Essays in Dynamic Macroeconomics and Fiscal Policy

by

Pawan Gopalakrishnan

Thesis submitted to the Indian Statistical Institute
in partial fulfilment of the requirements for the degree of
Doctor of Philosophy

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Contents

List of Figures

Chapter 1

Introduction

In the last two decades, several events in the global economy, like the recession in Japan (1990-2010), the dot com crisis in the US, and the recent global economic and financial crisis (which started in 2007-08), to name a few, have brought issues related to fiscal policy back into the limelight. If we look back into the history of policy making, fiscal policy was first brought onto the centre-stage in the US during the Great Depression years of the 1930s (see Kopcke et al. (2005) and Feldstein (2009)). It was believed that expansionary fiscal policy would serve as an automatic stabilizer and therefore enable the US economy to recover from massive unemployment and negative growth. In fact till the period of the 1960s-1970s, both fiscal and monetary policy received equal importance.

During the "Great Moderation" years of 1980s to the 2000s however, fiscal policy appeared to "take a back seat to monetary policy" (see Blanchard et al. (2010)). Given that maintaining stability and low inflation was the priority for advanced economies, policy makers preferred implementing counter-cyclical monetary policy instruments for the purpose of macroeconomic stabilization, instead of counter-cyclical fiscal policy, as the latter affects the real economy after considerable lags, and often faces political constraints (see Feldstein (2009) and Blanchard et al. (2010)).

Fiscal policy has however not just served as a tool for macroeconomic stabilization during recession years. A vast literature on advanced economies, both empirical and theoretical, has analyzed the effects of fiscal policy on long-run growth, welfare, and overall productivity (see Aschauer (1989), Barro (1990), Jones et al. (1993), Kind and Rebelo (1990), Glomm and Ravikumar (1994), Futagami, Morita, and Shibata (1993), Jones (1995), McGrattan (1998), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)).[1] The find-

[1] In an empirical study on OECD countries, Kneller et al. (1999) show that productive government expenditures enhance long run growth (also see Kormendi and Meguire (1985), Landau (1986), and Ram (1986) for some earlier empirical analysis on the effects of government spending on output growth and

ings are mixed, and therefore the issue of the effectiveness of fiscal policy – even in advanced economies – still remain un-resolved. In contrast to this, recent developments in the literature on dynamic stochastic general equilibrium models suggest that fiscal policy could serve as a stabilization tool even in emerging economies (see Male (2010) and Frankel et al. (2013)). This is contrary to previous literature which shows that fiscal policy is less stabilizing in emerging economies compared to advanced economies, because of financial frictions, underdeveloped financial markets, and poor economic institutions (see Agénor et al. (2000), Talvi and Vegh (2005), and Cuadra et al. (2010)).

A plethora of research has also analyzed the effectiveness of fiscal policy in the context of economic development (see Dréze and Sen (1989), Gupta et al. (1999, 2001), Chu et al. (2000), Lopes (2002), and Laframboise and Trumbic (2003)). Established by the United Nations, the Millennium Development Goals (MDGs) focuses on eight development issues. One of these deals with the issue nutrition and food security. Several emerging economies have passed the Right to Food as a constitutional amendment (see Knuth and Vidar (2011)). There are both economic and social benefits from implementing such a program, although these come at a cost. Governments have to either raise funds through taxation or through deficit financing.

Understanding the transmission effects of fiscal policy on the real economy is often analytically complicated and tractably challenging. In light of the above, this thesis is motivated to study the effect of fiscal policy on three different aspects – the effect of taxation and productive government spending on long-run growth in advanced economies, the stabilization effects of government expenditures on emerging economy business cycles, and the costs and benefits of implementing a welfare measure in a heterogenous agent developing economy. The purpose of this thesis therefore, is to understand the effectiveness of fiscal policy. In particular, it attempts to provide insights into conditions under which fiscal policy is effective, under specific environments of each chapter, which we briefly discuss in the following paragraphs.

Chapter 2 of this thesis deals with the effect of a change in factor income tax combinations on balanced growth in advanced economies. Several OECD economies exhibiting different factor income tax combinations are observed to have identical output growth rates. To explain this puzzling observation, this chapter builds a model with endogenous investment specific technological change where fiscal policy has offsetting effects on output growth. The model explains how the presence of externalities affects the magnitude of the factor income tax gap, and generates distinct rankings across different factor income tax rates, as observed across countries.

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productivity).

Chapter 3 deals with an emerging market economy (EME) real business cycle model, where the economy is subject to contemporaneous productivity and interest rate shocks. The model is also calibrated to qualitatively match the Indian data during the post-reform period. The chapter shows how fiscal policy can serve as a stabilization tool via counter-cyclical government expenditures which in turn make real interest rates a-cyclical or pro-cyclical, a feature true in India and many other EMEs.

Chapter 4 analyzes the effect of introducing a food subsidy program in an economy consisting of two agents – a farmer and an entrepreneur. Who gains, who loses and under what conditions – are some questions that are addressed. Further, the food subsidy program may also have differential impact on the output of the agricultural sector vis-à-vis the manufacturing sector. As a result, the impact of the subsidy program on sectoral outputs and relative prices are also analyzed. The chapter also shows that overall welfare gains are marginal, holding only for select combinations of food subsidies and not for every agent, and come at the cost of lowering the long run capital accumulation and output of the manufacturing sector.

Finally, chapter 5 discusses scope for future work.

The following sections broadly outline the main motivation and results obtained in each chapter.

1.1 Factor Income Taxation, Growth, and Investment Specific Technological change

Many studies, both theoretical and empirical, have analyzed whether fiscal policy affects long run growth in advanced economies. The standard theoretical literature on endogenous (AK) growth models predicts that fiscal policy has large growth effects through its impact on the economy's investment rate (e.g., see Barro (1990), Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). Results obtained from empirical studies, however, do not uniformly support these theoretical predictions (see Jones (1995) and Kneller et al. (1999)). The literature has also tried to find extensions to the standard endogenous growth model that can explain the apparent absence of growth effects of fiscal policy (see McGrattan (1998), Glomm and Ravikumar (1998), and Jaimovich and Rebelo (2012)).

Motivated by the empirical observation that several advanced economies with different factor income tax combinations exhibit the same long-run output growth rate, this chapter provides an alternative, but compatible, explanation because fiscal policy in this framework has offsetting effects on growth. Building on Huffman (2007, 2008), this chapter builds a model of endogenous investment specific technological change (ISTC) where externalities from the stock of public and private capital augment ISTC, and a specialized labor input that augments ISTC also exerts a positive spillover in final good production.

As in Huffman (2008), in this model there are two sectors: a final good sector and a research sector. The final good sector produces a final good, using private capital and labor. The second sector (the research sector) captures the effect of public and private capital stock spillovers, and research activity on ISTC. A social planner maximizes the growth rate of the economy by obtaining a fixed growth maximizing tax rate to finance the public investment. This yields the socially efficient solution. The corresponding competitive decentralized equilibrium growth rate – which can be decomposed into a labor and a capital factor – is obtained by taking the externality from public and private capital on ISTC, and the spillover from specialized labor on final goods production as given. Changes in factor income taxes, by affecting these factors, can have opposite effects on growth. Therefore, several factor income tax combinations can decentralize the planner's socially efficient growth rate. Further, the presence of externalities from the public and private capital, and the specialized labor input, affect the magnitude of the factor income tax gap, and generate distinct factor income tax rankings, as observed across several OECD economies.

1.2 Fiscal Policy in an Emerging Market Business Cycle Model

Fiscal policy plays an important role in macroeconomic stabilization in many developing countries and EMEs. This aspect has however received little attention in recent literature on EME business cycle models. Over the last decade, several EMEs have "graduated" from having pro-cyclical fiscal policy to counter-cyclical fiscal policy. This "graduation" has been attributed to improvements in institutional quality (see Frankel et al. (2013)). Another aspect that is particularly missing in the literature is the role of fiscal policy for macroeconomic stabilization when a small open economy is hit with an interest rate shock. Some recent empirical work (Male (2010) and Ghate et al. (2013)) show that real interest rates need not be counter-cyclical with respect to output across all EMEs. Ghate et al. (2013) show that government expenditures are counter-cyclical and real interest rates are pro-cyclical in India, in the post reform period, as is true in many other EMEs. To reconcile these mixed stylized facts, particularly related to the cyclical properties of government expenditure and real interest rates, the chapter builds a small open economy real business cycle model, extending the seminal work of Neumeyer and Perri (2005) by adding a crucial role for fiscal policy.

Building on Neumeyer and Perri (2005), the model incorporates two different roles for fiscal policy: the government provides public consumption with the private and public components of consumption substitutable; and, the government also lends a portion of the working capital constraint faced by the firm at a subsidized interest rate. These are financed using time invariant distortionary taxes on consumption, labor income, and capital income. The government also balances its budget in every time period. Second, unlike Neumeyer and Perri (2005), in this framework, agents are assumed to have Cobb-Douglas (CD) utility functions. The assumption of CD preferences permits a shock to the real interest rate to have income effects on labor supply through consumption. We show that these features can make the real interest rate less counter-cyclical or even pro-cyclical.

Fiscal policy affects the transmission of interest rate shocks onto the real economy through a standard inter-temporal substitution effect, and a time varying wedge which we denote as the *fiscal policy wedge*. The main theoretical contribution of this framework is that we characterize the fiscal policy wedge in closed-form under a variety of assumptions on fiscal policy, and show how this affects movements in labor supply adversely. We also show that by subsidizing a firm's working capital requirement, fiscal policy is able to dampen the reduction in labor demand due to a positive interest rate shock. Thus, both labor supply and labor demand channels make the real interest rate a-cyclical, and under certain cases, pro-cyclical, matching the qualitative features of wider EME data. We then use the model to replicate qualitatively some of the key features of the Indian business cycle.

1.3 Tax Policy and Food Security

This chapter builds a model to assess the effect of introducing a food subsidy program in an economy with two types of agents – a farmer and an entrepreneur. The subsidy program was passed as an act in several economies such as India, with the purpose of enabling the poor to increase their nutritional intake so that they can work more efficiently and contribute positively to the country's GDP. Implementing such a subsidy program however implies that the wealthier sections of the society would be taxed, which may in turn curb capital investment and long run growth of the economy. Further, the food subsidy program can also have a differential impact on the output of the agricultural sector as compared to (say) the manufacturing sector. In this chapter, we therefore build a model to analyze some of these questions.

In this model, the entrepreneur is endowed with capital while the other agent, the farmer, is not. Both agents are consumer-producers, and there is no occupational mobility. The farmer uses his labor to produce a food crop and a cash crop, where the former is a final good and the latter is an intermediate good. The entrepreneur employs cash crop, his labor, and capital to produce the manufacturing output, which is another final good. The manufacturing good is consumed by both the farmer and the entrepreneur, and is also accumulated as capital by the entrepreneur.

A key feature in the model is that each agent's total labor capacity endogenously depends on his intake of the food crop since consumption of the food crop provides nutrition (see Dasgupta and Ray (1986) and Dasgupta (1997)). This is a novel feature of this paper which ensures that the subsidy on food translates into "security", especially under low levels of productivity.

In particular, the government provides a per-unit subsidy on food consumption to both agents at an exogenous rate. The government may finance this program by either levying a direct tax or an indirect tax. Under the direct tax regime, the entrepreneur has to pay taxes proportional to his income, while in the indirect tax regime, a per-unit consumption tax is imposed on both the farmer and the entrepreneur on the manufacturing goods consumption. The tax rates are fixed so that the government balances its budget.

In both tax regimes, the subsidy program increases the steady state agriculture output but lowers the steady state manufacturing output, since the taxes levied negatively affect either the supply or the demand of the manufacturing good. The effects on relative prices are however different in the two tax regimes. In the income tax regime, the long run price of the food crop relative to the price of the manufacturing good declines with subsidies, while in the consumption tax regime it increases with subsidies. This is because, in the income tax regime, the supply of the manufacturing output falls, whereas in the consumption tax regime, the consumption of the manufacturing output becomes more expensive, as a result of which, agents substitute away from consumption of the manufacturing output.

In terms of welfare, compared to the no subsidy regime, the agents' steady state welfare improves only for a certain range of subsidies. There also may not exist any subsidy combination for which both the farmer and the entrepreneur are better off. Between the two tax regimes however, we find that financing the subsidy program using an indirect consumption tax regime, compared to a direct income tax regime, is welfare improving. On normative grounds, this suggests that whereas such a subsidy program may only have limited gains in a heterogeneous agent economy, it is best to implement the program by sharing the tax burden between both agents, i.e., by imposing an indirect tax on both agents, rather than a direct tax on only one agent.

Chapter 2

Factor Income Taxation, Growth, and Investment Specific Technological Change

2.1 Introduction

Why do countries with different factor income tax combinations exhibit similar growth rates? In this paper, we develop an endogenous growth model with endogenous investment specific technological change to understand this question.

Figure 2.1 plots the average aggregate annual real GDP growth rate from 1990 to 2007 against the factor income tax ratio for several advanced economies.[1] Average growth for all countries (excluding Ireland) falls between 0.875% and 2.462%. The standard deviation of the average real GDP growth rates is low at 0.878 (excluding Ireland, the standard deviation is 0.4756). Figure 2.2 plots the range of individual factor income taxes for these countries where the tax on capital and labor income have been averaged over 1990–2007. What is striking is that the range in the ratios of the average capital income tax rate to the average labor income tax rate in these economies is much more pronounced: 0.3951 to 1.725.[2] Also

[1]The growth rates are calculated from the OECD (2012) database: see Table ($VXVOB$). The countries are: Austria (AUS), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Ireland (IRE), Italy (ITA), Japan (JPN), Netherlands (NET), Portugal (PRT), Spain (SP), Sweden (SWE), United Kingdom (UK) and United States of America (USA). The base year is 2000

[2]Canada and Japan have data on capital and labor income tax estimates based on the approach used in Mendoza et al. (1994) and Trabandt and Uhlig (2009) from 1965 to 1996. For Germany, United Kingdom and United States of America, data is from 1965 to 2007. For France, the data is from 1970 to 2007. For Italy, the data is from 1980 to 2007. For Austria, Belgium, Denmark, Finland, Netherlands, Portugal and Sweden, the data is from from 1995 to 2007. For Spain and Greece, the data is from 2000 to 2007. Finally, for Ireland, the data is from 2002 to 2007.

whereas the difference between factor income taxes is large in some countries, it is quite small in others.[3] Figure 2.1 and Figure 2.2 suggest that countries with almost similar growth rates are accompanied by totally different factor income tax combinations.

[Insert Figure 2.1 and 2.2]

Figure 2.3 plots the levels of factor income tax rates across the G7 countries. The incidence of factor income taxation is quite disparate. In the US, UK, Canada, and Japan, the tax on capital income is greater than the tax on labor income. In contrast, for Germany, Italy, and France, the reverse is true.

[Insert Figure 2.3]

In other evidence, Jones (1995) also shows in a sample of 15 OECD countries from 1950 to 1987, that changes in investment rates do not have any significant long run growth effects. He shows that shocks to investments – both total and durables and in particular durable equipment – have only a short-run growth effect with no significant effect on long run growth.

Figures 2.1 - 2.3 and the evidence from Jones (1995) are suggestive of a "growth-tax" puzzle since countries with different factor income tax combinations exhibiting similar growth rates is incompatible with a standard model of endogenous growth.[4] The standard endogenous (AK) growth model predicts that fiscal policy has a large growth effect through its impact on the economy's investment rate. Taken to the data, these models would predict a high correlation between the investment rate and the growth rate. The above evidence therefore suggests that changes in fiscal policy (or factor income taxes) must have offsetting changes in investments such that growth rates do not change.

The literature has tried to find extensions to the standard endogenous growth model that can explain the apparent absence of growth effects of fiscal policy. McGrattan (1998) develops a theoretical framework where government policy can be incorporated into a standard AK growth model by incorporating two types of capital: structures and equipment capital. She shows that the equilibrium growth rate depends on the investment rate and the capital-output ratio. The reason why fiscal policy has no growth effects is because its effect on the investment rate is offset by the effect of fiscal policy on the capital-output ratio. Because of these offsetting effects, total investment does not change that much. Jaimovich and Rebelo (2012) show that changes in tax rates can have non-linear effects on long-run output growth.

[3]The data on factor income taxes are from Mendoza et al. (1994) and Trabandt and Uhlig (2009). The latter have used the approach in Mendoza et al. (1994) to estimate the tax rates for 17 OECD nations till 2007.

[4]Stokey and Rebelo (1995) also show in a numerical exercise that big changes in tax on capital income (up to the order 30%) do not have large growth effects on the US economy.

To capture this non-linearity, they construct a model where low tax rates have negligible effects on growth but when disincentives to invest are large, larger tax rates have a strong negative effect on output growth. The mechanism in their model is based on a skewed distribution of agents between workers and innovators, which results in a small number of highly productive workers in equilibrium. In a related literature, Glomm and Ravikumar (1998) build a growth model where public education spending, financed by distortionary taxes affect human capital accumulation. Again, they find that despite being distortionary in nature, tax rates have negligible effects on growth rates.

2.1.1 Description of our model and main results

Our paper provides an alternative, but compatible, explanation for the above growth-tax puzzle, i.e., the fact that different combinations of factor income taxes can generate the same growth rate. We construct an endogenous growth model with endogenous investment specific technological change with three types of externalities: an externality from the stock of private and public capital in the process of innovation; and an externality from labor allocated to research in final good production. In particular, the public capital stock – financed by distortionary taxes – and the private capital stock augment investment specific technological change (ISTC) as a positive externality.[5] Typically in the literature, the public input is seen as directly affecting final production directly either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). We show that embedding varying magnitudes of these externalities into a model of endogenous growth with endogenous ISTC leads to offsetting effects of factor income taxes on growth.[6]

Our basic model follows Huffman (2008).[7] There are two sectors in the model: a final goods sector and a research sector. The final good sector produces a final good, using private

[5] Our setup *also* allows investment specific technological change to enhance the accumulation of public capital. For instance, providing better infrastructure today reduces the cost of providing public capital in the future.

[6] To the best of our knowledge, we are not aware of any paper in the literature in which public capital affects ISTC, either directly or as an externality. In a different context, Harrison and Weder (2000) build a two sector representative agent model with increasing returns to scale driven by externalities that come from sector specific as well as aggregate economic activity. Benhabib and Farmer (1996) show that small empirically plausible external effects lead to indeterminacy. Neither of these papers has a role for public capital. Lloyd-Braga, Modesto, and Seegumuller (2008) introduce positive government spending externalities in preferences. In our model, externalities from the public stock influence ISTC directly.

[7] A growing literature has attributed the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Investment specific technological change refers to technological change which reduces the real price of capital goods. Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity.

capital and labor. Labor supply is composite in the sense that one type of labor activity is devoted to final good production, and the other to research which directly reduces the real price of capital goods in the next period. The second sector (the research sector) captures the effect of public capital and private capital stock spillovers and research activity on reducing the real price of capital goods. The agent optimally chooses each labor activity. We assume two types of labor activities: one type is labor allocated for final goods production, or current production, and another type is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production. Crucially, the agent might not be aware that his allocation of labor towards research also influences productivity of the current period's final goods production. Therefore, although research labor allocation is done from the point of future capital accumulation and hence future output we will assume that the agent might be unaware of the spillover it has on current production. This implies that the process of augmenting knowledge - which is designed to influence the price of capital in the future - may affect present output too. Effectively, this means that the process of augmenting knowledge may make routine labor (in the final goods sector) more effective.

The planner maximizes the utility of the representative agent and internalizes the externalities in the research sector and final good sector. In the planner's problem, we assume that public investment is financed by a *fixed* proportional income tax as in Barro (1990). Given a fixed tax rate, the planner's problem yields the socially efficient allocation. *Corresponding to this allocation*, we characterize the steady state balanced growth path and show that the growth rate depends on 1) a labor input devoted to research (the labor factor) and 2) the contribution to growth from public and private capital (the capital factor).

We then ask under what conditions can the planner's allocations be implemented in the competitive decentralized equilibrium with identical and different factor income taxes. We assume that public investment is financed by distortionary factor income taxes on capital and labor income. We show that the growth rate corresponding to the socially efficient allocation can be implemented in the competitive equilibrium by a combination of capital tax rates and labor tax rates through a 1) capital factor, and 2) a labor factor. Our definition of indeterminacy is as follows: there is *no unique combination* of factor income taxes on capital *and* labor income that implements growth rate corresponding to the efficient allocation for a fixed set of parameters. In other words, we show that multiple factor income tax combinations - and therefore factor income tax gaps - can implement the efficient growth rate. This finding is consistent with the empirical evidence documented in Figures 2.1 - 2.3. In a numerical section we show that for a fixed set of parameters a wide range of tax rates imply the same growth rate.

Indeterminacy obtains because the planner's allocations yield a constant growth rate, and factor income taxes have offsetting effects on the capital factor and labor factor. In particular, an increase in the capital income tax reduces the capital factor, and *reduces* growth. However, an increase in the labor income tax exerts both offsetting income and substitution effects. We show that with ISTC, the income effect is stronger than the substitution effect, and so increases in the labor income tax increase labor supply. The increase in labor supply increases the labor factor which augments capital accumulation and growth. We also show that the strength of the income effect is made stronger the larger the extent of ISTC. We are also able to analytically characterize the implementation of the growth rate corresponding to the socially efficient allocation.

How do the externalities affect the factor income tax gaps that implement the planner's allocations? We first consider the case of a positive spillover from the specialized research labor activity on final good production. In this case, an increase in the spillover increases the planner's allocation towards specialized labor. This increases the growth rate corresponding to the socially efficient allocation. To implement this higher growth rate, this requires an increase in the labor income tax, which raises the labor factor from the competitive growth rate, or a reduction in the capital income tax, which raises the capital factor. Implementing either leads to a widening of the equilibrium factor income tax gap.

