A NOTE ON THE CONSTRUCTION OF ORTHOGONAL LATIN SQUARES

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Mann (1949) has given a method of constructing a set of $\min_i (p_i^{n_i}-1)$ mutually orthogonal s-sided latin squares when s is of the form $s = \prod_{i=1}^{k} p_i^{n_i}$, the p_i's being different primes. Mann's method utilises properties of Galois fields. The following illustrates an alternative method of construction which is quite general.

2. Consider two latin squares, one 3-sided and the other 4-sided. Let the three-sided square be, say,

$$\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{matrix} = L_1 .$$

Associated with this are the following three latin squares L_2, L_3, and L_4, obtained by adding 3, 6, and 9 respectively to each element of L_1 :

$$L_2 = \begin{matrix} 4 & 5 & 6 \\ 5 & 6 & 4 \\ 6 & 4 & 5 \end{matrix} ; \quad L_3 = \begin{matrix} 7 & 8 & 9 \\ 8 & 9 & 7 \\ 9 & 7 & 8 \end{matrix} ; \quad L_4 = \begin{matrix} 10 & 11 & 12 \\ 11 & 12 & 10 \\ 12 & 10 & 11 \end{matrix}$$

Let the 4-sided latin square be, say,

$$\begin{matrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{matrix}$$

3. If now we arrange the four 3-sided squares L_1, L_2, L_3 and L_4 in the way in which the numbers 1, 2, 3, and 4 respectively are occurring in the 4-sided square we arrive at the following 12-sided latin square :

1	2	3	4	5	6	7	8	9	10	11	12
2	3	1	5	6	4	8	9	7	11	12	10
3	1	2	6	4	5	9	7	8	12	10	11
4	5	6	7	8	9	10	11	12	1	2	3
5	6	4	8	9	7	11	12	10	2	3	1
6	4	5	9	7	8	12	10	11	3	1	2
7	8	9	10	11	12	1	2	3	4	5	6
8	9	7	11	12	10	2	3	1	5	6	4
9	7	8	12	10	11	3	1	2	6	4	5
10	11	12	1	2	3	4	5	6	7	8	9
11	12	10	2	3	1	5	6	4	8	9	7
12	10	11	3	1	2	6	4	5	9	7	8

4. If we start from another pair of latin squares, one (L_1') 3-sided and the other 4-sided, which are orthogonal to those used in above, we shall get a 12-sided latin square orthogonal to that obtained above. For when the two latin squares are superposed, any pair of numbers say 4 in the first square and 8 in the second will coincide only when the subsquare L_2 (containing 4) of the first latin square falls upon subsquare L_3' (containing 8) of the second, which they do only once, in virtue of the orthogonality of the two 4-sided latin squares used. Also when L_2 and L_3' are superimposed, 4 in L_2 and 8 in L_3' come together in the *one* cell whose position is similar to the *one* cell (in virtue again of orthogonality) where $1(=4-3)$ in L_1 and $2(=8-2\times3)$ in L_1' coincide when L_1 and L_1' are superposed.

5. If $s = p_1^{e_1}p_2^{e_2}$, we get by the methods of Bose and Stevens outlined by Mann (1949), a set of $(p_1^{e_1}-1)$ orthogonal squares of side $p_1^{e_1}$ and another set of $(p_2^{e_2}-1)$ orthogonal squares of side $p_2^{e_2}$ We can go on taking from these two sets one square of side $p_1^{e_1}$ and another of side $p_2^{e_2}$ and "combining" them in the manner indicated to get min $(p_1^{e_1}, p_2^{e_2})-1$ orthogonal s-sided latin squares. This can be easily extended to the case $s = \prod_{i=1}^{k} p_i^{e_i}$.

REFERENCE

MANN, H. B. (1949): *Analysis and Design of Experiments : Analysis of variance and Analysis of variance designs.* Dover Publications, New York.

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