  ***
 *****
*******
```

4+8=12

(b) If $X_1, X_2, \ldots, X_n$ are i.i.d. observations from a Poisson distribution with parameter λ, determine a likelihood ratio test at level α for $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$. (5+5=10)

6. (a) Explain with suitable example the concepts of type I and type II errors and power of a test.

   (b) Let $X_1, X_2, \ldots, X_n$ be an i.i.d. random sample from $N(\mu_1, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_n$ be another i.i.d. random sample from $N(\mu_2, \sigma^2)$ and the two samples are independent of one another. Develop a test at level α for testing $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

7. What do you mean by a sequence of random variables to (i) converge in Probability and (ii) converge in distribution? Show that convergence in probability implies convergence in distribution. Is the converse true? Give support in favour of your opinion. (2+2+3+3=10)

Indian Statistical Institute

M.S.Q.E. 1st Year : 2013–2014
Semester Examination
Subject: Mathematical Methods

Date: 18/11/2013 Time: 3 hours Marks : 100

Answer Group-A and Group-B on separate answer scripts.
Notations used are as explained in the class.

Group-A

1. For any two matrices A and B for which AB can be defined, show that

   (a) $\mathcal{N}(AB) \supseteq \mathcal{N}(B)$,

   (b) $\mathcal{N}((AB)^T) \supseteq \mathcal{N}(A^T)$,

   (c) $\mathcal{C}(AB) \subseteq \mathcal{C}(A)$,

   (d) $\mathcal{R}(AB) \subseteq \mathcal{R}(B)$. [10]

2. Let $T : V \to W$ be a linear transformation from a vector space V to a vector space W. Then prove that the kernel $Ker(T)$ and the image $Im(T)$ are subspaces of V and W, respectively. [5]

3. Let $T : V \to W$ be a linear transformation from a vector space V to a vector space W. Prove that, T is one-to-one if and only if $Ker(T) = \{0\}$. [5]

4. If x and y are vectors in an inner product space V, then prove that $\langle x, y\rangle^2 \leq \langle x, x\rangle\langle y, y\rangle$. [5]

5. Let A be an $n \times n$ matrix. Then prove that

   (a) the determinant of A is the product of the n eigenvalues, and

   (b) the trace of A is the sum of the n eigenvalues.

   [9]

6. Find the matrices A and B such that $det(A) = det(B)$, $tr(A) = tr(B)$, but A is not similar to B. [6]

7. Show that the characteristic polynomial of the $n \times n$ matrix

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{pmatrix}$$

   is $f(\lambda) := a_0 + a_1\lambda + \cdots + a_{n-1}\lambda^{n-1} + \lambda^n$. [5]

8. Show that a real symmetric $n \times n$ matrix A is positive definite if and only if all the eigenvalues of A are positive. [5]

Group-B

1. Prove that the set of real numbers in the interval $[2, 5]$ is not countable. [10]

2. Let $x_1 \geq 2$ and $x_{n+1} = 1 + \sqrt{(x_n - 1)}$, $n \geq 1$. Show that $\lim_{n \to \infty} x_n$ exists and find it. [10]

3. Using ϵ-δ method, prove that the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ is continuous at every point. [10]

4. Give an example of a function ($\mathbb{R} \to \mathbb{R}$) that is continuous everywhere, differentiable everywhere except for threee points. [10]

5. Find a solution to the equation $x^3 - x - 2 = 0$, correct upto two places of decimal. [10]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2013-2014)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 20.11.2013 Maximum marks: 60 Duration: 3 Hours

Please answer Sections I and II on separate answer books. Each part is worth 30 marks.

Section I

Throughout this section, $\mathbb{R}^L$ is the L-dimensional Euclidean space. Let

$$\mathbb{R}^L_+ = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i \geq 0 \text{ for all } 1 \leq i \leq L\}$$

and

$$\mathbb{R}^L_{++} = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i > 0 \text{ for all } 1 \leq i \leq L\}.$$

Q1. Consider an economy with only one consumer and $\mathbb{R}^L$ as the commodity space. The demand function of the consumer is denoted by $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to X$, where $X \subset \mathbb{R}^L$ denotes the consumption set of the consumer.

(i) State the weak axiom of revealed preference ($WARP$) for x. Show that if x satisfies the $WARP$ then for any compensated price change from (p, w) to $(p'. w') = (p', \; p' \cdot x(p, w))$, the following inequality holds:

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0. \qquad [1 + 5]$$

(ii) Take $L \geq 3$ and $X = \{(x, y, z, 0, \cdots, 0) \in \mathbb{R}^L : x, z \geq 0 \text{ and } y < 0\}$. Define $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to X$ by

$$x(p, w) = \left(\frac{p_2}{p_3}, -\frac{p_1}{p_3}, \frac{w}{p_3}, 0, \cdots, 0\right).$$

Prove that x is homogeneous of degree zero and satisfies Walras' law. Show further that x violates the $WARP$. [2+2]

Q2. Answer any **two** questions.

(i) For $(x_1, y_1), (x_2, y_2) \in \mathbb{R}^2_+$, define the lexicographic preference relation by letting $(x_1, y_1) \succeq (x_2, y_2)$ if and only if "$x_1 > x_2$ or ($x_1 = x_2$ and $y_1 \geq y_2$)". Show that it is not continuous. Prove that there is no utility function that can represent the lexicographic preference relation. [2+3]

(ii) Consider a preference relation over $\mathbb{R}^2_+$ represented by the utility function $U(x,y) = \sqrt{x} + \sqrt{y}$. Find the demand functions for the commodities 1 and 2 as they depend on prices and wealth. [5]

(iii) Given a $(p,w) \in \mathbb{R}^L_{++} \times \mathbb{R}_{++}$, the *utility maximization problem* (UMP) of the consumer is the following:

$$\sup\{U(x) : x \in B(p,w)\}$$

where $B(p,w) = \{x \in \mathbb{R}^L_+ : p \cdot x \leq w\}$ is the budget set for (p,w) and $U : \mathbb{R}^L_+ \to \mathbb{R}$ is a utility function representing a preference relation $\succeq$.

(a) Show that the solution of this problem exists if U is continuous. [2]

(b) Prove that the function $v : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}$, defined by

$$v(p,w) = \sup\{U(x) : x \in B(p,w)\},$$

is strictly increasing in w whenever $\succeq$ is locally non-satiated and U is continuous. [3]

Q3. Answer **all** questions.

(a) Suppose that f is the production function associated with a single-output technology, and let Y be the production set of this technology. Show that Y satisfies constant returns to scale if and only if f is homogeneous of degree one. [5]

(b) A firm has a production function $y = x_1 x_2$, where x_1 and x_2 denotes the amount of inputs. If the prices of inputs are equal to 1, then the minimum cost of production is 4. Find the value of y when the minimum occurs. [3]

(c) Recall that the *addition closure* of a production set Y is the smallest production set that is additive and contains Y. For a single-output technology with only one input, let $f(x) = x^2$ denote the production function. Find the addition closure of the production set of this technology. [2]

Section II

Q.1. Suppose we have a single good in a market with I buyers, each of whom wants at most one unit of the good. Buyer i is willing to pay up to v_i for his unit, with $v_1 > \cdots > v_I$. There are a total of $q < I$ units available. Suppose that buyers simultaneously submit bids, and that the units of the good go to the q highest bidders, who pay the amounts of their bids.