In contrast, when the weight on the positive spillover from the public and private capital stock falls, this leads to a higher contribution of the existing stock of ISTC on the future level of ISTC. That is, a lower weight on the stock externalities implies that the weight on the persistence of ISTC is higher. Therefore, the growth rate of the planner is higher. To raise the competitive equilibrium growth rate, a reduction in the tax on capital income raises the capital factor and an increase in the labor income tax raises the labor factor. An increase in both factors raise growth which requires an increase in the factor income tax gap to implement the planner's growth rate.

Our general result is that to the extent that spillovers from a specialized labor input and the public and private capital stocks exist, an increase in these spillovers from the specialized labor input, and a decrease in the spillover from public and private capital, increase the planner's growth rate, and therefore increase the factor income tax gap required to implement the growth rate corresponding to the efficient allocation. Conversely, for a given level of externalities, maintaining the constancy of growth also requires different combinations of factor income taxes as in McGrattan (1998). We also show that when there are no externalities, *equal* factor income taxes always yield the optimal growth rate from the planner's problem. Hence, the factor income tax gap is zero. Finally, we also conduct a simple numerical exercise to show that equilibrium factor income taxes generated by our model are in accordance with Figures 2.1 - 2.3.

Empirical Evidence on Externalities

With respect to the private capital stock, DeLong and Summers (1991) show that investment in machinery is associated with very strong positive externalities, and that increases in investments in equipment implies higher growth. Hamilton and Monteagudo (1998) find that capital is associated with positive external effects in an estimated Solow growth model. Greenwood et al. (1997), show that the real price of capital equipment in the US – since 1950 – has fallen alongside a rise in the investment-GNP ratio. This suggests that the private capital stock exhibits a positive externality in investment specific technological change through the aggregate capital stock. Importantly, Greenwood et al. (1997, p. 342) say: "The negative co-movement between price and quantity.....can be interpreted as evidence that there has been significant technological change in the production of new equipment. Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and long run."

With respect to the nexus between public expenditures, R&D, and growth, Griliches (1979) examines how the indirect effects of research and development affect future output through induced changes in factor inputs. In his model, the accumulation of private capital is driven by the aggregate stock of knowledge and current and past stocks of research and development (R&D). Scott (1984) and Levin and Reiss (1984) estimate that the high spillovers from federal research and development spending dominates the crowding-out effect it has on private spending on R&D. The net effect is that public spending has a positive effect on productivity. Finally, David et al. (2000), show that public R&D spending is complementary to private R&D spending.

We also assume that the specialized labor input in the research sector exerts a positive externality in the production of the first sector, the final good. This assumption is motivated by both anecdotal evidence as well as the academic literature. For instance, Davidson (2012) documents evidence on the extent to which skills required for advanced manufacturing jobs. He argues that skilled factory workers these days are typically "hybrid-workers": they are both machinists (engaging in final good production) as well as computer programmers (engaging in research). For instance, in the US metal-fabricating sector, workers not only use cutting tools to shape a raw piece of metal, but they also write the computer code that instructs the machine to increase the speed of such operations. Globerman (1975) describes a class of machinists in the manufacturing sector called "tool and die makers", or also "mold makers" (see Bryce (1997)).[8] The machinist therefore receives on-the-job training which

[8] Primarily a skilled artisan, a tool and die maker works in an industrial environment where producing

enables him to work with machines and computers, which makes him multi-skilled. Even though on-the-job training is costly, Park (1996) shows, from an empirical study on manufacturing industries in Korea that employing "multi-skilled workers" makes a firm's production more efficient in comparison to employing "single-skilled "or specialized workers to handle each individual activity.[9] On the job training is undertaken for future benefits but it may also augment the efficiency of standard labor that has been assigned to produce output in the current period.

Related Literature

The setup of our model is technically similar to Huffman (2007, 2008) who explicitly models the mechanism by which the real price of capital falls when investment specific technological change occurs. Huffman (2008) builds a neoclassical growth model with investment specific technological change. Labor is used in research activities in order to increase investment specific technological change. In particular, the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.[10] Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector. His model includes capital taxes, labor taxes, and investment subsidies that are used to finance a lump-sum transfer. Huffman (2008) finds that a positive capital tax that is larger than a positive investment subsidy along with zero labor tax can replicate the first best allocation. Huffman's models however do not incorporate public capital - a feature we show that is important in explaining the growth-tax puzzle in our paper.

Our paper is also related to the literature on fiscal policy and long run growth in the neoclassical framework. The literature started by Barro (1990) and Futagami, Morita, and Shibata (1993) – incorporate a public input – such as public infrastructure – that directly augments production. In Barro (1990), public services are a flow; while in Futagami, Morita, and Shibata (1993), public capital accumulates. However, in the large literature on public capital and its impact on growth spawned by these papers, the public input, whether it is

the final good requires two different skills – creative skills and machine knowledge. An example of such an activity, crucial to the manufacturing sector, is engineering drawing.

[9] Even though labor productivity in final good production is typically seen to be a function of the stock of knowledge (and therefore the externality comes from the level of ISTC), we assume that there is no difference in skills and ability in the labor force in the two productive activities, so that labor allocated to research is not an exact proxy for the stock of knowledge.

[10] Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

modeled as a flow or a stock, doesn't directly influence the real price of capital goods.[11] Since public capital affects the real price of capital explicitly in our model, this means that the public input affects future output through its effect on both future investment specific technological change, as well as future private capital accumulation.

2.2 The Model

Consider an economy that is populated by identical infinitely lived agents with unit mass, who at each period t, derive utility from consumption of the final good C_t and leisure $(1-n_t)$. There is no population growth which implies that aggregate variables are also per-capita variables. The term n_t represents the fraction of time spent at time t in employment. The discounted life-time utility, U, of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)]. \tag{2.1}$$

where $\beta \in (0, 1)$ denotes the period-wise discount factor. The total supply of labor for the agent at any time t is given by n_t such that

$$n_t \equiv n_{1t} + n_{2t}, \tag{2.2}$$

where n_{1t} is labor allocated for final goods production, or current production, and n_{2t} is labor allocated for enhancing investment specific technological change, or future capital accumulation, and therefore future production. Crucially, the agent is not aware that his allocation of labor towards n_{2t} also influences productivity of the current period's final goods production.[12] Therefore, although n_{2t} is employed from the point of future capital accumulation and hence future output the agent is unaware of the spillover it has on current production.

The final good is therefore produced by a neoclassical production function with capital K_t, n_{1t}, and n_{2t}. An important point is that the planner internalizes the effect of n_{2t} on final goods production, while the agent will not. The production function is given by

$$Y_t = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \left(n_{2t}^{1-\alpha} \right)^{\xi} \tag{2.3}$$

where $\underline{A} > 0$ is a scalar that denotes the exogenous level of productivity, $\alpha \in (0, 1)$ is the

[11] For instance, in Ott and Turnovsky (2006) - who use the flow of public services to model the public input - and Chen (2006), Fischer and Turnovsky (1998) - who use stock of public capital - the shadow price of private capital is a function of public and private capital.

[12] This assumption is motivated by the empirical evidence on "multi-skilled" workers mentioned in the introduction.

share of output paid to capital and $\xi > 0$ is the externality parameter capturing the effect that n_2 has on direct production. When $\xi > 0$, the planner internalizes the effect that n_2 has on direct production. When $\xi = 0$, there is no externality from n_2 on the production of the final good. Note, in this framework, as in Huffman (2008) the two labor activities n_{1t} and n_{2t} are assumed to be equally skilled, but are optimally allocated across different activities by households.[13]

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

$$K_{t+1} = (1-\delta)K_t + I_t Z_t, \tag{2.4}$$

where $\delta \in [0,1]$ denotes the rate of depreciation of capital and I_t represents the amount of total output allocated towards private investment at time period t. We assume that, $\delta = 1$, to keep the model tractable. Z_t represents investment-specific technological change. The higher the value of Z_t, the lower is the cost of accumulating capital in the future. Hence Z_t can also be viewed as the inverse of the price of per-unit private capital at time period t. The term, $I_t Z_t$, therefore represents the effective amount of investment driving capital accumulation in time period $t+1$.

In addition to labor time deployed by the representative firm towards R&D, the public capital stock, G, plays a crucial role in lowering the price of capital accumulation. Typically the public input is seen as directly affecting final production – either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). Instead, here we assume that the public input facilitates investment specific technological change. This means that the public input affects future output through future private capital accumulation directly. In the above literature, the public input affects current output directly. This is our point of departure. We therefore formalize the link between fiscal policy and growth *through the effect that* fiscal policy has on ISTC.

We assume that in every period, public investment is funded by total tax revenue. Public capital therefore evolves according to

$$G_{t+1} = (1-\delta)G_t + I_t^g Z_t, \tag{2.5}$$

where G_{t+1} denotes the public capital stock in $t+1$, and I_t^g denotes the level of public

[13] Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor.

investment made by the government in time period t:

$$I_t^g = \tau Y_t, \tag{2.6}$$

where $\tau \in (0,1)$ is the proportional tax rate.[14] We assume that Z_t augments I_t^g in the same way as I_t since it enables us to analyze the joint endogeneity of Z and G. To derive the balanced growth path, we further assume that the period wise depreciation rate $\delta \in [0,1]$ is same for both private capital and public capital.

2.2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that Z grows according to the following law of motion,

$$Z_{t+1} = B{n_{2t}}^{\theta} Z_t^{\gamma} \left\{ \left(\frac{G_t}{Y_{t-1}}\right)^{\mu} \left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu} \right\}^{1-\gamma}. \tag{2.7}$$

Here, $B > 0$ stands for an exogenously fixed scale productivity parameter and $\mu \in (0,1)$ captures the impact of public investments on investment specific technological change. We assume that the parameters, $\theta \in (0,1)$ and $\gamma \in (0,1)$, where θ stands for the weight attached to research effort and γ is the level of persistence the current year's level of technology has on reducing the price of capital accumulation in the future.[15] The term $\frac{G_t}{Y_{t-1}}$ represents the externality from public capital in enhancing investment specific technological change in time period $t+1$. The aggregate capital-output ratio, $\frac{K_t}{Y_{t-1}}$, is also assumed to exert a positive externality effect on investment specific technological change. In particular, a higher aggregate stock of capital in t, K_t, relative to Y_{t-1}, raises Z_{t+1}. Like the externality from n_2, the planner internalizes the effect that stock of public capital and private capital has on investment specific technological change, while agents treat the effect of $\frac{G_t}{Y_{t-1}}$ and $\frac{K_t}{Y_{t-1}}$ on Z_{t+1}– the bracketed term – as given.[16] Note that when $\gamma = 1, \theta = 0$, ISTC is exogenous.

[14] Since $\delta = 1$, equation (2.5) implies that $G_{t+1} = I_t^g Z_t$, i.e., the ISTC adjusted public investment (flow) at period t equals the public capital stock in $t+1$.

[15] This contrasts with Huffman (2008) where $\gamma = 1$ is required for growth rates of Z and output to be along the balanced growth path. We require $\gamma \in (0,1)$ for the equilibrium growth rate to adjust to the steady state balanced growth path.

[16] We assume that $\delta = 1$ for analytical tractability. Our assumption of $\frac{G_t}{Y_{t-1}}$ augmenting Z_{t+1} is for two reasons. First, if G_t augmented output Y_t instead, we can show that in equilibrium, the only possible balanced growth path is when the gross growth rate of all endogenous variables is 1 that is, all variables are at their steady state. This means, public capital will not affect the growth rate. Hence, allowing for ISTC to depend on the public input enables the balanced growth path to be affected by tax policy *through* ISTC.

2.2.2 The Planner's Problem

We first solve the planner's problem who internalizes all the externalities. This yields the socially efficient allocation for a fixed tax rate. This is not a "full blown" planner's problem since the planner takes the fixed tax rate as given. This is equivalent to a constrained planning problem, an approach that is common in the literature.[17]

The aggregate resource constraint the economy faces in each time period t is given by

$$C_t + I_t \equiv Y_t(1-\tau) = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \left(n_{2t}^{1-\alpha}\right)^{\xi} (1-\tau) \tag{2.8}$$

where agents consume C_t at time period t and invest I_t at time period t. Aggregate consumption and investment add up to after-tax levels of output, $Y_t(1-\tau)$, where $\tau \in [0,1]$ is the proportional tax rate that is assumed to be fixed in every time period.

Since the planner internalizes the size of public expenditure given by

$$\frac{G_{t+1}}{Y_t} = \tau Z_t, \tag{2.9}$$

which follows from (2.5) and (2.6) after imposing $\delta = 1$, he takes the following law of motion of ISTC as a restriction:

$$Z_{t+1} = B {n_{2t}}^{\theta} Z_t^{\gamma} Z_{t-1}^{(1-\gamma)\mu} \tau^{\mu(1-\gamma)} \left(\frac{K_t}{Y_{t-1}}\right)^{(1-\mu)(1-\gamma)}, \tag{2.10}$$

which is obtained by substituting (2.9) in (2.7).

To obtain the efficient allocation, the planner maximizes the lifetime utility of the representative agent – given by (2.1) – subject to the economy wide resource constraint given by (2.8), the law of motion (2.4), the equation describing investment specific technological change (2.10) and the identity for total supply of labor given by (2.2).[18]

Second, if Z_{t+1} was instead parametrized as

$$Z_{t+1} = B {n_{2t}}^{\theta} Z_t^{\gamma} \left\{G_t^{\mu} K_t^{1-\mu}\right\}^{1-\gamma},$$

i.e., G and K are not normalized by Y, we can show that the growth rate if Z will never converge to a balanced growth path.

[17] We justify this assumption because of the main goal of our paper: to explain constant growth rates with positive and varying factor income taxes in the data. While we don't show this here, the competitive equilibrium growth rate always falls short of the (unconstrained) first best growth rate. These results are available from the authors on request. However, as we will see later, we can implement the growth rate corresponding to the constrained planner's problem by allowing the planner to tax factor incomes differentially. Differential taxes allows the planner to correct for the under-provision of private inputs in the competitive equilibrium.

[18] Clearly, $C_t + I_t + I_t^g = Y_t$.

First Order Conditions

The Lagrangian for the planner's problem is given by,

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t\left[\log C_t+\log(1-n_{1t}-n_{2t})\right]+\sum_{t=0}^{\infty}\beta^t\lambda_{1t}\left[\underline{A}K_t^{\alpha}n_{1t}^{1-\alpha}\left(n_{2t}^{1-\alpha}\right)^{\xi}(1-\tau)-C_t-\frac{K_{t+1}}{Z_t}\right]\\ &\quad+\sum_{t=0}^{\infty}\beta^t\lambda_{2t}\left[B{n_{2t}}^{\theta}Z_t^{\gamma}Z_{t-1}^{(1-\gamma)\mu}\tau^{\mu(1-\gamma)}\left(\frac{K_t}{Y_{t-1}}\right)^{(1-\mu)(1-\gamma)}-Z_{t+1}\right].\end{aligned}$$

where λ_{1t} and λ_{2t} are the Lagrangian multipliers. Because our focus is on the balanced growth path corresponding to the efficient allocation, we assume that $\delta = 1$[19].

The following first order conditions obtain with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, respectively[20]:

$$\{C_t\}:\tfrac{1}{C_t}=\lambda_{1t} \tag{2.11}$$

$$\{K_{t+1}\}:\frac{1}{C_tZ_t}=\frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}+\beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}}-\beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}} \tag{2.12}$$

$$\{Z_{t+1}\}:\lambda_{2t}=\beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}}+\frac{\beta}{Z_{t+1}}\left(\frac{I_{t+1}}{C_{t+1}}\right)+\beta^2\lambda_{2t+2}\mu(1-\gamma)\frac{Z_{t+3}}{Z_{t+1}} \tag{2.13}$$

$$\{n_{1t}\}:\frac{1}{1-n_t}=\frac{(1-\alpha)Y_t(1-\tau)}{C_tn_{1t}}-\beta\lambda_{2t+1}(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}} \tag{2.14}$$

and,

$$\{n_{2t}\}:\frac{1}{1-n_t}=\frac{(1-\alpha)\xi Y_t(1-\tau)}{C_tn_{2t}}+\lambda_{2t}\theta\frac{Z_{t+1}}{n_{2t}}-\beta\lambda_{2t+1}(1-\gamma)\xi(1-\alpha)\frac{Z_{t+2}}{n_{2t}}. \tag{2.15}$$

Equation (2.11) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (2.12) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. Equation (2.13) is the Euler equation with respect to Z_{t+1}. Equation (2.14) denotes the optimization condition with respect to labor supply (n_{1t}). Since $0 < \gamma < 1$, the second term in the RHS is positive which constitutes a reduction in the marginal utility of leisure. This reduces n_1 relative to the standard case in which there is no investment specific technological change. Finally, equation (2.15) is the first order condition with respect to n_{2t}.

[19] While assuming $\delta = 1$ is restrictive, we do this for analytical tractability. We get closed form solutions for our allocations using the method of undetermined coefficients. See Appendix E for all results with $\delta < 1$.

[20] See Appendix A for derivations.

Decision Rules

We now derive the closed form decision rules based on the above first order conditions using the method of undetermined coefficients, as shown in the following Lemma 2.1.

Lemma 2.1 *C_t, $I_t, n_t, n_{1t,} n_{2t}$ are given by (2.16), (2.17), (2.18), where $0 < \Phi < 1$ is given by (2.19), and $0 < x < 1$ given by (2.20) is a constant.*

$$C_t = \Phi_P Y_t(1-\tau), I_t = (1-\Phi_P)Y_t(1-\tau) \tag{2.16}$$

$$n_t = n_P = \frac{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)]}{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)] + \Phi_P x_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}, \tag{2.17}$$

$$n_{1P} = x_P n_P, n_{2P} = (1-x_P)n_P, \tag{2.18}$$

where Φ_P is given by

$$\Phi_P = 1 - \frac{\alpha\beta\left[(1-\beta\gamma) - \beta^2\mu(1-\gamma)\right]}{(1-\beta\gamma) - \beta^2(1-\gamma) + \alpha\beta^3(1-\gamma)}, \tag{2.19}$$

and x_P is given by

$$x_P = \frac{(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\} + \beta\theta(1-\Phi_P)}. \tag{2.20}$$

Proof. See Appendix A for derivations. ■

While decision rules for consumption and investment given by (2.16) suggest that levels of consumption and investment would fall if the proportional tax rate τ increases, the share of after tax income spent on consumption given by Φ_P increases when μ rises, and thereby for investment it falls. Intuitively, when μ rises the weight on the ratio of public capital to output, $\frac{G_t}{Y_{t-1}}$ in augmenting investment specific technological change increases and so the weight on the ratio $\frac{K_t}{Y_{t-1}}$ falls. Since the planner solves the optimization problem for the representative agent, the effect of increases in μ on private investments is therefore expected.

The Balanced Growth Path

We can obtain the balanced growth path (BGP) corresponding to the efficient allocation - and a fixed tax rate - by substituting (2.16), (2.17), (2.18), (2.19), and (2.20) into (2.7).

Define $\widehat{M_P}$ a constant as

$$\widehat{M_P} = B((1 - x_P)n_P)^{\theta}(1 - \Phi_P)^{(1-\mu)(1-\gamma)}. \tag{2.21}$$

Given the assumptions it is easy to show that we can obtain a constant growth rate for Z, K, G and Y. This condition necessarily implies $0 < \Phi_P,\ x_P,\ n_P < 1$ which always holds true. We therefore have the following Lemma 2.2.

Lemma 2.2 *On the steady state balanced growth path, the gross growth rate of Z, K, G and Y are given by (2.22), and (2.23)*[21]

$$\widehat{g_{z_P}} = [\widehat{M_P}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} \tag{2.22}$$

$$\widehat{g_{k_P}} = \widehat{g_{g_P}} = \widehat{g_{z_P}}^{\frac{1}{1-\alpha}}, \widehat{g_{y_P}} = \widehat{g_{k_P}}^{\alpha} = \widehat{g_{z_P}}^{\frac{\alpha}{1-\alpha}}. \tag{2.23}$$

There are several aspects of the equilibrium growth rate worth mentioning.[22] First, the growth rate corresponding to the socially efficient allocation is independent of the technology parameter, $\underline{A}$, but not B, as in Huffman (2008). Second, the growth rate of output, $\widehat{g_{y_P}}$, is less than $\widehat{g_{k_P}}$ along the balanced growth path because equation (2.7) is homogenous of degree $1 + \theta$. Lemma (2) therefore clearly establishes that the effect of the stock of public capital on Z affects not just marginal productivity of factor inputs but also growth rate at the balanced growth path.

Finally, from (2.22), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in models with public capital. The mechanism here is however different. For small values of the tax rate, a rise in τ leads to higher public capital relative to output, Y_{t-1}. This raises the future value of ISTC. An increase in ISTC reduces the real price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax rate depresses after tax income, and investment. This reduces G relative to Y, lowering Z, and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate although the planner may not necessarily choose it since the tax rate is arbitrary[23].

[21] See Bishnu, Ghate and Gopalakrishnan (2011).

[22] With $\delta < 1$ $\widehat{g_{z_P}}$ is given by

$$\widehat{g_{z_P}} = \left\{ Bn_2^{\theta} \left[(\tau\Delta_1)^{\mu} (\chi_4 (1-\tau))^{1-\mu} \right]^{1-\gamma} \right\}^{\frac{1}{2-\gamma}},$$

where Δ_1 and χ_4 are constants. The form is therefore identical to (2.22). In fact the growth rankings given by (2.23) also remain unchanged with $\delta < 1$. See Appendix E.

[23] Equation (2.22) implies that that $\widehat{g_{z_P}}$ is maximized at $\tau = \mu$. See Appendix A.

