Skilled-Unskilled Wage Inequality: Economic Theory and Policy

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Dedicated to my Sir

Professor Manash Ranjan Gupta

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Chapter 1

Introduction and Literature Survey

1.1 Globalization and Skilled-unskilled wage inequality

Globalization is the process of integration various economies of the world without creating any hindrances in the free flow of goods and services, technology, capital and even labour of human capital. The term, globalization has, therefore, following parameters: (i) Reduction of trade barriers to permit free flow of goods and services among nation-states; (ii) Creation of environment in which free flow of capital can take place among nation states; (iii) Creation of environment, permitting free flow of technology; and (iv) Last, but not least, from the point of view of developing countries, creation of environment in which free movement of labour can take place in different countries of the world.

Thus, basically globalization signifies a process of internationalization plus liberalization. Since the formation of the World Trade Organization (WTO), there have been revolutionary changes in liberalizing international trade across countries whether developed or developing. Liberalization of trade involves removal of quantitative restrictions as well as reduction in tariffs on trade and also some other measures to facilitate trade and input flows across the world as they would help to integrate domestic market with the world market.

According to the traditional theory, globalization leads to an improvement in welfare both from the aggregative and distributive perspectives. If globalization improves welfare from distributive perspective, then it should improve skilled-unskilled wage inequality. Here, skilled-unskilled wage inequality means, wage inequality between two different groups of people; one who have skill and one who don't have skill. Developed and less developed countries, who generally play opposite roles on international factor movements and face opposite type of changes in the relative price structure of traded goods due to trade liberalisation, should face opposite movements in the degree of skilled-unskilled wage inequality. This is the outcome of Stolper Samuelson theorem in a Heckscher-Ohlin-Samuelson (HOS) set up.

However, empirical works fail to point out this asymmetric movement of skilled-unskilled wage inequality and find a symmetric rise in skilled-unskilled wage income inequality all most in every part of the world except for East Asian countries. There exists a vast empirical literature pointing out that income inequality has grown in various countries in the form of a decline in income and employment of unskilled labourers compared to those of skilled labourers. This growing income inequality has been observed in U.S.A. during 1960s[1] and in European countries between 1978 and 1988[2]. We find similar observations in many developing countries too. Wage inequality has gone up in many Latin American and South Asian countries in the mid 1980s[3]. However, the experience of East Asian countries between 1960s and 1970s advocates the conventional theory that a greater openness to the rest of the world leads to a decrease in the skilled-unskilled wage gap[4]. Different empirical studies provide different explanations for this growing income inequality. Trade liberalization and technological

[1] See, for example, Bound and Johnson (1992), Juhn et. al. (1993), Autor et. al. (1998) Leamer (2000), Bils and Klenow (2000), Paul and Siegel (2001), Acemoglu (2002b) etc. Accoding to Lee and Wolpin (2010) wage differentials by education increased during the period 1968-2000. Acemoglu (2002b) makes an empirical analysis with the U.S. data of college premium and supply of college skills for 1964 to 1997. Here, college premium, i.e, average wage rate of the college graduates or more relative to that of high school passed students, represents the skilled-unskilled relative wage and the supply of college graduates represents the supply of skilled labour. Panel data is collected for different age groups with their corresponding wages. The behavior of skilled-unskilled wage inequality in the United States indicates that the technical change was skill biased during the survey period.

[2] See, for example, Lawrence (1994), Katz et. al. (1992), Winchester and Greenaway (2007) etc. According to Winchester and Greenaway (2007) when technology is skilled labour biased, there is increase in UK wage inequality over the 1980-1997 period due to changes in factor endowments.

[3] See, for example, Mollick (2009), Wood (1997), Dev (2000), Borjas and Ramey (1993), Banga (2005), Beyer et. al. (1999) etc. According to Mollick (2009), wage differentials by skilled labour actually increased in Mexico during the period 1990-2006.

[4] See, for example, Wood (1997). According to Wood (1997), this conflict of evidence is probably not the result of differences between East Asia and Latin America. Instead, the conflict is probably the result of differences between the 1960s and the 1980s, specifically, the entry of China into the world market and, perhaps, the advent of new technology biased against unskilled workers.

development are the main two controversial reasons of this phenomenon. According to Wood (1998), Beyer et. al. (1999), Green et. al. (2001), Behrman et. al. (2000), Isgut (2001) etc. trade liberalization causes wage inequality; but Wood (1997, 1998), Dev (2000) and Gorg and Strobl (2002) show that technological change increases demand for skilled labour and thus worsens wage inequality. Esquivel and Lopez (2003) shows that technological change worsens but trade liberalization improves wage inequality in Mexico. Many other empirical studies show other causes of this increasing inequality. These are international outsourcing[5], increase in the relative price of skill intensive good[6], entry of overpopulated less developed countries like Bangladesh, China, India, Indonesia, and Pakistan in the international market[7] etc.

The previous two paragraphs focus on worldwide increase in wage inequality which mainly started during sixties and continued till nineties. However, many recent empirical works confirm the existence of this rising trend of wage inequality even in the last decade of the twentieth century. Ho et. al. (2005) shows that wage inequality has increased in Hong Kong during the period of 1991–2002. Reenen (2011) shows that wage inequality has increased in the US and UK during the period, 1990-2008. Meschi et. al. (2011), using firm-level data, shows that wage inequality has increased during the period, 1980-2001, in the Turkish Manufacturing sector. Villrreal and Sakamoto (2011) shows that foreign investment and export promotions have caused increases in wage inequality in Mexico during 1992-2001. Popli (2010) also shows rise in skilled unskilled wage inequality in Mexico during 1984-2002 as a result of rapid trade liberalization. Mehta and Hasan (2012) shows that wage inequality has increased in India between 1993 and 2004. Han et. al. (2012) shows evidences of increases in wage inequality in China from 1988 to 2008.

[5] See Feenstra and Hanson (1997) in this context.

[6] See Harrison and Hanson (1999), Hanson and Harrison (1999) and Beyer et. al. (1999) in this context.

[7] See Wood (1997) in this context.

1.2 Survey of the Existing Theoretical Literature

Many theoretical models deal with the problem of growing wage inequality. We can divide the existing theoretical literature into two groups. One group of models is static in nature and the other group consists of dynamic models. Also static models can be divided into two parts. One set of models are built on competitive general equilibrium framework while the other set of models have a product variety structure with monopolistic competition in markets of different varieties. All these models consider two types of labour-skilled and unskilled; and consider the ratio of wage rate of the skilled worker to that of the unskilled worker as the measure of wage inequality.

1.2.1 Static Models

1.2.1.1 Competitive Equilibrium Models

These types of models are basically static models with competitive equilibrium in all product markets and factor markets; and these models analyse the effect of exogenous changes in different globalization related parameters like inflow and outflow of different factors and/or changes in prices of different commodities on the degree of skilled-unskilled wage inequality. We shall now briefly summarize main features of some of important static competitive general equilibrium models in this section. These models can be divided into two groups-models of small open economies and North-South models.

1.2.1.1.1Models of small open economies

A small open economy is a price taker in the international market. These models as available in the existing literature can be further classified into two groups: Full employment models and models with unemployment.

1.2.1.1.1.1Full employment models

In full employment models, all the factors are fully employed and this full employment is ensured by perfect flexibility of factor prices. This group of models include the works of Yabuuchi and Chaudhuri (2007), Chaudhuri and Yabuuchi (2007), Marjit and Kar (2005), Marjit et. al. (2004), Chaudhuri and Yabuuchi (2008), Kar and Beladi (2004), Yabuuchi and Chaudhuri (2009), Oladi and Beladi (2009), Zhu and Trefler (2005), Xu (2003), Glazer and Ranjan (2003), Deardorff and Staiger (1988), Panagariya (2000), Meckl and Zink (2002), Chao, Laffargue and Sgro (2010, 2012) etc.

Yabuuchi and Chaudhuri (2007) develops a static three sector full employment general equilibrium model of a small open economy with four factors: skilled labour, unskilled labour, capital and land. Skilled labour and land are specific to high skill manufacturing sector and primary agricultural sector, respectively. Capital is mobile between the high skill manufacturing sector and the low skill manufacturing sector; and unskilled labour is mobile between the primary agricultural sector and the low skill manufacturing sector. There is a distortion in the unskilled labour market because unskilled labourers earn higher unionized wage in the low skill manufacturing sector; and this unionized wage rate varies positively with the wage rate of the unskilled labour in the primary agricultural sector and with the bargaining strength of the labour union. Yabuuchi and Chaudhuri (2007) analyses the effect of international migration of either type of labour on the skilled-unskilled wage ratio; and the nature of this effect depends on the capital intensity rankings between the high skill manufacturing sector and the low skill manufacturing sector and on the institutional features of the unskilled labour market.

Chaudhuri and Yabuuchi (2007) adopts the structure of Yabuuchi and Chaudhuri (2007) and analyzes the effect of a reduction in tariff on the import of low skilled manufacturing product and the effect of foreign capital inflow on the skilled-unskilled wage ratio. Reduction in import tariff on low skill manufacturing sector products worsens the skilled-unskilled wage inequality problem. However, given some restrictions on the capital intensity ranking between the low skill manufacturing sector and the high skill manufacturing sector, wage inequality problem is improved by the inflow of foreign capital.

Marjit and Kar (2005) develops a static two sector three factor full employment general equilibrium model of a small open economy in which capital is intersectorally mobile but skilled labour and unskilled labour are specific to two sectors. This paper analyses the effect of international migration of either type of labour on the skilled-unskilled wage ratio; and the nature of this effect depends on the capital intensity ranking between these two sectors. The structure of the model is borrowed from Jones (1971) in which labour is intersectorally mobile but two capitals are specific to two sectors.

Marjit et. al. (2004) develops a static four sector full employment general equilibrium model of a small open economy with three factors: skilled labour, unskilled labour and capital. Skilled labour is mobile between a traded good sector and a non-traded intermediate good sector whose product is used as an input in the skilled labour using traded good sector. Out of the two unskilled labour using traded good sectors, one uses capital and another uses land as sector specific inputs. However, capital is mobile between the skilled labour using intermediate good sector and one of the two unskilled using traded good sectors. The traded good sector using intermediate good does not use capital directly while the intermediate good sector uses it. Marjit et. al. (2004) analyses the effects of changes in price of the traded good and of capital inflow on the degree of skilled-unskilled wage inequality. The nature of the effect depends on the effective capital intensity ranking between the skilled labour using traded good sector and the unskilled labour using traded good sector.

Chaudhuri and Yabuuchi (2008) develops a static four sector full employment general equilibrium model of a small open economy with four factors: capital, skilled labour, unskilled labour and land. This paper is basically an extension of Yabuuchi and Chaudhuri (2007) and of Chaudhuri and Yabuuchi (2007) with an additional sector producing a nontraded good. In one section of this paper, this nontraded good is used as an intermediate input in the low skill manufacturing sector; and, in another section, this nontraded good plays the role of a final commodity. The nontraded intermediate good is produced by unskilled labour and capital; and the non-traded final good uses land in place of capital. Chaudhuri and Yabuuchi (2008) analyses the effect of foreign capital inflow on the skilled-unskilled wage ratio. In the case of nontraded final good, the nature of this effect depends on the capital intensity ranking between the high skill manufacturing sector and the low skill manufacturing sector as well as on the land intensity ranking between the agricultural sector and the nontraded final good sector. However, in the case of nontraded intermediate good, the nature of the effect depends only on the capital intensity ranking between the high skill manufacturing sector and the low skill manufacturing sector.

Kar and Beladi (2004) develops a four sector full employment static general equilibrium model of a small open economy in which one sector produces skill with unskilled labour and capital as inputs and other three sectors produce traditional goods. Out of these three sectors one sector produces intermediate good with skilled labour and capital and another sector produces a final manufacturing good with skilled labour, capital and the intermediate input. The third sector produces agricultural product using unskilled labour and land. Skilled wage rate is fixed; and international migration rates of both skilled labour and unskilled labour are endogenous in this model. Capital is mobile among all three sectors except for the agricultural sector. Skilled labour is mobile between the intermediate good producing sector and the final manufacturing good sector but unskilled labour is mobile between the skill formation sector and the agricultural sector and land is specific to the agricultural sector. Kar and Beladi (2004) studies the impact of trade liberalization and trade union activity on the rate of skill formation, degree of skilled-unskilled wage inequality, pattern of migration of either type of labour and on national income.

Yabuuchi and Chaudhuri (2009) develops a three-sector three factor full-employment general equilibrium model of a small open economy in which one sector produces skill with unskilled labour and capital as inputs and other two sectors produces traded final goods. Capital is mobile among all three sectors. The unskilled labour is mobile between a traded good sector and the skill formation sector but skilled labour is specific to another traded good sector. The skill formation sector has to face a capital adjustment cost for which the effective unit cost of capital varies positively with the amount of capital employed in that sector. Yabuuchi and Chaudhuri (2009) examines effects of an exogenous infrastructure development scheme to the education sector and of an exogenous inflow of foreign capital on the skill formation and on the skilled-unskilled wage inequality; and shows that both would promote the skill formation though the effect on the degree of skilled-unskilled wage inequality would depend on the effective capital intensity ranking among all these three sectors.

Oladi and Beladi (2009) develops a static two sector full employment general equilibrium model of an open economy in which one sector produces a nontraded good and the other sector produces a pure exportable. The export earning is used to finance the consumption of an imported good. Unskilled labour is specific to the non-traded good sector but skilled labour is specific to the exportable sector. Capital is mobile between these two sectors. It is shown that the nature of various comparative static effects on the degree of skilled-unskilled wage inequality would depend on the elasticity of import demand with respect to relative price of imported good to nontraded good and with respect to income.

Zhu and Trefler (2005) develops a static full employment model that combines features of both Ricardian comparative advantage theory and factor endowment theory. In this model, South (less developed economy) tries to produce less skill intensive goods of North (Developed economy). However, these products become most skill intensive goods in South. This raises relative demand for skilled labour and skilled-unskilled wage ratio in the South. North also specializes in the production of goods of higher skill intensity. This raises relative demand for skilled labour and skilled-unskilled relative wage in the North. Zhu and Trefler (2005) provides empirical evidence that strongly supports this causal mechanism: Southern catch-up exacerbates Southern wage inequality by redirecting Southern export shares towards more skill intensive goods.

Xu (2003) shows that, in a two-country two-factor continuum-good full employment model, high protective tariffs generate a range of nontraded goods. The trade liberalization (tariff reduction) policy has two effects: a direct effect of import promotion and an indirect effect of export promotion through terms of trade improvement. If the export expansion dominates the import expansion, then this raises the relative demand for that factor which is more intensively used in production. This raises skilled-unskilled wage ratio in developing countries where skilled labour is now more intensively used.

Glazer and Ranjan (2003) introduces preference heterogeneity assuming that skilled workers prefer to consume skill intensive goods in a static product variety model. Glazer and

Ranjan (2003) shows that under plausible conditions an increase in the relative size of the skilled labour force raises the skilled-unskilled relative wage. In a two-country model of trade, an increase in the relative supply of skilled labour in one of the two countries raises the skilled-unskilled relative wage in both the countries.

Deardorff and Staiger (1988) develops a multi-commodity multi-factor static general equilibrium model and shows that, if all production functions and utility functions are Cobb–Douglas, then the factor content of trade can be used to derive the effect of trade on wage inequality with given tastes and technology. Factor prices in two trading equilibria can be compared by comparing their two 'equivalent autarky equilibria' which are constructed changing factor endowments by the factor content of trade. Using relationships between autarky factor prices and factor endowments, several relationships are derived between relative factor price under trade and its factor content. A positive correlation is found between relative changes in the factor content of trade, appropriately normalized, and the relative changes in factor prices.

Panagariya (2000) develops a simple static general equilibrium model with two commodities and two factors: skilled labour and unskilled labour; and modifies the model of Deardorff and Staiger (1988) introducing CES utility functions with identical elasticities of substitution. Like Deardorff and Staiger (1988), this model also shows that factor content of trade can be used to derive the effect of trade on wage inequality in a given year with given tastes and technology. A similar result is obtained even in a two period framework where tastes and technology are allowed to change.

Meckl and Zink (2002) develops a static model; and provides a theoretical explanation for a non-monotonic increase in wage differentials by skills even when the relative supply of skilled labour is steadily increased. People who differ among themselves with respect to their inherent abilities have to incur some fixed cost in order to acquire skill. Technological progress affects the unskilled wage rate as well as the skilled wage rate; and thus affects the individuals' investment incentives in skill formation. This rise in skill formation can be accompanied by a non-monotone behavior of the skilled-unskilled relative wage under fairly general conditions. The proposed mechanism should be seen as complementary to the widely discussed demand-side models, as it explains the U-shaped relation between the skill premium and the relative labour supply in the absence of any exogenous shock in total labour supply.

Chao, Laffargue and Sgro (2010) develops a model of a small open economy with two commodities and two factors and examines the effect of foreign aid on the distribution wage income in a static model. Two factors-skilled labour and unskilled labour-are specific to two sectors producing two commodities-one traded and the other non-traded. It is shown that an increase in foreign aid (in the form of tied aid) lowers the relative price of the nontraded good and thus widens the skilled-unskilled wage inequality. Chao, Laffargue and Sgro (2012) uses a similar model to examine the effects of stricter environmental regulation on wage income inequality. Production of both goods in this model emits pollution and needs purchase of permits. Emissions are considered as a production input which is mobile between two sectors. A decrease in the rate of pollution due to imposition of sticker environment laws can reduce skilled-unskilled relative wage by lowering wage of skilled labor in the traded good sector and raising the wage of unskilled labor in the non-traded good sector if the tourism terms of trade effect dominates.

1.2.1.1.1.2Models with unemployment

These models assume unemployment equilibrium in atleast one of the two labour markets. The small set of literature includes the works of Chaudhuri (2004), Beladi et. al. (2008), Chaudhuri (2008), Chaudhuri and Banerjee (2010) etc.

Chaudhuri (2004) develops a static two sector three factor general equilibrium model of a small open economy in which unskilled labour and capital are intersectorally mobile between these two sectors but skilled labour is specific to one of them, called the urban sector. Chaudhuri (2004) also introduces Harris-Todaro type of unemployment of unskilled labour in the urban sector. It is shown that an emigration of skilled and/or unskilled labour lowers the level of urban unemployment of unskilled labour and widens the skilled-unskilled wage-gap.
Beladi et. al. (2008) develops a static two sector general equilibrium model of a dual economy with three mobile factors- skilled labour, unskilled labour and capital, and with Harris-Todaro

(1970) type of unemployment in the unskilled labour market. Beladi et. al. (2008) analyses the effects of international factor movements on the skilled–unskilled wage inequality; and the nature of the effect crucially depends on the factor intensity ranking between these two sectors.

Chaudhuri (2008) also adopts the structure of Yabuuchi and Chaudhuri (2007) and of Chaudhuri and Yabuuchi (2007) but introduces Hariss-Todaro (1970) type of unemployment in the unskilled labour market. Chaudhuri (2008) analyses the effect of international migration of either type of labour and of foreign capital inflow not only on the skilled-unskilled wage ratio but also on the level of urban unemployment. The nature of the effect on wage ratio is conditional on the capital intensity ranking between the high skill manufacturing sector and the low skill manufacturing sector. However, an immigration of unskilled labour (an immigration of skilled labour and/or foreign capital inflow) unambiguously raises (lowers) the urban unemployment level in the unskilled labour market.

Chaudhuri and Banerjee (2010) develops a static three-sector general equilibrium model of a small open economy with four factors: skilled labour, unskilled labour, land and capital. The model considers unemployment of skilled labour as well as of unskilled labour. Unemployment of unskilled labour is of Harris–Todaro (1970) type while unemployment of skilled labour is explained by the efficiency wage hypothesis. Here unskilled labour is mobile between the agricultural sector and the low skill manufacturing sector but land and skilled labour are specific to agricultural sector and high skill manufacturing sector respectively. Capital is mobile among all three sectors. Unskilled labour earns an exogenously given high wage rate in the low skill manufacturing sector. The efficiency of skilled labour depends on relative rates of returns to all four factors and on the unemployment rate of skilled labour. Chaudhuri and Banerjee (2010) shows that the effect of an increase in capital and/or land on skilled-unskilled wage inequality depends on the degree of responsiveness of the efficiency function due to change in relative returns of all factors. However, this increase reduces the unemployment problem of unskilled labour.

1.2.1.1.2North-South models

North-South models are basically two country models of world economy where the two countries are structurally dual to each others. In these models, we can analyse asymmetric movements in the skilled-unskilled relative wage in different countries simultaneously. These group of North-South models includes the works of Feenstra and Hanson (1996 and 1997), Rowthorn et. al. (1997), Marjit and Acharyya (2006), Acharyya (2011) etc.

Feenstra and Hanson (1996 and 1997) develop static North-South models to analyse the effects of foreign direct investment in the form of outsourcing of production activity from developed to developing countries on the skilled-unskilled relative wage in both developed and developing countries. Each of these two models assumes a production technology with a continuum of production stages differing in their skilled–unskilled labor intensities. Depending on comparative advantages, some of these production stages are located in the North and others in the South. The South (North) has a comparative advantage in more unskilled (skilled) labor intensive stages of production. An outsourcing of some stages of production from the North to the South reduces the relative demand for unskilled labour in the North and hence raises the skilled-unskilled relative wage there. This relative wage also goes up in the South because the southern average skilled-labor intensity in the relatively unskilled labor intensive stages of production also goes up after outsourcing.

Rowthorn et. al. (1997) develops a static North-South model with two kinds of labour-skilled and unskilled. There are three sectors in each of these two countries. Sector 1 (2) produces a 'skill-intensive' (unskill labour intensive) manufacturing good; and sector 3 produces a nontradeable. In every sector, the elasticity of technical substitution between the two kinds of labour is constant though technologies are different in different sectors. It is shown that substantial gains from trade openness can be accompanied by a significant increase in skilled-unskilled wage inequality in the North. Increasing labour market flexibility can not correct this problem; and protectionist measures, such as enforcing labour standards on the South, may reduce global income. Less conventional labour market policies, such as employment subsidies for unskilled workers, may be a more effective solution.

Marjit and Acharyya (2006) develops a static North South full employment general equilibrium model of the world economy. Both countries have two sectors with two factors: skilled labour and unskilled labour. The home country produces a final consumption good and an intermediate good using skilled labour and unskilled labour as inputs; and the entire production of the intermediate good is imported to the foreign country who produces a final good using the intermediate good and skilled labour. Both the factors are intersectorally mobile in the home country. However, the unskilled labour is specific to the production of the intermediate good in the foreign country while skilled labour is intersectorally mobile there. Liberalisation in the trade policy of the foreign country (South) raises the world price of this intermediate good but lowers its tariff-inclusive price in the foreign country. These asymmetric movements of prices of intermediate goods in two countries raise the skilled-unskilled relative wage in both the countries.

Acharyya (2011) develops a static 2×2×2 HOS model where two countries, home and foreign, produce two goods with two internationally immobile but intersectorally mobile factors of production, skilled labour and unskilled labour. The home (foreign) country is relatively unskilled-labour (skilled-labour) abundant according to the physical definition of factor abundance; and production technologies of two goods differ in terms of factor intensities. Thus, the home (foreign) country has a comparative advantage to produce the relatively unskilled-labour (skilled labour) intensive good. This paper shows that the conversion of an import-quota into an equivalent voluntary export restraint raises (lowers) skilled-unskilled relative wage in the country importing the unskilled-labour (skilled labour) intensive good; and this result is independent of which good is initially subject to import quota. However, conversion of an import-quota into an equivalent import tariff, on the other hand, may lead to a rise in the skilled-unskilled wage ratio in both the countries. The driving force behind these results is the real income effect caused by the conversion of one type of trade restriction instrument into the other.

1.2.1.2 Product variety models

These models are also static general equilibrium models with a product variety structure and with monopolistic competition in markets of different varieties. These models also analyse the effect of change in different globalization related parameter on the degree of skilled-unskilled wage inequality. The literature includes the works of Anwar (2009, 2006a, 2008, 2006b) and Anwar and Rice (2009).

Anwar (2009) considers a static product variety general equilibrium model of a small open economy that assumes full employment in the labour market and produces two final products - one industrial good and one agricultural good. The industrial good is produced with skilled labour and a large number of varieties of intermediate goods which, in turn, are produced with unskilled labour and skilled labour. However, the agricultural good is produced with unskilled labour alone. While markets for primary factors and final products are competitive, monopolistic competition prevails in markets for varieties of intermediate goods. Anwar (2009) analyses the effect of downsizing which is defined as a decrease in the fixed cost of producing varieties. It is shown that downsizing raises wage inequality but produces a positive effect on welfare.

Anwar (2006a) also considers a static product variety full employment general equilibrium model of a small open economy that produces one industrial good and one agricultural good. However, capital is assumed to be an additional input and it is mobile among all sectors. The industrial good is produced by varieties of intermediate goods, capital and skilled labour. Intermediate goods are produced by capital and skilled labour but the agricultural good is produced by unskilled labour and capital. Assumptions regarding market structure in this model are also same as in Anwar (2009). Anwar (2006a) shows that emigration of skilled labour and/or of unskilled labour raises the degree of wage inequality even if income shares of capital are identical across industrial and agricultural sectors. However, an outflow of capital lowers the degree of wage inequality in this paper.

Anwar and Rice (2009) adopts the structure of Anwar (2006a) but considers two types of perfect substitute capital: foreign and domestic. The supply of domestic capital is exogenous but the supply of foreign capital is endogenous in this model. The model analyses the effect of immigration of either type of labour on the degree of skilled-unskilled wage inequality as well as on the level of foreign investment both in the short run and in the long run. Short run is defined as a situation where number of firms in markets for varieties is fixed. However, in the long run, there is no restriction on entry and exit of those firms. Anwar and Rice (2009) shows that, in the short run, inflow of either skilled labour or unskilled labour has no effect on the degree of wage inequality though it raises the supply of foreign capital. However, in the long run, inflow of skilled labour raises both wage inequality and the level of foreign investment.

Anwar (2008) develops a static product variety full employment general equilibrium model of a small open economy that produces two final goods with capital, labour and public infrastructure. The infrastructure is produced with capital and labour; and its cost of production is financed by non-distortionary taxation. The provision of public infrastructure involves fixed cost as well as variable cost; and the presence of public infrastructure gives rise to external economies of scale. Anwar (2008) analyses the effect of a change in the labour inflow on welfare. However, this model can not discuss its effect on skilled-unskilled wage inequality because it considers only one type of labour.

Anwar (2006b) develops a simple model of a small open economy that produces one industrial good, one agricultural good and one intermediate public good. The industrial good is produced by foreign capital, domestic labour and a large number of varieties of non-traded private intermediate goods which, in turn, are produced by foreign capital and domestic labour. The public intermediate good and the agricultural good are produced by domestic capital and domestic labour. The role of the public intermediate good is to reduce the fixed cost of production of varieties of private intermediate goods. Markets for all goods except for varieties of private intermediate goods are perfectly competitive. However, varieties of private intermediate goods are produced under monopolistic competition. The presence of internal economies in the private intermediate goods sector gives rise to specialisation-based external economies in the industrial good sector. Anwar (2006b) shows that an increase in the supply of the public intermediate good decreases foreign investment as long as the public intermediate good is not less capital intensive as compared to the agricultural good and the industrial good is not less capital intensive as compared to the private intermediate good. In the absence of specialisation-based external economies, an increase in the supply of the public good leads to an unambiguous decrease in welfare. Like Anwar (2008), this paper also can not analyse the effect on skilled-unskilled wage inequality because it considers only one type of labour.

1.2.1.3 Models with oligopolistic markets

Das (2002) develops a two sector two factor static general equilibrium model in which one of the two sectors is oligopolistic in nature. It is an extension of Feenstra–Hanson (1995, 1996, and 1997) models. Unlike in Feenstra–Hanson models, Foreign Direct Investment (FDI) takes place in an oligopolistic final-good producing sector in which foreign firms compete with local firms. The economy consists of this oligopolistic sector as well as a competitive sector which is less skilled labor intensive than the oligopolistic sector. It is shown that an increase in FDI activity, defined as an exogenous increase in the number of foreign firms, has three effects on the skilled-unskilled relative wage. First, there is a direct effect of an increase in the total number of firms in the skilled labor intensive sector; and this tends to widen the skilled-unskilled wage differential though an increase in the relative demand for skilled labour. Secondly, there is a technology gap effect, which would tend to lower the relative wage. Finally, there is a transfer effect via changes in foreign firms' profits to be repatriated; and this effect turns out to be ambiguous. The overall effect remains ambiguous too. However, Das (2002) also considers a case where entrepreneurial choice is endogenous and skilled workers are potential entrepreneurs of domestic firms. In this case, an increase in FDI activity lowers the number of domestic firms and the entrepreneurs tend to work as skilled labourers. This increase in the supply of skilled workers for production activity tends to lower the skilled-unskilled relative wage.

Das (2001) develops a two country static general equilibrium model with two types of labour-skilled and unskilled. In this model, skilled labour is used not only for production but also for supervision of unskilled labour to prevent them from shirking. However, unskilled labour is used only for production. Das (2001) shows that the effect of trade on skilled-unskilled relative wage is greater than that predicted by the Stolper-Samuelson theorem when trade is based on endowment differences. However, when trade is based on technological differences, a movement to trade tends to reduce the skilled-unskilled relative wage in each of the two countries.

1.2.1.4 Applied Model

Santis (2002) uses two alternative Applied General Equilibrium (AGE) models calibrated to the UK economy of the late 1970s to explain how endogenous and exogenous technical change affect wage inequality within labour groups in a two-sector general equilibrium setting. Both models are characterised by two primary factors of production (skilled labour and unskilled labour), the skilled labour-intensive modern service sector, the unskilled labour-intensive traditional manufacturing sector, input-output linkages and intra-industry trade in the traditional sector. The first model introduces endogenous sector-bias technical change where trade, by fostering the development of new goods to be produced by foreign partners, favours the diffusion of technology in the domestic economy. The second model assumes exogenous skill-augmenting technical change whose benefit is restricted within the originating country. The results of the numerical simulations are consistent with the stylised facts of the UK economy, though the model with endogenous technical change yields better results because it can also explain the decline in the wage rate of unskilled workers and the relatively large increase in the import of capital goods.

1.2.2 Dynamic Models

Dynamic models focus on the intertemporal accumulation of skilled labour, physical capital, technology etc. and analyse the behaviour of skilled-unskilled relative wage in the long run equilibrium and/or in the transitional phase of economic growth. The set of models developing dynamic intertemporal structure to analyse skilled-unskilled wage inequality in the long run equilibrium includes the works of Kiley (1999), Wang et. al. (2009), Fang et. al. (2008), Acemoglu (1998, 2003, 2002a), Galor and Moav (2000), Beladi and Chakrabarti (2008), Ripoll (2005), Grossman and Helpman (1991), Thoeing and Verdier (2003), Boedo (2010), He and Liu (2008), Turrini (1998), Weiss (2008), Aghion (2002), Eicher (1996), Meckl and Zink (2004), Shi (2002), Moore and Ranjan (2005) etc.

Kiley (1999) develops a two sector one commodity dynamic model with two types of labour-skilled and unskilled. One of these two sectors uses unskilled labour and its complementary intermediate goods to produce the final product and the other sector requires skilled labour and its complementary intermediate goods to produce the same product. The cost of developing a new specific intermediate good depends on the number of varieties of those specific intermediate goods available and on the level of existing research. However, there does not exist any inter sectoral knowledge spillover effect in these cost functions. The question of international knowledge spillover effect does not arise in this model because Kiley (1999) considers a closed economy. Kiley (1999) shows that the skilled-unskilled wage ratio in the steady-state growth equilibrium varies positively with the skilled-unskilled labour endowment ratio.

Wang et. al. (2009) develops a dynamic model of an economy, endowed with two types of labour - unskilled and skilled and producing two final goods - traditional goods and advanced goods. Skilled labour is either combined with sector specific capital to produce advanced final goods or used in the advanced R&D sector to design blueprints of new advanced final goods. The same is true for unskilled labour which is employed in the traditional final good sector and in the traditional R&D sector. There exists an international knowledge spillover effect from the advanced R&D sector of the foreign country to the advanced R&D sector of the home country and also a localized knowledge spillover effect from the advanced R&D sector to the traditional R&D sector of the home country. It is shown that two opposite effects play important roles to determine the degree of skilled unskilled wage inequality in the steady state equilibrium of the small open home country after it has been opened up to trade. These are the price effect and the skill discrepancy effect of which the former reduces the skilled-unskilled wage ratio and the latter raises it. Hence these two competing forces lead to ambiguity to analyse the effect of trade openness on wage inequality.

Fang et. al. (2008) develops a dynamic model very similar to Wang et. al. (2009) with the difference that only one final commodity is produced in both the sectors. The possibility of any international knowledge spillover is ruled out here though there exists localized knowledge spillover from the advanced R&D sector to the traditional R&D sector. It is shown that the skilled-unskilled wage ratio in the steady state equilibrium may move in any direction with an increase in the relative supply of skilled labour; and the direction of the movement depends upon the size of the relative supply of skilled labour and the magnitude of the efficiency parameter in the localized technology spillover function.

Acemoglu (1998) develops a dynamic quality ladder model of a closed economy to explain the growing wage inequality with the help of endogenous adoption of skill biased and unskilled biased technologies. If the proportion of skilled workers in the labour force is high, then this leads to a relatively larger size of skill-complementary technologies; and this, in turn, encourages faster upgrading of the productivity of skilled workers. As a result, an increase in the relative supply of skilled labour reduces the skilled-unskilled wage ratio in the short run. However, in the long run it induces skill biased technical change and thus raises the skilled-unskilled wage ratio, through an increase in the relative demand for skilled labour.

Acemoglu (2003) extends the model of Acemoglu (1998) introducing international trade; and, in this extended model, the degree of skilled-unskilled wage inequality is determined not only by the technology and by the relative supply of skilled labour but also by the nature of international trade. The most important result of the paper is that increased international trade induces skill-biased technical change. As a result, trade opening can cause a rise in the degree of skilled unskilled wage inequality both in the developed and in the less developed countries.

Acemoglu (2002a) develops a dynamic product variety model with two factors-skilled labour and unskilled labour; and attempts to explain the growing skilled unskilled wage inequality with the help of endogenous adoption of skill-biased and unskilled biased technologies. One final good is produced with two intermediate goods and its production function satisfies CES property. One (other) intermediate good is produced with skilled (unskilled) labour and with skill (unskill) complementary varieties of intermediate inputs. Monopolistic competition prevails in the markets for varieties but both the labour markets are competitive. It is shown that there are two forces affecting the nature of the bias in equilibrium: the price effect and the market size effect. While the price effect encourages innovations biased in favour of the scarce factor, the market size effect leads to technical change favouring abundant factors. The relative strength of these two effects finally determines the nature of the effect that a change in the relative supply of skill labour would produce on the relative wage; and this net effect finally depends on the aggregate elasticity of technical substitution between two factors in the production of the final good.

Galor and Moav (2000) develops a over-lapping generation model of a small open economy with skilled biased technological change that operates in a perfectly competitive market. Capital movements are unrestricted and economic activity extends over infinite discrete time. The economy produces a single homogeneous good with physical capital and a composite labour input (measured in efficiency units) that consists of skilled labour and unskilled labour. The relative supply of skilled labour to unskilled labour is determined by occupational choices of individuals within a generation as well as by the state of technology. The stock of physical capital accumulates through investment given by economy's aggregate saving net of international lending. It is shown that technological progress raises the skilled-unskilled wage inequality as well as the within group inequality.

Beladi and Chakrabarti (2008) develops a two period model to compare effects of immigration and of international outsourcing on the skilled-unskilled wage ratio in a model of outsourcing subject to contractual incompleteness. The paper shows that the skilled-unskilled wage inequality, while affected by frictions in immigration, is sensitive to variations in contractual frictions in intermediates that affect international outsourcing. In particular, Beladi and Chakrabarti (2008) predicts that a fall in the friction in immigration would cause the skilled-unskilled wage gap to widen while this gap would be dampened by a decline in the contractual frictions in the low-tech intermediates. A decline in the contractual frictions in the high-tech intermediates relative to the frictions in low-tech intermediates will increase the wage gap.

Ripoll (2005) studies a simple dynamic general equilibrium model of trade in which differences in initial endowments across developing countries play a key role in explaining the nature of skilled-unskilled wage inequality. The main finding of this paper is that the skilled-unskilled relative wage would fall with trade liberalization when the developing country is initially abundant in the ratio of skilled to unskilled workers and scarce in physical capital.

Grossman and Helpman (1991), in chapter 3, develop a dynamic product variety model in which only one production sector produces varieties of innovated products and a R&D sector gives birth of new varieties. This model neither makes any distinction between skilled labour and unskilled labour nor considers the problem of imitation. In chapter 5 of Grossman and Helpman (1991), the basic model is extended to introduce skilled labour and unskilled labour with zero elasticity of technical substitution between them in all production sectors with the assumption that the high technology final good sector is more skilled labour intensive than the traditional sector. An increase in the skilled (unskilled) labour endowment does not affect the relative demand for skilled labour; and hence lowers (raises) the skilled-unskilled wage ratio. However, in the model developed in chapter 6 of Grossman and Helpman (1991), elasticity of technical substitution between skilled labour and unskilled labour is assumed to be positive in each of all these production sectors and the R&D sector is assumed to use only skilled labour as input and not unskilled labour. So the change in skilled labour endowment affects the demand for skilled labour but a change in unskilled labour endowment has no effect on its demand in this model. Hence the skilled-unskilled wage ratio is increased with increase in either skilled or unskilled labour endowment.

Thoeing and Verdier (2003) develops a dynamic model of innovations in which firms can endogenously bias the direction of technological change; and then attempts to analyse the effects of imitation on the skilled-unskilled wage inequality. When there is an increased threat of imitation, innovating firms use skill intensive technology to get rid of the threat of imitation. This technological change raises the relative demand for skilled labour and thus worsens the problem of skilled-unskilled wage inequality.

Boedo (2010) develops a dynamic general equilibrium model that endogenizes the technology adoption decision with factor accumulation decision. The factor accumulation decision is taken over the stocks of skilled labour and physical capital; and the technology adoption decision focuses on the optimal choice of the level of skill bias in the production technology in the presence of a convex technology adoption cost that can be interpreted as an accelerated obsolescence (due to technological change) in the stocks of skilled labour and physical capital. The model explains why poor countries do not adopt advanced technologies even though they are readily available and have been implemented in rich countries. This is so because the cost of adoption may be very high and the transition phase may be very long. Boedo (2010) also shows that intertemporal changes in the degree of skilled-unskilled wage inequality would be observed in the transitional phase but not in the steady state equilibrium.

He and Liu (2008) develops a dynamic model to describe how investment-specific technological change generates a time path of skill accumulation and wage inequality. As technology improves over time, the relative price of capital equipments falls; and this encourages investment in new equipments. Given equipment–skill complementarity, this also encourages investment in skill accumulation because otherwise increases in equipments would raise (lower) the marginal productivity of skilled (unskilled) workers and thereby would drive up the skill premium. This consequent increase in the relative supply of skilled labour partly dampens the rise in the skill premium. It is shown that the revenue-neutral elimination of a capital income tax leads to a modest increase in the degree of wage inequality and to a sizable welfare gain. However, the revenue-neutral increase in the progressiveness of a labour income tax is not effective to reduce wage income inequality because it discourages skill accumulation and, in turn, leads to a large decline in the average productivity of skilled labour and consequently in the welfare. In contrast, a policy that provides direct subsidies to human capital accumulation raises the skilled–unskilled labour ratio, lowers the skill premium, and improves welfare.

Turrini (1998) develops a two period (generation), two sector, small open economy model where human capital accumulation is financed by household expenditure as well as by tax financed public expenditure. Workers are heterogeneous in skills; and only relatively more skilled workers are employable in high skill export sector. Turrini (1998) shows how endogenous public investments in human capital can enhance the skilled-unskilled income differentials arising from exogenous trade-related shocks and technology shocks. Median voter equlibria leads to underinvestment in publicly-provided component of human capital; and this, in turn, leads to an increase in the degree of income inequality across generations.

Weiss (2008) develops a dynamic two sector two factor general equilibrium model in which two consumption goods, services and manufacturing are produced using two types of labor-unskilled and skilled. Technological change is exogenous, factor-augmenting and skill-biased; and it only affects the manufacturing sector. The service sector does not derive any benefit from technological progress. Both types of labour are perfectly mobile. Weiss (2008) shows that an improvement in the skill biased technical progress does not necessarily imply an increase in the skilled-unskilled wage inequality in the long run because the nature of movement in the skilled-unskilled relative wage also depends on the relative price of the consumption good as well as on the preference parameters of the representative consumer.

Aghion (2002) develops a dynamic quality ladder Schumpeterian Growth model to explain two important puzzles of growing wage inequality in developed economies. The first puzzle concerns wage inequality between educational groups; and the second puzzle concerns wage inequality within educational groups. The model assumes only one final good to be produced with two intermediate goods one of which is produced by skilled labour and the other by unskilled labour. There is continuum of potential producers in each of the two intermediate goods sector; but, in any period, only one firm knows how to make technological advance. Technological lead is made in skilled (unskilled) labour using intermediate good sector by allocating larger R&D investments to that sector; and this raises the size of the monopoly rent in that sector. Innovations are always imitated after one period and hence an innovator gets monopoly rent only for a single period. Marginal revenue productivities of the R&D input in the two sectors must be same in equilibrium. It is shown that an increase in skilled labour endowment raises the relative productivity of skilled labour to that of unskilled labour; and this, in turn, raises the skilled-unskilled wage ratio. Aghion (2002) also explains the rise in the degree of within group wage inequality in an infinite horizon discrete time model with sequential productivity-improving innovations occurring in every period. This allows a new vintage of machine to be produced and used for final good production.

Eicher (1996) develops a dynamic two period model with three sectors and two factors; and attempts to analyse how interaction between endogenous human capital accumulation and technological change affects the behavior of skilled-unskilled relative wage and the process of economic growth. The education sector produces new technology with skilled labour; and two production sectors produce the same good with different technological sophistications and skill intensities. The high tech production sector uses new technology and both skilled labour and unskilled labour as inputs; but the low tech production sector uses old technology with unskilled labour as the only input. Private investments in human capital formation finances the employment of skilled labour in the education sector. The absorption of new skill intensive technologies into the high tech production sector raises the relative demand for skill labour; and thus raises the skilled-unskilled relative wage. In contrast to recent models of endogenous economic growth, higher rates of technological change and of economic growth in the steady-state equilibrium of this model may be accompanied by a higher level of skilled-unskilled relative wage and by a lower relative supply of skilled labour.

Meckl and Zink (2004) analyses the time behaviour of the skilled-unskilled wage inequality within a one sector simple neoclassical growth model where labour is heterogeneous in terms of abilities. The accumulation of physical capital causes changes in relative factor prices and thus in incentives to acquire skills. This, in turn, alters the composition of the labour force. Without relying on any exogenous shocks, this model generates dynamics of capital accumulation; and these, in turn, leads to important results related to the behaviour of skilled-unskilled wage inequality. For example, skilled-unskilled relative wage may grow in a non-monotone way. Additional incorporation of wage rigidities in the form of a restriction that people below certain ability would not get employment shows the trade off between skilled-unskilled wage inequality and employment opportunities for unskilled labour.

Shi (2002) analyses the directed search and matching problem in an economy with heterogeneous skills and skill-biased technology. It is shown that a unique symmetric equilibrium exists and is socially efficient; and matching is partially mixed in the equilibrium. A high-tech firm receives both skilled and unskilled applicants with positive probability, and favours skilled workers, while a low-tech firm receives only unskilled applicants. The model generates wage inequality among unskilled workers as well as skilled-unskilled wage inequality. Since high-tech firms favour skilled applicants, they must compensate unskilled applicants for the low employment probability by offering them a higher wage than low-tech firms do. This within-group inequality does not rely on the traditional assumptions of innate ability differences and the consequent productivity differences because unskilled workers perform the same task and have the same productivity in these two types of firms. Shi (2002) shows that skill-biased technological progress generates concurrent increases in within-group inequality as well as in the skilled-unskilled wage inequality, with the latter rising more sharply. A general productivity slowdown raises the degree of within-group wage inequality and reduces that of the skilled-unskilled wage inequality.

Moore and Ranjan (2005) investigates the static and dynamic effects of globalisation and skill-biased technological change on unemployment across skill classes using a model of search unemployment with two traded intermediate goods used in the production of a non-traded final good. Here search unemployment exists in the markets of skilled labour as well as of unskilled labour. One of the two intermediate goods is produced with skilled labour and the other is produced with unskilled labour. Although factors are sector specific the unemployment rate and the real wage rate in each sector respond to the relative price of intermediate goods. Countries open to trade are linked by world relative prices. In each country, unemployment substitutes for factor mobility in the transmission of relative price changes to sectoral output changes. In addition, the autarky relative price of the intermediate goods is determined by the relative supply of skilled labour. Therefore, relative factor endowments determine comparative advantage. It is shown that both globalisation and skill-biased technological change lead to increases in wage inequality. However, these shocks may have different effects on unemployment.

1.3 Existing research gaps

There may exist various types of research gaps to be fulfilled in the literature. However, following existing research gaps are to be addressed in the present thesis. The static competitive equilibrium models described in the section 1.2.1.1 assume that the non-traded good is produced by unskilled labour. Hence these group of models can not analyze the role played by inter-sectoral mobility of skilled labour and by the change in demand for skilled labour from the non-traded good sector on the skilled-unskilled wage inequality. All of them assume supply of skilled labour to be exogenously given except for Marjit and Acharyya (2003) and Yabuuchi and Chaudhuri (2009). However, Marjit and Acharyya (2003) and Yabuuchi and Chaudhuri (2009) assume that skilled labour is produced only with capital but not with skilled labour. The problem of unemployment of labour is ignored by most of the existing theoretical works expect for Marjit and Acharyya (2003), Beladi et. al. (2008) and Chaudhuri (2008, 2004) who consider Harris-Todaro (1970) type unemployment of unskilled labour. However, these models assume full employment of skilled labour. Chaudhuri and Banerjee (2010) explain unemployment of skilled labour with the help of efficiency wage hypothesis[8] and unemployment of unskilled labour using Harris-Todaro (1970) migration mechanism. However, they do not consider the role of non-traded good. The ratio of the wage rate of the skilled worker to that of the unskilled worker is taken as the measure of wage inequality in all these models. However, none of them considers Gini-Coefficient of wage income distribution as a measure of inequality.

Among the existing static product variety models described in section 1.2.1.2, Anwar (2005, 2006b) introduce a public input producing sector in their models in the presence of specialization-based external economics. However, they have only one type of labour in their models; and hence can not explain the skilled-unskilled wage inequality. Anwar (2006a, 2009) and Anwar and Rice (2009) analyse the problem of wage inequality using endogenous product variety framework with specialization-based external economics but do not consider the role of public input. Glazer and Ranjan (2003) introduces preference heterogeneity assuming that skilled workers prefer to consume skill intensive goods but does not consider the role of public intermediate good. Also, none of the existing models described in this section considers the problem of unemployment.

The dynamic models like Kiley (1999), Acemoglu (2002a), Fang et. al. (2008) summarized in the section 1.2.2 develop two sector dynamic models with a single final commodity. So they can not analyse the effects of trade on wage inequality. Acemoglu (2003) analyses the effect of

[8] The literature on efficiency wage hypothesis inclued works of Solow (1979), Agell and Lundborg (1992, 1995), Feher (1991) and Akerlof and Yellen (1990) etc.

trade. However, even Acemoglu (2003) does not analyse the effect of international and inter sectoral knowledge spillover on skilled-unskilled wage inequality.

North-South models of Grossman and Helpman (1991) analyse the role of imitation on the long run rate of growth and on the North-South relative wage. However, these models do not distinguish between skilled labour and unskilled labour. No model in the existing literature, except Thoeing and Verdier (2003), has analysed the effects of a change in the imitation rate on this skilled-unskilled wage inequality. In Thoeing and Verdier (2003), increased threat of imitation worsens the problem of skilled-unskilled wage inequality. Unfortunately, this result is not consistent with the findings of empirical works like Kanwar and Evenson (2003, 2009), Park (2008), Ginarte and Park (1997) etc. who show that there is significant improvement in the worldwide patent protection during the period 1960-2005.

1.4 A summary of the present thesis

In addition to the present introductory chapter, the thesis consists of four other chapters in which we develop different theoretical models attempting to fill up the research gaps pointed out in section 1.3.

The chapter 2 is devoted to analyse skilled-unskilled wage inequality in a static competitive general equilibrium framework with special emphasis on the role of non-traded final good sector that uses skilled labour.

In section 2.2 we develop a static competitive general equilibrium model of a small open economy. Here skilled labour is mobile between a traded good sector and the non-traded good sector. Unskilled labour is specific to another traded good sector. Capital is perfectly mobile among all these three sectors. This non-traded good is a non-inferior final good but not an intermediate one. The level of demand for the non-traded good is assumed to vary positively with the national income of the country; and thus increases in factor prices and/or factor endowments produce positive effects on the demand for non-traded good and consequently on its price. The skilled unskilled wage ratio is changed due to this change in the equilibrium price of the non-traded good. Existing theoretical literature does not adequately focus on the role of non-traded final good sector on the determination of skilled-unskilled relative wage.

We derive interesting results from our model. Capital intensity ranking between the skilled labour using non-traded good sector and the skilled labour using traded good sector appears to be the most important factor determining the nature of the effect on skilled-unskilled relative wage. A capital exporting country as well as a capital importing country may experience a similar effect on wage inequality when this inter-sectoral capital intensity ranking in these two countries are opposite to each others. The same is also true for a labour exporting country and a labour importing country in the case of this opposite inter-sectoral factor intensity ranking. Opening of trade may also produce similar effects in this case. Also the nature of the effect on wage inequality as measured by skilled-unskilled relative wage depends on the sign of the marginal effect of excess demand for non-traded good with respect to the change in parameter. This sign of this marginal demand effect may be different in different countries. Thus two countries, whose roles are dual to each others in the context of exchange of goods or movement of factors, may experience similar movements in wage inequality with different signs of marginal demand effects even if their capital intensity ranking between the traded good sector and the non-traded good sector are identical. Models of existing literature fails to put emphasis on these points because a skilled labour using non-traded good sector does not exist there and hence the role of intersectoral mobility of skilled labour can not be studied.

We extend the basic model introducing endogenous supply of skilled labour in section 2.3. Here the unskilled labour is transformed into skilled labour by the education sector and this transformation is instantaneous. The addition of this education sector makes the model a four sector competitive general equilibrium model. Capital is also perfectly mobile among the education sector, skilled labour using traded good sector and the nontraded good sector. However, land and unskilled labour are specific to another traded good sector in which capital does not get any entry. Otherwise, this extended model is identical to the basic model. Here also the effect of a change in different parameters on wage inequality depends on the factor intensity ranking between two skilled labour using sectors and on the relative strength of the marginal effects on demand for and supply of nontraded final good. We also analyse the effects of changes in different parameters on the endogenous supply of skilled labour.

We consider another extension of the basic model in section 2.4. In this extension, we introduce involuntary unemployment equilibrium in both the labour markets in an otherwise identical basic model of section 2.1 and explain unemployment using efficiency wage hypothesis. We examine the effects of change in different factor endowments on unemployment and on skilled-unskilled wage inequality. Also, we introduce Gini-Coefficient of wage income distribution as a measure of wage income inequality replacing skilled-unskilled relative wage. It is shown that Gini-coefficient is a monotonically increasing function of skilled-unskilled relative wage in a full employment model. However, in the presence of unemployment, this is not true. It is shown that a comparative static effect with respect to change in capital endowment may force the skilled-unskilled relative wage and the Gini-Coefficient of wage income distribution to move in opposite directions in the presence of unemployment.

Chapter 3 is devoted to explain skilled-unskilled wage inequality in a static general equilibrium model with product variety structure and with monopolistic competition in markets of different varieties.

In section 3.2, we develop a four sector small open economy model with two traded final good sectors, a public intermediate good producing sector and a nontraded good sector producing varieties of private intermediate goods. Production functions of all these sectors, except for varieties of private intermediate goods sector, satisfy all standard neo-classical properties including constant returns to scale (CRS). However, in the private intermediate goods producing sector, production function of each of these varieties satisfies increasing returns to scale (IRS). The public intermediate good plays the role of reducing the fixed cost of production of nontraded private intermediate goods. There are three primary factors: capital, skilled labour and unskilled labour. Industrial sector producing a traded good uses capital, intermediate goods and skilled labour as inputs. Private intermediate goods producing sector also uses capital and skilled labour. Public input producing sector and the agricultural sector producing the other traded good use capital and unskilled labour as inputs. It is shown that, if production technologies are same for the agricultural sector and the public input producing sector and if the scale elasticity of output is very low, then an increase in capital stock (unskilled labour endowment) raises (lowers) the skilled-unskilled wage ratio. However, an increase in skilled labour endowment does not produce any unambiguous effect. On the other hand, an increase in the tax rate on industrial output and/or an increase in the price of the agricultural product, armed with same set of assumptions, lowers the skilled-unskilled wage ratio.

In section 3.3, we develop a three sector small open economy model with two traded final good sectors and a nontraded good sector producing varieties of intermediate goods. Public intermediate good producing sector does not exist here; and the exclusion is made only for the sake of simplicity. The efficiency wage hypothesis is introduced to explain unemployment in each of these two labour markets. We again attempt to analyse the effect on unemployment as well as on the Gini-coefficient of wage income distribution. It is shown that an increase in either type of labour endowment (capital endowment) raises (lowers) the unemployment rate of either type of labour if the scale elasticity of output is very small. On the other hand, if the industrial sector is more capital intensive than the agricultural sector and if efficiency functions of both types of labour are identical, then an increase in either type of labour endowment (capital endowment) lowers (raises) the skilled-unskilled wage ratio. However, the effect of a change in capital endowment on the Gini Coefficient of wage income distribution is ambiguous in sign.

Chapter 4 of this thesis extends the one commodity two sector dynamic model of Kiley (1999) introducing two different commodities to be produced in two sectors and introducing international knowledge spill over from the rest of the world to the home country and localized knowledge spillover from the more advanced modern sector to the less advanced traditional sector. We analyse the effect of opening of international trade on the skilled unskilled wage inequality in the long run equilibrium. We show that the relationship between the skilled unskilled wage ratio and the skilled unskilled labour endowment ratio under autarky is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. However, when international trade is opened, its effect on this skilled unskilled wage ratio in the long run equilibrium depends not only on the degree of consumer’s indifference substitution between the two final goods but also on the intensity of spillover effects as well as on the inter country difference in factor endowments.

Chapter 5 of this thesis analyses the effect of skilled-unskilled wage inequality in long run equilibrium using a Helpman (1993) framework.

In section 5.2, we develop a dynamic three sector product variety model to analyse the role of imitation on skilled-unskilled wage inequality. One of these sectors produces varieties of innovated products with skilled labour as well as unskilled labour; and an other sector produces varieties of imitated products with only unskilled labour. Also there is a R&D sector developing blue-prints of new products with skilled labour as the only input. However, imitation is costless. It is shown that an increase in skilled (unskilled) labour endowment raises (lowers) the rate of growth, raises (lowers) the skilled-unskilled wage ratio, and lowers (raises) the level of social welfare. However, an increase in the rate of imitation raises this growth rate, lowers the skilled-unskilled wage ratio, and raises the level of social welfare.

In section 5.3, we extend the basic model developed in section 4.1. We here introduce endogenous imitation and assume the existence of a social institution that has control over this endogenous imitation. This social institution produces an imitation preventing public good with skilled labour as the only input. It is shown that an increase in skilled (unskilled) labour endowment raises (has no effect on) the rate of growth and raises (lowers) the skilled-unskilled wage ratio. However, an improvement in the imitation preventing efficiency of the public good raises the skilled-unskilled wage ratio though it has no effect on growth rate. A change in skilled labour endowment or a change in unskilled labour endowment has no effect on the imitation rate. However, an improvement in the imitation prevention efficiency of the public good lowers the imitation rate. We also analyse the effects of change in different parameters on the level of social welfare.

Concluding remarks are in made in chapter 6.

Chapter 2

Static competitive general equilibrium model

2.1 INTRODUCTION

In this chapter, we develop a static competitive general equilibrium model of a small open economy in which skilled labour is mobile between a traded good sector and a non-traded good sector. In reality, skilled workers are employed in various sectors; and a substantial part of skilled labour is employed in sectors like education, health, and legal services etc. which produce non-traded services. However, skilled workers are also employed in various manufacturing units producing technologically sophisticated traded products and in various service sectors providing different types of consultancy services to international organization. The picture of employment distribution of skilled labour is common to different parts of the globe. This motivates us to introduce inter-sectoral mobility of skilled labour between the traded good sector and the non-traded good sector in this theoretical model.

A small set of existing works deals with the mobility of skilled labour between the traded good sector and the non-traded intermediate good sector; and this set includes works of Marjit et. al. (2004), Marjit and Acharyya (2006) and Kar and Beladi (2004). Marjit and Acharyya (2006) develops a north south model with mobile skilled labour and non-traded intermediate good but does not consider capital as a factor of production. Kar and Beladi (2004) assumes the wage rate of skilled labour to be institutionally fixed and the non-traded intermediate good to be used in fixed proportion to produce the final traded good. On the contrary, our model analyses the role of inter-sectoral capital mobility and endogenous determination of skilled wage rate. The model of Marjit et. al. (2004) is closest to ours but it assumes that the traded good sector using intermediate good does not use capital as an input directly.

This is a three sector three factor model in which skilled labour is mobile between a traded good sector and a non-traded good sector and unskilled labour is specific to another traded good sector. Capital is perfectly mobile among all these three sectors. The degree of skilled unskilled wage inequality is measured by the ratio of skilled wage to unskilled wage. We examine the effects of change in different factor endowments and of globalization on the degree of skilled-unskilled wage inequality. We find that the effect of a change of a factor endowment on the skilled-unskilled relative wage depends on the factor intensity ranking between two skilled labours using sectors and on the relative strength of the marginal effects on demand for and supply of non-tradable good. We also find that a decrease in the price of the traded good produced by skilled (unskilled) labour lowers (raises) the skilled-unskilled wage ratio.

This chapter is organized as follows. Section 2.2 presents the basic model with full employment of all factors and with exogenous supply of skilled labour. Sub-section 2.2.1 describes the model and sub-section 2.2.2 analyzes the effects of changes in factor endowments. In sub-section 2.2.3, we analyze the effects of exogenous changes in prices of traded goods. In section 2.3, the model is extended with endogenous supply of skilled labour; and in section 2.4; we introduce unemployment in both the labour markets. Limitations of the model are described in section 2.5.