Show that every buyer making a bid of v_{q+1} and the good being assigned to buyers $1, \cdots, q$ is a Nash equilibrium of this game. [5]

Show also that in any pure strategy Nash equilibrium, buyers 1 through q receive a unit, while buyers $q+1$ through I do not. [5]

Q.2. Recall in the linear city model, we analysed equilibrium under the assumption $v > c + 3t$, where v is consumer valuation, c is cost of production per unit, and t is transport cost. Suppose instead we look at the case where $v \in (c + 2t, c + 3t)$.

Derive the best-response functions. Find the unique symmetric Nash equilibrium of this game. [5 +5]

Q.3. Consider an infinitely repeated Cournot duopoly with discount factor $\delta < 1$, unit costs of $c > 0$, and inverse demand function $p(q) = a - bq$, with $a > c$, and $b > 0$.

Under what conditions can the symmetric joint monopoly outputs $(q_1, q_2) = (\frac{q^m}{2}, \frac{q^m}{2})$ be sustained as an equilibrium of the infinitely repeated game with strategies that call for $(\frac{q^m}{2}, \frac{q^m}{2})$ to be played if no one has yet deviated and for the static Cournot equilibrium to be played otherwise? [10]

Indian Statistical Institute
Semester Examination: 2013-2014
MS(QE) I/ M.Stat.II: 2013-2014

Game Theory I

Date: 23/11/2013   Maximum Marks: 60   Duration: 3 Hours

Answer FIVE questions taking at least two questions from each group.

Group A

1. Consider any Two-Person-Zero-Sum Game. Define $v_i = \max\limits_{s_i \in S_i} (\min\limits_{s_j \in S_j} u_i(s_i, s_j))$; $i \neq j$. Prove the following.

   (a) $v_2 = -\min\limits_{s_2 \in S_2} (\max\limits_{s_1 \in S_1} u_1(s_1, s_2))$.

   (b) $-v_2 \geq v_1$.

   (c) If $-v_2 = v_1$ holds for the strategy pair (s_1^*, s_2^*), then show that it is a Nash equilibrium pair.

   (d) If v_1^m be the security level of player 1 under mixed strategy, prove that $v_1^m > v_1$.

   [2+3+3+4=12]

2. (a) Define the set of feasible and individually rational payoff vectors in the context of a repeated game.

   (b) Consider the following stage game:

	L	R
T	6, 6	2, 7
B	7, 2	0, 0

   Show that the payoff vector (4, 4) is feasible and individually rational.

   (c) Frame suitable strategy to show that (4, 4) can be achieved as the (discounted) average payoff of the infinitely repeated game, provided that the discount factor is large enough.

   [2+2+8=12]

3. There are two players to play a simultaneous move game. Player 1 has two actions T and B, and player 2 has actions L and R. Nature determines which of the following two games they will play when each game occurs with probability ½. [Consider only pure strategy equilibrium.]

	L	R
T	2, 1	0, 0
B	0. 0	1, 2

(Game 1)

	L	R
T	2, 0	0, 2
B	0, 1	1, 0

(Game 2)

(a) If Nature does not reveal the information to any player before the actions are chosen, find the optimal choice of actions of the players.

(b) If Nature reveals the information only to player 2, and not to player 1, find the optimal strategy (Bayes Nash equilibrium) of the players.

[4+8=12]

Group B

4. (a) Consider a variant of the bargaining game of alternating offers involving two players where each player i loses $c_i > 0$ during each period of delay (rather than discounting her payoff). Show that if $c_1 < c_2$, then this game has a subgame perfect Nash equilibrium in which player 1 always proposes (1,0).

(b) Show that if $c_1 = c_2 = c < 1$ then the game has many subgame perfect equilibrium outcomes.

[7+5=12]

5. Consider an N-bidder auction which is a "mixture" of a first and second-price auction in the sense that the highest bidder wins and pays a convex combination of his own bid and the second highest bid. Precisely, there is a fixed $\alpha \in (0,1)$, such that upon winning, bidder i pays $\alpha b_i + (1-\alpha)\, max_{j \neq i} b_j$. Find a symmetric equilibrium bidding strategy in such an auction when all bidders' values are distributed according to F with a continuous density f over the interval [0,1]. Also calculate the symmetric equilibrium bidding strategy when the values are uniformly distributed over [0,1].

[6+6=12]

6. A firm and a union representing L workers negotiate a wage-employment contract. The firm produces $f(l)$ units of output when it employs l workers, where f is an increasing function. Each worker not hired by the firm obtains the payoff w_0 (the wage in another job, or perhaps the unemployment benefit). The contract (w, l), in which the firm pays the wage w and employs l workers, yields payoffs of $f(l) - wl$ to the firm and $lw + (l - L)w_0$ to the union. In the event of disagreement, the firm's payoff is zero and that of the union is Lw_0; let $d = (0, Lw_0)$. Assume that f is such that the set U of possible pairs of payoffs to agreements satisfies the assumptions on a bargaining problem (i.e. U is compact and convex, $d \in U$ and some agreement is better for both players than disagreement). The Nash bargaining solution (U, d) involves the employment level l^* that maximizes $f(l) + (l - L)w_0$. Find the wage level in the Nash bargaining solution of (U, d).

[12]

MSTAT II - Theory of Finance I

Final Exam. / Semester I 2013-14

Time - 3 hours/ Maximum Score - 50

ate : 26.11.2013

NOTE : SHOW ALL YOUR WORK. RESULTS USED MUST BE CLEARLY STATED.

1. (a) (2 marks) Let $\{X_t\}$ be a continuous time stochastic process. Write down the exact conditions under which $\{X_t\}$ can be said to be a standard Brownian motion.

   (b) (4 marks) Let $\{B_t\}_{t\geq 0}$ be a standard Brownian motion. Define a process $\{X_t\}_{t\geq 0}$ as follows:

   $$X_t = \begin{cases} tB_{\frac{1}{t}} & \text{for } t \neq 0 \\ 0 & \text{for } t = 0. \end{cases}$$

   Show that $\{X_t\}$ is a standard Brownian motion.

   (c) (4 marks) Let $a > 0$. Define, $\Lambda = \sup\{t > 0 : B_t = at\}$. Here Λ is the last time, the (standard) Brownian motion $\{B_t\}$ touches/crosses a line at. Find the distribution of Λ.

   (d) (5 marks) Let $\sigma > 0$. For $t \geq 0$, define,

   $$M_t = e^{\sigma B_t - \frac{\sigma^2 t}{2}}.$$

   Let M_t be the price of a US dollar in terms of Euro at time t (assuming both of their domestic interest rate same). Let $\sigma = 0.35$ be the volatility per annum for the process. Find the probability that the price of a US dollar would never be more than one Euro after 4 years from its birth.