2.2.3 The Competitive Decentralized Equilibrium

We now solve the competitive decentralized equilibrium. Consider an economy that is populated by a set of homogenous and infinitely lived agents of unit mass with the aggregate population normalized to unity. There is no population growth and the representative firms are completely owned by agents. Firms pay taxes on capital income $\tau_k \in (0,1)$ while agents pay taxes on labor income $\tau_n \in (0,1)$. Agents derive utility from consumption of the final good and leisure given in equation (2.1). The wage payment w_t for both kinds of labor are the same since there is no skill difference assumed between both activities. Agents fund consumption and investment decisions from their after tax wages which they receive for supplying labor n_1 and n_2, and capital income earned from holding assets, which essentially equals the returns to capital lent out for production at each time period t.

Importantly, we assume that the planner can tax factor incomes at different rates which may or may not be equal to τ. This is because spillovers from labor and capital affect factor accumulation differentially. This gives the planner a wider set of instruments to implement the growth rate corresponding to the socially efficient allocation. Therefore, to fund public investment I_t^g, at each time period t a distortionary tax is imposed on labor, $\tau_n \in (0,1)$, and capital, $\tau_k \in (0,1)$ respectively. The following is therefore the government budget constraint:

$$I_t^g = w_t(n_{1t} + n_{2t})\tau_n + \{Y_t - w_t(n_{1t} + n_{2t})\}\tau_k.$$

The Firm's Dynamic Profit Maximization Problem

The representative firm produces the final good based on (2.3). Hence, the production function is given by

$$Y_t = \underline{A} K_t^{\alpha} n_{1t}^{1-\alpha} \underbrace{\left(\overline{n}_{2t}^{1-\alpha}\right)^{\xi}}_{\text{Externality}}$$

where the law of motion of private capital is given by (2.4). To determine the demand for factor inputs, competitive firms solve their dynamic profit maximization problems which, at time t, have capital stock, K_t, and the level of ISTC, Z_t. The firm chooses K_{t+1}, n_{1t}, and n_{2t} optimally, taking all externalities and factor prices as given. As noted before, the firm might not be aware that n_{2t}, employed from the point of lowering the price of future capital accumulation and hence future output, also has a spillover on current production. Let $v(K_t, Z_t)$ denote the value function of the firm at time t. The returns to investment in the credit markets are given by r_t and the wage is given by w_t at time period t. The firm's value function is given by:

$$v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ \left[Y_t - w_t\left(n_{1t} + n_{2t}\right)\right](1 - \tau_k) - \frac{K_{t+1}}{Z_t} + \frac{1}{1 + r_{t+1}} v(K_{t+1}, Z_{t+1}) \right\}, \tag{2.24}$$

which it maximizes subject to (2.7).

The firm's maximization exercise yields:[24]

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1 + r_{t+1}}\right) \frac{\alpha Y_{t+1}(1 - \tau_k)}{K_{t+1}}$$

$$\{n_{1t}\} : w_t = \frac{(1 - \alpha) Y_t}{n_{1t}}$$

$$\{n_{2t}\} : w_t(1 - \tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1 + r_{t+k+1}}\right] I_{t+j+1}.$$

The Agents Problem

Since agents completely own the firms, they receive profits π_t as dividends $\forall t$. Agents are also allowed to borrow and lend at the rate r_t by participating in the credit market. The agent maximizes (2.1) subject to the consumer budget constraint[25],

$$a_{t+1} = \pi_t + (1 + r_t)a_t + w_t n_t(1 - \tau_n) - c_t, \tag{2.25}$$

and takes factor prices w_t and r_t, profits π_t, and all externalities as given.[26] Agents choose how much to consume, how much labor to supply, and their assets in period $t + 1$. Finally, the labor market clearing condition is given by

$$n_t = n_{1t} + n_{2t}.$$

[24] See Appendix B.

[25] Because there is an unit mass of agents, any aggregate variable is equal to its per-capita magnitude.

[26] Note that we are not taxing the dividends, π_t, in the consumer budget constraint, but corporate capital income, $\left[Y_t - w_t\left(n_{1t} + n_{2t}\right)\right]$, as in Huffman (2008). Strictly speaking, τ_k is therefore a corporate (profit) tax and not a tax on capital income. Taxing the firm's corporate income at source, i.e., $\left[Y_t - w_t\left(n_{1t} + n_{2t}\right)\right]$, or at the level of the household, i.e., the dividend, π_t, does not change the qualitative results of the model. These results are available from the authors on request.

First Order Conditions

The following is the Lagrangian for the agent,

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t [\log c_t + \log(1 - n_t) + \lambda_t \{\pi_t + (1 + r_t)a_t + w_t n_t(1 - \tau_n) - c_t - a_{t+1}\}]. \tag{2.26}$$

The optimization conditions with respect to c_t, a_{t+1}, and n_t, are given by equations (2.27), (2.28), and (2.29) respectively:

$$\{c_t\} : \tfrac{1}{C_t} = \lambda_t \tag{2.27}$$

$$\{a_{t+1}\} : \frac{\beta(1 + r_{t+1})}{c_{t+1}} = \frac{1}{c_t} \tag{2.28}$$

$$\{n_t\} : \frac{w_t(1 - \tau_n)}{c_t} = \frac{1}{1 - n_t} \tag{2.29}$$

Once we substitute out for factor prices into the firm's problem (equations (2.27), (2.28), and (2.29)), we obtain the following first order conditions for the competitive equilibrium:

$$\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1 - \tau_k)}{c_{t+1} K_{t+1}} \tag{2.30}$$

$$\{n_{1t}\} : \frac{1}{1 - n_t} = \frac{(1 - \alpha)Y_t(1 - \tau_n)}{c_t n_{1t}} \tag{2.31}$$

$$\{n_{2t}\} : \frac{1}{1 - n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1 - \tau_n}{1 - \tau_k}\right) \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+1}}{c_{t+j+1}}. \tag{2.32}$$

Equation (2.30) is the standard Euler equation for the household. Compared to equation (2.12) in the planner's problem, the effect of the stock-externalities because of K and G on the inter-temporal savings decision is absent. This is because agents do not internalize this externality. Equations (2.31) and (2.32) equate the after tax wage to the MRS between consumption and leisure. Compared to equations (2.14) and (2.15) respectively, the additional terms due to the externalities are also absent because the agents take the externality from n_2 as given.

Decision Rules

Based on the above first order conditions, Lemma 2.3 states the optimal decision rules for the agents.

Lemma 2.3 *C_t, $I_t, n_t, n_{1t,}n_{2t}$ are given by (2.33), (2.34), (2.35), where $0 < \Phi_{CE} < 1$ is given by (2.36), and $0 < x_{CE} < 1$ given by (2.37) is a constant.*

$$C_t = \Phi_{CE} A Y_t, I_t = (1 - \Phi_{CE}) A Y_t \tag{2.33}$$

$$\textit{where, } A = \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha\beta^2\theta(\tau_n - \tau_k)}{(1 - \beta\gamma)}$$

$$n_t = n_{CE} = \frac{(1 - \alpha)(1 - \tau_n)}{(1 - \alpha)(1 - \tau_n) + x_{CE}\Phi_{CE}A}, \tag{2.34}$$

$$n_{1CE} = x_{CE} n_{CE}, n_{2CE} = (1 - x_{CE}) n_{CE}, \tag{2.35}$$

where Φ_{CE} is given by

$$\Phi_{CE} = 1 - \frac{\alpha\beta(1 - \tau_k)}{A}, \tag{2.36}$$

and x_{CE} is given by

$$x_{CE} = \frac{(1 - \alpha)(1 - \beta\gamma)}{\alpha\beta^2\theta + (1 - \alpha)(1 - \beta\gamma)}. \tag{2.37}$$

Proof. See Appendix B for details. ■

The above decision rules imply that depending upon the parameter values, there exists a feasible range of values that τ_k and τ_n can take such that

$$0 < A, \Phi_{CE}, n_{CE} < 1,$$

are true.[27] The relationship between growth rates at the balanced growth path for private capital, public capital, output and investment specific technological change are identical to that for the planner's version, as given in Lemma 2.2.

The Competitive Equilibrium Growth Rate

We would like to ascertain under what conditions the growth rate corresponding to the competitive equilibrium allocation implement the growth corresponding to the socially efficient allocation. From equations (2.33), (2.34), (2.35), (2.36), and (2.37), the growth rate under the competitive equilibrium is given by:

$$g_{z_{CE}} = \left[B \underbrace{n_{2CE}^{\theta}}_{\text{Labor factor}} \underbrace{\left\{ (1 - A)^{\mu} (A)^{1-\mu} (1 - \Phi_{CE})^{1-\mu} \right\}^{1-\gamma}}_{\text{Capital factor}} \right]^{\frac{1}{2-\gamma}}. \tag{2.38}$$

[27] Restriction (2.53) in Appendix B is required on τ_n and τ_k for $0 < A, \Phi_{CE}, n_{CE} < 1$.

The growth rate, $g_{z_{CE}}$, depends on two factors: a labor factor, n_{2CE}^{θ}, and a capital factor given by $\Upsilon = \left\{(1-A)^{\mu}(A)^{1-\mu}(1-\Phi_{CE})^{1-\mu}\right\}^{1-\gamma}$, both of which depend on factor income taxes, τ_k and τ_n.

The capital factor In Appendix C we show that

$$\Upsilon = \left\{\left[\frac{(1-\beta\gamma)\left[(1-\alpha)(\tau_n-\tau_k)+\tau_k\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma}\right]^{\mu}\left[\alpha\beta(1-\tau_k)\right]^{1-\mu}\right\}^{1-\gamma}, \quad (2.39)$$

i.e., the capital factor,Υ, unambiguously increases in τ_n and the tax gap $(\tau_n-\tau_k)$. We also show that Υ also decreases in τ_k as long as the following sufficient condition is satisfied:

$$1-\beta\gamma < \beta^2\theta. \quad (2.40)$$

Importantly, when $\tau_k = 1, \Upsilon = 0$, and there is no growth.[28]

The labor factor The research labor input n_{2CE} is given by

$$n_{2CE} = (1-x_{CE})n_{CE}, \quad (2.41)$$

where

$$\begin{aligned}(1-x_{CE}) &= \frac{\alpha\beta^2\theta}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)},\\ n_{CE} &= \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A}.\end{aligned}$$

Clearly, $(1-x_{CE})$ is independent on factor income taxes. Hence, a change in taxes therefore affects n_{2CE} only through n_{CE}. In Appendix C, we show that

$$n_{CE} = \frac{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]}{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]+\Psi}, \quad (2.42)$$

where

$$\Psi = \frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\left\{\alpha(1-\beta)+(1-\alpha)+\alpha(1-\beta)(\tau_n-\tau_k)-(1-\alpha\beta)\tau_n\right\}-\alpha\beta^2\theta(\tau_n-\tau_k)\right].$$

[28] Equation (2.40) can be re-written as, $\beta\theta+\gamma > \frac{1}{\beta}$, which implies that if the returns from allocating resources to ISTC are greater than the returns from investing in an asset (which equals $\frac{1}{\beta}$ in the steady state), an increase in the tax on capital income will depress the capital factor.

As shown in Appendix C, if condition (2.40) holds, Ψ decreases in the tax gap $(\tau_n - \tau_k)$ and τ_n, and increases in τ_k. As a result, n_{CE} increases in $(\tau_n - \tau_k)$ and τ_n, and decreases in τ_k. The effect of a change in the factor income tax gap $(\tau_n - \tau_k)$ and τ_n on labor supply, and therefore the labor factor, can be summarized by Lemma 2.4.

Lemma 2.4 *Suppose*

$$1 - \beta\gamma < \beta^2\theta.$$

Then, (i) An increase in τ_k lowers the capital factor, i.e., $\frac{\partial \Upsilon}{\partial \tau_k} < 0$. (ii) A rise in the labor income tax rate, τ_n, and the factor income tax gap, $(\tau_n - \tau_k)$, increases the labor factor, i.e., $\frac{\partial n_{CE}}{\partial(\tau_n - \tau_k)} > 0$, $\frac{\partial n_{CE}}{\partial \tau_n} > 0$, and $\frac{\partial n_{CE}}{\partial \tau_k} < 0 \Longrightarrow \frac{\partial n^{\theta}_{2CE}}{\partial(\tau_n - \tau_k)} > 0$ and $\frac{\partial n^{\theta}_{2CE}}{\partial \tau_n} > 0$.

Proof. See Appendix C. ■

Lemma 2.4 implies that a smaller γ makes n_{CE} increase by more for an increase in τ_n. Proposition 2.1 summarizes the effect of tax rates on the competitive equilibrium growth rate.

Proposition 2.1 *Since the labor factor and capital factor are increasing in τ_n and decreasing in τ_k, the competitive equilibrium growth rate, $g_{z_{CE}}$, is increasing in the factor income tax gap, $(\tau_n - \tau_k)$. An increase in $g_{z_{CE}}$, is obtained by increasing $(\tau_n - \tau_k)$. The factor income tax gap must be increased by either raising τ_n, or lowering τ_k, or both.*

Proof. Follows from $\frac{\partial \Upsilon}{\partial \tau_n} > 0$, $\frac{\partial \Upsilon}{\partial(\tau_n - \tau_k)} > 0$, and Lemma 2.4. ■

The intuition behind the above proposition is as follows. Assume that the sufficient condition, (2.40), holds, because of a high value of θ.[29] Since the competitive equilibrium growth rate $g_{z_{CE}}$ increases in the factor income tax gap $(\tau_n - \tau_k)$, an increase in τ_k requires a higher τ_n to decentralize the same growth rate g_{z_P}. This suggests that fiscal policy has an offsetting effect on the agent's growth rate. A higher τ_k lowers the capital factor Υ. To mitigate the negative effect of τ_k on Υ, we have to raise τ_n which not only has a positive effect on the labor factor n^{θ}_{2CE}, but also on Υ. This happens because although the substitution effect induces an increase in leisure,$1 - n_{CE}$, due to an increase in τ_n (the after tax wage has gone down), labor supply (and therefore the labor factor) increases because of the strong(er) income effect induced by ISTC. In particular, ISTC leads to an additional income effect, through consumption, compared to a case where ISTC is not endogenous. This can be seen from the below equation for, $\Phi_{CE}A$,

$$\Phi_{CE}A = \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) - \frac{\alpha\beta^2\theta(\tau_n - \tau_k)}{(1 - \beta\gamma)} - \alpha\beta\,(1 - \tau_k)\,.$$

[29] We can implement the planner's allocations even if equation (2.40) is violated. However we assume this to be our main case because it is satisfied with reasonable parameter values. In the numerical section, we explore both possibilities.

When $\theta > 0$, an increase in τ_n lowers after-tax labor income and lowers consumption even more. Relative to the case where there is no endogenous ISTC, the after tax fraction of income allocated for private consumption, $\Phi_{CE}A$, is lowered by the term, $\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}$. The drop in consumption causes leisure to fall more (relative to case when $\theta = 0$) and labor supply to increase by more (which follows from equation (2.29), where $c_t = w_t(1-\tau_n)(1-n_{CE})$). An increase in n_{CE} in turn implies a higher n_{2CE}, from equation (2.41) and noting that $1 - x_{CE}$ is also increasing in θ. Hence the labor factor rises. A rise in the labor factor increases Z_{t+1} which increases capital accumulation and therefore future output and future consumption. Without ISTC, it could be possible that labor supply falls if the substitution effect dominates the income effect. However with ISTC, the income effect dominates the substitution effect and labor supply, n_{CE}, rises.

Fiscal policy also offsets the effect of taxes because public capital crowds out private capital in our model. This is because, from (2.39) we know that $(1 - A)$ increases in τ_k whereas, $A(1 - \Phi_{CE})$ decreases. Proposition 2.1 therefore suggest that we can raise $g_{z_{CE}}$ to match the socially efficient growth rate by increasing the factor income tax gap $(\tau_n - \tau_k)$ from an initial point where $g_{z_{CE}} < g_{z_P}$. Further, since ISTC in our model is endogenous, a higher θ causes a bigger increase in n_{CE} and therefore n_{2CE}. This translates into a bigger increase in $g_{z_{CE}}$ for a given increase in τ_n. In terms of the capital factor, since the representative agent under-accumulates private capital because of taking the effect of Υ on Z as given, τ_k must be lowered. As a result, an increase in the tax gap by raising τ_n and lowering τ_k increases $g_{z_{CE}}$.

In sum, as to which effect dominates depends on the sufficient condition, (2.40), identified in Proposition 2.1. For instance, the sufficient condition, (2.40) is also satisfied for higher values of γ, which in turn strengthens the income effect channel because of ISTC on labor supply, for an increase in τ_n. A higher γ also means that the weight on the capital stock externalities is weaker. As a result, the net effect is that a high γ and a high θ makes the labor factor increase for an increase in τ_n. Since condition (2.40), which is satisfied for a high γ and θ, causes the capital factor to fall when τ_k increases, the planner's growth rate is decentralized using a combination of a high τ_n and a low τ_k.

The Effect of γ and ξ Given the sufficient condition, (2.40), we graphically characterize the implementation of the socially efficient growth rate, g_{z_P} to illustrate the effect of a change in the externality parameters on the factor income tax gap required to decentralize the planner's equilibrium growth rate. First, as ξ increases, the spillover from n_2 in final goods production increases. The planner therefore allocates more labor towards n_2, which increases the socially efficient growth rate, g_{z_P}. This is shown in Figure 2.4, where we assume

$\tau_k = \overline{\tau}_k$, which yields a zero factor income tax gap. Starting with $\xi = 0$, the factor income tax gap required to decentralize g_{z_P} corresponds to point 'a'. Now suppose ξ increases arbitrarily. Since the agent's allocations do not depend, on ξ, the competitive equilibrium growth rate $g_{z_{CE}}$ does not change. We know from Proposition 2.1 that in order to match a higher g_{z_P}, the labor income tax must be increased for a given $\overline{\tau}_k$, which causes an increase in the factor income tax gap. The new factor income tax gap corresponds to point 'b'.

[Insert Figure 2.4]

Now suppose γ is arbitrarily increased from a low to a high value. The spillover from the capital factor for a higher γ is low. This makes ISTC more persistent, which increases g_{z_P}. At the same time, the growth rate of the agent also increases because the weight on the externality from the capital factor is lower for a higher γ. This reduces the extent of under-accumulation of capital. As a result, the equilibrium factor income tax gap $(\tau_n - \tau_k)$ decreases. This is illustrated in Figure 2.5. Point 'a' corresponds to $\gamma = 0.5$ and point 'b' corresponds to $\gamma = 0.8$. The crucial difference is that both γ and ξ raise the planner's growth rate, whereas only γ raises the competitive equilibrium growth rate.

[Insert Figure 2.5]

2.3 Numerical Examples

In this section, we consider a few numerical examples to show how different factor income tax combinations may implement the growth rate corresponding to the socially efficient allocation. We also analyze how the magnitude of externalities (γ, ξ) affect the factor income tax gap. To do this, we consider a benchmark value for the socially efficient growth rate, g_{z_P}, calculated at $\tau = \mu$.[30] In particular, we consider two examples: one where the sufficient condition given by equation (2.40) holds and another where the condition is violated. Our main result is to numerically show that for a fixed set of parameters a wide range of tax rates implement the same growth rate.

We first calibrate out factor income tax gaps that are broadly consistent with Figures 2.1 - 2.3. We start with two arbitrary values of $\gamma = \{0.1, 0.9\}$ corresponding to the case where

[30] Note from equation (2.22), $\tau = \mu$ also maximizes the socially efficient growth rate, g_{z_P}. Therefore this is a useful benchmark growth rate to be implemented by the competitive decentralized equilibrium using different factor income tax combinations. There is a large literature on political economy and institutional motives for designing fiscal policy. In this literature, the policy setter is assumed to set fiscal policy to maximize the efficient growth rate to maintain constituent support (see Key (1966), Tufte (1978), Fiorina (1981), Kiewiet and Rivers (1985), Lewis-Beck (1990), Harrington (1993), Ghate (2003)).

the externality from the stock externalities are high and low, respectively. Then, starting with $\xi = 0$, we gradually raise ξ to make it arbitrarily large, and calibrate out the factor income tax gap, $(\tau_n - \tau_k)$, for each change in ξ. In all the numerical experiments we fix $\alpha = 0.35$ and $\beta = 0.95$ as in Huffman (2008).

Suppose we set $\gamma = 0.9$.[31] Other parameters are arbitrarily chosen as: $\mu = 0.5, \theta = 0.8$, and $B = 1.46$ which yields a growth rate of 2.5% as in Figure 2.1. This set of parameters satisfy condition (2.40). Table 1 summarizes the values of τ_n for each value of τ_k such that $g_{z_{CE}} = g_{z_P}$ across different values of $\xi = \{0, 0.1, 0.2\}$ and range $\tau_k = \{0.1, 0.2, 0.3, 0.4\}$.

Two observations emerge. First, as can be seen from the second column of Table 1, with a fixed set of parameters (and assuming $\xi = 0$) a wide range of tax rates implement the same growth rate. For instance, when $\xi = 0$, $\{\tau_k = 0.1, \tau_n = 0.335\}$ yields the same growth rate of 2.5% as $\{\tau_k = 0.2, \tau_n = 0.415\}$. This holds for columns 3 and 4 as well where the cases of $\xi = 0.1$ and $\xi = 0.2$, are considered respectively.

Second, as ξ increases, the equilibrium factor income tax gap needed to decentralize the planners growth increases as in Figure 2.4. This is because, an increase in ξ increases the spillover from n_2 in final goods production. The planner therefore allocates more labor towards n_2. This increases g_{z_P}. To match a higher g_{z_P}, the labor income tax must be increased for a given τ_k, which causes an increase in the factor income tax gap. This requires $\tau_n > \tau_k$ to decentralize g_{z_P}.