2.2. The Basic Model: [9]

2.2.1 Description:

We consider a small open economy with three sectors and three factors- unskilled labour, skilled labour and capital. Sectors 1 and 2 produce products using skilled labour and capital as inputs; and sector U uses unskilled labour and capital as inputs. Production function of each of the three sectors satisfies all standard neo-classical properties including CRS. Sectors 1 and U produce traded goods but sector 2 produces non-traded goods which is normal to the consumers. All factor endowments are exogenously given. Capital is mobile among all three

[9] Gupta and Dutta (2010a) is partly based on the materials presented in this section.

sectors; and skilled labour is mobile between sectors 1 and 2. However, unskilled labour is specific to sector U. Factor prices in each of the three sectors are perfectly flexible and this flexibility ensures full employment of all the factors. All markets are competitive. The representative firm maximizes profit; and the representative consumer maximizes utility subject to the budget constraint. If the sector 2 producing non-tradables is dropped then the present model is reduced to Marjit and Kar (2005) model which is equivalent to a Jones (1971) model with sector specific labour and capital mobility. If our sector 2 is dropped and if land is introduced as another specific factor in sector U, this model is reduced to Yabuuchi and Chaudhuri (2007) model.

We use the following notations.

P_i	=	Effective producer's price of ith commodity for $i = 1,2,U$.
W_S	=	Wage rate of skilled labour.
W_U	=	Wage rate of unskilled labour.
r	=	Common rate of return to capital in all the sectors.
D_2	=	Demand function for commodity 2.
Y	=	Total factor Income.
X_i	=	Level of output of ith sector for $i = 1,2,U$.
K_i	=	Capital used in sector i for $i = 1,2,U$.
S_i	=	Skilled labour used in sector i for $i = 1,2$.
S	=	Exogenously given endowment of skilled labour.
L	=	Exogenously given total labour endowment.
K	=	Exogenously given capital endowment.
a_{Ki}	=	$\frac{K_i}{X_i}$ for $i = 1,2,U$.
a_{Si}	=	$\frac{S_i}{X_i}$ for $i = 1,2$.
a_{LU}	=	$\frac{L-S}{X_U}$.
θ_{ji}	=	$\frac{a_{ji}W_j}{P_i}$ for $j = S,K,L$ and $i = 1,2,U$.
λ_{ji}	=	$\frac{a_{ji}X_i}{j}$ for $j = S,K,L$ and $i = 1,2,U$.

S^h_{ji} = Elasticity of factor output coefficient of jth factor in hth sector with respect to price of ith factor, for $\text{j}, \text{i} = \text{S}, \text{K}, \text{L}$ and $\text{h} = 1,2, \text{U}$. For example, $S^1_{SK} = \left(\frac{r}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial r}\right), S^1_{SS} = \left(\frac{W_S}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial W_S}\right)$ etc. $S^h_{ji} > 0$ for $j \neq i$; and $S^h_{jj} < 0$.

$\hat{x}$ = $\frac{dx}{x}$ = Relative change in x.

Following equations describe the model.

$$P_1 = a_{S1}W_S + a_{K1}r \tag{2.2.1}$$

$$P_2 = a_{S2}W_S + a_{K2}r \tag{2.2.2}$$

$$P_U = a_U W_U + a_{KU}r \tag{2.2.3}$$

$$D_2(P_2, Y) = X_2 \tag{2.2.4}$$

$$Y = W_S S + rK + W_U(L - S) \tag{2.2.5}$$

$$a_{S1}X_1 + a_{S2}X_2 = S \tag{2.2.6}$$

$$a_U X_U = L - S \tag{2.2.7}$$

and

$$a_{K1}X_1 + a_{K2}X_2 + a_{KU}X_U = K \tag{2.2.8}$$

Here equations (2.2.1), (2.2.2) and (2.2.3) represent profit maximizing conditions of competitive firms in sectors 1, 2 and U. Equation (2.2.4) implies the supply-demand equality in the market of the nontraded good. Equation (2.2.5) represents total factor income (national income at factor cost in the absence of taxes and subsidies on factor income) and equations (2.2.6), (2.2.7), and (2.2.8) stand for equilibrium conditions in factor markets.

In this model, P_1 and P_U are internationally given, but P_2 is endogenously determined by demand-supply mechanism. There are eight unknowns in the model: W_S, W_U, r, P_2, X_1, X_2, X_U and Y. Parameters of this system are: P_1, P_U, L, S and K. There are eight independent equations with eight unknowns. The production structure does not possess the decomposition property; and so factor prices cannot be solved independent of factor endowments.

The working of the general equilibrium model is described as follows. Two input prices W_S and r are determined from equations (2.2.1) and (2.2.2) simultaneously as functions of P_2. Then, from equation (2.2.3), we obtain W_U as a function of P_2. As factor prices are determined, so all the factor output coefficients are also determined as functions of P_2. Now, from equation (2.2.7), we can obtain X_U; and then equations (2.2.6) and (2.2.7) simultaneously solve for X_1

and X_2 as functions of P_2 given L, S and K. Then, from equation (2.2.5), we can find Y as function of P_2. Since Y and X_2 are determined as functions of P_2, so, from equation (2.2.4), we can solve for P_2.

Differentiating equations (2.2.1), (2.2.2), (2.2.3) and using profit maximizing conditions, we obtain following equations.

$$\theta_{S1}\widehat{W}_S + \theta_{K1}\hat{r} = \widehat{P}_1 \quad (2.2.1\text{-A});$$

$$\theta_{S2}\widehat{W}_S + \theta_{K2}\hat{r} = \widehat{P}_2 \quad (2.2.2\text{-A});$$

and

$$\theta_U\widehat{W}_U + \theta_{KU}\hat{r} = \widehat{P}_U \quad (2.2.3\text{-A}).$$

Using equations (2.2.1), (2.2.2), (2.2.3), (2.2.5), (2.2.6), (2.2.7) and (2.2.8), it can be easily shown that

$$Y = P_1X_1 + P_2X_2 + P_UX_U \, .$$

Here the RHS represents the aggregate sales revenue (national income at product prices in the absence of commodity taxes and subsidies).

2.2.2 <u>Changes in factor endowments</u>:-

We do not consider any change in trade and fiscal policies in this section. So $\widehat{P}_1 = \widehat{P}_U = 0$. We analyze the effects of changes in factor endowments. We consider the followings: (i) An exogenous increase in capital stock resulting either from foreign capital inflow or from domestic capital accumulation. (ii) An exogenous expansion of the education sector transforming unskilled labour into skilled labour; and (iii) An exogenous decrease in labour endowment caused by international labour migration. Changes in factor endowments, with given product prices, affect the output composition of the economy on the basis of intersectoral factor intensity ranking among different sectors; and these effects are known as Rybczynski effects in the literature. These effects are important here too. However, we do not highlight these effects in this chapter because our focus is limited to the analysis of skilled-unskilled wage inequality. Using equations (2.2.1-A) and (2.2.2-A), we obtain

$$\widehat{W}_S = -\frac{\widehat{P}_2\theta_{K1}}{|\theta|} \quad (2.2.9);$$

and

$$\hat{r} = \frac{\hat{P}_2\theta_{S1}}{|\theta|} \tag{2.2.10}$$

Here, $|\theta| = \theta_{S1}\theta_{K2} - \theta_{S2}\theta_{K1}$.Mathematical sign of $|\theta|$ indicates the capital intensity ranking between the two skilled labour using sectors.

Then, using equations (2.2.3-A) and (2.2.10), we obtain

$$\widehat{W}_U = -\frac{\theta_{KU}}{\theta_U}\frac{\theta_{S1}}{|\theta|}\hat{P}_2 \tag{2.2.11}$$

Hence, using equations (2.2.9) and (2.2.11), we have

$$\widehat{W}_S - \widehat{W}_U = -\frac{\hat{P}_2}{|\theta|\theta_U}[\theta_{K1}\theta_U - \theta_{KU}\theta_{S1}] \tag{2.2.12}$$

Equation (2.2.12) shows that the magnitude of the rate of change of the skilled-unskilled wage ratio depends on the magnitude of the rate of change of the price of the non-traded good; and the nature of their relationship is conditional on the capital-intensity ranking of the three sectors. If $|\theta|$ and $(\theta_{K1}\theta_U - \theta_{KU}\theta_{S1})$ are of same (opposite) sign, skilled-unskilled wage ratio varies inversely (directly) with the price of the non-traded good. Effect of any parametric change on $\left(\frac{W_S}{W_U}\right)$ operates through its effect on P_2.

Now putting the expression of X_U from equation (2.2.7) in equation (2.2.8) and then differentiating equations (2.2.6) & (2.2.8) and solving the differentials by cramers rule, we obtain[10]

$$\hat{X}_2 = \frac{1}{|\lambda|}[\lambda_{S1}\hat{K} - \frac{a_{KU}L}{a_U K}\lambda_{S1}\hat{L} + \frac{\hat{S}}{a_U X_U}\{\lambda_{S1}\lambda_{KU}S - \lambda_{K1}(L-S)\}] + \hat{P}_2 M \tag{2.2.13}$$

where,

$|\lambda| = \lambda_{S1}\lambda_{K2} - \lambda_{K1}\lambda_{S2}$

and

$$M = \frac{1}{|\theta||\lambda|}\Big[\theta_{K1}\{\lambda_{S1}(\lambda_{K1}S_{KS}^1 + \lambda_{K2}S_{KS}^2) - \lambda_{K1}(\lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2)\}$$
$$+\theta_{S1}\{\lambda_{K1}(\lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2) - \lambda_{S1}(\lambda_{K1}S_{KK}^1 + \lambda_{K2}S_{KK}^2 + \lambda_{KU}S_{KK}^U - \lambda_{KU}S_{UK}^U)\}$$
$$+\lambda_{S1}(\lambda_{KU}S_{KU}^U - S_U^U\lambda_{KU})\frac{\theta_{KU}\theta_{S1}}{\theta_U}\Big] > 0\ .$$

[10] Detailed derivation of equation (2.2.13) is given in Appendix (2.A).

Mathematical sign of $|\lambda|$ indicates the capital intensity ranking between the two skilled labours using sectors. Interpretations of $|\lambda|$ and $|\theta|$ are identical. As a factor endowment is changed, given P_2, factor prices are also changed; and this leads to changes in factor output coefficients. This brings a further change in the production of the non-traded good which in turn alters equilibrium value of P_2; and this change in P_2 alters the level of production of the non-traded good again altering the factor prices and factor output coefficients. For example, when capital endowment, K, is increased, given other factor endowments and prices, r falls but both W_S and W_U rise. So the production processes of all three sectors become more capital intensive. Thus a_{Ki} will rise and a_{Si} will fall for $i = 1,2,U$. So the level of production of X_2 goes up (down) when sector 2 is more capital intensive than sector 1; and this is followed by a change in P_2. This increase in K may cause the excess demand for the non-traded good move in any direction because it raises its supply as well as its demand at given P_2. So the equilibrium value of P_2 may go either way. This change in P_2 alters the level of production of X_2 again altering the factor prices and factor-output coefficients. Similar effects are obtained when other factor endowments are increased. So the change in the factor endowment causes change in the production of the non-traded good directly and also indirectly through change in its price. M captures the indirect effect on change in its production through change in its price, P_2. Following three terms $\theta_{K1}\{\lambda_{S1}(\lambda_{K1}S_{KS}^1 + \lambda_{K2}S_{KS}^2) - \lambda_{K1}(\lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2)\}$, $\theta_{S1}\{\lambda_{K1}(\lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2) - \lambda_{S1}(\lambda_{K1}S_{KK}^1 + \lambda_{K2}S_{KK}^2 + \lambda_{KU}S_{KK}^U - \lambda_{KU}S_{UK}^U)\}$ and $\lambda_{S1}(\lambda_{KU}S_{KU}^U - S_U^U\lambda_{KU})\frac{\theta_{KU}\theta_{S1}}{\theta_U}$ capture the indirect effects on the change in production of the non-traded good due to change in W_S, r and W_U respectively caused by the change in P_2.

Now, differentiating equation (2.2.4), we obtain

$$e_{P_2}\hat{P}_2 + e_{M_2}\hat{Y} = \hat{X}_2 \qquad (2.2.4\text{-A}).$$

Here $e_{P_2} < 0$ represents price elasticity of demand for the non-traded good and $e_{M_2} > 0$ represents its income elasticity of demand. Then, differentiating equation (2.2.5), we obtain

$$\hat{Y} = \frac{W_S S}{Y}(\hat{S} + \hat{W}_S) + \frac{rK}{Y}(\hat{K} + \hat{r}) + \frac{W_U L}{Y}(\hat{L} + \hat{W}_U) - \frac{W_U S}{Y}(\hat{S} + \hat{W}_U) \qquad (2.2.5\text{-A}).$$

Putting the expressions of $\hat{W}_s$, $\hat{W}_u$ & $\hat{r}$ from equations (2.2.9), (2.2.10), and (2.2.11) in equation (2.2.5-A) we find that

$$\widehat{Y} = \frac{W_s s}{Y}\left(1 - \frac{W_U}{W_S}\right)\widehat{S} + \frac{W_U L}{Y}\widehat{L} + \frac{rK}{Y}\widehat{K} - \widehat{P}_2 T \qquad (2.2.5\text{-B}).$$

Here

$$T = \frac{1}{|\theta|}\left[\frac{W_S S}{Y}\theta_{K1} - \frac{rk}{Y}\theta_{S1} + \frac{W_U (L-S)\theta_{KU}}{Y\theta_U}\theta_{S1}\right].$$

captures the effect of the change in production of the non-traded good on the factor income through change in its price; and it is always ambiguous in sign.

Now, putting expressions of $\widehat{Y}$ & $\widehat{X}_2$ from equations (2.2.5-B) & (2.2.13) in equation (2.2.4-A) we find that

$$\widehat{P}_2 = \frac{1}{D}\left[\widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \widehat{L}\left(\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right) + \widehat{S}\left\{\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L-S)\right) - \frac{e_{M_2} s W_S}{Y}\left(1 - \frac{W_U}{W_S}\right)\right\}\right] \qquad (2.2.14);$$

where

$$D = e_{P_2} - e_{M_2}T - M \qquad (2.2.15).$$

Finally we use the stability condition in the market for commodity 2 to show that $D < 0$; and this stability condition, with $D_2 = X_2$, is given by

$$\frac{\widehat{D}_2}{\widehat{P}_2} - \frac{\widehat{X}_2}{\widehat{P}_2} < 0 \qquad (2.2.15\text{-A}).$$

Equation (2.2.15-A) implies that $D < 0$; and this can be shown using equations (2.2.13), (2.2.5-B) and (2.2.15) for $\widehat{K} = \widehat{L} = \widehat{S} = 0$.

Equation (2.2.14) shows how the equilibrium price of the non-traded good is affected by the exogenous changes in the capital stock, K, labour endowment, L, and skilled labour endowment, S. Combining equations (2.2.12) and (2.2.14) we can analyze the effects of parametric changes on $\left(\frac{W_S}{W_U}\right)$. According to the factor price equalization theorem, changes in factor endowments have no effects on factor prices. However, in this model, validity of this theorem is lost due to the presence of a specific factor and of a non-traded good. Any parametric change in factor endowments affects the price of the non-traded good; and this, in turn, affects the skilled-unskilled wage ratio. We assume that $\left(\frac{W_S}{W_U}\right) > 1$ in the initial equilibrium; and the comparative static effects do not reverse this inequality.

2.2.2.1 Effects on Wage inequality:-

Case-1-

It is assumed that $\Theta_{KU} > \Theta_{K1}$. This means that the unskilled labour using sector is more capital intensive than the skilled labour using sector producing traded goods. This assumption is borrowed from Marjit and Kar (2005) and Chaudhuri and Yabuuchi (2007) model. This is divided into two sub-cases.

Sub case- 1(A):-

Here we consider that $|\lambda| > 0$, or, equivalently, $|\theta| > 0$.This implies that the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector.

We first consider $\widehat{K} > 0$ with $\widehat{L} = \widehat{S} = 0$ .In this case, the effect on $(\widehat{W}_S - \widehat{W}_U)$ depends on the mathematical sign of $\left(\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)$. Here equations (2.2.12) and (2.2.14) show that

$\widehat{K} > 0$ with $\widehat{L} = \widehat{S} = 0$ and with $\left(\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) \gtreqless 0 \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \lesseqgtr 0$.

Here, $\frac{\lambda_{K1}}{|\lambda|}$ represents the marginal supply response on the non-tradable sector with respect to a change in capital stock given its product price. $e_{M_2}\frac{rk}{Y}$ is the corresponding marginal demand effect that takes place through an increase in rental income. If the increase in capital stock takes place through foreign capital inflow and if the entire foreign capital income is repatriated, then its marginal demand effect is nil. However, this marginal demand effect is positive in the case of domestic capital accumulation.

Equations (2.2.12) and (2.2.14), with $\widehat{K} = \widehat{S} = 0$, show that

$\widehat{L} < 0 \Rightarrow \widehat{P}_2 < 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$.

These two equations, with $\widehat{L} = \widehat{K} = 0$, show that

$\widehat{S} > 0$ with $\lambda_{S1}\lambda_{KU}S < \lambda_{K1}(L - S) \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$.

Here $\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L - S)\right)$ represents the marginal supply response on the non-traded good sector due to change in the skilled labour endowment with given P_2. Here, $|\lambda| > 0$ and $\lambda_{S1}\lambda_{KU}S < \lambda_{K1}(L - S)$ implies that this marginal supply effect is negative.

$\frac{e_{M_2 S W_S}}{Y}\left(1-\frac{W_U}{W_S}\right)$ represents the marginal demand effect due to change in S; and this is positive when $W_U < W_S$.

So we can establish the following proposition.

PROPOSITION-2.2.1: *If the skilled labour using traded good sector is most labour intensive of all the three sectors, then, given other parameters, (i) a decrease in labour endowment lowers the skilled unskilled wage ratio; (ii) an increase in skilled labour endowment, with an initial low level of skilled labour endowment, raises the skilled unskilled wage ratio; and (iii) an increase in capital stock raises (lowers) the skilled unskilled wage ratio if the marginal demand effect of capital accumulation on the non-traded good sector exceeds (falls short of) its corresponding supply effect.*

A decrease in the labour endowment, which basically implies a fall in unskilled labour endowment as the skilled labour endowment is fixed, results from an emigration of unskilled labour. This leads to the contraction of the unskilled labour using sector U; and hence the demand for capital in this sector is reduced. This, in turn, raises the capital availability for the two skilled labour using sectors. As sector 2 producing non-traded is more capital intensive than sector 1 producing traded good, the level of output of this capital intensive sector is increased given the price of the non-traded good. This is consistent with the Rybczynski effect. This decrease in labour endowment lowers the total factor income; and hence the demand for the non-traded good falls given its price. So an excess supply is created at the initial equilibrium price; and the price of the non-traded good falls in the new equilibrium. This leads to a decrease in the rental rate on capital, r, following a Stolper-Samuelson effect because the non-traded goods sector is more capital intensive than the traded good sector. Then both W_S and W_U should rise to satisfy the competitive equilibrium conditions. The effect on the skilled-unskilled wage ratio crucially depends on the differences between rates of increase in W_S and W_U. Wage rate, being equal to the marginal productivity of labour of the representative profit maximizing firm, varies positively with the capital labour ratio. By assumption, sector U is more capital intensive than sector 1. So the rate of increase in W_U is more than that in W_S; and this implies that the skilled unskilled wage ratio is reduced in this case.

On the other hand, an increase in the skilled labour endowment caused by educational development policies produces two opposite effects on the level of production of the non-traded good, given the product price. Directly, this leads to an expansion of sector 1 and a contraction of sector 2 at given P_2 because sector 2 is more capital intensive than sector 1. This takes place following the Rybczynski effect. However the increase in skilled labour endowment implies a decrease in the unskilled labour endowment because total labour endowment is given; and this implies a contraction of sector U. So the demand for capital is reduced in this sector; and this, in turn, raises the capital availability for the two skilled labour using sectors. As sector 2 is more capital intensive than sector 1, the level of production of sector 2 is increased following the Rybczynski effect. If initially the skilled labour endowment is very low, then the proportionate increase in skilled labour endowment is more than the proportionate decrease in the unskilled labour endowment. So finally we find a contraction of sector 2. The increase in skilled labour endowment also raises the total factor income because the skilled wage rate is always higher than the unskilled wage rate in equilibrium. So the demand for the non-traded good rises; and an excess demand is created at the initial equilibrium price. So the price of the non-traded good rises in the new equilibrium; and this raises the skilled-unskilled wage ratio.

An increase in capital endowment raises the level of output of sector 2 because the sector 2 is more capital intensive than sector 1. This is consistent with the Rybczynski effect. If the increase in capital stock takes place through foreign capital inflow and if the entire foreign capital income is repatriated, then its marginal demand effect is nil. However, if it is increased through domestic capital accumulation, then the rental income goes up and the demand for non-tradable is increased. If the marginal effect on demand for non-tradables due to capital accumulation exceeds (falls short of) its corresponding supply effect, then price of the non-traded good rises (falls); and accordingly, the skilled-unskilled wage ratio goes up (down).

Sub case- 1(B):-

Here we consider that $|\lambda| < 0$, or equivalently, $|\theta| < 0$. This implies that the skilled labour using non-traded goods sector is more labour intensive than the skilled labour using traded goods sector.

Then equations (2.2.12) and (2.2.14) show that

$\widehat{\mathrm{K}} > 0$ with $\widehat{\mathrm{L}} = \widehat{\mathrm{S}} = 0 \Rightarrow \widehat{\mathrm{P}}_2 > 0 \Rightarrow \widehat{\mathrm{W}}_\mathrm{S} - \widehat{\mathrm{W}}_\mathrm{U} < 0.$

Here $|\lambda| < 0$; and so $\left(\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) < 0$. This implies that the marginal supply effect of capital accumulation on the non-traded good sector is always negative. However, its marginal demand effect is positive; and hence the marginal effect on excess demand is always positive. So the price of the non-tradable must rise in this case.

While analyzing the effects of the change in labour endowment, we find that the nature of the effect depends on the mathematical sign of $\left(\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right)$. It represents the marginal effect on the excess demand for non-tradables with respect to change in the labour endowment, given the product price, P_2. Here, $\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1}$ represents the marginal effect on its supply; and $e_{M_2}\frac{W_U L}{Y}$ stands for the marginal effect on its demand.

Here,

$\widehat{\mathrm{L}} < 0$, with $\widehat{\mathrm{K}} = \widehat{\mathrm{S}} = 0$ and with $-\frac{a_{KU}L}{a_u K|\lambda|}\lambda_{S1} \lesseqgtr e_{M_2}\frac{W_U L}{Y}$, $\Rightarrow \widehat{\mathrm{P}}_2 \lesseqgtr 0 \Rightarrow \widehat{\mathrm{W}}_\mathrm{S} - \widehat{\mathrm{W}}_\mathrm{U} \gtreqless 0.$

Also, using equations (2.2.12) and (2.2.14), it can be shown that

$\widehat{\mathrm{S}} > 0$, with $\widehat{\mathrm{L}} = \widehat{\mathrm{K}} = 0$ and with $\lambda_{S1}\lambda_{KU}S > \lambda_{K1}(L - S), \Rightarrow \widehat{\mathrm{P}}_2 > 0 \Rightarrow \widehat{\mathrm{W}}_\mathrm{S} - \widehat{\mathrm{W}}_\mathrm{U} < 0.$

This leads to the following proposition.

PROPOSITION-2.2.2: *If the skilled labour using traded good sector is more labour intensive than the unskilled labour using traded good sector and is less labour intensive than the skilled labour using non-traded goods sector, then, given other parameters, (i) a decrease in labour endowment raises (lowers) the skilled-unskilled wage ratio if its marginal demand effect on non-traded goods sector exceeds (falls short of) its corresponding supply effect, (ii) an increase in skilled labour endowment, with a high initial level of skilled labour endowment, lowers the skilled-unskilled wage ratio, and (iii) an increase in capital stock lowers the skilled-unskilled wage ratio.*

A decrease in the labour endowment causes contraction of the sector U and raises the capital availability for the two skilled labour using sectors. Sector 2 being less capital intensive than sector 1 experiences a contraction given the price following the Rybczynski effect. This also lowers the factor income and consequently the demand for the non-traded good at its given price. If the marginal effect on demand for non-tradables due to the decrease in labour endowment exceeds (falls short of) its corresponding supply effect, then the price of the non-traded good falls (rises) and hence the skilled-unskilled wage ratio is increased (decreased) accordingly.

Here also an increase in the skilled labour endowment leads to an expansion of the less capital intensive sector 2 at given P_2 following the Rybczynski effect. However, the associated decrease in the unskilled labour endowment lowers the demand for capital in sector U and raises the capital availability for sectors 1 and 2. So the level of production of the skilled labour intensive sector 2 is decreased. If initially the skilled labour endowment is very high, finally we find a contraction of sector 2. The factor income is also increased because the skilled wage rate is higher than the unskilled wage rate. So the demand for the non-traded good rises given its price; and hence the price of the non-traded good rises in the new equilibrium. This, in turn, lowers the skilled-unskilled wage ratio.

An increase in capital endowment lowers the level of output of the relatively labour intensive sector 2 following the Rybczynski effect. In the case of foreign capital inflow, the demand effect on the non-traded good sector is nil. However, this demand effect is positive in the case of domestic capital accumulation. So an excess demand is created given the price; and the price of the non-traded good rises in the new equilibrium. As a result, the skilled unskilled wage ratio is reduced in this case; and the rate of reduction is higher in the case of domestic capital accumulation.

Case-2-

Now we turn to the opposite assumption which states that $\theta_{KU} < \theta_{K1}$. This implies that the unskilled labour using traded good sector is less capital intensive than the skilled labour using traded good sector. Marjit and Kar (2005) consider this case but Yabuuchi and Chaudhuri (2007) do not consider this case. However, this is more sensible assumption from the view point of empirical reality. Agricultural sector and urban informal sector in a less developed economy are dependent on unskilled labour; and they are more labour intensive than the organized industrial sector that employs skilled labour. This case is also divided into two sub-cases.

Sub case- 2(A):-

Here $|\lambda| > 0$, or, equivalently $|\theta| > 0$. So equations (2.2.12) and (2.2.14) show that

$\widehat{K} > 0$ with $\widehat{L} = \widehat{S} = 0$ and with $\left(\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) \gtreqless 0 \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \gtreqless 0$;

$\widehat{L} < 0$ with $\widehat{K} = \widehat{S} = 0 \Rightarrow \widehat{P}_2 < 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$;

and

$\widehat{S} > 0$ with $\widehat{K} = \widehat{L} = 0$ and with $\lambda_{S1}\lambda_{KU}S < \lambda_{K1}(L - S) \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$.

So we have the following proposition.

PROPOSITION-2.2.3: *If the skilled labour using traded good sector is more capital intensive than the unskilled labour using traded good sector but is less capital intensive than the skilled labour using non-traded good sector, then, given other parameters, (i) a decrease in labour endowment raises the skilled unskilled wage ratio; (ii) an increase in skilled labour endowment, with a low initial level of skilled labour endowment, lowers the skilled unskilled wage ratio; and (iii) an increase in capital stock lowers (raises) the skilled unskilled wage ratio if the marginal demand effect of capital accumulation on the non-traded good sector exceeds (falls short of) its corresponding supply effect.*

A reduction in the labour endowment raises the capital availability for the two skilled labour using sectors through the contraction of sector U. So the level of output of the capital intensive sector 2 is increased given its price. The decrease in factor income lowers the demand for the non-traded good given its price. So the price of the non-traded good falls in the new equilibrium. Rate of increase in W_U is less than that in W_S because sector U is less capital intensive than sector 1. Hence the skilled unskilled wage ratio is increased.

An increase in the skilled labour endowment leads to a contraction of the capital intensive sector 2 at given P_2. However, the associated decrease in the unskilled labour endowment causes contraction of sector U and raises the capital availability for the two skilled labour using sectors; and this, in turn, causes expansion of the capital intensive sector 2. If initially the skilled labour endowment is very low, finally we find a contraction of sector 2. The increase in factor income takes place due to skilled unskilled wage gap; and this raises the demand for the non-traded good. So the price of the non-traded good rises in the new equilibrium causing the skilled-unskilled wage ratio to fall.

An increase in capital endowment raises the level of output of the relatively capital intensive sector 2. In the case of foreign capital inflow (domestic capital accumulation), its demand effect is nil (positive). If the marginal effect on demand exceeds (falls short of) its corresponding marginal supply effect, then price of the non-traded good rises (falls) lowering (raising) the skilled-unskilled wage ratio.

Sub case- 2(B):-

Here $|\lambda| < 0$,or, equivalently, $|\theta| < 0$. So equations (2.2.12) and (2.2.14) show that

$\widehat{K} > 0$ with $\widehat{L} = \widehat{S} = 0 \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$.

The nature of the effect of a change in L on $\left(\frac{W_S}{W_U}\right)$ depends on the mathematical sign of $\left(\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right)$. Here equations (2.2.12) and (2.2.14) show that

$\widehat{L} < 0$ with $\widehat{K} = \widehat{S} = 0$ and with $-\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} \lesseqgtr e_{M_2}\frac{W_U L}{Y} \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \lesseqgtr 0$.

Also equations (2.2.12) and (2.2.14) show that

$\widehat{S} > 0$ with, $\widehat{K} = \widehat{L} = 0$ and with $\lambda_{S1}\lambda_{KU}S > \lambda_{K1}(L - S) \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$.

This leads to the following proposition.

PROPOSITION-2.2.4: *If the skilled labour using traded good sector has the highest capital intensity, then, given other parameters, (i) a decrease in labour endowment lowers (raises) the skilled-unskilled wage ratio if its marginal demand effect on the non-traded goods sector exceeds (falls short of) its corresponding marginal supply effect; (ii) an increase in skilled labour endowment, with a high initial level of skilled labour endowment, raises the skilled-unskilled wage ratio; and (iii) an increase in capital stock raises the skilled-unskilled wage ratio.*

A decrease in the labour endowment causes contraction of sector U and raises the capital availability for the two skilled labour using sectors. Sector 2 being less capital intensive than sector 1 experiences a contraction given the price. This also lowers the factor income; and consequently the demand for the non-tradable is reduced at the given price. If the marginal effect on demand for non-tradable exceeds (falls short of) its corresponding marginal supply effect, then price of the non-traded good falls (rises) causing the skilled-unskilled wage ratio to decrease (increase).

Here also an increase in the skilled labour endowment leads to an expansion of the skilled labour intensive sector 2 at given P_2. However, the associated decrease in the unskilled labour endowment lowers the demand for capital in sector U and raises the capital availability for sectors 1 and 2. So the level of production of the skilled labour intensive sector 2 is decreased given P_2. If initially the skilled labour endowment is high, then finally we find a contraction of sector 2. The factor income is also increased because the skilled wage rate is higher than the unskilled wage rate; and so the demand for the non-traded good rises given its price. Hence the price of the non-traded good rises in the new equilibrium raising the skilled-unskilled wage ratio.

An increase in capital endowment lowers the level of output of the relatively labour intensive sector 2. In the case of foreign capital inflow (domestic capital accumulation) the demand effect is nil (positive). So an excess demand is created given the price; and the price of the non-traded good rises in the new equilibrium raising the skilled unskilled wage ratio.

It should be noted that neither Marjit and Kar (2005) nor Yabuuchi and Chaudhuri (2007) considers a skilled labour using non traded good sector and the intersectoral mobility of skilled labour. Skilled labour is specific to a traded good sector in each of these two models. So comparative static results in those two models do not depend on factor intensity ranking between two skilled labour using sectors.

2.2.2.2 <u>Effects on total factor income</u>:-

Now we consider the effects of changes in factor endowments on the total factor income. Total factor income is the most important component of national income; and is identical to the latter in the absence of taxes and subsidies. Hence it is the most important determinant of social welfare in a small open economy given the policy parameters. Effects of changes in factor endowments on social welfare are qualitatively similar to corresponding effects on national income.

Using equations (2.2.14) and (2.2.5-B), we obtain

$$\hat{Y} = \left[\frac{rK}{Y} - \frac{T}{D}\left\{\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right\}\right]\hat{K} + \left[\frac{W_U L}{Y} + \frac{T}{D}\left\{\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right\}\right]\hat{L} + \left[\frac{W_S S}{Y}\left(1 - \frac{W_U}{W_S}\right) - \frac{T}{D}\left\{\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L - S)\right) - \frac{e_{M_2}SW_S}{Y}\left(1 - \frac{W_U}{W_S}\right)\right\}\right]\hat{S} \quad (2.2.5\text{-C}).$$

Equation (2.2.5-C) shows how total factor income is affected by exogenous changes in the capital stock, K, labour endowment, L, and skilled labour endowment, S. Here, the sign of T is always ambiguous.

Here equation (2.2.5-C) shows that

$\hat{K} > 0$ with $\hat{L} = \hat{S} = 0$ and with $\left[\frac{rK}{Y} \gtrless \frac{T}{D}\left\{\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right\}\right] \Rightarrow \hat{Y} \gtrless 0.$

Here, $\frac{T}{D}\left\{\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right\}$ represents the indirect effect of an increase in K on Y through change in P_2; and $\frac{rk}{Y}$ shows the corresponding direct effect. As T is ambiguous in sign, so is also $\frac{T}{D}\left\{\frac{\lambda_{K1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right\}$. If the direct effect of the increase in K on Y is stronger (weaker) than the indirect effect, then Y is increased (decreased).

Equation (2.2.5-C) also shows that

$\hat{L} < 0$, with $\hat{K} = \hat{S} = 0$ and $\left[\frac{W_U L}{Y} \gtrless -\frac{T}{D}\left\{\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right\}\right]$, $\Rightarrow \hat{Y} \lesseqgtr 0.$

Here, $\frac{T}{D}\left\{\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right\}$ represents the indirect effect of an increase in L on Y through change in P_2; and $\frac{W_U L}{Y}$ shows the corresponding direct effect. Here also the sign of $\frac{T}{D}\left\{\frac{a_{KU}L}{a_U K|\lambda|}\lambda_{S1} + e_{M_2}\frac{W_U L}{Y}\right\}$ is ambiguous. So, if the direct effect of a decrease in L is stronger (weaker) than the indirect effect, then Y falls (rises).

The same equation, with $\hat{L} = \hat{K} = 0$, shows that

$\hat{S} > 0$ with $\frac{W_S S}{Y}\left(1 - \frac{W_U}{W_S}\right) \gtrless \frac{T}{D}\left\{\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L - S)\right) - \frac{e_{M_2}SW_S}{Y}\left(1 - \frac{W_U}{W_S}\right)\right\} \Rightarrow \hat{Y} \gtrless 0.$

Here, $\frac{T}{D}\left\{\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L - S)\right) - \frac{e_{M_2}SW_S}{Y}\left(1 - \frac{W_U}{W_S}\right)\right\}$ represents the indirect effect of an increase in S on Y through change in P_2; and $\frac{W_S S}{Y}\left(1 - \frac{W_U}{W_S}\right)$ shows the corresponding direct effect. Here also the sign of $\frac{T}{D}\left\{\frac{1}{a_U X_U|\lambda|}\left(\lambda_{S1}\lambda_{KU}S - \lambda_{k1}(L - S)\right) - \frac{e_{M_2}SW_S}{Y}\left(1 - \frac{W_U}{W_S}\right)\right\}$ is ambiguous; and so a stronger (weaker) direct effect makes Y increase (decrease) with increase in S.

How strong is the possibility of the direct effect dominating the indirect effect? It depends on the mathematical sign of T, which, in turn, depends on the nature of the capital intensity ranking between the two skilled labour using sectors, i.e., on the mathematical sign of $|\theta|$.

2.2.3 Effects of changes in product prices:-

In this section, we want to analyze the effects of various trade and fiscal policies. Changes in fiscal instruments affect the system through changes in prices of traded goods. Any globalization programme, that lowers the tariff rate on imports, also lowers the effective producers' price of the import-competing product. We do not consider any change in factor endowments in this section. So $\hat{L} = \hat{S} = \hat{K} = 0$. Changes in product prices of traded goods affect the factor prices. When factors are perfectly mobile among sectors, these effects are known as Stolper-Samuelson effects. Equations (2.2.1-A) and (2.2.2-A) clearly show that there is a Stolper-Samuelson subsystem in this model and these effects on W_S and r can be easily derived. Then, using equations (2.2.1-A) and (2.2.2-A), we obtain

$$\hat{W}_S = \frac{\hat{P}_1\theta_{K2} - \hat{P}_2\theta_{K1}}{|\theta|} \qquad (2.2.9.1);$$

and

$$\hat{r} = \frac{\hat{P}_2\theta_{S1} - \hat{P}_1\theta_{S2}}{|\theta|} \qquad (2.2.10.1).$$

Using equations (2.2.3-A) and (2.2.10.1), we obtain

$$\hat{W}_U = \frac{\hat{P}_U}{\theta_U} - \frac{\theta_{KU}}{\theta_U|\theta|}\left(\theta_{S1}\hat{P}_2 - \theta_{S2}\hat{P}_1\right) \qquad (2.2.11.1).$$

Hence, using equations (2.2.9.1) and (12.2.1.1), we have

$$\hat{W}_S - \hat{W}_U = \frac{\hat{P}_1}{|\theta|}\left(\theta_{K2} - \frac{\theta_{KU}\theta_{S2}}{\theta_U}\right) - \frac{\hat{P}_2}{|\theta|}\left[\theta_{K1} - \frac{\theta_{KU}\theta_{S1}}{\theta_U}\right] - \frac{\hat{P}_U}{\theta_U} \qquad (2.2.12.1).$$

Now putting the expression of X_U from equation (2.2.7) in equation (2.2.8) and then differentiating equations (2.2.6) and (2.2.8) and solving the differentials by cramers rule, we obtain[11]

[11] Detailed derivation of equation (2.2.13.1) is given in Appendix (2.B).

$$\widehat{X}_2 = M_2\widehat{P}_1 + N_2\widehat{P}_2 - O_2\widehat{P}_U \quad (2.2.13.1);$$

where,

$$M_2 = \frac{1}{|\theta||\lambda|}\Big[\theta_{K2}\{\lambda_{K1}(\lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2) - \lambda_{S1}(\lambda_{K1}S_{KS}^1 + \lambda_{K2}S_{KS}^2)\}$$
$$+\theta_{S2}\{\lambda_{S1}(\lambda_{K1}S_{KK}^1 + \lambda_{K2}S_{KK}^2 + \lambda_{KU}S_{KK}^U - \lambda_{KU}S_{UK}^U) - \lambda_{K1}(\lambda_{S2}S_{SK}^1 + \lambda_{S2}S_{SK}^2)\}$$
$$-\lambda_{S1}\lambda_{KU}(S_{KU}^U - S_U^U)\frac{\theta_{KU}\theta_{S2}}{\theta_U}\Big] < 0,$$

$$N_2 = \frac{1}{|\theta||\lambda|}\Big[\theta_{K1}\{\lambda_{S1}(\lambda_{K1}S_{KS}^1 + \lambda_{K2}S_{KS}^2) - \lambda_{K1}(\lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2)\}$$
$$+\theta_{S1}\{\lambda_{K1}(\lambda_{S2}S_{SK}^1 + \lambda_{S2}S_{SK}^2) - \lambda_{S1}(\lambda_{K1}S_{KK}^1 + \lambda_{K2}S_{KK}^2 + \lambda_{KU}S_{KK}^U - \lambda_{KU}S_{UK}^U)\}$$
$$+\lambda_{S1}\lambda_{KU}(S_{KU}^U - S_U^U)\frac{\theta_{KU}\theta_{S1}}{\theta_U}\Big] > 0,$$

and,

$$O_2 = \frac{1}{|\lambda|\theta_U}\lambda_{S1}\lambda_{KU}(S_{KU}^U - S_U^U) > 0.$$

Existing mathematical Sign of O_2 holds provided $|\theta| > 0$ or equivalently $|\lambda| > 0$. If $|\theta| < 0$, then it's mathematical sign is reversed. But signs of M_2 and O_2 hold regardless of the mathematical sign of $|\theta|$ because $|\theta|$ and $|\lambda|$ are always of same sign.

We can easily show that $(M_2 + N_2 - O_2) = 0$[12]. It means that the general equilibrium supply function of the non-traded good is homogenous of degree zero in terms of absolute prices of all the commodities. Here, N_2 is the own price elasticity of its supply; and M_2 and O_2 are two cross price elasticities of its supply.

Then, differentiating equation (2.2.5) and assuming that $\widehat{L} = \widehat{S} = \widehat{K} = 0$, we obtain

$$\widehat{Y} = \frac{W_S S}{Y}\widehat{W}_S + \frac{rK}{Y}\hat{r} + \frac{W_U(L-S)}{Y}\widehat{W}_U \quad (2.2.5\text{-A-1}).$$

Putting the expressions of $\widehat{W}_s$, $\widehat{W}_U$ & $\hat{r}$ from equations (2.2.9.1), (2.2.10.1), and (2.2.11.1) in equation (2.2.5-A.1) we find that

$$\widehat{Y} = Q_2\widehat{P}_1 - R_2\widehat{P}_2 + T_2\widehat{P}_U \quad (2.2.5\text{-B.1});$$

where

$$Q_2 = \frac{1}{|\theta|}\Big[\frac{W_S S}{Y}\theta_{K2} - \frac{rk}{Y}\theta_{S2} + \frac{W_U(L-S)\theta_{KU}}{Y\theta_U}\theta_{S2}\Big],$$

$$R_2 = \frac{1}{|\theta|}\Big[\frac{W_S S}{Y}\theta_{K1} - \frac{rk}{Y}\theta_{S1} + \frac{W_U(L-S)\theta_{KU}}{Y\theta_U}\theta_{S1}\Big],$$

[12] It is proved in Appendix (2.C).

and

$T_2 = \frac{W_U(L-S)}{Y\theta_U}$.

We can easily show that $(Q_2 - R_2 + T_2) = 1$[13]. This means that national income (total sales revenue) is linearly homogenous in terms of all commodity prices. This must be true because Walras law is satisfied here and the supply function of every commodity is homogenous of degree zero in terms of all absolute prices. Here, Q_2, R_2 and T_2 represent elasticities of aggregate revenue function with respect to prices – P_1, P_2 and P_U. Q_2 and R_2 are ambiguous in sign and $T_2 > 0$.

Putting the expressions of $\widehat{Y}$ & $\widehat{X}_2$ from equations (2.2.5-B.1) and (2.2.13.1) in equation (4-A), we find that

$$\widehat{P}_2 = Z_2\widehat{P}_1 + V_2\widehat{P}_U \quad (2.2.14.1);$$

where

$Z_2 = \frac{-e_{M_2}Q_2 + M_2}{\overline{D}}$,

$V_2 = \frac{-e_{M_2}T_2 - O_2}{\overline{D}}$,

and

$$\overline{D} = e_{P_2} - e_{M_2}R_2 - N_2 \quad (2.2.15.1).$$

Here Z_2 and V_2 represent general equilibrium elasticities of excess demand for the non-traded good with respect to P_1 and P_U respectively.

Finally we use the stability condition in the market for commodity 2 to show that $\overline{D} < 0$; and this can be shown using equations (2.2.13.1), (2.2.5-B.1) and (2.2.15.1) for $\widehat{P}_1 = \widehat{P}_U = 0$.

Here, $Z_2 + V_2 = 1$ if $e_{P_2} + e_{M_2} = 0$. This can be easily shown by using expressions of Z_2, V_2, Q_2, T_2, M_2 and O_2[14]. Here, $e_{P_2} + e_{M_2} = 0$ because the demand function for commodity 2 is homogenous of degree zero in terms of its arguments; and such a demand function with zero cross price elasticity of demand can be derived from a Cobb-Douglas utility function.

Here $Q_2 > 0$ implies that the total sales revenue falls with the decrease in P_1. When $|\theta| > 0$ with $Q_2 > 0$, it can be shown using expressions of Q_2, T_2, M_2 and O_2 that $Z_2 > 0$ and

[13] It is shown in Appendix (2.C).
[14] It is shown in Appendix (2.C).

$V_2 > 0$. This, in turn, proves that both Z_2 and V_2 lie between zero and unity because $Z_2 + V_2 = 1$. However, when $|\theta| < 0$, then along with $Q_2 > 0$, we need to assume that $e_{M_2}T_2 > -O_2$ in order to prove that both Z_2 and V_2 lie between zero and unity. Here $e_{M_2}T_2 > -O_2$ implies that the marginal decrease in demand for the non-traded good exceeds the marginal decrease in its supply when P_U is reduced. If $0 \leq Z_2, V_2 \leq 1$, equation (2.2.14.1) implies that the rate of change in the price of the non-traded good is a weighted average of the rates of changes in prices of traded goods. Similar results are shown by Jones (1974) and Marjit (2003).

Also using equations (2.2.9.1), (2.2.11.1) and (2.2.14.1), we obtain

$$\widehat{W}_S = \frac{\widehat{P}_1(\theta_{K2} - \theta_{K1}Z_2)}{|\theta|} - \frac{\theta_{K1}V_2\widehat{P}_U}{|\theta|} \quad ;$$

and

$$\widehat{W}_U = -\frac{\theta_{KU}}{\theta_U|\theta|}\{\theta_{S1}Z_2 - \theta_{S2}\}\widehat{P}_1 + \left\{\frac{1}{\theta_U} - \frac{\theta_{KU}V_2}{\theta_U|\theta|}\right\}\widehat{P}_U \, .$$

Now using equations (2.2.14.1) and (2.2.12.1), and using the relations

$$|\theta| = \theta_{S1}\theta_{K2} - \theta_{S2}\theta_{K1} = \theta_{K2} - \theta_{KU},$$

and

$$\theta_{K1}\theta_U - \theta_{S1}\theta_{KU} = \theta_{K1} - \theta_{KU},$$

we can finally obtain the following.

$$\widehat{W}_S - \widehat{W}_U = \frac{\widehat{P}_1}{|\theta|\theta_U}[(\theta_{K2} - \theta_{KU}) - Z_2(\theta_{K1} - \theta_{KU})] - \frac{\widehat{P}_U}{\theta_U}\left[\frac{V_2}{|\theta|}(\theta_{K1} - \theta_{KU}) + 1\right] \qquad (2.2.16.1).$$

Equations (2.2.14.1) and (2.2.16.1) are keys to analyze the effects of various trade and fiscal policies on the skilled-unskilled wage inequality. Equation (2.2.14.1) shows how the equilibrium price of the non-traded good is affected due to exogenous changes in prices of traded goods. Equation (2.2.16.1) helps us to analyze the effects of parametric changes on the skilled-unskilled wage ratio. Here the Stolper-Samuelson effects of changes in prices of traded goods work through the indirect effects of the change in the price of the nontraded good.

2.2.3.1 Effects on Wage inequality:-

We consider the following alternative cases:

(i) $|\theta| > 0$and $\theta_{K2} > \theta_{KU} > \theta_{K1}$;

(ii) $|\theta| < 0$and $\theta_{KU} > \theta_{K1} > \theta_{K2}$;

(iii) $|\theta| > 0$ and $\theta_{K2} > \theta_{K1} > \theta_{KU}$;

and

(iv) $|\theta| < 0$ and $\theta_{K1} > \theta_{KU} > \theta_{K2}$.

If $0 \leq Z_2, V_2 \leq 1$, then, in each of the four alternative cases, equation (16) shows that

$\widehat{P}_1 < 0$ with $\widehat{P}_U = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$;

and

$\widehat{P}_U < 0$ with $\widehat{P}_1 = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$.

So, combining the results obtained in all these four cases we can establish the following proposition.

PROPOSITION-2.2.5: *If $0 \leq Z_2, V_2 \leq 1$, then, given other parameters, a decrease in the price of the product produced by skilled (unskilled) labour using traded good sector lowers (raises) the skilled unskilled wage ratio.*

If $0 \leq Z_2, V_2 \leq 1$, and if $Z_2 + V_2 = 1$, then the rate of change in the price of the non-traded good is an weighted average of the rates change in prices of two traded goods. So a decrease in the price of any of the two traded goods reduces the price of the non-traded good at a lower rate when price of the other traded good is given. The condition that $0 \leq Z_2, V_2 \leq 1$, is satisfied regardless of capital intensity ranking between the two skilled labour using sectors.

So, a decrease in P_1 lowers P_2 though the rate of change in P_2 is less than that in P_1. This leads to the contraction of each of the two skilled labour using sectors. So the demand for skilled labour and capital are decreased in these two sectors. However, the demand for unskilled labour rises because P_U remains same and mobile capital move towards sector U. So the skilled unskilled wage ratio is reduced.

On the other hand, the fall in P_U also lowers P_2 at a lower rate; and this leads to the contraction of both the sectors U and 2. So demand for unskilled labour is decreased. Demand for skilled labour is reduced in sector 2 at a lower rate. Given P_1, capital moves towards sector 1 and the demand for skilled labour is increased in this sector. So total demand for skilled labour is either increased or reduced at a lower rate than the rate of decrease in the demand for unskilled labour. So the skilled unskilled wage ratio is increased.

The globalization programme leads to a decrease in the effective producers' price of the import competiting domestic product through reduction in the tariff rate on imports. Thus globalization programme may lower (raise) the degree of wage inequality if the small open economy is a net importer of the product produced by skilled (unskilled) labour; and this result may be valid regardless of the capital intensity ranking among these sectors.

So globalization cannot explain simultaneous increase in wage inequality in all the countries because, with the opening of trade, the relative price of the product produced by skilled labour rises (falls) for the country who is net exporter (importer) of the product. However, when $0 \leq Z_2, V_2 \leq 1$ is not satisfied, the effects of a change in the terms of trade on the skilled-unskilled wage ratio are conditional on the capital intensity ranking among different sectors.

If $0 \leq Z_2, V_2 \leq 1$ is not satisfied, then we have following different cases summarized in table 1.

Table: 1:- Effects on skilled unskilled wage ratio.

<table>
<tr><td rowspan="6">P_1 falls given P_U</td><td rowspan="2">Capital intensity rankings</td><td rowspan="2">$Z_2 > 1; V_2 < 0$</td><td colspan="2">$Z_2 < 0; V_2 > 1$</td></tr>
<tr><td>$|Z_2| > 1$</td><td>$|Z_2| < 1$</td></tr>
<tr><td>$\theta_{K2} > \theta_{KU} > \theta_{K1}$</td><td>Decrease</td><td>Ambiguous</td><td>Decrease</td></tr>
<tr><td>$\theta_{KU} > \theta_{K1} > \theta_{K2}$</td><td>Increase</td><td>Decrease</td><td>Ambiguous</td></tr>
<tr><td>$\theta_{K2} > \theta_{K1} > \theta_{KU}$</td><td>Ambiguous</td><td>Decrease</td><td>Decrease</td></tr>
<tr><td>$\theta_{K1} > \theta_{KU} > \theta_{K2}$</td><td>Decrease</td><td>Increase</td><td>Ambiguous</td></tr>
<tr><td rowspan="6">P_U falls given P_1</td><td rowspan="2">Capital intensity rankings</td><td rowspan="2">$Z_2 < 0; V_2 > 1$</td><td colspan="2">$Z_2 > 1; V_2 < 0$</td></tr>
<tr><td>$|V_2| > 1$</td><td>$|V_2| < 1$</td></tr>
<tr><td>$\theta_{K2} > \theta_{KU} > \theta_{K1}$</td><td>Ambiguous</td><td>Increase</td><td>Increase</td></tr>
<tr><td>$\theta_{KU} > \theta_{K1} > \theta_{K2}$</td><td>Increase</td><td>Decrease</td><td>Ambiguous</td></tr>
<tr><td>$\theta_{K2} > \theta_{K1} > \theta_{KU}$</td><td>Increase</td><td>Ambiguous</td><td>Increase</td></tr>
<tr><td>$\theta_{K1} > \theta_{KU} > \theta_{K2}$</td><td>Decrease</td><td>Increase</td><td>Increase</td></tr>
</table>

Results summarized in table 1 clearly show that we can explain the simultaneous increase in the skilled-unskilled wage ratio in both the trading countries when the capital intensity ranking between the skilled labour using traded good sector and the skilled labour using non-traded good sector in those two countries are different and when the values of Z_2 and V_2 are also different for them.

2.2.3.2 **<u>Effects on total factor income</u>:-**

We now analyze the effects of changes in prices of traded goods on the total real factor income.

Using equations (2.2.14.1) and (2.2.5-B.1), we obtain

$$\widehat{Y} = (Q_2 - R_2 Z_2)\widehat{P}_1 + (T_2 - R_2 V_2)\widehat{P}_U \quad (2.2.5\text{-C.1}).$$

When $\widehat{P}_U = 0$, and $\widehat{P}_1 \neq 0$,

$$\widehat{Y} - \widehat{P}_1 = [R_2(1 - Z_2) - T_2]\widehat{P}_1 \quad (2.2.5\text{-D.1})$$

is a measure of the change in total real factor income.

Here Q_2 and R_2 are ambiguous in sign. Hence $(\widehat{Y} - \widehat{P}_1)$ is also so.

When $\widehat{P}_1 = 0$, and $\widehat{P}_U \neq 0$,

$$\widehat{Y} - \widehat{P}_U = [R_2(1 - V_2) - Q_2]\widehat{P}_U \quad (2.2.5\text{-E.1})$$

is also ambiguous in sign.

A decrease in the price of the traded good produced by skilled labour lowers the price of the non-traded good at a lower rate. When, $|\theta| > 0$, the rate of decrease in W_S is more than the rate of change in r. Accordingly, W_U can move in either direction. However, the final effect on the total real factor income is conditional on the initial income share of different factors and hence is ambiguous. Similar ambiguity remains even when $|\theta| < 0$ though the effects on factor prices are reversed.

2.3 The Model with endogenous supply of skilled labour: [15]

In this section, we extend the model of section 2.2 by introducing endogenous supply of skilled labour. The supply of skilled labour is produced within the model using capital and skilled labour. Here production of skilled labour means the transformation of unskilled labour into skilled labour; and, in this static model, this transformation takes place instantaneously[16]. There is an education sector in this model and it transforms the unskilled labour into a skilled one; and this transformation technology requires capital and skilled labour as inputs.

This section is organized as follows. Sub-section 2.3.1 describes the model and sub-section 2.3.2 analyzes effects of changes in factor endowments. In section 2.3.3, we analyze the effects of exogenous changes in price of traded goods.

2.3.1 Description:

We consider a small open economy with four sectors and three primary factors- labour, capital and land. Sectors 1 and 2 produce products using skilled labour and capital as inputs; and sector U uses unskilled labour and land as inputs. Here skilled labour is also produced within the system using skilled labour and capital as inputs; and this is done in sector S. Here production of skilled labour means the transformation of unskilled labour into skilled labour; and, in this static model, this transformation takes place instantaneously. Production function in each of these four sectors satisfies all standard neo-classical properties including CRS. Sectors 1 and U produce traded goods but sector 2 produces a non-traded good. All factor endowments are exogenously given. Capital is mobile among sectors 1, 2 and S; and skilled labour is also mobile among those three sectors. However, unskilled labour and land are specific to sector U. Factor prices in each of these four sectors are perfectly flexible; and this flexibility ensures full employment of all these factors. All markets are competitive. The representative firm

[15] Gupta and Dutta (2010b) is partly based on the materials presented in this section.

[16] Skill formation takes place over time; and a dynamic model is more appropriate to deal with this issue.

maximizes profit in each of these four sectors; and the representative consumer maximizes utility subject to the budget constraint.

We use following additional notations. Other notations have their usual meanings as found in section 2.2.

a_{Si} = Skilled labour output ratio in ith sector for i= 1, 2, S.

a_{KU} = Land output ratio in sector U.

a_U = Unskilled labour output ratio in sector U.

P_i = Effective producer's price of ith commodity for i=1, 2, S, U.

r_U = Rate of return on land.

S = Level of skilled labour produced in the skilled labour producing sector.

L = Exogenously given total labour endowment.

K_U = Exogenously given amount of land.

Following equations describe the model

$$P_1 = a_{S1}W_S + a_{K1}r \quad (2.3.1);$$

$$P_2 = a_{S2}W_S + a_{K2}r \quad (2.3.2);$$

$$P_S = W_S - W_U = a_{ss}W_S + a_{KS}r \quad (2.3.3)$$

$$P_U = a_U W_U + a_{KU}r_U \quad (2.3.4);$$

$$D_2(P_2, Y) = X_2 \quad (2.3.5);$$

$$Y = W_S S + rK + W_U(L - S) + r_U K_U \quad (2.3.6);$$

$$a_{S1}X_1 + a_{S2}X_2 + a_{SS}S = S \quad (2.3.7);$$

$$a_U X_U = L - S \quad (2.3.8);$$

$$a_{K1}X_1 + a_{K2}X_2 + a_{KS}S = K \quad (2.3.9);$$

and

$$a_{KU}X_U = K_U \quad (2.3.10)$$

Equations (2.3.1), (2.3.2), (2.3.3) and (2.3.4) represent profit maximizing competitive equilibrium conditions in sectors 1, 2, S and U respectively. $P_S = W_S - W_U$ implies that the competitive education sector charges a price of educational service equal to the individuals marginal benefit of acquiring education which, in turn, is equal to the skilled-unskilled wage gap. Equation (2.3.5) stands for the supply demand equality in the market for non-traded good.

Equation (2.3.6) represents total factor income; and equations (2.3.7), (2.3.8), (2.3.9) and (2.3.10) indicate full employment equilibrium in factor markets.

In fact, equation (2.3.3) provide the micro foundation of skill formation in this static set up. Using these two equations, we obtain

$W_S = a_{ss}W_S + a_{KS}r + W_U$.

Here W_U is the opportunity cost of being educated and $(a_{ss}W_S + a_{KS}r)$ is average cost of acquiring education. Hence, an unskilled worker sacrifices an amount of $(a_{SS}W_S + a_{KS}r + W_U)$ while acquiring education. W_S is the amount he earns after being educated. The representative worker prefers to be skilled (unskilled) if the skilled wage rate exceeds (falls short of) his combined cost of acquiring education; and these two are equal in equilibrium. If the supply of skilled labour is very low, then the marginal productivity of skilled labour in the skilled labour using sector is very high. So the skilled wage rate will also be very high in the competitive skilled labour market. On the other hand, supply of unskilled labour will be very high leading to a low unskilled wage rate. So there will remain an incentive to acquire skill in this case. This mechanism prevents the equilibrium supply of skilled labour from being zero.

In this model also, P_1 and P_U are internationally given and P_2 is endogenously determined. There are ten unknowns in the model: W_S, W_U, r, r_U, P_2 , X_1, X_2, $X_{U,}$ S and Y. The parameters of this system are: P_1, P_U, L, K_U and K. There are ten independent equations. The production structure does not possess the decomposition property; and so factor prices cannot be solved independent of factor endowments.

The description of working of this general equilibrium model is similar to that done in earlier section. Two input prices W_S and r_U are determined from equations (2.3.1) and (2.3.2) simultaneously as functions of P_2. From equation (2.3.3), we obtain W_U as a function of P_2 .Then, from equation (2.3.4), we solve for r_U as a function of P_2. As factor prices are determined, so all the factor output coefficients are also determined as functions of P_2.Now, from equation (2.3.10), we find X_U and from equation (2.3.8), we obtain S in terms of P_2. Then, from equations (2.3.7) and (2.3.9), we simultaneously determine X_1 and X_2 as functions of P_2 given L, K and K_U. Then, from equation (2.3.6), we obtain Y as function of P_2. Since Y and X_2 are determined as functions of P_2, so, from equation (2.3.5), we solve for P_2.

Differentiating equations (2.3.1), (2.3.2), (2.3.3), (2.3.4) and using profit maximizing conditions we obtain following equations.