2. A financial institution in India plans to buy stocks in the European market for their customers, where stocks are denominated in Euro. Assume that the price of one such stocks at time t is S_t and it follows the model,

   $$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

   where $\{W_t\}$ is a standard Brownian motion, $\mu = r_e$ is the risk-free interest rate in the Euro-zone and σ is the constant volatility of the stock during the period considered. Since the company is India-based and so are their customers they would like to get their profit in rupees only. Let P_t be the price of a Euro in rupees and it follows the model,

   $$dP_t = \mu_0 P_t dt + \sigma_0 P_t dB_t,$$

where $\{B_t\}$ is a standard Brownian motion and for $s < u$, $COV((B_s, W_s)', (B_t, W_t)) = \begin{pmatrix} s & \rho s \\ \rho s & s \end{pmatrix}$ where $\rho \in (-1, 1)$. Here $\mu_0 = r_i - r_e$ = difference between the risk-free interest rate in India and the risk-free interest rate in Euro-zone, whereas σ_0 is the constant volatility of the Euro (with respect to the rupees) during the period considered.

(a) (7 marks) Let $U_t = P_t S_t$ be the price of the stock in rupees. Use Itô's formula to show that U_t satisfies,

$$dU_t = \mu_1 U_t dt + \sigma_1 U_t d\hat{W}_t,$$

for some μ_1, σ_1 and a standard Brownian motion $\{\hat{W}_t\}$. Write down μ_1 and σ_1 and $\{\hat{W}_t\}$ in terms of the above parameters and standard Brownian motions $\{B_t\}$ and $\{W_t\}$.

(b) (7 marks) A derivative on the stock pays off $Rs.50$ at maturity time T if $U_T \geq 100$, $Rs.25$ if $U_T \leq 50$ and pays nothing otherwise. Draw a diagram for the profit (against U_T). Use the risk-neutral valuation to calculate the price of the derivative at time 0, when $r_i = 9\%$, $r_e = 3\%$, $\sigma = 0.3$, $\sigma_0 = 0.4$, $\rho = 0.1$, $S_0 = 100$ Euro, $P_0 = Rs.60$ and $T = 3$ months.

3. ($4 \times 4 = 16$) Write TRUE or FALSE and justify your answer (the justification should be Mathematical as much as possible).

(a) If the log price ($\log S$) follow a Standard Brownian motion then expected time for the log price to hit certain barrier $a > \log S_0$ is finite.

(b) Exponential ARCH is worse than GARCH(1,1) when it comes to capturing the property of negative correlation between returns and change in volatility.

(c) Suppose two portfolios have equal expected return. Then the portfolio with more volatility is preferable to the portfolio with less volatility, whether the expected return is positive or negative.

(d) Binomial model gives the unique martingale pricing formula whereas the trinomial model (that you get, through the explicit difference method with log transformation, or otherwise) gives pricing through infinitely many equivalent martingales.

4. (a) (5 marks) A risky asset A is said to dominate second degree stochastically dominate (SSD) risky asset B if all risk averse individuals (having utility function whose first derivatives are continuous except on atmost a countable set) prefer A to B. Prove that it is equivalent to $r_B = r_A + \epsilon$, with $E(\epsilon | r_A) = 0$.

(b) Assume that the distribution of risky assets are multivariate Normal.

(i) (3 marks) Argue that the market portfolio is on the efficient frontier.

(ii) (4 marks) If the two fund separation holds (and if there is no riskless asset) then show (using part (4a) or otherwise) that for any feasible portfolio r_q can be expressed as

$$r_q = r_{zc(m)} + \beta_{qm}(r_m - r_{zc(m)}) + \epsilon$$

where r_m is the market portfolio and $r_{zc(m)}$ is the corresponding zero-covariance portfolio and β_{qm} is such that

$$E(r_q) = E(r_{zc(m)}) + \beta_{qm}(E(r_m) - E(r_{zc(m)})) \quad \text{and} \quad E(\epsilon \mid r_{zc(m)} + \beta_{qm}(r_m - r_{zc(m)})) = 0.$$

Argue that $\beta_{qm} = Cov(r_q, r_m)/Var(r_m)$ and $E(r_m) > E(r_{zc(m)})$.

(iii) (3 marks) When riskless asset (r_f) exists and it is in strictly positive supply and assume that borrowing at the riskless rate r_f is prohibited, then show that for any feasible portfolio q,

$$E[r_q] = E[r_{zc(m)}] + \beta_{qm}(E[r_m] - E[r_{zc(m)}]), \quad E[r_m] > E[r_{zc(m)}] \quad \text{and} \quad E[r_{zc(m)}] \geq r_f.$$

All the best.

(Back Paper)

Indian Statistical Institute

Semester Examination: 2013-2014

MS(QE) I/ M.Stat.II: 2013-2014

Game Theory I

Date: 05/02/2014 ~~/11/2013~~ Maximum Marks: 100 Duration: 3 Hours

Group A

Answer TWO questions

1. (a) Consider the game of Matching Pennies. Derive the mixed strategy Nash equilibrium of this game.

   (b) Provide a pure strategy Bayesian Nash equilibrium of a corresponding game of incomplete information such that as the incomplete information disappears, the players' behavior in the Bayesian Nash equilibrium approaches their behavior in the mixed strategy Nash equilibrium in the original game of complete information.

   [10+15=25]

2. Consider the following infinitely repeated Cournot game of two players. The market demand function is: $P(X) = 1 - X,\ 0 \le X \le 1$. The cost function of each firm is: $C(x_i) = (1/2)x_i;\ x_1 + x_2 = X$.

   (a) What is 'Cournot-Nash reversion' strategy?

   (b) Let the common discount factor be $\delta = (1/2)$. Will the collusive output be sustained as a subgame perfect Nash equilibrium under the Cournot-Nash reversion strategy?

   (c) If there is one-period lag in detecting deviation by the rival, will the collusive outcome still be sustained?

   [3+15+7=25]

3. (a) Explain the concept of pure strategy, mixed strategy and behavior strategy. State the relation between behavior strategy and mixed strategy. Establish the relation with the help of an example.

(b) Consider the following problem. Each of two firms has one job opening. Suppose the firms offer wages w_1 and w_2 where $\frac{1}{2} w_1 < w_2 < 2 w_1$. Imagine that there are two workers each of who can apply to only one firm. The workers simultaneously decide whether to apply to firm 1 or firm 2. If only one worker applies to a firm, he gets the job with probability one. If both apply, then each can get selected with probability ½. Find the Nash equilibrium of the game.

[(7+4+6)+8=25]

Group B

Answer all questions

4. (a) Consider the following strategies in the infinite horizon version of Rubinstein's bargaining model. According to notational conventions, the offer $(s, 1-s)$ means that player 1 will get s and player 2 gets $(1-s)$ irrespective of who makes the offer. Let $s^* = \frac{1}{1+\delta}$. Player 1 always offers $(s^*, 1-s^*)$ and accepts offer $(s, 1-s)$ only if $s > \delta s^*$. Player 2 always offers $(s^*, 1-s^*)$ and accepts offer $(s, 1-s)$ only if $1-s > \delta s^*$. Show that these strategies are a Nash equilibrium as well as a subgame-perfect equilibrium.