τ_k	$\tau_n - \tau_k$ $(\xi = 0)$	$\tau_n - \tau_k$ $(\xi = 0.1)$	$\tau_n - \tau_k$ $(\xi = 0.2)$
0.1	0.235	0.24	0.24
0.2	0.215	0.22	0.22
0.3	0.19	0.19	0.19
0.4	0.165	0.17	0.17

Table 1: Equilibrium factor income tax gaps under $\gamma = 0.9$

When γ is high, the spillover from the capital factor is low. This also makes ISTC more persistent. This increases the growth rate of the planner. To raise the competitive equilibrium growth rate, a reduction in the tax on capital income raises the capital factor and an increase in the labor income tax raises the labor factor. At the same time, since the effect of the externality from the capital factor is low, and the effect of public capital is low, $(\tau_n - \tau_k)$ is narrower.[32]

Suppose now $\gamma = 0.1$. Other parameters are arbitrarily chosen to be: $\mu = 0.9, \theta = 0.01$,

[31] We have chosen the parameters such that n_2 has a large weight on Z and the externality from public and private capital on Z has a small weightage. In addition, the effect of public capital to output ratio on Z is moderate.

[32] We show in Appendix D that when there are no externalities, *equal* factor income taxes always yield the optimal growth rate from the planner's problem. Hence, the factor income tax gap is zero.

and $B = 1.74$ which yields a growth rate of 2.5% which is roughly equal to the average growth rate for our sample of OECD countries in Figure 2.1.[33] This set of parameters violates condition (2.40).

Table 2 summarizes the values of τ_n for each value of τ_k.such that $g_{z_{CE}} = g_{z_P}$ across different values of $\xi = \{0, 0.1, 0.2\}$,and different values of $\tau_k = \{0.3, 0.5, 0.7, 0.9\}$. Observe that not only are the individual factor income tax combinations higher than in Table 1, for lower τ_n, the tax gaps $(\tau_n - \tau_k)$ are also higher. The tax gaps also become negative, i.e., $\tau_k > \tau_n$, for higher values of τ_n.

τ_k	$\tau_n - \tau_k$ $(\xi = 0)$	$\tau_n - \tau_k$ $(\xi = 0.1)$	$\tau_n - \tau_k$ $(\xi = 0.2)$
0.3	0.58	0.59	0.59
0.5	0.31	0.33	0.34
0.7	0.07	0.08	0.09
0.9	−0.09	−0.07	−0.06

Table 2: Equilibrium factor income tax gaps under $\gamma = 0.1$

First, similar to Table 1, the factor income tax gap in each column corresponds to a fixed set of parameter values. As can be seen from column 2, for $\xi = 0$, both $\{\tau_k = 0.3, \tau_n = 0.88\}$ and $\{\tau_k = 0.9, \tau_n = 0.81\}$ implement a 2.5% growth rate. In other words, for a fixed set of externality and non-externality deep parameters a reversal in the factor income tax ranking implies the same growth rate. From columns 3 and 4 we again observe that for an increase in ξ, there is a marginal increase in the tax gap $(\tau_n - \tau_k)$.

Second, as τ_k increases, the value of τ_n that decentralizes the planner's growth rate for the given value of τ_k also increases. We also observe that as τ_k increases, the tax gap $(\tau_n - \tau_k)$ starts narrowing. For very high values of τ_k the corresponding value of τ_n could be smaller, such that the rankings get reversed and $\tau_n - \tau_k$ becomes negative. This is because the condition given by equation (2.40) is now violated**.** The intuition is as follows. For a low value of θ, the income effect channel because of ISTC on labor supply is weakened, for an increase in τ_n. Therefore, an increase in τ_n on the net, may not increase the labor factor. In addition, a low value of γ also means that the weight on the capital stock externalities is stronger. Since the capital stock externalities consist of public and private capital, a higher τ_k may not have offsetting effects on the labor and capital factor, as in the previous case where the sufficient condition (2.40) is satisfied. As a result, a high τ_k and a low τ_n may implement g_{z_P}. This is consistent with Figure 2.2 where we generally observe high τ_k economies also have a lower τ_n (e.g., US, UK, Japan, and Denmark). Thus Table 1 is able to qualitatively match the factor income tax gaps in these economies even though the calibrated

[33] Our choice of parameters are now such that n_2 has a small weightage on Z while the externality from public and private capital on Z has a high weightage. In addition, the effect of public capital to output ratio on Z is very high while that of private capital to output ratio is very small.

factor income tax gaps are smaller in magnitude in this experiment.

The numerical results above identify why the externalities are crucial for our results. While our model yields equilibrium factor income tax gaps that implement g_{z_P} under a fixed set of parameters, i.e., indeterminacy, we also show that a change in the magnitude of the externalities widen/narrows the equilibrium factor income tax gaps required to implement the planner's growth rate. These results are consistent with the growth-tax puzzle identified in Figures 2.1 - 2.3.

2.4 Conclusion

This paper constructs a simple and tractable endogenous growth model with endogenous investment specific technological change. Our theoretical model is motivated by the empirical observation that advanced economies – which are presumed to be on their balanced growth paths and therefore experience similar or identical growth rates – have widely varying factor income tax combinations. This observation is puzzling since it is incompatible with a standard model of endogenous growth: in the standard model, fiscal policy can have large growth effects through its impact on the economy's investment rate. We see our contribution as providing an alterative, but compatible, explanation based on the fact that different combinations of taxes can generate the same growth rate. Our innovation is to incorporate aggregate public and private capital stock externalities in ISTC, as well as positive spillovers driven by specialized labor in the research sector to explain this puzzle.

We characterize the balanced growth path of the economy corresponding to the socially efficient allocation for a fixed tax rate and derive conditions under which the competitive equilibrium can implement this growth rate. Our general result is that to the extent that spillovers from a specialized labor input and the public and private capital stocks exist, an increase in these spillover from specialized labor, and a decrease in the spillover from public and private capital, increases the growth rate corresponding to the socially efficient allocation, and therefore increases the factor income tax gap required to implement the higher planner's growth rate. Conversely, for a given level of externalities, maintaining the constancy of growth also requires different combinations of factor income taxes. Finally, when there are no externalities, *equal* factor income taxes always yield the socially efficient growth rate. Hence, the factor income tax gap is zero. In the numerical section, we show that we can qualitatively match the factor income tax gaps observed in the data.

In the future, we hope to extend our framework by comparing the growth and welfare effects of optimal tax policy on research and development versus funding public investment. In addition, our model characterizes the optimal tax rate along the balanced growth path.

Future work can model the transitional dynamics.

2.5 Appendix

Appendix A: Planner's problem

The following first order conditions are therefore obtained with respect to C_t, K_{t+1}, Z_{t+1}, n_{1t}, and n_{2t}:

$$\{C_t\} : \tfrac{1}{C_t} = \lambda_{1t}$$

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}} - \beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}}$$

$$\{Z_{t+1}\} : \lambda_{2t} = \beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}} + \frac{\beta}{Z_{t+1}}\left(\frac{I_{t+1}}{C_{t+1}}\right) + \beta^2\lambda_{2t+2}\mu(1-\gamma)\frac{Z_{t+3}}{Z_{t+1}}$$

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}}$$

and,

$$\{n_{2t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)\xi Y_t(1-\tau)}{C_t n_{2t}} + \lambda_{2t}\theta\frac{Z_{t+1}}{n_{2t}} - \beta\lambda_{2t+1}(1-\gamma)\xi(1-\alpha)\frac{Z_{t+2}}{n_{2t}}.$$

We will use the method of undetermined coefficients. We have assumed,

$$C_t = \Phi_P Y_t(1-\tau),\; I_t = (1-\Phi_P)Y_t(1-\tau),\; I_t^g = \tau Y_t$$

and

$$n_1 = xn, n_2 = (1-x)n.$$

From $\{Z_{t+1}\}$,

$$\{Z_{t+1}\} : Z_{t+1}\lambda_{2t} = \beta\lambda_{2t+1}\gamma Z_{t+2} + \beta^2\lambda_{2t+2}\mu(1-\gamma)\frac{Z_{t+3}}{Z_{t+1}} + \beta\left(\frac{1-\Phi_P}{\Phi_P}\right).$$

From $\{n_{1t}\}$,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}},$$

which implies

$$\frac{x_P n_P}{1-n_P} = \frac{(1-\alpha)}{\Phi_P} - \beta(1-\gamma)(1-\alpha)\lambda_{2t+1}Z_{t+2}.$$

Therefore,

$$\lambda_{2t+1}Z_{t+2}=\frac{\frac{(1-\alpha)}{\Phi_P}-\frac{x_Pn_P}{1-n_P}}{\beta(1-\gamma)\left(1-\alpha\right)}.$$

This also implies for constant decision rules and a constant labor supply in every time period,

$$\lambda_{2i-1}Z_i=\frac{\frac{(1-\alpha)}{\Phi_P}-\frac{x_Pn_P}{1-n_P}}{\beta(1-\gamma)\left(1-\alpha\right)},\quad \text{for all } i=t.$$

Substituting in $\{Z_{t+1}\}$,

$$\frac{\left[\frac{(1-\alpha)}{\Phi_P}-\frac{x_Pn_P}{1-n_P}\right]\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]}{\beta(1-\gamma)\left(1-\alpha\right)}=\beta\left(\frac{1-\Phi_P}{\Phi_P}\right).$$

This on rearranging gives

$$\frac{n_P}{1-n_P}=\frac{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]}.$$

Hence,

$$n_P=\frac{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]}{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]+x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]}.$$

Using

$$\frac{n_P}{1-n_P}=\frac{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]},$$

we get

$$\lambda_{2i-1}Z_i=\left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)}\right).$$

From $\{n_{2t}\}$

$$\{n_{2t}\}:\frac{\left(1-x_P\right)n_P}{1-n_P}=\frac{(1-\alpha)\xi}{\Phi_P}+\theta\lambda_{2t}Z_{t+1}-\beta(1-\gamma)\xi\left(1-\alpha\right)\lambda_{2t+1}Z_{t+2}.$$

This implies

$$\frac{\left(1-x_P\right)n_P}{1-n_P}=\frac{(1-\alpha)\xi}{\Phi_P}+\left[\theta-\beta(1-\gamma)\xi\left(1-\alpha\right)\right]\left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)}\right).$$

Since

$$\frac{n_P}{1-n_P}=\frac{\left(1-\alpha\right)\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)-\beta^2\left(1-\gamma\right)\left(1-\Phi_P\right)\right]}{x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu\left(1-\gamma\right)\right]},$$

we get

$$\begin{aligned}
&\left(\frac{1-x_P}{x_P}\right)\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)-\beta^2\,(1-\gamma)\,(1-\Phi_P)\right]}{\Phi_P\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)\right]} \\
= &\quad \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)\right]\xi+\beta\left[\theta-\beta(1-\gamma)\xi\,(1-\alpha)\right](1-\Phi_P)}{\Phi_P\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)\right]} \\
= &\quad \frac{(1-\alpha)\xi\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)-\beta^2(1-\gamma)\,(1-\Phi_P)\right]+\beta\theta\,(1-\Phi_P)}{\Phi_P\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)\right]}.
\end{aligned}$$

Hence,

$$x_P=\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)-\beta^2\,(1-\gamma)\,(1-\Phi_P)\right]}{(1-\alpha)\,(1+\xi)\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)-\beta^2(1-\gamma)\,(1-\Phi_P)\right]+\beta\theta\,(1-\Phi_P)}.$$

Finally, from $\{K_{t+1}\}$,

$$\{K_{t+1}\}:\frac{1}{C_tZ_t}=\frac{\alpha\beta Y_{t+1}\,(1-\tau)}{C_{t+1}K_{t+1}}+\beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}}-\beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}}$$

$$\begin{aligned}
\frac{1}{\Phi_PY_tZ_t} \quad = &\quad \frac{\alpha\beta}{\Phi_P\,(1-\Phi_P)\,Y_tZ_t}+\frac{\beta\,(1-\gamma)\,(1-\mu)}{(1-\Phi_P)\,Y_tZ_t}\left(\frac{1-\Phi_P}{\Phi_P}\right) \\
&+\frac{\beta^2\,(1-\gamma)\,(1-\mu)\,\gamma}{(1-\Phi_P)\,Y_tZ_t}\lambda_{2t+2}Z_{t+3}-\frac{\beta^2\alpha\,(1-\gamma)}{(1-\Phi_P)\,Y_tZ_t}\lambda_{2t+2}Z_{t+3}.
\end{aligned}$$

Since

$$\lambda_{2i-1}Z_i=\left(\frac{1-\Phi_P}{\Phi_P}\right)\left(\frac{\beta}{1-\beta\gamma-\beta^2\mu\,(1-\gamma)}\right),$$

we get

$$1=\frac{\alpha\beta}{(1-\Phi_P)}+\beta\,(1-\gamma)\,(1-\mu)-\frac{\beta^3\,(1-\gamma)\,\alpha}{\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)\right]}.$$

On simplifying we get

$$1-\Phi_P=\frac{\alpha\beta\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)\right]}{(1-\beta\gamma)-\beta^2(1-\gamma)+\alpha\beta^3\,(1-\gamma)}.$$

Conditions

As long as $(1-\Phi_P)<1$, we will get

$$0<x_P<1.$$

We know,

$$(1-\Phi_P) = \frac{\alpha\beta\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)\right]}{(1-\beta\gamma)-\beta^2(1-\gamma)+\alpha\beta^3\,(1-\gamma)}$$

Since,

$$0<(1-\beta\gamma)-\beta^2(1-\gamma)=(1-\beta)\left[1+\beta(1-\gamma)\right],$$

$$(1-\Phi_P)>0.$$

To show

$$(1-\Phi_P) = \frac{\alpha\beta\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)\right]}{(1-\beta\gamma)-\beta^2(1-\gamma)+\alpha\beta^3\,(1-\gamma)}<1,$$

we require,

$$\begin{aligned} & (1-\beta\gamma)-\beta^2(1-\gamma)+\alpha\beta^3\,(1-\gamma) \\ > \; & \alpha\beta\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)\right], \end{aligned}$$

or,

$$(1-\beta\gamma)\,(1-\alpha\beta)-\beta^2(1-\gamma)+\alpha\beta^3\,(1-\gamma)+\alpha\beta^3\mu\,(1-\gamma)>0.$$

Rewriting the above LHS we get

$$(1-\beta\gamma)\,(1-\alpha\beta)-\beta^2(1-\gamma)\left[1-\alpha\beta\,(1+\mu)\right].$$

Since,

$$(1-\beta\gamma)>\beta^2(1-\gamma)$$

and

$$1-\alpha\beta>1-\alpha\beta\,(1+\mu)\,,$$

therefore

$$(1-\Phi_P)\in(0,1)\,.$$

Since,

$$x_P=\frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu\,(1-\gamma)-\beta^2\,(1-\gamma)\,(1-\Phi_P)\right]}{(1-\alpha)\,(1+\xi)\left[(1-\beta\gamma)-\beta^2\mu\,(1-\gamma)-\beta^2(1-\gamma)\,(1-\Phi_P)\right]+\beta\theta\,(1-\Phi_P)}$$

Therefore

$$0<x_P,\Phi_P<1.$$

Finally, since

$$n_P = \frac{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]}{(1-\alpha)\left[1-\beta\gamma-\beta^2\mu(1-\gamma)-\beta^2(1-\gamma)(1-\Phi_P)\right]+x_P\Phi_P\left[1-\beta\gamma-\beta^2\mu(1-\gamma)\right]}$$

and,

$$0 < x_P, \Phi_P < 1,$$

therefore,

$$0 < n_P < 1.$$

Growth rate at the BGP

$$Y_t = \underline{A}.\left(n_{2t}^{1-\alpha}\right)^{\xi} K_t^{\alpha} n_{1t}^{1-\alpha}$$

On the balanced growth path (BGP),

$$g_{y_P} = g_{y_{Pt+1}} = \frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}^{\alpha}}{K_t^{\alpha}} = g_{k_{Pt+1}}^{\alpha} = g_{k_P}^{\alpha},$$

$$\text{and } g_{k_P} = \frac{K_{t+1}}{K_t} = \frac{I_t Z_t}{I_{t-1}Z_{t-1}} = g_{y_P}.g_{z_P}.$$

Hence,

$$g_{y_P} = g_{z_P}^{\frac{\alpha}{1-\alpha}}, g_{k_P} = g_{g_P} = g_{z_P}^{\frac{1}{1-\alpha}}.$$

Comparative statics of the growth rate with respect to τ

The growth rate, $\widehat{g_{z_P}}$ is maximized at $\tau = \mu$. To see this, we first take logs, such that

$$\ln \widehat{g_{z_P}} = \frac{1}{2-\gamma}\left[\ln \widehat{M_P} + (1-\gamma)\mu \ln \tau + (1-\gamma)(1-\mu)\ln(1-\tau)\right].$$

Since $\widehat{M_P}$ is independent of τ, at the point of maximum,

$$\begin{aligned}
\frac{\partial \ln \widehat{g_{z_P}}}{\partial \tau} &= \frac{(1-\gamma)\mu}{(2-\gamma)}\frac{\partial \ln \tau}{\partial \tau} + \frac{(1-\gamma)(1-\mu)}{(2-\gamma)}\frac{\partial \ln(1-\tau)}{\partial \tau} = 0 \\
&\Longrightarrow \frac{(1-\gamma)\mu}{(2-\gamma)\tau} - \frac{(1-\gamma)(1-\mu)}{(2-\gamma)(1-\tau)} = 0 \\
&\Longrightarrow \frac{1-\tau}{\tau} = \frac{(1-\mu)}{\mu}, \\
&\Longrightarrow \tau = \mu.
\end{aligned}$$

Therefore, $\widehat{g_{zP}}$ is maximized at $\tau = \mu$. The second order condition is also negative, as follows:

$$
\begin{aligned}
& \frac{(1-\gamma)\,\mu}{(2-\gamma)} \frac{\partial\left(\frac{1}{\tau}\right)}{\partial \tau} - \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)} \frac{\partial\left(\frac{1}{1-\tau}\right)}{\partial \tau} \\
= \quad & -\frac{(1-\gamma)\,\mu}{(2-\gamma)} \left(\frac{1}{\tau^2}\right) - \frac{(1-\gamma)\,(1-\mu)}{(2-\gamma)} \left(\frac{1}{1-\tau}\right)^2 < 0.
\end{aligned}
$$

Appendix B: Competitive decentralized equilibrium

We assume $\delta = 1$. The FOCs are:

$$\{K_{t+1}\} : \frac{-1}{Z_t} + \left(\frac{1}{1+r_{t+1}}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}} = 0.$$

$$\Rightarrow \{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r_{t+1}}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}. \tag{2.43}$$

$$\{n_{1t}\} : \frac{(1-\alpha)Y_t(1-\tau_k)}{n_{1t}} - w_t(1-\tau_k) = 0$$

$$\Rightarrow \{n_{1t}\} : w_t = \frac{(1-\alpha)Y_t}{n_{1t}}. \tag{2.44}$$

Finally,

$$\{n_{2t}\} : w_t(1-\tau_k) = \left(\frac{\theta}{n_{2t}}\right) \sum_{j=0}^{\infty} \gamma^j \left[\prod_{k=0}^{j} \frac{1}{1+r_{t+k+1}}\right] I_{t+j+1}. \tag{2.45}$$

The Consumer's Problem

$$
\begin{aligned}
\{c_t\} \quad &: \quad \frac{1}{c_t} = \lambda_t, \\
\{a_{t+1}\} \quad &: \quad \frac{\beta(1+r_{t+1})}{c_{t+1}} = \frac{1}{c_t} \\
\{n_t\} \quad &: \quad \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t}
\end{aligned}
$$

From the firm's FOC $\{K_{t+1}\}$:

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r_{t+1}}\right) \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}.$$

Substituting for $(1+r_{t+1})$ from $\{a_{t+1}\}$

$$\Rightarrow \quad \frac{1}{Z_t} = \frac{\beta c_t}{c_{t+1}}\left[\frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}}\right]$$
$$\Rightarrow \quad \{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}}$$

Similarly,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}}$$

and,

$$\{n_{2t}\} : \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.$$

To summarize all FOCs,

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} \\
\{n_{1t}\} &: \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{c_t n_{1t}} \\
\{n_{2t}\} &: \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

When

$$\tau_k = \tau_k = \tau,$$

we have

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau)}{c_{t+1}K_{t+1}} \\
\{n_{1t}\} &: \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau)}{n_{1t}} \\
\{n_{2t}\} &: \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

The Decision Rules

We use the method of undetermined coefficients to obtain the decision rules

$$\begin{aligned}
C_t &= \Phi_{CE} A Y_t, \\
I_t &= (1-\Phi_{CE}) A Y_t \\
n_{1t} &= x_{CE} n_{CE} \\
n_{2t} &= (1-x_{CE}) n_{CE} \\
n_t &= n_{CE},
\end{aligned}$$

where,

$$\{Y_t - w_t(n_{1t}+n_{2t})\}(1-\tau_k) + w_t(n_{1t}+n_{2t})(1-\tau_n) = AY_t.$$

$$\Rightarrow [\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t + w_t n_{2t}(\tau_k-\tau_n) = AY_t$$

$$\Rightarrow [\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)]Y_t + \left\{\frac{\beta\theta A Y_t\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}\right\}(\tau_k-\tau_n) = AY_t$$

$$\Rightarrow \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n) + \frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n) = A$$

$$\Rightarrow Y_t\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n) + \frac{\beta\theta A\,(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right] = AY_t,$$

$$\Rightarrow A = \left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n) + \frac{\beta\theta\,(1-\Phi)\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right]. \tag{2.46}$$

From the FOC of $\{K_{t+1}\}$

$$\{K_{t+1}\} : \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}}$$

This implies,

$$\frac{1}{\Phi_{CE} A Y_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{\Phi A Y_{t+1}(1-\Phi_{CE}) A Y_t Z_t}$$

$$\Rightarrow (1-\Phi_{CE}) = \frac{\alpha\beta(1-\tau_k)}{A}. \tag{2.47}$$

Substituting for $(1-\Phi_{CE})A$ from 2.47 into 2.46,

$$\Rightarrow \quad A = \left[\alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) + \frac{\beta\theta\,(1-\Phi_{CE})\,A}{(1-\tau_k)(1-\beta\gamma)}(\tau_k-\tau_n)\right] \tag{2.48}$$
$$= \quad \alpha(1-\tau_k) + (1-\alpha)(1-\tau_n) - \frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}.$$