$\theta_{S1}\widehat{W}_S + \theta_{K1}\hat{r} = \widehat{P}_1$ (2.3.1-A);

$\theta_{S2}\widehat{W}_S + \theta_{K2}\hat{r} = \widehat{P}_2$ (2.3.2-A);

$W_S\widehat{W}_S - W_U\widehat{W}_U = (W_S - W_U)\left[\theta_{SS}\widehat{W}_S + \theta_{KS}\hat{r}\right]$ (2.3.3-A);

and

$\theta_U\widehat{W}_S + \theta_{KU}\hat{r}_U = \widehat{P}_U$ (2.3.4-A).

Using equations (2.3.1), (2.3.2), (2.3.3), (2.3.4), (2.3.6), (2.3.7), (2.3.8), (2.3.9) and (2.3.10), it can be easily shown that

$Y = P_1X_1 + P_2X_2 + P_UX_U + P_SS$.

Here also the R.H.S. represents the aggregate sales revenue (national income at product prices in the absence of commodity taxes and subsidies).

2.3.2. Changes in factor endowments:-

Here also we do not consider any change in trade and fiscal policies. So $\widehat{P}_1 = \widehat{P}_U = 0$ in a small open economy. We consider the effects of the followings one by one: (i) an exogenous increase in capital stock resulting either from foreign capital inflow or from domestic capital accumulation, (ii) an exogenous reduction in labour endowment caused by international labour migration, and (iii) a land augmenting technical change in the form of irrigation development and multiple cropping in the unskilled labour using agricultural sector. Then, using equations (2.3.1-A) and (2.3.2-A), we obtain

$\widehat{W}_S = -\frac{\widehat{P}_2\theta_{K1}}{|\theta|}$ (2.3.11);

and

$\hat{r} = \frac{\widehat{P}_2\theta_{S1}}{|\theta|}$ (2.3.12).

Here, $|\theta| = \theta_{S1}\theta_{K2} - \theta_{S2}\theta_{K1}$.Mathematical sign of $|\theta|$ indicates the capital intensity ranking between the two skilled labour using sectors, 1 and 2.

Then, using equations (2.3.3-A), (2.3.11) and (2.3.12), we obtain

$$\widehat{W}_U = \frac{\widehat{P}_2}{|\theta|}\left[-\theta_{KS}\left(\frac{W_S}{W_U}-1\right)-\theta_{K1}\right] \qquad (2.3.13);$$

and, using equations (2.3.4-A) and (2.3.13), we obtain

$$\hat{r}_U = \frac{\widehat{P}_2}{|\theta|}\frac{\theta_U}{\theta_{KU}}\left[\theta_{KS}\left(\frac{W_S}{W_U}-1\right)+\theta_{K1}\right] \qquad (2.3.14).$$

Hence, using equations (2.3.11) and (2.3.13) we have

$$\widehat{W}_S - \widehat{W}_U = \frac{\widehat{P}_2\theta_{KS}}{|\theta|}\left[\frac{W_S}{W_U}-1\right] \qquad (2.3.15).$$

Equation (2.3.15) shows that the magnitude of the rate of change of the skilled-unskilled wage ratio depends on the magnitudes of the initial skilled-unskilled wage ratio and of the rate of change in the price of the non-traded good. The nature of their relationship is conditional on the capital-intensity ranking between sector 1 and sector 2. If $|\theta|$ is positive (negative), skilled-unskilled wage ratio varies directly (inversely) with the price of the non-traded good, P_2. Effect of any parametric change in factor endowments on $\left(\frac{W_S}{W_U}\right)$ operates through its effect on P_2.

Using equations (2.3.10) and (2.3.8), we obtain

$$S = L - \frac{a_U K_U}{a_{KU}} \qquad (2.3.16).$$

Then using equations (2.3.7), (2.3.9) and (2.3.16), we obtain[17]

$$\widehat{X}_2 = H_1\widehat{K} - I_1\widehat{L} + J_1\widehat{K}_U + \widehat{P}_2\varphi_1 \qquad (2.3.17);$$

where,

$$|\lambda| = \lambda_{S1}\lambda_{K2} - \lambda_{K1}\lambda_{S2}\,,$$

$$H_1 = \frac{\lambda_{S1}}{|\lambda|} > 0,$$

$$I_1 = \frac{L}{|\lambda|}\left\{\lambda_{K1}\frac{(1-\lambda_{SS})}{S} + \lambda_{S1}\frac{a_{KS}}{K}\right\} > 0,$$

$$J_1 = \frac{(L-S)}{|\lambda|}\left\{\lambda_{K1}\frac{(1-\lambda_{SS})}{S} + \lambda_{S1}\frac{a_{KS}}{K}\right\} > 0,$$

and

$$\varphi_1 = \frac{1}{|\lambda||\theta|}[\lambda_{S1}\{\left(\lambda_{S1}S_{KS}^1 + \lambda_{S2}S_{KS}^2 + \lambda_{SS}S_{KS}^S\right)\theta_{K1}$$
$$-\left(\lambda_{S1}S_{KK}^1 + \lambda_{S2}S_{KK}^2 + \lambda_{SS}S_{KK}^S\right)\theta_{S1}$$
$$-\frac{a_{KS}(L-S)}{S}\left(S_U^U - S_{KU}^U\right)\left(\theta_{KS}\left(\frac{W_S}{W_U}-1\right)+\theta_{K1}\right)\left(1-\frac{\theta_U}{\theta_{KU}}\right)\}$$

[17] Detailed derivation of equation (2.3.17) is given in Appendix (2.D).

$$-\lambda_{K1}\{(\lambda_{S1}S_{SS}^{1}+\lambda_{S2}S_{SS}^{2}+\lambda_{SS}S_{SS}^{S})\theta_{K1}-(\lambda_{S1}S_{SK}^{1}+\lambda_{S2}S_{SK}^{2}+\lambda_{SS}S_{SK}^{S})\theta_{S1}$$

$$-\frac{(1-\lambda_{SS})(L-S)}{S}\left(S_{U}^{U}-S_{KU}^{U}\right)\left\{\theta_{KS}\left(\frac{W_S}{W_U}-1\right)+\theta_{K1}\right\}\left(\frac{\theta_U}{\theta_{KU}}-1\right)\}\Big]>0$$

Existing mathematical signs of H_1, I_1 and J_1 are valid when $|\theta|>0$, or, equivalently, $|\lambda|>0$. If $|\theta|<0$, or, equivalently, $|\lambda|<0$, then their mathematical signs are reversed. However, the mathematical sign of φ_1 holds regardless of the mathematical sign of $|\theta|$ because $|\theta|$ and $|\lambda|$ are always of same sign.

Mathematical sign of $|\lambda|$ indicates the capital intensity ranking between the two skilled labour using sectors. As the factor endowments change, factor output coefficients also change through changes in factor prices; and this leads to a change in the general equilibrium supply of the non-traded good. Also the demand for the non-traded good is changed; and this finally leads to a change in P_2 which affects the supply again. So changes in factor endowments cause changes in the production of the non-traded good directly and also indirectly through change in the price of the non-traded good. Now, φ_1 captures the indirect effect of change in production of the non-traded good through change in the price of the non-traded good; and H_1, I_1 and J_1 capture direct effects of change in K, L and K_U respectively.

Now, differentiating equation (2.3.4), we obtain

$$e_{P_2}\widehat{P}_2+e_{M_2}\widehat{Y}=\widehat{X}_2 \qquad (2.3.5\text{-A}).$$

Here $e_{P_2}<0$ represents price elasticity of demand for the non-traded good and $e_{M_2}>0$ represents its income elasticity of demand. Then, differentiating equation (2.3.6), we obtain

$$\widehat{Y}=\frac{W_S S}{Y}\widehat{W}_S+\frac{W_S L}{Y}\widehat{L}+\frac{(L-S)W_U}{Y}\widehat{W}_U+\frac{(W_U-W_S)(L-S)}{Y}\widehat{K}_U+\frac{rK}{Y}(\hat{r}+\widehat{K})+\frac{r_U K_U}{Y}(\hat{r}_U+\widehat{K}_U)+$$
$$\frac{(W_U-W_S)(L-S)}{Y}\left[\left(S_U^U-S_{KU}^U\right)\widehat{W}_U+\left(S_{UK}^U-S_{KK}^U\right)\hat{r}_U\right] \qquad (2.3.6\text{-A}).$$

Putting the expressions of $\widehat{W}_s$, $\hat{r}$, $\widehat{W}_u$ & $\hat{r}_U$ from equations (2.3.11), (2.3.12), (2.3.13) and (2.3.14) in equation (2.3.6-A) we find that

$$\widehat{Y}=\frac{W_S L}{Y}\widehat{L}+\frac{rK}{Y}\widehat{K}+\frac{1}{Y}[r_U K_U+(W_U-W_S)(L-S)]\widehat{K}_U+N_1\widehat{P}_2 \qquad (2.3.6\text{-A-1});$$

where

$$N_1 = \frac{1}{|\theta|}\Big[-\frac{W_S S}{Y}\theta_{K1} + \frac{rk}{Y}\theta_{S1} - \frac{(W_U - W_S)(L-S)}{Y}\left(S_U^U - S_{KU}^U\right)\left\{\theta_{KS}\left(\frac{W_S}{W_U} - 1\right) + \theta_{K1}\right\}\left(1 + \frac{\theta_U}{\theta_{KU}}\right) - \frac{(L-S)W_U}{Y}\left\{\theta_{KS}\left(\frac{W_S}{W_U} - 1\right) + \theta_{K1}\right\} + \frac{r_U K_U}{Y}\frac{\theta_U}{\theta_{KU}}\left\{\theta_{KS}\left(\frac{W_S}{W_U} - 1\right) + \theta_{K1}\right\}\Big].$$

Changes in factor endowments cause change in the production of the non-traded good directly and indirectly through change in its price. N_1 captures the effect of its change in production on the total factor income through change in its price. The sign of N_1 is always ambiguous. An increase in P_2 leads to increase in factor prices and decrease in other factor prices depending on the factor intensity ranking among the sectors. So the net effect of a change in P_2 on total factor income is always ambiguous.

Now, putting the expressions of $\widehat{Y}$ & $\widehat{X}_2$ from equations (2.3.6-A-1) & (2.3.17) in equation (2.3.4-A), we find that

$$\widehat{P}_2 = \frac{1}{D^1}\left[\widehat{K}\left(H_1 - e_{M_2}\frac{rk}{Y}\right) - \widehat{L}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right) + \widehat{K}_U\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\}\right] \tag{2.3.18}$$

where

$$D^{(1)} = e_{P_2} + e_{M_2}N_1 - \varphi_1 \tag{2.3.19}$$

Finally, we use the stability condition in the market for commodity 2 which, with $D_2 = X_2$, is given by

$$\frac{\widehat{D}_2}{\widehat{P}_2} - \frac{\widehat{X}_2}{\widehat{P}_2} < 0\ .$$

This condition implies that $D^{(1)} < 0$; and this can be shown using equations (2.3.17), (2.3.6-A-1) and (2.3.19) for $\widehat{K} = \widehat{K}_U = \widehat{L} = 0$.

Equation (2.3.18) shows how the equilibrium price of the non-traded good is affected by exogenous changes in capital stock, K, total land endowment, $\widehat{K}_U$ and labour endowment, L.

Equations (2.3.15) and (2.3.18) are keys to analyze the comparative static effects on skilled-unskilled wage inequality with respect to changes in factor endowments. Any parametric change affects the price of the non-traded good; and this, in turn, affects the skilled-unskilled wage ratio. We assume that $\left(\frac{W_S}{W_U}\right) < 1$ in the initial equilibrium and comparative static effects are too small to reverse this inequality.

2.3.2.1 Effects on Wage inequality:-

Sub case-A:-

Here we consider that $|\lambda| > 0$, or, equivalently, $|\theta| > 0$. This implies that the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector.

We first consider $\widehat{K}>0$ with $\widehat{L} = \widehat{K}_U = 0$. In this case, the effect on $(\widehat{W}_S - \widehat{W}_U)$ depends on the mathematical sign of $\left(H_1 - e_{M_2}\frac{rk}{Y}\right)$. Here equations (2.3.15) and (2.3.18) show that

$\widehat{K} > 0$, with $\widehat{L} = \widehat{K}_U = 0$ and with $\left(H_1 - e_{M_2}\frac{rk}{Y}\right) \gtrless 0, \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \lesseqgtr 0$.

Here, H_1 represents the marginal supply response on the non-traded good sector with respect to a change in capital stock given its product price. $e_{M_2}\frac{rk}{Y}$ is the similar marginal effect on the demand for non-tradable; and this effect takes place through an increase in rental income. If the increase in capital stock takes place through foreign capital inflow and if the entire foreign capital income is repatriated, then its marginal demand effect is nil. However, this marginal demand effect is positive in the case of domestic investment.

Equations (2.3.15) and (2.3.18), with $\widehat{K} = \widehat{K}_U = 0$, show that

$\widehat{L} < 0 \Rightarrow \widehat{P}_2 < 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$.

Here the marginal supply effect on the non-traded good sector with respect to change in L is denoted by the term I_1 of equation (2.3.17); and the corresponding marginal demand effect is denoted by $e_{M_2}\frac{W_S L}{Y}$. Both these two effects are positive.

These two equations, with $\widehat{L} = \widehat{K} = 0$, show that

$\widehat{K}_U > 0$ with $\left\{J_1 - e_{M_2}\frac{r_U K_U}{Y} - e_{M_2}\frac{(W_U - W_S)(L-S)}{Y}\right\} \gtrless 0 \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \lesseqgtr 0$.

Here J_1 represents the marginal supply response on the non-traded goods sector due to change in the land endowment, given P_2. $e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)$ represents the corresponding marginal effect on its demand. Here $e_{M_2}\frac{r_U K_U}{Y}$ represents the direct positive marginal demand effect due to change in K_U. However, the change in K_U causes a change in S in

the opposite direction. $e_{M_2}\frac{(W_U-W_S)(L-S)}{Y}$ represents the indirect marginal demand effect due to change in K_U operating through change in S; and this effect is negative because $W_U < W_S$.
So we can establish the following proposition.

***PROPOSITION-2.3.1**: If the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector, then, given other parameters, (i) an increase in capital stock raises (lowers) the skilled unskilled wage ratio if the marginal demand effect of capital accumulation on the non-traded good sector exceeds (falls short of) its corresponding supply effect, (ii) a decrease in labour endowment lowers the skilled unskilled wage ratio; and, (iii) an improvement in land augmenting technology raises (lowers) the skilled unskilled wage ratio if its net positive marginal demand effect on the non-traded good sector exceeds (falls short of) its corresponding supply effect.*

An increase in capital endowment, given other factor endowments and the price of the non-traded good, raises the level of output of sector 2 which is more capital intensive than sector 1. If the increase in capital stock takes place through foreign capital inflow and if the entire foreign capital income is repatriated, then its demand effect is nil. However, if it is increased through domestic capital accumulation, then the rental income goes up and the demand for non-traded good rises at its given initial price. If the marginal effect on demand for non-tradables due to capital accumulation exceeds its corresponding marginal supply effect, then price of the non-traded good goes up; and this leads to an increase in the rental rate on capital, r, and a decrease in the skilled wage rate, W_S, following a Stolper-Samuelson effect. Then W_U should also decrease; and the decrease in W_U should be more than that in W_S to satisfy the competitive equilibrium condition. Hence the skilled-unskilled wage ratio is increased. The same logic explains why skilled-unskilled wage ratio falls when the marginal effect on demand is nil, or, is positive but weaker than the corresponding supply effect.

A decrease in labour endowment may take place from international labour migration. Given the stock of sector specific capital (land) used by the unskilled labour using sector, demand for unskilled labour remains unchanged at the given price of the non traded good. So the number of individuals available to acquire education is reduced in this case; and hence the supply of skilled labour is decreased in competitive equilibrium. This raises the level of output of sector 2 at the given price because sector 2 is more capital intensive than sector 1. This decrease in unskilled labour endowment also lowers the total factor income; and hence the demand for non-traded good falls given its price. So an excess supply of non-traded good is created at the initial equilibrium price; and hence the price of the non-traded good falls in the new equilibrium. So the skilled unskilled wage ratio is reduced in this case.

An improvement in land augmenting technology takes place in the form of land augmenting technical change. Irrigation development leading to multiple cropping is a land augmenting technical progress in agriculture which is an unskilled labour using sector. This raises the demand for unskilled labour and lowers the supply of skilled labour in the competitive equilibrium. Thus the level of output of sector 2 is increased. However, this increase in land endowment produces two opposite effects on total factor income. Rental income on land is increased; but the labour income is reduced due to a decrease in the supply of skilled labour. If the positive marginal direct effect on demand for the non-traded good operating through the increase in rental income exceeds the negative marginal indirect demand effect operating through the decrease in the consequent skilled labour supply, then the net demand for the non-traded good rises at the given price. Finally, if the net positive marginal effect on demand for the non-traded good exceeds (falls short of) its corresponding supply effect, then the price of the non-traded good increases (decreases), and accordingly the skilled-unskilled wage ratio rises (falls).

<u>Sub case- B</u>:-

Here we consider that, $|\lambda|< 0$ implying $|\theta|<0$. This implies that the skilled labour using non-traded goods sector is more labour intensive than the skilled labour using traded goods sector. Then equations (2.3.15) and (2.3.18) show that

$\widehat{K}>0$ with $\widehat{L} = \widehat{K}_U = 0 \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U<0$.

Here $|\lambda|< 0$; and hence $H_1 < 0$. This implies that the marginal supply effect of capital accumulation on the non-traded good sector is always negative. However, its marginal demand effect is non-negative; and hence the marginal effect on excess supply denoted by $\left(H_1 - e_{M_2}\frac{rk}{Y}\right)$ is always negative.

The effect of the change in labour endowment, L, depends on the mathematical sign of $\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)$ that represents its marginal effect on the excess demand for the non-tradable, given the product price, P_2. Here, I_1 represents the negative marginal effect on its supply because $|\lambda| < 0 \Rightarrow I_1 < 0$. $e_{M_2}\frac{W_S L}{Y}$ stands for the positive marginal effect on its demand. Here, $\widehat{L} < 0$, with $\widehat{K} = \widehat{K}_U = 0$ and with $-I_1 \lesseqgtr e_{M_2}\frac{W_U L}{Y}, \Rightarrow \widehat{P}_2 \lesseqgtr 0 \Rightarrow \widehat{W}_S - \widehat{W}_U \gtreqless 0$.

Here, $J_1 < 0$ because $|\lambda| < 0$. So the marginal effect on the supply of non-tradables due to change in land endowment is negative. Equations (2.3.15) and (2.3.18), with $\widehat{L} = \widehat{K} = 0$, show that

$\widehat{K}_U > 0$ with $e_{M_2}\left\{\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L - S)}{Y}\right\} > 0 \Rightarrow \widehat{P}_2 > 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$.

This leads to the following proposition.

PROPOSITION-2.3.2: *If the skilled labour using non-traded goods sector is more labour intensive than the skilled labour using traded goods sector, then, given other parameters, (i) an increase in capital stock lowers the skilled-unskilled wage ratio, (ii) a decrease in labour endowment raises (lowers) the skilled-unskilled wage ratio if its marginal demand effect on non-traded goods sector exceeds (falls short of) its corresponding supply effect, and (iii) an improvement in land augmenting technology raises the skilled unskilled wage ratio if its positive marginal direct demand effect on the non-traded good sector exceeds its negative marginal indirect demand effect.*

An increase in capital endowment lowers the level of output of the more labour intensive non-traded good sector. If the increase in capital stock takes place through foreign capital inflow, then its demand effect is nil. However, in the case of domestic capital accumulation, the demand for non-tradable is increased. So the excess demand created at initial price lowers its price in the new equilibrium; and hence the skilled unskilled wage ratio is reduced.

A decrease in labour endowment lowers the supply of skilled labour; and this lowers the level of output of the more labour intensive non-traded good sector at its initial price. The total factor income is also reduced causing the demand for non-traded good to fall at the given price. If the marginal effect on the demand for non-tradables exceeds (falls short of) its corresponding supply effect, then the price of the non-traded good falls (rises); and hence the skilled-unskilled wage ratio is increased (decreased).

An improvement in land augmenting technology raises demand for unskilled labour and hence lowers the supply of skilled labour in competitive equilibrium. This lowers the level of output of the non-traded good. However, this produces two opposite effects on total factor income raising landlords' rental income and lowering labourers' wage income; and hence the net effect on demand for the non-traded good is ambiguous. If the net marginal demand effect is positive, then an excess demand is created at the initial price; and hence the price of the non-traded good rises in the new equilibrium. As a result, the skilled unskilled wage ratio is reduced.

2.3.2.2 Effects on skill formation and total factor income:-

2.3.2.2.1 Supply of Skilled Labour:-

We now analyze the effects of exogenous changes in factor endowments on the supply of skilled labour. Differentiating equation (2.3.16) and then putting the expressions of $\widehat{W}_s, \hat{r}, \widehat{W}_u, \hat{r}_U$ & $\widehat{P}_2$ from equations (2.3.11), (2.3.12), (2.3.13), (2.3.14) and (2.3.18), we obtain

$$\hat{S} = -\frac{R_1}{D^{(1)}}\left(H_1 - e_{M_2}\frac{rk}{Y}\right)\widehat{K} + \left[\frac{L}{S} + \frac{R_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)\right]\widehat{L} - \left[\frac{L-S}{S} + \frac{R_1}{D^{(1)}}\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\}\right]\widehat{K}_U \qquad (2.3.20);$$

where,

$$R_1 = -\frac{1}{|\theta|}\frac{L-S}{S}\left(S_U^U - S_{KU}^U\right)\left\{\theta_{KS}\left(\frac{W_S}{W_U} - 1\right) + \theta_{K1}\right\}\left(1 + \frac{\theta_U}{\theta_{KU}}\right).$$

Here, $|\theta| \gtreqless 0 \Rightarrow R_1 \gtreqless 0$. An increase in one of the factor endowments, given others, causes a change in the price of the non-traded good. R_1 captures the effect of this change in the price of the non-traded good on the supply of skilled labour. Direct effect of a change in factor endowment is obtained without altering P_2; and the indirect effect works through the change in P_2.

Sub case- A:-

Here $|\lambda| > 0$ implying $|\theta| > 0$; and hence $R_1 > 0$.

Then equation (2.3.20) shows that

$\widehat{K} > 0$ with $\widehat{L} = \widehat{K}_U = 0$ and with $\left(H_1 - e_{M_2}\frac{rk}{Y}\right) \gtreqless 0 \Rightarrow \widehat{S} \gtreqless 0.$

Here, $-\frac{R_1}{D^{(1)}}\left(H_1 - e_{M_2}\frac{rk}{Y}\right)$ represents the indirect effect of an increase in K on S operating through change in P_2; and the corresponding direct effect is nil.

Equation (2.3.20), with $\widehat{K} = \widehat{K}_U = 0$, shows that

$\widehat{L}<0$ with $\left[\frac{L}{S} + \frac{R_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)\right] \gtreqless 0 \Rightarrow \widehat{S} \lesseqgtr 0.$

Here, $\frac{R_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)$ represents the indirect effect of a change in L on S operating through change in P_2; and $\frac{L}{S}$ shows the corresponding direct effect. Here the sign of $\frac{R_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)$ is negative. So, if the direct effect of a decrease in L is stronger than the corresponding indirect effect, then S is decreased.

Again, with $\widehat{L} = \widehat{K} = 0$, this equation shows that

$\widehat{K}_U > 0$ with $J_1 < e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)$ and $r_U K_U > (W_U - W_S)(L - S) \Rightarrow \widehat{S} < 0.$

Here, $\frac{R_1}{D^{(1)}}\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\}$ represents the indirect effect of an increase in K_U on S operating through change in P_2; and $\frac{L-S}{S}$ shows the corresponding direct effect.

So we can establish the following proposition.

__PROPOSITION-2.3.3__*: If the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector, then, given other parameters, (i) an increase in capital stock raises (lowers) the supply of skilled labour if the marginal demand effect of capital accumulation on the non-traded good sector falls short of (exceeds) its corresponding marginal supply effect, (ii) a decrease in labour endowment lowers (raises) the supply of skilled labour if its positive direct effect exceeds (falls short of) its corresponding indirect effect; and, (iii) A land augmenting technical progress lowers the supply of skilled labour if its net positive marginal demand effect on the non-traded good sector exceeds (falls short of) its corresponding marginal supply effect.*

This proposition can be intuitively explained as follows. An increase in capital endowment has no direct effect on the supply of skilled labour. However, it has an indirect effect taking place through change in price of the non-traded good. If the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector, then the price of the non-traded good is decreased (increased) in this case when the marginal demand effect of capital accumulation on the non-traded good sector falls short of (exceeds) its corresponding marginal supply effect. This fall (rise) in price lowers (raises) the supply of skilled labour.

A decrease in labour endowment, given other factor endowments, lowers the supply of skilled labour directly. However, it also causes a change in the price of the non-traded good in the opposite direction; and thus supply of skilled labour is increased. Now, it is obvious that the final effect will depend on the relative strength of the direct and indirect effects.

A land augmenting technical progress given other factor endowments, lowers the supply of skilled labour directly. However, it also makes the price of non-traded good move in the same direction if the net positive marginal effect on demand for non-tradables exceeds its corresponding supply effect; and thus the supply of skilled labour is reduced indirectly.

<u>Sub case- B</u>:-

Here $|\lambda| < 0$ implying $|\theta| < 0$; and hence $R_1 < 0$. Then equation (2.3.20) shows that

$\widehat{K} > 0$ with $\widehat{L} = \widehat{K}_U = 0 \Rightarrow \widehat{S} > 0$.

Equation (20), with $\widehat{K} = \widehat{K}_U = 0$, also shows that

$\widehat{L} < 0$ with $-I_1 < e_{M_2} \frac{W_S L}{Y} \Rightarrow \widehat{S} < 0$.

Again, with $\widehat{L} = \widehat{K} = 0$, this equation shows that

$\widehat{K}_U > 0$ with $\frac{L-S}{S} > \frac{R_1}{D^{(1)}}\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\} \Rightarrow \widehat{S} < 0.$

So we can establish the following proposition.

<u>PROPOSITION-2.3.4</u>: *If the skilled labour using non-traded goods sector is less capital intensive than the skilled labour using traded goods sector, then, given other parameters, (i) an increase in capital stock raises the supply of skilled labour, (ii) a decrease in labour endowment lowers the supply of skilled labour if its marginal demand effect on the non-traded goods sector exceeds its corresponding marginal supply effect; and, (iii) a land augmenting technical* *progress lowers the supply of skilled labour if its direct effect exceeds corresponding indirect effect through change in the price of the non-traded good.*

This proposition can be intuitively explained as follows. An increase in capital endowment has no direct effect on the supply of skilled labour. However, it has an indirect effect taking place through change in the price of the non-traded good. Now, an increase in capital endowment raises the price of the non-traded good which, in turn, raises the supply of skilled labour. A decrease in labour endowment lowers supply of the skilled labour directly. It also lowers the price of the non-traded good if its marginal demand effect on the non-traded goods sector exceeds its corresponding supply effect. This, in turn, lowers the supply of skilled labour. A land augmenting technical progress lowers the supply of skilled labour directly but its indirect effect through change in the price of the non-traded good is ambiguous. So, if the direct effect exceeds its corresponding indirect effect, then the supply of skilled labour is decreased due to a land augmenting technical progress.

2.3.2.2.2 Total Factor Income:-

Let us now consider the effects of changes in factor endowments on the total factor income which is the most important component of national income. There is no difference between national income and total factor income in this model in the absence of taxes and subsidies. Effects of changes in factor endowment on social welfare are qualitatively similar to those on national income.

Putting the expression of $\widehat{P}_2$ from equation (2.3.18) in equation (6-A-1), we obtain

$$\widehat{Y} = \left[\frac{W_S L}{Y} - \frac{N_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)\right]\widehat{L} + \left[\frac{rK}{Y} + \frac{N_1}{D^{(1)}}\left(H_1 - e_{M_2}\frac{rk}{Y}\right)\right]\widehat{K} +$$
$$\left[\frac{1}{Y}\{r_U K_U + (W_U - W_S)(L-S)\} + \frac{N_1}{D^{(1)}}\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\}\right]\widehat{K}_U \quad (2.3.6\text{-A-2});$$

Here, $\frac{N_1}{D^{(1)}}\left(H_1 - e_{M_2}\frac{rk}{Y}\right)$represents the indirect effect of an increase in K on Y through change in P_2; and $\frac{rK}{Y}$ represents the corresponding direct effect. $\frac{N_1}{D^{(1)}}\left(I_1 + e_{M_2}\frac{W_S L}{Y}\right)$ represents the indirect effect of an increase in L on Y through change in P_2; and $\frac{W_S L}{Y}$ shows the corresponding direct effect. $\frac{N_1}{D^{(1)}}\left\{J_1 - e_{M_2}\left(\frac{r_U K_U}{Y} + \frac{(W_U - W_S)(L-S)}{Y}\right)\right\}$ represents the indirect effect of an increase in K_U on Y through change in P_2; and $\frac{1}{Y}\{r_U K_U + (W_U - W_S)(L-S)\}$ shows the corresponding direct effect. If the direct effect of an increase in K or L exceeds the corresponding indirect effect, then total factor income is increased. However, this is not true for K_U where the direct effect is also ambiguous in sign.

2.3.3. <u>Changes in prices of traded goods</u>:-

We now want to analyze the effects of various trade and fiscal policies. Changes in fiscal instruments affect the system through changes in effective producers' prices of traded goods. Any globalization programme, that lowers the tariff rate on imports, also lowers the effective producers' price of the import-competing product. We do not consider any change in factor endowments i.e.; L, K_U and K in this section. So $\hat{L} = \hat{K}_U = \hat{K} = 0$. Then, using equations (2.3.1-A) and (2.3.2-A), we obtain

$$\hat{W}_S = \frac{\hat{P}_1\theta_{K2} - \hat{P}_2\theta_{K1}}{|\theta|} \qquad (2.3.21);$$

and

$$\hat{r} = \frac{\hat{P}_2\theta_{S1} - \hat{P}_1\theta_{S2}}{|\theta|} \qquad (2.3.22).$$

Here, $|\theta|$is defined as in section 2.3.2. Then, using equations (2.3.3-A), (2.3.21) & (2.3.22), we obtain

$$\hat{W}_U = \frac{\hat{P}_1}{|\theta|}[\theta_{KS}A_2 + \theta_{K2}] - \frac{\hat{P}_2}{|\theta|}[\theta_{KS}A_2 + \theta_{K1}] \qquad (2.3.23);$$

where,

$$A_2 = \left(\frac{W_S}{W_U} - 1\right) > 0 \text{ .}$$

Using equations (2.3.4-A) and (2.3.23), we obtain

$$\hat{r}_u = \frac{\hat{P}_U}{\theta_{KU}} - \frac{\theta_U}{\theta_{KU|\theta|}}\left[\hat{P}_1 B_2 - \hat{P}_2 C_2\right] \qquad (2.3.24);$$

where,

$$B_2 = (\theta_{KS}A_2 + \theta_{K2}) > 0$$

and

$C_2 = (\theta_{KS}A_2 + \theta_{K1}) > 0.$

Using equations (2.3.21) and (2.3.23), we have

$$\widehat{W}_S - \widehat{W}_U = \frac{\theta_{KS}A_2}{|\theta|}\left[\widehat{P}_2 - \widehat{P}_1\right] \qquad (2.3.25).$$

Differentiating equation (2.3.16) and then putting the expressions of $\widehat{W}_s$, $\hat{r}$, $\widehat{W}_u$ & $\hat{r}_U$ from equations (2.3.21), (2.3.22), (2.3.23) and (2.3.24), we obtain

$$\hat{S} = F_2\widehat{P}_1 - G_2\widehat{P}_2 - E_2\widehat{P}_U \qquad (2.3.26);$$

where,

$$E_2 = \frac{(L-S)}{S}\frac{(S_{KU}^U - S_U^U)}{\theta_{KU}} > 0,$$

$$F_2 = \frac{E_2B_2}{|\theta|} > 0,$$

and,

$$G_2 = \frac{E_2C_2}{|\theta|} > 0.$$

Here, E_2, F_2 and G_2 capture the effects on the relative change in S due to relative changes in P_1, P_2 and P_U respectively.

Using equations (2.3.7), (2.3.9) and (2.3.16) we obtain[18]

$$\widehat{X}_2 = \alpha_2\widehat{P}_1 + T_2\widehat{P}_2 + \beta_2\widehat{P}_U \qquad (2.3.27)\ ;$$

where,

$$\alpha_2 = \frac{1}{|\lambda||\theta|}\left[\lambda_{S1}\left\{-\lambda_{KS}E_2B_2 - \left(\lambda_{SS}S_{KS}^S + \lambda_{S1}S_{KS}^1 + \lambda_{S2}S_{KS}^2\right)\theta_{K2}\right.\right.$$
$$\left.+\left(\lambda_{SS}S_{KK}^S + \lambda_{S1}S_{KK}^1 + \lambda_{S2}S_{KK}^2\right)\theta_{S2}\right\}$$
$$-\lambda_{K1}\left\{(1-\lambda_{SS})E_2B_2 - \left(\lambda_{SS}S_{SS}^S + \lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2\right)\theta_{K2}\right.$$
$$\left.\left.\left(\lambda_{SS}S_{SK}^S + \lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2\right)\theta_{S2}\right\}\right] < 0,$$

$$T_2 = \frac{1}{|\lambda||\theta|}\left[\lambda_{S1}\left\{\lambda_{KS}E_2C_2 + \left(\lambda_{SS}S_{KS}^S + \lambda_{S1}S_{KS}^1 + \lambda_{S2}S_{KS}^2\right)\theta_{K1}\right.\right.$$
$$\left.-\left(\lambda_{SS}S_{KK}^S + \lambda_{S1}S_{KK}^1 + \lambda_{S2}S_{KK}^2\right)\theta_{S1}\right\}$$
$$-\lambda_{K1}\left\{-(1-\lambda_{SS})E_2C_2 + \left(\lambda_{SS}S_{SS}^S + \lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2\right)\theta_{K1}\right.$$
$$\left.\left.-\left(\lambda_{SS}S_{SK}^S + \lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2\right)\theta_{S1}\right\}\right],$$

[18] Detailed derivation of equation (2.3.27) is given in Appendix (2.E).

and,

$$\beta_2 = \frac{1}{|\lambda|}[\lambda_{S1}\lambda_{KS}E_2 + \lambda_{K1}(1-\lambda_{SS})E_2] > 0.$$

Sign of β_2 holds provided $|\theta| > 0$, or, equivalently, $|\lambda| > 0$. If $|\theta| < 0$, then it's sign is reversed. However, signs of α_2 and T_2 hold regardless of the sign of $|\theta|$.

We can easily show that $(\alpha_2 + T_2 + \beta_2) = 0$[19]. It means that the general equilibrium supply function of the non-traded good is homogenous of degree zero in terms of absolute prices of all the commodities. Here, T_2 is its own price elasticity of supply; and α_2 and β_2 are two cross price elasticities of supply.

Then, differentiating equation (2.3.6), we obtain

$$\widehat{Y} = \frac{W_S S}{Y}\widehat{W}_S + \frac{rK}{Y}\hat{r} + \frac{W_U(L-S)}{Y}\widehat{W}_U + \frac{r_U K_U}{Y}\hat{r}_U + \frac{W_S S}{Y}\widehat{S} \qquad (2.3.6\text{-B}).$$

Putting the expressions of $\widehat{W}_s$, $\hat{r}$, $\widehat{W}_u$ & $\hat{r}_U$ from equations (2.3.21), (2.3.22), (2.3.23) and (2.3.24) in equation (2.3.6-B) we find that

$$\widehat{Y} = V_2\widehat{P}_1 - W_2\widehat{P}_2 + \psi_2\widehat{P}_U \qquad (2.3.6\text{-B-1});$$

where,

$$V_2 = \frac{W_S S}{Y|\theta|}\theta_{K2} + \frac{W_S S}{Y}F_2 - \frac{rk}{Y|\theta|}\theta_{S2} + \frac{W_U(L-S)}{Y|\theta|}B_2 - \frac{r_U K_U}{Y}\frac{\theta_U}{\theta_{KU}|\theta|}B_2,$$

$$W_2 = \frac{W_S S}{Y|\theta|}\theta_{K1} + \frac{W_S S}{Y}G_2 - \frac{rk}{Y|\theta|}\theta_{S1} + \frac{W_U(L-S)}{Y|\theta|}C_2 - \frac{r_U K_U}{Y}\frac{\theta_U}{\theta_{KU}|\theta|}C_2\,,$$

and

$$\psi_2 = -\frac{W_S S}{Y}E_2 + \frac{r_U K_U}{Y\theta_{KU}}.$$

We can easily show that $(V_2 - W_2 + \psi_2) = 1$[20]. Here total factor income is equal to the aggregate sales revenue. This means that the aggregate revenue function is linearly homogenous in terms of all commodity prices. This must be true because walras law is satisfied here and supply function of every commodity is homogenous of degree zero in terms of all absolute product prices. Here, V_2, W_2 and ψ_2 represent elasticities of aggregate revenue with respect to prices – P_1, P_2 and P_U, respectively. Signs of V_2, W_2 and ψ_2 are ambiguous because, with change in the price of any commodity, some factor prices move in the same direction and

[19] It is proved in Appendix (2.F).
[20] It is shown in Appendix (2.F).

other factor prices move in the opposite direction. So the final effect on total factor income is conditional on the initial income share of different factors.

Now, putting the expressions of $\widehat{Y}\,\&\,\widehat{X}_2$ from equations (2.3.6-B-1) & (2.3.27) in equation (2.3.5-A), we find that

$$\widehat{P}_2 = Y_2\widehat{P}_1 + Z_2\widehat{P}_U \qquad (2.3.28);$$

where

$Y_2 = \frac{-e_{M_2}V_2 + \alpha_2}{D}$,

$Z_2 = \frac{-e_{M_2}\psi_2 + \beta_2}{D}$,

and

$$D^{(2)} = e_{P_2} - e_{M_2}W_2 - T_2 \qquad (2.3.29).$$

Finally we use the stability condition in the market for commodity 2 which, with $D_2 = X_2$, is given by

$$\frac{\widehat{D}_2}{\widehat{P}_2} - \frac{\widehat{X}_2}{\widehat{P}_2} < 0\ .$$

This condition implies that $D^{(2)} < 0$; and this can be shown using equations (2.3.27), (2.3.6-B-1) and (2.3.29) for $\widehat{P}_1 = \widehat{P}_U = 0$.

Now, $Y_2 + Z_2 = 1$ if $e_{P_2} + e_{M_2} = 0$; and this can be easily shown by using expressions of $Y_2,\ Z_2, V_2, \alpha_2, \psi_2$ and β_2[21]. Here $e_{P_2} + e_{M_2} = 0$ because the demand function for commodity 2 is homogenous of degree zero in terms of its arguments; and such a demand function with zero cross price elasticity can be derived from a Cobb-Douglas utility function.

Here V_2 and ψ_2 are assumed to be positive. Hence $Y_2 > 0$ because $\alpha_2 < 0$. $Z_2 > 0$ if $e_{M_2}\psi_2 > \beta_2$. If $|\theta| < 0$, then $\beta_2 < 0$ and hence $(e_{M_2}\psi_2 - \beta_2)$ must be positive. However, for $|\theta| > 0$, $(e_{M_2}\psi_2 - \beta_2) > 0$ appears to be a necessary condition to prove that $Z_2 > 0$. Then both Y_2 and Z_2 should lie between zero and unity in this case if $V_2 > 0$, $\psi_2 > 0$ and $e_{M_2}\psi_2 > \beta_2$. Then equation (2.3.28) implies that the rate of change in the price of the non-traded good is an weighted average of the rates of changes in prices of traded goods. Similar results are shown by Jones (1974) and Marjit (2003).

[21] It is shown in Appendix (2.F).

Also using equations (2.3.21), (2.3.22) and (2.3.28), we obtain

$\widehat{W}_S = \frac{\widehat{P}_1(\theta_{K2} - Y_2\theta_{K1})}{|\theta|} - \frac{\theta_{K1}Z_2\widehat{P}_U}{|\theta|}$;

and

$\widehat{W}_U = \frac{\widehat{P}_1}{|\theta|}[\theta_{KS}A_2(1 - Y_2) + (\theta_{K2} - Y_2\theta_{K1})] - \frac{Z_2\widehat{P}_U}{|\theta|}(\theta_{KS}A_2 + \theta_{K1})$.

Now, putting the expression of $\widehat{P}_2$ from equation (2.3.28) in equation (2.3.25), and using the relations mentioned above, we can finally obtain

$$\widehat{W}_S - \widehat{W}_U = \frac{\theta_{KS}A_2}{|\theta|}\left[(Y_2 - 1)\widehat{P}_1 + Z_2\widehat{P}_U\right] \qquad (2.3.30).$$

Equations (2.3.28) and (2.3.30) are keys to analyze the effects of various trade and fiscal policies on skilled-unskilled wage inequality. Equation (2.3.28) shows how the equilibrium price of the non-traded good is affected due to exogenous changes in the prices of traded goods. Equation (2.3.30) establishes the link between the change in the price of a traded good and the change in the skilled-unskilled wage ratio. $\left(\frac{W_S}{W_U}\right) < 1$ in the initial equilibrium and the comparative static effects do not reverse this inequality.

2.3.3.1 <u>Effects on Wage inequality</u>:-

<u>Sub case-A</u>:-

Here we consider $|\lambda| > 0$, implying $|\theta| > 0$. Equation (2.3.30) shows that

$\widehat{P}_1 < 0$ with $\widehat{P}_U = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$ when $0 < Y_2 < 1$;

and

$\widehat{P}_U < 0$ with $\widehat{P}_1 = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$ when $0 < Z_2 < 1$.

This leads to the following proposition.

<u>PROPOSITION-2.3.5</u>: *If the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector and if 0 < Z_2, Y_2 < 1, then, given other parameters, a decrease in the price of the traded good produced by skilled (unskilled) labour raises (lowers) the skilled unskilled wage ratio.*

If $0 < Z_2$, $Y_2 < 1$, and $Y_2 + Z_2 = 1$, then the price of the non-traded good, P_2, is an weighted average of prices of two traded goods. Hence the change in the price of any of these two traded goods leads to a change in the price of the non-traded good in the same direction but at lower rate.

A decrease in P_1 also lowers P_2. However, the rate of change in P_2 is less than that in P_1. The decrease in P_1 given P_2 leads to a decrease in the skilled wage rate, W_S and an increase in the rental rate on capital, r, following a Stolper-Samuelson effect because the non-traded goods sector is more capital intensive than the traded good sector. Then W_U should also fall to satisfy the competitive equilibrium condition in the education sector; and accordingly the skilled-unskilled wage ratio should rise. On the other hand, a decrease in P_2 with given P_1 leads to an increase in W_S and a decrease in r; and thus W_U rises. The increase in W_U is more than that in W_S and so the skilled-unskilled wage ratio is reduced. As the rate of change in P_2 is less than that in P_1, the skilled-unskilled wage ratio is finally increased.

The fall in P_U also lowers P_2 at a lower rate. The decrease in P_2 given P_1 leads to an increase in W_S and a decrease in r following the Stolper-Samuelson effect. Then W_U should also increase to satisfy the competitive equilibrium condition in the education sector; and the increase in W_U should be more than that in W_S. So the skilled-unskilled wage ratio is reduced.

The globalization programme leads to a decrease in the effective producers' price of the import competiting domestic product through reduction in the tariff rate on imports. Thus globalization programme lowers (raises) the degree of wage inequality if the small open economy is a net importer of the product produced by unskilled (skilled) labour and if the skilled labour using non-traded good sector is more capital intensive than the skilled labour using traded good sector.

Sub case- B:-

Here $|\lambda| < 0$, or equivalently, $|\theta| < 0$. Equation (2.3.30) shows that

$\widehat{P}_1 < 0$ with $\widehat{P}_U = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U < 0$ when $0 < Y_2 < 1$;

and

$\widehat{P}_U < 0$ with $\widehat{P}_1 = 0 \Rightarrow \widehat{W}_S - \widehat{W}_U > 0$ when $0 < Z_2 < 1$.

This leads to the following proposition.

PROPOSITION-2.3.6: *If the skilled labour using non-traded goods sector is more labour intensive than the skilled labour using traded goods sector and if $0 < Z_2, Y_2 < 1$, then, given other parameters, a decrease in the price of the traded good produced by skilled (unskilled) labour lowers (raises) the skilled unskilled wage ratio.*

A decrease in P_1 reduces P_2 at a lower rate. The decrease in P_1 with given P_2 leads to an increase in W_S and a decrease in r because the non-traded goods sector is less capital intensive now. Then W_U should also increase at a higher rate than W_S to satisfy the competitive equilibrium condition in the education sector. Accordingly the skilled-unskilled wage ratio should fall. However, the decrease in P_2 with given P_1 should produce an opposite effect on the skilled-unskilled wage ratio following the same logic. As the rate of change in P_2 is less than the rate of change in P_1 in this case, the skilled-unskilled wage ratio falls.

The fall in P_U also lowers P_2 at a lower rate. The decrease in P_2 with given P_1 makes W_S fall and r rise and the rate of decrease in W_U to be higher that that of W_S. Accordingly the skilled-unskilled wage ratio is increased.

Thus the globalization programme reducing the tariff rate on imports lowers (raises) the degree of wage inequality if the importable is produced by skilled (unskilled) labour when the skilled labour using non-traded good sector is more labour intensive than the skilled labour using traded good sector.

With the opening of trade between two countries, $\frac{P_1}{P_U}$ rises for one country and falls for the other. However, if we combine propositions 2.3.5 and 2.3.6, we can explain the simultaneous increase in wage inequality in both the countries when the capital intensity ranking between the skilled labour using traded good sector and the skilled labour using non-traded good sector in one country is exactly opposite to that in the other country.

2.3.3.2 Effects on skill formation and total factor income:-

2.3.3.2.1 Supply of Skilled Labour:-

Now we analyze the effect of the change in the price of a traded good on the supply of skilled labour.

Putting the expression of $\widehat{P}_2$ from equation (2.3.28) in equation (2.3.26), we obtain

$$\widehat{S} = \frac{E_2}{|\theta|}(B_2 - C_2Y_2)\widehat{P}_1 - E_2\left(1 + \frac{C_2Z_2}{|\theta|}\right)\widehat{P}_U \qquad (2.3.31).$$

Now, if $|\theta| > 0$, then $B_2 > C_2$. Also $0 < Y_2, Z_2 < 1$; and $E_2 > 0$. So equation (2.3.30) shows that $\widehat{P}_1 < 0$, with $\widehat{P}_U = 0$ and $|\theta| > 0, \Rightarrow \widehat{S} < 0$;

and

$\widehat{P}_U < 0$, with $\widehat{P}_1 = 0$ and $|\theta| > 0, \Rightarrow \widehat{S} > 0$.

However, if $|\theta| < 0$, then the effect of a change in P_U or P_1 on S is ambiguous. When $|\theta| < 0$, we have $B_2 < C_2$. However, we can not say anything about the sign of $(B_2 - C_2Y_2)$ because $0 < Y_2 < 1$ (according to our assumption). So the effect of a change in P_1 on S is indeterminate here. Also $\frac{C_2Z_2}{|\theta|} < 0$ but the sign of $\left(1 + \frac{C_2Z_2}{|\theta|}\right)$ is not known. So the effect of a change in P_U on S is also ambiguous.

Now we can establish the following proposition.

<u>PROPOSITION-2.3.7</u>: *If the skilled labour using non-traded goods sector is more capital intensive than the skilled labour using traded goods sector and if $0 < Y_2, Z_2 < 1$, then, given other parameters, a decrease in the price of the traded good produced by skilled (unskilled) labour lowers (raises) the supply of skilled labour. However, the effect is ambiguous in both these cases when the factor intensity ranking is reversed.*

Proposition-2.3.7 can be intuitively explained as follows. When the skilled labour using non-traded good sector is more capital intensive than the skilled labour using traded good sector, a decrease in P_1 also lowers the price of the non-traded good, P_2. However, the rate of change in P_2 is less than that in P_1. So the rate of decrease in W_S is more than that in r. However, W_U falls and r_U rises. The decrease in W_U raises the demand for unskilled labour. This lowers the supply of skilled labour.

On the other hand, a decrease in P_U also lowers P_2 at a lower rate. The decrease in P_2, given P_1, leads to an increase in W_S and a decrease in r following a Stolper-Samuelson effect. Then W_U is increased to satisfy the competitive equilibrium condition in the education sector.

Thus the real cost of unskilled labour, $\left(\frac{W_U}{P_U}\right)$, is increased at a higher rate. This lowers the demand for unskilled labour which, in turn, raises the supply of skilled labour.

However, when the factor intensity ranking is reversed, the decrease in the price of a traded good accompanied by a decrease in the price of the non-traded good at a lower rate, makes the decrease in r more than that in W_S, due to magnification effect of price change. However, the direction of change in W_U is indeterminate; and the same is true to the direction of change in the supply of skilled labour.

2.3.3.2.2 **Total Factor Income**:-

Now, we turn to analyze the effects of changes in P_1 or P_U on total real factor income.

Putting the expression of $\widehat{P}_2$ from equation (2.3.28) in equation (2.3.6-B-1), we obtain

$$\widehat{Y} = (V_2 - W_2Y_2)\widehat{P}_1 + (\psi_2 - W_2Z_2)\widehat{P}_U \qquad (2.3.6\text{-A-2}).$$

We have already shown, using equation (2.3.6-B-1), that $(V_2 - W_2 + \psi_2) = 1$.

When $\widehat{P}_U = 0$,

$$\widehat{Y} - \widehat{P}_1 = [W_2(1 - Y_2) - \psi_2]\widehat{P}_1 \qquad (2.3.5\text{-D.1})$$

is a measure of the change in total real factor income.

As V_2, W_2 and T_2 are ambiguous in sign, $(\widehat{Y} - \widehat{P}_1)$ is also so.

When $\widehat{P}_1 = 0$, then

$$\widehat{Y} - \widehat{P}_U = [W_2(1 - Z_2) - V_2]\widehat{P}_U \qquad (2.3.5\text{-E.1})$$

is a measure of the change in total real factor income.

Here also, $(\widehat{Y} - \widehat{P}_U)$ is ambiguous in sign because V_2, W_2 and T_2 are also so.

A decrease in P_1 also lowers the price of the non-traded good, P_2. However, the rate of change in P_2 is less than the rate of change in P_1. Now when, $|\theta| > 0$, with given P_U, we find that the decrease in W_S is more than that in r. However, W_U always falls and r_U rises; and initial endowments of different factors are also given. So the final effect on factor income is ambiguous depending on endowment levels of different factors. Thus, the effect on real factor income is also ambiguous. Similar ambiguity remains when $|\theta| < 0$. Ambiguity of the effects on factor income due to change in P_U can also be explained in the same manner.

2.4 The Model with unemployment:[22]

This section is also an extension of the model developed in section 2.2. Here we consider unemployment of both types of labour; i.e., skilled labour and unskilled labour; and introduce efficiency wage hypothesis to explain unemployment in each of the two labour markets. In reality, there is unemployment of labour; and, in theory, this unemployment can be explained by wage rigidity. For employers, the efficiency of the labourer is very crucial; and so labourers are paid according to their efficiency. Involuntary unemployment of both skilled labour and unskilled labour can be explained by efficiency wage hypothesis. Here, labour is measured in efficiency unit, where efficiency of labour depends on wage, unemployment and some other related factors; and one important determinant of efficiency is the relative wage compared to average wage in the society. A low relative wage lowers the efficiency.

The seminal work on efficiency wage hypothesis appeared in the late 1950s (Leibenstein (1957a, 1957b, 1958)); and since the mid-1970s we have witnessed a huge of interest in this area (Mirrlees (1975), Rodgers (1975), Stiglitz (1976), Bliss and Stern (1978), Agarwala (1979), Dasgupta and Ray (1986)). The period in between was not totally barren. Mazumdar (1959), Ezekiel (1960), Wonnacott (1962) and also parts of Myrdal (1968) discuss this problem. In the recent time efficiency wage literature includes works of Agell and Lundborg (1992, 1995), Feher (1991) and Akerlof and Yellen (1990), Gupta (2000), Gupta and Gupta (2001) etc. In their models, efficiency of labour depends also on rental rate of capital, unemployment benefit, stock of knowledge along with wage rate and unemployment. But as they consider only one type of labour, so they can’t explain skilled-unskilled wage inequality. Chaudhuri and Banerjee (2010) introduce unemployment of both types of labour, where unemployment of skilled labour is explained by efficiency wage hypothesis and that of unskilled labour by Harris-Todaro (1970) type of migration mechanism. Micro foundations of such efficiency functions are

[22] Gupta and Dutta (2011) is partly based on the materials presented in this section.

available in Copeland (1989), Shapiro and Stiglitz (1984), Pisauro (1991), Gupta and Gupta (2001) etc. Chaudhuri and Banerjee (2008) provides a micro foundation of nutritional efficiency function which we do not consider here.

In this section our goal is to explain unemployment of labourer; we therefore consider a simple efficiency function of labourer where efficiency of a labourer varies positively with its wage rate and the unemployment rate in the labour market. The same efficiency function has been considered for skilled labour as well as for unskilled labour only from the view point of simplicity. Gini-Coefficient of wage income distribution is also considered as a measure of wage income inequality in addition to the skilled-unskilled wage ratio.

We derive some interesting results from this model. First, when identical efficiency functions are introduced in two labour markets to explain unemployment, then two dissimilar countries may face similar movements in the skilled-unskilled average income ratio due to opposite type of changes in a factor endowment or in the price of a traded good only if either the sign of the effect on the excess demand function for the non-traded good or the capital intensity ranking between the skilled labour using traded good sector and the non-traded good sector in one country is opposite to that in the other country. However, when efficiency functions are not identical for these two types of labourers, then we may succeed to explain the simultaneous increase in the skilled-unskilled average income ratio in those two cases even if these countries have identical demand functions for non-traded goods and identical factor intensity rankings among different sectors. Secondly, different comparative static effects may force the skilled-unskilled relative wage and the Gini-Coefficient of wage income distribution to move in opposite directions in the presence of unemployment. However, in a full employment model, these two measures always move in same direction. So, our present work justifies why the skilled-unskilled relative wage may give us misleading ideas about the change in the degree of wage income inequality in the presence of unemployment even though existing full employment models rightly assume this relative wage as the appropriate measure of wage income inequality.

This section is organized as follows. Sub-section 2.4.1 describes the model; and sub-section 2.4.2 analyzes effects of changes in factor endowments on unemployment rate, skilled-unskilled relative wage, skilled-unskilled average income ratio and Gini-Coefficient of wage-income distribution. In section 2.4.3, we analyze similar effects of exogenous changes in prices of traded goods.

2.4.1 Description:

The description of this model is otherwise identical to that described in section 2.2 with the exception that each of these two types of labour is measured in efficiency unit. There exists unemployment in both these two labour markets; and these are explained by the efficiency wage hypothesis[23] which states that the efficiency of either type of labourer varies positively with its wage rate and unemployment rate[24].

We use following additional notations.

h = Efficiency of the skilled worker.

f = Efficiency of the unskilled worker.

$\frac{W_S}{h}$ = Wage rate per efficiency unit of skilled labour.

$\frac{W_U}{f}$ = Wage rate per efficiency unit of unskilled labour.

S = Exogenously given endowment of skilled workers.

N = Exogenously given endowment of unskilled workers.

v_S = Unemployment rate of skilled workers.

v_U = Unemployment rate of unskilled workers.

Following equations describe the model

$$P_1 = a_{S1}\left(\frac{W_S}{h}\right) + a_{K1}r \quad (2.4.1);$$

$$P_2 = a_{S2}\left(\frac{W_S}{h}\right) + a_{K2}r \quad (2.4.2);$$

$$P_3 = a_{N3}\left(\frac{W_U}{f}\right) + a_{K3}r \quad (2.4.3);$$

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[23] See works of Agell and Lundborg (1992, 1995), Gupta (2000), Gupta and Gupta (2001) and Chuadhuri and Banerjee (2010).

[24] Our efficiency function is a special case of the more general efficiency function considered in the fair wage hypothesis developed by Agell and Lundborg (1992, 1995) where rental rate on capital also appears as an argument. Chaudhuri and Banerjee (2010) use this more general efficiency function.

$h = h(W_S, v_S)$, with $h_1 > 0, h_2 > 0$ and $h_{11} < 0, h_{22} < 0$ (2.4.4);

$f = f(W_U, v_U)$, with $f_1 > 0, f_2 > 0$ and $f_{11} < 0, f_{22} < 0$ (2.4.5);

$$\frac{\partial h}{\partial W_S}\frac{W_S}{h} = 1 \quad (2.4.6);$$

$$\frac{\partial f}{\partial W_U}\frac{W_U}{f} = 1 \quad (2.4.7);$$

$$a_{K1}X_1 + a_{K2}X_2 + a_{K3}X_3 = K \quad (2.4.8);$$

$$a_{S1}X_1 + a_{S2}X_2 = S(1 - v_S)h \quad (2.4.9);$$

$$a_{N3}X_3 = N(1 - v_U)f \quad (2.4.10);$$

$$Y = W_S S(1 - v_S) + rK + W_U N(1 - v_U) \quad (2.4.11);$$

and

$$D_2(P_2, Y) = X_2 \quad (2.4.12).$$

Here equations (2.4.1), (2.4.2) and (2.4.3) represent profit maximizing conditions of competitive firms in sectors 1, 2 and 3. Equations (2.4.4) and (2.4.5) represent efficiency functions of skilled labour and unskilled labour, respectively. Each of these two efficiency functions is a positive and concave function in terms of every argument. Effective unit costs of employing skilled labour and unskilled labour are $\left(\frac{W_S}{h}\right)$ and $\left(\frac{W_U}{f}\right)$, respectively. $\left(\frac{W_S}{h}\right)$ is minimized with respect to W_S and the first-order condition of minimization is given by equation (2.4.6). Similarly $\left(\frac{W_U}{f}\right)$ is minimized with respect to W_U and the first-order minimization condition is given by equation (2.4.7). Equations (2.4.6) and (2.4.7) are basically two modified Solow (1979) conditions implying that wage elasticities of efficiency are equal to unity in these two labour markets. Equation (2.4.8) stands for equilibrium condition in the capital market. Equations (2.4.9) and (2.4.10) are unemployment adjusted equilibrium conditions in the skilled labour market and in the unskilled labour market, respectively. Equation (2.4.11) represents total factor income (national income at factor cost in the absence of taxes and subsidies) and equation (2.4.12) implies the supply-demand equality in the market of the non-traded good.

In this model, P_1 and P_3 are internationally given but P_2 is endogenously determined by the demand-supply mechanism. There are twelve unknowns in the model: W_S, W_U, r, h, f, v_S, v_U, P_2, X_1, X_2, X_3 and Y. Parameters of this system are: P_1, P_3, N, S and K. There are twelve independent equations with twelve unknowns; and so the system is determinate. The production structure does not possess the decomposition property; and so factor prices cannot be solved independent of factor endowments.