(b) Consider a variant of the bargaining game of alternating offers in which only one player, say player 1 makes proposals; in every period, player 1 makes a proposal, which player 2 either accepts, ending the game, or rejects, leading to the next period, in which player 1 always proposes $(x_1, 1-x_1)$ and player 2 always accepts a proposal (y_1, y_2) if and only if $y_2 \geq 1 - x_1$. Find the value(s) of x_1 for which this strategy pair satisfies the one-deviation property. Comment whether the same strategy pair is also a subgame perfect Nash equilibrium.

[(8+5)+12=25]

5. Suppose there are n bidders with private values that are distributed independently according to the distribution $F(x) = x^a$ over $[0,1]$, where $a > 0$. Derive the symmetric equilibrium bidding strategies in first-price auction and second price auction. Comment on the ranking of expected revenues from the two formats and argue in favour of your comment.

[10+10+5=25]

MS QE-I / MSAT II - Theory of Finance I

Back Paper Exam. / Semester I 2013-14

Time - 3 hours/ Maximum Score - 100

7.2.14

1. (12 marks) Let p be any frontier and let q be any other feasible portfolio. Assume that there is no risk-free asset and $E(r_p) \neq E(r_{mvp})$.

   (a) Then,

   (i) find the best possible linear fit for r_q (i.e., find α_1 and α_2) that minimises $E(r_q - (\alpha_1 r_p + \alpha_2 r_{zc(p)}))^2$ with respect to α_1 and α_2, subject to $E(r_q - (\alpha_1 r_p + \alpha_2 r_{zc(p)})) = 0$;
   (ii) argue briefly that it is equivalent to finding the best possible linear fit for r_q that minimises $E[(r_q - E(r_q)) - (\alpha_1(r_p - E(r_p)) + \alpha_2(r_{zc(p)} - E(r_{zc(p)})))]^2$ with respect to α_1 and α_2.

   (b) Would $\alpha_1 + \alpha_2 = 1$ hold? Interpret briefly your findings in terms of SSD (and portfolio frontier).

2. (12 marks) A risky asset A is said to third degree stochastically dominate (TSD) a risky asset B if all investors exhibiting decreasing absolute risk aversion prefer A to B. Provide a sufficient condition on the distribution function for $A \geq_{TSD} B$ that is strictly weaker than that for the SSD. Show all your work while deriving the sufficient condition.

3. (a) (5 marks) Assuming one step model, find the value of Δ in terms of future share prices (S_u and S_d) and future payoffs (f_u, f_d), so that the portfolio would be riskless. Let r be the risk-free interest rate per annum. Find the risk-neutral probability p, (in terms of S_0, u, d and r). Show that, assuming $S_d < K < S_u$, it satisfies the equation,

$$f_0(1 + rT) = [pf_u + (1-p)f_d].$$

   (b) (16 marks) Derive the n-step model and then the limiting distribution of S_T (i.e., Log-Normal distribution). Find the value of f_0, the price of the derivative.

   (c) (3 marks) Calculate the price of this derivative at time zero, taking $S_0 = 60$, $r = 6\%$, the strike price $K = 63$, $\sigma = 0.35\%$ and the maturity time $T = 6$ months.

4. (a) (12 marks) Let $\{X_i\}$ be a sequence of independent $N(0, \sigma^2)$ random variables. For $n \geq 1$, define $S_n = X_1 + \cdots + X_n$, with $S_0 = 0$. Then show that (i) $\{S_n\}_{n\geq 0}$

and (ii) $\{W_n = \{(S_n^2 - n\sigma^2)\}_{n\geq 0}$ are both martingales with respect to the filtration $\{\mathcal{F}_n = \sigma(S_0, X_1, \ldots, X_n)\}_{n\geq 0}$.

(b) (12 marks) Let the daily price of a currency move according to $\{\exp(S_n)\}$ in (a) above, with $\sigma = 0.25$. Define, $T = \inf\{n \geq 0 : S_n \geq \log(1.3),\ or,\ S_n \leq \log(0.8)\}$. Find the probability that starting at 1, the price of the currency reaches 1.3 before reaching 0.8. Calculate the expected number of days it takes to reach the upper or lower boundary (i.e., 1.3 or 0.8), starting from 1.

In calculating your answer you may assume the process hits the boundary exactly, as in continuous time case, and $E(W_T) = E(W_1),\ \ E(S_T) = E(S_1)$.

5. A trader buys a European put option on a stock for $Rs.3$ with a maturity in 3 months. The initial stock price was $Rs.42$ and the strike price was $Rs.40$. Assume the price used risk-neutral valuation and the stock follows the Black-Scholes model with the risk-free interest rate $r = 3\%$.

   (a) (7 marks) Would the Put-Call parity hold for options for any t? Justify your answer.

   (b) (6 marks) Would the price of the put options increase if the volatility decreases? Give a mathematical justification to your answer.

   (c) (7 marks) Find the probability that the trader would make a profit on the Put options. What is the probability that the option would be exercised?

   (d) (4 marks) Draw a diagram plotting the trader's profit against the stock price at the maturity of the option.

   (e) (4 marks) What happens to the price of the option (at time $t = 0$) if risk-free interest rate is increasing or decreasing? Give a mathematical justification for your answer.

All the best.

INDIAN STATISTICAL INSTITUTE

BACK PAPER: (2013-2014)

MSQE I and M.Stat II

Microeconomic Theory I

Date: 7.2.14 Maximum marks: 100 Duration: 3 Hours

Please answer Sections I and II on separate answer books. Each part is worth 50 marks.

Section I

Throughout this section, $\mathbb{R}^L$ is the L-dimensional Euclidean space. Let

$$\mathbb{R}^L_+ = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i \geq 0 \text{ for all } 1 \leq i \leq L\}$$

and

$$\mathbb{R}^L_{++} = \{x = (x_1, \cdots, x_L) \in \mathbb{R}^L : x_i > 0 \text{ for all } 1 \leq i \leq L\}.$$

The symbols $x \geq y$ and $x \gg y$ are used to denote the facts that $x - y \in \mathbb{R}^L_+$ and $x - y \in \mathbb{R}^L_{++}$ respectively.

Q1. Answer **all** questions.

(i) Let $\succeq$ be a rational preference relation over a non-empty countable set of alternatives X. Show that the preference relation has a utility representation. [10]

(ii) A preference relation $\succeq$ over $\mathbb{R}^L_+$ is called *monotone* (resp. *weakly monotone*) if $x, y \in \mathbb{R}^L_+$ and $x \gg y$ (resp. $x \geq y$) together imply $x \succ y$ (resp. $x \succeq y$). Show that if $\succeq$ is locally non-satiated, rational and weakly monotone then it is monotone. Prove or disprove: the preference relation $\succeq$ over $\mathbb{R}^2_+$, defined by $(x, y) \succeq (x', y')$ if and only if $xy' \geq x'y$, is monotone. [7+3]

Q2. Consider the demand function $x : \mathbb{R}^L_{++} \times \mathbb{R}_{++} \to \mathbb{R}^L_+$.