When $\tau_n = \tau_k = \tau$

$$\begin{aligned} A &= [\alpha(1-\tau) + (1-\alpha)(1-\tau)] \\ &= (1-\tau). \end{aligned}$$

From $\{n_{1t}\}$ we get

$$\begin{aligned} \{n_{1t}\} \quad : \quad \frac{x_{CE}n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)Y_t(1-\tau_n)}{\Phi_{CE}AY_t} \\ \Rightarrow \quad \frac{x_{CE}n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A} \\ \Rightarrow \quad \frac{n_{CE}}{1-n_{CE}} &= \frac{(1-\alpha)(1-\tau_n)}{x_{CE}\Phi_{CE}A} \end{aligned}$$

$$\Rightarrow n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n) + x_{CE}\Phi_{CE}A}. \tag{2.49}$$

From $\{n_{2t}\}$

$$\{n_{2t}\} : \frac{(1-x)n_{CE}}{1-n_{CE}} = \frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}}$$

$$\begin{aligned} \Rightarrow \quad \frac{(1-\alpha)(1-\tau_n)}{\Phi_{CE}A}\frac{(1-x_{CE})}{x_{CE}} &= \frac{\beta\theta}{(1-\beta\gamma)}\left(\frac{1-\tau_n}{1-\tau_k}\right)\frac{(1-\Phi_{CE})}{\Phi_{CE}} \\ \Rightarrow \quad \frac{(1-x_{CE})}{x_{CE}} &= \frac{A\beta\theta(1-\Phi_{CE})}{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}. \end{aligned}$$

$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)(1-\tau_k)}{A\beta\theta(1-\Phi_{CE}) + (1-\alpha)(1-\tau_k)(1-\beta\gamma)}. \tag{2.50}$$

Since,

$$A(1-\Phi_{CE}) = \alpha\beta(1-\tau_k),$$
$$\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta + (1-\alpha)(1-\beta\gamma)}.$$

From (2.36), we need

$$0 < 1 - \frac{\alpha\beta(1-\tau_k)}{A} < 1,$$

which gives us

$$0 < \frac{\alpha\beta(1-\tau_k)}{A} < 1,$$

or

$$A > \alpha\beta(1-\tau_k). \tag{2.51}$$

In addition, we also need

$$0 < A < 1 \tag{2.52}$$

to be satisfied. If equations (2.51) and (2.52) hold, we obtain

$$0 < A, \Phi_{CE}, n_{CE} < 1.$$

Equations (2.51) and (2.52) gives us a lower limit and an upper limit on τ_n, such that

$$\frac{-\alpha\left[1-\beta\theta-\beta^2\theta\right]}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta}\tau_k < \tau_n < \frac{(1-\beta\gamma)(1-\alpha\beta)}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta} - \alpha\frac{\left[(1-\beta\gamma)(1-\beta)-\beta^2\theta\right]}{(1-\alpha)(1-\beta\gamma)+\alpha\beta^2\theta}\tau_k. \tag{2.53}$$

In other words, for each τ_k the lower and the upper bound on τ_n must satisfy Restriction (2.53).

Appendix C

$$\begin{aligned}
1-A &= 1-\left[\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}\right] \\
&= \frac{(1-\beta\gamma)-\{\alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)\}(1-\beta\gamma)+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)} \\
&= \frac{(1-\beta\gamma)\left[\tau_n-\alpha(\tau_n-\tau_k)\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)} \\
&= \frac{(1-\beta\gamma)\left[\tau_k+(1-\alpha)(\tau_n-\tau_k)\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta\gamma)}.
\end{aligned}$$

Since

$$\begin{aligned}
A(1-\Phi_{CE}) &= \alpha\beta(1-\tau_k) \\
1-A &= \frac{(1-\beta\gamma)\left[(1-\alpha)(\tau_n-\tau_k)+\tau_k\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma}.
\end{aligned}$$

This implies,

$$\Upsilon = \left\{ \left[\frac{(1-\beta\gamma)\left[(1-\alpha)(\tau_n-\tau_k)+\tau_k\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma} \right]^{\mu} \left[\alpha\beta(1-\tau_k)\right]^{1-\mu} \right\}^{1-\gamma}.$$

In Υ, $\alpha\beta(1-\tau_k)$ decreases in τ_k. Further, suppose

$$\begin{aligned} M_1 &= \left[\frac{(1-\beta\gamma)\left[(1-\alpha)(\tau_n-\tau_k)+\tau_k\right]+\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma} \right] \\ M_2 &= \left[\alpha\beta(1-\tau_k)\right]. \end{aligned}$$

Therefore,

$$\frac{\partial\Upsilon}{\partial\tau_k} = (1-\gamma)\,\Upsilon^{-\frac{\gamma}{1-\gamma}} \left[M_2\mu\alpha \left\{ \frac{1-\beta\gamma-\beta^2\theta}{1-\beta\gamma} \right\} - M_1(1-\mu)\,\alpha\beta \right] M_1^{\mu-1} M_2^{-\mu}.$$

Since, $M_1 > 0$ because $1-A>0$ and $M_2>0$ by assumption,

$$(1-\beta\gamma)-\beta^2\theta<0,$$

implies that Υ will fall with an increase in τ_k.

From the labor supply term

$$\begin{aligned} n_{CE} &= \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A} \\ &= \frac{(1-\alpha)}{(1-\alpha)+\frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)}}. \end{aligned}$$

Note that

$$x_{CE}\Phi_{CE}A = \frac{(1-\alpha)(1-\beta\gamma)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)}\left[A-\alpha\beta(1-\tau_k)\right].$$

But

$$A-\alpha\beta(1-\tau_k) = \frac{(1-\beta\gamma)\left[\alpha(1-\beta)(1-\tau_k)+(1-\alpha)(1-\tau_n)\right]-\alpha\beta^2\theta(\tau_n-\tau_k)}{1-\beta\gamma}.$$

Hence,

$$x_{CE}\Phi_{CE}A = \frac{(1-\alpha)\left[(1-\beta\gamma)\left\{\alpha(1-\beta)(1-\tau_k)+(1-\alpha)(1-\tau_n)\right\}-\alpha\beta^2\theta(\tau_n-\tau_k)\right]}{\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]}.$$

The term

$$(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)\left(1-\tau_k\right)+(1-\alpha)(1-\tau_n)\right\}$$

can be re-written as

$$(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+\left(1-\alpha\right)-\alpha\left(1-\beta\right)\tau_k-(1-\alpha)\tau_n\right\},$$

$$\begin{aligned}
&= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+\left(1-\alpha\right)-\alpha\tau_k+\alpha\beta\tau_k-\tau_n+\alpha\tau_n\right\} \\
&= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+\left(1-\alpha\right)+\alpha\left(\tau_n-\tau_k\right)-\alpha\beta\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\} \\
&= (1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+\left(1-\alpha\right)+\alpha\left(1-\beta\right)\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\}.
\end{aligned}$$

Hence,

$$n_{CE}=\frac{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]}{(1-\alpha)\left[\alpha\beta^2\theta+(1-\alpha)(1-\beta\gamma)\right]+\Psi},$$

where

$$\Psi=\frac{(1-\alpha)}{(1-\tau_n)}\left[(1-\beta\gamma)\left\{\alpha\left(1-\beta\right)+\left(1-\alpha\right)+\alpha\left(1-\beta\right)\left(\tau_n-\tau_k\right)-\left(1-\alpha\beta\right)\tau_n\right\}-\alpha\beta^2\theta\left(\tau_n-\tau_k\right)\right].$$

Proof of Lemma 2.4

Note that

$$\begin{aligned}
\frac{x_{CE}\Phi_{CE}A}{1-\tau_n} &= x_{CE}\left[\frac{\alpha\left(1-\beta\right)\left(1-\tau_k\right)}{\left(1-\tau_n\right)}+\left(1-\alpha\right)-\frac{\alpha\beta^2\theta\left(\tau_n-\tau_k\right)}{\left(1-\beta\gamma\right)\left(1-\tau_n\right)}\right] \\
&= x_{CE}\left[\frac{\alpha\left(1-\beta\right)\left(1-\tau_k\right)}{\left(1-\tau_n\right)}+\left(1-\alpha\right)-\frac{\alpha\beta^2\theta\tau_n}{\left(1-\beta\gamma\right)\left(1-\tau_n\right)}+\frac{\alpha\beta^2\theta\tau_k}{\left(1-\beta\gamma\right)\left(1-\tau_n\right)}\right]
\end{aligned}$$

Therefore,

$$\frac{\partial\frac{x_{CE}\Phi_{CE}A}{1-\tau_n}}{\partial\tau_n}=x_{CE}\left[\frac{\alpha\left(1-\beta\right)\left(1-\tau_k\right)}{\left(1-\tau_n\right)^2}-\frac{\alpha\beta^2\theta\left(1-\tau_k\right)}{\left(1-\beta\gamma\right)\left(1-\tau_n\right)^2}\right],$$

which will be negative if

$$\left(1-\beta\gamma\right)\left(1-\beta\right)<\beta^2\theta.$$

This condition will be satisfied if equation (2.40) holds. And this implies

$$\frac{\partial n_{CE}}{\partial\tau_n}>0.$$

Further, since x_{CE} is independent of taxes,

$$\frac{\partial n_{2CE}}{\partial \tau_n} > 0.$$

Similarly, since

$$\Psi = \frac{(1-\alpha)}{(1-\tau_n)} \left[(1-\beta\gamma)\left\{\alpha(1-\beta) + (1-\alpha) + \alpha(1-\beta)(\tau_n - \tau_k) - (1-\alpha\beta)\tau_n\right\} - \alpha\beta^2\theta(\tau_n - \tau_k)\right],$$

$$\frac{\partial \Psi}{\partial(\tau_n - \tau_k)} = \frac{(1-\alpha)}{(1-\tau_n)} \left[\alpha(1-\beta)(1-\beta\gamma) - \alpha\beta^2\theta\right](1-\tau_k) < 0,$$

if equation (2.40) holds, which further implies,

$$\frac{\partial n_{CE}}{\partial(\tau_n - \tau_k)} > 0.$$

Finally,

$$\frac{\partial \Psi}{\partial \tau_k} = -\frac{(1-\alpha)}{(1-\tau_n)} \left[(1-\beta\gamma)\alpha(1-\beta) - \alpha\beta^2\theta\right] < 0,$$

if equation (2.40) holds.

Appendix D

We know that,

$$\begin{aligned}
(1-\Phi_P) &= \frac{\alpha\beta\left[(1-\beta\gamma) - \beta^2\mu(1-\gamma)\right]}{(1-\beta\gamma) - \beta^2(1-\gamma) + \alpha\beta^3(1-\gamma)} \\
x_P &= \frac{(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\}}{(1+\xi)(1-\alpha)\{(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi_P)\} + \beta\theta(1-\Phi_P)} \\
n_P &= \frac{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi)]}{(1-\alpha)[(1-\beta\gamma) - \beta^2\mu(1-\gamma) - \beta^2(1-\gamma)(1-\Phi)] + \Phi x\left[1-\beta\gamma - \beta^2\mu(1-\gamma)\right]}.
\end{aligned}$$

When $\gamma = 1$ and when $\xi = 0$,

$$\begin{aligned}
1-\Phi_P &= \alpha\beta \\
x_P &= \frac{(1-\alpha)(1-\beta)}{(1-\alpha)(1-\beta) + \alpha\beta^2\theta} \\
n_P &= \frac{(1-\alpha)}{(1-\alpha) + \Phi_P x_P}.
\end{aligned}$$

In the competitive equilibrium under equal factor income taxes,

$$\begin{aligned}
A &= 1-\tau. \\
&\Rightarrow (1-\Phi_{CE}) = \alpha\beta \\
&\Rightarrow n_{CE} = \frac{(1-\alpha)}{(1-\alpha)+x_{CE}\Phi_{CE}} \\
&\Rightarrow x_{CE} = \frac{(1-\alpha)(1-\beta)}{\alpha\beta^2\theta+(1-\alpha)(1-\beta)}.
\end{aligned}$$

Clearly, when $\gamma = 1$ and $\xi = 0$, and $\tau_n = \tau_k = \tau$,

As $\gamma \to 1$,

$$\begin{aligned}
1-\Phi_P &= 1-\Phi_{CE} \\
x_P &= x_{CE} \\
n_P &= n_{CE}
\end{aligned}$$

$$\Rightarrow g_{z_{CE}} = g_{z_P}.$$

Only equal factor income taxes under the no externality case, yields the planner's growth rate, except under a very restrictive parametric restriction,

$$\left(\frac{1-\beta}{\beta}\right)^2 = \theta.$$

Under this equal factor income taxes are *one* among infinitely many factor income tax combinations that decentralize the planner's growth rate. We can show this as follows.

For growth equalization, we need

$$n_{CE} = \frac{(1-\alpha)(1-\tau_n)}{(1-\alpha)(1-\tau_n)+x_{CE}\Phi_{CE}A} = n_P.$$

$$\begin{aligned}
&\Rightarrow \frac{x_{CE}\Phi_{CE}A}{(1-\tau_n)} = \Phi_P x_P \\
&\Rightarrow \frac{\Phi_{CE}A}{(1-\tau_n)} = \Phi_P \\
&\Rightarrow \frac{A-\alpha\beta(1-\tau_k)}{(1-\tau_n)} = 1-\alpha\beta \\
&\Rightarrow A-\alpha\beta(1-\tau_k) = (1-\alpha\beta)(1-\tau_n)
\end{aligned}$$

$$\Rightarrow \alpha(1-\tau_k)+(1-\alpha)(1-\tau_n)-\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}-\alpha\beta(1-\tau_k)=(1-\alpha\beta)(1-\tau_n).$$

Hence,

$$(\alpha-\alpha\beta)(1-\tau_k)-(\alpha-\alpha\beta)(1-\tau_n)=\frac{\alpha\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}$$

which implies

$$(1-\beta)(\tau_n-\tau_k)=\frac{\beta^2\theta(\tau_n-\tau_k)}{(1-\beta)}.$$

Clearly, as long as $\frac{(1-\beta)}{\beta}\neq\sqrt{\theta}$, $\tau_n=\tau_k$ always decentralizes planner's growth rates. When $\frac{(1-\beta)}{\beta}=\sqrt{\theta}$, any factor income tax combination decentralizes planner's growth rate. As noted in the text, for $\theta=0.2$, (or $\theta=0.5$, as we have used in our numerical exercise) as in Huffman, the value of $\beta=0.69098$ is very small and is not consistent with the literature. (When or $\theta=0.5$, $\beta=0.585\,79$ which is even smaller.). We therefore rule out the possibility of equality.

Appendix E: Planner's problem without full depreciation

The following first order conditions are therefore obtained with respect to C_t, K_{t+1}, Z_{t+1}, n_{1t}, and n_{2t} (with $\delta<1$):

$$\{C_t\}:\tfrac{1}{C_t}=\lambda_{1t}$$

$$\{K_{t+1}\}:\frac{1}{C_tZ_t}=\frac{\beta\,(1-\delta)}{C_{t+1}Z_{t+1}}+\frac{\alpha\beta Y_{t+1}\,(1-\tau)}{C_{t+1}K_{t+1}}+\beta(1-\gamma)(1-\mu)\lambda_{2t+1}\frac{Z_{t+2}}{K_{t+1}}-\beta^2\lambda_{2t+2}(1-\gamma)\alpha\frac{Z_{t+3}}{K_{t+1}} \tag{2.54}$$

$$\{Z_{t+1}\}:\lambda_{2t}=\beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}}+\beta\lambda_{1t+1}\left(\frac{K_{t+2}-(1-\delta)\,K_{t+1}}{Z_{t+1}^2}\right)+\beta^2\lambda_{2t+2}\mu\,(1-\gamma)\,\tau\frac{Z_{t+3}}{\frac{G_{t+2}}{Y_{t+1}}} \tag{2.55}$$

$$\{n_{1t}\}:\frac{1}{1-n_t}=\frac{(1-\alpha)Y_t\,(1-\tau)}{C_tn_{1t}}-\beta\lambda_{2t+1}(1-\gamma)\,(1-\alpha)\,\frac{Z_{t+2}}{n_{1t}} \tag{2.56}$$

and,

$$\{n_{2t}\}:\frac{1}{1-n_t}=\frac{(1-\alpha)\xi Y_t\,(1-\tau)}{C_tn_{2t}}+\lambda_{2t}\theta\frac{Z_{t+1}}{n_{2t}}-\beta\lambda_{2t+1}(1-\gamma)\xi\,(1-\alpha)\,\frac{Z_{t+2}}{n_{2t}}. \tag{2.57}$$

We use the method of undetermined coefficients in order to characterize the BGP. As in the case with $\delta = 1$,

$$C_t = \Phi_P Y_t (1-\tau), \;\; I_t = (1-\Phi_P) Y_t (1-\tau), \;\; I_t^g = \tau Y_t$$

and

$$n_1 = xn, n_2 = (1-x)n.$$

We know from $\{n_{1t}\}$,

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{(1-\alpha) Y_t (1-\tau)}{C_t n_{1t}} - \beta\lambda_{2t+1}(1-\gamma)(1-\alpha)\frac{Z_{t+2}}{n_{1t}},$$

which implies

$$\frac{x_P n_P}{1-n_P} = \frac{(1-\alpha)}{\Phi_P} - \beta(1-\gamma)(1-\alpha)\lambda_{2t+1} Z_{t+2}.$$

Therefore,

$$\lambda_{2t+1} Z_{t+2} = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)(1-\alpha)}.$$

This also implies for constant decision rules and a constant labor supply in every time period,

$$\lambda_{2i-1} Z_i = \frac{\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}}{\beta(1-\gamma)(1-\alpha)}, \quad \text{for all } i = t.$$

From $\{Z_{t+1}\}$,

$$\{Z_{t+1}\} : \lambda_{2t} = \beta\lambda_{2t+1}\gamma\frac{Z_{t+2}}{Z_{t+1}} + \beta\lambda_{1t+1}\left(\frac{K_{t+2} - (1-\delta) K_{t+1}}{Z_{t+1}^2}\right) + \beta^2\lambda_{2t+2}\mu(1-\gamma)\tau\frac{Z_{t+3}}{G_{t+2}/Y_{t+1}}.$$

On rearranging, this gives us

$$\lambda_{2t} Z_{t+1} = \beta\lambda_{2t+1}\gamma Z_{t+2} + \beta\lambda_{1t+1}\left(\frac{K_{t+2} - (1-\delta) K_{t+1}}{Z_{t+1}}\right) + \beta^2\lambda_{2t+2} Z_{t+3}\mu(1-\gamma)\tau\frac{Z_{t+1}}{G_{t+2}/Y_{t+1}}$$

Substituting in $\{Z_{t+1}\}$,

$$\begin{aligned}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]\frac{[1-\beta\gamma]}{\beta(1-\gamma)(1-\alpha)} &= \beta\left(\frac{I_{t+1}}{C_{t+1}}\right) \\ &+ \frac{\tau\beta^2\mu(1-\gamma)\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]}{\beta(1-\gamma)(1-\alpha)}\frac{Z_{t+1}}{G_{t+2}/Y_{t+1}}.\end{aligned}$$

This is of the form

$$\chi_1 = \chi_2 \left(\frac{I_{t+1}}{C_{t+1}} \right) + \chi_3 \frac{Z_{t+1}}{G_{t+2}/Y_{t+1}},$$

where

$$\begin{aligned}
\chi_1 &= \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right] \frac{[1-\beta\gamma]}{\beta(1-\gamma)(1-\alpha)} \\
\chi_2 &= \beta \\
\chi_3 &= \frac{\tau\beta^2\mu(1-\gamma)\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]}{\beta(1-\gamma)(1-\alpha)}.
\end{aligned}$$

Since

$$\left(\frac{I_{t+1}}{C_{t+1}} \right) = \left(\frac{1-\Phi_P}{\Phi_P} \right),$$

substituting, we get

$$\frac{Z_{t+1}}{G_{t+2}/Y_{t+1}} = \frac{\chi_1 - \chi_2 \left(\frac{1-\Phi_P}{\Phi_P}\right)}{\chi_3} = \text{constant}. \tag{2.58}$$

In equation (2.58) equality between the LHS and the RHS will not be restored if the LHS is not a constant. Therefore, on the BGP, equation (2.58) must be true.