The working of the general equilibrium model is described as follows. W_S, W_U, r, h, f, v_S and v_U are determined simultaneously from equations (2.4.1) to (2.4.7) as functions of P_2. Now, from equation (2.4.10), we can obtain X_3; and then equations (2.4.8) and (2.4.9) simultaneously solve for X_1 and X_2 as functions of P_2 given N, S and K. Then, from equation (2.4.11), we can find Y as a function of P_2. Since Y and X_2 are determined as functions of P_2, we can solve for P_2 from equation (2.4.12).

Differentiating equations (2.4.1), (2.4.2), (2.4.3) and using profit maximizing conditions, we obtain following equations.

$$\theta_{S1}(\widehat{W}_S - \hat{h}) + \theta_{K1}\hat{r} = \widehat{P}_1 \qquad (2.4.1\text{-A});$$

$$\theta_{S2}(\widehat{W}_S - \hat{h}) + \theta_{K2}\hat{r} = \widehat{P}_2 \qquad (2.4.2\text{-A});$$

and

$$\theta_{N3}(\widehat{W}_U - \hat{f}) + \theta_{K3}\hat{r} = \widehat{P}_3 \qquad (2.4.3\text{-A}).$$

Differentiating equation (2.4.12), we obtain

$$e_{P_2}\widehat{P}_2 + e_{M_2}\widehat{Y} = \widehat{X}_2 \qquad (2.4.12\text{-A}).$$

Here $e_{P_2} < 0$ and $e_{M_2} > 0$; and these two represent price elasticity of demand and income elasticity of demand for the non-traded good, respectively.

Using equations (2.4.1), (2.4.2), (2.4.3), (2.4.8), (2.4.9), (2.4.10) and (2.4.11), it can be easily shown that

$$Y = P_1X_1 + P_2X_2 + P_3X_3 .$$

This is the aggregate sales revenue (national income at product prices in the absence of commodity taxes and subsidies).

2.4.2 <u>Changes in factor endowments</u>:-

Here also we assume that $\widehat{P}_1 = \widehat{P}_3 = 0$; and then analyze effects of changes in factor endowments.

The relative rate of change in the price of the non-traded good is derived as follows[25].

$$\widehat{P}_2 = \frac{1}{D}\left[\widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \widehat{N}\left\{\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right\} + \widehat{S}\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}\right] \quad (2.4.13);$$

where

$$D = e_{P_2} + e_{M_2}E - C \quad (2.4.14).$$

Here a change in a factor endowment affects the disposable income of the representative consumer; and thus affects the demand function for the non-traded good. Similarly, this change affects the supply function of the non-traded good through reallocation of factors among different sectors. Here $\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)$ represents the effect of a change in capital stock on the excess supply of the non-traded good. $-\left\{\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right\}$ and $\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}$ represent similar effects with respect to changes in unskilled labour endowment and skilled labour endowment respectively.

Here, $|\theta|$ and $|\lambda|$ are defined as follows:

$|\theta| = \theta_{S1}\theta_{K2} - \theta_{S2}\theta_{K1}$ and $|\lambda| = \lambda_{S1}\lambda_{K2} - \lambda_{K1}\lambda_{S2}$.

Mathematical signs of $|\theta|$ and $|\lambda|$ indicate the capital intensity ranking between the two skilled labour using sectors. So they are of same sign.

We use the stability condition in the market for the non-traded good to show that $D < 0$; and this stability condition, with $D_2 = X_2$, is given by

$$\frac{\widehat{D}_2}{\widehat{P}_2} - \frac{\widehat{X}_2}{\widehat{P}_2} < 0 \quad (2.4.14\text{-A}).$$

Equation (2.4.14-A) implies that $D < 0$.

Equation (2.4.13) shows how exogenous changes in capital stock, K, unskilled labour endowment, N, and skilled labour endowment, S, affect the equilibrium price of the non-traded good. The direction of change in this equilibrium price depends on the sign of the corresponding factor endowment effect on its excess demand function.

2.4.2.1 <u>Effects on unemployment rate</u>:-

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[25] Derivation of equations (2.4.13) and (2.4.14) are given in the Appendix (2.G). Mathematical expressions of C and E are also formally defined there.

Relative rates of change in unemployment rate of skilled workers and unskilled workers are given by following two equations[26].

$$\hat{v}_S = \frac{\hat{P}_2\theta_{K1}}{\varepsilon_{v_S}|\theta|} \quad (2.4.15);$$

and

$$\hat{v}_U = \frac{\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}}\frac{\theta_{S1}}{|\theta|}\hat{P}_2 \quad (2.4.16).$$

Here, ε_{v_U} (ε_{v_S}) is the elasticity of the efficiency function of the unskilled (skilled) labour with respect to unskilled (skilled) unemployment rate.

Using equations (2.4.13), (2.4.15) and (2.4.16), we obtain

$$\hat{v}_S = \frac{\theta_{K1}}{D\varepsilon_{v_S}|\theta|}\left[\hat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \hat{N}\left(\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right) + \hat{S}\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}\right] \quad (2.4.17);$$

and

$$\hat{v}_U = \frac{\theta_{K3}}{D\varepsilon_{v_U}\theta_{N3}}\frac{\theta_{S1}}{|\theta|}\left[\hat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \hat{N}\left(\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right) + \hat{S}\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}\right] \quad (2.4.18).$$

Here, equations (2.4.17) and (2.4.18) show how exogenous changes in capital stock, K, unskilled labour endowment, N, and, skilled labour endowment, S, affect unemployment rates of skilled workers and unskilled workers respectively. The sign of each of these endowment effects depends on two features: (i) the capital intensity ranking between the two skilled labour using sectors and (ii) the sign of the effect on the excess demand function for the non-traded good.

2.4.2.2 <u>Effects on skilled-unskilled relative wage</u>:-

Relative rates of change in wage rates of skilled workers and of unskilled workers are given by following two equations[27].

$$\hat{W}_S = \frac{\varepsilon_{v_S}h}{h_{11}(W_S)^2}\hat{v}_S \quad (2.4.19);$$

[26] Derivations of equations (2.4.15) and (2.4.16) are given in the Appendix (2.G).

[27] Derivations of equations (2.4.19) and (2.4.20) are given in the Appendix (2.G).

and

$$\widehat{W}_U = \frac{\varepsilon_{v_U} f}{f_{11}(W_U)^2}\hat{v}_U \qquad (2.4.20).$$

Using equations (2.4.19) and (2.4.20), we obtain

$$\hat{\Delta} = \widehat{W}_S - \widehat{W}_U = \frac{\varepsilon_{v_S} h}{h_{11}(W_S)^2}\hat{v}_S - \frac{\varepsilon_{v_U} f}{f_{11}(W_U)^2}\hat{v}_U \qquad (2.4.21).$$

where, $\Delta = \frac{W_S}{W_U}$.

Using equations (2.4.17), (2.4.18) and (2.4.21), we have

$$\hat{\Delta} = \frac{\left\{\left(-\frac{f\theta_{K3}\theta_{S1}}{f_{11}(W_U)^2}\right)-\left(-\frac{h\theta_{K1}\theta_{N3}}{h_{11}(W_S)^2}\right)\right\}}{D|\theta|\theta_{N3}}\left[\widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \widehat{N}\left(\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right) + \hat{S}\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}\right] \qquad (2.4.22).$$

Here equation (2.4.22) shows how exogenous changes in K, N, and, S affect the skilled-unskilled relative wage, Δ. The sign of the effect depends on three features: (i) capital intensity ranking between sectors 1 and 2; (ii) efficiency adjusted capital intensity ranking between sectors 1 and 3 who use two different types of labour with different efficiency functions; and (iii) the sign of the effect on the excess demand function for the non-traded good. $\left\{\left(-\frac{f\theta_{K3}}{f_{11}(W_U)^2\theta_{N3}}\right) - \left(-\frac{h\theta_{K1}}{h_{11}(W_S)^2\theta_{S1}}\right)\right\}$ represents the efficiency adjusted capital intensity ranking between sector 1 and sector 3.

Existing full employment models take skilled-unskilled relative wage as the only measure of income inequality of workers. However, in the presence of unemployment, relative wage is not an appropriate measure of wage income inequality.

2.4.2.3 Effects on skilled-unskilled average income ratio:-

The degree of skilled-unskilled wage income inequality may be measured by the skilled-unskilled average income ratio; and the ratio of average income of skilled workers to that of unskilled workers is defined as

$$R = \frac{W_S(1-v_S)}{W_U(1-v_U)}.$$

Here, $W_S(1-v_S)$ is the average income of skilled workers and $W_U(1-v_U)$ is the average income of unskilled workers. The average income differs from the wage rate due to the presence of unemployment. Chaudhuri (2004, 2008) and Beladi et. al. (2008), who consider Harris-Todaro (1970) type of unemployment in the unskilled labour market and full employment in the skilled labour market correctly measure the degree of income inequality by the skilled wage to unskilled average income ratio. However, Chaudhuri and Banerjee (2010), who introduce unemployment in both the labour markets, surprisingly use skilled wage to unskilled average income ratio as the measure of income inequality ignoring the distinction between the skilled wage rate and the average income of skilled workers.

The relative rate of change of the skilled-unskilled average income ratio is given as follows[28].

$$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\hat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\hat{v}_U =$$

$$\frac{\widehat{P}_2}{|\theta|\theta_{N3}}\left[\left\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right\}\theta_{K3}\theta_{S1} - \left\{\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right\}\theta_{K1}\theta_{N3}\right] \quad (2.4.23).$$

where, $\widehat{P}_2$ is given by equation (2.4.13).

Equation (2.4.23) shows that the magnitude of the relative rate of change of the skilled-unskilled average income ratio depends on the magnitude of the relative rate of change of the price of the non-traded good; and the direction of their relationship is conditional on the capital-intensity ranking among the three sectors and on the magnitude and the sign of the following two crucial terms:

$\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)$ and $\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$.

Combining equations (2.4.13) and (2.4.23) we can analyze the effects of parametric changes on skilled-unskilled average income ratio. Any parametric change affects the price of the non-traded good; and this, in turn, affects the skilled-unskilled average income ratio.

Here $\frac{v_U}{(1-v_U)\varepsilon_{v_U}}$ is the reciprocal of the elasticity of the efficiency function of the unskilled labour with respect to unskilled labour employment rate and this always takes a positive sign. However, $\frac{f}{f_{11}(W_U)^2}$ is the reciprocal of the elasticity of marginal efficiency of unskilled labour, i.e., $\frac{\partial f}{\partial W_U}$, with respect to unskilled wage rate and this always takes a negative sign. Similarly,

[28] Derivation of equation (2.4.23) is given in the Appendix (2.G).

$\frac{v_S}{(1-v_S)\varepsilon_{v_S}}$ is the reciprocal of the elasticity of the efficiency function of the skilled labour with respect to skilled labour employment rate which is always positive; and $\frac{h}{h_{11}(W_S)^2}$ is the reciprocal of the elasticity of marginal efficiency of skilled labour, i.e., $\frac{\partial h}{\partial W_S}$, with respect to skilled wage rate which is always negative. So both $\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)$ and $\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$ are always positive.

We consider a special case where

$$\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right) = \left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right) = \alpha > 0.$$

This special case arises when the efficiency functions of two types of labour are identical.

Then equation (2.4.23) is reduced to the following.

$$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\hat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\hat{v}_U = \frac{\widehat{P}_2}{|\theta|\theta_{N3}}[\theta_{K3}\theta_{S1} - \theta_{K1}\theta_{N3}]\alpha \qquad (2.4.23.R).$$

where, $\widehat{P}_2$ is given by equation (2.4.13).

Equation (2.4.23.R) shows that the sign of the relative rate of change of skilled-unskilled wage ratio, $\widehat{R}$, depends on the sign of the relative rate of change of the price of the non-traded good, $\widehat{P}_2$, and on the capital-intensity ranking among the three sectors.

Equation (2.4.13) has already shown that the sign of $\widehat{P}_2$ depends on how a change in a factor endowment affects the excess demand function for the non-traded good. So, in this special case, countries with identical factor intensity rankings among different sectors and with identical demand functions for non-traded goods must face similar (opposite) effects on skilled-unskilled average income ratio with respect to change in factor movements if they play similar (opposite) roles on international factor mobility. So, in this special case, we cannot explain the simultaneous increase in the degree of wage inequality of a factor receiving country and a factor exporting country when they have identical demand functions for non-traded good and identical factor intensity rankings among different sectors. Developed and less developed countries generally play opposite roles on international factor movements but empirical data show that both have experienced increase in wage inequality. However, these empirical findings can be explained in this special case of our model when either of these two conditions is satisfied: (i) the sign of the effect of a change in a factor endowment on the excess demand function for the non-traded good in a factor exporting country is opposite to that in a factor receiving country. (ii) the capital intensity ranking between skilled labour using traded good sector and the non-traded good sector in one country is opposite to that in the other country[29]. However, when efficiency functions are not identical for these two types of labourers, we analyse the effect of change in K or N or S on R using equation (2.4.23). In this case, $\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right) \neq \left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$; and these two crucial terms may take different values for different countries. So even if two countries are identical in terms of capital intensity ranking among different sectors and in terms of properties of the excess demand function for the non-traded good, they may have similar mathematical signs of $\widehat{R}$ in this case as a consequence of opposite mathematical signs of $\widehat{K}$ or $\widehat{N}$ or $\widehat{S}$ when $\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$ exceeds $\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)$ in one country but falls short of the latter in the other country. So, in this model, we may succeed to explain the simultaneous increase in wage inequality in dissimilar countries playing opposite roles on international factor mobility even when they have identical demand functions for non-traded goods and identical factor intensity rankings among different sectors; and the root of our success lies in the difference of efficiency functions in two labour markets[30].

2.4.2.4 Effects on Gini Coefficient:-

We consider the Gini Coefficient of wage income distribution as a measure of income inequality of working population; and this Gini Coefficient, denoted by G, is obtained as follows[31].

$$G = \frac{(Nv_U+Sv_S)\{N(1-v_U)+S(1-v_S)\Delta\}+NS(1-v_U)(1-v_S)(\Delta-1)}{(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}} \qquad (2.4.24);$$

[29] Gupta and Dutta (2010a) have already emphasized on these two conditions to explain wage inequality in dissimilar countries in a full employment model.

[30] Gupta and Dutta (2010a) can not show this because they do not introduce efficiency wage hypothesis and unemployment equilibrium.

[31] Derivation of equation (2.4.24) is given in the Appendix (2.H).

where

$\Delta = \frac{W_S}{W_U}$.

If there is no unemployment, i.e., if $v_U = v_S = 0$, then, from equation (2.4.24), we have

$G = \frac{NS(\Delta-1)}{(N+S-1)(N+S\Delta)}$;

and, from this expression, we have

$\frac{dG}{d\Delta} = \frac{NS(N+S)}{(N+S-1)(N+S\Delta)^2} > 0.$

So the Gini-Coefficient varies positively with the skilled-unskilled relative wage in a full employment model. This justifies why full employment models in the existing literature use relative wage as the measure of wage income inequality.

This expression of G as given by equation (2.4.24) also represents the degree of inequality in national income (wage income plus capital income) distribution of the entire economy when capital stock is equally distributed among all workers.

Using equation (2.4.24), we obtain[32]

$$\widehat{G} = A_3\hat{v}_U + B_3\hat{v}_S + C_3\widehat{\Delta} \qquad (2.4.25);$$

where,

$A_3 = \frac{v_U[NS^2(1-v_S)^2\Delta+2N^2S(1-v_U)(1-v_S)\Delta+N^3(1-v_U)^2]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0,$

$B_3 = \frac{v_S[N^2S(1-v_U)^2+2NS^2(1-v_U)(1-v_S)\Delta+S^3(1-v_S)^2\Delta^2]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0,$

$C_3 = \frac{\Delta[NS(1-v_U)(1-v_S)\{N(1-v_U)+S(1-v_S)\}]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0,$

and $\hat{v}_S$, $\hat{v}_U$ and $\widehat{\Delta}$ are given by equations (2.4.17), (2.4.18) and (2.4.22) respectively. Equation (2.4.25) implies that the relative rate of change in the degree of income inequality of the working class is explained not only by the relative rate of change in the skilled-unskilled wage ratio but also by relative rates of change in unemployment rates of skilled workers and unskilled workers. Here A_3, B_3 and C_3 represent elasticities of Gini-Coefficient of wage income distribution with respect to unemployment rate in the unskilled labour market, unemployment rate in the skilled labour market and skilled-unskilled relative wage respectively.

In a full employment model, $v_U = v_S = 0$; and these imply that $A_3 = B_3 = 0$. Also,

[32] Derivation of equation (2.4.25) is given in the Appendix (2.H).

$C_3 = \frac{\Delta NS(N+S)}{G(N+S-1)(N+S\Delta)^2}$.

So, in this case, change in inequality is explained by change in relative wage only.

We analyse the effect of a change in K only and ignore the effects of changes in N and S. These two effects are highly complicated because changes in N and S affect all frequencies of wage income distribution in addition to unemployment rates and relative wages. So putting $\widehat{N} = \widehat{S} = 0$ and using equations (2.4.17), (2.4.18), (2.4.22) and (2.4.25), we obtain[33]

$$\widehat{G} = \frac{\widehat{K}}{D|\theta|\theta_{N3}}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)\left[A_3\frac{\theta_{K3}\theta_{S1}}{\varepsilon_{v_U}} + B_3\frac{\theta_{K1}\theta_{N3}}{\varepsilon_{v_S}}\right] + C_3\widehat{\Delta} \qquad (2.4.26),$$

where $\widehat{\Delta}$ is given by equation (2.4.22) for $\widehat{N} = \widehat{S} = 0$.

So a change in the capital stock affects the degree of income inequality of the workers in two different ways: (i) through changes in unemployment rates in two labour markets and (ii) through a change in the skilled-unskilled relative wage. The combined effect operated through changes in unemployment rates of two types of workers is represented by the first term of the R.H.S. of equation (2.4.26); and its second term shows the effect operated through change in the skilled-unskilled relative wage. The effect of a change in K on the degree of wage income inequality, as measured by the value of G, is not necessarily unambiguous in sign. Here the sign of the effects on unemployment rates is independent of the capital intensity ranking between sector 1 and sector 3 but the sign of the relative wage effect is dependent on this capital intensity ranking. So the effect on G and Δ may move in opposite directions due to a change in K.

No earlier work except Chaudhuri (2004, 2008), Beladi el. al. (2008) and Chaudhuri and Banerjee (2010) analyses the problem of growing skilled-unskilled wage inequality in the presence of unemployment using a static competitive equilibrium framework. However, none of them uses Gini-Coefficient as the measure of income inequality. Our present work justifies why the skilled-unskilled relative wage as a measure of wage income inequality may give us misleading results in the presence of unemployment even though existing full employment models rightly use this relative wage as the only measure of inequality. Our analysis suggests

[33] Derivation of equation (2.4.26) is given in the Appendix (2.H).

that it should be replaced by the Gini-Coefficient of income distribution in the presence of unemployment.

2.4.3 Effects of changes in prices of traded goods:-

We now turn to analyze effects of various trade and fiscal policies which affect the system through exogenous changes in prices of traded goods. We assume that $\hat{N} = \hat{S} = \hat{K} = 0$.

Here, we find that[34]

$$\hat{P}_2 = \overline{\alpha}\hat{P}_1 + \overline{\beta}\hat{P}_3 \qquad (2.4.13.1);$$

where

$\overline{\alpha} = \frac{J_2 - e_{M_2}M_2}{\overline{D}}$,

$\overline{\beta} = \frac{L_2 - e_{M_2}O_2}{\overline{D}}$,

and

$$\overline{D} = e_{P_2} + e_{M_2}\delta_2 - \gamma_2 \qquad (2.4.14.1).$$

Here, $\overline{\alpha}$ represents the net effect of a change in P_1 on the excess demand for the non-traded good and $\overline{\beta}$ represents the similar effect of a change in P_3. Changes in prices of traded goods cause reallocation of resources among different sectors and this affects the supply function of the non-traded good. Similarly they affect factor prices and thus total disposable income of consumers which in turn affects the demand function for the non-traded good.

We use the stability condition in the market for commodity 2 to show that $\overline{D} < 0$.

2.4.3.1 Effects on unemployment rate:-

Relative rates of change in unemployment rates of skilled workers and unskilled workers are given by following two equations[35].

[34] Derivation of equations (2.4.13.1) and (2.4.14.1) are given in the Appendix (2.I). Mathematical notations like J_2, γ_2, L_2, M_2, δ_2 and O_2 are also formally defined in the Appendix (2.I).

[35] Derivations of equations (2.4.15.1) and (2.4.16.1) are given in the Appendix (2.I).

$$\hat{v}_S = \frac{\hat{P}_2\theta_{K1} - \hat{P}_1\theta_{K2}}{\varepsilon_{v_S}|\theta|} \qquad (2.4.15.1);$$

and

$$\hat{v}_U = \frac{\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}|\theta|}\left(\theta_{S1}\hat{P}_2 - \theta_{S2}\hat{P}_1\right) - \frac{\hat{P}_3}{\varepsilon_{v_U}\theta_{N3}} \qquad (2.4.16.1).$$

Using equations (2.4.13.1), (2.4.15.1) and (2.4.16.1), we obtain

$$\hat{v}_S = \frac{\hat{P}_1(\overline{\alpha}\theta_{K1} - \theta_{K2}) + \hat{P}_3\overline{\beta}\theta_{K1}}{\varepsilon_{v_S}|\theta|} \qquad (2.4.17.1);$$

and

$$\hat{v}_U = \frac{\hat{P}_1\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}|\theta|}\{\overline{\alpha}\theta_{S1} - \theta_{S2}\} + \frac{\hat{P}_3}{\varepsilon_{v_U}\theta_{N3}}\left\{\frac{\overline{\beta}\theta_{S1}\theta_{K3}}{|\theta|} - 1\right\} \qquad (2.4.18.1).$$

Here, equations (2.4.17.1) and (2.4.18.1) show how changes in prices of traded goods affect unemployment rates in the skilled labour market and in the unskilled labour market. So how changes in P_1 and P_3 affect v_S and v_U would depend on the following two features: (i) capital intensity ranking between sectors 1 and 2 and (ii) the sign and magnitude of the effects on the excess demand function for the non-traded good.

2.4.3.2 <u>Effects on skilled-unskilled relative wage</u>:-

Using equations (2.4.21), (2.4.17.1) and (2.4.18.1), we have

$$\hat{\Delta} = \frac{\hat{P}_1}{|\theta|}\left[\frac{h(\overline{\alpha}\theta_{K1} - \theta_{K2})}{h_{11}(W_S)^2} - \frac{f}{f_{11}(W_U)^2}\frac{\theta_{K3}}{\theta_{N3}}\{\overline{\alpha}\theta_{S1} - \theta_{S2}\}\right] + \hat{P}_3\left[\frac{\overline{\beta}\theta_{K1}}{|\theta|}\frac{h}{h_{11}(W_S)^2} - \frac{1}{\theta_{N3}}\frac{f}{f_{11}(W_U)^2}\left\{\frac{\overline{\beta}\theta_{S1}\theta_{K3}}{|\theta|} - 1\right\}\right] \qquad (2.4.19.1).$$

Here equation (2.4.19.1) shows how changes in prices of traded goods affect the skilled-unskilled relative wage; and the nature of this effect is determined by the followings: (i) the nature of the capital intensity ranking between sectors 1 and 2 (ii) the nature of the capital intensity ranking between sectors 1 and 3 and (iii) the sign and magnitudes of the effects on excess demand function for the non-traded good.

2.4.3.3 <u>Effects on skilled-unskilled average income ratio</u>:-

We obtain the relative rate of change in the skilled-unskilled average income ratio, R, as follows[36].

$$\widehat{\mathrm{R}} = \frac{\widehat{\mathrm{P}}_1}{|\theta|\theta_{N3}}\left[\left\{\theta_{K2}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)-\theta_{K3}\theta_{S2}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)\right\}-\overline{\alpha}\left\{\theta_{K1}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)-\theta_{K3}\theta_{S1}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)\right\}\right]-\frac{\widehat{\mathrm{P}}_3}{\theta_{N3}}\left[\frac{\overline{\beta}}{|\theta|}\left\{\theta_{K1}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)-\theta_{K3}\theta_{S1}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)\right\}+\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)\right] \quad (2.4.20.1).$$

Equation (2.4.20.1) is used to analyze effects of various trade and fiscal policies on the skilled-unskilled average income ratio because these trade and fiscal policies affect the system through changes in effective prices of traded goods.

Here also, if efficiency functions of two types of labourers are identical, i.e., if

$$\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)=\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)=\alpha>0,$$

then, equation (2.4.20.1) is reduced to the following:

$$\widehat{\mathrm{R}} = \frac{\alpha\widehat{\mathrm{P}}_1}{|\theta|\theta_{N3}}\left[\{\theta_{K2}\theta_{N3}-\theta_{K3}\theta_{S2}\}-\overline{\alpha}\{\theta_{K1}\theta_{N3}-\theta_{K3}\theta_{S1}\}\right]-\frac{\alpha\widehat{\mathrm{P}}_3}{\theta_{N3}}\left[\frac{\overline{\beta}}{|\theta|}\{\theta_{K1}\theta_{N3}-\theta_{K3}\theta_{S1}\}+1\right] \quad (2.4.20.1.R).$$

This equation (2.4.20.1.R) also shows that, when efficiency functions of two types of labourers are identical, the sign and magnitude of the rate of change of skilled-unskilled average income ratio, $\widehat{\mathrm{R}}$, depends on the sign and magnitude of rates of change of prices of traded goods, $\widehat{\mathrm{P}}_1$ and $\widehat{\mathrm{P}}_3$, and on the capital-intensity ranking of the three sectors, and on numerical values of $\overline{\alpha}$ and $\overline{\beta}$. Opening of trade produces opposite effects on P_1 and P_3 in the two countries between whom trade is opened. Hence, if these two countries have identical excess demand functions for the non-traded good and identical factor intensity rankings among different sectors, then we can not explain the trade induced increase in wage inequality in both the countries simultaneously when efficiency functions are same for both types of labour. Since α always takes a positive value, inter-country differences in values of α does not affect the

[36] Derivation of equation (2.4.20.1) is given in the Appendix (2.I).

mathematical sign of $\widehat{R}$. So, in this special case, the opening of trade can worsen the problem of wage income inequality in both these two countries simultaneously only if they differ either in respect of the sign of the effect on the excess demand function for the non-traded good or, in respect of the factor intensity ranking between the traded good sector and the non-traded good sector.

However, when efficiency functions are not identical for these two types of labourers, then we analyse effects of changes in P_1 and P_3 on R using equation (2.4.20.1). In this case, $\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right) \neq \left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$; and these two crucial terms may take different values for different countries. So, in this case, even if the two countries are identical in terms of all θ_{ij} coefficients and in terms of the values of $\overline{\alpha}$ and $\overline{\beta}$, they may have similar mathematical signs of $\widehat{R}$ as a consequence of opposite mathematical signs of $\widehat{P}_1$ and $\widehat{P}_3$ when $\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)$ exceeds $\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)$ in one country but falls short of the latter in the other country. So we may succeed to explain the simultaneous increase in wage inequality caused by the opening of trade in both the trading countries in this model even when they have identical demand functions for non-traded goods and identical factor intensity rankings among different sectors; and the necessary condition to attain this success is that efficiency functions must be different in these two labour markets.

2.4.3.4 <u>Effects on Gini Coefficient</u>:-

Using equations (2.4.25), (2.4.17.1) and (2.4.18.1), we obtain

$$\widehat{G} = \frac{\widehat{P}_1}{|\theta|}\left[A_3 \frac{\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}}\{\overline{\alpha}\theta_{S1} - \theta_{S2}\} + B_3 \frac{(\overline{\alpha}\theta_{K1} - \theta_{K2})}{\varepsilon_{v_S}}\right] + \widehat{P}_3\left[\frac{A_3}{\varepsilon_{v_U}\theta_{N3}}\left\{\frac{\overline{\beta}\theta_{S1}\theta_{K3}}{|\theta|} - 1\right\} + B_3 \frac{\overline{\beta}\theta_{K1}}{\varepsilon_{v_S}|\theta|}\right] + C_3\widehat{\Delta} \quad (2.4.21.1).$$

where, $\widehat{\Delta}$ is given by equation (2.4.19.1).

Equation (2.4.21.1) shows how changes in prices of traded goods affect the Gini Coefficient of the wage income distribution of workers. The sign of effect of changes in P_1 and

P_3 on the degree of wage income inequality, as measured by the value of G, depends on the following three features: (i) capital intensity ranking between sectors 1 and 2; (ii) capital intensity ranking between sectors 1 and 3; and (iii) the sign and magnitudes of effects on the excess demand function for the non-traded good. Here also changes in P_1 and P_3 affect G through changes in unemployment rates in two labour markets and through a change in the skilled-unskilled relative wage; and the combined effect on G may be opposite to the individual effect on the relative wage, Δ. This again justifies why the skilled-unskilled relative wage, often used as the measure of income inequality in models of full employment, should be replaced by the Gini- Coefficient of income distribution in the presence of unemployment.

2.5 LIMITATIONS

The model developed in the present chapter suffers from a set of limitations. It is a static model where skilled labour and capital do not accumulate over time. In the basic model developed in section 2.2, skilled labour and unskilled labour are assumed to be two different primary factors of production. So the skilled unskilled wage ratio is basically a relative price of two different primary factors of production. This is not the perfect way of modeling the observed empirical phenomenon because, in the existing empirical discussions, the skilled unskilled premium has been taken to be either graduate/non-graduate income difference or non-production sector/ production sector wage difference[37]. There are other limitations of this model. The problem of imperfections of markets, though exists in reality, is not considered here. We ignore cross price effects on the demand for the non-traded good. We do not analyze the role of sector specific capital. Role of backward institutions on unskilled labour using sectors is also ignored. We rule out the possibility of induced migration caused by interregional or rural-urban wage gap as analysed by Harris and Todaro (1970), Corden and Findlay (1975) etc. This is an important point because, in reality, there is inter regional variation in the wage rate of either type of labour. We rule out the possibility of unemployment of labour both in section 2.2 and section 2.3.

[37] All other theoretical models face same limitation.

In section 2.4 of this chapter, we introduce involuntary unemployment equilibrium in both the labour markets and explain unemployment using efficiency wage hypothesis. In that section, identical efficiency function has been considered for skilled labour as well as for unskilled labour only from the view point of technical simplicity. Few authors[38] think that nutritional efficiency functions are relevant for unskilled workers who often lie below the poverty line. According to the nutritional efficiency function, efficiency varies positively with level of consumption of food. If food is a normal good, then efficiency should vary positively with the wage rate and inversely with the unemployment rate; and an inverse relationship between unemployment rate and efficiency must affect the results of the present model to some extent. Major results of the model would also change marginally if we introduce Harris-Todaro type of unemployment in the unskilled labour market[39].

[38] See, for example, Chaudhuri and Banerjee (2008).
[39] See, for example, Chaudhuri and Banerjee (2008,2010).

Appendix (2.A):

Derivation of equation (2.2.13)

Totally differentiating equation (2.2.6), we have

$$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = \widehat{S} - \lambda_{S1}\hat{a}_{S1} - \lambda_{S2}\hat{a}_{S2} \qquad (2.2.A.1).$$

Each of the factor output coefficients is a function of prices of factors employed in that sector, for example, $a_{S1} = a_{S1}(W_S, r)$. Thus, the rate of change of that coefficient is expressed as

$\hat{a}_{S1} = S^1_{SS}\widehat{W}_S + S^1_{SK}\hat{r}$,

where, $S^1_{SS} = \left(\frac{W_S}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial W_S}\right)$ and $S^1_{SK} = \left(\frac{r}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial r}\right)$. Similarly we can derive $\hat{a}_{S2}$.

Using all these expressions of the rate of change of factor output coefficients and equation 2.2.A.1), we obtain

$$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = \widehat{S} - (\lambda_{S1}S^1_{SS} + \lambda_{S2}S^2_{SS})\widehat{W}_S - (\lambda_{S1}S^1_{SK} + \lambda_{S2}S^2_{SK})\hat{r} \qquad (2.2.A.2).$$

Now, putting the expressions of $\widehat{W}_S$ and $\hat{r}$ from equations (2.2.9) and (2.2.10) in equation (2.2.A.2), we obtain

$$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = \widehat{S} + \frac{\widehat{P}_2}{|\theta|}(\theta_{K1}A - \theta_{S1}B) \qquad (2.2.A.3).$$

Here,

$A = (\lambda_{S1}S^1_{SS} + \lambda_{S2}S^2_{SS}) < 0$;

and

$B = (\lambda_{S1}S^1_{SK} + \lambda_{S2}S^2_{SK}) > 0.$

Now, from equation (2.2.7), we obtain

$$X_U = \frac{(L-S)}{a_U} \qquad (2.2.A.4).$$

Putting this expression of X_U in equation (2.2.8) and then totally differentiating equation (2.2.8) and thereafter using the expressions of the rate of change of factor output coefficients and finally putting the expressions of $\widehat{W}_S$ and $\hat{r}$ from equations (2.2.9) and (2.2.10), we obtain

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} + \frac{\widehat{P}_2}{|\theta|}\left(\theta_{K1}F - \theta_{S1}G + \frac{\theta_{KU}\theta_{S1}}{\theta_U}H\right) - \frac{a_{KU}L}{a_U K}\widehat{L} + \frac{a_{KU}L}{a_U K}\widehat{S} \qquad (2.2.A.5).$$

Here,

$F = (\lambda_{K1}S^1_{KS} + \lambda_{K2}S^2_{KS}) > 0;$

$G = \left(\lambda_{K1}S_{KK}^{1} + \lambda_{K2}S_{KK}^{2} + \lambda_{KU}S_{KK}^{U} - \lambda_{KU}S_{UK}^{U}\right) < 0$;

and,

$H = \left(\lambda_{KU}S_{KU}^{U} - S_{U}^{U}\lambda_{KU}\right) > 0.$

Now, solving equations (2.2.A.3) and (2.2.A.5) simultaneously, we obtain

$\widehat{X}_2 = \frac{1}{|\lambda|}[\lambda_{S1}\widehat{K} - \frac{a_{KU}L}{a_U K}\lambda_{S1}\widehat{L} + \frac{\widehat{S}}{a_U X_U}\{\lambda_{S1}\lambda_{KU}S - \lambda_{K1}(L - S)\}] + \widehat{P}_2 M$ (2.2.A.6);

where,

$|\lambda| = \lambda_{S1}\lambda_{K2} - \lambda_{K1}\lambda_{S2}$

and

$M = \frac{1}{|\theta||\lambda|}[\lambda_{s1}\left(F\Theta_{K1} - G\Theta_{S1} + H\frac{\theta_{KU}\theta_{S1}}{\theta_U}\right) - \lambda_{K1}(\Theta_{K1}A - \Theta_{S1}B)] > 0.$

This equation (2.2.A.6) is same as equation (2.2.13) in the body of the chapter.

Appendix (2.B):

Derivation of equation (2.2.13.1)

Totally differentiating equation (2.2.6) and assuming that factor endowments do not change, we have

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = -\lambda_{S1}\widehat{a}_{S1} - \lambda_{S2}\widehat{a}_{S2}$ (2.2.B.1).

Now repeating the similar exercise as done in Appendix (2.A), we obtain

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = -G_2\widehat{W}_S - H_2\widehat{r}$ (2.2.B.2).

Here,

$G_2 = (\lambda_{S1}S_{SS}^{1} + \lambda_{S2}S_{SS}^{2}) < 0$;

and

$H_2 = (\lambda_{S2}S_{SK}^{1} + \lambda_{S2}S_{SK}^{2}) > 0.$

Now, putting expressions of $\widehat{W}_S$ and $\widehat{r}$ from equations (2.2.9.1) and (2.2.10.1) in equation (2.2.B.2), we obtain

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = I_2\widehat{P}_1 + J_2\widehat{P}_2$ (2.2.B.3);

where,

$I_2 = \frac{1}{|\theta|}[-G_2\theta_{K2} + H_2\theta_{S2}],$

and

$J_2 = \frac{1}{|\theta|}[G_2\theta_{K1} - H_2\theta_{S1}]$.

Then putting the expression of X_U from equation (2.2.A.4) in equation (2.2.8) and then totally differentiating equation (2.2.8), and thereafter using the expressions of the rate of change of factor output coefficients and finally putting the expressions of $\widehat{W}_S$ and $\hat{r}$ from equations (2.2.9) and (2.2.10), we obtain

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \varpi_2\widehat{P}_1 + E_2\widehat{P}_2 - F_2\widehat{P}_U \qquad (2.2.B.4).$$

Here,

$\varpi_2 = \frac{1}{|\theta|}\left[-A_2\theta_{K2} + B_2\theta_{S2} - \frac{C_2\theta_{Ku}}{\theta_U}\theta_{S2}\right]$;

$E_2 = \frac{1}{|\theta|}\left[A_2\theta_{K1} - B_2\theta_{S1} + \frac{C_2\theta_{Ku}}{\theta_U}\theta_{S1}\right]$;

$F_2 = \frac{C_2}{\theta_U} > 0$;

$A_2 = (\lambda_{K1}S^1_{KS} + \lambda_{K2}S^2_{KS}) > 0$;

$B_2 = \left(\lambda_{K1}S^1_{KK} + \lambda_{K2}S^2_{KK} + \lambda_{KU}S^U_{KK} - \lambda_{KU}S^U_{UK}\right) < 0$;

and

$C_2 = \lambda_{KU}\left(S^U_{KU} - S^U_{U}\right) > 0$.

Now, solving equations (2.2.B.3) and (2.2.B.4) simultaneously, we obtain

$$\widehat{X}_2 = M_2\widehat{P}_1 + N_2\widehat{P}_2 - O_2\widehat{P}_U \qquad (2.2.B.5);$$

where,

$M_2 = \frac{1}{|\lambda|}(\lambda_{S1}\varpi_2 - \lambda_{K1}I_2) < 0$,

$N_2 = \frac{1}{|\lambda|}(\lambda_{S1}E_2 - \lambda_{K1}J_2) > 0$,

and,

$O_2 = \frac{1}{|\lambda|}\lambda_{S1}F_2$.

This equation (2.2.B.5) is same as equation (2.2.13.1) in the body of the chapter.

Appendix (2.C):

Proof of $\mathbf{Z_2 + V_2 = 1}$

$Z_2 + V_2 = 1 \Rightarrow e_{P_2} + e_{M_2}(Q_2 - R_2 + T_2) + (-M_2 - N_2 + O_2) = 0$ (2.2.C.1).

Here,

$(Q_2 - R_2 + T_2) = \frac{1}{|\theta|}\left[\frac{W_S S}{Y}\Theta_{K2} - \frac{rK}{Y}\Theta_{S2} + \frac{W_U(L-S)\theta_{KU}}{Y\theta_U}\Theta_{S2}\right]$

$-\frac{1}{|\theta|}\left[\frac{W_S S}{Y}\Theta_{K1} - \frac{rk}{Y}\Theta_{S1} + \frac{W_U(L-S)\theta_{KU}}{Y\theta_U}\Theta_{S1}\right] + \frac{W_U(L-S)}{Y\theta_U}$

$\Rightarrow (Q_2 - R_2 + T_2) = \frac{(\theta_{K2}-\theta_{K1})}{|\theta|}\left[\frac{W_S S}{Y} + \frac{rK}{Y} - \frac{W_U(L-S)\theta_{KU}}{Y\theta_U}\right] + \frac{W_U(L-S)}{Y\theta_U}$

$\Rightarrow (Q_2 - R_2 + T_2) = \frac{1}{Y}\left[W_S S + rK + \frac{W_U(L-S)(1-\theta_{KU})}{\theta_U}\right]$

$\Rightarrow (Q_2 - R_2 + T_2) = 1$ (2.2.C.2).

and,

$(-M_2 - N_2 + O_2) = \frac{1}{|\lambda|}[\lambda_{S1}(F_2 - \varpi_2 - E_2) + \lambda_{K1}(I_2 + J_2)]$ (2.2.C.3)

Again,

$(F_2 - \varpi_2 - E_2) = \frac{C_2}{\theta_U} - \frac{(\theta_{K2}-\theta_{K1})}{|\theta|}\left[-A_2 - B_2 + C_2\frac{\theta_{KU}}{\theta_U}\right]$

$\Rightarrow (F_2 - \varpi_2 - E_2) = A_2 + B_2 + \frac{C_2}{\theta_U}(1 - \Theta_{KU})$

$\Rightarrow (F_2 - \varpi_2 - E_2) = A_2 + B_2 + C_2$

$\Rightarrow (F_2 - \varpi_2 - E_2) = (\lambda_{K1}S_{KS}^1 + \lambda_{K2}S_{KS}^2) + \left(\lambda_{K1}S_{KK}^1 + \lambda_{K2}S_{KK}^2 + \lambda_{KU}S_{KK}^U - \lambda_{KU}S_{UK}^U\right) +$

$\lambda_{KU}\left(S_{KU}^U - S_U^U\right)$

$\Rightarrow (F_2 - \varpi_2 - E_2) = \lambda_{K1}(S_{KS}^1 + S_{KK}^1) + \lambda_{K2}(S_{KS}^2 + S_{KK}^2) + \lambda_{KU}\left(S_{KK}^U + S_{KU}^U\right) - \lambda_{KU}\left(S_{UK}^U + S_{UU}^U\right)$

$\Rightarrow (F_2 - \varpi_2 - E_2) = 0$ (2.2.C.4).

$\left[\because S_{KS}^1 + S_{KK}^1 = S_{KS}^2 + S_{KK}^2 = S_{KK}^U + S_{KU}^U = S_{UK}^U + S_{UU}^U = 0\right]$.

Similarly, $(I_2 + J_2) = 0$ (2.2.C.5).

Now, using equations (2.2.C.3), (2.2.C.4) and (2.2.C.5), we obtain

$\therefore (-M_2 - N_2 + O_2) = 0$ (2.2.C.6).

Finally, using equations (2.2.C.1), (2.2.C.2) and (2.2.C.6), we obtain

$e_{P_2} + e_{M_2} = 0$

Hence, we have

$Z_2 + V_2 = 1.$

Appendix (2.D):

Derivation of equation (2.3.17)

From equations (2.3.8) and (2.3.10), we obtain

$$S = L - \frac{a_U K_U}{a_{KU}} \qquad (2.3.A.1).$$

Putting this expression of S in equation (2.3.7) and then totally differentiating equation (2.3.7), we have

$$\lambda_{S1}\hat{X}_1 + \lambda_{S2}\hat{X}_2 = -\lambda_{SS}\hat{a}_{SS} - \lambda_{S1}\hat{a}_{S1} - \lambda_{S2}\hat{a}_{S2} + \frac{(1-\lambda_{SS})}{S}\left[L\hat{L} - (L-S)\{\hat{K}_U + \hat{a}_U - \hat{a}_{KU}\}\right]$$

(2.3.A.2).

Each of the factor output coefficients is a function of prices of factors employed in that sector, for example, $a_{S1} = a_{S1}(W_S, r)$. Thus, the rate of change of that coefficient is expressed as

$\hat{a}_{S1} = S_{SS}^1\hat{W}_S + S_{SK}^1\hat{r}$;

where, $S_{SS}^1 = \left(\frac{W_S}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial W_S}\right)$ and $S_{SK}^1 = \left(\frac{r}{a_{S1}}\right)\left(\frac{\partial a_{S1}}{\partial r}\right)$. Similarly we can derive $\hat{a}_{S2}$, $\hat{a}_U$ and $\hat{a}_{KU}$.

Using all these expressions of rates of change of factor output coefficients and equation (2.3.A.2), we obtain

$$\lambda_{S1}\hat{X}_1 + \lambda_{S2}\hat{X}_2 = \frac{(1-\lambda_{SS})L}{S}\hat{L} - \frac{(1-\lambda_{SS})(L-S)}{S}\hat{K}_U - \left(\lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2 + \lambda_{SS}S_{SS}^S\right)\hat{W}_S$$
$$-\left(\lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2 + \lambda_{SS}S_{SK}^S\right)\hat{r} - \frac{(1-\lambda_{SS})(L-S)}{S}\left(S_U^U - S_{KU}^U\right)\hat{W}_U - \frac{(1-\lambda_{SS})(L-S)}{S}\left(S_{UK}^U - S_{KK}^U\right)\hat{r}_U$$

(2.3.A.3).

Putting the expressions of $\hat{W}_S$, $\hat{r}$, $\hat{W}_U$ and $\hat{r}_U$ from equations (2.3.11), (2.3.12), (2.3.13) and (2.3.14) in equation (2.3.A.3), we obtain

$$\lambda_{S1}\hat{X}_1 + \lambda_{S2}\hat{X}_2 = A_1\hat{L} - B_1\hat{K}_U + C_1\hat{P}_2 \qquad (2.3.A.4).$$

Here,

$A_1 = \frac{(1-\lambda_{SS})L}{S} > 0$,

$B_1 = \frac{(1-\lambda_{SS})(L-S)}{S} > 0$,

and,

$$C_1 = \frac{1}{|\theta|}\Big[\big(\lambda_{S1}S_{SS}^{1} + \lambda_{S2}S_{SS}^{2} + \lambda_{SS}S_{SS}^{S}\big)\theta_{K1} - \big(\lambda_{S1}S_{SK}^{1} + \lambda_{S2}S_{SK}^{2} + \lambda_{SS}S_{SK}^{S}\big)\theta_{S1}$$

$$-\frac{(1-\lambda_{SS})(L-S)}{S}\big(S_U^U - S_{KU}^U\big)\Big\{\theta_{KS}\Big(\frac{W_S}{W_U} - 1\Big) + \theta_{K1}\Big\}\Big(\frac{\theta_U}{\theta_{KU}} - 1\Big)\Big] < 0.$$

Putting the expression of S from equation (2.3.A.1) in equation (2.3.9) and then totally differentiating equation (2.3.9) and thereafter using the expressions of the rates of change of factor output coefficients and finally putting the expressions of $\widehat{W}_S$, $\hat{r}, \widehat{W}_U$ and $\hat{r}_U$ from equations (2.3.11), (2.3.12), (2.3.13) and (2.3.14), in the equation derived from equation (9), we obtain

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} - E_1\widehat{L} + F_1\widehat{K}_U + G_1\widehat{P}_2 \qquad (2.3.A.5).$$

Here,

$$E_1 = \frac{a_{KS}L}{K} > 0,$$

$$F_1 = \frac{a_{KS}(L-S)}{K} > 0,$$

and,

$$G_1 = \frac{1}{|\theta|}\Big[\big(\lambda_{S1}S_{KS}^{1} + \lambda_{S2}S_{KS}^{2} + \lambda_{SS}S_{KS}^{S}\big)\theta_{K1} - \big(\lambda_{S1}S_{KK}^{1} + \lambda_{S2}S_{KK}^{2} + \lambda_{SS}S_{KK}^{S}\big)\theta_{S1}$$

$$-\frac{a_{KS}(L-S)}{S}\big(S_U^U - S_{KU}^U\big)\Big\{\theta_{KS}\Big(\frac{W_S}{W_U} - 1\Big) + \theta_{K1}\Big\}\Big(1 - \frac{\theta_U}{\theta_{KU}}\Big)\Big] > 0.$$

Now, solving equations (2.3.A.4) and (2.3.A.5) simultaneously, we obtain

$$\widehat{X}_2 = H_1\widehat{K} - I_1\widehat{L} + J_1\widehat{K}_U + \widehat{P}_2\varphi_1 \qquad (2.3.A.6);$$

where,

$$|\lambda| = \lambda_{S1}\lambda_{K2} - \lambda_{K1}\lambda_{S2}\ ,$$

$$H_1 = \frac{\lambda_{S1}}{|\lambda|} > 0,$$

$$I_1 = \frac{1}{|\lambda|}(\lambda_{K1}A_1 + \lambda_{S1}E_1) > 0,$$

$$J_1 = \frac{1}{|\lambda|}(\lambda_{K1}B_1 + \lambda_{S1}F_1) > 0,$$

and,

$$\varphi_1 = \frac{1}{|\lambda|}(\lambda_{S1}G_1 - \lambda_{K1}C_1) > 0,$$

This equation (2.3.A.6) is same as equation (2.3.17) in the body of the chapter.

Appendix (2.E):

Derivation of equation (2.3.27)

Totally differentiating equation (2.3.7) and assuming that factor endowments do not change, we have

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = (1-\lambda_{SS})\widehat{S} - \lambda_{S1}\hat{a}_{S1} - \lambda_{S2}\hat{a}_{S2} - \lambda_{SS}\hat{a}_{SS}$ (2.3.B.1).

Repeating the similar exercise as done in Appendix (2.A), we obtain

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = (1-\lambda_{SS})\widehat{S} + H_2\widehat{W}_S + I_2\hat{r}$ (2.3.B.2).

Here,

$H_2 = -(\lambda_{SS}S_{SS}^{S} + \lambda_{S1}S_{SS}^{1} + \lambda_{S2}S_{SS}^{2}) > 0,$

and,

$I_2 = -(\lambda_{SS}S_{SK}^{S} + \lambda_{S1}S_{SK}^{1} + \lambda_{S2}S_{SK}^{2}) < 0$.

Putting expressions of $\widehat{W}_S$, $\hat{r}$and $\widehat{S}$ from equations (2.3.21), (2.3.22) and (2.3.26) in equation (2.3.B.2), we obtain

$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = J_2\widehat{P}_1 + \gamma_2\widehat{P}_2 - \varphi_2\widehat{P}_U$ (2.3.B.3);

where,

$J_2 = \left[(1-\lambda_{SS})F_2 + \frac{H_2\theta_{K2}}{|\theta|} - \frac{I_2\theta_{S2}}{|\theta|}\right] > 0,$

$\gamma_2 = \left[-(1-\lambda_{SS})G_2 - \frac{H_2\theta_{K1}}{|\theta|} + \frac{I_2\theta_{S1}}{|\theta|}\right] < 0,$

and,

$\varphi_2 = (1-\lambda_{SS})E_2 > 0$.

Then totally differentiating equation (2.3.9), and thereafter using the expressions of the rates of change of factor output coefficients and finally putting the expressions of$\widehat{W}_S$, $\hat{r}$, $\widehat{W}_U$, $\hat{r}_U$ and $\widehat{S}$ from equations (2.3.21), (2.3.22), (2.3.23), (2.3.24) and (2.3.26), in the equation derived from equation (2.3.9), we obtain

$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = N_2\widehat{P}_1 + O_2\widehat{P}_2 + Q_2\widehat{P}_U$ (2.3.B.4).

Here,

$N_2 = \left[-\lambda_{KS}F_2 + \frac{M_2^1\theta_{K2}}{|\theta|} - \frac{M_2^2\theta_{S2}}{|\theta|}\right] < 0,$

$$0_2 = \left[\lambda_{KS}G_2 - \frac{M_2^1\theta_{K1}}{|\theta|} + \frac{M_2^2\theta_{S1}}{|\theta|}\right] > 0,$$

$$Q_2 = \lambda_{KS}E_2 > 0$$

$$M_2^1 = -\left(\lambda_{SS}S_{KS}^S + \lambda_{S1}S_{KS}^1 + \lambda_{S2}S_{KS}^2\right) < 0,$$

and,

$$M_2^2 = -\left(\lambda_{SS}S_{KK}^S + \lambda_{S1}S_{KK}^1 + \lambda_{S2}S_{KK}^2\right) > 0\ .$$

Now, solving equations (2.3.B.3) and (2.3.B.4) simultaneously, we obtain

$$\widehat{X}_2 = \alpha_2\widehat{P}_1 + T_2\widehat{P}_2 + \beta_2\widehat{P}_U \qquad (2.3.B.5)\ ;$$

where,

$$\alpha_2 = \frac{1}{|\lambda|}(\lambda_{S1}N_2 - \lambda_{K1}J_2) < 0,$$

$$T_2 = \frac{1}{|\lambda|}(\lambda_{S1}0_2 - \lambda_{K1}\gamma_2) > 0,$$

and,

$$\beta_2 = \frac{1}{|\lambda|}(\lambda_{S1}Q_2 + \lambda_{K1}\varphi_2) > 0,$$

This equation (2.3.B.5) is same as equation (2.3.27) in the body of the chapter.

Appendix (2.F):

Proof of $\mathbf{Y_2 + Z_2 = 1}$

$$Y_2 + Z_2 = 1 \Rightarrow e_{P_2} + e_{M_2}(V_2 - W_2 + \psi_2) + (\alpha_2 + T_2 + \beta_2) = 0 \qquad (2.3.C.1).$$

Here,

$$(V_2 - W_2 + \psi_2) = \frac{W_S S}{Y|\theta|}\theta_{K2} + \frac{W_S S}{Y}F_2 - \frac{rk}{Y|\theta|}\theta_{S2} + \frac{W_U(L-S)}{Y|\theta|}B_2 - \frac{r_U K_U}{Y}\frac{\theta_U}{\theta_{KU}|\theta|}B_2 - \frac{W_S S}{Y|\theta|}\theta_{K1} - \frac{W_S S}{Y}G_2 + \frac{rk}{Y|\theta|}\theta_{S1} - \frac{W_U(L-S)}{Y|\theta|}C_2 + \frac{r_U K_U}{Y}\frac{\theta_U}{\theta_{KU}|\theta|}C_2 - \frac{W_S S}{Y}E_2 + \frac{r_U K_U}{Y\theta_{KU}}$$

$$\Rightarrow (V_2 - W_2 + \psi_2) = \frac{1}{Y}\left[W_S S + W_S S(F_2 - G_2 - E_2) + rk + \frac{W_U(L-S)}{|\theta|}(B_2 - C_2) + r_U K_U\left\{-\frac{(B_2 - C_2)}{|\theta|}\frac{\theta_U}{\theta_{KU}} + \frac{1}{\theta_{KU}}\right\}\right] \qquad (2.3.C.2).$$

Now,

$$(B_2 - C_2) = \theta_{K2} - \theta_{K1} = |\theta| \qquad (2.3.C.3);$$

and,

$(F_2 - G_2 - E_2) = \frac{E_2 B_2}{|\theta|} - \frac{E_2 C_2}{|\theta|} - E_2 = 0$ (2.3.C.4).

Using equations (2.3.C.2), (2.3.C.3) and (2.3.C.4), we obtain

$(V_2 - W_2 + \psi_2) = \frac{1}{Y}[W_S S + rk + W_U(L - S) + r_U K_U]$

$\Rightarrow (V_2 - W_2 + \psi_2) = 1$ (2.3.C.5);

and,

$(\alpha_2 + T_2 + \beta_2) = \frac{1}{|\lambda|}[\lambda_{S1}(N_2 + O_2 + Q_2) - \lambda_{K1}(J_2 + \gamma_2 + \varphi_2)]$ (2.3.C.6).

Again,

$(N_2 + O_2 + Q_2) = \lambda_{KS}(F_2 - G_2 - E_2) + M_2^1 + M_2^1$ (2.3.C.7).

Now,

$M_2^1 + M_2^1 = 0$ (2.3.C.8).

$\left[\because\ S_{KS}^1 + S_{KK}^1 = S_{KS}^2 + S_{KK}^2 = S_{KS}^S + S_{KK}^S = 0\right]$

Again,

$(J_2 + \gamma_2 + \varphi_2) = (1 - \lambda_{SS})(F_2 - G_2 - E_2) + H_2 + I_2$ (2.3.C.8).

Now,

$H_2 + I_2 = 0$ (2.3.C.9).

Using equations (2.3.C.4), (2.3.C.6), (2.3.C.7), (2.3.C.8) and (2.3.C.9), we obtain

$(\alpha_2 + T_2 + \beta_2) = 0$ (2.3.C.10)

Finally, using equations (2.3.C.1), (2.3.C.5) and (2.3.C.10), we obtain

$Y_2 + Z_2 = 1$

$\therefore\ e_{P_2} + e_{M_2} = 0.$

Appendix (2.G):

<u>Derivation of equations (2.4.13), (2.4.14), (2.4.15), (2.4.16), (2.4.19), (2.4.20) and (2.4.23):</u>

Differentiating both sides of equation (2.4.6), we obtain

$\frac{\partial^2 h}{\partial W_S^2}\frac{dW_S}{W_S}\frac{(W_S)^2}{h} + \frac{\partial h}{\partial W_S}\frac{dW_S}{W_S}\frac{W_S}{h} - \frac{\partial h}{\partial W_S}\frac{W_S}{h^2}\left[\frac{\partial h}{\partial W_S}\frac{dW_S}{W_S}W_S + \frac{\partial h}{\partial v_S}\frac{dv_S}{v}v_S\right] = 0$ (2.4.A.1).

Using equations (2.4.6) and (2.4.A.1), we have

$h_{11}\frac{(W_S)^2}{h}\widehat{W}_S - \varepsilon_{v_S}\hat{v}_S = 0$

$$\Rightarrow \widehat{W}_S = \frac{\varepsilon_{v_S} h}{h_{11}(W_S)^2}\hat{v}_S \qquad (2.4.A.2),$$

where

$\varepsilon_{v_S} = \frac{\partial h}{\partial v_S}\frac{v_S}{h} > 0$ and $h_{11} = \frac{\partial^2 h}{\partial {W_S}^2} < 0.$

Equation (2.4.A.2) is same as equation (2.4.19) in the body of the chapter.

Similarly, differentiating both sides of equation (2.4.7), we obtain

$$\widehat{W}_U = \frac{\varepsilon_{v_U} f}{f_{11}(W_U)^2}\hat{v}_U \qquad (2.4.A.3),$$

where

$\varepsilon_{v_U} = \frac{\partial f}{\partial v_U}\frac{v_U}{f} > 0$ and $f_{11} = \frac{\partial^2 f}{\partial {W_U}^2} < 0.$

Equation (2.4.A.3) is same as equation (2.4.20) in the body of the chapter.

From equations (2.4.4) and (2.4.5), we obtain

$$\hat{h} = \widehat{W}_S + \varepsilon_{v_S}\hat{v}_S \qquad (2.4.A.4).$$

and

$$\hat{f} = \widehat{W}_U + \varepsilon_{v_U}\hat{v}_U \qquad (2.4.A.5).$$

With $\widehat{P}_1 = 0$, from equations (2.4.1-A) and (2.4.2-A), we obtain

$$\widehat{W}_S - \hat{h} = -\frac{\widehat{P}_2\theta_{K1}}{|\theta|} \qquad (2.4.A.6);$$

and

$$\hat{r} = \frac{\widehat{P}_2\theta_{S1}}{|\theta|} \qquad (2.4.A.7).$$

Using equations (2.4.A.4) and (2.4.A.6), we obtain

$$\hat{v}_S = \frac{\widehat{P}_2\theta_{K1}}{\varepsilon_{v_S}|\theta|} \qquad (2.4.A.8).$$

Equation (2.4.A.8) is same as equation (2.4.15) in the body of the chapter.

With $\widehat{P}_3 = 0$, from equation (2.4.3-A), we obtain

$$\widehat{W}_U - \hat{f} = -\frac{\theta_{K3}}{\theta_{N3}}\frac{\theta_{S1}}{|\theta|}\widehat{P}_2 \qquad (2.4.A.9).$$

Using equations (2.4.A.5) and (2.4.A.9), we have

$$\hat{v}_U = \frac{\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}}\frac{\theta_{S1}}{|\theta|}\widehat{P}_2 \qquad (2.4.A.10).$$

Equation (2.4.A.10) is same as equation (2.4.16) in the body of the chapter.