(i) Show that if x satisfies the *weak axiom of revealed preference* ($WARP$), then x is homogeneous of degree zero. [5]

(ii) Show that if x is generated by a rational preference relation, then it must satisfy $WARP$. [6]

(iii) Consider an economy with two commodities. Suppose that the demand function x of a consumer satisfies the Walras's law and $x_1(p, w) = \frac{\alpha w}{p_1}$, where $0 < \alpha < 1$. Find his demand function for the second commodity. Is his demand function homogeneous of degree zero? [4]

Q3. Answer **all** questions.

(i) The production set is additive and satisfies the non-increasing returns to scale condition if and only if it is a convex cone. [8]

(ii) Show that for a single-output technology, the production set Y is convex if and only if the production function f is concave. [7]

Section II

Q.1. Recall in the unique equilibrium of the Bertrand duopoly model, either firm sets price equal to c, the marginal cost. Consider a variant of the model where prices must be named in some discrete unit of account of size s. Show that both firms setting prices equal to the smallest multiple of s that is strictly greater than c is a pure strategy equilibrium of this game. [5]

Q.2. Consider a J-firm Cournot oligopoly model in which firms' costs differ. Let $c_j(q_j) = \alpha_j \tilde{c}(q_j)$ denote firm j's cost function, and assume that $\tilde{c}(\cdot)$ is strictly increasing and convex. Assume also that $\alpha_1 > \cdots > \alpha_J$.

Show that if more than one firm is making positive sales in a Nash equilibrium of this model, then we cannot have productive efficiency, i.e., the equilibrium aggregate output is produced inefficiently. [15]

Q.3. Recall in the linear city model, we assumed that a consumer's travel cost was linear in distance, i.e., total cost of purchasing from a firm j was $p_j + td$, where $t > 0$ was the cost parameter and d was the consumer's distance from the firm. Consider a variant of the model where the travel cost is quadratic in the distance, i.e., total cost of purchasing from a firm j is $p_j + td^2$.

Derive the symmetric Nash equilibrium of this model, assuming that v, consumer valuation, is large enough so that the possibility of non-purchase can be ignored. [30]

INDIAN STATISTICAL INSTITUTE
Mid-Semestral Examination: (2013-2014)
MS (Q.E.) I Year
Macroeconomics I

Date: 24.02.14 | Maximum Marks 40 | Duration 3 hours

1. Show the condition that perceived demand curve be more elastic than the market share demand curve is sufficient to guarantee that the market equilibrium in the Dixit- Stiglitz model is stable. (10)

2. Show that a coordinated reduction in all prices and wages, beginning from a situation of monopolistic equilibrium, will raise real profits and also the utility. (10)

3. Analyse how inflation is viewed as a quasi-equilibrium phenomenon in the model of Bent Hansen. (10)

or

Derive the dynamic aggregate supply curve from the Phillip's curve. How does the long run dynamic aggregate supply curve differ from a short run dynamic aggregate supply curve? Distinguish, in this context, between short run equilibrium and long run equilibrium. (4+3+3)

4. Consider the Mundell Fleming model

a) For a small open economy with perfect capital mobility, under fixed exchange rate regime. What will be the impact of an exogenous decline in the interest rate in the world financial market in this model? Explain your answer.

b) Suppose in the above model, y_y^{∂} denoting marginal propensity to spend in the commodity market is 0.8, while marginal propensity to import is 0.3 and $\frac{\partial L}{\partial y} = (L_y) = 0.25$. Suppose initially the system is in full equilibrium with net inflow of foreign exchange = 0.

Now following an autonomous increase in demand for commodity by 50 units, the system attains a new full equilibrium. Compute the change in capital account balance from the old to the new equilibrium. (Assume all money to be H.P.M. and NFY + NFT = 0)

(6+4)

INDIAN STATISTICAL INSTITUTE
MID-SEMESTRAL EXAMINATION, 2013-2014
M.S.(Q.E.) I & II year
Theory of Finance I

Date: 25.02.2014 Maximum Marks: 40 Time: 90 minutes

1. State and prove a result which shows that if the relative risk aversion measure is increasing, then the wealth elasticity of demand for the underlying risky prospect is less than one.

(12)

2. Define certainty equivalent. Show that the certainty equivalent is a continuous and increasing function of state contingent outputs.

(1+5 = 6)

3. (a) Explain the usefulness of writing a covered call and show its profit function graphically.

   (b) Show that the prices of on American call option and a European call option has an upper bound. Establish explicitly the relation between these three quantities.

(5+5 = 10)

4. When do you say that a

   (a) call option is out the money,

   (b) put option is out-of-the money?

(2)

5. (a) Does the certainty equivalent of a prospect remain invariant owner affine transformations of the underlying utility function? Justify your answer.

   (b) Do you agree or disagree with the statement that $0_{EC}(K_1) - 0_{EC}(K_2) > (K_2 - K_1)e^{-\lambda T}$, where $K_2 > K_1$? Justify your answer.

(5 + 5 = 10)

INDIAN STATISTICAL INSTITUTE

Mid-semester Examination: 2013-14

M.S. (Q.E.) I

Environmental Economics

25

Date: [redacted] February, 2014 **Maximum Marks: 65** **Duration: 2 hrs.**

1. (a). Suppose a paper mill located by the side of a river disposes off wastewater, generated during production, in the river. A fishery firm operates its fishing activities in the downstream of the river. Assuming zero transaction cost of bargaining, examine if it is possible to regulate externalities through bargaining if property rights are clearly defined between polluter and victim. Explain your answer.

   (b). Suppose in the downstream a Municipality supplies water from that river, which is required to be purified. The cost function of the paper mill is $C_P = Q^2$, when Q stands for output of the firm. The price per unit of paper output is Rs. 12.0. The level of pollution generated by the firm $E = 0.01Q^2$. Municipality's cost of supply of pure water is $C_W = 20 + 50E$.

   (i). Find both social and private optimal level of outputs.

   (ii). How much compensation is to be paid by the firm to the Municipality if it has the right to have clean water and how much bribe may be claimed by the firm if it has the right to pollute.

   [12+8=20]

2. (a) Explain the concept of total economic value of environmental services.

   (b). Suppose a consumer derives utility from consumption of a good x and personal hygiene H. H depends on air pollution level α and medical care M which is the defensive expenditure to reduce the effect of air pollution on health. Consumption of both x and M involves money and time. Suppose market prices of x and M are p_x and p_M, respectively. Consumer's total income consists of wage income W (for L hours, available for work, at wage rate w per hour) and asset income A. Assuming no change in prices explain how you can estimate empirically the marginal cost of change in air pollution level.

   [8+12 = 20]

3. Consider a small country facing market failure for producing a good with adverse environmental impact leaving uncontrolled and prospect of trade liberalization, in which its own actions do not affect the rest of the world.