Now, using the FOC with respect to K_{t+1},

$$\begin{aligned}
\frac{K_{t+1}}{C_t Z_t} &= \frac{K_{t+1}\beta(1-\delta)}{C_{t+1}Z_{t+1}} + \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}} + \beta(1-\gamma)(1-\mu)\lambda_{2t+1}Z_{t+2} - \beta^2\lambda_{2t+2}Z_{t+3}(1-\gamma)\alpha \\
&= \frac{K_{t+1}\beta(1-\delta)}{C_{t+1}Z_{t+1}} + \frac{\alpha\beta Y_{t+1}(1-\tau)}{C_{t+1}} + \frac{(1-\mu-\alpha\beta)}{(1-\alpha)} \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right].
\end{aligned}$$

On rearranging, we get

$$\frac{K_{t+1}}{C_t Z_t} \left[1 - \beta(1-\delta) \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{Z_t}{Z_{t+1}} \right) \right] = \frac{(1-\mu-\alpha\beta)}{(1-\alpha)} \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right].$$

This implies

$$\frac{K_{t+1}}{C_t Z_t} = \frac{\frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]}{\left[1-\beta(1-\delta)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{Z_t}{Z_{t+1}}\right)\right]}.$$

Again this implies Z_t is growing at the same rate at $\frac{K_{t+1}}{C_t}$, or Z_{t+1} is growing at the same rate at $\frac{K_{t+2}}{C_{t+1}}$. Since, $C_{t+1} = \Phi_P Y_{t+1}(1-\tau)$, Z_{t+1} is growing at the same rate at $\frac{K_{t+2}}{Y_{t+1}}$. This is because, on the BGP the RHS is constant. In fact,

$$\frac{K_{t+2}}{Y_{t+1}Z_{t+1}} = \chi_4(1-\tau), \tag{2.59}$$

where

$$\chi_4 = \frac{\frac{(1-\mu-\alpha\beta)}{(1-\alpha)}\left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P}\right]\Phi_P}{\left[1-\beta\left(1-\delta\right)\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{Z_t}{Z_{t+1}}\right)\right]}.$$

As in equation (2.58), in equation (2.59) the equality between the LHS and the RHS will not be restored if the LHS is not a constant. Therefore, on the BGP, equation (2.59) must be true. Using equation (2.58) and (2.59), we conclude that on the BGP,

$$\begin{aligned} g_{z_P} &= \frac{g_{k_P}}{g_{y_P}}, \text{ and} \\ g_{z_P} &= \frac{g_G}{g_{y_P}}. \end{aligned}$$

We know

$$Z_{t+1} = BZ_t^{\gamma} n_2^{\theta}\left[\left(\frac{G_t}{Y_{t=1}}\right)^{\mu}\left(\frac{K_t}{Y_{t=1}}\right)^{1-\mu}\right]^{1-\gamma}.$$

This implies

$$\begin{aligned} \frac{Z_{t+1}}{Z_t} &= \frac{Z_t^{\gamma}}{Z_{t-1}^{\gamma}}\left[\frac{\left(\frac{G_t}{Y_{t=1}}\right)^{\mu}\left(\frac{K_t}{Y_{t=1}}\right)^{1-\mu}}{\left(\frac{G_{t-1}}{Y_{t=2}}\right)^{\mu}\left(\frac{K_{t-1}}{Y_{t=2}}\right)^{1-\mu}}\right]^{1-\gamma} \\ g_{z_P} &= g_{z_P}^{\gamma} g_{z_P}^{1-\gamma} = g_{z_P}. \end{aligned}$$

Growth rate at the BGP

Since

$$\frac{K_{t+2}}{Y_{t+1}} = \chi_4\left(1-\tau\right)Z_{t+1},$$

$$\begin{aligned} g_{k_P} &= g_{z_P} g_{y_P} \\ &= g_{z_P} g_{k_P}^{\alpha} \end{aligned}$$

Therefore,

$$\begin{aligned} g_{k_P} &= g_{z_P}^{\frac{1}{1-\alpha}}, \\ \text{and therefore, } g_{y_P} &= g_{z_P}^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

We therefore obtain qualitatively identical results to the $\delta = 1$ case.

Growth rate of ISTC

The expression for Z_{t+1} is given by

$$
\begin{aligned}
Z_{t+1} &= BZ_t^{\gamma} n_2^{\theta} \left[\left(\frac{G_t}{Y_{t=1}} \right)^{\mu} \left(\frac{K_t}{Y_{t=1}} \right)^{1-\mu} \right]^{1-\gamma} \\
&= BZ_t^{\gamma} n_2^{\theta} \left[\left(\frac{G_t}{Y_{t=1}} \right)^{\mu} \left(\frac{K_t}{Y_{t=1}} \right)^{1-\mu} \right]^{1-\gamma} \\
&= BZ_t^{\gamma} n_2^{\theta} \left[\left(\frac{\chi_3 Z_{t-1}}{\chi_1 - \chi_2 \left(\frac{1-\Phi_P}{\Phi_P} \right)} \right)^{\mu} \left(\chi_4 \left(1-\tau \right) Z_{t-1} \right)^{1-\mu} \right]^{1-\gamma} \\
&= BZ_t^{\gamma} n_2^{\theta} Z_{t-1}^{1-\gamma} \left[\left(\tau \frac{\frac{\beta^2 \mu (1-\gamma) \left[\frac{(1-\alpha)}{\Phi_P} - \frac{x_P n_P}{1-n_P} \right]}{\beta (1-\gamma)(1-\alpha)}}{\chi_1 - \chi_2 \left(\frac{1-\Phi_P}{\Phi_P} \right)} \right)^{\mu} \left(\chi_4 \left(1-\tau \right) \right)^{1-\mu} \right]^{1-\gamma} .
\end{aligned}
$$

We can then summarize the growth rate of Z_{t+1} on the BGP

$$
g_z = \left\{ B n_2^{\theta} \left[\left(\tau \Delta_1 \right)^{\mu} \left(\chi_4 \left(1-\tau \right) \right)^{1-\mu} \right]^{1-\gamma} \right\}^{\frac{1}{2-\gamma}} .
$$

2.6 Figures

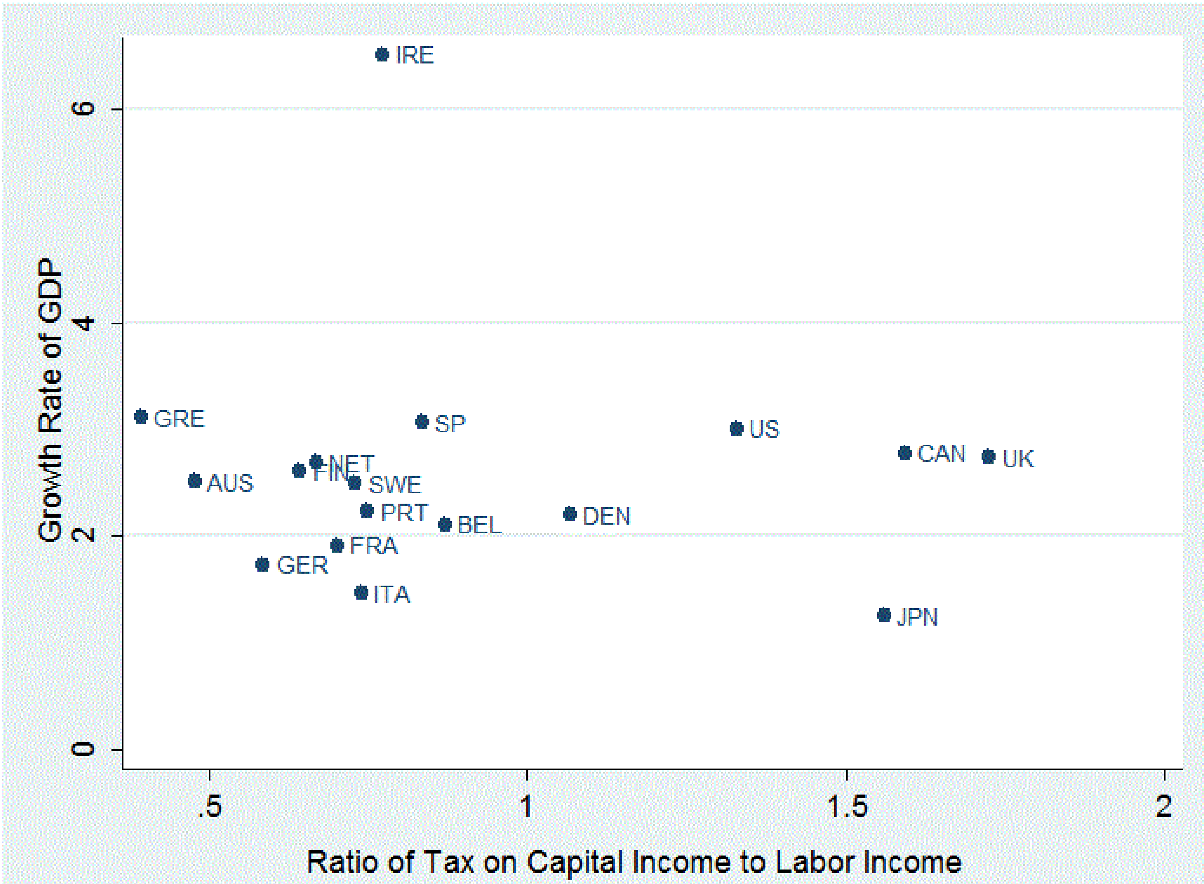

Figure 2.1: Average growth rates for select OECD economies versus the ratio of tax on capital income to tax on labor income

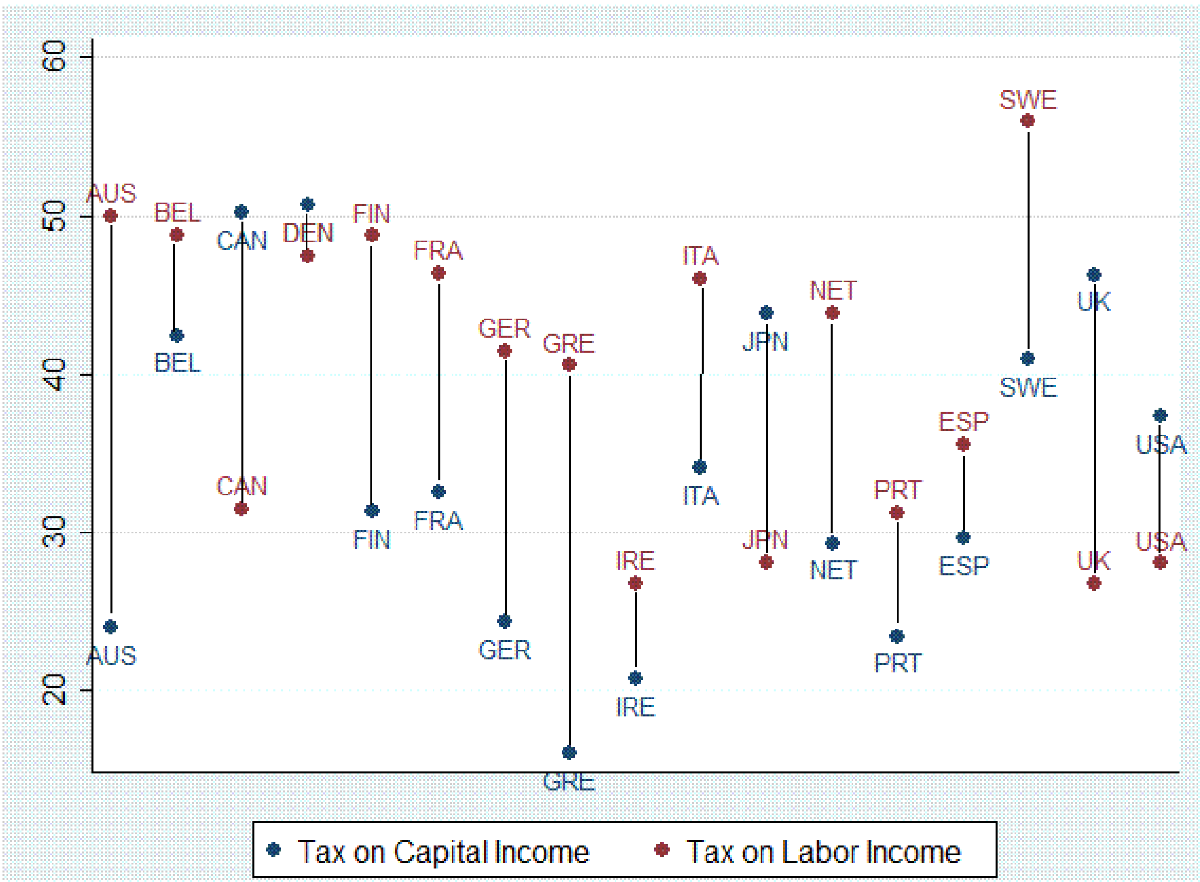

Figure 2.2: Average factor income tax rates for select OECD economies

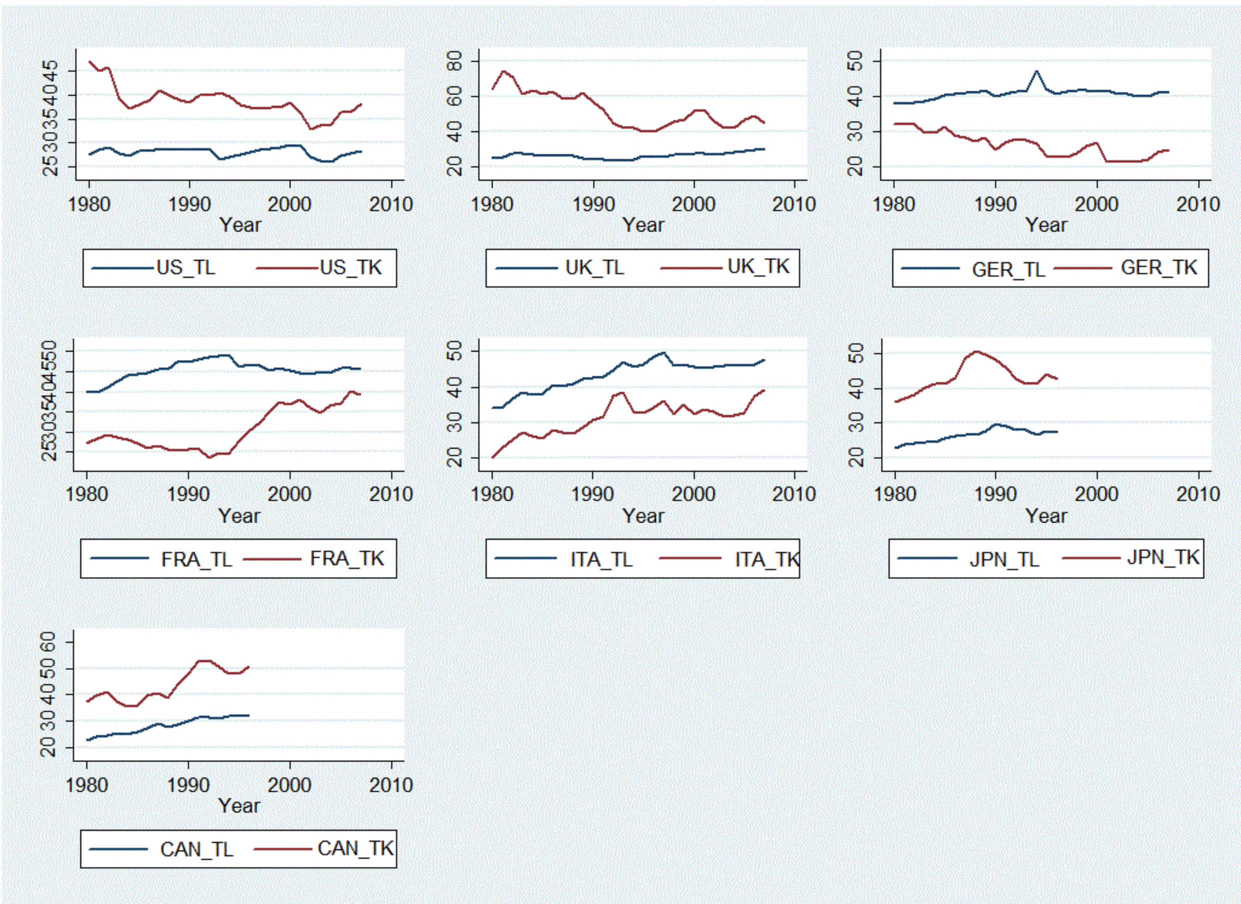

Figure 2.3: Time trend of factor income taxes for G7 economies

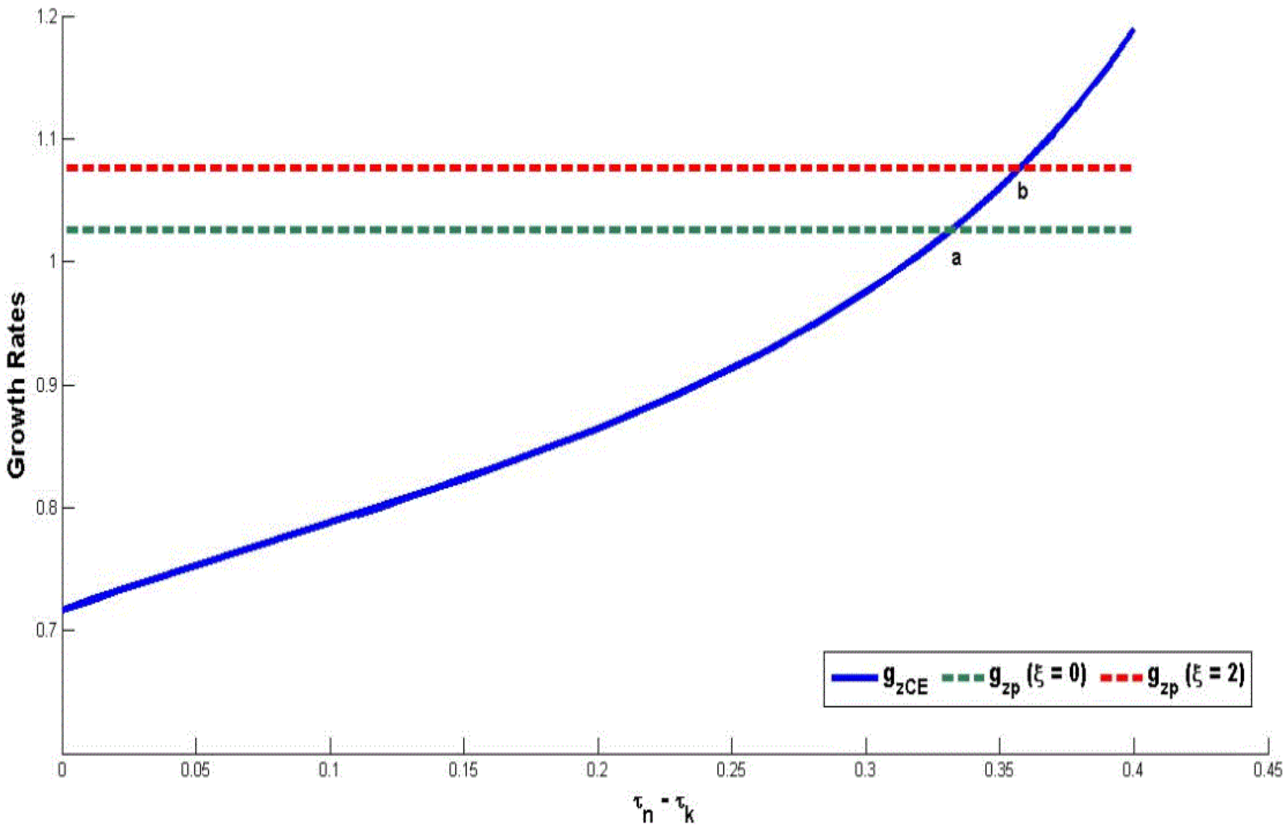

Figure 2.4: The effect of a change in ξ on $(\tau_n - \tau_k)$

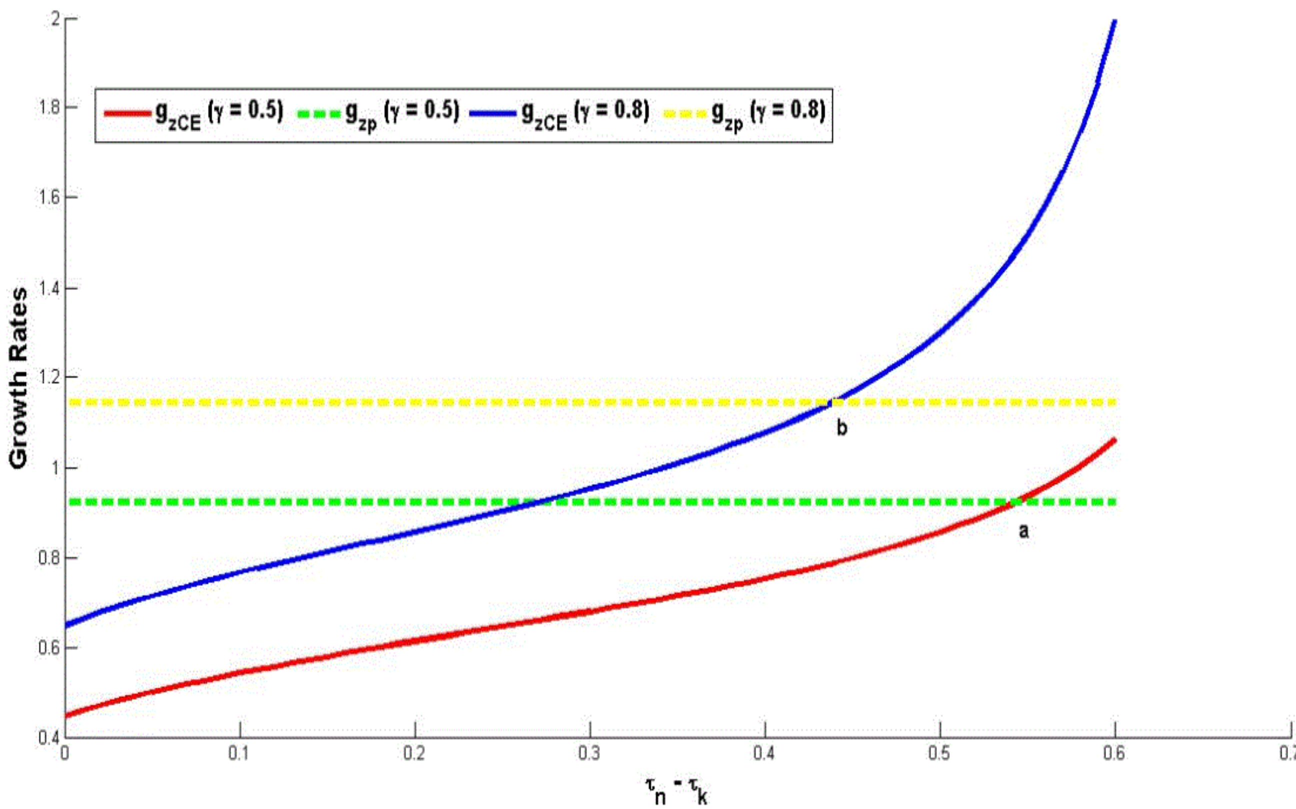

Figure 2.5: The effect of a change in γ on $(\tau_n - \tau_k)$

Chapter 3

Fiscal Policy in an Emerging Market Business Cycle Model

3.1 Introduction

Building dynamic stochastic general equilibrium (DSGE) models of emerging market economies (EMEs) has become an important area of research in macroeconomics. A recent empirical literature has identified key stylized facts in emerging market economy business cycles to see how these differ from the main features of advanced economy (AE) business cycles (see Agénor et al. (2000), Rand and Tarp (2002), Neumeyer and Perri (2005), Loayza et al. (2007), Male (2010), and Ghate et al. (2013)). The key stylized facts that these papers have identified are as follows. First, output in EMEs tends to be more volatile than output in advanced economies.[1] Second, EMEs have counter-cyclical real interest rates. In AEs, real interest rates are typically a-cyclical or at the most mildly pro-cyclical. Third, consumption is pro-cyclical and more volatile than output in EMEs, whereas in AEs it is pro-cyclical but is less volatile than output. Fourth, net exports are much more counter-cyclical with respect to output in EMEs in comparison to the AEs.