Here,

$R = \frac{W_S(1-v_S)}{W_U(1-v_U)}$.

Hence,

$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\hat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\hat{v}_U$.

Using equations (2.4.A.2), (2.4.A.3), (2.4.A.8) and (2.4.A.10), we obtain

$$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\hat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\hat{v}_U$$

$$= \frac{\widehat{P}_2}{|\theta|\theta_{N3}}\left[\left\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right\}\theta_{K3}\theta_{S1} - \left\{\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right\}\theta_{K1}\theta_{N3}\right] \quad (2.4.A.11).$$

This equation (2.4.A.11) is same as equation (2.4.23) in the body of the chapter.

Differentiating equation (2.4.10), we obtain

$$\widehat{X}_3 = \hat{f} + \widehat{N} - \hat{a}_{N3} - \frac{v_U}{(1-v_U)}\hat{v}_U \quad (2.4.A.12).$$

Each of all optimum factor output coefficients is a function of prices of factors employed in that sector. For example, $a_{N3} = a_{N3}\left(\left(\frac{W_U}{f}\right), r\right)$. Hence,

$$\hat{a}_{N3} = S_{NN}^3 \widehat{\left(\frac{W_U}{f}\right)} + S_{NK}^3\hat{r} \,. \quad (2.4.A.13);$$

where, $S_{NN}^3 = \left(\frac{\left(\frac{W_U}{f}\right)}{a_{N3}}\right)\left(\frac{\partial a_{N3}}{\partial\left(\frac{W_U}{f}\right)}\right)$ and $S_{NK}^3 = \left(\frac{r}{a_{N3}}\right)\left(\frac{\partial a_{N3}}{\partial r}\right)$. Similarly, we can derive rates of change in other factor output coefficients.

Differentiating equation (2.4.8) and using expressions of rates of change of all these factor output coefficients, we have

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} - \lambda_{K1}\left(S_{KK}^1\hat{r} - S_{KS}^1\varepsilon_{v_S}\hat{v}_S\right) - \lambda_{K2}\left(S_{KK}^2\hat{r} - S_{KS}^2\varepsilon_{v_S}\hat{v}_S\right) - \lambda_{K3}\left(S_{KK}^3\hat{r} - S_{KN}^3\varepsilon_{v_U}\hat{v}_U\right) - \lambda_{K3}\widehat{X}_3 \quad (2.4.A.14).$$

Using equations (2.4.A.3), (2.4.A.5), (2.4.A.7), (2.4.A.8), (2.4.A.10), (2.4.A.12) and (2.4.A.14), we have

$$\lambda_{K1}\widehat{X}_1 + \lambda_{K2}\widehat{X}_2 = \widehat{K} - \lambda_{K3}\widehat{N} + A\widehat{P}_2 \quad (2.4.A.15);$$

where,

$$A = \frac{1}{|\theta|}\Bigg[\{\lambda_{K1}S_{KS}^{1} + \lambda_{K2}S_{KS}^{2}\}\theta_{K1} - \{\lambda_{K1}S_{KK}^{1} + \lambda_{K2}S_{KK}^{2} + \lambda_{K3}(S_{KK}^{3} - S_{NK}^{3})\}\theta_{S1} - \lambda_{K3}\Bigg\{(1 + S_{NN}^{3} - S_{KN}^{3}) - \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\Bigg\}\frac{\theta_{K3}\theta_{S1}}{\theta_{N3}} - \lambda_{K3}\frac{\theta_{K3}\theta_{S1}f}{\theta_{N3}f_{11}(W_U)^2}\Bigg].$$

Similarly, from equation (2.4.9), we have

$$\lambda_{S1}\widehat{X}_1 + \lambda_{S2}\widehat{X}_2 = \widehat{S} + B\widehat{P}_2 \qquad (2.4.A.16);$$

where,

$$B = \frac{1}{|\theta|}\Bigg[\Bigg\{1 - \Bigg(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\Bigg)\Bigg\}\theta_{K1} + (\lambda_{S1}S_{SS}^{1} + \lambda_{S2}S_{SS}^{2})\theta_{K1} - (\lambda_{S1}S_{SK}^{1} + \lambda_{S2}S_{SK}^{2})\theta_{S1}\Bigg].$$

Now, solving equations (2.4.A.15) and (2.4.A.16) simultaneously, we obtain

$$\widehat{X}_2 = \frac{1}{|\lambda|}[\lambda_{S1}\widehat{K} - \lambda_{S1}\lambda_{K3}\widehat{N} - \lambda_{K1}\widehat{S}] + \widehat{P}_2 C \qquad (2.4.A.17);$$

where,

$$C = \frac{1}{|\lambda|}[\lambda_{s1}A - \lambda_{K1}B].$$

Totally differentiating equation (2.4.11), we have

$$\widehat{Y} = \frac{W_S S(1-v_S)}{Y}\left(\widehat{S} + \widehat{W}_S\right) + \frac{rK}{Y}\left(\widehat{K} + \widehat{r}\right) + \frac{W_U N(1-v_U)}{Y}\left(\widehat{N} + \widehat{W}_U\right) - \frac{W_S S v_S}{Y}\widehat{v}_S - \frac{W_U N v_U}{Y}\widehat{v}_U \qquad (2.4.A.18).$$

Using equations (2.4.A.2), (2.4.A.3), (2.4.A.7), (2.4.A.8) and (2.4.A.10), we have

$$\widehat{Y} = \frac{W_S S(1-v_S)}{Y}\widehat{S} + \frac{rK}{Y}\widehat{K} + \frac{W_U N(1-v_U)}{Y}\widehat{N} + \widehat{P}_2 E \qquad (2.4.A.19);$$

where,

$$E = \frac{1}{|\theta|}\Bigg[\frac{rK}{Y}\theta_{S1} - \frac{W_U N(1-v_U)}{Y}\Bigg\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\Bigg\}\frac{\theta_{K3}\theta_{S1}}{\theta_{N3}} - \frac{W_S S(1-v_S)}{Y}\Bigg\{\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\Bigg\}\theta_{K1}\Bigg].$$

Now, using equations (2.4.12-A), (2.4.A.17) and (2.4.A.19), we obtain

$$\widehat{P}_2 = \frac{1}{D}\Bigg[\widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) - \widehat{N}\left(\frac{\lambda_{S1}\lambda_{K3}}{|\lambda|} + e_{M_2}\frac{W_U N(1-v_U)}{Y}\right) + \widehat{S}\left\{\frac{\lambda_{k1}}{|\lambda|} - e_{M_2}\frac{W_S S(1-v_S)}{Y}\right\}\Bigg] \qquad (2.4.A.20);$$

where

$$D = e_{P_2} + e_{M_2}E - C \qquad (2.4.A.21).$$

These equations (2.4.A.20) and (2.4.A.21) are same as equations (2.4.13) and (2.4.14), respectively shown in the body of the chapter.

Equation (2.4.14-A) implies that $D < 0$; and this can be shown using equations (2.4.A.17), (2.4.A.19) and (2.4.A.21) for $\widehat{K} = \widehat{N} = \widehat{S} = 0$.

Appendix (2.H):

Derivation of equations (2.4.24), (2.4.25) and (2.4.26):

$$G = \frac{\bar{\Delta}}{2\mu} \qquad (2.4.A.22),$$

where

$$\bar{\Delta} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|$$

and

μ = Mean Income.

Here,

$$\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j| = 2[(Nv_U + Sv_S)N(1 - v_U)W_U + (Nv_U + Sv_S)S(1 - v_S)W_S + N(1 - v_U)S(1 - v_S)(W_S - W_U)]$$

$$\Rightarrow (N + S)(N + S - 1)\bar{\Delta} =$$

$$2[(Nv_U + Sv_S)\{N(1 - v_U)W_U + S(1 - v_S)W_S\} + NS(1 - v_U)(1 - v_S)(W_S - W_U)] \qquad (2.4.A.23),$$

and

$$\mu = \frac{[N(1-v_U)W_U + S(1-v_S)W_S]}{(N+S)} \qquad (2.4.A.24).$$

Using equations (2.4.A.22), (2.4.A.23) and (2.4.A.24), we have

$$G = \frac{(Nv_U + Sv_S)\{N(1-v_U)W_U + S(1-v_S)W_S\} + NS(1-v_U)(1-v_S)(W_S - W_U)}{(N+S-1)N(1-v_U)W_U + S(1-v_S)W_S}$$

$$\Rightarrow G = \frac{(Nv_U + Sv_S)\left\{N(1-v_U) + S(1-v_S)\frac{W_S}{W_U}\right\} + NS(1-v_U)(1-v_S)\left(\frac{W_S}{W_U} - 1\right)}{(N+S-1)\left\{N(1-v_U) + S(1-v_S)\frac{W_S}{W_U}\right\}}$$

$$\Rightarrow G = \frac{(Nv_U + Sv_S)\{N(1-v_U) + S(1-v_S)\Delta\} + NS(1-v_U)(1-v_S)(\Delta - 1)}{(N+S-1)\{N(1-v_U) + S(1-v_S)\Delta\}} \qquad (2.4.A.25),$$

where

$$\Delta = \frac{W_S}{W_U}.$$

Equation (2.4.A.25) is same as equation (2.4.24) in the body of the chapter.

Now, differentiating equation (2.4.A.25), we have

$$(N + S - 1)\{N(1 - v_U) + S(1 - v_S)\Delta\}dG - G(N + S - 1)Ndv_U - G(N + S - 1)S\Delta dv_S$$

$$G(N + S - 1)S(1 - v_S)d\Delta$$

$$= [-(Nv_U + Sv_S)N + \{N(1 - v_U) + S(1 - v_S)\Delta\}N - NS(1 - v_S)(\Delta - 1)]dv_U +$$
$$[-(Nv_U + Sv_S)S\Delta + \{N(1 - v_U) + S(1 - v_S)\Delta\}S - NS(1 - v_U)(\Delta - 1)]dv_S$$
$$+[(Nv_U + Sv_S)S(1 - v_S) + NS(1 - v_U)(1 - v_S)]d\Delta \qquad (2.4.A.26).$$

Now, using equations (2.4.A.25) and (2.4.A.26), we obtain

$$(N + S - 1)\{N(1 - v_U) + S(1 - v_S)\Delta\}^2 dG =$$
$$[\{-N^2 v_U - NSv_S + N^2(1 - v_U) + NS(1 - v_S)\Delta -$$
$$NS(1 - v_S)(\Delta - 1)\}\{N(1 - v_U) + S(1 - v_S)\Delta\}$$
$$+\{N^2 v_U + NSv_S\}\{N(1 - v_U) + S(1 - v_S)\Delta\} + N^2 S(1 - v_U)(1 - v_S)(\Delta - 1)]dv_U +$$
$$[\{-NS\Delta v_U - S^2\Delta v_S + NS(1 - v_U) + S^2(1 - v_S)\Delta$$
$$-NS(1 - v_U)(\Delta - 1)\}\{N(1 - v_U) + S(1 - v_S)\Delta\} +$$
$$\{NS\Delta v_U + S^2\Delta v_S\}\{N(1 - v_U) + S(1 - v_S)\Delta\} + NS^2(1 - v_U)(1 - v_S)\Delta(\Delta - 1)]dv_S +$$
$$[\{(Nv_U + Sv_S)S(1 - v_S) + NS(1 - v_U)(1 - v_S)\}\{N(1 - v_U) + S(1 - v_S)\Delta\}$$
$$-(Nv_U + Sv_S)\{N(1 - v_U) + S(1 - v_S)\Delta\}S(1 - v_S) - NS^2(1 - v_U)(1 - v_S)^2(\Delta - 1)]d\Delta$$

$$\Rightarrow dG =$$

$$\frac{[NS^2(1-v_S)^2\Delta + 2N^2S(1-v_U)(1-v_S)\Delta + N^3(1-v_U)^2]}{(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} dv_U + \frac{[N^2S(1-v_U)^2 + 2NS^2(1-v_U)(1-v_S)\Delta + S^3(1-v_S)^2\Delta^2]}{(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} dv_U$$
$$+ \frac{[NS(1-v_U)(1-v_S)\{N(1-v_U)+S(1-v_S)\}]}{(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} d\Delta$$

$$\Rightarrow \widehat{G} = A_3\widehat{v}_U + B_3\widehat{v}_S + C_3\widehat{\Delta} \qquad (2.4.A.27);$$

where,

$$A_3 = \frac{v_U[NS^2(1-v_S)^2\Delta + 2N^2S(1-v_U)(1-v_S)\Delta + N^3(1-v_U)^2]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0,$$

$$B_3 = \frac{v_S[N^2S(1-v_U)^2 + 2NS^2(1-v_U)(1-v_S)\Delta + S^3(1-v_S)^2\Delta^2]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0$$

and

$$C_3 = \frac{\Delta[NS(1-v_U)(1-v_S)\{N(1-v_U)+S(1-v_S)\}]}{G(N+S-1)\{N(1-v_U)+S(1-v_S)\Delta\}^2} > 0.$$

Equation (2.4.A.27) is same as equation (2.4.25) in the body of the chapter.

Using equations (2.4.17), (2.4.18), (2.4.22) and (2.4.25), we obtain

$$\widehat{G} = A_3 \frac{\theta_{K3}}{D\varepsilon_{v_U}\theta_{N3}} \frac{\theta_{S1}}{|\theta|} \widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right) + B_3 \frac{\theta_{K1}}{D\varepsilon_{v_S}|\theta|} \widehat{K}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)$$
$$+C_3 \frac{\widehat{K}}{D|\theta|\theta_{N3}} \left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)\left\{\left(-\frac{f\theta_{K3}\theta_{S1}}{f_{11}(W_U)^2}\right) - \left(-\frac{h\theta_{K1}\theta_{N3}}{h_{11}(W_S)^2}\right)\right\}$$

$$\Rightarrow \widehat{G} = \frac{\widehat{K}}{D|\theta|\theta_{N3}}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)\left[A_3\frac{\theta_{K3}\theta_{S1}}{\varepsilon_{v_U}} + B_3\frac{\theta_{K1}\theta_{N3}}{\varepsilon_{v_S}} + C_3\left\{\left(-\frac{f\theta_{K3}\theta_{S1}}{f_{11}(W_U)^2}\right) - \left(-\frac{h\theta_{K1}\theta_{N3}}{h_{11}(W_S)^2}\right)\right\}\right]$$

(2.4.A.28).

Using equations (2.4.22) and (2.4.A.28), we obtain

$$\Rightarrow \widehat{G} = \frac{\widehat{K}}{D|\theta|\theta_{N3}}\left(\frac{\lambda_{S1}}{|\lambda|} - e_{M_2}\frac{rk}{Y}\right)\left[A_3\frac{\theta_{K3}\theta_{S1}}{\varepsilon_{v_U}} + B_3\frac{\theta_{K1}\theta_{N3}}{\varepsilon_{v_S}}\right] + C_3\widehat{\Delta} \qquad (2.4.A.29).$$

Equation (2.4.A.29) is same as equation (2.4.26) in the body of the chapter.

Appendix (2.I):

Derivation of equations (2.4.13.1), (2.4.14.1), (2.4.15.1), (2.4.16.1) and (2.4.20.1):

From equations (2.4.1-A) and (2.4.2-A), we obtain

$$\widehat{W}_S - \widehat{h} = \frac{\widehat{P}_1\theta_{K2} - \widehat{P}_2\theta_{K1}}{|\theta|} \qquad (2.4.A.30);$$

and

$$\widehat{r} = \frac{\widehat{P}_2\theta_{S1} - \widehat{P}_1\theta_{S2}}{|\theta|} \qquad (2.4.A.31).$$

Using equations (2.4.A.4) and (2.4.A.30), we obtain

$$\widehat{v}_S = \frac{\widehat{P}_2\theta_{K1} - \widehat{P}_1\theta_{K2}}{\varepsilon_{v_S}|\theta|} \qquad (2.4.A.32).$$

Equation (2.4.A.32) is same as equation (2.4.15.1) in the body of the chapter.

From equation (2.4.3-A), we obtain

$$\widehat{W}_U - \widehat{f} = \frac{\widehat{P}_3}{\theta_{N3}} - \frac{\theta_{K3}}{\theta_{N3}|\theta|}\left(\Theta_{S1}\widehat{P}_2 - \Theta_{S2}\widehat{P}_1\right) \qquad (2.4.A.33).$$

Using equations (2.4.A.5) and (2.4.A.33), we have

$$\widehat{v}_U = \frac{\theta_{K3}}{\varepsilon_{v_U}\theta_{N3}|\theta|}\left(\Theta_{S1}\widehat{P}_2 - \Theta_{S2}\widehat{P}_1\right) - \frac{\widehat{P}_3}{\varepsilon_{v_U}\theta_{N3}} \qquad (2.4.A.34).$$

Equation (2.4.A.34) is same as equation (2.4.16.1) in the body of the chapter.

Finally, using equations (2.4.A.2), (2.4.A.3), (2.4.A.32) and (2.4.A.34), we obtain

$$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\widehat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\widehat{v}_U$$

$$= \frac{\widehat{P}_1}{|\theta|\theta_{N3}}\left\{\Theta_{K2}\Theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right) - \left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)\Theta_{K3}\Theta_{S2}\right\}$$

$$-\frac{\widehat{P}_2}{|\theta|}\left\{\theta_{K1}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)-\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)\theta_{K3}\theta_{S2}\right\}$$

$$-\frac{\widehat{P}_3}{\varepsilon_{v_U}\theta_{N3}}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}}-\frac{f}{f_{11}(W_U)^2}\right)$$

(2.4.A.35).

Differentiating, equation (2.4.10), we obtain

$$\widehat{X}_3=\hat{f}-\hat{a}_{N3}-\frac{v_U}{(1-v_U)}\hat{v}_U \qquad (2.4.A.36).$$

Totally differentiating equation (2.4.8) and putting the expressions of rates of change of different factor-output coefficients, we have

$$\lambda_{K1}\widehat{X}_1+\lambda_{K2}\widehat{X}_2=-\left(S^1_{KK}\hat{r}-S^1_{KS}\varepsilon_{v_S}\hat{v}_S\right)-\lambda_{K2}\left(S^2_{KK}\hat{r}-S^2_{KS}\varepsilon_{v_S}\hat{v}_S\right)-\lambda_{K3}\left(S^3_{KK}\hat{r}-S^3_{KN}\varepsilon_{v_U}\hat{v}_U\right)-\lambda_{K3}\widehat{X}_3 \qquad (2.4.A.37).$$

Using equations (2.4.A.3), (2.4.A.5), (2.4.A.31), (2.4.A.32), (2.4.A.34), (2.4.A.36) and (2.4.A.37), we have

$$\lambda_{K1}\widehat{X}_1+\lambda_{K2}\widehat{X}_2=E_2\widehat{P}_1+F_2\widehat{P}_2+G_2\widehat{P}_3 \qquad (2.4.A.38);$$

where,

$$E_2=\frac{1}{|\theta|}\left[-A_2\theta_{S2}-B_2\theta_{K2}-C_2\frac{\theta_{K3}\theta_{S2}}{\theta_{N3}}\right],$$

$$F_2=\frac{1}{|\theta|}\left[A_2\theta_{S1}+B_2\theta_{K1}+C_2\frac{\theta_{K3}\theta_{S2}}{\theta_{N3}}\right],$$

$$G_2=-\frac{C_2}{\theta_{N3}\varepsilon_{v_U}},$$

$$A_2=-\{\lambda_{K1}S^1_{KK}+\lambda_{K2}S^2_{KK}+\lambda_{K3}(S^3_{KK}-S^3_{NK})\}>0,$$

$$B_2=(\lambda_{K1}S^1_{KS}+\lambda_{K2}S^2_{KS})>0,$$

and

$$C_2=\left[\left\{(1+S^3_{NN}-S^3_{KN})-\frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right\}-\frac{f}{f_{11}(W_U)^2}\right].$$

Similarly, from equation (2.4.9), we have

$$\lambda_{S1}\widehat{X}_1+\lambda_{S2}\widehat{X}_2=H_2\widehat{P}_1+I_2\widehat{P}_2 \qquad (2.4.A.39);$$

where,

$$H_2=\frac{1}{|\theta|}\left[-(1+\lambda_{S1}S^1_{SS}+\lambda_{S2}S^2_{SS})\theta_{K2}+(\lambda_{S1}S^1_{SK}+\lambda_{S2}S^2_{SK})\theta_{S2}+\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}}-\frac{h}{h_{11}(W_S)^2}\right)\theta_{K2}\right],$$

and

$I_2 = \frac{1}{|\theta|}\left[(1 + \lambda_{S1}S_{SS}^1 + \lambda_{S2}S_{SS}^2)\theta_{K1} - (\lambda_{S1}S_{SK}^1 + \lambda_{S2}S_{SK}^2)\theta_{S1} - \left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right)\theta_{K1}\right].$

Solving equations (2.4.A.38) and (2.4.A.39) simultaneously, we obtain

$\hat{X}_2 = J_2\hat{P}_1 + \gamma_2\hat{P}_2 + L_2\hat{P}_3$ (2.4.A.40);

where,

$J_2 = \frac{1}{|\lambda|}[\lambda_{s1}E_2 - \lambda_{K1}H_2],$

$\gamma_2 = \frac{1}{|\lambda|}[\lambda_{s1}F_2 - \lambda_{K1}I_2],$

and

$L_2 = \frac{\lambda_{s1}G_2}{|\lambda|}.$

J_2, γ_2 and L_2 represent the elasticities of general equilibrium supply function of the non-traded good with respect to P_1, P_2 and P_3 respectively.

Totally differentiating equation (2.4.11), we have

$\hat{Y} = \frac{W_S S(1-v_S)}{Y}\hat{W}_S + \frac{rK}{Y}\hat{r} + \frac{W_U N(1-v_U)}{Y}\hat{W}_U - \frac{W_S S v_S}{Y}\hat{v}_S - \frac{W_U N v_U}{Y}\hat{v}_U$ (2.4.A.41).

Using equations (2.4.A.2), (2.4.A.3), (2.4.A.31), (2.4.A.32) and (2.4.A.34), we have

$\hat{Y} = M_2\hat{P}_1 + \delta_2\hat{P}_2 + \vartheta_2\hat{P}_3$ (2.4.A.42);

where,

$M_2 = \frac{1}{|\theta|}\left[-\frac{rK}{Y}\theta_{S2} + \frac{W_U N(1-v_U)}{Y}\left\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right\}\frac{\theta_{K3}\theta_{S2}}{\theta_{N3}}\right.$

$\left. + \frac{W_S S(1-v_S)}{Y}\left\{\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right\}\theta_{K2}\right],$

$\delta_2 = \frac{1}{|\theta|}\left[\frac{rK}{Y}\theta_{S1} - \frac{W_U N(1-v_U)}{Y}\left\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right\}\frac{\theta_{K3}\theta_{S1}}{\theta_{N3}}\right.$

$\left. - \frac{W_S S(1-v_S)}{Y}\left\{\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right\}\theta_{K1}\right],$

and,

$\vartheta_2 = \frac{W_U N(1-v_U)}{Y\theta_{N3}}\left\{\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right\} > 0.$

Here, M_2, δ_2 and ϑ_2 indicate relative changes in Y due to relative changes in P_1, P_2 and P_3 respectively.

Now, using equations (2.4.12-A), (2.4.A.40) and (2.4.A.42), we obtain

$\hat{P}_2 = \bar{\alpha}\hat{P}_1 + \bar{\beta}\hat{P}_3$ (2.4.A.43);

This equation (2.4.A.43) is same as equation (2.4.13.1) in the body of the chapter.

Here,

$\overline{\alpha} = \frac{J_2 - e_{M_2} M_2}{\overline{D}}$,

$\overline{\beta} = \frac{L_2 - e_{M_2} O_2}{\overline{D}}$,

and

$$\overline{D} = e_{P_2} + e_{M_2}\delta_2 - \gamma_2 \quad (2.4.A.44).$$

This equation (2.4.A.44) is same as equation (2.4.14.1) in the body of the chapter.

Finally we use the stability condition in the market for commodity 2 to show that $\overline{D} < 0$; and this can be shown using equations (2.4.A.40), (2.4.A.42) and (2.4.A.44) for $\widehat{P}_1 = \widehat{P}_3 = 0$.

Finally, using equations (2.4.A.35) and (2.4.A.43), we obtain

$$\widehat{R} = \widehat{W}_S - \frac{v_S}{(1-v_S)}\hat{v}_S - \widehat{W}_U + \frac{v_U}{(1-v_U)}\hat{v}_U$$

$$= \frac{\widehat{P}_1}{|\theta|\theta_{N3}}\left[\left\{\theta_{K2}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right) - \theta_{K3}\theta_{S2}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)\right\}\right.$$

$$\left. - \overline{\alpha}\left\{\theta_{K1}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right) - \theta_{K3}\theta_{S1}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)\right\}\right]$$

$$- \frac{\widehat{P}_3}{\theta_{N3}}\left[\frac{\overline{\beta}}{|\theta|}\left\{\theta_{K1}\theta_{N3}\left(\frac{v_S}{(1-v_S)\varepsilon_{v_S}} - \frac{h}{h_{11}(W_S)^2}\right) - \theta_{K3}\theta_{S1}\left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)\right\}\right.$$

$$\left. + \left(\frac{v_U}{(1-v_U)\varepsilon_{v_U}} - \frac{f}{f_{11}(W_U)^2}\right)\right] \quad (2.4.A.45).$$

This equation (2.4.A.45) is same as equation (2.4.20.1) in the body of the chapter.

Chapter 3

A static general equilibrium product variety model

3.1 INTRODUCTION

This chapter is devoted to explain skilled-unskilled wage inequality in a static general equilibrium model with product variety structure and with monopolistic competition in markets of different varieties. The chapter is an addition to the existing theoretical literature on static product variety models. Among the existing static product variety models, Glazer and Ranjan (2003) introduces preference heterogeneity assuming that skilled workers prefer to consume skill intensive goods. However, they do not introduce any public intermediate good in their model. Anwar (2006a, 2009) and Anwar and Rice (2009) analyse the problem of wage inequality using endogenous product variety framework with specialization-based external economics but ignore the role of public input in their model. Anwar (2005, 2006b) introduce a public input producing sector in their models in the presence of specialization-based external economics. However, those models have only one type of labour; and hence fail to explain the skilled-unskilled wage inequality. The model developed in the present chapter is an extension of the works of Anwar (2006a, 2009) and Anwar and Rice (2009) introducing a public input producing sector like that in Anwar (2005, 2006b) and consisting of two types of labour-skilled and unskilled. We develop a static four sector small open economy model with two traded good sectors, a public intermediate good producing sector and a private nontraded good sector producing varieties of intermediate goods. Its production is financed by a proportional tax on output of the industrial sector. There are three primary factors in this model- skilled labour, unskilled labour and capital. The public intermediate good plays the role of reducing the fixed cost of production of nontraded private intermediate goods. Production functions of all these sectors, except for varieties of private intermediate goods, satisfy all standard neo-classical properties including constant returns to scale (CRS). However, in the private intermediate goods producing sector, production function of each of these varieties satisfies increasing returns to scale (IRS).

In reality, public infrastructure plays a significant role to the development of market economics. The study of Ram (1986), based on data of many developed and developing countries, points out a positive relationship between the government size and the growth of national income. In the context of Korean economy, Kim (1998) shows that infrastructure investment leads to economic growth as well as inflation. Rioja (1999) argues that public infrastructure investment can lead to sizeable increase in GDP. Ang (2008), Hill (2007) and Appleyard et al. (2007) show that infrastructural development promotes foreign investment. On the other hand, Delorme et al. (1999) finds a negative relationship between the growth of public infrastructure and technical efficiency. So, in this chapter, we consider a public intermediate good producing sector in a static product variety model.

This chapter is organized as follows. Section 3.2 presents the basic model with full employment of all factors and with a public good producing sector. Sub-section 3.2.1 describes the model and sub-section 3.2.2 analyzes its various comparative static properties. Effects of changes in factor endowments on skilled-unskilled wage ratio are described in subsection 3.2.2.1. In subsection 3.2.2.2, we analyze effects of exogenous changes in prices of traded goods and in the income tax-rate on skilled-unskilled relative wage. In section 3.3; we introduce unemployment in both the labour markets. Limitations of this model are described in section 3.4.

3.2. The Basic Model:[40]

3.2.1 Description:

We consider a small open economy with two traded good sectors, Y and Z, and two nontraded good sectors, X and G. There are three primary factors- skilled labour, unskilled labour and capital. Sector Y produces an industrial good using skilled labour, capital and large

[40] Gupta and Dutta (2012) is partly based on the materials presented in this section.

number of varieties of intermediate goods produced by sector X with skilled labour and capital as inputs. Sector G produces a public input and sector Z produces an agricultural good; and each of these two is produced by unskilled labour and capital. The role of public input is to reduce the fixed cost of producing X. Production functions of all these sectors, except for sector X, satisfies all standard neo-classical properties including constant returns to scale (CRS). However, in sector X, production function of each of these varieties satisfies increasing returns to scale (IRS). All factor endowments are exogenously given. Capital is mobile among all these four sectors. However, skilled labour is mobile between sector Y and sector X; and unskilled labour is mobile between sector Z and sector G. Factor prices in each of these four sectors are perfectly flexible; and this flexibility ensures full employment of all these primary factors. All markets are competitive except for markets of varieties produced by sector X in which monopolistic competition exists. The representative firm maximizes profit in each of the three private goods sectors. The production of the public input is financed by a tax revenue obtained from the industrial sector; and the budget of the government is always balanced[41].

Production functions of sectors Y, Z and G are described as follows.

$$Y = \left(L_{SY}^{1-\beta} K_Y^{\beta}\right)^{1-\alpha} \left(\sum_{i=1}^{n} x_i^{\delta}\right)^{\frac{\alpha}{\delta}} \quad (3.2.1),$$

$$Z = L_{UZ}^{1-\gamma} K_Z^{\gamma} \quad (3.2.2),$$

and

$$G = L_{UG}^{1-\varphi} K_G^{\varphi} \quad (3.2.3).$$

Here, x_i is the quantity of the ith variety of intermediate good produced in sector X; and n is the number of these varieties. L_{SY} stands for the amount of skilled labour employed in sector Y; and L_{UZ} and L_{UG} represent amounts of unskilled labour employed in sectors Z and G respectively. K_Y, K_Z and K_G stand for amounts of capital used in sectors Y, Z and G respectively. α, β, γ, δ and φ are relevant elasticity parameters defined in the range $(0,1)$. Y, Z and G stand for levels of output of sectors Y, Z and G respectively.

[41] In reality, industrial sector is the most important source of collecting tax revenue even if it is not the only source.

Increasing returns to scale exists in the production of each of different varieties of intermediate goods because production of every variety involves fixed cost as well as variable cost. The total cost function of the ith variety produced in sector X is given as follows.

$$c(W_S, r, x_i) = \left(\frac{\mu}{G^{\sigma}} + \lambda x_i\right) {W_S}^{1-\theta} r^{\theta}; \qquad 0 < \theta, \sigma < 1;\ \mu, \lambda > 0. \qquad (3.2.4),$$

Here, W_S and r represent wage rate of skilled labour and rental rate on capital respectively; and θ stands for the capital elasticity of output. $\mu > 0$ implies the presence of the fixed cost; and σ represents the elasticity of fixed cost with respect to the public input.

Here, $\frac{\mu}{G^{\sigma}} {W_S}^{1-\theta} r^{\theta}$ is the fixed cost of producing an intermediate good; and $\sigma > 0$ implies that this fixed cost is reduced as the production of the public input is expanded. Holtz-Eakin and Lovely (1996) and Anwar (2001, 2006b) also make similar assumption about the role of the public input. However, they are not interested to analyse the problem of skilled unskilled wage inequality and so they consider only one kind of labour. $\lambda x_i {W_S}^{1-\theta} r^{\theta}$ represents the variable cost of producing the ith intermediate good. Fixed cost and unit variables costs are same for all i; and this means that all private intermediate goods are produced with identical production technology.

As all intermediate goods have identical production technologies and as their producers also face same prices to buy inputs, they are produced in equal quantities and are sold at equal prices in equilibrium. So, $x_i = x$ for all i; and the total production of sector X is nx. Hence, equation (3.2.1) can be rewritten as

$$Y = L_{SY}^{(1-\beta)(1-\alpha)} K_Y^{\beta(1-\alpha)} X^{\alpha} n^{\frac{\alpha(1-\delta)}{\delta}} \qquad (3.2.5),$$

where, $\frac{\alpha(1-\delta)}{\delta}$ indicates the scale elasticity of output. It is always positive; and, in the rest of the analysis, we assume that

$$\frac{\alpha(1-\delta)}{\delta} <$$

$$Min\left\{\left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right), \left(\frac{\psi(\beta(1-\alpha)+\alpha\theta)}{\sigma\gamma(1-\psi)-\theta\psi}\right), \left(\frac{(\beta(1-\alpha)+\alpha\theta)}{\frac{\sigma\left\{\frac{\theta W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}} + \frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}}{\left\{\frac{\psi}{(1-\psi)W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \left(\frac{\gamma}{(1-\gamma)}\right)^2 \frac{\psi}{1-\psi} W_U L_U\right\}} - \theta}\right)\right\}$$

........................(3.2.A).

Assumption (3.2.A) implies that scale elasticity of output is low in the industrial sector. This means that degree of increasing returns to scale is low; and this is always satisfied when δ is very close to unity; i.e., different varieties of intermediate inputs are highly imperfect substitute in the industrial production function. This scale effect does not exist when $\delta = 1$; i.e., all these intermediate inputs are perfect substitute.

Anwar (2006a, 2006b, 2009) and Anwar and Rice (2009) also assume this scale elasticity of output to be very small in order to ensure the stability of equilibrium in the market of intermediate goods. They borrow the assumption from Ethier (1982). However, the magnitude of its upper bound in our model is different from those in other models.

In this model, the production of G is financed by a proportional output tax imposed on sector Y. So, we have

$$\psi \mathrm{Y} = \mathrm{G} \qquad (3.2.6).$$

where, ψ is the exogenously given proportional tax rate on industrial output. In Holtz-Eakin and Lovely (1996) and Anwar (2001, 2006b), G is treated as exogenous. In this model, G is endogenized by equation (3.2.6). However, the interpretation of endogenity must be done carefully. Here G is endogenized satisfying consistency criterion only because G is solved from the balanced budget equation like that in Barro (1990). G is not optimally determined here.

Now, using equations (3.2.1)–(3.2.6), we obtain a set of equations. The first order profit maximizing condition in sector Y is given by

$$1 - \psi = \mathrm{M}\left(\frac{\mathrm{W_S}}{\mathrm{r}}\right)^{(-\beta+\alpha\beta)}\left(\frac{\mathrm{W_S}}{\mathrm{P_x}}\right)^{-\alpha}\frac{\mathrm{W_S}}{\mathrm{n}^{\frac{\alpha(1-\delta)}{\delta}}} \qquad (3.2.7),$$

where,

$$\mathrm{M} = \frac{\alpha^{-\alpha}\beta^{-\beta(1-\alpha)}}{(1-\alpha)^{(1-\alpha)}(1-\beta)^{(1-\alpha)(1-\beta)}} > 0;$$

and $\mathrm{P_x}$ stands for the price of the representative intermediate good.

Here the left hand side of equation (3.2.7) represents the effective per unit return from the production of good Y because price of Y is normalized to unity and ψ is the unit tax rate. The right hand side of equation (3.2.7) represents its marginal cost of production. An increase in the number of varieties, n, lowers the marginal cost of producing the industrial good and thus raises its level of production.

In the short run, the first order profit maximizing condition in sector X is given by

$$\delta P_x = \lambda {W_S}^{1-\theta} r^{\theta} \quad (3.2.8).$$

The left hand side of equation (3.2.8) represents the marginal revenue and its right hand side represents the marginal cost. However, this maximized profit must be equal to the fixed cost of production in the group equilibrium because entry of new firms (producers of new varieties of intermediate goods) takes place so long a positive supernormal profit is earned. This group equilibrium condition is given by

$$(1-\delta) P_x x = \frac{\mu}{G^{\sigma}} {W_S}^{1-\theta} r^{\theta} \quad (3.2.9).$$

The first order profit maximizing condition in the agricultural sector, Z, is given by

$$P_Z = \left[\frac{1}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}\right] {W_U}^{1-\gamma} r^{\gamma} \quad (3.2.10),$$

where P_Z and W_U represent price of the agricultural good, Z, and wage rate of unskilled labour respectively. The right hand side of equation (3.2.10) stands for the marginal cost of producing the agricultural good, Z.

Equilibrium conditions in markets of unskilled labour, skilled labour and capital are given by following equations.

$$\left[\frac{\gamma}{(1-\gamma)}\right]^{-\gamma} \left(\frac{W_U}{r}\right)^{-\gamma} Z + \left[\frac{\varphi}{(1-\varphi)}\right]^{-\varphi} \left(\frac{W_U}{r}\right)^{-\varphi} G = L_U \quad (3.2.11),$$

$$n\left(\frac{\mu}{G^{\sigma}} + \lambda x\right)(1-\theta)\left(\frac{W_S}{r}\right)^{-\theta} + M(1-\alpha)(1-\beta)\left(\frac{W_S}{r}\right)^{(-\beta+\alpha\beta)} \left(\frac{W_S}{P_x}\right)^{-\alpha} \frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = L_S (3.2.12),$$

and,

$$\left[\frac{\gamma}{(1-\gamma)}\right]^{(1-\gamma)} \left(\frac{W_U}{r}\right)^{(1-\gamma)} Z + \left[\frac{\varphi}{(1-\varphi)}\right]^{(1-\varphi)} \left(\frac{W_U}{r}\right)^{(1-\varphi)} G + n\left(\frac{\mu}{G^{\sigma}} + \lambda x\right)\theta\left(\frac{W_S}{r}\right)^{(1-\theta)}$$

$$+ M\beta(1-\alpha)\left(\frac{W_S}{r}\right)^{(1-\beta+\alpha\beta)} \left(\frac{W_S}{P_x}\right)^{-\alpha} \frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = K \quad (3.2.13).$$

Here, left sides of equations (3.2.11), (3.2.12) and (3.2.13) represent total demand for unskilled labour, skilled labour and capital, respectively. On the other hand, right hand sides of those equations represent given endowments of those factors. The first and second terms in the left hand side of equation (3.2.11) represent demand for unskilled labour from the agricultural sector and that from the public input production sector respectively. Similarly the first and second terms in the left hand side of equation (3.2.12) stand for demand for skilled labour from all intermediate goods sector and that from the industrial sector respectively. Finally, first, second, third and fourth terms in the left hand side of equation (3.2.13) represents the amounts of capital demanded by the agricultural sector, by the public input producing sector, by the intermediate goods sector, and by the industrial sector respectively.

The market clearing condition in the intermediate goods sector is given by

$$M\alpha\left(\frac{W_S}{r}\right)^{(-\beta+\alpha\beta)}\left(\frac{W_S}{P_x}\right)^{(1-\alpha)}\frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = nx \qquad (3.2.14),$$

where the left hand side and the right hand side of this equation stand for total demand for and total supply of all intermediate goods respectively.

This model contains nine independent equations (3.2.6)-(3.2.14) with nine endogenous variables; W_S, W_U, r, n, x, P_x, Y, Z and G. The parameters of the model are L_U, L_S , K, P_Z and ψ.

The working of the model is described as follows. From equation (3.2.6), we obtain G in terms of Y. Equation (3.2.10) solves for r in terms of W_U; and equation (3.2.11) solves for Z in terms of G. From equations (3.2.8) and (3.2.9), we solve for x in terms of G. And, from equation (3.2.8), we solve for P_x in terms of W_S and r. Now, substituting these G, Z, x, P_x and r in equations (3.2.7), (3.2.12) and (3.2.14), we simultaneously solve for W_S, W_U and n in terms of Y. Equation (3.2.13) finally solves for Y.

Using equations (3.2.8) and (3.2.9), we obtain

$$x = \frac{\mu}{\lambda G^{\sigma}}\frac{\delta}{(1-\delta)} \qquad (3.2.15).$$

This equation (3.2.15) shows that all varieties are produced in equal quantities and the quantity of every variety falls as the level of production of the public input is expanded.

Here, we make two crucial assumptions which will remain valid in the rest of the analysis: (i) $(\beta(1-\alpha)+\alpha\theta) > \theta$. Here, $(\beta(1-\alpha)+\alpha\theta)$ and θ are interpreted as relative shares of capital in the industrial sector and in the intermediate goods sector respectively. $\beta(1-\alpha)$ and $\alpha\theta$ are direct and indirect relative output shares of capital in the industrial sector. So this assumption implies that the industrial sector is more capital intensive than the intermediate goods sector. (ii) $\beta(1-\alpha)+\alpha\theta > \gamma$. This means that the industrial sector is more capital intensive than the agricultural sector.

If equation (3.2.6) is dropped with G being exogenously given and if there is only one type of labour with two types of capital-foreign capital being used in sector Y and sector X and exogenously given domestic capital being used in sector Z and sector G, then the present model is reduced to the model of Anwar (2006b). If our equations (3.2.3) and (3.2.6) are dropped, i.e., sector G is eliminated from the present model, then it is reduced to the model of Anwar (2006a). Anwar (2006a), Anwar (2009) and Anwar and Rice (2009) analyse the problem of wage inequality using this static product variety framework but none of them considers the role of public input.

3.2.2. Comparative Statics:

3.2.2.1 Change in factor endowments:

We consider one of the followings taking place at a time: (i) an exogenous increase in capital stock, (ii) an exogenous expansion of unskilled labour endowment; and (iii) an exogenous expansion of skilled labour endowment. We do not consider any change in trade and fiscal policies in this section. So $\widehat{\psi} = \widehat{P}_Z = 0$. Using equations (3.2.6), (3.2.7), (3.2.8), (3.2.9), (3.2.11), (3.2.12) and (3.2.14), we obtain[42]

$$\widehat{W}_S = \widehat{Y} - \widehat{L}_S \quad (3.2.16),$$

$$\widehat{W}_U = \frac{\widehat{Y}\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)\right]-\widehat{L}_S\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta-1)\right]}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \quad (3.2.17),$$

and

$$\widehat{n} = \frac{\widehat{L}_S[\beta(1-\alpha)+\alpha\theta-\theta]+\widehat{Y}[\theta+\sigma\{\beta(1-\alpha)+\alpha\theta\}]}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \quad (3.2.18);$$

where, a circumflex is used to denote the proportionate change, i.e., $\widehat{x} = \frac{dx}{x}$.

Equation (3.2.16) shows that a change in the level of industrial output (skilled labour endowment) causes the skilled wage rate to change in the same (opposite) direction. On the other hand, due to the restriction given by $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right)$ in condition (3.2.A), we find

[42] Derivation of equations (3.2.16), (3.2.17) and (3.2.18) are given in the Appendix (3.A).

a direct relationship between the level of industrial output and the unskilled wage rate from equation (3.2.17). Also, equation (3.2.17) shows that a change in skilled labour endowment causes the unskilled wage rate to move in the opposite direction. Since $(\beta(1-\alpha)+\alpha\theta) > \theta$, equation (3.2.18) shows that a change in the level of industrial output and/or a change in the skilled labour endowment causes the number of varieties to change in the same direction.

Using equations (3.2.16) and (3.2.17), we have

$$\widehat{W}_S - \widehat{W}_U = \frac{\widehat{Y}\left[\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right]-\widehat{L}_S\left[\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1-\frac{\alpha(1-\delta)}{\delta}\right]}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \quad (3.2.19).$$

Here, $\left(\widehat{W}_S - \widehat{W}_U\right)$ denotes the rate of change in the skilled-unskilled wage ratio. Here, $\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right) > 0$ because $0 < \gamma, \alpha < 1$. γ is interpreted as the relative share of capital in in the agricultural sector.

We have already assumed that $\beta(1-\alpha)+\alpha\theta > \gamma$; and, with this assumption, we find $\left\{\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma} - 1 + \frac{\sigma\alpha(1-\delta)}{\delta}\right\}$ and $\left\{\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma} - 1 - \frac{\alpha(1-\delta)}{\delta}\right\}$ to be positive. So a change in the level of industrial output (skilled labour endowment) causes the skilled-unskilled relative wage to move in the same (opposite) direction.

However, the level of production of industrial good, Y, itself is endogenously determined; and its rate of change is obtained by using equations (3.2.6), (3.2.11), (3.2.13), (3.2.14) and (3.2.15). It is derived as follows[43].

$$\widehat{Y} = \frac{rK\widehat{K} - \frac{\gamma}{(1-\gamma)}W_U L_U \widehat{L}_U + A\widehat{L}_S}{B} \quad (3.2.20).$$

Here,

$$A = \left[\left\{-\left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} W_U{}^{(1-\varphi)} r^{\varphi} \frac{(\varphi-\gamma)^2}{(1-\varphi)(1-\gamma)\gamma} + \right.\right.$$
$$\left. + \frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}\left\{\frac{1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta-1)}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right\}$$
$$\left. + \frac{W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}} \frac{\left\{(1-\alpha)(1-\beta)\theta+\frac{\alpha\theta}{\delta}(1-\theta)\right\}}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right];$$

and,

[43] Derivation of equation (3.2.20) is given in the Appendix (3.B).

$$B = \left[\left\{-\left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} W_U{}^{(1-\varphi)} r^{\varphi} \frac{(\varphi-\gamma)^2}{(1-\varphi)(1-\gamma)\gamma}\right.\right.$$

$$+\frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S}-\frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right)+\frac{\gamma}{(1-\gamma)} W_U L_U\Bigg\}\left\{\frac{1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right\}$$

$$+\left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} W_U{}^{(1-\varphi)} r^{\varphi} \frac{(\varphi-\gamma)}{(1-\varphi)(1-\gamma)}$$

$$\left.+\frac{W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\frac{\left\{\left(1-\alpha\theta-\frac{\alpha\theta(1-\delta)}{\delta}\right)\theta+(1-\alpha)(1-\theta)\beta+\frac{\alpha\theta}{\delta}(1-\sigma)+\alpha\theta\sigma\right\}}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right].$$

A and B are positive if

(i) $\frac{rK}{W_S L_S} > \frac{\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}$

(ii) $\left(1-\alpha\theta-\frac{\alpha\theta(1-\delta)}{\delta}\right) > 0$ and

(iii) $\varphi = \gamma$.

These are a set of sufficient conditions but not necessary to make A and B positive. Here condition (i) ensures that

$$\frac{rK}{W_S L_S} > \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}$$

because $0 < \theta < 1$. Also condition (i) implies that

$$\frac{rK}{W_S L_S} > \frac{\frac{r(K_Y+K_X)}{Y}}{\frac{W_S(L_{SY}+L_{SX})}{Y}},$$

$\Rightarrow rK > r(K_Y + K_X)$, $\qquad [\because L_{SY} + L_{SX} = L_S]$

$\Rightarrow K > (K_Y + K_X)$;

and the last inequality is always valid because capital is also used in sector Z and in sector G. The assumption that $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right)$ made in inequality (3.2.A) ensures condition (ii). Condition (iii) implies that the agricultural sector and public input producing sector have identical production technologies[44].

If A and B are positive, then, from equation (3.2.20), we find that an exogenous change in capital stock or skilled labour endowment causes the level of output of the industrial sector, Y

[44] This is a restrictive condition. However, it is sufficient but not at all necessary to make A and B positive. Hence none of the results crucially rely on the assumption. If $\varphi \neq \gamma$, then the first term in the expression of both A and B, will be negative. However, still both A and B can be positive because other terms are positive.

to move in the same direction but a change in the unskilled labour endowment makes Y move in the opposite direction.

Using equations (3.2.18) and (3.2.20), we obtain

$$\hat{n} = \frac{\hat{L}_S\left[\{\beta(1-\alpha)+\alpha\theta-\theta\}+\frac{A}{B}\{\theta+\sigma\{\beta(1-\alpha)+\alpha\theta\}\}\right]}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \tag{3.2.18.A}$$

Equation (3.2.18.A) shows a direct relationship between skilled labour endowment and number of varieties to be produced. An increase in skilled labour endowment causes an increase in the number of varieties to be produced because the intermediate goods producing sector uses skilled labour as an input. This is the direct effect and is also obtained in Anwar (2006a). However, there is an additional indirect effect; and it takes place through the expansion of the industrial sector leading to a consequent expansion in tax revenue and, in turn, to an expansion in the public input producing sector. This lowers the fixed cost of producing intermediate inputs and causes a further increase in its number of varieties to be produced. This indirect effect does not exist in Anwar (2006a) because there is no public input in his model. So the rate of increase in the number of varieties in the present model exceeds that obtained in Anwar (2006a), where $\sigma = 0$ and consequently

$$\hat{n} = \frac{\hat{L}_S\left[\{\beta(1-\alpha)+\alpha\theta-\theta\}+\frac{A\theta}{B}\right]}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}.$$

Finally, using equations (3.2.19) and (3.2.20), we obtain

$$\hat{W}_S - \hat{W}_U = \frac{rK\left[\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right]}{B\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\hat{K} - \frac{\frac{\gamma}{(1-\gamma)}W_U L_U\left[\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right]}{B\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\hat{L}_U$$
$$+\frac{\left[\frac{A}{B}\left\{\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right\}-\left\{\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1-\frac{\alpha(1-\delta)}{\delta}\right\}\right]}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\hat{L}_S \tag{3.2.21}$$

Equation (3.2.21) shows how skilled-unskilled wage ratio, $\frac{W_S}{W_U}$, is changed due to changes in different factor endowments. Here, a change in capital (unskilled labour) endowment causes the skilled-unskilled wage ratio to move in the same (opposite) direction and the magnitude of this comparative static effect is minimum when $\sigma = 0$, i.e., when we do not consider the role of public input. However, the effect of a change in the skilled labour endowment on this skilled-unskilled wage ratio is always ambiguous because A and B can take any positive values. So we can establish the following proposition.

<u>PROPOSITION-3.2.1</u>: *If production technologies are identical for the agricultural sector and for the public input producing sector, and if assumption (3.2.A) is valid, then given other parameters, (i) an increase in capital stock raises the skilled-unskilled wage ratio; (ii) an increase in unskilled labour endowment lowers the skilled-unskilled wage ratio; and (iii) an increase in skilled labour endowment makes the skilled-unskilled wage ratio move in any direction.*

We now provide intuitions behind this result. An increase in capital stock leads to an expansion in the industrial sector, Y, through an increased allocation of capital to this sector. So the demand for skilled labour is increased and this raises the skilled wage rate. This effect exists even in Anwar (2006a). On the other hand, this expansion of sector, Y, raises the tax revenue of the government which, in turn, expands the size of the public input, G. This lowers the fixed cost of producing intermediate goods. So the number of varieties of private intermediate goods are increased in the group equilibrium and the quantity of each variety produced falls. As the number of varieties of intermediate goods are increased, level of production of the industrial sector, Y, is improved; and this creates a further increase in the demand for skilled labour. This additional effect does not exist in Anwar (2006a). Capital allocations to unskilled labour using sectors, Z and G, are also increased; and this raises the demand for unskilled labour leading to an increase in the unskilled wage rate. However, sector Y is more capital intensive than sector Z and production technologies are same in sectors Z and G. So there should be a higher rate of allocation of capital to sector Y compared to corresponding rates in sectors Z and G. Hence the rate of increase in the skilled wage rate should be more than that in the unskilled wage rate; and so the skilled-unskilled wage ratio is increased. The rate of increase in skilled-unskilled wage ratio in this model is higher than that in Anwar (2006a).

On the other hand, an increase in the unskilled labour endowment lowers the unskilled wage rate. It causes expansions of sector Z and sector G. So capital moves from sector Y to sector Z and to sector G; and thus sector Y contracts. This lowers the demand for skilled labour leading to a decline in the skilled wage rate. As sector Y is more capital intensive than sectors Z and G, the rate of decrease in the skilled wage rate is more than that in the unskilled wage rate.

So the skilled-unskilled wage ratio is reduced; and the rate of decline in the present model is higher than that in Anwar (2006a).

When skilled labour endowment is increased, then skilled-unskilled wage ratio should fall directly and this direct effect is also obtained in Anwar (2006a). However, in the present model, there is an additional indirect effect that takes place through expansion of sector Y; and this raises number of intermediate inputs produced through expansion in the public input producing sector which lowers the fixed cost of producing varieties. So the demand for skilled labour is increased leading to a consequent increase in the skilled-unskilled wage ratio due to this additional effect. Hence the net effect on skilled-unskilled wage ratio remains ambiguous in this model while this ratio goes down in Anwar (2006a).

Anwar (2006a) shows that an increase in unskilled labour (capital) endowment lowers (raises) the skilled unskilled wage inequality; and the condition required to show this result is $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta}\right)$. We obtain a similar qualitative result in the present model with the condition given by $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right)$ and with the agricultural sector and the public input producing sector having identical production technologies. As in Anwar (2006a), a public input producing sector is absent, so $\sigma = 0$ there. So conditions required to obtain similar results in Anwar (2006a) are modified in the present model. In Glazer and Ranjan (2003), skilled workers prefer to consume skill intensive goods. So with an increase in skilled (unskilled) labour endowment, relative demand for skill intensive goods and consequently the relative demand for skilled labour are increased (decreased) leading to an increase (decrease) in the skilled-unskilled wage ratio. Anwar and Rice (2009) shows that an increase in either type of labour has no effect on skilled-unskilled relative wage in the short run when there is no entry or exit in the intermediate good producing sector. However, in the long run, with free entry and or exit in that sector, an increase in skilled labour endowment causes entries of new firms and thus raises the demand for skilled labour and consequently the skilled-unskilled wage ratio.

3.2.2.2 Change in fiscal policies:

In this section, we analyze effects of various trade and fiscal policies. We consider one of the followings given the others: (i) an exogenous increase in the tax rate on the production of sector Y; and (ii) an exogenous increase in price of the product produced by the agricultural sector, Z. Changes in fiscal instruments affect the system through changes in prices of traded goods. Subsidization to the agricultural sector raises the price of its product. Using equations (3.2.6), (3.2.7), (3.2.8), (3.2.9), (3.2.11), (3.2.12) and (3.2.14), we obtain[45]

$$\widehat{W}_S = \widehat{Y} - \frac{\psi}{1-\psi}\widehat{\psi} \qquad (3.2.16.1);$$

$$\widehat{W}_U = \frac{\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)\right]\widehat{Y}+\left[\frac{\psi}{1-\psi}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)-\frac{\alpha\sigma(1-\delta)}{\delta}\right]\widehat{\psi}+\frac{1}{\gamma}\left[\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right]\widehat{P}_Z}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.17.1);$$

and

$$\widehat{n} = \frac{\frac{\sigma(1-\gamma)}{\gamma}[\beta(1-\alpha)+\alpha\theta]\widehat{\psi}+[\theta+\sigma\{\beta(1-\alpha)+\alpha\theta\}]\widehat{Y}}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.18.1).$$

Equation (3.2.16.1) shows that a change in the industrial output level (tax rate on industrial output) causes the skilled wage rate to move in the same (opposite) direction. On the other hand, equation (3.2.17.1) shows that a change in the industrial output level makes the unskilled wage rate move in the same direction because assumption (3.2.A) states that $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right)$. Similarly, this equation (3.2.17.1) shows that a change in the tax rate on industrial output makes the unskilled wage rate move in the opposite direction because assumption (3.2.A) also implies that $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{\psi(\beta(1-\alpha)+\alpha\theta)}{\sigma\gamma(1-\psi)-\theta\psi}\right)$. Also this equation shows a direct relationship between the price of the agricultural product and the unskilled wage rate. Finally, equation (3.2.18.1) shows that a change in the industrial output level and/or a change in the tax rate on industrial output affects the number of varieties in the same direction.

Using equations (3.2.16.1) and (3.2.17.1), we have

$$\widehat{W}_S - \widehat{W}_U = \frac{\left[\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right]\widehat{Y}-\left[\frac{\psi}{1-\psi}\frac{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}{\gamma}-\frac{\alpha\sigma(1-\delta)}{\delta}\right]\widehat{\psi}-\frac{1}{\gamma}\left[\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right]\widehat{P}_Z}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.19.1).$$

[45] Derivation of equations (3.2.16.1), (3.2.17.1) and (3.2.18.1) are given in the Appendix (3.C).

We have already assumed that $\beta(1-\alpha)+\alpha\theta > \gamma$; and so, with inequality (3.2.A), equation (3.2.19.1) implies that a change in Y makes $\frac{W_S}{W_U}$ moves in the same direction but a change in P_Z or ψ makes $\frac{W_S}{W_U}$ move in the opposite direction.

Here Y is endogenously determined and, using equations (3.2.6), (3.2.11), (3.2.13), (3.2.14) and (3.2.15), we derive its rate of change as follows[46].

$$\widehat{Y} = \frac{C\widehat{P}_Z + D\widehat{\psi}}{B} \qquad (3.2.20.1),$$

where,

$$C = -\left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} W_U^{(1-\varphi)} r^{\varphi} \frac{(\varphi-\gamma)\phi}{(1-\varphi)(1-\gamma)\gamma} + \left(\frac{\varphi}{1-\varphi}\right)^{-\varphi} W_U^{(1-\varphi)} r^{\varphi} \frac{(\varphi-\gamma)^2}{(1-\varphi)(1-\gamma)^2\gamma} - \frac{\gamma}{(1-\gamma)^2} W_U L_U \, ;$$

and,

$$D =$$

$$-\frac{\psi}{(1-\psi)W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) - \left(\frac{\gamma}{(1-\gamma)}\right)^2 \frac{\psi}{1-\psi} W_U L_U +$$
$$\frac{\sigma\alpha(1-\delta)\gamma}{\delta(1-\gamma)\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\left\{\frac{\theta W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}} + \frac{(1-\gamma)\gamma}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}.$$

Using the sufficient condition, $\varphi = \gamma$, we find that $C < 0$.

Also assumption (3.2.A) implies that,

$$\frac{\alpha(1-\delta)}{\delta} < \left(\frac{(\beta(1-\alpha)+\alpha\theta)}{\dfrac{\sigma\left\{\frac{\theta W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}} + \frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}}{\left\{\frac{\psi}{(1-\psi)W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \left(\frac{\gamma}{(1-\gamma)}\right)^2 \frac{\psi}{1-\psi} W_U L_U\right\}} - \theta}\right);$$

and so sufficient conditions which make A and B positive can also ensure C and D to be negative.

If C and D are negative, then, from equation (3.2.20.1), we find that an exogenous change in price of the agricultural product, P_Z, and/or in the tax rate, ψ, induces the level of output of the industrial sector to move in the opposite direction.

Using equations (3.2.18.1) and (3.2.20.1), we obtain

$$\hat{n} = \frac{[\theta + \sigma\{\beta(1-\alpha)+\alpha\theta\}]\frac{C}{B}\widehat{P}_Z}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.18.1.A)$$

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[46] Derivation of equation (3.2.20.1) is given in the Appendix (3.D).