   (a). Show how does the welfare of small country change if the country shifts from autarchy to open trade.

   (b). Show the relative efficiency of trade and environmental policies in reducing the environmental degradation.

   [15+10=25]

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination: 2013-2014
MS (Q.E.) I Year
Econometric Methods I

Date: 27.02.14 Maximum Marks 30 Duration 2 hours

All notations are self-explanatory. This question paper carries a total of 30 marks. Marks allotted to each question are given within parentheses.

1. Consider the true classical linear regression regression model

$$Y = X\beta + e = X_1\beta_1 + X_2\beta_2 + e,$$

where X_1, X_2, β_1, and β_2 are appropriate sub-matrices of X and β, respectively.

Let X_1^* be the residuals of the regression of X_1 on X_2.

(a) Obtain an expression for the bias of the OLS estimator of β_1 in the linear model $Y = X_1\beta_1 + u$.

(b) Prove that the least square estimator of β_1 in the linear model $Y = X_1^*\beta_1 + \epsilon$ is unbiased. [6+ 7 =13]

2. Consider the simple linear regression model

$$\begin{aligned} Y_1 &= \alpha_1 + u_1 \\ Y_2 &= 2\alpha_1 - \alpha_2 + u_2 \\ Y_3 &= \alpha_1 + 2\alpha_2 + u_3, \end{aligned}$$

where u_i's are mean zero and i.i.d normal variables.

(a) Find the OLS estimator of α_i, $i = 1, 2$.

(b) Derive the $F-$ statistic for testing $H_0 : \alpha_1 = \alpha_2$. [Write down the statistic in terms of observations and the estimated parameters.] [5+ 12 =17]

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2013-2014
M.S. (Q.E.) I & II year
Theory of Finance I

Date: 05.05.14 Maximum Marks: 100 Time: 3 hours

Note: Answer Parts A and B in separate answer scripts. You can use a calculator in the examination. The paper carries 110 marks. The maximum you can score s 100.

A

1. Use the Black-Scholes-Merton partial differential equation to determine the Black-Scholes pricing formula for a European call option. Clearly explain all the symbols you use.

   (18)

2. Define second order stochastic dominance and demonstrate its expected utility equivalence by stating assumptions about the utility functions explicitly.

   (1+13 = 14)

3. (a) Give an example of a bullish price- spread- strategy and derive its payoff function analytically. Explain the usefulness of such a strategy.

   (2+ 5+2=9)

   (b) Using the rho of a European put option discuss the implication of an increase in the risk-free interest rate on the price of the option.

   (5)

4. Consider a two-period world in which $I \geq 2$ assets are traded in the financial market. The current prices of the assets are known but the prices in the future period are dependent on k states of nature. Develop a necessary and sufficient condition for the non-existence of a scope of an arbitrage in this world.

   (16)

5. Do you agree or disagree with the following statements? In either case justify your answer.
   (a) Completeness of an asset market demands attainability of any given contingent claim.
   (b) First order stochastic dominance is necessary but not sufficient for the second order stochastic dominance.

(c) The price of S_0 of a stock in period 0 is 80 dollars. The stock price S_1 in period 1 takes on values 96 dollars and 64 dollars with probabilities 0.6 and 0.4 respectively. Determine the risk-free rate of return for which risk neutral valuation holds.

(5+5+5=15)

6. Consider a portfolio consisting of one risky asset and one risk-free asset. State and prove a result which shows that if the absolute risk aversion measure is decreasing, then the amount invested in the risky prospect increases with an increase in the level of total invested wealth.

(13)

B

7. Consider the formula for bond price, P, where C is the coupon value (in $'s). Q is the par value and r is the yield to maturity

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{Q}{(1+r)^n}$$

Derive Macaulay Duration (i.e., Sensitivity of bond price to a change in interest rate. (5)

8. The XYZ stock price is currently Rs.80. The stock price will move up or down by 15% each year. Calculate the risk neutral probability of an up-movement in XYZ put option if the continuously compounded annual interest rate is 3.9%. Then, construct a 2-step binomial model to find the value of a 2-year European put option with an exercise price of Rs.62.

(5)

9. Assume an investor holds a portfolio of bonds as follows:

- $2,000,000 par value of 10-year bonds with a duration of 6.95 priced at $95.50
- $3,000,000 par value of 15 year bonds with a duration of 9.77 priced at 88.6275
- $5,000,000 par value of 30-year bonds with a duration of 14.81 priced at $114.875

What is the duration of this portfolio?

(5)

10. Companies A and B have been offered the following rates per annum on a $20 million

5- year loan:

	Fixed rate	*Floating rate*
Company A:	5.0%	LIBOR + 0.1%
Company B:	6.5%	LIBOR + 0.6%

Based on comparative advantage, identify who will be *Fixed rate player* and the *Floating rate player*. Design a swap deal between two companies that will be equally attractive to companies. (Hint: Assume that the fixed rate player pays LIBOR to the floating rate player)

(5)

INDIAN STATISTICAL INSTITUTE
SEMESTRAL EXAMINATION, 2013-2014
M.S. (Q.E.) I & II year
Theory of Finance I

Date: 05.05.14 Maximum Marks: 100 Time: 3 hours

Note: Answer Parts A and B in separate answer scripts. You can use a calculator in the examination. The paper carries 110 marks. The maximum you can score s 100.

A

1. Use the Black-Scholes-Merton partial differential equation to determine the Black-Scholes pricing formula for a European call option. Clearly explain all the symbols you use.
(18)

2. Define second order stochastic dominance and demonstrate its expected utility equivalence by stating assumptions about the utility functions explicitly.
(1+13 = 14)

3. (a) Give an example of a bullish price- spread- strategy and derive its payoff function analytically. Explain the usefulness of such a strategy.
(2+ 5+2=9)

(b) Using the rho of a European put option discuss the implication of an increase in the risk-free interest rate on the price of the option.
(5)

4. Consider a two-period world in which $l \geq 2$ assets are traded in the financial market. The current prices of the assets are known but the prices in the future period are dependent on k states of nature. Develop a necessary and sufficient condition for the non-existence of a scope of an arbitrage in this world.
(16)

5. Do you agree or disagree with the following statements? In either case justify your answer.
(a) Completeness of an asset market demands attainability of any given contingent claim.
(b) First order stochastic dominance is necessary but not sufficient for the second order stochastic dominance.

(c) The price of S_0 of a stock in period 0 is 80 dollars. The stock price S_1 in period 1 takes on values 96 dollars and 64 dollars with probabilities 0.6 and 0.4 respectively. Determine the risk-free rate of return for which risk neutral valuation holds.

(5+5+5=15)

6. Consider a portfolio consisting of one risky asset and one risk-free asset. State and prove a result which shows that if the absolute risk aversion measure is decreasing, then the amount invested in the risky prospect increases with an increase in the level of total invested wealth.