This research has motivated new theoretical models to understand the propagation and amplification of shocks in EME business cycles. One branch of the literature builds upon the seminal work of Neumeyer and Perri (2005).[2] These authors build a small open economy (SOE) real business cycle (RBC) model with interest rate shocks and working capital

[1] Male also (2010) estimates output to be on average twice as volatile in EMEs in comparison to AEs. Rand and Tarp (2002) on the contrary state that output is no more than 20% more volatile in EMEs compared to AEs.

[2] Recently, Tiryaki (2012) estimates Neumeyer and Perri's (2005) model to replicate Turkey's business cycle properties.

constraints.[3] A higher interest rate implies that a firm's borrowing costs to meet its working capital constraint increases. This leads to a decline in the labor demanded by firms, and since this is a full employment model, a reduction in labor demand leads to a reduction in output. This channel makes real interest rates counter-cyclical. A crucial feature of this model is that households have GHH preferences (see Greenwood et al. (1988)). GHH preferences shut off the income effect, making labor supply invariant to the income effects associated with an interest rate shock (see Li (2011)). Consumption drops instantaneously and falls more than output due to the inter-temporal substitution effect from a rise in real interest rates. At the same time private investments also fall since the demand for private capital falls because of higher interest rates. As a result, net exports (defined as the savings-investment gap) displays counter-cyclicality with respect to output.

Another branch of theoretical models of EME business cycles builds on the seminal work of Aguiar and Gopinath (2007). These authors explain the key stylized facts of EME business cycles discussed above by allowing for both permanent trend shocks and transitory changes in productivity. Trend shocks affect both current income and future income. They justify this assumption by noting that emerging markets are characterized by a large number of policy regime shifts, which can be viewed as shocks to the trend productivity growth rate. Using the permanent income hypothesis as the identification mechanism, a shock to trend productivity implies a boost to both current output and also future output. Since a shock to the trend productivity increases permanent income, consumption increases more than income. This reduces savings, and generates a counter-cyclical current account deficit.

One aspect that is missing in the above theoretical literature is that there is no explicit role for fiscal policy. This is puzzling since fiscal policy plays an important role in macroeconomic stabilization in many developing countries and EMEs.[4] For instance, Male (2010) finds that government expenditures tend to be significantly more volatile than output in EMEs. She also reports that there is no robust stylized observation on the correlation between

[3] More specifically, in their model, firms face a working capital constraint, i.e., firms have to pay a fraction of the wage bill before actual production takes place. In order to finance this working capital constraint, firms issue corporate bonds to agents in international capital markets at a market determined interest rate on bonds. The interest rate has two different components – an international interest rate component and a country spread component driven by a shock to the country spread risk with the latter varying according to an individual country's sovereign risk.

[4] Fiscal policy can serve as a stabilizing instrument if government expenditures are counter-cyclical along with pro-cyclicality of government revenues. One explanation for pro-cyclical government expenditure in EMEs is that governments often face political pressures or temptations to avoid budgetary surpluses during boom-time thereby constraining themselves from lowering expenditures or raising taxes. During recessions, governments in EMEs are forced to reduce spending because of lack of access to credit (see Talvi and Vegh (2005)). In the post great financial crisis period, there is also a renewed interest in fiscal policy in small open economies.

real government expenditure and output.[5] In other evidence (see Talvi and Vegh (2005)), government expenditures have tended to be more pro-cyclical in EMEs than in AEs although there are countries where government expenditures are counter-cyclical.[6] Our takeaway from this literature is that in some EMEs, government expenditures are counter-cyclical with respect to output yet in others it is pro-cyclical.

Another aspect that has typically not received sufficient attention is the role of fiscal policy for macroeconomic stabilization when an economy is hit with an interest rate shock. For instance, Male (2010) finds that a typical feature of these EMEs is that both government expenditures and real interest rates are more volatile than output. The contemporaneous correlation of the government expenditure and the real interest rate with respect to output however, is positive or negative.[7] This is in contrast to advanced economies where real interest rates are observed to be a-cyclical or mildly pro-cyclical (see Agénor et al. (2000), Neumeyer and Perri (2005), and Male (2010)). In Table 1, we summarize the estimates of the relative standard deviation and contemporaneous correlations of government expenditures (G) and real interest rates (R) for twelve EMEs from Male (2010).[8] In countries for which data is available, five countries have counter-cyclical real interest rates, while six have pro-cyclical interest rates. Further, while government expenditure is counter-cyclical in five countries, it is pro-cyclical in four.

[5] Agénor et al. (2000) state that government expenditures tend to be more counter-cyclical in AEs as compared to EMEs.

[6] This is at odds with a volumnious literature that has found that fiscal policies are pre-dominantly pro-cyclical in EMEs (Talvi and Vegh (2005), Cuadra el al. (2010)). Over the last decade, however, several EMEs have "graduated" from having pro-cyclical fiscal policy to having counter-cyclical fiscal policy. This "graduation" has been attributed to improvements in institutional quality (see Frankel et al. (2013)).

[7] While Neumeyer and Perri (2005) and Uribe and Yue (2005) state that interest rates are generally counter-cyclical in EMEs, Male (2010) finds this observation not to be universally true particularly among EMEs in Africa, Asia, and Eastern Europe.

[8] $\sigma(Z)$ denotes the standard deviation of variable Z and $\rho(Z, Y)$ is the contemporaneous correlation of variable Z with output, Y. For India, we obtain the moments from Ghate et al. (2013).

Country	Sample	$\frac{\sigma(G)}{\sigma(Y)}$	$\frac{\sigma(R)}{\sigma(Y)}$	$\rho(G, Y)$	$\rho(R, Y)$
Chile	1980:1-2004:4	11.3	1.7	–	−0.22
Colombia	1980:1-2004:4	2.2	3.7	0.35	0.27
Hong Kong	1980:1-2004:4	2.5	3.1	−0.21	0.33
Hungary	1980:1-2004:4	1.7	2.6	−0.63	−0.01
Israel	1980:1-2004:4	20.7	8.7	–	−0.02
Korea	1980:1-2004:4	2.4	2.1	−0.04	−0.36
Mexico	1980:1-2004:4	4.0	8.5	−0.11	−0.48
Slovak Rep.	1980:1-2004:4	2.3	5.1	–	0.45
Slovenia	1980:1-2004:4	1.5	11.1	0.27	0.25
South Africa	1980:1-2004:4	1.9	3.9	0.04	0.13
Turkey	1980:1-2004:4	8.3	–	0.74	–
India	1999:2-2010:2	5.53	1.77	−0.35	0.38

Table 1 : Real interest rates and government expenditures in EMEs from Male (2010)

Drawing on the evidence from Table 1, we summarize the stylized facts that are the focus of the theoretical literature on EME business cycles (Column 2), and the wider EME evidence in Column 3 in Table 2.

Variables	Evidence from NP and AG[9]	Wider evidence from Male[10]
$\frac{\sigma(C)}{\sigma(Y)}$	> 1	> 1
$\rho(\frac{NX}{Y}, Y)$	< 0	< 0
$\rho(R, Y)$	< 0	$\gtrless 0$
$\rho(G, Y)$	No Role	$\gtrless 0$

Table 2: Facts based on the wider literature on EME business cycles

Given Table 1 and Table 2, we build a small open economy RBC model which allows us to understand the causal link between the nature of counter-cyclical or a-cyclical fiscal policy, pro-cyclical or counter-cyclical real interest rates, counter-cyclical net exports, and higher relative consumption volatility. Ours is therefore a more general framework to understand the wider EME evidence on business cycles.

[9] NP stands for Neumeyer and Perri (2005) and AG stands for Aguiar and Gopinath (2007).

[10] See Male (2010)

3.1.1 Description and Main Results

We develop a small open economy (SOE) real business cycle (RBC) model along the lines of Neumeyer and Perri (2005) with two crucial differences.

First, we extend their framework by incorporating fiscal policy. We incorporate two different roles for fiscal policy: the government provides public consumption with the private and public components of consumption substitutable; and, the government lends a portion of the working capital constraint faced by the firm at a subsidized interest rate. We assume that the government imposes time invariant distortionary taxes on consumption, labor income and capital income, and maintains a balanced budget at every time period.

Second, unlike Neumeyer and Perri (2005), where agents have GHH preferences, in our framework, agents are assumed to have Cobb-Douglas (CD) utility functions. The assumption of CD preferences *permits* a shock to the real interest rate to have income effects on labor supply through consumption.

In this paper we show that these added features make the real interest rate less counter-cyclical or even pro-cyclical at times. Fiscal policy affects the transmission of interest rate shocks onto the real economy through a standard inter-temporal substitution effect, and a time varying wedge which we denote as the *fiscal policy wedge*. We show that the fiscal policy wedge is a more general version of the simple intra-temporal tax wedge that distorts labor hours in the standard stochastic growth model. Our theoretical contribution is two-fold: first, we characterize the fiscal policy wedge in closed-form under a variety of assumptions on fiscal policy, and show how this affects movements in labor supply adversely; and second, we show that because the fiscal policy wedge is time varying and increases with a positive interest rate shock, the impact of an increase of the wedge on labor supply is higher when there is a higher weight on government consumption in the utility function. This happens because of two effects. First, when an economy is hit with an interest rate shock labor supply falls due to an increase in the fiscal policy wedge. The fiscal policy wedge increases more when households value public consumption highly. Second, a higher weight on public consumption in utility induces a strong standard inter-temporal substitution effect which reduces private consumption and increases labor supply. The net effect on labor market outcomes of a positive interest rate shock therefore depends on the relative strength of these two individual effects. In general, the net effect will be positive (i.e., equilibrium employment and output increase).[11]

We also show that fiscal policy's second role in our model - to subsidize working capital -

[11] The counter-cyclicality of government spending is also consistent with the theoretical prediction of government spending in the neo-classical framework where we would expect to see government consumption move counter-cyclically, if public and private components are substitutes. See Lane (2003).

dampens the reduction in labor demand due to a positive interest rate shock in the standard Neumeyer and Perri (2005) setup. Thus, both labor supply and labor demand channels make the real interest rate a-cyclical, and under certain cases, pro-cyclical, matching the qualitative features of the EME data in Table 2.[12]

Indian Business Cycle

We calibrate our model to India.[13] We choose India because India typifies the broader EME business cycle experience listed in Column 3 (Table 2). The key Indian stylized facts are as follows: higher relative consumption volatility, higher relative investment volatility, counter-cyclical net exports, counter-cyclical government expenditures, and a pro-cyclical interest rate (see Ghate et al. (2013), Table 5).[14] The counter-cyclicality of government expenditures has been coupled with pro-cyclical interest rates and counter-cyclical net exports, consistent with the evidence on other EMEs reported in Table 1 and Table 2. There is no robust estimate for labor hours on the Indian economy. However, as we will show later, because equilibrium output depends on labor market outcomes, analyzing changes in equilibrium output are sufficient from the standpoint of determining co-movements. We also believe that the specification of fiscal policy in this paper is particularly relevant for India (and generally some other EMEs). For instance, although there have been major financial sector reforms, public sector banks still own 70% of the banking sector's assets in India.[15] These banks extend priority sector lending to certain sectors such as agriculture, exports, infrastructure and small and medium enterprises at a subsidized lending rate. Government consumption expenditures, in recent years, has also approximated 12% of GDP[16] suggesting its role as a plausible channel through which interest rate shocks are propagated in the model.

3.2 The Model

3.2.1 The Firm's Problem

The economy consists of firms, a government, and households. At any given time t a representative firm produces final output using labor employed at time t and capital carried

[12] Our results are consistent with many papers in the literature which argue that the final effect of the simple inter-temporal tax wedge on hours worked depends crucially on whether public consumption is perceived as highly substitutable by agents (see Prescott, 2002).

[13] We calibrate our model using Dynare Version 4.3.0.

[14] These tables have been generated using quarterly data from 1999-Q2 - 2010-Q2.

[15] Table no. 3.1, statistical tables relating to banks of India, Handbook of Statistics of the Indian Economy, 2012 (http://www.rbi.org.in/scripts/PublicationsView.aspx?id=14672).

[16] See the 2013 World Development Indicators: http://data.worldbank.org/indicator/NE.CON.GOVT.ZS

forward from time period $t-1$. However, prior to actual production, the firm needs to pay a portion $\theta \in [0,1]$ of its total wage bill in advance. To meet this working capital constraint, the firm borrows from the government and from households by issuing debt.[17] The firm issues corporate bonds to households to whom they promise a return of R^P_{t-1} which is a mark-up over the existing international interest rate R^*_{t-1}. Firms can also borrow from the government at a subsidized interest rate $R^P_{t-1}(1-s)$ where $0 \leq s < 1$ is the subsidy. We assume however that only a fixed portion, θ_G, of the total firm's working capital constraint, θ, such that $\theta_G \leq \theta$ can be borrowed from the government at the subsidized rate. The rest of the working capital constraint $(\theta - \theta_G \geq 0)$ has to be covered by issuing bonds in international capital markets at R^P_{t-1}.

The firm hires labor (l_t) and uses capital (k_{t-1}) accumulated in time period $t-1$ to produce the final output y_t such that

$$\begin{aligned} y_t &= A_t k^{\alpha}_{t-1} l^{1-\alpha}_t (1+\gamma)^{t(1-\alpha)} \\ &= A_t k^{\alpha}_{t-1} \left[(1+\gamma)^t l_t\right]^{1-\alpha}, \quad 0 < \alpha < 1 \end{aligned} \tag{3.1}$$

where $(1+\gamma)^t$ is labor augmenting technical progress in time period t. We assume that the production technology, y_t, exhibits constant returns to scale (CRS). The firm's profits are given by:

$$\pi_t = y_t - w_t l_t - r_t k_{t-1} - \left(R^G_{t-1} - 1\right)\theta_G w_t l_t - \left(R^P_{t-1} - 1\right)(\theta - \theta_G) w_t l_t, \tag{3.2}$$

The last two terms in (3.2) denote the interest costs for working capital loans from the government and households, respectively, where R^P_{t-1} is the country specific gross interest rate at which firms borrow from international capital markets, and R^G_{t-1} is the subsidized gross interest rate offered by the government to lend 'θ_G' portion of the firm's total working capital constraint. No-arbitrage implies:

$$R^G_{t-1} = R^P_{t-1}(1-s) > 1, \quad 0 \leq s < 1 \tag{3.3}$$

Here $R^G_{t-1} > 1$ since it is the gross interest rate. We can therefore re-write equation (3.2) as

$$\pi_t = y_t - r_t k_{t-1} - (1-\theta) w_t l_t - w_t l_t R^P_{t-1}\left[\theta - s\theta_G\right]. \tag{3.4}$$

The partially subsidized loan provided by the government to cover the firm's working capital constraint therefore effectively creates a wedge $[\theta - s\theta_G]$ on the interest payment. If subsidy

[17] In Neumeyer and Perri (2005), firms cannot borrow from the government.

s is zero, we go back to the standard Neumeyer and Perri (2005) model.[18]

Timing of Events The timing of events and decisions is given in Figure 3.1. In the beginning of period t, which we denote as t^-, firms borrow, $\theta w_t l_t$, to make advance payments to labor prior to actual production (which occurs at t). Firms then produce output and repay the loan borrowed at the end of time period (t^+), with workers receiving the rest of their wage bill, $(1-\theta) w_t l_t$, at time t^+ also. Since the time gap between t^- and t, and between t and t^+ is very small, we drop these superscripts and consider the entire period as time period t.

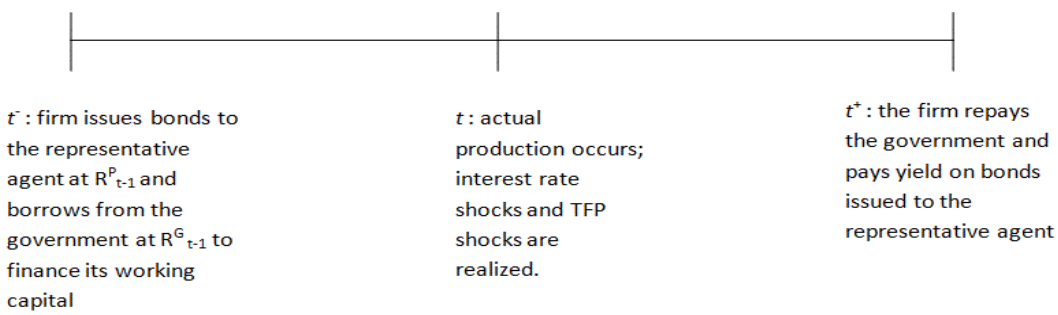

Figure 3.1: Timing of Events and Decisions

We transform output y_t to it's stationary value $\widetilde{y}_t$, as follows[19]

$$\begin{aligned}
\widetilde{y}_t &= \frac{y_t}{(1+\gamma)^t} = \frac{A_t k_{t-1}^{\alpha} l_t^{1-\alpha} (1+\gamma)^{t(1-\alpha)}}{(1+\gamma)^t} \\
&= A_t \frac{k_{t-1}^{\alpha}}{\left[(1+\gamma)^t\right]^{\alpha}} l_t^{1-\alpha} \\
&= \frac{A_t}{(1+\gamma)^{\alpha}} \widetilde{k}_{t-1}^{\alpha} l_t^{1-\alpha}.
\end{aligned}$$

[18] As in Neumeyer and Perri (2005), country specific interest rates depend on the international interest rate and country specific spread component which measures the economy's riskiness.

[19] For any variable m_t, we define it's stationary transformation as $\widetilde{m}_t$ such that,

$$\widetilde{m}_t = \frac{m_t}{(1+\gamma)^t}.$$

All variables in our model grow at the same exogenous rate $(1+\gamma)$. All variables are therefore transformed to their corresponding stationary values except l_t, which is assumed to be stationary.

Hence, equation (3.4) can be re-written as

$$\widetilde{\pi}_t = \widetilde{y}_t - \frac{r_t \widetilde{k}_{t-1}}{(1+\gamma)} - (1-\theta)\,\widetilde{w}_t l_t - \widetilde{w}_t l_t R^P_{t-1}\left[\theta - s\theta_G\right].$$

Firms Profit Maximizing Conditions

The firm's profit maximization yields the following first order conditions $\forall t$, for labor, l_t, and capital, $\widetilde{k}_{t-1}$, respectively.

$$\begin{aligned} \{l_t\} \quad &: \quad \frac{(1-\alpha)\widetilde{y}_t}{l_t} = \widetilde{w}_t\left[(1-\theta) + R^P_{t-1}\left(\theta - s\theta_G\right)\right] \\ \left\{\widetilde{k}_{t-1}\right\} \quad &: \quad \frac{\alpha \widetilde{y}_t}{\widetilde{k}_{t-1}} = \frac{r_t}{(1+\gamma)}. \end{aligned} \tag{3.5}$$

Without any working capital constraints, $\theta = \theta_G = 0$, and the standard first order condition for labor demand, $\frac{(1-\alpha)\widetilde{y}_t}{l_t} = \widetilde{w}_t$, obtains. The presence of the working capital constraint therefore modifies this condition by changing the effective wage payment to, $\widetilde{w}_t\left[(1-\theta) + R^P_{t-1}\left(\theta - s\theta_G\right)\right]$. For given values of θ and θ_G, interest rate shocks affect wage payments with a lag since effective wage payments depend on R^P_{t-1}.

3.2.2 Government

The government collects tax revenue by imposing time invariant distortionary taxes on consumption $\tau_c \in [0,1]$, wage income $\tau_w \in [0,1]$, and capital income $\tau_k \in [0,1]$. It also receives interest income from financing the 'θ_G' component of a firm's working capital constraint. The interest income is given by, $R^G_{t-1}\theta_G w_t l_t$. The government allocates G_t of it's total revenue towards government consumption. We assume that net of G_t, the government lends S_t to firms at time period t at a subsidized interest rate given by (3.3). The government is assumed to balance it's budget at every time period t such that

$$TR_t + R^G_{t-1}\theta_G w_t l_t = G_t + S_t.$$

TR_t denotes the total tax revenue collected by the government at every time period such that

$$TR_t = \tau_c c_t + \tau_w w_t l_t + \tau_k r_t k_{t-1}. \tag{3.6}$$

As discussed above, due to the timing of the firm's problem, we have

$$S_t = \theta_G w_t l_t.$$

Clearly, this implies[20]

$$G_t = \tau_c c_t + \left\{ \left[R^P_{t-1}(1-s) - 1 \right] \theta_G + \tau_w \right\} w_t l_t + \tau_k r_t k_{t-1}. \tag{3.7}$$

3.2.3 The Household's Problem

The economy is populated by infinitely lived households with a mass normalized to 1. Each representative household consumes and invests a homogenous good and supplies labor and capital to firms. The representative household has the following expected discounted lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c^*_t, l_t), \tag{3.8}$$

where $\beta \in (0,1)$ denotes the households subjective discount factor. We assume that

$$c^*_t = c_t + \Theta G_t, \tag{3.9}$$

where household consumption, c_t is augmented by government consumption, G_t. Following Barro (1981), Ni (1995), Roche (1996), Ambler and Paquet (1996), and Christiano and Eichenbaum (1992) the parameter Θ captures the weight of public consumption in household utility, where $\Theta > 0$. Given our specification in equation (3.9), c_t and G_t are assumed to be perfect substitutes.[21] We assume that agents treat G_t as given. l_t denotes hours worked.