Equation (3.2.18.1.A) shows an inverse relationship between the price of the agricultural product and number of varieties to be produced because $\mathrm{B} > 0$ and $\mathrm{C} < 0$. This is qualitatively similar to Anwar (2006a). An increase in the price of the agricultural product raises the demand for unskilled labour and capital. Capital moves from other sectors to agricultural sector. So number of varieties of intermediate goods produced is reduced. This direct effect is also obtained in Anwar (2006a). However, in our model, the total negative effect is stronger than that in Anwar (2006a) because there is an additional indirect effect that takes place through the contraction of the industrial sector and, in turn, leads to a contraction of the public input producing sector. This raises the fixed cost of producing intermediate inputs and thus lowers their numbers further. This second effect does not exist in Anwar (2006a) where $\sigma = 0$ and consequently

$$\hat{n} = \frac{\theta\frac{C}{B}\hat{P}_Z}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}.$$

Using equations (3.2.19.1) and (3.2.20.1), we obtain

$$\widehat{W}_S - \widehat{W}_U =$$

$$\frac{-\left[\frac{\psi}{1-\psi}\frac{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}{\gamma}-\frac{\alpha\sigma(1-\delta)}{\delta}-\frac{D}{B}\left(\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right)\right]\hat{\psi}-\frac{1}{\gamma}\left[\beta(1-\alpha)+\frac{\alpha\theta}{\delta}-\frac{C}{B}\left(\frac{\beta(1-\alpha)+\alpha\theta+\frac{\alpha\theta(1-\delta)}{\delta}}{\gamma}-1+\frac{\sigma\alpha(1-\delta)}{\delta}\right)\right]\hat{P}_Z}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}$$

(3.2.21.1).

This equation shows how skilled-unskilled wage ratio, $\frac{W_S}{W_U}$, is altered due to exogenous change in the price of the agricultural product, P_Z, and due to exogenous change in the tax rate, ψ. Both these changes cause the skilled-unskilled wage ratio to move in the opposite direction. So we can establish the following proposition.

PROPOSITION-3.2.2: *If the agricultural product producing sector and the public input producing sector have identical production technologies, and if assumption (3.2.A) is valid, then given other parameters, an increase (decrease) in the tax rate on industrial output and/or an increase (decrease) in the price of the agricultural product lowers (raises) the skilled-unskilled wage ratio.*

An increase in the tax rate on industrial output lowers the level of demand for skilled labour and the level of demand for capital in the industrial sector. So the skilled wage rate falls and capital moves from this sector to other sectors. Capital allocations to unskilled labour using sectors, Z and G, are increased; and this raises the demand for unskilled labour leading to an increase in the unskilled wage rate. Sector G expands and the fixed cost of producing intermediate goods is reduced. Sector Y contracts, and the demand for intermediate goods is also reduced. So the final effect on the number of varieties of intermediate goods to be produced in group equilibrium remains ambiguous. However, the skilled-unskilled relative wage is reduced. No other model in the existing literature analyses the effect of an exogenous change in the tax rate.

On the other hand, an increase in the price of the agricultural product raises the demand for unskilled labour and capital. So the unskilled wage rate is increased. Capital moves from other sectors to agricultural sector. So sector Y contracts and hence demand for skilled labour is reduced leading to a decline in the skilled wage rate. So the skilled-unskilled wage ratio is reduced.

Anwar (2009) also shows that an increase in the price of the agricultural product lowers the skilled-unskilled wage ratio even though his model does not include physical capital as an input and a public input producing sector. The condition required to show that result in Anwar (2009) is $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\alpha\theta}{\theta}\right)$. We obtain the similar qualitative result in the present model if $\frac{\alpha(1-\delta)}{\delta} < \left(\frac{1-\beta(1-\alpha)-\alpha\theta}{\theta+\sigma}\right)$ and if the agricultural sector and public input producing sector have identical production technologies. $\beta = 0$ implies that capital is not used as an input; and $\sigma = 0$ implies that public input can not affect the fixed cost of producing varieties. With $\beta = 0$ and $\sigma = 0$, our sufficient condition is identical to that in Anwar (2009). Since $\beta > 0$ and $\sigma > 0$, in our model the rate of decline in skilled unskilled relative wage is higher here.

3.3. The Model with unemployment:[47]

In this section, we consider unemployment of either type of labour. However, we do not consider any public good producing sector in this section for simplicity. This section introduces efficiency wage hypothesis to explain an unemployment equilibrium in each of these two

[47]Dutta (2012) is partly based on the materials presented in this section.

labour markets. According to the efficiency wage hypothesis, efficiency of a labourer varies positively with its wage rate and the unemployment rate in the labour market. There is enough empirical support in the literature to justify introducing unemployment. Marjit and Acharyya (2003) shows that there is growing unemployment problem with rising wage inequality in European countries[48] and the effect on unemployment due to trade liberalization is mixed in developing countries[49]. Gini-Coefficient of wage income distribution is also considered as the measure of wage income inequality in addition to the skilled-unskilled wage ratio[50].

This section develops a three sector small open economy model with two traded final good sectors and a nontraded good sector producing varieties of intermediate goods. There are three primary factors: capital, skilled labour and unskilled labour. Industrial sector producing a traded good uses capital, intermediate goods and skilled labour as inputs. Intermediate goods producing sector also uses capital and skilled labour. The efficiency wage hypothesis is introduced to explain unemployment in each of these two labour markets. It is shown that an increase in either type of labour endowment (capital endowment) raises (lowers) the unemployment rate of either type of labour if the scale elasticity of output is very small. On the other hand, if the industrial sector is more capital intensive than the agricultural sector and if efficiency functions of both types of labour are identical, then an increase in either type of labour endowment (capital endowment) lowers (raises) the skilled-unskilled wage ratio. However, the effect of a change in capital endowment on the Gini Coefficient of wage income distribution is ambiguous in sign.

This section is organized as follows. Sub-section 3.3.1 describes the model; and section 3.3.2 analyzes effects of changes in factor endowments on unemployment rate, skilled-unskilled relative wage, skilled-unskilled average income ratio and Gini-Coefficient of wage-income distribution.

3.3.1 <u>Description:</u>

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[48] See Table 2.2 in page 11 of Marjit and Acharyya (2003).

[49] See Table 2.11 in page 31 of Marjit and Acharyya (2003).

[50] Gupta and Dutta (2011) have also considered Gini-Coefficient of wage income distribution as a measure of wage inequality. But, they have considered unemployment of both types of labour in a competitive general equilibrium model.

We consider a small open economy consisting of two traded good sectors, Y and Z, and a nontraded good sector, X. However, there is no public input producing sector. All other assumptions made in section 3.2 are also valid here. Skilled labour is mobile between sector Y and sector X; and unskilled labour is specific to sector Z. Each of these two types of labour is measured in efficiency unit; and both wage rates are perfectly flexible. There exist unemployment in both these two labour markets; and the existence of unemployment equilibrium is explained by the efficiency wage hypothesis which states that the efficiency of either type of labourer varies positively with its wage rate as well as with its unemployment rate[51].

Production functions in sectors Y and Z are same as in section 3.2 and are given by followings.

$$\mathrm{Y} = \left(\mathrm{L}_{\mathrm{SY}}^{1-\beta}\mathrm{K}_{\mathrm{Y}}^{\beta}\right)^{1-\alpha}\left(\sum_{i=1}^{n} x_i^{\delta}\right)^{\frac{\alpha}{\delta}} \qquad (3.3.1);$$

and

$$\mathrm{Z} = \mathrm{L}_{\mathrm{U}}^{1-\gamma}\mathrm{K}_{\mathrm{Z}}^{\gamma} \qquad (3.3.2).$$

Here, L_{SY} and L_{U} stand for the level of employment of skilled labour in sector Y and of unskilled labour in sector Z, respectively and not endowment because there exists unemployment in both the labour markets. Other notations indicate the same as in section 3.2.

The total cost function of the ith variety produced in sector X is given by

$$c(\mathrm{W}_{\mathrm{S}}, r, x_i) = (\mu + \lambda x_i)\left(\frac{\mathrm{W}_{\mathrm{S}}}{h}\right)^{1-\theta} r^{\theta} \text{ with } 0 < \theta < 1;\ \mu, \lambda > 0. \qquad (3.3.3).$$

Here, all the notations have their usual meanings used in section 3.2. Putting $\sigma = 0$ and replacing W_{S} by $\left(\frac{\mathrm{W}_{\mathrm{S}}}{h}\right)$ in equation (3.2.4) we obtain this equation (3.3.3). Effective unit cost of employing skilled labour is given by $\left(\frac{\mathrm{W}_{\mathrm{S}}}{h}\right)$.

[51] Our efficiency function is a special case of the more general efficiency function considered in the fair wage hypothesis developed by Agell and Lundborg (1992, 1995) where rental rate on capital also appears as an argument. Chaudhuri and Banerjee (2010) use this more general efficiency function.

Here $\mu\left(\frac{W_S}{h}\right)^{1-\theta} r^{\theta}$ and $\lambda x_i\left(\frac{W_S}{h}\right)^{1-\theta} r^{\theta}$ represent the fixed cost and variable cost of producing the ith intermediate good, respectively. Here also, $x_i = x$ for all i; and hence the total production of sector X is nx. Hence, equation (3.3.1) can be rewritten as

$$Y = L_{SY}^{(1-\beta)(1-\alpha)} K_Y^{\beta(1-\alpha)} X^{\alpha} n^{\frac{\alpha(1-\delta)}{\delta}} \qquad (3.3.4).$$

Here also, $\frac{\alpha(1-\delta)}{\delta}$ indicates the scale elasticity of output.

Using equations (3.3.1)–(3.3.4), we find that the first order condition of profit maximization in sector Y is given by

$$1 = M\left(\frac{W_S}{h}\right)^{(1-\alpha)(1-\beta)} r^{\beta(1-\alpha)} P_x^{\alpha} n^{-\frac{\alpha(1-\delta)}{\delta}} \qquad (3.3.5),$$

where,

$$M = \frac{\alpha^{-\alpha}\beta^{-\beta(1-\alpha)}}{(1-\alpha)^{(1-\alpha)}(1-\beta)^{(1-\alpha)(1-\beta)}} > 0.$$

Here P_x and h stand for the price of the representative intermediate good and the efficiency of the skilled worker respectively.

In the short run, the first order condition of profit maximizing in sector X is given by

$$\delta P_x = \lambda\left(\frac{W_S}{h}\right)^{1-\theta} r^{\theta} \qquad (3.3.6).$$

The group equilibrium condition is given by

$$(1-\delta)P_x x = \mu\left(\frac{W_S}{h}\right)^{1-\theta} r^{\theta} \qquad (3.3.7).$$

The first order condition of profit maximizing in the agricultural sector, Z, is given by

$$P_Z = \left[\frac{1}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}}\right]\left(\frac{W_U}{f}\right)^{1-\gamma} r^{\gamma} \qquad (3.3.8),$$

where P_Z, W_U and f stand for price of the agricultural good, wage rate of unskilled labour and efficiency of unskilled labour respectively. $\left(\frac{W_U}{f}\right)$ represents the effective unit cost of employing unskilled labour. Efficiency functions of skilled labour and of unskilled labour are same as defined in section 2.4 of chapter 2 and are given by following two equations.

$$h = h(W_S, v_S)\text{ , with } h_1 > 0, h_2 > 0 \text{ and } h_{11} < 0, h_{22} < 0 \qquad (3.3.9),$$

and

$$f = f(W_U, v_U)\text{ , with } f_1 > 0, f_2 > 0 \text{ and } f_{11} < 0, f_{22} < 0 \qquad (3.3.10).$$

Each of these two efficiencies is a positive and concave function in terms of every argument.

$\left(\frac{W_S}{h}\right)$ and $\left(\frac{W_U}{f}\right)$ are minimized with respect to W_S and W_U, respectively; and the two first-order conditions of minimization are given by following two equations.

$$\frac{\partial h}{\partial W_S}\frac{W_S}{h} = 1 \qquad (3.3.11),$$

and

$$\frac{\partial f}{\partial W_U}\frac{W_U}{f} = 1 \qquad (3.3.12).$$

Equations (3.3.11) and (3.3.12) are basically two modified Solow (1979) conditions. These imply that wage elasticities of efficiency are equal to unity in these two labour markets.

Equilibrium conditions in markets of unskilled labour, skilled labour and capital are given by following equations.

$$\left[\frac{\gamma}{(1-\gamma)}\right]^{-\gamma}\left(\frac{W_U/f}{r}\right)^{-\gamma} Z = L_U(1-v_U)f \qquad (3.3.13);$$

$$n(\mu+\lambda x)(1-\theta)\left(\frac{W_S/h}{r}\right)^{-\theta} + M(1-\alpha)(1-\beta)\left(\frac{W_S/h}{r}\right)^{(-\beta+\alpha\beta)}\left(\frac{W_S/h}{P_x}\right)^{-\alpha} Yn^{-\frac{\alpha(1-\delta)}{\delta}} =$$

$$L_S(1-v_S)h \qquad (3.3.14);$$

and

$$\left[\frac{\gamma}{(1-\gamma)}\right]^{(1-\gamma)}\left(\frac{W_U/f}{r}\right)^{(1-\gamma)} Z + n(\mu+\lambda x)\theta\left(\frac{W_S/h}{r}\right)^{(1-\theta)} +$$

$$M\beta(1-\alpha)\left(\frac{W_S/h}{r}\right)^{(1-\beta+\alpha\beta)}\left(\frac{W_S/h}{P_x}\right)^{-\alpha}\frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = K \qquad (3.3.15).$$

Here, v_S and v_U stand for unemployment rates of skilled labour and unskilled labour, respectively. Equations (3.3.13) and (3.3.14) are unemployment adjusted equilibrium conditions in the skilled labour market and in the unskilled labour market, respectively. Equation (3.3.15) stands for the equilibrium condition in the capital market.

The market clearing condition in the intermediate goods sector is given by

$$M\alpha\left(\frac{W_S/h}{r}\right)^{(-\beta+\alpha\beta)}\left(\frac{W_S/h}{P_x}\right)^{(1-\alpha)}\frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = nx \qquad (3.3.16),$$

This model consists of twelve independent equations given by (3.3.5)-(3.3.16) with twelve endogenous variables given by W_S, W_U, r, h, f, v_S, v_U, n, x, P_x, Y and Z. The parameters of the model are L_U, L_S , K and P_Z.

The working of the model is described as follows. From equations (3.3.6) and (3.3.7), we solve for x and r in terms of W_S, h and P_x. Then, using equation (3.3.5) and equations (3.3.8) to (3.3.12), we simultaneously solve for W_S, W_U, h, f, v_S, v_U in terms of P_x and n. Finally equations (3.3.13) – (3.3.16) simultaneously solve for n, P_x, Y and Z.

Using equations (3.3.6) and (3.3.7), we obtain

$$x = \frac{\mu}{\lambda}\frac{\delta}{(1-\delta)} \qquad (3.3.17);$$

and this equation shows that all varieties are produced in equal quantities.

If the present model excludes equations (3.3.9), (3.3.10), (3.3.11) and (3.3.12), i.e., if it is a full employment model, then it is identical to that of Anwar (2006a). It should be noted that Anwar (2006a), Anwar (2009) and Anwar and Rice (2009) analyse the problem of skilled-unskilled wage inequality using this static endogenous product variety framework without considering the problem of unemployment.

3.3.2. Change in factor endowments:

We consider exogenous change in factor endowments one by one given P_Z.

The relative rates of change in unemployment rates and number of varieties of intermediate goods are obtained as follows[52].

$$\hat{v}_S = \frac{\varepsilon_{v_U}\left[-\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}\left(B\hat{L}_S-\hat{K}+A\hat{L}_U\right)-\frac{\alpha(1-\delta)}{\delta}\left[\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\hat{L}_S-\left(\hat{K}-A\hat{L}_U\right)\frac{\theta(1-\gamma)}{\gamma}\right]\right]}{D} \qquad (3.3.18),$$

$$\hat{v}_U = \frac{\varepsilon_{v_S}\left[-(1-\beta(1-\alpha)-\alpha\theta)\left(B\hat{L}_S-\hat{K}+A\hat{L}_U\right)-\frac{\alpha(1-\delta)}{\delta}\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\left(\hat{K}-A\hat{L}_U\right)+B(1-\theta)\hat{L}_S\right]\right]}{D} \qquad (3.3.19),$$

[52] Derivations of equations (3.3.18), (3.3.19) and (3.3.20) are given in the Appendix (3.E).

and

$$\hat{n} = \frac{-\varepsilon_{v_U}\varepsilon_{v_S}\left[\left\{(1-\beta(1-\alpha)-\alpha\theta)\frac{\theta(1-\gamma)}{\gamma}+\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)\right\}(\hat{K}-A\hat{L}_U)\right.}{D}$$

$$\frac{\left.+\varepsilon_{v_U}\varepsilon_{v_S}\left\{(1-\beta(1-\alpha)-\alpha\theta)\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}-\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}B(1-\theta)\right\}\hat{L}_S\right]}{D} \quad (3.3.20).$$

Here,

$$A = \frac{\frac{\gamma}{(1-\gamma)}\left(\frac{W_U/f}{r}\right)L_U(1-v_U)f}{K} > 0;$$

$$B = \frac{\frac{\mu n}{(1-\delta)}\left(\frac{W_S/h}{r}\right)^{(1-\theta)}\left(\theta+\frac{\beta(1-\alpha)}{\alpha}\right)}{K} > 0;$$

and

$$D = -\varepsilon_{v_U}\varepsilon_{v_S}\left[(1-\beta(1-\alpha)-\alpha\theta)\left[B\frac{\theta(1-\gamma)}{\gamma}-\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\right]+\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}B\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}+(1-\theta)\right]\right.$$
$$\left.+\frac{\alpha(1-\delta)}{\delta}\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}+B(1-\theta)\frac{\theta(1-\gamma)}{\gamma}\right]\right].$$

ε_{v_U} (ε_{v_S}) represents the elasticity of the efficiency of unskilled (skilled) labour with respect to unskilled (skilled) unemployment rate.

If the efficiency function of the skilled labour is Cobb-Douglas type, then it can be shown that $\left(\frac{h}{h_{11}(W_S)^2}+1\right) < 0$. If the scale elasticity of output is very small, i.e., if $\frac{\alpha(1-\delta)}{\delta} \to 0$, then $D < 0$ because $(1-\beta(1-\alpha)-\alpha\theta)\left[B\frac{\theta(1-\gamma)}{\gamma}-\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\right]$ as well as $\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}B\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}+(1-\theta)\right]$ is positive. So, from equations (3.3.18), (3.3.19) and (3.3.20), we can conclude that, if $\frac{\alpha(1-\delta)}{\delta}$ is very small, then both v_S and v_U vary positively with L_U and L_S and inversely with K. This leads to the following proposition.

PROPOSITION 3.3.1: *An increase in either type of labour (capital) endowment raises (lowers) the unemployment rate of either type of labour if the scale elasticity of output is very small.*

The intuition behind this result is the following. When one type of labour endowment is increased then unemployment rate of that type of labour is also increased; and perfect inter sectoral mobility of capital explains the increase in unemployment rate of another type of labour. If capital endowment is increased, then demand for both types of labour are increased due to perfect inter sectoral mobility of capital; and so unemployment rates of both type of labour are reduced. On the other hand, an increase in skilled labour endowment and/or capital (unskilled labour) endowment raises (lowers) the number of varieties of intermediate inputs. This is so because only capital and skilled labour and not unskilled labour are used as inputs to produce intermediate goods.

3.3.2.1. Skilled-unskilled relative wage:

Relative rates of change in wage rates of skilled workers and of unskilled workers are given by

$$\widehat{W}_S = \frac{\varepsilon_{v_S} h}{h_{11}(W_S)^2}\hat{v}_S \qquad (3.3.21);$$

and

$$\widehat{W}_U = \frac{\varepsilon_{v_U} f}{f_{11}(W_U)^2}\hat{v}_U \qquad (3.3.22).$$

Using equations (3.3.21) and (3.3.22), we obtain

$$\hat{\Delta} = \widehat{W}_S - \widehat{W}_U = \frac{\varepsilon_{v_S} h}{h_{11}(W_S)^2}\hat{v}_S - \frac{\varepsilon_{v_U} f}{f_{11}(W_U)^2}\hat{v}_U \qquad (3.3.23).$$

where, $\Delta = \frac{W_S}{W_U}$ represents the skilled-unskilled relative wage.

Using equations (3.3.18), (3.3.19) and (3.3.23), we have

$$\hat{\Delta} = \frac{\varepsilon_{v_U}\varepsilon_{v_S}\left(B\hat{L}_S - \hat{K} + A\hat{L}_U\right)}{D}\left[-\frac{h}{h_{11}(W_S)^2}\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma} + \frac{f}{f_{11}(W_U)^2}(1-\beta(1-\alpha)-\alpha\theta)\right]$$
$$-\frac{\varepsilon_{v_U}\varepsilon_{v_S}}{D}\frac{\alpha(1-\delta)}{\delta}\left[\left\{\frac{h}{h_{11}(W_S)^2}\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma} - \frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\right.\right.$$
$$\left.\left. + \frac{f}{f_{11}(W_U)^2}B(1-\theta)\right\}\hat{L}_S - \left\{\frac{\theta(1-\gamma)}{\gamma}\frac{h}{h_{11}(W_S)^2} + \frac{f}{f_{11}(W_U)^2}\left\{\left(\theta - \frac{h}{h_{11}(W_S)^2} - 1\right) + \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\right\}\right.$$

$\left(\widehat{K}-A\widehat{L}_U\right)]$ (3.3.24).

Here, $\frac{f}{f_{11}(W_U)^2}$ is the reciprocal of the elasticity of marginal efficiency of unskilled labour, i.e., $\frac{\partial f}{\partial W_U}$, with respect to unskilled wage rate; and this always takes a negative sign. Similarly, $\frac{h}{h_{11}(W_S)^2}$ is the reciprocal of the elasticity of marginal efficiency of skilled labour, i.e., $\frac{\partial h}{\partial W_S}$, with respect to skilled wage rate and this is also always negative. So both $\frac{f}{f_{11}(W_U)^2}$ and $\frac{h}{h_{11}(W_S)^2}$ are always negative.

We consider a special case where

$\frac{f}{f_{11}(W_U)^2}=\frac{h}{h_{11}(W_S)^2}=\vartheta<0.$

This special case arises when the efficiency functions of two types of labour are identical.

Then equation (3.3.24) is reduced to the following.

$$\widehat{\Delta}=\frac{\varepsilon_{v_U}\varepsilon_{v_S}\left(B\widehat{L}_S-\widehat{K}+A\widehat{L}_U\right)\vartheta}{D}[\gamma-\beta(1-\alpha)-\alpha\theta]$$

$$-\frac{\varepsilon_{v_U}\varepsilon_{v_S}}{D}\frac{\alpha(1-\delta)\vartheta}{\delta}\left[\left\{\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{\vartheta}{\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}+B(1-\theta)\right\}\widehat{L}_S-\left\{\frac{\theta(1-\gamma)}{\gamma}+\right.\right.$$

$$\left.\left.\left\{(\theta-\vartheta-1)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\right\}\left(\widehat{K}-A\widehat{L}_U\right)\right] \qquad (3.3.25).$$

Equation (3.3.25) shows that, when the scale elasticity of output is weak, i.e., when $\frac{\alpha(1-\delta)}{\delta}\rightarrow 0$, the effects of changes in different factor endowments on the skilled-unskilled wage ratio depends on the nature of capital intensity ranking between the industrial sector and the agricultural sector. Even if capital intensities are same in both the sectors, i.e., $\gamma=\beta(1-\alpha)+\alpha\theta$, then also we find a change in the skilled-unskilled wage ratio due to changes in factor endowments and this phenomenon is explained by the presence of specialization based external economics. So, from equation (3.3.25), we can conclude that the effect of changes in different factor endowments on the skilled-unskilled wage ratio depends on: (i) capital intensity ranking between the industrial sector and the agricultural sector, (ii) the magnitude of the scale elasticity of output and (iii) properties of efficiency functions of two types of labour.

We can establish the following proposition.

PROPOSITION-3.3.2: *If the industrial sector is more capital intensive than the agricultural sector and if efficiency functions of both type of labourer are identical, then an increase in either type of labour (capital) endowment lowers (raises) the skilled-unskilled wage ratio when scale elasticity of output is very low.*

The intuition behind this result is as follows. An increase in the skilled labour endowment is not accompanied by corresponding increases in the demand for skilled labour in the industrial sector and in the intermediate good producing sector. So the skilled wage rate falls and the rental rates on capital in those sectors go up. Hence capital moves from the agricultural sector to the industrial sector and to the intermediate goods producing sector. So the demand for unskilled labour is also reduced; and this leads to a fall in the unskilled wage rate. However, as the industrial sector is more capital intensive than the agricultural sector, so the rate of decline in the skilled wage rate exceeds that of the unskilled wage rate. Hence skilled-unskilled wage ratio falls. Same mechanism explains why an increase in unskilled labour endowment also lowers the skilled-unskilled wage ratio. Finally, when capital endowment is increased, demand for skilled labour as well as the demand for unskilled labour goes up. Since both labour endowments remain unchanged, the two wage rates go up. Once again capital intensity ranking assumption between the industrial sector and the agricultural sector explains why skilled-unskilled wage ratio should be increased.

3.3.2.2. Skilled-unskilled average income ratio:

In the presence of unemployment, the skilled-unskilled average income ratio can be derived in similar fashion as in section 2.4 of chapter 2. Here,

$\mathrm{R} = \frac{\mathrm{W_S}(1-\mathrm{v_S})}{\mathrm{W_U}(1-\mathrm{v_U})}$;

and $\mathrm{W_S}(1-\mathrm{v_S})$ and $\mathrm{W_U}(1-\mathrm{v_U})$ represent average incomes of skilled workers and of unskilled workers respectively.

The relative rate of change in the skilled-unskilled average income ratio is given as follows.

$$\widehat{\mathrm{R}} = \widehat{\mathrm{W}}_\mathrm{S} - \frac{\mathrm{v_S}}{(1-\mathrm{v_S})}\hat{\mathrm{v}}_\mathrm{S} - \widehat{\mathrm{W}}_\mathrm{U} + \frac{\mathrm{v_U}}{(1-\mathrm{v_U})}\hat{\mathrm{v}}_\mathrm{U} \qquad (3.3.26).$$

Using equations (3.3.18), (3.3.19), (3.3.21) and (3.3.22), we obtain

$$\widehat{R} = \frac{\varepsilon_{v_U}\varepsilon_{v_S}(B\widehat{L}_S - \widehat{K} + A\widehat{L}_U)}{D}\Big[-\left(\frac{h}{h_{11}(W_S)^2} - \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right)\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma} +$$

$$\left(\frac{f}{f_{11}(W_U)^2} - \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right)(1-\beta(1-\alpha)-\alpha\theta)\Big]$$

$$-\frac{\varepsilon_{v_U}\varepsilon_{v_S}}{D}\frac{\alpha(1-\delta)}{\delta}\Bigg[\Bigg\{\left(\frac{h}{h_{11}(W_S)^2} - \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right)\Bigg\{\{A + B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}$$

$$-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\Bigg\} + \left(\frac{f}{f_{11}(W_U)^2} - \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right)B(1-\theta)\Bigg\}\widehat{L}_S$$

$$-\Bigg\{\frac{\theta(1-\gamma)}{\gamma}\left(\frac{h}{h_{11}(W_S)^2} - \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right) + \left(\frac{f}{f_{11}(W_U)^2} - \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right)$$

$$\Bigg\{\left(\theta - \frac{h}{h_{11}(W_S)^2} - 1\right) + \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\Bigg\}\Bigg\}(\widehat{K} - A\widehat{L}_U)\Bigg] \qquad (3.3.27).$$

In the special case, where

$$\left(\frac{f}{f_{11}(W_U)^2} - \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right) = \left(\frac{h}{h_{11}(W_S)^2} - \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right) = \vartheta < 0,$$

equation (3.3.27) is reduced to the following.

$$\widehat{R} = \frac{\varepsilon_{v_U}\varepsilon_{v_S}(B\widehat{L}_S - \widehat{K} + A\widehat{L}_U)\vartheta}{D}[\gamma - \beta(1-\alpha) - \alpha\theta]$$

$$-\frac{\varepsilon_{v_U}\varepsilon_{v_S}}{D}\frac{\alpha(1-\delta)\vartheta}{\delta}$$

$$\Bigg[\Bigg\{\Bigg\{\{A + B(1-\theta)(1-\gamma)\}\frac{\left(\vartheta + \frac{v_U}{(1-v_U)\varepsilon_{v_U}}\right)}{\gamma} - \frac{Av_U}{(1-v_U)\varepsilon_{v_U}} + B(1-\theta)\Bigg\}\widehat{L}_S -$$

$$\left\{\frac{\theta(1-\gamma)}{\gamma} + (\theta - 1 - \vartheta)\right\}(\widehat{K} - A\widehat{L}_U)\Bigg] \qquad (3.3.28).$$

So equation (3.3.28) is very similar to equation (3.3.25). While defining skilled-unskilled relative wage, Δ, we do not incorporate the unemployment rates. However, the definition of R includes unemployment in two labour markets; and hence the second and the fourth terms appear in equation (3.3.26) and not in equation (3.3.23). This is how equation (3.3.28) differs from equation (3.3.25). If $\frac{\alpha(1-\delta)}{\delta} \to 0$, then these two equations are identical. So, in this special case, comparative static results on relative wage and on average income ratio with respect to changes in factor endowments are qualitatively similar.

3.3.2.3. Gini Coefficient of wage income distribution:

Here we introduce Gini-coefficient as a measure of wage inequality like that in section 2.4 of chapter 2; and this Gini-coefficient, denoted by G, is same as that in equation (2.4.24) found in section 2.4. Here

$$\mathrm{G} = \frac{(\mathrm{N}v_\mathrm{U}+\mathrm{S}v_\mathrm{S})\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\Delta\}+\mathrm{NS}(1-v_\mathrm{U})(1-v_\mathrm{S})(\Delta-1)}{(\mathrm{N}+\mathrm{S}-1)\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\Delta\}} \quad (3.3.29);$$

where

$\Delta = \frac{\mathrm{W_S}}{\mathrm{W_U}}$.

Using equation (3.3.29), we obtain

$$\widehat{\mathrm{G}} = \mathrm{A}_2\hat{v}_\mathrm{U} + \mathrm{B}_2\hat{v}_\mathrm{S} + \mathrm{C}_2\widehat{\Delta} \quad (3.3.30);$$

where,

$$\mathrm{A}_2 = \frac{v_\mathrm{U}\left[\mathrm{NS}^2(1-v_\mathrm{S})^2\Delta+2\mathrm{N}^2\mathrm{S}(1-v_\mathrm{U})(1-v_\mathrm{S})\Delta+\mathrm{N}^3(1-v_\mathrm{U})^2\right]}{\mathrm{G}(\mathrm{N}+\mathrm{S}-1)\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\Delta\}^2} > 0,$$

$$\mathrm{B}_2 = \frac{v_\mathrm{S}\left[\mathrm{N}^2\mathrm{S}(1-v_\mathrm{U})^2+2\mathrm{NS}^2(1-v_\mathrm{U})(1-v_\mathrm{S})\Delta+\mathrm{S}^3(1-v_\mathrm{S})^2\Delta^2\right]}{\mathrm{G}(\mathrm{N}+\mathrm{S}-1)\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\Delta\}^2} > 0,$$

$$\mathrm{C}_2 = \frac{\Delta[\mathrm{NS}(1-v_\mathrm{U})(1-v_\mathrm{S})\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\}]}{\mathrm{G}(\mathrm{N}+\mathrm{S}-1)\{\mathrm{N}(1-v_\mathrm{U})+\mathrm{S}(1-v_\mathrm{S})\Delta\}^2} > 0.$$

This equation is same as equation (2.4.25); in section 2.4.

Here, $\hat{v}_\mathrm{S}$, $\hat{v}_\mathrm{U}$ and $\widehat{\Delta}$ are given by equations (3.3.18), (3.3.19) and (3.3.24) respectively. Equation (3.3.30) implies that the relative rate of change in the Gini Coefficient of wage income distribution is explained not only by the relative rate of change in the skilled-unskilled relative wage but also by relative rates of change in unemployment rates of two types of workers. Here A_2, B_2 and C_2 represent elasticities of Gini-Coefficient with respect to unemployment rate in the unskilled labour market, unemployment rate in the skilled labour market and skilled-unskilled wage ratio respectively.

We analyse the effect of a change in capital endowment, K, on the Gini Coefficient in the same way as done in section 2.4. So putting $\widehat{\mathrm{N}} = \widehat{\mathrm{S}} = 0$ and using equations (3.3.18), (3.3.19) and (3.3.30), we obtain

$$\widehat{G}=\left[\frac{A_2\varepsilon_{v_S}\left\{(1-\beta(1-\alpha)-\alpha\theta)-\frac{\alpha(1-\delta)}{\delta}\left(\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right)\right\}+B_2\varepsilon_{v_U}\left\{\{\beta(1-\alpha)+\alpha\theta\}+\frac{\alpha(1-\delta)}{\delta}\theta\right\}\frac{(1-\gamma)}{\gamma}}{D}\right]\widehat{K}+$$

$$C_2\widehat{\Delta} \qquad (3.3.31),$$

where $\widehat{\Delta}$ is given by equation (3.3.24) for $\widehat{N}=\widehat{S}=0$.

An exogenous change in the capital stock affects the degree of wage income inequality of the workers in two different ways. First, it alters unemployment rates in two labour markets. Secondly, it causes a change in the skilled-unskilled relative wage. The combined effect operated through changes in unemployment rates in two labour markets is represented by the first term of the R.H.S. of equation (3.3.31); and its second term shows the effect working through change in the skilled-unskilled wage ratio. If $\frac{\alpha(1-\delta)}{\delta}\rightarrow 0$, then the first term in the R.H.S. of equation (3.3.31) is negative. This implies that an increase in capital stock lowers the combined unemployment of both types of labour. On the other hand, since the industrial sector is more capital intensive than the agricultural sector and efficiency function of both types of labour are identical, second term of R.H.S. of equation (3.3.31) is positive. This means that an increase in capital endowment raises the skilled-unskilled wage ratio. So, the final effect of a change in K on the value of G is ambiguous in sign. We can establish the following proposition.

<u>PROPOSITION-3.3.3</u>: *If the industrial sector is more capital intensive than the agricultural sector and if efficiency functions of both type of labourer are identical, then an increase in capital endowment may cause the Gini-coefficient of wage income distribution and the skilled-unskilled relative wage to move in opposite directions when scale elasticity of output is very low.*

3.4 <u>LIMITATIONS</u>

We consider a static model where skilled labour and capital do not accumulate over time. There are many restrictive assumptions in this model. There is no education sector that produces skilled labour and also there is no nontraded final good in this chapter. Different varieties of intermediate goods are assumed to be produced with identical technologies. Industrial sector does not use unskilled labour.

In section 3.2, we have a public intermediate good producing sector and our assumption that this public intermediate good producing sector does not use skilled labour as input is a restrictive one. More importantly, this public intermediate good is specific to the sector producing varieties of private intermediate goods; and is neither used in the agricultural sector nor used in the final good producing industrial sector. We also do not consider Lindahal pricing and optimum provision of the public input. While deriving results from this model; technical complications compel us to assume that agricultural sector and public intermediate goods sector have identical production technologies.

In section 3.3 of this chapter, we introduce involuntary unemployment equilibrium in both the labour markets and explain unemployment using efficiency wage hypothesis. So the model developed in this section is subject to the same set of limitations as applied in the section 2.4 of chapter 2.

Appendix (3.A):

<u>Derivation of equations (3.2.16), (3.2.17) and (3.2.18):</u>

From equation (3.2.10), we obtain

$$\hat{r} = -\frac{(1-\gamma)}{\gamma}\widehat{W}_U \tag{3.2.A.1}$$

Using equations (3.2.14) and (3.2.15), we have

$$M\alpha\left(\frac{W_S}{r}\right)^{(-\beta+\alpha\beta)}\left(\frac{W_S}{P_x}\right)^{(1-\alpha)}\frac{Y}{n^{\frac{\alpha(1-\delta)}{\delta}}} = n\frac{\mu}{\lambda G^{\sigma}}\frac{\delta}{(1-\delta)}$$

$$\Rightarrow -\beta(1-\alpha)\left(\widehat{W}_S-\hat{r}\right)+(1-\alpha)\left(\widehat{W}_S-\widehat{P}_x\right)+\widehat{Y}-\frac{\alpha(1-\delta)}{\delta}\hat{n} = \hat{n}-\sigma\widehat{G} \tag{3.2.A.2}$$

Using equations (3.2.6), (3.2.8), (3.2.A.1) and (3.2.A.2), we have

$$\{\theta-\beta(1-\alpha)-\alpha\theta\}\widehat{W}_S+\frac{\{\theta-\beta(1-\alpha)-\alpha\theta\}(1-\gamma)}{\gamma}\widehat{W}_U-\left(1+\frac{\alpha(1-\delta)}{\delta}\right)\hat{n} = -(3.2.1+\sigma)\widehat{Y} \tag{3.2.A.3}$$

Using equations (3.2.2), (3.2.8) and (3.2.A.1), we have

$$\{1-\beta(1-\alpha)-\alpha\theta\}\widehat{W}_S-\frac{\{\beta(1-\alpha)+\alpha\theta\}(1-\gamma)}{\gamma}\widehat{W}_U-\frac{\alpha(1-\delta)}{\delta}\hat{n} = 0 \tag{3.2.A.4}$$

Using equations (3.2.12), (3.2.14) and (3.2.15), we have

$$\frac{n\mu}{G^{\sigma}}\left(1+\frac{\delta}{(1-\delta)}\right)(1-\theta)\left(\frac{W_S}{r}\right)^{-\theta}+\frac{(1-\alpha)(1-\beta)}{\alpha}\frac{P_x n\mu\delta}{W_S\lambda G^{\sigma}(1-\delta)} = L_S \tag{3.2.A.5}$$

Using equations (3.2.8) and (3.2.A.5), we obtain

$$\frac{n\mu}{(1-\delta)}{W_S}^{1-\theta}r^{\theta}\left\{(1-\theta)+\frac{(1-\alpha)(1-\beta)}{\alpha}\right\} = G^{\sigma}W_S L_S \tag{3.2.A.6}$$

Using equations (3.2.6), (3.2.A.1) and (3.2.A.6), we obtain

$$-\theta\widehat{W}_S-\frac{\theta(1-\gamma)}{\gamma}\widehat{W}_U+\hat{n} = \widehat{L}_S+\sigma\widehat{Y} \tag{3.2.A.7}$$

Using equations (3.2.A.3), (3.2.A.4) and (3.2.A.7), we have

$$\widehat{W}_S = \widehat{Y}-\widehat{L}_S \tag{3.2.A.8}$$

$$\widehat{W}_U = \frac{\widehat{Y}\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)\right]-\widehat{L}_S\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta-1)\right]}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \tag{3.2.A.9}$$

and

$$\hat{n} = \frac{\widehat{L}_S[\beta(1-\alpha)+\alpha\theta-\theta]+\widehat{Y}[\theta+\sigma\{\beta(1-\alpha)+\alpha\theta\}]}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \tag{3.2.A.10}$$

Equations (3.2.A.8), (3.2.A.9) and (3.2.A.10) are same as equations (3.2.16), (3.2.17) and (3.2.18) respectively in the body of the chapter.

Appendix (3.B):

Derivation of equation (3.2.20):

Using equation (3.2.11), we have

$$\left(\frac{\gamma}{1-\gamma}\right)^{(1-\gamma)}\left(\frac{W_U}{r}\right)^{(1-\gamma)} Z = \left(\frac{\gamma}{1-\gamma}\right)\frac{W_U L_U}{r} - \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma}\left(\frac{W_U}{r}\right)^{(1-\gamma)} G \qquad (3.2.A.11).$$

Using equations (3.2.13), (3.2.14), (3.2.15) and (3.2.A.11), we have

$$\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{1-\gamma} r^{\gamma}\left\{\left(\frac{\gamma}{1-\gamma}\right) - \left(\frac{\gamma}{1-\gamma}\right)\right\} + \frac{n\mu}{G^{\sigma}(1-\delta)} {W_S}^{1-\theta} r^{\theta}\left\{\theta + \frac{\beta(1-\alpha)}{\alpha}\right\} = rK - \left(\frac{\gamma}{1-\gamma}\right) W_U L_U$$

$$\Rightarrow \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{1-\gamma} r^{\gamma}\left\{\left(\frac{\gamma}{1-\gamma}\right) - \left(\frac{\gamma}{1-\gamma}\right)\right\}\left\{(1-\gamma)\widehat{W}_U + \gamma\hat{r} + \widehat{Y}\right\}$$

$$+ \frac{n\mu}{G^{\sigma}(1-\delta)} {W_S}^{1-\theta} r^{\theta}\left\{\theta + \frac{\beta(1-\alpha)}{\alpha}\right\}\left\{\hat{n} + (1-\theta)\widehat{W}_U + \theta\hat{r} - \sigma\widehat{Y}\right\} =$$

$$rK\left(\hat{r} + \widehat{K}\right) - \left(\frac{\gamma}{1-\gamma}\right) W_U L_U\left(\widehat{W}_U + \widehat{L}_U\right) \text{(3.2.A.12).}$$

Using equations (3.2.6), (3.2.A.1) and (3.2.A.12), we have

$$\widehat{Y} = \frac{rK\widehat{K} - \frac{\gamma}{(1-\gamma)} W_U L_U \widehat{L}_U + A\widehat{L}_S}{B} \qquad (3.2.A.13),$$

where,

$$A = \left[\left\{-\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{(1-\gamma)} r^{\gamma}\frac{(\gamma-\gamma)^2}{(1-\gamma)(1-\gamma)\gamma} + \right.\right.$$

$$\left. + \frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}\left\{\frac{1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta-1)}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right\}$$

$$\left. + \frac{W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\frac{\left\{(1-\alpha)(1-\beta)\theta+\frac{\alpha\theta}{\delta}(1-\theta)\right\}}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right];$$

and,

$$B = \left[\left\{-\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{(1-\gamma)} r^{\gamma}\frac{(\gamma-\gamma)^2}{(1-\gamma)(1-\gamma)\gamma}\right.\right.$$

$$\left. + \frac{(1-\gamma)}{\gamma W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}\left\{\frac{1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\right\}$$

$$+ \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{(1-\gamma)} r^{\gamma}\frac{(\gamma-\gamma)}{(1-\gamma)(1-\gamma)}$$

$$+\frac{W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\frac{\left\{\left(1-\alpha\theta-\frac{\alpha\theta(1-\delta)}{\delta}\right)\theta+(1-\alpha)(1-\theta)\beta+\frac{\alpha\theta}{\delta}(1-\sigma)+\alpha\theta\sigma\right\}}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)}\Bigg].$$

Equation (3.2.A.13) is same as equation (3.2.20) in the body of the chapter.

Appendix (3.C):

Derivation of equations (3.2.16.1), (3.2.17.1) and (3.2.18.1):

From equation (3.2.10), we obtain

$$\hat{r}=\frac{1}{\gamma}\widehat{P}_Z-\frac{(1-\gamma)}{\gamma}\widehat{W}_U \qquad (3.2.A.1.1).$$

Using equations (3.2.14) and (3.2.15), we have

$$-\beta(1-\alpha)\left(\widehat{W}_S-\hat{r}\right)+(1-\alpha)\left(\widehat{W}_S-\widehat{P}_x\right)+\widehat{Y}-\frac{\alpha(1-\delta)}{\delta}\hat{n}=\hat{n}-\sigma\widehat{G} \qquad (3.2.A.2.1).$$

Using equations (3.2.6), (3.2.8), (3.2.A.1.1) and (3.2.A.2.1), we have

$$\{\theta-\beta(1-\alpha)-\alpha\theta\}\widehat{W}_S+\frac{\{\theta-\beta(1-\alpha)-\alpha\theta\}(1-\gamma)}{\gamma}\widehat{W}_U-\left(1+\frac{\alpha(1-\delta)}{\delta}\right)\hat{n}=-(1+\sigma)\widehat{Y}+$$
$$\frac{\{\theta-\beta(1-\alpha)-\alpha\theta\}}{\gamma}\widehat{P}_Z-\sigma\widehat{\psi} \qquad (3.2.A.3.1).$$

Using equations (3.2.2), (3.2.8) and (3.2.A.1.1), we have

$$\{1-\beta(1-\alpha)-\alpha\theta\}\widehat{W}_S-\frac{\{\beta(1-\alpha)+\alpha\theta\}(1-\gamma)}{\gamma}\widehat{W}_U-\frac{\alpha(1-\delta)}{\delta}\hat{n}=-\frac{\{\beta(1-\alpha)+\alpha\theta\}}{\gamma}\widehat{P}_Z-\frac{\psi}{1-\psi}\widehat{\psi}$$
$$(3.2.A.4.1).$$

Using equations (3.2.6), (3.2.A.1.1) and (3.2.A.6), we obtain

$$-\theta\widehat{W}_S-\frac{\theta(1-\gamma)}{\gamma}\widehat{W}_U+\hat{n}=\sigma\widehat{Y}-\frac{\theta}{\gamma}\widehat{P}_Z+\sigma\widehat{\psi} \qquad (3.2.A.5.1).$$

Using equations (3.2.A.3.1), (3.2.A.4.1) and (3.2.A.5.1), we have

$$\widehat{W}_S=\widehat{Y}-\frac{\psi}{1-\psi}\widehat{\psi} \qquad (3.2.A.6.1),$$

$$\widehat{W}_U=\frac{\left[1-\beta(1-\alpha)-\alpha\theta-\frac{\alpha(1-\delta)}{\delta}(\theta+\sigma)\right]\widehat{Y}+\left[\frac{\psi}{1-\psi}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)-\frac{\alpha\sigma(1-\delta)}{\delta}\right]\widehat{\psi}+\frac{1}{\gamma}\left[\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right]\widehat{P}_Z}{\frac{(1-\gamma)}{\gamma}\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.A.7.1),$$

and

$$\hat{n}=\frac{\frac{\sigma(1-\gamma)}{\gamma}[\beta(1-\alpha)+\alpha\theta]\widehat{\psi}+\widehat{Y}[\theta+\sigma\{\beta(1-\alpha)+\alpha\theta\}]}{\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \qquad (3.2.A.8.1).$$

Equations (3.2.A.6.1), (3.2.A.7.1) and (3.2.A.8.1) are same as equations (3.2.16.1), (3.2.17.1) and (3.2.18.1) respectively in the body of the chapter.

Appendix (3.D):

Derivation of equation (3.2.20.1):

Using equations (3.2.13), (3.2.14), (3.2.15) and (3.2.A.11), we have

$$\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{1-\gamma} r^{\gamma} \left\{\left(\frac{\gamma}{1-\gamma}\right) - \left(\frac{\gamma}{1-\gamma}\right)\right\} + \frac{n\mu}{G^{\sigma}(1-\delta)} {W_S}^{1-\theta} r^{\theta} \left\{\theta + \frac{\beta(1-\alpha)}{\alpha}\right\} = rK - \left(\frac{\gamma}{1-\gamma}\right) W_U L_U\ ,$$

$$\Rightarrow \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{1-\gamma} r^{\gamma} \left\{\left(\frac{\gamma}{1-\gamma}\right) - \left(\frac{\gamma}{1-\gamma}\right)\right\} \left\{(1-\gamma)\widehat{W}_U + \gamma\hat{r} + \widehat{Y}\right\}$$

$$+ \frac{n\mu}{G^{\sigma}(1-\delta)} {W_S}^{1-\theta} r^{\theta} \left\{\theta + \frac{\beta(1-\alpha)}{\alpha}\right\} \left\{\hat{n} + (1-\theta)\widehat{W}_U + \theta\hat{r} - \sigma\widehat{Y}\right\} = rK\hat{r} - \left(\frac{\gamma}{1-\gamma}\right) W_U L_U \widehat{W}_U$$

(3.2.A.9.1).

Using equations (3.2.6), (3.2.A.1.1) and (3.2.A.9.1), we have

$$\widehat{Y} = \frac{C\widehat{P}_Z + D\widehat{\psi}}{B} \qquad (3.2.A.10.1)$$

where,

$$C = -\left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{(1-\gamma)} r^{\gamma} \frac{(\gamma-\gamma)\phi}{(1-\gamma)(1-\gamma)\gamma} + \left(\frac{\gamma}{1-\gamma}\right)^{-\gamma} {W_U}^{(1-\gamma)} r^{\gamma} \frac{(\gamma-\gamma)^2}{(1-\gamma)(1-\gamma)^2\gamma} - \frac{\gamma}{(1-\gamma)^2} W_U L_U\ ;$$

and,

$$D =$$

$$-\frac{\psi}{(1-\psi)W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) - \left(\frac{\gamma}{(1-\gamma)}\right)^2 \frac{\psi}{1-\psi} W_U L_U +$$

$$\frac{\sigma\alpha(1-\delta)\gamma}{\delta(1-\gamma)\left(\beta(1-\alpha)+\frac{\alpha\theta}{\delta}\right)} \left\{\frac{\theta W_S L_S\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}} + \frac{\gamma}{(1-\gamma)W_S L_S}\left(\frac{rK}{W_S L_S} - \frac{\theta\{\beta(1-\alpha)+\alpha\theta\}}{\{(1-\beta)(1-\alpha)+\alpha(1-\theta)\}}\right) + \frac{\gamma}{(1-\gamma)} W_U L_U\right\}.$$

Equation (3.2.A.10.1) is same as equation (3.2.20) in the body of the chapter.

Appendix (3.E):

Derivation of equations (3.3.18), (3.3.19) and (3.3.20):

From equations (3.3.9) and (3.3.10), we obtain

$$\hat{h} = \widehat{W}_S + \varepsilon_{v_S}\hat{v}_S \qquad (3.3.A.4).$$

and

$$\hat{f} = \widehat{W}_U + \varepsilon_{v_U}\hat{v}_U \qquad (3.3.A.5).$$

From equation (3.3.8), we obtain

$$\hat{r} = -\frac{(1-\gamma)}{\gamma}\left(\widehat{W}_U - \hat{f}\right) \qquad (3.3.A.6).$$

Using equations (3.3.5), (3.3.6), (3.3.A.2), (3.3.A.3), (3.3.A.4), (3.3.A.5) and (3.3.A.6), we obtain

$$-\varepsilon_{v_S}(1-\beta(1-\alpha)-\alpha\theta)\hat{v}_S + \varepsilon_{v_U}\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}\hat{v}_U - \frac{\alpha(1-\delta)}{\delta}\hat{n} = 0 \qquad (3.3.A.7).$$

Using equations (3.3.16) and (3.3.17), we have

$$Y = \frac{\mu\delta}{\lambda(1-\delta)}\frac{n^{\left(1+\frac{\alpha(1-\delta)}{\delta}\right)}}{M\alpha\left(\frac{W_S/_h}{r}\right)^{(-\beta+\alpha\beta)}\left(\frac{W_S/_h}{P_x}\right)^{(1-\alpha)}} \qquad (3.3.A.8).$$

Using equations (3.3.14) and (3.3.A.8), we obtain

$$\frac{\mu n}{(1-\delta)}\left(\frac{W_S/_h}{r}\right)^{-\theta}\left((1-\theta)+\frac{(1-\beta)(1-\alpha)}{\alpha}\right) = L_S(1-v_S)h \qquad (3.3.A.9).$$

Using equations (3.3.A.2), (3.3.A.3), (3.3.A.4), (3.3.A.5), (3.3.A.6) and (3.3.A.9), we get

$$\varepsilon_{v_S}\left\{\left(\theta - \frac{h}{h_{11}(W_S)^2} - 1\right) + \frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\hat{v}_S + \varepsilon_{v_U}\frac{\theta(1-\gamma)}{\gamma}\hat{v}_U + \hat{n} = \hat{L}_S \qquad (3.3.A.10).$$

Using equations (3.3.13), (3.3.15) and (3.3.A.8), we obtain

$$A + B = 1 \qquad (3.3.A.11),$$

where,

$$A = \frac{\frac{\gamma}{(1-\gamma)}\left(\frac{W_U/_f}{r}\right)L_U(1-v_U)f}{K} > 0;$$

and

$$B = \frac{\frac{\mu n}{(1-\delta)}\left(\frac{W_S/_h}{r}\right)^{(1-\theta)}\left(\theta+\frac{\beta(1-\alpha)}{\alpha}\right)}{K} > 0.$$

Using equations (3.3.A.2), (3.3.A.3), (3.3.A.4), (3.3.A.5), (3.3.A.6) and (3.3.A.11), we get

$$-\varepsilon_{v_S}B(1-\theta)\hat{v}_S + \varepsilon_{v_U}\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma} - \frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\hat{v}_U + B\hat{n} = \widehat{K} - A\hat{L}_U \qquad (3.3.A.12).$$

Using equations (3.3.A.7), (3.3.A.10) and (3.3.A.12), we have

$\hat{v}_S =$

$$\frac{\varepsilon_{v_U}\left[-\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}\left(B\hat{L}_S-\hat{K}+A\hat{L}_U\right)-\frac{\alpha(1-\delta)}{\delta}\left[\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\hat{L}_S-\left(\hat{K}-A\hat{L}_U\right)\frac{\theta(1-\gamma)}{\gamma}\right]\right]}{D}$$

(3.3.A.13),

$$\hat{v}_U = \frac{\varepsilon_{v_S}\left[-(1-\beta(1-\alpha)-\alpha\theta)\left(B\hat{L}_S-\hat{K}+A\hat{L}_U\right)-\frac{\alpha(1-\delta)}{\delta}\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\left(\hat{K}-A\hat{L}_U\right)+B(1-\theta)\hat{L}_S\right]\right]}{D}$$

(3.3.A.14),

and

$$\hat{n} = \frac{-\varepsilon_{v_U}\varepsilon_{v_S}\left[\left\{(1-\beta(1-\alpha)-\alpha\theta)\frac{\theta(1-\gamma)}{\gamma}+\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)\right\}\left(\hat{K}-A\hat{L}_U\right)\right.}{D}$$

$$\frac{\left.+\varepsilon_{v_U}\varepsilon_{v_S}\left\{(1-\beta(1-\alpha)-\alpha\theta)\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}-\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}B(1-\theta)\right\}\hat{L}_S\right]}{D}$$

(3.3.A.15);

where,

$$D = -\varepsilon_{v_U}\varepsilon_{v_S}\left[(1-\beta(1-\alpha)-\alpha\theta)\left[B\frac{\theta(1-\gamma)}{\gamma}-\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\right.\right.\right.$$

$$\left.\left.\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}\right]+\{\beta(1-\alpha)+\alpha\theta\}\frac{(1-\gamma)}{\gamma}B\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}+(1-\theta)\right]$$

$$+\frac{\alpha(1-\delta)}{\delta}\left[\left\{\left(\theta-\frac{h}{h_{11}(W_S)^2}-1\right)+\frac{v_S}{(1-v_S)\varepsilon_{v_S}}\right\}\left\{\{A+B(1-\theta)(1-\gamma)\}\frac{f}{f_{11}(W_U)^2\gamma}-\frac{Av_U}{(1-v_U)\varepsilon_{v_U}}\right\}+\right.$$

$$\left.\left.B(1-\theta)\frac{\theta(1-\gamma)}{\gamma}\right]\right].$$

Equations (3.3.A.13), (3.3.A.14) and (3.3.A.15) are same as equations (3.3.18), (3.3.19) and (3.3.20) respectively in the body of the chapter.

Chapter 4

A dynamic model with international trade and international knowledge spillover

4.1 INTRODUCTION

This chapter presents an open economy version of the closed economy model developed by Kiley (1999). It is a two commodity model with international trade and international and inter-sectoral technology spillover. In Kiley (1999), the cost of developing a new specific intermediate good depends on the number of varieties of those specific intermediate goods available and on the level of existing research but not on the intensity of inter-sectoral knowledge spillover effect. The question of international knowledge spillover does not arise in that model because Kiley (1999) considers a closed economy. However, in the present model, this cost also depends on the intensity of international as well as inter sectoral knowledge spillover effect. The motivation of the present research comes from the existence of several empirical studies which support the incidence of this international knowledge spillover. This empirical literature includes works of Branstetter (2001), Coe and Helpman (1995), Coe et. al. (1997), Lichtenberg and Potterie (1998), Griliches (1992), Keller (2002) etc. They show that foreign R&D has beneficial effects on domestic productivity and the strength of these positive effects vary positively with the degree of openness. According to Keller (2002), the contribution of R&D in the mother industry itself is about 50%, and in other domestic industries is about 30%. The remaining 20% is contributed to foreign industries.

In our model, the cost of developing new intermediate goods is reduced due to positive international and intersectoral knowledge spillover effects. We have two final goods in this model which are not perfect substitutes. We derive many interesting results. The skilled unskilled wage ratio is affected by the intensity of international knowledge spillover directly as well as through inter country difference in relative factor endowments. In the long run equilibrium of a closed economy, the relationship between the skilled unskilled wage ratio and the skilled unskilled labour endowment ratio is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. A direct relationship may (must) be obtained only if these two goods are highly (perfect) substitutes. In the one commodity model of Kiley (1999), we always obtain such a positive relationship. However, when international trade is opened, we always find a positive relationship between the wage ratio and the domestic factor endowment ratio in the long run equilibrium of a small open economy. The effect of the opening of trade on the long run equilibrium skilled unskilled wage ratio depends not only on the inter-country difference of factor endowment ratios but also on the intensity of spillover effects. If factor endowment ratios in both the countries are equal and if the foreign country has a larger endowment of each of the two factors, then the globalization policy leads to a rise in the skilled-unskilled wage ratio in the home country in the presence of spillover effects.

This chapter is organized as follows. Section 4.2 describes the model and section 4.3 analyses properties of it's the balanced growth equilibrium. The equilibrium under autarky is described in subsection 4.3.1 and the effect of opening of trade is analysed in sub section 4.3.2. Limitations of the model are described in section 4.4.

4.2. The Model:

There are two countries in the world; and each of them has two factors of production-unskilled labour and skilled labour. The unskilled labour is used to produce a traditional final good, $\mathrm{Y^U}$; and an advanced final good, $\mathrm{Y^S}$, is produced by skilled labour. These two goods are not perfect substitutes to consumers. In Kiley (1999), both the sectors produce the same commodity. Some intermediate goods complement skilled labour and other intermediate goods complement unskilled labour. Intermediate goods complementing skilled labour are denoted by X; and those complementing unskilled labour are denoted by Z. New intermediate goods are developed by the R&D sector in each of the two countries. Production technologies in the two final good sectors are identical in both the countries. However, these two countries may differ in terms of their factor endowments. $\mathrm{H}(\mathrm{H}^*)$ and $\mathrm{L}(\mathrm{L}^*)$ stand for skilled labour endowment and unskilled labour endowment of the home (foreign) country respectively.

The home country is a small open economy and thus is a price taker in the world market. However, the foreign country, which is basically the rest of the world, is essentially a closed economy. Due to international knowledge spillover from the advanced R&D sector in the rest of the world to that in the home country, the advanced R&D sector in the home country enjoys a cost advantage in the production of new intermediate goods used for the production of the advanced final good. Also there exists localized knowledge spillover from the advanced R&D sector to the traditional R&D sector in the home country; and this gives a cost advantage to the traditional R&D sector of the home country. However, there is neither any international knowledge spillover nor any intersectoral knowledge spillover in the model of Kiley (1999). Markets for final goods and primary factors in both the countries are perfectly competitive. Intermediate goods are rented; and every intermediate good producer is a monopolist in the rental market. Factors are internationally immobile; and consumers in both the countries have identical tastes.