(13)

B

7. Consider the formula for bond price, P, where C is the coupon value (in $'s). Q is the par value and r is the yield to maturity

$$P = \sum_{t=1}^{n} \frac{C}{(1+r)^t} + \frac{Q}{(1+r)^n}$$

Derive Macaulay Duration (i.e., Sensitivity of bond price to a change in interest rate. (5)

8. The XYZ stock price is currently Rs.80. The stock price will move up or down by 15% each year. Calculate the risk neutral probability of an up-movement in XYZ put option if the continuously compounded annual interest rate is 3.9%. Then, construct a 2-step binomial model to find the value of a 2-year European put option with an exercise price of Rs.62.

(5)

9. Assume an investor holds a portfolio of bonds as follows:

- $2,000,000 par value of 10-year bonds with a duration of 6.95 priced at $95.50
- $3,000,000 par value of 15 year bonds with a duration of 9.77 priced at 88.6275
- $5,000,000 par value of 30-year bonds with a duration of 14.81 priced at $1 14.875

What is the duration of this portfolio?

(5)

10. Companies A and B have been offered the following rates per annum on a $20 million

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(5)

INDIAN STATISTICAL INSTITUTE

SEMESTRAL EXAMINATION: (2013-2014)

MSQE I and M.Stat II

Microeconomic Theory II

Date: 10.05.2014 Maximum marks: 60 Duration: 3 Hours

Note: Answer all questions.

Note: $\mathbb{R}^\ell$ denotes the ℓ-dimensional Euclidean space. Assume that

$$\mathbb{R}^\ell_+ = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i \geq 0 \text{ for all } 1 \leq i \leq \ell\}$$

and

$$\mathbb{R}^\ell_{++} = \{x = (x^1, \cdots, x^\ell) \in \mathbb{R}^\ell : x^i > 0 \text{ for all } 1 \leq i \leq \ell\}.$$

Q1. Let $\mathcal{O} = \{A_1, \cdots, A_K\}$ be a finite set of outcomes and $\hat{\mathscr{L}}$ be the set of compound lotteries over $\mathcal{O}$. Suppose that $A_K \succeq \cdots \succeq A_1$, and U, V are two linear utility representations of the preference relation $\succeq$ over $\hat{\mathscr{L}}$. Show that

$$(U(A_K) - U(A_j))(V(A_j) - V(A_1)) = (V(A_K) - V(A_j))(U(A_j) - U(A_1))$$

for all $1 \leq j \leq K$. [10]

Q2. Let $\mathbb{R}_+$ be the set of wealth and $U : \mathbb{R}_+ \to \mathbb{R}$ be a utility function such that $U(\omega) = \ln(\omega)$. Find the certainty equivalence and risk premium of a lottery L defined by

$$L = \left[\frac{1}{2}(\omega_0 + h), \frac{1}{2}(\omega_0 - h)\right],$$

where ω_0, h are elements of $\mathbb{R}_+$ such that $\omega_0 - h \in \mathbb{R}_+$. [5]

Q3. Consider an economy $\mathscr{E} = \{N; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in N}\}$, where N is the set of agents containing n many elements; $\mathbb{R}^\ell_+$ is the consumption set of each agent; and $\succeq_i$ and ω_i are the preference and initial endowment of agent i, respectively. Suppose further that $\succ_i$ and $\sim_i$ are the strict preference and indifference relations associated with the rational preference relation $\succeq_i$ for all $i \in N$. A price is an element of $\mathbb{R}^\ell \setminus \{0\}$, and $B_i(p)$ is the budget set of agent i for price p. Assume

$\mathscr{W}(\mathscr{E})$: the set of Walrasian allocations of $\mathscr{E}$;

$\mathscr{C}(\mathscr{E})$: the core of $\mathscr{E}$;

$\mathscr{I}(\mathscr{E})$: the set of individually rational allocations of $\mathscr{E}$;

$\mathscr{P}(\mathscr{E})$: the set of Pareto optimal allocations of $\mathscr{E}$.

(i) Suppose that $x \in \mathbb{R}^\ell_{++} \cap B_i(p)$ and $p \cdot y \geq p \cdot \omega_i$ for all $y \in \mathbb{R}^\ell_+$ satisfying $y \succ_i x$. If $\succeq_i$ is continuous and strictly monotone, then show that $p \in \mathbb{R}^\ell_{++}$. [5]

(ii) Prove or disprove: (a) $\mathscr{W}(\mathscr{E}) \subseteq \mathscr{P}(\mathscr{E})$; and (b) $\mathscr{C}(\mathscr{E}) = \mathscr{I}(\mathscr{E}) \cap \mathscr{P}(\mathscr{E})$. [10]

(iii) If $\succeq_i$ is strictly convex and non-satiated for each $i \in N$, then show that every Pareto optimal allocation can be supported by a price. [7]

(iv) Assume that $N = \{1, 2\}$ and $\ell = 2$. Suppose that $\succeq_i$ is represented by a utility function for $i = 1, 2$. Let

$$\begin{cases} \omega_1 = (0,4), & U_1(x,y) = \sqrt{x} + \sqrt{y}; \\ \omega_2 = (2,2), & U_2(x,y) = x. \end{cases}$$

Show that $((0,6),(2,0)) \notin \mathscr{W}(\mathscr{E})$. [6]

(v) Let $\mathscr{E}_3$ be the 3-fold replicated economy of $\mathscr{E}$. Suppose that

$$x = (x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, \cdots, x_{n1}, x_{n2}, x_{n3}) \in \mathscr{C}(\mathscr{E}_3).$$

Suppose that $\succeq_i$ is continuous, convex and strictly monotone for all $i \in N$. Show that $x_{ij} \sim x_{ik}$ for all $i \in N$ and $1 \leq j, k \leq 3$. [7]

Q4. Consider a neoclassic private ownership production economy

$$\mathscr{E} = \left\{N; K; \mathbb{R}^\ell_+; (\succeq_i, \omega_i)_{i \in N}; (Y_j)_{j \in K}; (\theta_{ij})_{i \in N, j \in K}\right\},$$

where N, $\mathbb{R}^\ell_+$ and $(\succeq_i, \omega_i)_{i \in N}$ are the same as in Q3. In addition, K denotes the set of producers containing k many elements; Y_j is the production set of producer $j \in K$; and θ_{ij} represents consumer i's share of producer j's profit. Assume that $\omega_i \in \mathbb{R}^\ell_+ \setminus \{0\}$, Y_j is strictly convex for all $j \in K$, and $\succeq_i$ is rational, continuous, strictly monotone and strictly convex for each $i \in N$. Show that agent i's demand function $x_i : \mathbb{R}^\ell_{++} \to \mathbb{R}^\ell_+$ is continuous. [10]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2013-2014
MS (Q.E.) I Year
Econometric Methods I
Date:May 13, 2014 Maximum Marks 100
Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 100 marks. Marks allotted to each question are given within parentheses.

1. (a) Suppose consumption expenditure (y) depends on income (x); but different levels of income produce different values of marginal propensity to consume, say β_i, for the i-th level of income. In particular,

$$\frac{dE(y|x)}{dx} = \begin{cases} \beta_1 & \text{if } x < Rs.\ 100 \\ \beta_2 & \text{if } Rs.\ 100 \leq x < Rs.\ 500 \\ \beta_3 & \text{if } x \geq Rs.\ 500. \end{cases}$$

Using 'dummy' variables, estimate the marginal propensities to consume for the three groups (assuming that there is no intercept term), given that $\sum yx$ is 30,000; 5,00,000 and 10,00,000 for the three groups, respectively, and $\sum x^2$ is 90,000; 25,00,000 and 100,00,000 for the three groups, respectively.

(b) Consider the classical linear regression model

$$Y_i = \beta X_i + e_i, \quad i = 1, 2, \ldots, n, \text{ where } n \text{ is even.}$$

Divide the sample into two groups each having equal (i.e., $\frac{n}{2}$) number of observations.

(i) Show that $\hat{\beta}$ defined as

$$\hat{\beta} = \frac{\bar{Y}_2 - \bar{Y}_1}{\bar{X}_2 - \bar{X}_1}$$

is a consistent estimator of β, where $\bar{X}_j$ and $\bar{Y}_j$, $j = 1, 2$, are the group means of X and Y, respectively.

(ii) Show that the ordinary least square estimator of β is more efficient than $\hat{\beta}$.

[10 + (5+10)=25]

2. Consider the linear regression model

$$Y = \beta X + e,$$

where X is an $n \times k_1$ stochastic matrix and is known to be exogenous. Assume that all other CLRM assumptions hold. Let Z_1 be an $n \times k_1$ stochastic matrix with full column rank; and also with $plim(Z_1'X)$, a finite non-singular matrix. Furthermore, let $Z = (Z_1, Z_2)$ be an $n \times k$, $k = (k_1 + k_2)$, stochastic matrix with full column rank. Let $\hat{\beta}_{ols}$, $\hat{\beta}_{IV1}$, and $\hat{\beta}_{IV2}$ are the ols estimator, IV estimator based on Z_1 and IV estimator based on Z, respectively.

Show that $Var(\hat{\beta}_{ols}) \leq Var(\hat{\beta}_{IV2}) \leq Var(\hat{\beta}_{IV1})$. **[10+15 =25]**

3. Consider the simple linear regression model $Y_i = \mu + \beta i + u_i$, u_i ***independent*** $\sim N(0, \sigma_i^2), i = 1, 2, \ldots, n$.

(a) Find the OLS estimators of μ and β. Prove the consistency of these estimators.

(b) Now suppose that β is known to be zero. How will you estimate the variance of $\hat{\mu}$ consistently? Give analytical derivations. **[15+ 10 =25]**

4. Consider the linear regression model

$$Y_t = \alpha + \delta Y_{t-1} + \beta X_t + e_t, \quad Y_0 = 0, \quad t = 1, 2, \ldots, T \text{ and}$$
$$e_t = \rho e_{t-1} + u_t, \; e_0 = 0, |\delta| < 1, |\rho| < 1,$$

where each of $\{X_t\}$ and $\{u_t\}$ is independently and identically distributed with mean zero and variances σ_x^2 and σ_u^2, respectively. Further, X_t and u_t are mutually independent random variables. Let $\theta = (\alpha, \delta, \beta)'$.

(a) Show that $\hat{\theta}_{ols}$ is inconsistent. Propose a consistent estimator of θ, and prove its consistency analytically.

(b) How would you test for the null hypothesis, $H_0 : \delta = 0$? Provide analytical derivations.

[(5+10)+10 =25]

INDIAN STATISTICAL INSTITUTE
Second Semestral Examination: 2013-2014 (Back paper)
MS (Q.E.) I Year
Econometric Methods I

Date: 17·07·14 Maximum Marks 100 Duration 3 hours

All notations are self-explanatory. This question paper carries a total of 100 marks. Marks allotted to each question are given within parentheses.

1. Consider the linear regression model

$$y_i = \alpha + \beta x_i + e_i, \quad i = 1, 2, \ldots, n,$$

where $X = (x_1, x_2, \ldots, x_n)'$ is non-stochastic with finite second moment and $\{e_i\}$ is independently distributed random variable with mean zero and variances σ_i^2. Consider the quantity $S^2 = (Y - \alpha\mathbf{1} - \beta\mathbf{X})'\mathbf{W}(\mathbf{Y} - \alpha\mathbf{1} - \beta\mathbf{X})$, where $Y = (y_1, y_2, \ldots, y_n)'$, and W is $n \times n$ known symmetric matrix. Let Σ be the variance covariance matrix of $e = (e_1, e_2, \ldots, e_n)'$. Define $\theta = (\alpha, \beta)', \mathbf{1} = (\mathbf{1}, \mathbf{1}, \ldots, \mathbf{1})'$.

(a) Let $W = XX'$. Minimize S^2 with respect to θ. Show that this minimizer is a consistent estimator of θ.

(b) Let $W = \Sigma^{-1}$. Minimize S^2 with respect to θ. Show that this minimizer is a consistent estimator of θ.

(c) Compare the relative efficiency of the above two estimators, *viz.* the estimator from (a) and the estimator from (b), respectively.

(d) Suppose you want to test the null hypothesis, $H_0 : \beta = 0$. How would you test the hypothesis using the estimator you obtained in (a)? **[8+7+ 10+10 =35]**

2. Consider the 'true' classical linear regression regression model

$$Y = X\beta + e = X_1\beta_1 + X_2\beta_2 + e,$$

where X_1, X_2, β_1, and β_2 are appropriate sub-matrices of X and β, respectively.

Let X_1^* be the residuals of the regression of X_1 on X_2.

(a) Obtain an expression for the bias of the OLS estimator of β_1 in the linear model $Y = X_1\beta_1 + u$.

(b) Prove that the least square estimator of β_1 in the linear model $Y = X_1^*\beta_1 + \epsilon$ is unbiased. [10+ 10 =20]

3. Consider the simple linear regression model $Y_i = \mu + u_i,\ u_i \sim N(0, \sigma_i^2), i = 1, 2, \ldots, n$.

(a) Find the OLS estimator of μ. Prove the consistency of this estimator.

(b) How will you estimate the variance of your estimator consistently? Give analytical derivations. [10+ 10 =20]

4. Consider the linear regression model

$$Y_t = \alpha + \delta Y_{t-1} + \beta X_t + e_t,\ \ Y_0 = 0,\ \ t = 1, 2, \ldots, T \text{ and}$$
$$e_t = \rho e_{t-1} + u_t,\ e_0 = 0, |\delta| < 1, |\rho| < 1,$$

where each of $\{X_t\}$ and $\{u_t\}$ is independently and identically distributed with mean zero and variances σ_x^2 and σ_u^2, respectively. Further, X_t and u_t are mutually independent random variables. Let $\theta = (\alpha,\ \delta.\ \beta)'$.

(a) Show that $\hat{\theta}_{ols}$ is inconsistent.

(b) How would you test for the null hypothesis, $H_0 : \delta = 0$? Provide analytical derivations.

(c) How would you test for the null hypothesis, $H_0 : \rho = 0$? [5+15+5 =25]