4.2.1. Final goods:

The description of two final goods producing sectors is identical to that found in Kiley (1999). The production functions of two final goods produced by firm i at time t are specified as follows.

$$\mathrm{Y}_{\mathrm{it}}^{\mathrm{U}} = (\mathrm{L}_{\mathrm{it}})^{(1-\mathrm{a})} \int_0^{\mathrm{M}(\mathrm{t})} \left(\mathrm{Z}_{\mathrm{itj}}\right)^{\mathrm{a}} \mathrm{dj} \qquad (4.1);$$

and

$$\mathrm{Y}_{\mathrm{it}}^{\mathrm{S}} = (\mathrm{H}_{\mathrm{it}})^{(1-\mathrm{a})} \int_0^{\mathrm{N}(\mathrm{t})} \left(\mathrm{X}_{\mathrm{itj}}\right)^{\mathrm{a}} \mathrm{dj}, \text{ with } 0 < a < 1 \qquad (4.2).$$

Here, Z_{itj} and X_{itj} are the quantities of jth variety of intermediate good and L_{it} and H_{it} are the quantities of unskilled labour and skilled labour used by ith competitive firm in the production of traditional final good and advanced final good respectively at the time point t. Numbers of varieties of intermediate goods that complement skilled labour and unskilled labour at time point t are denoted by $N(t)$ and $M(t)$ respectively.

The instantaneous profit functions of the ith competitive firm in sectors producing Y^U and Y^S are given by

$$\Pi_i^U = (L_{it})^{(1-a)} \int_0^{M(t)} \left(Z_{itj}\right)^a dj - \int_0^{M(t)} P_{jt}^U Z_{itj} dj - W_t^U L_{it} \quad (4.3);$$

and,

$$\Pi_i^S = (H_{it})^{(1-a)} \int_0^{N(t)} \left(X_{itj}\right)^a dj - \int_0^{N(t)} P_{jt}^S X_{itj} dj - W_t^S H_{it} \quad (4.4).$$

Π_i^U and Π_i^S stand for profit of the ith firm in the U sector and in the S sector respectively in terms of its own sectors product. Π_i^U is maximized with respect to L_{it} and Z_{itj}; and Π_i^S is maximized with respect to H_{it} and X_{itj} given the input prices. Here, P_{jt}^U and P_{jt}^S are rental prices of the jth intermediate good used as input in the traditional final good sector and in the advanced final good sector, respectively; and these are expressed in terms of the final product of the corresponding sector. P_F is the relative price of the advanced final good in terms of the traditional final good. W_t^U and W_t^S are wage rates of unskilled labour and skilled labour, respectively, at time point t expressed in terms of the product of the corresponding sector. First order conditions of profit maximization in both the sectors to be valid at each t are given by followings.

$$P_{jt}^U = (L_{it})^{(1-a)} a \left(Z_{itj}\right)^{(a-1)} \quad (4.5);$$

$$P_{jt}^S = (H_{it})^{(1-a)} a \left(X_{itj}\right)^{(a-1)} \quad (4.6);$$

$$W_t^S = (1-a) P_F N(t) H^{-a} \left(X_{itj}\right)^a \quad (4.7);$$

and,

$$W_t^U = (1-a) M(t) L^{-a} \left(Z_{itj}\right)^a \quad (4.8).$$

4.2.2. Intermediate goods:

Intermediate goods are nontraded and are durable in nature without having any depreciation; and these are rented to final good sectors. One unit of intermediate good of either type is required to produce one unit of the corresponding final good. The intermediate good can be used from the period in which it is developed. Then the discounted present value of profit of the jth intermediate good producer, who supplies it to the final good sectors, over the infinite time horizon are given by followings.

$$V_j^S(t) = \int_t^{\infty} e^{-r(\tau-t)}\left(P_{j\tau}^S - r\right)X_{j\tau}d\tau \qquad (4.9);$$

and,

$$V_j^U(t) = \int_t^{\infty} e^{-r(\tau-t)}\left(P_{j\tau}^U - r\right)Z_{j\tau}d\tau \qquad (4.10).$$

Here, $V_j^S(t)$ and $V_j^U(t)$ are the discounted present values of profits earned from renting; and r is the constant real interest rate that plays the role of marginal cost of renting as well as of the rate of discounting future return.

Now, from equations (4.5) and (4.6), we derive demand functions for jth intermediate good of the ith firm in the two final goods sectors as follows.

$$Z_{itj} = L_{it}\left(\frac{a}{P_{jt}^U}\right)^{\left(\frac{1}{1-a}\right)} \qquad (4.11);$$

and,

$$X_{itj} = H_{it}\left(\frac{a}{P_{jt}^S}\right)^{\left(\frac{1}{1-a}\right)} \qquad (4.12).$$

The producer cum reinter of each of these intermediate goods is a monopolist maximizing its corresponding rental income subject to the demand constraint. Both the demand functions shown by equations (4.11) and (4.12) have constant price elasticities of demand denoted by $\left(\frac{1}{1-a}\right)$; and these imply that monopoly profit maximizing prices of intermediate goods are also constant and given as follows.

$$P_{jt}^S = P_{jt}^U = \frac{r}{a} \qquad (4.13).$$

So these monopoly prices vary neither across varieties nor over time. Now, using the assumption that labour endowments are fixed and time independent and also using equations (4.11), (4.12) and (4.13), we obtain equilibrium quantities of intermediate goods given by

$$X = H\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} \tag{4.14;}$$

and

$$Z = L\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)} \tag{4.15.}$$

So aggregate uses of intermediate goods are linear in terms of specific labour endowments in each of the two sectors.

Using equations (4.1), (4.2), (4.14) and (4.15), we obtain aggregate output of two final good producing sectors as follows.

$$Y_t^U = M(t)L\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} \tag{4.16;}$$

and

$$Y_t^S = N(t)H\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} \tag{4.17.}$$

So level of outputs of final goods are linear in terms of number of varieties of specific intermediate goods; and this ensures that the rate of growth of final output in a particular sector is equal to the rate of expansion of the number of varieties of intermediate goods specific to that sector, given the factor endowment.

Now, using equations (4.9), (4.10), (4.13), (4.14) and (4.15), we obtain infinite time horizon discounted present values of profit of intermediate good producers for advanced and traditional final good sectors as follows:

$$V^S(t) = \left(\frac{1-a}{a}\right)X \tag{4.18;}$$

and

$$V^U(t) = \left(\frac{1-a}{a}\right)Z \tag{4.19.}$$

4.2.3. R&D sector:

Expansions of the number of skill augmenting or unskill augmenting intermediate goods cause growth in output of the corresponding final good sector. However, this expansion process is costly. New intermediate goods are developed through R&D activities. We follow Kiley (1999) to assume that the cost of development of new specific intermediate goods varies positively with the number of varieties of specific intermediate goods and inversely with the level of existing research $\left(R(t)\right)$. It is also assumed that existing research level, $R(t)$, is same in both the countries. However, there exists positive inter-sectoral spillover effect from the skill augmenting R&D activities to the unskill augmenting R&D activities in the home country because unskilled workers learn how to improve their efficiency while working with skilled workers. Also there exists positive international spillover effect from skill augmenting R&D activity in the foreign country to that in the home country because knowledge capital whose accumulation generates skill is always internationally mobile. However, there is no international spillover effect between unskilled labour augmenting activities of two countries because they are not at all connected to each others. Kiley (1999) does not consider any spillover effect. Costs of developing skill augmenting new intermediate goods and unskill augmenting new intermediate goods are denoted by Γ^S and Γ^U, respectively; and these cost functions are given as follows

$$\Gamma^{S} = \frac{\delta_1\left[\frac{N(t)}{R(t)}\right]^{K_1}}{\left[\frac{N^*(t)}{R(t)}\right]^{K_2}}, \text{ with} K_1 > 1, \delta_1 > 0 \textit{ and } K_2 > 0 \qquad (4.20);$$

and,

$$\Gamma^{U} = \frac{\delta_2\left[\frac{M(t)}{R(t)}\right]^{K_1 q}}{\left[\frac{N(t)}{R(t)}\right]^{K_3}} \text{ with} K_1 q > 1, \delta_2 > 0, q \neq 1, K_3 > 0 \text{and } K_1 q > K_3 \qquad (4.21).$$

These cost functions are increasing and convex in terms of $N(t)$ and $M(t)$ respectively because $K_1 > 1$ and $K_1 q > 1$. However, the presence of knowledge spillover always produces a downward effect on these cost functions because $K_2 > 0$ and $K_3 > 0$. Here δ_1 and δ_2 are the reciprocals of the productivity parameters in the two R&D sectors.

$K_2 > 0$ in equation (4.20) implies that the positive international knowledge spillover is allowed from the advanced R & D sector of the foreign country to that of the home country. Here $N^*(t)$ is the number of varieties of advanced intermediate goods in the foreign country and the numerical value of K_2 denotes the magnitude of international knowledge spillover efficiency.

$K_3 > 0$ in equation (4.21) implies that the positive inter sectoral knowledge spillover takes place from the advanced R & D sector to the traditional R & D sector in the home country; and the numerical value of K_3 denotes the magnitude of inter sectoral knowledge spillover efficiency. This is less than the own technical efficiency parameter of the traditional R&D sector, denoted by $K_1 q$. We go back to Kiley (1999) when $K_2 = K_3 = 0$ and $q = 1$. It should also be noted that results of this model may be changed substantially if restrictions imposed on these parameters are altered. We follow Kiley (1999) to assume $K_1 > 1$. However, what River-Batiz and Romer (1991) and Wang et. al. (2009) assume is equivalent to assuming $K_1 < 0$.

Markets for R & D designs are perfectly competitive. In competitive equilibrium of the R & D sector, we also have

$$\Gamma^S = V^S \quad (4.22);$$

and,

$$\Gamma^U = V^U \quad (4.23).$$

If the value of the firm producing the intermediate good is greater than cost of developing the R & D design entry would occur until the cost equals the value. So equations (4.22) and (4.23) are the conditions of no entry and no exit in the advanced and traditional R & D sector respectively.

We follow Kiley (1999) to assume that the level of existing research, denoted by $R(t)$, grows over time at a constant rate.

So, we have

$$\dot{R} = gR \quad (4.24).$$

where g is the exogenous growth rate.

4.2.4. Consumers equilibrium:

The representative consumer consumes each of the two final goods; and her problem is to maximize the discounted present value of instantaneous utility over the infinite time horizon. It is given by

$\int_0^\infty e^{-\gamma t}\left[\beta C_U^{-\rho} + (1-\beta)C_S^{-\rho}\right]^{-\frac{1}{\rho}} dt$, with $-1 < \rho < \propto$ and with $\rho \neq 0$.

This is maximized subject to the intertemporal budget constraint given by

$\int_0^\infty e^{-rt}[C_U + P_F C_S]dt = \int_0^\infty e^{-rt}[W_L L + W_H H]dt.$

Here $W_L L$ and $W_H H$ can be added because equation (4.13) shows that prices of intermediates are same in two sectors. γ is the constant consumption rate of discount; C_U and C_S are the consumption levels of two goods of the representative consumer; ρ is the substitution parameter of the two goods in the utility function; r is the interest rate; and $(W_L L + W_H H)$ is the total income of the representative consumer[53].

Now, solving the consumer's utility maximization problem, we obtain

$$P_F = \frac{(1-\beta)C_U^{(\rho+1)}}{\beta C_S^{(\rho+1)}} \quad (4.25).$$

Here the R.H.S. of equation (4.25) represents the marginal rate of indifferent substitution between two final goods.

4.3. Balanced growth equilibrium:

Along a balanced growth path, levels of output of the two sectors (Y^U and Y^S), consumption levels of the two goods (C_U and C_S), the stock of existing research level (R), the number of varieties of skill and unskilled complements (M and N) and the wage rates of two types of labour (W_U and W_S) grow at same rate g[54].

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[53] If we consider two different representative consumers- one for skilled workers and the other for unskilled, then also we should have equation (4.25) for the equilibrium of each of them provided that their preferences are identical.

[54] This rate of growth is obtained from equation (4.24)

Now, we want to examine the effect of the opening of trade on skilled-unskilled relative wage in the home country in the balanced growth equilibrium. So we derive the skilled unskilled wage ratio in the balanced growth equilibrium in the following two cases: (i) the home country is closed to international trade and (ii) the home country is a small open economy whose relative product price P_F is equal to that obtained in the competitive equilibrium in the rest of the world. Kiley (1999) can not analyse the effect of the opening of trade on skill-unskilled wage ratio with the one final good model. We now use superscripts tr and au to denote the variables of the home country under free trade and under autarky respectively.

4.3.1. Wage inequality under autarky:

Under autarky, there is no effect of international knowledge spillover. So $K_2 = 0$. Supply equals to demand in the competitive equilibrium for each of two final goods market in the home country. Hence we have following two equations

$$\mathrm{Y^S} = \mathrm{C_S} + \frac{\mathrm{d}\left(\int_0^{\mathrm{N(t)}} \mathrm{x_{itj}dj}\right)}{\mathrm{dt}} \qquad (4.26);$$

and,

$$\mathrm{Y^U} = \mathrm{C_U} + \frac{\mathrm{d}\left(\int_0^{\mathrm{M(t)}} \mathrm{z_{itj}dj}\right)}{\mathrm{dt}} \qquad (4.27).$$

Equations (4.26) and (4.27) show that total supply of each of the two products is equal to total consumption demand plus total investment demand for that product. Here intermediate goods are modeled as durables as in Romer (1990). So only the newly invented intermediate goods use resources.

Using $K_2 = 0$ and also using equations (4.14), (4.15), (4.18), (4.19), (4.22) and (4.23), we obtain the following equation[55]

$$\left(\frac{N}{M}\right)^{au} = \left[\frac{1}{\delta_1}\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_1(q-1)-K_3}{K_1^2 q}\right]} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]} \qquad (4.28).$$

[55] Derivation of equation (4.28) in detail is obtained in Appendix (4.A).

Then, using equations (4.14), (4.15), (4.16), (4.17), (4.25), (4.26) and (4.27), we obtain the competitive equilibrium relative price of the advanced final good under autarky as follows[56].

$$P_F^{au} = \frac{(1-\beta)}{\beta}\left(\frac{ML}{NH}\right)^{(\rho+1)} \qquad (4.29).$$

Using equations, (4.7), (4.8), (4.14) and (4.15), we have[57]

$$\left(\frac{W^S}{W^U}\right)^{au} = P_F^{au}\left(\frac{N}{M}\right)^{au} \qquad (4.30).$$

Using equations (4.29) and (4.30), we obtain

$$\left(\frac{W^S}{W^U}\right)^{au} = \frac{(1-\beta)}{\beta}\left(\frac{L}{H}\right)^{(\rho+1)}\left(\frac{M}{N}\right)^{\rho} \qquad (4.31).$$

Finally, using equations (4.28) and (4.31), we have

$$\left(\frac{W^S}{W^U}\right)^{au} =$$

$$\frac{(1-\beta)}{\beta}\left[\delta_1\right]^{\rho\left(\frac{K_1 q-K_3}{K_1^2 q}\right)}\left[\frac{1}{\delta_2}\right]^{\frac{\rho}{K_1 q}}\left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\rho\left[\frac{K_3-K_1(q-1)}{K_1^2 q}\right]}\left(\frac{L}{H}\right)^{\rho\left(\frac{K_1 q-K_3}{K_1^2 q}\right)+\rho+1}(L)^{\rho\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]}$$

(4.32).

This equation (4.32) shows how the skilled unskilled wage ratio in the long run equilibrium of the closed economy is determined by various parameters. If $\rho \geq 0$, then the relationship between L and $\left(\frac{W^S}{W^U}\right)^{au}$, as given by equation (4.32), is ambiguous; but the relationship between H and $\left(\frac{W^S}{W^U}\right)^{au}$ is negative. However, if $\rho < 0$, then each of these two relationships is ambiguous. A positive relationship between H and $\left(\frac{W^S}{W^U}\right)^{au}$ can be obtained only if $\left\{\rho\left(\frac{K_1 q-K_3}{K_1^2 q}\right)+\rho+1\right\}<0$; and this necessary condition is likely to be satisfied when two goods are highly substitutes.

If the consumer's utility function is Cobb-Douglas, i.e., if $\rho = 0$, then equation (4.32) is reduced to

$$\left(\frac{W^S}{W^U}\right)^{au} = \frac{(1-\beta)}{\beta}\left(\frac{L}{H}\right) \qquad (4.32A);$$

[56] Derivation of equation (4.29) in detail is obtained in Appendix (4.B).
[57] Derivation of equation (4.30) in detail is obtained in Appendix (4.C).

and this equation (4.32A) gives an inverse relationship between $\left(\frac{W^S}{W^U}\right)^{au}$ and $\left(\frac{H}{L}\right)$ along a rectangular hyperbola. This is important because Kiley (1999) who develops an one final good model always obtains a positive relationship between $\left(\frac{W^S}{W^U}\right)^{au}$ and $\left(\frac{H}{L}\right)$. Also the expression of the relative wage is independent of productivity parameters in the R&D sector and of technology parameters in the final good sector in this special case with $\rho = 0$; but this is not true in Kiley (1999). However, equation (4.32A) is identical to the corresponding equation[58] obtained in Wang. et. al. (2009) who also solves a similar problem with $\rho = 0$.

Actually, the model of Kiley (1999) is a special case of the present model with $\rho = -1$, $K_3 = 0$ and $q = 1$. In this special case, equation (4.32) is modified as follows.

$$\left(\frac{W^S}{W^U}\right)^{au} = \frac{(1-\beta)}{\beta}\left[\frac{\delta_2}{\delta_1}\right]^{\frac{1}{K_1}}\left(\frac{H}{L}\right)^{\frac{1}{K_1}}; \qquad (4.32B);$$

and this equation with $\beta = \frac{1}{2}$ is identical to the expression of skilled unskilled wage ratio derived in Kiley (1999)[59] when $\beta = \frac{1}{2}$. This equation (4.32B) clearly shows a positive relationship between $\left(\frac{W^S}{W^U}\right)^{au}$ and $\left(\frac{H}{L}\right)$. Here $\rho = -1$ implies that the two final goods are perfect substitutes; and this is analytically equivalent to Kiley (1999) assumption that both the sectors produce the same good.

We can now establish the following proposition.

<u>PROPOSITION-4.1:</u> *Skilled unskilled wage ratio varies positively with the stock of domestic skilled labour in the long run equilibrium of the closed economy only if the degree of substitutability between two goods is sufficiently high[60].*

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[58] See equation (18) in Wang. et. al. (2009).

[59] The expression of wage inequality is derived in Kiley (1999) using equations (11), (12) and (18c) in his model; and is shown as follows.

$$\frac{W^S}{W^U} = \left[\frac{\delta_2}{\delta_1}\right]^{\frac{1}{K_1}}\left(\frac{H}{L}\right)^{\frac{1}{K_1}}.$$

[60] Here we require ρ to be very close to minus unity. We do not require $\rho = -1$; which is a stronger assumption.

4.3.2. Wage inequality under trade:

When the home country is open to international trade, we have $K_2 > 0$ because a positive international knowledge spillover effect is present. Then, adopting a similar process as used in the previous sub-section and using equations (4.14), (4.15), (4.18), (4.19), (4.22) and (4.23), we obtain the following equation [61]

$$\left(\frac{N}{M}\right)^{tr} =$$

$$\left[\frac{1}{\delta_1}\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_1(q-1)-K_3}{K_1^2 q}\right]} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]}$$

$$\mathrm{X} \left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_2\{K_1 q - K_3\}}{K_1^3 q}\right]} (H^*)^{\left[\frac{K_2\{K_1 q - K_3\}}{K_1^3 q}\right]} \quad (4.33).$$

Equations (4.28) and (4.33) are identical when $K_2 = 0$. Equations to be satisfied in the free trade equilibrium of the foreign country (rest of the world), which is assumed to be a closed economy, are as follows

$$Y^{S^*} = C_S{}^* + \frac{d\int_0^{N^*(t)} X_{itj}{}^* dj}{dt} \quad (4.26F);$$

and,

$$Y^{U^*} = C_U{}^* + \frac{d\int_0^{M^*(t)} Z_{itj}{}^* dj}{dt} \quad (4.27F).$$

So, using equations (4.14), (4.15), (4.16), (4.17), (4.25), (4.26F) and (4.27F), we obtain competitive equilibrium relative price of the advanced good in the rest of world as follows[62].

$$P_F^* = P_F^{tr} = \frac{(1-\beta)}{\beta}\left(\frac{M^* L^*}{N^* H^*}\right)^{(\rho+1)} \quad (4.34).$$

The small open home country is a taker of this relative price. So, under trade, equation (4.30) showing skilled-unskilled relative wage of the home country is modified as follows.

$$\left(\frac{W^S}{W^U}\right)^{tr} = P_F^{tr}\left(\frac{N}{M}\right)^{tr} \quad (4.35).$$

Then, using equations (4.34) and (4.35), we obtain the following equation

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[61] Detailed derivation of equation (4.33) is given in Appendix (4.D).

[62] Derivation of equation (4.34) in detail is obtained in Appendix (4.E).

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)}{\beta}\left(\frac{L^*}{H^*}\right)^{(\rho+1)} \frac{\left(\frac{N}{M}\right)^{tr}}{\left(\frac{N^*}{M^*}\right)^{(\rho+1)}} \tag{4.36}$$

As we take the rest of the world to be a closed economy, equation (4.28) is modified as follows.

$$\left(\frac{N}{M}\right)^* = \left[\frac{1}{\delta_1}\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_1(q-1)-K_3}{K_1^2 q}\right]} \left(\frac{H^*}{L^*}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L^*}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]} \tag{4.37}$$

Using equations (4.33) and (4.37), we derive the following equation.

$$\frac{\left(\frac{N}{M}\right)^{tr}}{\left(\frac{N^*}{M^*}\right)^{(\rho+1)}} =$$

$$[\delta_1]^{\rho\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left[\frac{1}{\delta_2}\right]^{\frac{\rho}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\rho\left[\frac{K_3-K_1(q-1)}{K_1^2 q}\right]} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]} \left(\frac{L^*}{H^*}\right)^{\frac{(\rho+1)(K_1 q - K_3)}{K_1^2 q}}$$

$$\text{X}\ (L^*)^{\frac{(\rho+1)[K_1(1-q)+K_3]}{K_1^2 q}} \left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} H^*\right]^{\frac{K_2(K_1 q - K_3)}{K_1^3 q}} \tag{4.38}$$

Using equations (4.36) and (4.38), we obtain the following equation.

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)}{\beta}$$

$$\text{X}[\delta_1]^{\rho\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left[\frac{1}{\delta_2}\right]^{\frac{\rho}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\rho\left[\frac{K_3-K_1(q-1)}{K_1^2 q}\right]} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]}$$

$$\text{X}\left(\frac{L^*}{H^*}\right)^{\frac{(\rho+1)[K_1 q(1+K_1)-K_3]}{K_1^2 q}} (L^*)^{\frac{(\rho+1)[K_1(1-q)+K_3]}{K_1^2 q}} \left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} H^*\right]^{\frac{K_2(K_1 q - K_3)}{K_1^3 q}} \tag{4.39}$$

If $\rho = 0$, then equation (4.39) is reduced to the following equation.

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)}{\beta}\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} H^*\right]^{\frac{K_2(K_1 q - K_3)}{K_1^3 q}} \left(\frac{L^*}{H^*}\right)^{1+\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{H}{L}\right)^{\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{L^*}{L}\right)^{\frac{K_1(1-q)+K_3}{K_1^2 q}} \tag{4.39A}$$

Now, with $\rho = 0$, we find a positive relationship between $\left(\frac{W^S}{W^U}\right)^{tr}$ and $\left(\frac{H}{L}\right)$ because $K_1 q > K_3$; and, in fact, equation (4.39) shows that the nature of the relationship between $\left(\frac{W^S}{W^U}\right)^{tr}$ and $\left(\frac{H}{L}\right)$

is independent of the value of ρ. So the positive relationship between the skilled unskilled wage ratio and the domestic skilled unskilled labour endowment ratio obtained in the one sector model of Kiley (1999) is always valid in this two commodity model when the economy is small and open to international trade but is not necessarily valid when it is closed. This is so because the effect of a change in $\left(\frac{H}{L}\right)$ is lost through the movement of P_F^* in the reverse direction in the closed economy equilibrium but it does not happen in the case of a small open economy where P_F^* is determined in the rest of the world and hence is independent of $\left(\frac{H}{L}\right)$. It should also be noted from equation (4.39) that $\left(\frac{W^S}{W^U}\right)^{tr}$ varies inversely with $\left(\frac{H^*}{L^*}\right)$ when two goods are imperfectly substitutes because an increase in $\left(\frac{H^*}{L^*}\right)$ induces P_F^* to move in the opposite direction[63]. We can now establish the following proposition.

<u>PROPOSITION-4.2</u>*: In the long run equilibrium, the skilled-unskilled wage ratio of the small open home country always varies positively (inversely) with the skilled unskilled labour endowment ratio in the home (foreign) country.*

Here equation (4.39) shows that the relative wage in the home country is affected by factor endowments of the foreign country. This is so because the home country is a taker of the relative product price determined in the competitive international market and because there is an international knowledge spillover effect. Also both the parameters, K_2 and K_3, enter into the R.H.S. expression of equation (4.39). So not only the magnitude of localized knowledge spillover but also the magnitude of international knowledge spillover affects the skilled-unskilled relative wage of the small open home country. However, in Wang. et. al (2009), this relative wage is only subject to the localized knowledge spillover effect and not to the globalized knowledge spillover effect. This is so because, in Wang et. al. (2009), the effect of international knowledge spillover comes only through the difference in relative factor endowments between the home country and the rest of the world. However, in our model, it comes directly as well as indirectly through the endowment difference. However, if $\rho + 1 = 0$ and $K_2 = 0$, then equation (4.39) is reduced to the following.

[63] See equation (4.34).

$$\left(\frac{W^S}{W^U}\right)^{tr} = \frac{(1-\beta)}{\beta}\left[\frac{1}{\delta_1}\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\rho\left[\frac{K_1(q-1)-K_3}{K_1^2 q}\right]} \left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)} \left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]} \qquad (4.39B).$$

This equation (4.39B) implies that, when the two final goods are perfect substitutes and when there is no international knowledge spillover effect, then the relative wage of the small open home country is independent of factor endowments of the foreign country. If, further, we assume that $K_3 = 0$ and $q = 1$, then equation (4.39B) is reduced to equation (4.32B). So, in this very special case where we get back the model of Kiley (1999), there is no effect of international trade on wage inequality in the home country. If two commodities are perfect substitutes, then a two commodity system works like an one commodity system because the consumer, in equilibrium, consumes one of the two commodities. However, there is the effect of trade on wage inequality when $\rho + 1 \neq 0$.

Using equations (4.32A) and (4.39A), we have

$$\Delta = \left(\frac{W^S}{W^U}\right)^{tr} - \left(\frac{W^S}{W^U}\right)^{au} =$$

$$\frac{(1-\beta)}{\beta}\frac{L}{H}\left[\left\{\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} H^*\right\}^{\frac{K_2(K_1 q - K_3)}{K_1^3 q}} \left(\frac{L^*}{H^*}\right)^{1+\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{H}{L}\right)^{1+\frac{(K_1 q - K_3)}{K_1^2 q}} \left(\frac{L^*}{L}\right)^{\frac{K_1(1-q)+K_3}{K_1^2 q}} - 1\right] \qquad (4.40).$$

Here Δ is a measure of the change in wage inequality in the home country caused by the opening of trade when $\rho = 0$. Equation (4.40) shows that the effect of trade on skilled-unskilled wage ratio depends on the inter-country differences in the levels of factor endowments and on the intensity of localized and international spillover effects. If factor endowment ratios are same in both the countries, i.e., $\frac{H}{L} = \frac{H^*}{L^*}$ then, with a high value of H^* and with $L^* > L$, we find that $\Delta > 0$ when at least one of the parameters $K_1(1-q)$, K_2 and K_3 takes a positive value. Hence we can establish the following proposition.

<u>PROPOSITION-4.3:</u> *If factor endowment ratios in both the countries are equal and if the foreign country has a larger endowment of each of the two factors, then opening of trade raises skilled-*

unskilled wage ratio in the home country in the presence of international and/or intra-sectoral spillover effect.

Proposition 3 implies that if home country is very small compared to the rest of the world then opening of trade worsens its skilled-unskilled wage inequality problem.

Now, equation (4.40) also shows that, if there is no international or intra-sectoral spillover i.e. $\mathrm{K}_2 = \mathrm{K}_3 = 0$ and $\mathrm{q} = 1$, then $\Delta = 0$. In this case, we go back to Kiley (1999) model and here skilled-unskilled wage inequality remains unchanged even after the opening of trade. So trade alone can not affect the skilled-unskilled wage inequality in this model. Trade can aggravate this problem only in the presence of spillover effect. In Acemoglu (2003), trade does not cause international technology spillover. In Wang et. al. (2009), trade leads to international technology spillover but the effect of trade on the change in wage inequality is independent of the magnitude of the international technology spillover effect parameter.

4.4. LIMITATIONS

However, the model developed in this chapter is subject to a set of limitations. Only final goods are traded in this model but intermediate goods are non-traded. The level of existing research is assumed to grow over time at an exogenously given rate. So, along the balanced growth path, all other endogenous variable of the model grow at the same exogenous rate. So the long run equilibrium growth rate in this model is exogenous; and this problem also exists in Kiley (1999). We assume the foreign country (rest of the world) to be a closed economy. Identical production technologies are assumed to exist in both the skilled labour using sector and the unskilled labour using sector. Also, the problems of imperfections in the markets for the final goods and the problem of international factor motilities are not considered in this model. The assumption of a representative household consisting of skilled labour as well as of unskilled labour is also a restrictive one. The possibility of unemployment is also ruled out.

Appendix (4.A):

Derivation of equation (4.28)

Under autarky, $K_2 = 0$. So, using equations (4.18) and (4.22), we obtain

$$\left[N(t) \big/ R(t) \right] = \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{1}{K_1}} \qquad (4.A.1).$$

Similarly, from equations (4.19) and (4.23), we have

$$\left[M(t) \big/ R(t) \right] = \left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1 q}} \left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\frac{K_3}{K_1^2 q}} \qquad (4.A.2).$$

Finally, from equations (4.A.1) and (4.A.2), we obtain

$$\frac{N}{M} = \frac{\left[\frac{1}{\delta_1} \left(\frac{1-a}{a} \right) X \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)}}{\left[\frac{1}{\delta_2} \left(\frac{1-a}{a} \right) Z \right]^{\frac{1}{K_1 q}}} \qquad (4.A.3).$$

Then, using equations (4.14), (4.15) and (4.A.3), we obtain

$$\left(\frac{N}{M} \right)^{au} = \left[\frac{1}{\delta_1} \right]^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} [\delta_2]^{\frac{1}{K_1 q}} \left[\left(\frac{1-a}{a} \right) \left(\frac{a^2}{r} \right)^{\left(\frac{a}{1-a} \right)} \right]^{\left[\frac{K_1 (q-1) - K_3}{K_1^2 q} \right]} \left(\frac{H}{L} \right)^{\left(\frac{K_1 q - K_3}{K_1^2 q} \right)} \left(\frac{1}{L} \right)^{\left[\frac{K_1 (1-q) - K_3}{K_1^2 q} \right]} \qquad (4.A.4).$$

This equation (4.A.4) is same as equation (4.28) in the body of the chapter.

Appendix (4.B):

Derivation of equation (4.29)

Using equations (4.25), (4.26) and (4.27), we obtain

$$P_F^{au} = \frac{(1-\beta)}{\beta} \left(\frac{Y^U - \frac{d\left(\int_0^{M(t)} Z_{itj} dj \right)}{dt}}{Y^S - \frac{d\left(\int_0^{N(t)} X_{itj} dj \right)}{dt}} \right)^{(\rho+1)} ,$$

$$\Rightarrow P_F^{au} = \frac{(1-\beta)}{\beta} \left(\frac{Y^U - MZg}{Y^S - NXg} \right)^{(\rho+1)} \qquad (4.B.1).$$

Using equations (4.14), (4.15), (4.16), (4.17) and (4.B.1), we obtain

$$P_F^{au} = \frac{(1-\beta)}{\beta}\left(\frac{ML\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - ML\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}{NH\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - NH\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}\right)^{(\rho+1)},$$

$$\Rightarrow P_F^{au} = \frac{(1-\beta)}{\beta}\left(\frac{ML}{NH}\right)^{(\rho+1)}\left(\frac{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}\right)^{(\rho+1)},$$

$$\Rightarrow P_F^{au} = \frac{(1-\beta)}{\beta}\left(\frac{ML}{NH}\right)^{(\rho+1)} \qquad (4.B.2).$$

This equation (4.B.2) is same as equation (4.29) in the body of the chapter.

Appendix (4.C):

Derivation of equation (4.30)

Using equations (4.7) and (4.8), we have

$$\frac{W^S}{W^U} = P_F\left(\frac{N}{M}\right)\left(\frac{X}{Z}\right)^a\left(\frac{H}{L}\right)^{-a} \qquad (4.C.1).$$

Using equations (4.14), (4.15) and (4.C.1), we obtain

$$\frac{W^S}{W^U} = P_F\left(\frac{N}{M}\right)\left(\frac{H\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}}{L\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}}\right)^a\left(\frac{H}{L}\right)^{-a},$$

$$\Rightarrow \frac{W^S}{W^U} = P_F\left(\frac{N}{M}\right) \qquad (4.C.2).$$

Now, in autarky equation (4.C.2) becomes

$$\left(\frac{W^S}{W^U}\right)^{au} = P_F^{au}\left(\frac{N}{M}\right)^{au} \qquad (4.C.3).$$

This equation (4.C.3) is same as equation (4.30) in the body of the chapter.

Appendix (4.D):

Derivation of equation (4.33)

Under trade, $K_2 \neq 0$. So, using equations (4.18) and (4.22), we obtain

$$\left[{}^{N(t)}\!/_{R(t)}\right] = \left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X\right]^{\frac{1}{K_1}}\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X^*\right]^{\frac{K_2}{K_1^2}} \qquad (4.D.1).$$

Similarly, from equations (4.19) and (4.23), we have

$$\left[{}^{M(t)}\!/_{R(t)}\right] = \left[\frac{1}{\delta_2}\left(\frac{1-a}{a}\right)Z\right]^{\frac{1}{K_1 q}}\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X\right]^{\frac{K_3}{K_1^2 q}}\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X^*\right]^{\frac{K_3 K_2}{K_1^3 q}} \qquad (4.D.2).$$

Finally, from equations (4.D.1) and (4.D.2), we obtain

$$\frac{N}{M} = \frac{\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)}\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)X^*\right]^{K_2\left(\frac{K_1 q - K_3}{K_1^3 q}\right)}}{\left[\frac{1}{\delta_2}\left(\frac{1-a}{a}\right)Z\right]^{\frac{1}{K_1 q}}} \qquad (4.D.3).$$

Then, using equations (4.14), (4.15) and (4.D.3), we obtain

$$\left(\frac{N}{M}\right)^{tr} =$$

$$\left[\frac{1}{\delta_1}\right]^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)}[\delta_2]^{\frac{1}{K_1 q}}\left[\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_1(q-1)-K_3}{K_1^2 q}\right]}\left(\frac{H}{L}\right)^{\left(\frac{K_1 q - K_3}{K_1^2 q}\right)}\left(\frac{1}{L}\right)^{\left[\frac{K_1(1-q)-K_3}{K_1^2 q}\right]}$$

$$X\left[\frac{1}{\delta_1}\left(\frac{1-a}{a}\right)\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)}\right]^{\left[\frac{K_2\{K_1 q - K_3\}}{K_1^3 q}\right]}(H^*)^{\left[\frac{K_2\{K_1 q - K_3\}}{K_1^3 q}\right]} \qquad (4.D.4).$$

This equation (4.D.4) is same as equation (4.33) in the body of the chapter.

Appendix (4.E):

Derivation of equation (4.34)

Using equations (4.25), (4.26F) and (4.27F), we obtain

$$P_F^* = \frac{(1-\beta)}{\beta}\left(\frac{Y^{U^*} - \frac{d\int_0^{M^*(t)} Z_{itj}{}^* dj}{dt}}{Y^{S^*} - \frac{d\int_0^{N^*(t)} X_{itj}{}^* dj}{dt}}\right)^{(\rho+1)},$$

$$\Rightarrow P_F^* = \frac{(1-\beta)}{\beta}\left(\frac{Y^{U^*} - M^* Z^* g^*}{Y^{S^*} - N^* X^* g^*}\right)^{(\rho+1)} \qquad (4.E.1).$$

Using equations (4.14), (4.15), (4.16), (4.17) (4.modified under trade) and (4.E.1), we obtain

$$P_F^* = \frac{(1-\beta)}{\beta}\left(\frac{M^*L^*\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - M^*L^*\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}{N^*H^*\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - N^*H^*\left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}\right)^{(\rho+1)},$$

$$\Rightarrow P_F^* = \frac{(1-\beta)}{\beta}\left(\frac{M^*L^*}{N^*H^*}\right)^{(\rho+1)}\left(\frac{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}{\left(\frac{a^2}{r}\right)^{\left(\frac{a}{1-a}\right)} - \left(\frac{a^2}{r}\right)^{\left(\frac{1}{1-a}\right)}g}\right)^{(\rho+1)},$$

$$\Rightarrow P_F^* = \frac{(1-\beta)}{\beta}\left(\frac{M^*L^*}{N^*H^*}\right)^{(\rho+1)} \qquad (4.E.2).$$

This equation (4.E.2) is same as equation (4.34) in the body of the chapter.

Chapter 5

The role of imitation in a dynamic product variety model

5.1 INTRODUCTION

This chapter is developed to analyse the effect of imitation on skilled-unskilled wage inequality problem; and the model developed in this chapter is an extension of the product variety model developed in chapter 3 of Grossman and Helpman (1991). Grossman and Helpman (chapter 3, 1991) develop a dynamic product variety model in which only one production sector produces varieties of innovated products and a R&D sector gives birth of new varieties. That model neither makes any distinction between skilled labour and unskilled labour nor considers the problem of imitation. North-South models of Grossman and Helpman (1991) and Helpman (1993) analyse the role of imitation on the long run rate of growth and on the North-South relative wage. However, these models do not distinguish between skilled labour and unskilled labour.

There is no empirical work based on cross-section evidence focusing on the relationship between IPR protection and wage inequality. Empirical works like Kanwar and Evenson (2003, 2009), Park (2008), Ginarte and Park (1997) etc. show that there is significant improvement in the worldwide patent protection during the period 1960-2005. Ginarte and Park (1997), who presents an index of patent rights for 110 countries for the period of 1960-1990, shows that the degree of strength of patent laws and composition of patent rights vary across countries and this variation is related to the variation in the level of economic development. A country with a larger size of innovating sector has a greater incentive to provide patent laws due to large fixed costs of establishing a patent system. Park (2008) extends the study of Ginarte and Park (1997) for 122 countries and for the period upto 2005; and obtains results similar to Ginarte and Park (1997). Using panel data for 1981-2000, Kanwar and Evenson (2009) shows that, due to shortage of financial capital and skilled labour, developing countries offer weaker protection for potential gain. This policy of strengthening patent protection should give incentives to innovation leading to an increase in the relative demand for skilled labour because the R & D sector is highly skilled labour intensive. This, in turn, should be followed by a rise in skilled-unskilled relative wage. That an increase in the degree of skilled-unskilled wage inequality is empirically found worldwide from 1960's is already mentioned in the page 2 of chapter 1[64]. The relationship between the imitation rate and skilled-unskilled wage inequality appears to be an important one because existing substantial inter country variations in the degree of effective implementation of Intellectual Property Rights (IPR) [65] leads to inter country variations in the imitation rate. Imitation is a serious problem for a less developed country. On the one hand, it discourages innovation; and, on the one other hand, it affects the relative demand for skilled labour because imitation sector is more unskilled labour intensive than the innovation sector. These two evidences justify that there should be some correlation between the strength of IPR protection and the degree of wage inequality; and we plan to explain this theoretically in this chapter. No model in the existing theoretical literature, except Thoeing and Verdier (2003), has attempted to analyse the effects of imitation on this skilled-unskilled wage inequality problem.

In the present model, we consider a closed economy with skilled labour as well as unskilled labour. The economy consists of two producing sectors of which one sector produces varieties of innovated products with skilled labour as well as unskilled labour. However, the other sector imitates those innovated products without bearing any cost of imitation[66] and then produces those imitated products using unskilled labour as the only input. Also, like Grossman and Helpman (1991) and Helpman (1993), we introduce a R&D sector in this model to develop blue-prints of new products using skilled labour as the only input.

[64] In this context see footnote 1, 2 and 3 of chapter 1.
[65] Inter-country variations in IPR are shown in Park (2008), Ginarte and Park (1997).
[66] We assume this following Helpman (1993) but we are fully aware that imitation activity is not at all cost less in the real world.

We derive many interesting results from the basic model. First, there exists a constant rate of growth in this model and it is independent of the attainment of steady-state equilibrium. Secondly, an increase in skilled (unskilled) labour endowment raises (lowers) the rate of expansion of product varieties. Thirdly, an increase in skilled (unskilled) labour endowment raises (lowers) the skilled-unskilled wage ratio and an increase in the imitation rate lowers it. Fourthly, the steady-state equilibrium is stable in this model. Finally, in the steady state equilibrium, an increase in skilled (unskilled) labour endowment lowers (raises) the level of welfare of the representative consumer but an increase in the imitation rate raises it.

We extend the basic model introducing endogenous imitation and assume the existence of a social institution that has control over this endogenous imitation rate. This social institution produces an imitation preventing public good with skilled labour as the only input. It is shown that an increase in skilled (unskilled) labour endowment raises (has no effect on) the rate of growth and raises (lowers) the skilled-unskilled wage ratio. However, an improvement in the imitation preventing efficiency of the public good raises the skilled-unskilled wage ratio though it has no effect on growth rate.

Our results related to effects of imitation on skilled-unskilled wage ratio are interesting compared to corresponding results obtained in Thoeing and Verdier (2003). In our model, an exogenous increase in the imitation rate lowers the skilled-unskilled wage ratio by raising the relative demand for unskilled labour because production of imitated goods requires only unskilled labour but the change in the imitation rate has no effect on the technology of the innovating firms. However, in Thoeing and Verdier (2003), innovating firms use skill intensive technology to meet the increased threat of imitation; and thus the relative demand for skilled labour is increased leading to an increase in the skilled-unskilled wage ratio as a consequence of an exogenous increase in the threat of imitation.

The rest of the chapter is organized as follows. Section 5.2 describes the basic model model with exogenous imitation rate. Sub-section 5.2.1 describes the model and sub-section 5.2.2 analyzes working of the model. Rate of growth is derived in subsection 5.2.2.1 and the stability of the steady-state equilibrium is analysed in subsection 5.2.2.2. Effects of parametric changes on the degree of wage inequality in the steady-state growth equilibrium are described in subsection 5.2.2.3. The rate of interest is determined in subsection 5.2.2.4; and comparative static effects on welfare are analysed in sub section 5.2.2.5. In section 5.3; we introduce endogenous imitation rate. Limitations of this model are described in section 5.4.

5.2. The Basic Model:[67]

5.2.1 Description:

We consider a closed economy with three sectors and two primary factors- skilled labour and unskilled labour. Sector 1 produces varieties of innovated products with skilled labour as well as unskilled labour as inputs; and sector 2 produces varieties of imitated products with only unskilled labour[68,69]. Also there is a R&D sector developing blue-prints of new products and it uses skilled labour as the only input.

Let the rate of innovation of new products per unit time be denoted by $\dot{n}$. Then the production function in the R&D sector is given by

$$\dot{n} = \frac{lK_S}{a} \qquad (5.2.1);$$

where, l is the amount of skilled labour employed in the R&D sector; K_S is the existing stock of knowledge and a is the per unit skilled labour requirement in the R&D sector. Following Grossman and Helpman (1991), Helpman (1993) etc. we assume that $K_S = n$ where n is the total number of varieties innovated as well as imitated. So we can modify equation (5.2.1) as follows.

$$g = \frac{\dot{n}}{n} = \frac{l}{a} \qquad (5.2.2).$$

where g is the rate of growth of new products.

[67] Gupta and Dutta (2013) is partly based on the materials presented in this section.

[68] Generally varieties innovated in a country are imitated in other countries. This model may also represent the world economy with free trade, perfect mobility of factors, identical production technology across countries and with intercountry variations in the degree of implementation of intellectual Property Right (IPR) protection Acts.

[69] None of the imitated products, in reality, is produced without the use of skilled labour. However, unskilled workers acquire some production specific skill through learning by doing and can replace skilled workers in many skill intensive stages of production once products are imitated. Our concept of skilled labour does not include this learning by doing skill of unskilled workers.

Firms in sector 2 do imitations without bearing any cost. Rate of imitation done by sector 2 in this basic model is assumed to be exogenous; and this rate, denoted by m, is defined as follows.

$$\mathrm{m} = \frac{\dot{\mathrm{n}}^{\mathrm{U}}}{\mathrm{n}^{\mathrm{S}}} \qquad (5.2.3).$$

Here n^{S} and n^{U} represent total number of varieties produced by sector 1 and sector 2, respectively. Sector 1 does not produce any variety already imitated by sector 2.

So we have

$$\mathrm{n}^{\mathrm{S}} + \mathrm{n}^{\mathrm{U}} = \mathrm{n} \qquad (5.2.4).$$

The fraction of goods not imitated by sector 2 is denoted by ξ. Hence

$$\xi = \frac{\mathrm{n}^{\mathrm{S}}}{\mathrm{n}} \qquad (5.2.5).$$

Now, from equation (5.2.5), we obtain[70]

$$\dot{\xi} = \mathrm{g} - (\mathrm{g} + \mathrm{m})\xi \qquad (5.2.6).$$

Equation (5.2.6) shows the rate of change in the fraction of unimitated (innovated) products.

In the steady-state equilibrium, the fraction of unimitated goods remains unchanged over time. Hence $\dot{\xi} = 0$. So we obtain

$$\xi = \frac{\mathrm{g}}{(\mathrm{g}+\mathrm{m})} \qquad (5.2.7).$$

So equation (5.2.7) implies that the fraction of innovated products in the steady-state equilibrium varies positively with the growth rate.

All individuals have identical preferences. The representative household maximizes the discounted present value of instantaneous utility over the infinite horizon; and it is given by

$$\mathrm{U}(\mathrm{t}) = \int_{\mathrm{t}}^{\infty} \mathrm{e}^{-\rho(\tau-\mathrm{t})} \log \mathrm{u}(\tau) \mathrm{d}\tau \qquad (5.2.8).$$

The intertemporal budget equation of that representative household[71] is given by

$$\int_{\mathrm{t}}^{\infty} \mathrm{e}^{-\mathrm{r}(\tau-\mathrm{t})} \mathrm{E}(\tau) \mathrm{d}\tau = \int_{\mathrm{t}}^{\infty} \mathrm{e}^{-\mathrm{r}(\tau-\mathrm{t})} \mathrm{I}(\tau) \mathrm{d}\tau + \mathrm{A}(\mathrm{t}) \quad \forall \mathrm{t} \quad (5.2.9).$$

[70] Detailed derivation of equation (5.2.6) is given in the Appendix (5.A).

[71] We assume that the representative household has both skilled labour endowment and unskilled labour endowment. Even if we consider two representative households- one with skilled labour endowment and the other with unskilled labour endowment, aggregate demand functions for varieties would remain unchanged provided that their preferences are identical.

Here, $u(\tau)$, $E(\tau)$, $I(\tau)$ and $A(\tau)$ represent levels of instantaneous utility, instantaneous expenditure, instantaneous income and current assets respectively at the time point τ. ρ and r denote the subjective discount rate and the nominal interest rate respectively.

The instantaneous utility function of the representative consumer is given by the following.

$$u(\tau) = \left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}}, \ 0 < \alpha < 1, \qquad (5.2.10),$$

where, $x(j)$ is the level of consumption of jth variety. This instantaneous utility function is of CES type satisfying all standard properties and being symmetric in its arguments. Maximizing the discounted present value of instantaneous utility defined over the infinite time horizon subject to the intertemporal budget constraint, we obtain the following optimality condition[72].

$$\frac{\dot{E}}{E} = r - \rho \qquad (5.2.11).$$

We can also derive the aggregate demand function for jth variety as follows.

$$x(j) = p(j)^{-\varepsilon}\frac{E}{P^{1-\varepsilon}} \qquad (5.2.12).$$

Here $\varepsilon = \frac{1}{1-\alpha} > 1$ is the price elasticity of demand for the representative variety. Here, $p(j)$ is the price of the jth variety, E is the aggregate spending on all these varieties, and P is a price index defined as

$$P = \left[\int_0^n p(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \qquad (5.2.13).$$

Sector 1 produces each of these innovated products with skilled as well as unskilled labour as inputs; and labour-output coefficient of each of these two types of labour is assumed to be unity. So $(W^S + W^U)$ is the marginal cost of production of each of these innovated varieties. Producer of each of all innovated varieties is a monopolist. So it charges a monopoly price of its product; and it is given by

$$P^S = \frac{1}{\alpha}(W^S + W^U) \qquad (5.2.14).$$

Here, P^S is the price of the representative innovated variety produced in sector 1; and W^S and W^U are wage rates of skilled labour and unskilled labour, respectively.

[72] Detailed derivations of equations (5.2.11) and (5.2.12) are given in the Appendix (5.C) and Appendix (5.B), respectively.

Sector 2, that produces varieties of imitated products with unskilled labour as the only input, faces a competitive market for each of those varieties; and hence charges a price equal to the marginal cost of production which, in turn, is equal to the wage rate of unskilled labour. So we have

$P^U = W^U$ (5.2.15).

Here, P^U is the price of the representative imitated variety. We assume that $W^S > W^U$ in the initial equilibrium and comparative static effects are too small to reverse this inequality. Then, from equations (5.2.14) and (5.2.15), we also have $P^S > P^U$.

Out of total n products, n^S products are sold at the price, P_S, and n^U products are sold at the price, P_U. Hence, using equations (5.2.4) and (5.2.5), equation (5.2.13) can be expressed as follows.

$P = n^{\frac{1}{1-\varepsilon}}\left[\xi(P^S)^{(1-\varepsilon)} + (1-\xi)(P^U)^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}}$ (5.2.16).

Let x^S and x^U be levels of output of the representative varieties to be produced in sector 1 and sector 2, respectively. L^S and L^U denote endowments of total skilled labour and total unskilled labour respectively. Markets for each of these two types of labour are assumed to be competitive. So market clearing conditions of these two types of labour, who are perfectly mobile among their using sectors, are given by following two equations.

$ag + n^S x^S = L^S$ (5.2.17);

and,

$n^S x^S + n^U x^U = L^U$ (5.2.18).

We assume free entry of firms of sector 1 into the R&D sector. The return from this R&D activity, denoted by v^S, is basically the value of the blue print; and this is equal to the discounted present value of profit of the producer of the representative innovated variety defined over the infinite time horizon. Under competitive equilibrium, return from this R&D activity must be equal to its cost; and $\frac{W^S a}{n}$ is the cost of developing a blueprint because only skilled labour is used in the R&D sector. So we have

$v^S = \frac{W^S a}{n}$ (5.2.19).

Firms of sector 1 issue equities to finance their R&D investments. $\frac{\Pi^S}{v^S}$ represents the rate of dividend and $\frac{\dot{v}^S}{v^S}$ is the rate of growth of the value of the firm. Since m stands for the rate of imitation, $\left(\frac{\Pi^S}{v^S}+\frac{\dot{v}^S}{v^S}-m\right)$ is the net rate of return from investment in the stock market. This net rate of return should not fall short of the interest rate obtained from the loan market. Hence we have

$$\frac{\Pi^S}{v^S}+\frac{\dot{v}^S}{v^S}\geq r+m \qquad (5.2.20).$$

If condition (5.2.20) does not hold, then firms of sector 1 would not produce and would lend their capital at the interest rate, r.

5.2.2. Working of the model

5.2.2.1. Rate of growth

Using equations (5.2.5), (5.2.12), (5.2.17) and (5.2.18), we obtain[73]

$$\frac{P^S}{P^U}=\left\{\left(\frac{L^U}{L^S-ag}-1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}} \qquad (5.2.21).$$

Using equations (5.2.14), (5.2.15) and (5.2.21), we obtain[74]

$$\Delta=\frac{W^S}{W^U}=\alpha\left\{\left(\frac{L^U}{L^S-ag}-1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}}-1 \qquad (5.2.22).$$

Using equations (5.2.5), (5.2.12), (5.2.14), (5.2.15), (5.2.16), (5.2.18) and (5.2.22), we obtain[75]

$$E=W^U(L^S-ag)\left[\left\{\left(\frac{L^U}{L^S-ag}-1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\}+1\right] \qquad (5.2.23).$$

Finally, using equations (5.2.12), (5.2.14), (5.2.15), (5.2.16), (5.2.17), (5.2.22) and (5.2.23), we obtain[76]

$$g=\frac{2L^S-L^U}{2a} \qquad (5.2.24).$$

[73] Derivation of equation (5.2.21) is given in the Appendix (5.D).
[74] Derivation of equation (5.2.22) is given in the Appendix (5.D).
[75] Derivation of equation (5.2.23) is given in the Appendix (5.E).
[76] Derivation of equation (5.2.24) is given in the Appendix (5.E).

Equation (5.2.24) shows the constant rate of product development (economic growth) in this model; and this rate is independent of whether the economy is in the steady-state growth equilibrium or not. While deriving equation (5.2.24), we do not use equation (5.2.7) i.e., the steady-state equilibrium condition of this model. In Helpman (1993) or in Grossman and Helpman (1991), the rate of product development is constant only in the steady-state growth equilibrium. Here the rate of expansion of varieties (rate of growth) is determined by exogenously given values of some parameters like skilled labour endowment, $\mathrm{L^S}$ unskilled labour endowment, $\mathrm{L^U}$, and the productivity parameter in the R&D sector, a. We need appropriate restrictions on the values of those parameters to ensure that $\mathrm{g} > 0$[77]. However, equation (5.2.24) shows that g varies positively with $\mathrm{L^S}$ and inversely with $\mathrm{L^U}$ and a. So we can establish the following proposition.

<u>PROPOSITION-5.2.1</u>: *An increase in skilled (unskilled) labour endowment raises (lowers) the rate of expansion of varieties.*

We now provide the intuition behind this proposition. As skilled labour endowment is increased with unskilled labour endowment remaining unchanged, demand for skilled labour falls in sector 1; and so the wage rate of skilled labor is reduced in that sector. So skilled labour moves from sector 1 to the R&D sector; and hence the supply of skilled labour is increased in the R&D sector. The R&D sector, with a linear production function, can employ the entire labour force. So the growth rate is increased. The same mechanism works in the opposite direction when unskilled labour endowment is increased with skilled labour endowment remaining unchanged.

5.2.2.2.<u>The stability of steady-state equilibrium</u>

Using equations (5.2.6) and (5.2.24), we obtain

$$\dot{\xi} = \left(\frac{2\mathrm{L^S} - \mathrm{L^U}}{2\mathrm{a}}\right) - \left(\frac{2\mathrm{L^S} - \mathrm{L^U} + 2\mathrm{am}}{2\mathrm{a}}\right)\xi \qquad (5.2.25).$$

In the steady-state growth equilibrium, $\dot{\xi} = 0$. Hence, the steady-state growth equilibrium value of ξ is given by

[77] The rate of growth is positive if $2L^S > L^U$.

$\xi^* = \frac{2L^S - L^U}{2L^S - L^U + 2am}$.

Since ξ^* is a constant, then $\frac{n^S}{n^U} = \frac{\xi}{1-\xi}$ is also so. Hence, in the steady-state growth equilibrium, we have $\frac{\dot{n}^S}{n^S} = \frac{\dot{n}^U}{n^U} = \frac{\dot{n}}{n} = g$. This equation (5.2.25) shows that $\dot{\xi}$ is a negative function of ξ. So the equilibrium is stable. If the economy initially starts with a higher (lower) fraction of goods not imitated, then that fraction falls (rises) over time and converges to its steady-state growth equilibrium value. We can establish the following proposition.

PROPOSITION-5.2.2: *The steady-state growth equilibrium is stable.*

In models of Helpman (1993), Grossman and Helpman (1991) etc., the steady-state equilibrium is a saddle point because g is a constant in none of those models. In each of these models, we find another differential equation like

$\dot{g} = g(g, \xi)$;

and the stability property of the dynamic equilibrium in that model is to be investigated by solving the time path of ξ and g simultaneously. In our model, equation (5.2.24) shows that $\dot{g} \equiv 0$; and so the stability property is analyzed using only the time path of ξ.

5.2.2.3.Wage inequality

Using equations (5.2.7), (5.2.22) and (5.2.24), we obtain

$$\Delta = \frac{W_S}{W_U} = \alpha \left(\frac{2L^S - L^U}{2am}\right)^{\frac{1}{\varepsilon}} - 1 \qquad (5.2.26).$$

Here $\Delta > 0$; and so we need appropriate restrictions on the values of parameters to ensure this. This equation (5.2.26) shows how the skilled-unskilled wage ratio in the long run equilibrium varies with values of different parameters. Here $\alpha > 0$ and $\varepsilon > 0$. Hence Δ varies positively with L^S and inversely with L^U and m**.** This leads to the following proposition.

PROPOSITION-5.2.3: *An increase in the level of skilled (unskilled) labour endowment raises (lowers) the skilled-unskilled wage ratio in the steady state equilibrium; and an increase in the imitation rate lowers it.*

We now provide intuitive explanations for this result. An increase in the skilled labour endowment has two effects. The direct effect implies a fall in the skilled wage rate. However, the growth rate is also increased implying that more blue prints are produced in the R&D sector. This leads to an increase in the demand for skilled labour as well as for unskilled labour in sector 1 that produces innovated products. Thus both the skilled wage rate and the unskilled wage rate are increased following the indirect effect. However, the increase in the skilled wage rate obtained from the indirect effect outweighs the combined effects of the increase in the unskilled wage rate obtained from the indirect effect and the decrease in the skilled wage rate obtained from the direct effect. Hence the skilled-unskilled wage ratio is increased. The same mechanism works in the opposite direction when unskilled labour endowment is increased with skilled labour endowment remaining unchanged; and so the skilled-unskilled wage ratio is reduced in that case.

On the other hand, as the imitation rate is increased, the proportion of innovated (not imitated) products is reduced in sector 1 and fraction of imitated products produced by sector 2 is increased in the new steady state equilibrium. So the demand for unskilled labour and consequently the unskilled wage rate are increased in sector 2. So unskilled labour moves from sector 1 to sector 2. So the demand for skilled labour falls in sector 1 because the production function in that sector is of fixed coefficient type; and, as a result, the skilled wage rate falls. So the skilled-unskilled wage ratio is decreased.

Our result related to effects of imitation on skilled-unskilled wage ratio is interesting compared to the corresponding result obtained in Thoeing and Verdier (2003). In our model, an increase in the imitation rate lowers the skilled-unskilled wage ratio by raising the relative demand for unskilled labour because production of imitated goods requires only unskilled labour and the change in the imitation rate has no effect on the technology of producing innovated goods. In Thoeing and Verdier (2003), firms producing innovated products use skill intensive technology to meet the increased threat of imitation; and thus the relative demand for skilled labour is increased leading to an increase in the skilled-unskilled wage ratio when there is an increased threat of imitation.

5.2.2.4. Interest rate:

In the steady-state growth equilibrium, ξ takes a constant value and g is always a constant. Hence, from equations (5.2.19), (5.2.22) and (5.2.23), we have

$$\frac{\dot{v}^S}{v^S} + g = \frac{\dot{W}^S}{W^S} = \frac{\dot{W}^U}{W^U} = \frac{\dot{E}}{E}.$$

Here the value of the firm, v^S, is normalized to unity following Lai (1998), Mondal and Gupta (2008) etc. Hence, $\dot{v}^S = 0$; and so we have

$$\frac{\dot{E}}{E} = g \qquad (5.2.27).$$

Using equations (5.2.11) and (5.2.27) we have $r = \rho + g$; and then using equation (5.2.24), we can solve for r in the steady-state growth equilibrium. Obviously r and g behave in similar ways with respect to changes in parameters.

On the other hand, using equations (5.2.7), (5.2.23) and (5.2.24), we obtain

$$W^U\left[\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}} + 1\right] = \frac{2E}{L^U} \qquad (5.2.28).$$

Using equations (5.2.5), (5.2.7), (5.2.11), (5.2.14), (5.2.19), (5.2.20), (5.2.24) and (5.2.28), we derive the following condition[78].

$$\Delta \leq \frac{(1-\alpha)L^U\left(2L^S - L^U + 2am\right)}{\alpha 2a\rho\left(2L^S - L^U\right) + \left(2L^S - L^U + 2am\right)\left(\alpha 2L^S - L^U\right)} \qquad (5.2.29).$$

This inequality (5.2.29) is the condition necessary as well as sufficient for firms in sector 1 to continue production and to finance R&D expenditure by issuing equities. If this condition is not satisfied, these firms would not produce and would lend their capital in the loan market. Inequality (5.2.29) basically implies an upper limit on the skilled-unskilled wage ratio which is necessary for our results to hold though our purpose is to explain the increase in wage inequality. Skilled labour is an essential factor of production for firms in sector 1 but is not so for firms in sector 2. So firms producing innovated goods are in difficulties when skilled-unskilled wage ratio is very high; and hence they then do not find production profitable. Obviously, the present model or any variant of Helpman (1993) or of Grossman and Helpman (1991) model does not work when innovating firms stop production.

[78] Detailed derivation of equation (5.2.29) is given in Appendix (5.F).

5.2.2.5. Effect on Welfare

The instantaneous utility function of the representative household given by equation (5.2.10) is an index of social welfare because all households are identical here. Using equations (5.2.7), (5.2.10), (5.2.12) - (5.2.16), (5.2.24) and (5.2.26), we obtain following modified form of this utility function[79].

$$u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{1}{\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \left[\left\{\frac{1}{\alpha}(\Delta+1)\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \qquad (5.2.30).$$

In Appendix (5.H), it is shown that equation (5.2.30) implies an inverse relationship between the degree of wage inequality, Δ, and the level of utility, u, of the household. So, using proposition 3, we can show that an increase in the level of skilled (unskilled) labour endowment lowers (raises) the level of social welfare, u, through an increase (decrease) in the skilled-unskilled wage ratio; and an increase in the imitation rate raises the level of social welfare through a decrease in the skilled-unskilled wage ratio. So we establish the following proposition.

PROPOSITION-5.2.4: *In the steady state growth equilibrium, an increase in the level of skilled (unskilled) labour endowment lowers (raises) the level of social welfare; and an increase in the imitation rate raises it.*

5.3. The Model with endogenous imitation rate:

This section presents an extension of the basic model developed in section 5.2 of this chapter. Here we introduce endogenous imitation and assume that a social institution has control over this endogenous imitation rate. We introduce an imitation preventing public good producing sector in addition to the sectors described in the basic model. This social institution or imitation preventing public good producing sector uses skilled labour as the only input; and the rate of imitation varies inversely with the size of this institution which is also endogenously determined. However, the efficiency parameter of this sector is exogenous and its magnitude

[79] Detailed derivation of equation (5.2.30) is given in the Appendix (5.G).

stands for the efficiency of the institution. Also, we introduce another modification assuming that innovated good producing sector now uses only unskilled labour. So both the production sectors in this extended model use only unskilled labour as input. In general, manufacturing sectors are unskilled labour intensive relative to R&D sector and imitation preventing public good sector. In the light of this empirical fact, we consider an extreme example here assuming that both the production sectors uses only unskilled labour as input while the R&D sector and the imitation preventing public good sector uses only skilled labour as input. However, the innovated product producing manufacturing sector that derives benefits from this social institution must bear the burden of financing the cost of production of this public good. So this cost is financed by lump sum tax imposed on all firms producing innovated varieties.

We now turn to explain the motivation behind this extension. According to Acemoglu and Verdier (1998), property rights are never perfect in terms of implementation. Social infrastructure is very crucial for monitoring of these written laws. Difference in institutional framework can have huge impact on the effective implementation of these laws. Many empirical studies focus on the relationship between the presence of the appropriate social institution and the strength of the intellectual property right. Magge (1992) estimates significant benefits to strong legal systems. His empirical approach implicitly assumes an endogenous institutions model where a fraction of population is hired to build and maintain those institutions. Khan (2003), in the context of British patent system, argues that patent laws are regarded only when they are monitored. Khan and Sokoloff (2001) provide extensive evidence to justify that early development of broad access to IPR institutions with strict enforcement was crucial for USA to move from a net importer to a net exporter of patents. Hall and Jones (1999) and Grigorian and Martinez (2002) argue that social institutions as measured by quality, corruption, risk of appropriation and repudiation of contracts of government bureaucracy are important factors to explain cross-country differences in output per worker. North and Thomas (1973) shows that social infrastructure or Government institutions help social agents to capture the full returns of their actions by reducing uncertainty and transaction costs. According to Rodrik (2000), social institutions play an important role to protect intellectual property rights. So threat of imitation can not be reduced only by introducing laws.

This motivates us to introduce endogenous imitation in the model and to assume that there exists a social institution to control this endogenous imitation rate.

We derive many interesting results from this extended model. First, there exists a constant rate of growth in this model and it is independent of the attainment of steady-state equilibrium. Secondly, an increase in skilled labour endowment raises the rate of growth (expansion of varieties) but a change in unskilled labour endowment has no effect on it. Thirdly, the change in skilled labour endowment or in unskilled labour endowment has no effect on the imitation rate. An improvement in the imitation prevention efficiency of the public good (social institution) lowers the imitation rate. Fourthly, an increase in unskilled labour endowment and/or an improvement in the imitation preventing efficiency of the public good (social institution) raises the skilled-unskilled wage ratio in our model. If the monopoly power of each firm in the innovated sector is very low, then an increase in skilled labour endowment lowers the skilled-unskilled wage ratio. Finally, in the steady state equilibrium, an increase in the level of unskilled labour endowment raises the level of social welfare but an increase in skilled labour endowment and an improvement in the imitation prevention efficiency of the public good has an ambiguous effect on it.

The paper is organized as follows. Section 5.3.1 describes the model and section 5.3.2 analyses its results. Rate of growth and rate of imitation are derived in subsections 5.3.2.1 and 5.3.2.2 respectively; and the stability of the steady-state equilibrium is analysed in subsection 5.3.2.3. The rate of interest is determined in subsection 5.3.2.4. Effects of parametric changes on the degree of wage inequality in the steady-state growth equilibrium are described in subsection 5.3.2.5; and comparative static effects on welfare are analysed in sub section 5.3.2.6.

5.3.1. <u>Description:</u>

The production function of the imitation preventing public good producing sector is given as follows.

$\mathrm{y_m} = \mathrm{nl_m}^{\beta}$, with $0 < \beta < 1$ (5.3.1).

Here y_m stands for the level of output of this public good and l_m is the amount of skilled labour employed in this public good sector. β is the labour elasticity of output. $0 < \beta < 1$ implies that there is diminishing returns to labour in this sector.[80] Productivity of skilled labour in this sector also varies proportionately with the stock of knowledge, n, because expansion of the stock of knowledge enhances the level of skill of the worker required to control imitation.

In equilibrium, real wage rate of skilled labour is equal to its average physical productivity in the public good producing sector because the objective of the social institution providing the public good is to maintain a no profit no loss equilibrium, i.e., its budget must be balanced. So

$$n{l_m}^{(\beta-1)} = W_S \qquad (5.3.2).$$

Here W_S represents the wage rate of the skilled labour in the public good (social institution) sector. Firms in sector 2 do imitations without bearing any direct cost. The rate of imitation is assumed to vary inversely with the size of the imitation preventing public good sector and positively with the existing stock of knowledge, n. So the imitation rate, denoted by m, is defined as follows.

$$m = \frac{\dot{n}^U}{n^S} = \frac{n}{y_m b} \qquad (5.3.3).$$

Here b is a parameter measuring the efficiency of imitation prevention done by the social institution. m varies inversely with b; and a higher value of b implies a greater efficiency to prevent imitation.

In the basic model developed in section 5.2, the innovated good producing sector uses both skilled labour and unskilled labour as inputs; and labour-output coefficient in this sector is assumed to be unity for each type of labour. So marginal cost of production in that sector is given by $(W_S + W_U)$. However, in this extended model, sector 1 produces each of these innovated products with unskilled labour as only input; and so the wage rate of unskilled labour, W_U, is the marginal cost of production of each of these innovated varieties. The producer of each of these innovated varieties is a monopolist. So it charges a monopoly price of its product which is given by

[80] It does not mean that the assumption of constant returns is empirically rejected. We assume constant returns in the R&D sector and so an interior allocation of skilled labour can not be obtained with constant returns in both the R&D sector and in the public good (social institution) sector.

$$P^S = \frac{1}{\alpha} W_U \qquad (5.3.4).$$

In this section, skilled labour is used in the R&D sector and in the imitation preventing public good sector. So market clearing condition of skilled labour is different from that in section 5.2 and is given by the following equation.

$$ag + l_m = L^S \qquad (5.3.5).$$

Also, we modify the stock market clearing condition of section 5.2; which is given by equation (5.2.20) as follows

$$\frac{\Pi^S}{v^S} + \frac{\dot{v}^S}{v^S} = r + m \qquad (5.3.6).$$

This equation (5.3.6) implies that the net rate of return from investment in the stock market is equal to the interest rate obtained from the loan market.

Π^S is the level of net profit of the representative firm in sector 1. All firms in sector 1, who produce innovated varieties, have to bear the cost of producing the public good as it protects imitation. This cost takes the form of a lump sum tax imposed by the government. So using equation (5.3.4), Π^S is defined as follows.

$$\Pi^S = (1-\alpha)P^S x_S - \frac{W_S l_m}{n^S} \qquad (5.3.7).$$

Here, $W_S l_m$ is the cost of producing the imitation preventing public good because skilled labour is the only input in that sector; and this amount is taken by the government in the form of lump sum taxes.

However equations (5.2.1), (5.2.2), (5.2.4)-(5.2.13) of the basic model remain unchanged here.

5.3.2. Working of the model

5.3.2.1. Rate of growth

Using equations (5.3.2) and (5.3.5), we obtain[81]

$$g = \frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} \qquad (5.3.8).$$

[81] Detailed derivation of equation (5.3.8) is given in the Appendix (5.I).

Equation (5.3.8) shows the constant rate of product development (growth) in this modified model; and like that in section 5.2, this rate is also independent of whether the economy is in the steady-state growth equilibrium or not. While deriving equation (5.3.8), we never use equation (5.2.7) i.e., the steady-state equilibrium condition of this model. Here the rate of growth is determined by values of different parameters like skilled labour endowment, the productivity parameter in the R&D sector and the labour elasticity of output parameter in the imitation preventing public good sector. We need appropriate restrictions on the values of those parameters to ensure that $g > 0$[82].However, equation (5.3.8) shows that g varies positively with L^S and inversely with a and β. Also, g is independent of change in L^U and b. Here b is the efficiency parameter of the imitation prevention of the public good. So we can establish the following proposition.

<u>PROPOSITION-5.3.1</u>: *An increase in skilled labour endowment raises the rate of growth (expansion of varieties) but a change in unskilled labour endowment or an improvement in the imitation prevention efficiency of the public good has no effect on it.*

5.3.2.2. Rate of imitation

Using equations (5.3.1), (5.3.3), (5.3.5) and (5.3.8), we obtain[83]

$$m = \frac{1}{(a)^{\frac{\beta}{(1-\beta)}} b} \qquad (5.3.9).$$

Equation (5.3.9) shows that imitation rate is independent of L^S and L^U. However, it changes with respect to change in other parameters, a, b and β. Here m varies inversely with a, b and β. So we can establish the following proposition.

<u>PROPOSITION-5.3.2</u>: *The long run rate of imitation is independent of changes in skilled labour endowment and unskilled labour endowment. However, an improvement in the imitation prevention efficiency of the public good (social institution) and/or an improvement in the productivity in the R & D sector lowers the imitation rate.*

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[82] The rate of growth is positive if $L^S > (a)^{\frac{1}{(1-\beta)}}$.

[83] Detailed derivation of equation (5.3.9) is given in the Appendix (5.J).

5.3.2.3. <u>The stability of steady-state equilibrium</u>

Using equations (5.2.6), (5.3.8) and (5.3.9), we obtain

$$\dot{\xi} = \frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a} - \left(\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{\left(L^S-ag\right)^{\beta} b}\right)\xi \qquad (5.3.10).$$

In the steady-state growth equilibrium, $\dot{\xi} = 0$. Hence, the steady-state growth equilibrium value of ξ is given by

$$\xi^* = \frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{\left(L^S-ag\right)^{\beta} b}}.$$

Since ξ^* is a constant, then $\frac{n^S}{n^U} = \frac{\xi}{1-\xi}$ is also so. This equation (5.3.10) shows that $\dot{\xi}$ is a negative function of ξ; and so the steady state growth equilibrium is stable.

5.3.2.4. <u>Interest rate:</u>

Using equations (5.2.5), (5.2.12), (5.2.15), (5.2.16), (5.2.18) and (5.3.4), we obtain[84]

$$E = W_U L^U \left[\frac{\xi\alpha^{(\varepsilon-1)}+(1-\xi)}{\alpha^{\varepsilon}\xi+(1-\xi)}\right] \qquad (5.3.11).$$

In the steady-state growth equilibrium, ξ takes a constant value and g is always a constant. Hence, from equations (5.2.19) and (5.3.11), we have

$$\frac{\dot{v}^S}{v^S} + g = \frac{\dot{W}^S}{W^S} = \frac{\dot{W}^U}{W^U} = \frac{\dot{E}}{E}.$$

Here $\dot{v}^S = 0$ because v^S is normalized to unity; and so we have

$$\frac{\dot{E}}{E} = g \qquad (5.3.12).$$

Using equations (5.2.11) and (5.3.12) we have $r = \rho + g$; and then using equation (5.3.10), we can solve for r in the steady-state growth equilibrium.

[84] Derivation of equation (5.3.11) is given in the Appendix (5.K).

5.3.2.5. Wage inequality

Here also $\Delta = \frac{W_S}{W_U}$. Using equations (5.2.7), (5.2.11), (5.2.12), (5.2.15), (5.2.16), (5.2.19), (5.3.4), (5.3.6)-(5.3.10) and (5.3.12), we derive[85]

$$\Delta = \frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\rho + \frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)\left(1 - \frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)} \quad (5.3.13).$$

Here, $\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}} < 1$ because $\alpha^{\varepsilon} < 1$ and $L^S > (a)^{\frac{1}{(1-\beta)}}$. So equation (5.3.13) ensures that $\Delta > 0$. This equation (5.3.13) shows how the skilled-unskilled wage ratio $\frac{W_S}{W_U}$ varies with changes in different parameters in the long run equilibrium. Here, Δ varies positively with L^U and b. The effect of change in L^S on Δ is ambiguous. If the value of α is very large, then Δ varies inversely with respect to change in L^S[86]. This leads to the following proposition.

PROPOSITION-5.3.3 *(i) An increase in the level of unskilled labour endowment and/or an improvement in the imitation prevention efficiency of the public good (social institution) raises the skilled-unskilled wage ratio. (ii) If the monopoly power of the representative firm in the innovated good producing sector is very low, then an increase in skilled labour endowment lowers the skilled-unskilled wage ratio.*

We now provide intuitive explanations for this result. As unskilled labour endowment is increased, there is no effect on growth rate, imitation rate and on the demand for unskilled labour in sector 1 and sector 2. So unskilled wage rate falls and the skilled-unskilled wage ratio rises. Similarly, as the imitation prevention efficiency of the public good is improved, rate of imitation falls. This lowers the demand for unskilled labour in the imitated goods producing sector. However, demand for unskilled labour in the innovated goods producing sector remains

[85] Derivation of equation (5.3.13) is given in the Appendix (5.L).

[86] Detailed analysis is given in the Appendix (5.M).

unchanged because innovation rate is independent of the imitation prevention efficiency of the public good. So the aggregate demand for unskilled labour falls and hence the unskilled wage rate is also reduced; and thus the skilled-unskilled wage ratio is increased.

An increase in the skilled labour endowment has two effects. The direct effect implies a fall in the skilled wage rate. However, the innovation rate is also increased implying that more blue prints are produced in the R&D sector. So the proportion of innovated goods is increased and the proportion of imitated goods is reduced in the new steady state equilibrium. Unskilled labour moves from the imitated good producing sector to the innovated sector. However, excess demand for unskilled labour in the innovated good producing sector is less than its excess supply in the imitated good producing sector. So the unskilled wage rate is also reduced. This is the indirect effect. So we have a net ambiguous effect on the skilled-unskilled wage ratio. If the monopoly power of each producer in the innovated good producing sector is very low, then excess demand for unskilled labour in the innovated good producing sector is almost same as its excess supply in the imitated good producing sector. So the decrease in the skilled wage rate is more than the decrease in the unskilled wage rate in this special case.

Our results related to effects of imitation on skilled-unskilled wage ratio is interesting compared to the corresponding result obtained in Thoeing and Verdier (2003). In our model, an improvement in the efficiency of imitation preventing public good implies a reduction in the threat of imitation. This efficiency improvement lowers the relative demand for unskilled labour and raises the skilled-unskilled wage ratio because production of imitated goods requires only unskilled labour and the change in the threat of imitation has no effect on the technology of producing innovated goods. In Thoeing and Verdier (2003), firms producing innovated products use skill intensive technology to meet the increased threat of imitation; and thus the relative demand for skilled labour is increased leading to an increase in the skilled-unskilled wage ratio when there is an increased threat of imitation.

5.3.2.6. <u>Effect on Welfare</u>

Using equations (5.2.7), (5.2.10), (5.2.11)-(5.2.13), (5.2.15), (5.2.16), (5.3.4), (5.3.8) and (5.3.9) we obtain following modified form of this utility function[87].

$$u=\frac{L^U}{2}n^{\left(\frac{1}{\varepsilon-1}\right)}\left\{\frac{a}{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}\right\}^{\left(\frac{1}{\varepsilon-1}\right)}\frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]} \quad (5.3.14).$$

Here, we normalize the utility function with respect to love for variety effect; and the normalized utility function is given by the following.

$$u^*=\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}=\frac{L^U}{2}\left\{\frac{1}{\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1}\right\}^{\left(\frac{1}{\varepsilon-1}\right)}\frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]} \quad (5.3.15).$$

In Appendix (5.O), it is shown that if $\alpha=\frac{1}{2}$, then equation (5.3.15) implies that the nature of relationship between u^* and L^S or b depends on the value of $\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b$. If $\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b>(<)2$ then u^* varies directly (inversely) with both L^S and/or b. Also, equation (5.3.32) implies a direct relationship between the unskilled labour endowment and the level of utility of the household. So we can establish the following proposition.

<u>PROPOSITION-5.3.4</u>: *In the steady state growth equilibrium, an increase in the level of unskilled labour endowment raises the level of social welfare; and with $\alpha=\frac{1}{2}$, an increase in skilled labour endowment and/or an improvement in the efficiency of imitation prevention of the public good (social institution) raises (lowers) the welfare level if $\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b>(<)2$.*

[87] Detailed derivation of equation (5.3.14) is given in the Appendix (5.N).

Here the quantity of skilled labour that social institution employs is given by $l_m = (a)^{\frac{1}{(1-\beta)}}$. So l_m varies directly with β. Thus the welfare effect of an improvement in the efficiency of imitation prevention of the social institution is qualitatively similar to an increase in the level of skilled labour employment in that sector.

5.4 LIMITATIONS

The model developed in this chapter has following limitations. We assume a closed economy and hence can not analyse the role of international trade on the skilled-unskilled wage inequality. The possibility of unemployment in any of these two labour markets is also ruled out; and both the labour markets are assumed to be competitive. Symmetry assumption in the utility function and the linearity assumption in all production functions are also simplifying ones. Imitation is also assumed to be cost-less. It may be a weak excuse to say that all models built on Helpman (1993) product variety structure suffer from these common limitations.

Appendix (5.A):

Derivation of equation (5.2.6):

Differentiating both sides of equation (5.2.5), with respect to t, we obtain

$$\frac{\dot{\xi}}{\xi} = \frac{\dot{n}^S}{n^S} - \frac{\dot{n}}{n}$$

$$\Rightarrow \dot{\xi} = \left(\frac{\dot{n}}{n^S} - \frac{\dot{n}^U}{n^S} - \frac{\dot{n}}{n}\right) \xi$$

$$\Rightarrow \dot{\xi} = \frac{\dot{n}}{n^S}\frac{n^S}{n} - (g + m)\, \xi$$

$$\Rightarrow \dot{\xi} = g - (g + m)\, \xi \qquad (5.2.A.1).$$

Equation (5.2.A.1) is same as equation (5.2.6) in the body of the chapter.

Appendix (5.B):

Derivation of equation (5.2.12):

The consumer maximizes instantaneous utility function given by equation (5.2.10) subject to the instantaneous budget constraint which is given by

$$E = \int_0^n p(j)\, x(j) dj \qquad (5.2.A.2).$$

So, the Lagrange function is given by

$$\mathcal{L} = \left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}} + \lambda\left[E - \int_0^n p(j)\, x(j) dj\right] \qquad (5.2.A.3).$$

where, λ is the Lagrangian multiplier.

The f.o.c.'s of utility maximization is given by

$$\left[\int_0^n x(i)^\alpha dj\right]^{\frac{1}{\alpha}-1} x(i)^{\alpha-1} = \lambda p(i) \qquad (5.2.A.4),$$

and,

$$\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}-1} x(j)^{\alpha-1} = \lambda p(j) \qquad (5.2.A.5).$$

Using equations (5.2.A.4) and (5.2.A.5), we obtain

$$\left[\frac{x(i)}{x(j)}\right]^{1-\alpha} = \frac{p(j)}{p(i)} \qquad (5.2.A.6);$$

and from equation (5.2.A.6), we obtain

$$\frac{\left[\widehat{\frac{x(i)}{x(j)}}\right]}{\left[\widehat{\frac{p(j)}{p(i)}}\right]} = \frac{1}{1-\alpha}$$

$$\Rightarrow \varepsilon = \frac{1}{1-\alpha}. \qquad (5.2.A.7).$$

This ε is the price elasticity of demand for the representative variety.

Multiplying both sides of equation (5.2.A.5) by $x(j)$ and summing over all j, we obtain

$$\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}} = \lambda \int_0^n p(j)\, x(j) dj$$

$$\Rightarrow \lambda = \frac{\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}}}{\int_0^n p(j) x(j) dj} = \frac{1}{P} \qquad (5.2.A.8).$$

Finally, using equations (5.2.A.2), (5.2.A.5) and (5.2.A.8), we obtain

$$x(j) = p(j)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} \qquad (5.2.A.9).$$

Equation (5.2.A.9) is same as equation (5.2.12) in the body of the chapter.

Appendix (5.C):

Derivation of equation (5.2.11):

Substituting the demand functions given by (5.2.12) into equation (5.2.10) and then using equation (5.2.13), we obtain the indirect utility function

$$\log(u) = \log(E) - \log(P) \qquad (5.2.A.10).$$

Differentiating both sides of equation (5.2.9), we obtain

$$\dot{A} = I - E + rA \qquad (5.2.A.11).$$

The current value Hamiltonian corresponding to this dynamic optimization problem is given by

$$H = \log(u) + h(I - E + rA)$$

$$\Rightarrow H = [\log(E) - \log(P)] + h(I - E + rA)$$

Here h is the co-state variable. The first order optimality condition with respect to E is given by

$$\frac{\partial H}{\partial E} = \frac{\partial u}{\partial E} - h = 0$$

$$\Rightarrow \frac{1}{E} = h$$

$$\Rightarrow \frac{\dot{h}}{h} = -\frac{\dot{E}}{E} \qquad (5.2.A.12).$$

The equation motion of the co-state variable, h, should satisfy the following differential equation along the optimal path.

$$\frac{\dot{h}}{h} = r - \rho \qquad (5.2.A.13).$$

Using equations (5.2.A.12) and (5.2.A.13), we obtain

$$\frac{\dot{E}}{E} = r - \rho \qquad (5.2.A.14).$$

Equation (5.2.A.14) is same as equation (5.2.11) in the body of the chapter.

Appendix (5.D):

<u>Derivation of equations (5.2.21) and (5.2.22):</u>

From equation (5.2.17), we obtain

$$x^S = \frac{(L^S - ag)}{n^S} \qquad (5.2.A.15).$$

Using equations (5.2.12) and (5.2.A.15), we obtain

$$(P^S)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} = \frac{(L^S - ag)}{n^S} \qquad (5.2.A.16).$$

Similarly using equations (5.2.12) and (5.2.18), we have

$$(P^U)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} = \frac{(L^U - n^S x^S)}{n^U} \qquad (5.2.A.17).$$

Using equations (5.2.17) and (5.2.A.17), we obtain

$$(P^U)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} = \frac{(L^U - L^S + ag)}{n^U} \qquad (5.2.A.18).$$

Using equations (5.2.A.16) and (5.2.A.18), we have

$$\frac{(P^S)^{-\varepsilon}}{(P^U)^{-\varepsilon}} = \frac{(L^S - ag)n^U}{(L^U - L^S + ag)n^S} \qquad (5.2.A.19).$$

Finally, using equations (5.2.5) and (5.2.A.19), we obtain

$$\frac{P^S}{P^U} = \left\{\left(\frac{L^U}{L^S - ag} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}} \qquad (5.2.A.20).$$

Equation (5.2.A.20) is same as equation (5.2.21) in the body of the chapter.

Using equations (5.2.14), (5.2.15) and (5.2.A.20), we obtain

$$\frac{1}{\alpha}\left(\frac{W^S}{W^U} + 1\right) = \left\{\left(\frac{L^U}{L^S - ag} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}}$$

$$\Rightarrow \Delta = \frac{W^S}{W^U} = \alpha\left\{\left(\frac{L^U}{L^S - ag} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}} - 1 \qquad (5.2.A.21).$$

Equation (5.2.A.21) is same as equation (5.2.22) in the body of the chapter.

Appendix (5.E):

<u>Derivation of equations (5.2.23) and (5.2.24):</u>

From equations (5.2.12) and (5.2.18), we obtain

$$\frac{n^S\left(P^S\right)^{-\varepsilon}E}{P^{1-\varepsilon}} + \frac{n^U\left(P^U\right)^{-\varepsilon}E}{P^{1-\varepsilon}} = L^U \qquad (5.2.A.22).$$

Using equations (5.2.5) and (5.2.A.22), we obtain

$$\frac{En}{P^{1-\varepsilon}}\left[\xi(P^S)^{-\varepsilon} + (1-\xi)(P^U)^{-\varepsilon}\right] = L^U \qquad (5.2.A.23).$$

Using equations (5.2.16) and (5.2.A.23), we obtain

$$\frac{E\left[\xi\left(P^S\right)^{-\varepsilon} + (1-\xi)\left(P^U\right)^{-\varepsilon}\right]}{\left[\xi\left(P^S\right)^{(1-\varepsilon)} + (1-\xi)(P^U)^{(1-\varepsilon)}\right]} = L^U$$

$$\Rightarrow \frac{E}{P^U}\left[\frac{\left[\xi\left(\frac{P^S}{P^U}\right)^{-\varepsilon} + (1-\xi)\right]}{\left[\xi\left(\frac{P^S}{P^U}\right)^{(1-\varepsilon)} + (1-\xi)\right]}\right] = L^U \qquad (5.2.A.24).$$

Using equations (5.2.14), (5.2.15) and (5.2.A.24), we obtain

$$\Rightarrow \frac{E}{P^U}\left[\frac{\left[\xi\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{-\varepsilon} + (1-\xi)\right]}{\left[\xi\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{(1-\varepsilon)} + (1-\xi)\right]}\right] = L^U \qquad (5.2.A.25)$$

Using equations (5.2.22) and (5.2.A.25), we obtain

$$\frac{E\left[\xi\left\{\left(\frac{L^U}{L^S-ag} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{-1} + (1-\xi)\right]}{\left[\xi\left\{\left(\frac{L^U}{L^S-ag} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\left(\frac{1-\varepsilon}{\varepsilon}\right)} + (1-\xi)\right]} = W^U L^U$$

$$\Rightarrow E = W^U(L^S - ag)\left[\left\{\left(\frac{L^U}{L^S-ag} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\} + 1\right] \qquad (5.2.A.26).$$

Equation (5.2.A.26) is same as equation (5.2.23) in the body of the chapter.

Using equations (5.2.12) and (5.2.17), we obtain

$$(\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{n^S}(\mathrm{P^S})^{-\varepsilon}\mathrm{E}}{\mathrm{P}^{1-\varepsilon}} \qquad (5.2.A.27).$$

Using equations (5.2.16) and (5.2.A.27), we obtain

$$(\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{n^S}(\mathrm{P^S})^{-\varepsilon}\mathrm{E}}{\mathrm{n}\left[\xi(\mathrm{P^S})^{(1-\varepsilon)} + (1-\xi)(\mathrm{P^U})^{(1-\varepsilon)}\right]}$$

$$\Rightarrow (\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{n^S}(\mathrm{P^S})^{-\varepsilon}\mathrm{E}}{\mathrm{n}(\mathrm{P^S})^{(1-\varepsilon)}\left[\xi + (1-\xi)\left(\frac{\mathrm{P^S}}{\mathrm{P^U}}\right)^{(\varepsilon-1)}\right]}$$

$$\Rightarrow (\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{n^S}\mathrm{E}}{\mathrm{n}\mathrm{P^S}\xi\left[1 + \left(\frac{\xi}{1-\xi}\right)\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{(\varepsilon-1)}\right]} \qquad (5.2.A.28).$$

Using equations (5.2.5), (5.2.22) and (5.2.A.28), we obtain

$$(\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{E}}{\mathrm{P^S}\left[\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{-1}{\varepsilon}}\right\} + 1\right]} \qquad (5.2.A.29).$$

Using equations (5.2.A.26) and (5.2.A.29), we obtain

$$(\mathrm{L^S} - \mathrm{ag}) = \frac{\mathrm{W^U}(\mathrm{L^S}-\mathrm{ag})\left[\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\} + 1\right]}{\mathrm{P^S}\left[\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{-1}{\varepsilon}}\right\} + 1\right]}$$

$$\Rightarrow \frac{\mathrm{P^S}}{\mathrm{W^U}} = \frac{\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\}\left[\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{-1}{\varepsilon}}\right\} + 1\right]}{\left[\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{-1}{\varepsilon}}\right\} + 1\right]}$$

$$\Rightarrow \frac{\mathrm{P^S}}{\mathrm{W^U}} = \left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\} \qquad (5.2.A.30).$$

Using equations (5.2.14) and (5.2.A.30), we obtain

$$\frac{1}{\alpha}(\Delta + 1) = \left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\} \qquad (5.2.A.31).$$

Finally, using equations (5.2.22) and (5.2.A.31), we obtain

$$\left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)\left(\frac{\xi}{1-\xi}\right)\right\}^{\frac{1}{\varepsilon}} = \left\{\left(\frac{\mathrm{L^U}}{\mathrm{L^S}-\mathrm{ag}} - 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\frac{\xi}{1-\xi}\right)^{\frac{1}{\varepsilon}}\right\}$$

$$\Rightarrow g = \frac{2L^S - L^U}{2a} \qquad (5.2.A.32).$$

Equation (5.2.A.32) is same as equation (5.2.24) in the body of the chapter.

Appendix (5.F):

Derivation of condition (5.2.29):

Differentiating both sides of equation (5.2.19) with respect to time, t, and using equation (5.2.2), we obtain

$$\frac{\dot{v}^S}{v^S} = \frac{\dot{W}^S}{W^S} - g \qquad (5.2.A.33).$$

Now, using condition (5.2.20) and equation (5.2.A.33), we have

$$\frac{\Pi^S}{v^S} + \frac{\dot{W}^S}{W^S} - g \geq r + m$$

$$\Rightarrow \frac{\dot{W}^S}{W^S} \geq r + m + g - \frac{\Pi^S}{v^S} \qquad (5.2.A.34).$$

We know that

$$\Pi^S = (1-\alpha)P^S x^S \qquad (5.2.A.35).$$

Using equations (5.2.19) and (5.2.A.35), we obtain

$$\frac{\Pi^S}{v^S} = \frac{(1-\alpha)P^S x^S n}{W^S a} \qquad (5.2.A.36).$$

Using equations (5.2.17) and (5.2.A.36), we have

$$\frac{\Pi^S}{v^S} = \frac{(1-\alpha)P^S (L^S - ag) n}{W^S a n^S} \qquad (5.2.A.37).$$

Using equations (5.2.5), (5.2.14) and (5.2.A.37), we obtain

$$\frac{\Pi^S}{v^S} = \frac{1-\alpha}{\alpha}\left(1 + \frac{1}{\Delta}\right)\left(\frac{L^S - ag}{a\xi}\right) \qquad (5.2.A.38).$$

Using equations (5.2.A.34) and (5.2.A.38), we have

$$\frac{\dot{W}^S}{W^S} \geq r + m + g - \frac{1-\alpha}{\alpha}\left(1 + \frac{1}{\Delta}\right)\left(\frac{L^S - ag}{a\xi}\right) \qquad (5.2.A.39).$$

Using equations (5.2.7) and (5.2.28), we obtain

$$W^U\left[\left(\frac{g}{m}\right)^{\frac{1}{\varepsilon}} + 1\right] = \frac{2E}{L^U} \qquad (5.2.A.40).$$

Using equations (5.2.24) and (5.2.A.40), we obtain

$$W^U\left[\left(\frac{2L^S-L^U}{2am}\right)^{\frac{1}{\varepsilon}}+1\right]=\frac{2E}{L^U} \qquad (5.2.A.41).$$

Differentiating both sides of equation (5.2.A.41) with respect to time, t, we obtain

$$\frac{\dot{W}^U}{W^U}=\frac{\dot{E}}{E} \qquad (5.2.A.42).$$

Using equations (5.2.11), (5.2.A.39) and (5.2.A.42), we have

$$\frac{\dot{W}^S}{W^S}-\frac{\dot{W}^U}{W^U}\geq r+m+g-\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^S-ag}{a\xi}\right)-r+\rho$$

$$\Rightarrow\frac{\dot{W}^S}{W^S}-\frac{\dot{W}^U}{W^U}\geq m+g+\rho-\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^S-ag}{a\xi}\right) \qquad (5.2.A.43).$$

In the steady-state equilibrium, $\frac{\dot{W}^S}{W^S}-\frac{\dot{W}^U}{W^U}=0.$

So, from equation (5.2.A.43), we have

$$\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^S-ag}{a\xi}\right)\geq m+g+\rho \qquad (5.2.A.44).$$

Using equations (5.2.7) and (5.2.24), we obtain

$$\xi=\frac{(2L^S-L^U)}{(2L^S-L^U+2am)} \qquad (5.2.A.45).$$

From equation (5.2.24), we obtain

$$L^S-ag=\frac{L^U}{2} \qquad (5.2.A.46).$$

From equations (5.2.A.44), (5.2.A.45) and (5.2.A.46), we obtain

$$\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^U}{2a}\right)\frac{(2L^S-L^U+2am)}{(2L^S-L^U)}\geq m+g+\rho \qquad (5.2.A.47).$$

Using equations (5.2.24) and (5.2.A.47), we obtain

$$\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^U}{2a}\right)\frac{(2L^S-L^U+2am)}{(2L^S-L^U)}\geq m+\frac{2L^S-L^U}{2a}+\rho$$

$$\Rightarrow\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)\left(\frac{L^U}{2a}\right)\frac{(2L^S-L^U+2am)}{(2L^S-L^U)}\geq\frac{2am+2a\rho+2L^S-L^U}{2a}$$

$$\Rightarrow\frac{1-\alpha}{\alpha}\left(1+\frac{1}{\Delta}\right)L^U\frac{(2L^S-L^U+2am)}{(2L^S-L^U)}\geq 2am+2a\rho+2L^S-L^U$$

$$\Rightarrow\left(1+\frac{1}{\Delta}\right)\geq\frac{\alpha(2am+2a\rho+2L^S-L^U)(2L^S-L^U)}{(1-\alpha)L^U(2L^S-L^U+2am)}$$

$$\Rightarrow\Delta\leq\frac{(1-\alpha)L^U(2L^S-L^U+2am)}{\alpha 2a\rho(2L^S-L^U)+(2L^S-L^U+2am)(\alpha 2L^S-L^U)} \qquad (5.2.A.48).$$

Condition (5.2.A.48) is same as condition (5.2.29) in the body of the chapter.

Appendix (5.G):

Derivation of equation (5.2.30):

From equation (5.2.A.10), we obtain

$$u = \frac{E}{P} \tag{5.2.A.49}$$

From equation (5.2.A.25), we obtain

$$E = L^U P^U \left[\frac{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + (1-\xi) \right]}{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{-\varepsilon} + (1-\xi) \right]} \right] \tag{5.2.A.50}$$

Using equations (5.2.14), (5.2.15) and (5.2.16), we obtain

$$P = n^{\frac{1}{1-\varepsilon}} P^U \left[\xi \left\{ \frac{1}{\alpha}(\Delta + 1) \right\}^{(1-\varepsilon)} + (1-\xi) \right]^{\frac{1}{1-\varepsilon}} \tag{5.2.A.51}$$

Using equations (5.2.A.49), (5.2.A.50) and (5.2.A.51), we obtain

$$u = \frac{L^U}{n^{\frac{1}{1-\varepsilon}}} \frac{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + (1-\xi) \right]}{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{-\varepsilon} + (1-\xi) \right] \left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + (1-\xi) \right]^{\frac{1}{1-\varepsilon}}}$$

$$\Rightarrow u = \frac{L^U}{n^{\frac{1}{1-\varepsilon}}} \frac{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + (1-\xi) \right]^{\frac{\varepsilon}{1-\varepsilon}}}{\left[\xi \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{-\varepsilon} + (1-\xi) \right]}$$

$$\Rightarrow u = L^U n^{\left(\frac{1}{\varepsilon-1}\right)} (1-\xi)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\left\{ \frac{\xi}{(1-\xi)} \right\} \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + 1 \right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\left\{ \frac{\xi}{(1-\xi)} \right\} \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{-\varepsilon} + 1 \right]} \tag{5.2.A.52}$$

Using equations (5.2.7) and (5.2.A.52), we have

$$u = L^U n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{m}{g+m} \right)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\left\{ \frac{g}{m} \right\} \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{(1-\varepsilon)} + 1 \right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\left\{ \frac{g}{m} \right\} \left\{ \frac{1}{\alpha}(\Delta+1) \right\}^{-\varepsilon} + 1 \right]} \tag{5.2.A.53}$$

Using equations (5.2.24) and (5.2.26), we obtain

$$\Delta = \alpha \left\{ \frac{g}{m} \right\}^{\frac{1}{\varepsilon}} - 1$$

$$\Rightarrow \frac{g}{m} = \left\{ \frac{1}{\alpha}(\Delta + 1) \right\}^{\varepsilon} \tag{5.2.A.54}$$

Using equations (5.2.A.53) and (5.2.A.54), we obtain

$$u = L^U n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{1}{\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{\varepsilon}\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{(1-\varepsilon)}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{\varepsilon}\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{-\varepsilon}+1\right]}$$

$$\Rightarrow u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{1}{\left\{\frac{1}{\alpha}(\Delta+1)\right\}^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \left[\left\{\frac{1}{\alpha}(\Delta+1)\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \quad (5.2.A.55).$$

Equation (5.2.A.55) is same as equation (5.2.30) in the body of the chapter.

Appendix (5.H):

Derivation of the relationship between consumers' utility and wage inequality:

Let,

$$\left\{\frac{1}{\alpha}(\Delta+1)\right\} = C \quad (5.2.A.56).$$

Using equations (5.2.A.55) and (5.2.A.56), we obtain

$$u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{1}{C^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} (C+1)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \quad (5.2.A.57).$$

From equation (5.2.A.57), we have

$$\frac{du}{dC} = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left[\left(\frac{1}{C^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{\varepsilon}{\varepsilon-1}\right)(C+1)^{\left(\frac{1}{\varepsilon-1}\right)} - (C+1)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)} \left(\frac{\varepsilon}{\varepsilon-1}\right) C^{(\varepsilon-1)} \left(\frac{1}{C^{\varepsilon}+1}\right)^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}\right]$$

$$\Rightarrow \frac{du}{dC} = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{\varepsilon}{\varepsilon-1}\right) \left(\frac{1}{C^{\varepsilon}+1}\right)^{\left(\frac{1}{\varepsilon-1}\right)} (C+1)^{\left(\frac{1}{\varepsilon-1}\right)} \left[1-(C+1)C^{(\varepsilon-1)}\left(\frac{1}{C^{\varepsilon}+1}\right)\right] \quad (5.2.A.58).$$

Here, $\varepsilon > 1$. Hence,

$\frac{du}{dC} < 0$ if and only if

$1 < (C+1)C^{(\varepsilon-1)}\left(\frac{1}{C^{\varepsilon}+1}\right)$

$\Rightarrow C^{(\varepsilon-1)} > 1$

$\Rightarrow C > 1$

However, equation (5.2.A.56) implies that this is always true because $\Delta > 1$ and $0 < \alpha < 1$.

So, we have

$\frac{du}{dC} < 0$.

Again from equation (5.2.A.56), we have

$\frac{dC}{d\Delta} > 0.$

$\therefore \frac{du}{d\Delta} = \frac{du}{dC}\frac{dC}{d\Delta} < 0.$

So, we have an inverse relationship between the level of utility of the representative consumer and the degree of wage inequality.

Appendix (5.I):

Derivation of equation (5.3.8):

Using equations (5.3.2) and normalizing v^S to unity, we obtain

$$l_m = (a)^{\frac{1}{(1-\beta)}} \qquad (5.3.A.1).$$

Using equations (5.3.5) and (5.3.A.1), we have

$$L_S - ag = (a)^{\frac{1}{(1-\beta)}}$$

$$\Rightarrow g = \frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} \qquad (5.3.A.2).$$

Equation (5.3.A.2) is same as equation (5.3.8) in the body of the chapter.

Appendix (5.J):

Derivation of equation (5.3.9):

Using equations (5.3.1) and (5.3.3), we obtain

$$m = \frac{1}{{l_m}^{\beta} b} \qquad (5.3.A.3).$$

Using equations (5.3.A.1) and (5.3.A.3), we have

$$m = \frac{1}{(a)^{\frac{\beta}{(1-\beta)}} b} \qquad (5.3.A.4).$$

Equation (5.3.A.4) is same as equation (5.3.9) in the body of the chapter.

Appendix (5.K):

Derivation of equation (5.3.11):

Using equations (5.2.15), (5.3.4) and (5.2.A.24), we obtain

$$\frac{E}{W_U}\left[\frac{\left[\xi\left(\frac{1}{\alpha}\right)^{-\varepsilon}+(1-\xi)\right]}{\left[\left[\xi\left(\frac{1}{\alpha}\right)^{(1-\varepsilon)}+(1-\xi)\right]\right]}\right] = L^U$$

$$\Rightarrow E = W_U L^U\left[\frac{\xi\alpha^{(\varepsilon-1)}+(1-\xi)}{\alpha^{\varepsilon}\xi+(1-\xi)}\right] \qquad (5.3.A.5)$$

Equation (5.3.A.5) is same as equation (5.3.11) in the body of the chapter.

Appendix (5.L):

Derivation of equation (5.3.13):

Using equations (5.3.6) and (5.3.7), we obtain

$$(1-\alpha)P^S x_S = r + m + \frac{l_m}{a\xi} \qquad (5.3.A.6).$$

Using equations (5.2.11), (5.2.12), (5.3.12) and (5.3.A.6), we get

$$\frac{(1-\alpha)P^{S^{(1-\varepsilon)}}E}{P} = g + \rho + m + \frac{(L_S - ag)}{a\xi}$$

$$\Rightarrow \frac{(1-\alpha)P^{S^{(1-\varepsilon)}}E}{P} = \frac{L^S}{a\xi} + \rho \qquad (5.3.A.7).$$

Using equations (5.2.15), (5.2.16), (5.3.4) and (5.3.A.7), we get

$$\frac{(1-\alpha)P^{S^{(1-\varepsilon)}}E}{n\left[\xi\left(P^S\right)^{(1-\varepsilon)}+(1-\xi)(P^U)^{(1-\varepsilon)}\right]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)E}{n\left[\xi+(1-\xi)\alpha^{(1-\varepsilon)}\right]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)E}{n\alpha^{(1-\varepsilon)}\left[\xi\alpha^{(\varepsilon-1)}+(1-\xi)\right]} = \frac{L^S}{a\xi} + \rho \qquad (5.3.A.8).$$

Using equations (5.3.11) and (5.3.A.8), we obtain

$$\frac{(1-\alpha)W_U L^U}{n\alpha^{(1-\varepsilon)}[\alpha^{\varepsilon}\xi+(1-\xi)]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)W_S L^U}{n\alpha^{(1-\varepsilon)}\Delta[\alpha^{\varepsilon}\xi+(1-\xi)]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)L^U}{a\Delta\alpha^{(1-\varepsilon)}[\alpha^{\varepsilon}\xi+(1-\xi)]} = \frac{L^S}{a\xi} + \rho \qquad [\because v^S = 1 \text{ and } \Delta = \frac{W_S}{W_U}]$$

$$\Rightarrow \Delta = \frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\frac{L_S}{a}+\rho+m\right)[1-\xi(1-\alpha^{\varepsilon})]} \qquad (5.3.A.9).$$

Using equations (5.2.7), (5.3.9) and (5.3.A.9), we obtain

$$\Delta = \frac{(1-\alpha)L^U}{a\alpha^{(\varepsilon-1)}\left(\frac{L^S\left(g+\frac{1}{(L^S-ag)^{\beta}b}\right)}{ag}+\rho\right)\left(1-\frac{g(1-\alpha^{\varepsilon})}{g+\frac{1}{(L^S-ag)^{\beta}b}}\right)} \qquad (5.3.A.10).$$

Using equations (5.3.8) and (5.3.A.10), we obtain

$$\Delta = \frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\rho+\frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)} \qquad (5.3.A.11).$$

Equation (5.3.A.11) is same as equation (5.3.13) in the body of the chapter.

Appendix (5.M):

<u>Relationship between skilled-unskilled wage inequality and the skilled labour endowment:</u>

In the denominator of the expression of Δ; $\left(\rho+\frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)$ varies positively with L^S and $\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)$ varies negatively with L^S. If, If the value of α is very large, then $\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)$ is very small and Δ varies inversely with respect to change in L^S.

Appendix (5.N):

Derivation of equation (5.3.14):

Using equations (5.2.15), (5.2.16) and (5.3.4), we obtain

$$P = n^{\frac{1}{1-\varepsilon}} P^{U} \left[\xi\alpha^{(\varepsilon-1)} + (1-\xi)\right]^{\frac{1}{1-\varepsilon}} \quad (5.3.A.12).$$

Using equations (5.2.A.49), (5.2.A.50) and (5.3.A.12), we obtain

$$u = \frac{L^{U}}{n^{\frac{1}{1-\varepsilon}}} \frac{\left[\xi\alpha^{(\varepsilon-1)}+(1-\xi)\right]}{\left[\alpha^{\varepsilon}\xi+(1-\xi)\right]\left[\xi\alpha^{(\varepsilon-1)}+(1-\xi)\right]^{\frac{1}{1-\varepsilon}}}$$

$$\Rightarrow u = L^{U} n^{\left(\frac{1}{\varepsilon-1}\right)} (1-\xi)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\left\{\frac{\xi}{(1-\xi)}\right\}\alpha^{(\varepsilon-1)}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\left\{\frac{\xi}{(1-\xi)}\right\}\alpha^{\varepsilon}+1\right]} \quad (5.3.A.13).$$

Using equations (5.2.7) and (5.3.A.13), we have

$$u = \frac{L^{U}}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{m}{g+m}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{g}{m}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left(\alpha^{\varepsilon}\left\{\frac{g}{m}\right\}+1\right)} \quad (5.3.A.14).$$

Using equations (5.3.8), (5.3.9) and (5.3.A.14), we have

$$u = \frac{L^{U}}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left\{\frac{1}{\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1}\right\}^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]} \quad (5.3.A.15).$$

Equation (5.3.A.15) is same as equation (5.3.14) in the body of the chapter.

Appendix (5.O):

Derivation of the relationship between consumers' utility and the parameters:

$$\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}=\frac{L^{U}}{2}\left\{\frac{1}{\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1}\right\}^{\left(\frac{1}{\varepsilon-1}\right)}\frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]}$$

Suppose that

$$\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}=C$$

Then we have

$$\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}=\frac{L^{U}}{2}(C+1)^{(1-\varepsilon)}\frac{\left[\alpha^{(\varepsilon-1)}C+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}C+1\right]}$$

$$\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{dC}=\frac{L^{U}(C+1)^{(-\varepsilon)}\left(\alpha^{(\varepsilon-1)}C+1\right)^{\left(\frac{1}{\varepsilon-1}\right)}}{2(\alpha^{\varepsilon}C+1)^{2}}\Big[(1-\varepsilon)\left(\alpha^{(\varepsilon-1)}C+1\right)(\alpha^{\varepsilon}C+1)+\alpha^{\varepsilon}(C+1)\Big\{\alpha^{(\varepsilon-2)}C+$$
$$\frac{1}{\alpha^{2}}-\alpha^{(\varepsilon-2)}C-1\Big\}\Big]$$

Let $\alpha=\frac{1}{2}$. Then $\varepsilon=2$;

and,

$$\Big[(1-\varepsilon)\left(\alpha^{(\varepsilon-1)}C+1\right)(\alpha^{\varepsilon}C+1)+\alpha^{\varepsilon}(C+1)\Big\{\alpha^{(\varepsilon-2)}C+\frac{1}{\alpha^{2}}-\alpha^{(\varepsilon-2)}C-1\Big\}\Big]=\frac{C-2}{8}.$$

We find that

$$\frac{dC}{dL^{S}}=\frac{(a)^{\frac{\beta}{(1-\beta)}}b}{a}>0;\text{ and }\frac{dC}{db}=\frac{\left(L^{S}-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}}{a}>0.$$

So, if $C>(<)2$, then

$$\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{dL^{S}}=>(<)0;\text{ and }\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{db}=>(<)0.$$

Chapter 6

Conclusion

In earlier chapters of this thesis, we have analysed a few theoretical problems related to effects of globalization on skilled-unskilled wage inequality in the context of a developing economy. In this chapter, we summarize major results obtained in earlier chapters and mention limitations of the work done as well as the scope for future research.

6.1 Major findings of the present thesis

In chapter 1 of the thesis, we have made a survey of the existing empirical and theoretical works on skilled-unskilled wage inequality; and have pointed out the research gaps in the existing theoretical literature.

The chapter 2 is devoted to analyse skilled-unskilled wage inequality problem in a static competitive general equilibrium framework with special emphasis on the role of non-traded final good sector using skilled labour. In sections 2.2 and 2.3, we develop full employment models; and, in section 2.4, we introduce unemployment. In section 2.2, the endowment of skilled labour is exogenously given but, in section 2.3, the supply of skilled labour is endogenously determined by the working of the education sector. We derive various comparative static results which have interesting policy implications.

Empirical literature points out symmetric movement in skilled unskilled relative wage for various countries who are asymmetric in various directions. Our theoretical works point out two explanations of this observed phenomenon: (i) Difference in capital intensity ranking and (ii) Difference in marginal effect on excess demand for the non-traded good.

Firstly, Capital intensity ranking between the skilled labour using non-traded good sector and the skilled labour using traded good sector appears to be the most important factor determining the nature of the effect on skilled-unskilled relative wage. A capital exporting country as well as a capital importing country may experience a similar effect on the skilled-unskilled relative wage when this inter-sectoral capital intensity ranking in these two countries are opposite to each others. The same is also true for a labour exporting country and a labour importing country in the case of this opposite inter-sectoral factor intensity ranking. Opening of trade may also produce similar effects in this case.

Secondly, the nature of the effect on this skilled-unskilled relative wage depends on the mathematical sign of the marginal effect of excess demand for non-traded good with respect to changes in parameters. This sign of this marginal effect on excess demand may be different in different countries. Thus two countries, whose roles are dual to each others in the context of exchange of goods or movement of factors, may experience similar movements in skilled-unskilled relative wage with different signs of marginal demand effects even if their capital intensity ranking between the traded good sector and the non-traded good sector are identical. Models of existing literature fail to put emphasis on these points because a skilled labour using non-traded good sector does not exist there; and hence the role of intersectoral mobility of skilled labour can not be studied in those models.

We have analysed mainly comparative static effects of changes in prices of traded goods and of changes in factor endowments on skilled-unskilled wage inequality. These comparative static exercises have the following policy implications. Exogenous changes in values of fiscal instruments affect the system through changes in effective prices of traded goods. Any globalization programme, that lowers the tariff rate on imports, also lowers the effective producers' price of the import-competing product. The policy of export subsidy raises the effective price of the exportable. Increase in capital stock takes place through a liberal policy to foreign capital inflow and direct foreign investment or, may result from an increase in domestic savings. International migration of labour leads to a change in labour endowment. Land augmenting technological progress like irrigation development in agriculture leads to an increase in effective land endowment in efficiency unit.

In section 2.4, where we introduce involuntary unemployment equilibrium using efficiency wage hypothesis, we introduce Gini-Coefficient of wage income distribution as a measure of wage income inequality replacing skilled-unskilled relative wage. No other existing model has used Gini-coefficient as the measure of wage income inequality. It is shown that the Gini-coefficient is a monotonically increasing function of the skilled-unskilled relative wage in a full employment model. However, in the presence of unemployment, this is not true; and Gini-coefficient and skilled unskilled relative wage may move in opposite directions due to policy changes depending on its nature of unemployment effect. This questions the theoretical justification of measuring the degree of inequality by relative wage in the presence of unemployment.

In chapter 3 of the thesis, we analyse skilled-unskilled wage inequality problem using a static general equilibrium product variety framework with monopolistic competition in markets of different varieties and with increasing returns to scale in their production technology. In section 3.2, we develop a full employment model where a public intermediate good producing sector plays the role of reducing the fixed cost of production of nontraded private intermediate goods. However, in section 3.3, we introduce involuntary unemployment equilibrium but drop the public intermediate good producing sector from this model.

It is shown that, if production technologies are same for the agricultural sector and the public input producing sector and if the scale elasticity of output is very low, then an increase in capital stock resulting either from the increase in domestic investment or from the increase in foreign investment raises the skilled-unskilled wage ratio. However, an increase in skilled labour endowment resulting from a policy of education development does not produce any unambiguous effect. On the other hand, an increase in the tax rate on industrial output and/or an increase in the price of the agricultural product, armed with same set of assumptions, lowers the skilled-unskilled wage ratio.

Chapter 4 of this thesis analyses the skilled-unskilled wage inequality problem using a two sector dynamic intertemporal framework with special focus on international knowledge spill over from the rest of the world and localized knowledge spillover from the advanced sector to the traditional sector. The cost of developing new intermediate goods is reduced due to positive knowledge spillover effects.

We analyse the effect of opening of international trade on the skilled unskilled relative wage in the long run equilibrium of this dynamic model. It appears that the relationship between the skilled unskilled wage ratio and the skilled unskilled labour endowment ratio under autarky is ambiguous; and the nature of this relationship depends on the degree of consumer's indifference substitution between the two final goods. However, when international trade is opened, the nature of this effect depends not only on the degree of consumer's indifference substitution between the two final goods but also on the intensity of spillover effects as well as on the inter country difference in factor endowments.

Chapter 5 of this thesis sheds light on the role of imitation on skilled-unskilled wage inequality problem in the long run equilibrium of a dynamic model; and this dynamic model is built on the Helpman (1993) framework. In section 5.2 of this chapter, we assume exogenous rate of imitation. It is shown that an increase in skilled (unskilled) labour endowment raises (lowers) the rate of growth, raises (lowers) the skilled-unskilled wage ratio, and lowers (raises) the level of social welfare. However, an increase in the exogenous rate of imitation raises this growth rate, lowers the skilled-unskilled wage ratio, and raises the level of social welfare. This result is opposite to that found in Thoeing and Verdier (2003) where innovating firms use skill intensive technology to meet the increased threat of imitation and thus the skilled-unskilled wage ratio is increased.

These results have interesting policy implications. A policy of strengthening Intellectual Property Protection Rights (IPPR) lowers the rate of imitation and thus aggravates the problem of skilled-unskilled wage inequality. Policies promoting higher education sector raise the level of skilled labour endowment and hence also aggravates the problem of wage inequality.

In section 5.3, we introduce endogenous imitation and assume the existence of a social institution that controls this endogenous imitation rate producing an imitation preventing public good. It is shown that an improvement in the imitation preventing efficiency of the public good raises the skilled-unskilled wage ratio though it has no effect on the growth rate.

6.2 Future plan

Firstly, we plan to analyse changes in wage inequality in two countries simultaneously using a North-South framework. North-South framework has been extensively used in the literature on international trade and economic development. Existing North-South models deal with problems of international terms of trade, international factor mobility and international technology transfer and the role of these problems on the development of a less developed economy. Wage inequality is also an international problem; and hence a North South model is more appropriate to analyse this problem. Secondly, we want to introduce the problem of imperfection of markets. Labour market is highly imperfect and labour unions play an important role on wage determination. Bargaining power of union not only varies from region to region but also is different in two labour markets. This difference in bargaining power should play an important role to determine the degree of wage inequality. Thirdly, we plan to analyze the role of backward institutions on unskilled labour using sectors. Various factor markets are often interlinked with each others in agricultural sectors and in urban informal sectors in less developed countries; and the role of interlinkage is also very important on the wage determination of unskilled labour in those sectors. Fourthly, it would be interesting to analyse the best way to reduce the inequality out of all possible alternative policies. One can analyse whether labour market intervention would be more efficient and direct than trade intervention. Fifthly, knowledge spillover and imitations are interesting issues on their own and may call for a different type of policy interventions. These could be explored in more details. The issue of intellectual property rights (IPR) is very much a part of DOHA round of the WTO, and these models may have something to say on this issue.

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