We parametrize the utility function, U, in (3.8) by assuming a Cobb-Douglas (CD) specification, i.e.,

$$U(c^*_t, l_t) = \frac{\left[(c^*_t)^{\mu} (1-l_t)^{(1-\mu)} \right]^{(1-\sigma)}}{(1-\sigma)}, \quad 0 < \mu < 1, \sigma > 0.$$

which is an important point of departure from Neumeyer and Perri (2005). These authors, instead, assume GHH preferences because their focus was to explain counter-cyclicality of interest rates in the select EMEs that they consider. Assuming GHH preferences ensures that labor supply is independent of consumption and therefore interest rates. A positive shock to the interest rates does not cause any shift in the labor supply while it reduces demand for labor thereby reducing equilibrium labor. This leads to a reduction in output

[20] The stationary transformation of equation (3.7) is:

$$\widetilde{G}_t = \tau_c \widetilde{c}_t + \left\{ \left[R^P_{t-1}(1-s) - 1 \right] \theta_G + \tau_w \right\} \widetilde{w}_t l_t + \frac{\tau_k r_t \widetilde{k}_{t-1}}{1+\gamma}.$$

[21] In an emerging market context, an example of G_t can be public health or public transportation services whose quality is typically seen as being superior to private alternatives. Higher provision of services elicits a strong reduction in the private consumption of these services. See Kuehlwein (1998).

which makes real interest rates counter-cyclical with respect to output. We focus on the Cobb-Douglas (CD) utility function to allow equilibrium labor to increase due to a single period interest rate shock for some parametric restrictions. The parameter σ is the coefficient of risk aversion and μ is the intra-temporal elasticity of substitution of labor supply.

The household maximizes expected lifetime discounted utility (3.8) subject to

$$(1+\tau_c)c_t + x_t + b_t + \kappa(b_t) \leq (1-\tau_w)w_t l_t + (1-\tau_k)r_t k_{t-1} + R^P_{t-1}b_{t-1}. \tag{3.10}$$

where b_t denotes bond holdings of the household, x_t denotes investment, and $\tau_c \in [0,1]$ is the tax on consumption, $\tau_w \in [0,1]$ is the tax on labor income, and $\tau_k \in [0,1]$ is the tax on capital income. Agents take the competitive factor awards, w_t, the wage rate, and r_t, the return to capital as given in deciding optimal choices. For bond holdings, b_t, the term $\kappa(b_t)$ in (3.10) is the bond holding cost such that

$$\kappa(b_t) = \frac{\kappa}{2} y_t \left[\left(\frac{b_t}{y_t} \right) - \left(\frac{b}{y} \right) \right]^2, \tag{3.11}$$

where $\frac{b}{y}$ is the steady state bond holding to output ratio.[22] The term x_t in (3.10) is the level of private investment such that

$$x_t = k_t - (1-\delta)k_{t-1} + \Phi(k_t, k_{t-1}), \tag{3.12}$$

where $\Phi(k_t, k_{t-1})$ is the investment adjustment costs such that[23]

$$\Phi(k_t, k_{t-1}) = \frac{\phi}{2} k_{t-1} \left[\left(\frac{k_t}{k_{t-1}} \right) - (1+\gamma) \right]^2. \tag{3.13}$$

Households First Order Conditions We obtain the following first order conditions with respect to all stationary transformed variables, $\forall t$, by maximizing (3.8) subject to consumer budget constraint (3.10), where equations (3.11), (3.12), and (3.13) have been substituted into the consumer budget constraint. This yields the first order conditions (3.14), (3.15), (3.16), and (3.17), with respect to consumption, labor, bond holdings ,and the capital stock,

[22] Introducing adjustment costs on international bonds is one of the many ways of guaranteeing stationarity in the model (see Schmitt-Grohe and Uribe (2003) and Uribe and Yue (2006)). This ensures in steady state, $\frac{b_t}{y_t} = \frac{b}{y}$, i.e., the bond-to-output ratio is constant and depends only on parameters. Our model, as in Neumeyer and Perri (2005). In this framework, a bond adjustment cost is analogous to operational mark-ups caused by financial intermediaries. However, the adjustment cost parameter parameter κ is very small and therefore the business cycle properties of our model are preserved.

[23] An investment adjustment cost is required to make the volatility of private investments relative to output match empirically observed values. This is also a standard procedure in RBC models.

respectively. The first order condition for consumption is given by

$$\{\widetilde{c}_t\} : \lambda_t(1+\tau_c) = \left[(\widetilde{c}_t^*)^{\mu}(1-l_t)^{(1-\mu)}\right]^{-\sigma} \mu(\widetilde{c}_t^*)^{\mu-1}(1-l_t)^{(1-\mu)}, \tag{3.14}$$

where $\widetilde{\beta} = \beta\,(1+\gamma)^{\mu(1-\sigma)}$ and λ_t is the Lagrangian multiplier. The first order condition for labor supply is given by

$$\{l_t\} : \lambda_t(1-\tau_w)\widetilde{w}_t = \left[(\widetilde{c}_t^*)^{\mu}(1-l_t)^{(1-\mu)}\right]^{-\sigma} (1-\mu)(\widetilde{c}_t^*)^{\mu}(1-l_t)^{-\mu}. \tag{3.15}$$

The first order condition with respect to $\widetilde{b}_t$ is given by

$$\left\{\widetilde{b}_t\right\} : 1+\kappa\left[\frac{\widetilde{b}_t}{\widetilde{y}_t}-\frac{b}{y}\right] = E_t\left[\frac{\widetilde{\beta}}{(1+\gamma)}\frac{\lambda_{t+1}}{\lambda_t}R_t^P\right]. \tag{3.16}$$

Finally the first order condition with respect to $\widetilde{k}_t$ (the Euler equation) is given by

$$\left\{\widetilde{k}_t\right\} : 1+\phi\,(1+\gamma)\left[\left(\frac{\widetilde{k}_t}{\widetilde{k}_{t-1}}\right)-1\right] = E_t\left[\widetilde{\beta}\frac{\lambda_{t+1}}{\lambda_t}\left\{\begin{array}{c}\frac{(1-\delta)+(1-\tau_k)r_{t+1}}{(1+\gamma)} \\ +\frac{\phi}{2}\,(1+\gamma)\left\{\left(\frac{\widetilde{k}_{t+1}}{\widetilde{k}_t}\right)^2-1\right\}\end{array}\right\}\right]. \tag{3.17}$$

From (3.14) and (3.15) we get the standard expression for the labor supply (3.18), i.e.,

$$(1-l_t) = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left(\frac{\widetilde{c}_t^*}{\widetilde{w}_t}\right). \tag{3.18}$$

Competitive Equilibrium A competitive equilibrium of our model is defined as follows.

Definition 3.1 *Given* $\{A_t \text{ and } R_t^P\}_{t=0}^{\infty}$, *a vector of fiscal policy parameters* $\{\tau_c, \tau_k, \tau_w, \theta_G, s, \Theta\}$, *and initial conditions* $\widetilde{k}_{-1}$, $\widetilde{b}_{-1}$, R_{-1}^P, *a competitive equilibrium is a vector of allocations of* $\left\{\widetilde{c}_t, \widetilde{k}_t, \widetilde{b}_t, l_t \text{ and } \widetilde{G}_t\right\}_{t=0}^{\infty}$ *and factor prices* $\{\widetilde{w}_t \text{ and } r_t\}_{t=0}^{\infty}$ *such that, for the given sequence of factor prices, (i)* $\left\{\widetilde{k}_t \text{ and } l_t\right\}_{t=0}^{\infty}$ *solves the firm's profit maximization problem (3.4) and (3.5), (ii)* $\left\{\widetilde{c}_t, \widetilde{k}_t, \widetilde{b}_t, l_t\right\}_{t=0}^{\infty}$ *maximizes the utility of the representative agent (3.8) subject to (3.1), (3.10), (3.9), (3.11), (3.12) and (3.13), together with* $\widetilde{c}_t, \widetilde{k}_t \geqslant 0$, *(iii)* $\widetilde{G}_t$ *satisfies (3.7), (iv) a no-Ponzi associated with the initial conditions* k_{-1} *and* b_{-1} *holds for the representative agent, and finally, (v) all markets clear* $\forall t$.

3.2.4 The steady state

From the firm's first order conditions (3.5), at the steady state we obtain

$$\frac{\overline{y}}{\overline{r}\overline{k}} = \frac{1}{(1+\gamma)\alpha} \tag{3.19}$$

and

$$(1-\alpha)\overline{y} = \overline{w}\overline{l}\left[(1-\theta) + \overline{R}^P(\theta - s\theta_G)\right]. \tag{3.20}$$

From equation (3.18), at the steady state, labor supply is given as

$$(1-\overline{l}) = \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left(\frac{\overline{c}+\Theta\overline{G}}{\overline{w}}\right)$$

which implies

$$\overline{l} = 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left(\frac{\overline{c}+\Theta\overline{G}}{\overline{w}}\right) \tag{3.21}$$

where $\overline{c}$, $\overline{G}$, $\overline{y}$ and $\overline{w}$ are steady levels of consumption, government consumption, output and the wage rate respectively.[24] From the first order conditions for the representative agent with respect to bonds (3.16), at the steady state, we obtain

$$\overline{R}^P = \frac{(1+\gamma)}{\widetilde{\beta}} > (1+\gamma), \tag{3.22}$$

since $\widetilde{\beta} \in (0,1)$.

From the first order conditions for the representative agent with respect to capital (3.17), at the steady state, we obtain the following arbitrage condition between $\overline{r}$ and $\overline{R}^P$

$$\overline{r} = \frac{\overline{R}^P - (1-\delta)}{(1-\tau_k)}. \tag{3.23}$$

In sum, the steady state of this economy is summarized by equations (3.19)–(3.23).[25]

3.2.5 The Fiscal Policy Wedge

This section derives the fiscal policy wedge for the model, and shows how the impact of interest rate shocks on labor market outcomes is mitigated by the presence of the wedge.

[24] See the Appendix for details on the steady state of the economy.

[25] See Appendix for other steady state equations.

The effect of interest rate shocks on labor supply

Note that from equation (3.18), and unlike the case with GHH preferences, labor supply depends on current levels of effective consumption, $\widetilde{c}_t^*$ because of the income effect, and $\widetilde{w}_t$. The following Proposition shows that l_t^S can be expressed as a function of consumption, wages and a time varying *fiscal policy wedge*, which we denote by, $\Gamma_t > 1$.

Proposition 3.1 *Labor supply, l_t^S, is given by:*

$$l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\Gamma_t \tag{3.24}$$

where

$$\Gamma_t = \left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\Psi_t}{D_{t-1}} \tag{3.25}$$

and

$$\begin{aligned} D_{t-1} &= 1+\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G+\tau_w\right\} \\ \Psi_t &= \left[1+\Theta\tau_c+\frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\widetilde{c}_t}+\frac{\Theta\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G+\tau_w\right\}\widetilde{w}_t}{\widetilde{c}_t}\right]. \end{aligned}$$

Further, suppose $\tau_c > \tau_w$, $\tau_c > \left[R_{t-1}^P(1-s)-1\right]\theta_G$, and $\mu > 0.5$. This implies that $\Gamma_t > 1$.

Proof. *See Appendix.* ■

The above proposition derives sufficient conditions under which $\Gamma_t > 1$. From equation (3.25), the presence of the fiscal policy wedge reduces labor supply relative to the case $\Gamma_t = 1$. Note also that from equation (3.25), Γ_t depends upon the fiscal policy parameters $\tau_k, \tau_w, \tau_c, s, \theta_G$, and Θ. This implies that the fiscal policy wedge is not just sensitive to the tax rates but also to the subsidy given to the firms.

The above proposition formalizes the mechanism through which interest rate shocks affect labor market outcomes. From equation (3.24), interest rate shocks affect labor supply through two channels in time period, t. First, a positive interest rate shock causes consumption, $\widetilde{c}_t$, to instantaneously fall due to the standard inter-temporal substitution effect (equation (3.16)). Figure 3.2 illustrates the effect of a single period shock to R_t^P on labor supply.

[INSERT FIGURE 3.2]

The second channel occurs through the fiscal policy wedge, Γ_t. As can be seen in Proposition 3.1, Γ_t, consists of two time varying variables D_{t-1} and Ψ_t. The variable D_{t-1} does not change on impact because it depends on R_{t-1}^P, that is, the interest rate that prevailed in time period $t-1$. The variable Ψ_t however increases in time period t due to a reduction in $\widetilde{c}_t$ and an increase in r_t (through the no arbitrage condition). As a result, the fiscal policy wedge, Γ_t, increases on impact due to a positive interest rate shock. Also, for a higher value of Θ, the increase in the fiscal policy wedge is higher for a given interest rate shock. We therefore obtain the following Proposition

Proposition 3.2 *For a positive shock to* R_t^P

$$\frac{\partial \widetilde{c}_t}{\partial R_t^P} < 0 \Longrightarrow \frac{\partial l_t^S}{\partial R_t^P} > 0$$

Further, a positive interest rate shock always increases the fiscal policy wedge, i.e., $\frac{\partial \Gamma_t}{\partial R_t^P} > 0$, *with the effect stronger for a higher* Θ. *An increase in* Γ_t *therefore dampens the outward shift of the labor supply:*

$$\left| \frac{\partial l_t^S}{\partial R_t^P} \right|_{\Gamma_t = 1} > \left| \frac{\partial l_t^S}{\partial R_t^P} \right|_{\Gamma_t > 1} > 0.$$

Figure 3.3 illustrates how Γ_t dampens the effect of a single period shock to R_t^P on labor supply.

[INSERT FIGURE 3.3]

Labor supply moves out to $L^{S''}$ instead of $L^{S'}$ due to an increase in Γ_t. With Cobb-Douglas preferences, the labor supply moves outward as a result of an interest rate shock because consumption drops instantaneously. This is the standard inter-temporal substitution effect, and it is strengthened with a higher value of Θ. This happens because a higher value of Θ implies a higher weight on government consumption in utility which allows households to reduce their private consumption more and push current consumption to the future. However, because of the simultaneous increase in Γ_t, labor supply reduces since an increase in Γ_t makes consumption more expensive in terms of leisure. The net increase in labor supply is therefore determined by the inter-temporal substitution effect and the fiscal policy wedge, Γ_t. When Θ is high, the inter-temporal substitution effect has a stronger effect on labor supply than the fiscal policy wedge. This causes a larger net increase in labor supply in time period, t. When Θ is small (< 1), the inter-temporal substitution effect has a smaller effect on labor supply than the fiscal policy wedge. This causes a smaller net increase in labor supply in time period, t. Importantly, in time period, t, equilibrium labor always increases.

Note that the fiscal policy wedge, Γ_t, is time varying because government consumption directly affects the household's effective consumption $\widetilde{c}_t^*$. Under certain conditions however, the fiscal policy wedge will be constant and greater than 1. This constancy of the wedge implies interest rate shocks will affect labor supply only through consumption, $\widetilde{c}_t$, and not the fiscal policy wedge. Remark 3.1 summarizes these conditions.

Remark 3.1 *Γ_t will be a constant under different specifications of fiscal policy. When there is no fiscal policy, i.e.,*

$$\begin{aligned} \tau_k &= \tau_w = \tau_c = 0 \\ s &= \theta_G = \Theta = 0, \end{aligned}$$

$$\Rightarrow \Gamma_t = \overline{\Gamma} = 1, \ \forall t.$$

When $\widetilde{G}_t$ does not affect $\widetilde{c}_t^*$ or if the government provided a lump-sum income transfer instead to the representative agent, in which case, the fiscal policy wedge is the standard intra-temporal tax wedge, i.e.,

$$\begin{aligned} \Theta &= 0, \\ \Rightarrow \quad & \Gamma_t = \overline{\Gamma} = \left(\frac{1+\tau_c}{1-\tau_w} \right), \ \forall t \end{aligned}$$

Under GHH preferences, Γ_t, is given by[26]

$$\Gamma_t = \overline{\Gamma} = \left(\frac{1-\tau_w}{1+\tau_c} \right)^{\frac{1}{v-1}} \forall t.$$

The effect of interest rate shocks on labor demand

From equation (3.4), we can show that the demand for labor is given by

$$l_t^D = \frac{(1-\alpha)\widetilde{y}_t}{\widetilde{w}_t \left[(1-\theta) + R_{t-1}^P (\theta - s\theta_G) \right]},$$

[26] In this case, $U(\widetilde{c}_t^*, l_t) = \frac{[\widetilde{c}_t^* - \psi l_t^v]^{(1-\sigma)}}{(1-\sigma)}$, and the first order conditions yields:

$$\left(\frac{1-\tau_w}{1+\tau_c} \right) \widetilde{w}_t = \psi v \left(l_t^s \right)^{v-1}.$$

which can be re-written as

$$l_t^D = \left[\frac{(1-\alpha)A_t}{\widetilde{w}_t \left[(1-\theta) + R_{t-1}^P \left(\theta - s\theta_G \right) \right]} \right]^{\frac{1}{\alpha}} \frac{\widetilde{k}_{t-1}}{(1+\gamma)}. \tag{3.26}$$

An increase in R_t^P causes the labor demand curve l^D to shift inwards only in time period $t+1$, that is,

$$\frac{\partial l_{t+1}^D}{\partial R_t^P} = -\frac{l_{t+1}^D \left(\theta - s\theta_G \right)}{\alpha \left[(1-\theta) + R_t^P \left(\theta - s\theta_G \right) \right]}. \tag{3.27}$$

This is shown in the following Figure 3.4.

[INSERT FIGURE 3.4]

The presence of the subsidy parameters θ_G and s however dampens the inward shift of l_{t+1}^D. As shown in Figure 3.5, if the government increases θ_G or s, the reduction in l_{t+1}^D is less, and the new labor demand curve is $L^{D''}$ and not $L^{D'}$.

[INSERT FIGURE 3.5]

Proposition 3.3 summarizes the effect of a single period shock R_t^P on labor demand.

Proposition 3.3 *A positive single period shock to interest rate R_t^P lowers labor demand only in time period $t+1$. However, the presence of θ_G and s, dampens the reduction in l_{t+1}^D. That is*

$$\left| \frac{\partial l_{t+1}^D}{\partial R_t^P} \right|_{s \neq 0, \theta_G \neq 0} < \left| \frac{\partial l_{t+1}^D}{\partial R_t^P} \right|_{s=0, \theta_G = 0}.$$

Proof. See Appendix. ■

Therefore from Proposition 3.2 and Proposition 3.3 we obtain the impact of a single period positive interest rate shock on equilibrium labor and output in time period t. This is shown in Proposition 3.4

Proposition 3.4 *Equilibrium labor l_t increases on impact due to a positive single period shock to R_t^P. This causes output y_t to increase on impact, that is,*

$$\frac{\partial \widetilde{y}_t}{\partial R_t^P} > 0.$$

Fiscal policy dampens the movements in equilibrium labor. This is because an increase in the fiscal policy wedge Γ_t dampens the outward movement of l_t^S and the subsidy parameters, θ_G and s, dampens the inward movement of l_{t+1}^D.

Proof. Follows from Proposition 3.2 and Proposition 3.3, and because private capital in time period $t-1$, $\widetilde{k}_{t-1}$, remains unchanged. ■

An impact of a single period interest rate shock on the fiscal policy wedge Γ_t can be shown in Figure 3.6.[27].

[INSERT FIGURE 3.6]

As can be seen in Figure 3.6 a single period interest rate shock causes an instantaneous increase in the fiscal policy wedge Γ_t. The import of Proposition 3.4 is that in time period t, on impact, equilibrium output increases. However, in $t+1$, the labor demand schedule moves downwards which implies that output will rise/fall depending on the magnitude of the fiscal policy wedge, and the policy parameters in the labor demand curve. If the fiscal policy wedge is strong and the subsidy parameters, (s, θ_G) are small, then a downward movement in labor demand will unambiguously decrease full employment output.

3.3 Calibration

In this section, we calibrate the model to Indian data. Based on the quarterly data available on the Indian macroeconomy documented in Ghate et al. (2013), the stylized facts relevant for India are[28], (a) higher relative consumption volatility, (b) counter-cyclical net exports, (c) counter-cyclical government expenditures, and (d) a pro-cyclical real interest rate. In this paper, we seek to replicate these facts qualitatively.

As noted in the introduction, while the first two facts are common to a wide variety of EMEs, there is no robust stylized observation on the correlation between real government expenditure and output. In some EMEs, government expenditures are counter-cyclical with respect to output and in others it is pro-cyclical. Also, while Neumeyer and Perri (2005) and Uribe and Yue (2005) state that interest rates are generally counter-cyclical in EMEs, Male (2010) finds this observation not to be universally true particularly among EMEs in Africa, Asia and Eastern Europe. In the small sample of EMEs in Neumeyer and Perri (2005), however, the interest rate is counter-cyclical, and there is no role for fiscal policy. Our theoretical model therefore be seen as providing a more general framework that produces a range of business cycle outcomes that are consistent with the broader EME experience.

[27] In Figure 3.6, we assume $\tau_w = \tau_k = 0$. As we increase the factor income tax rates, the magnitude of the increase in Γ_t for a single period shock increases. These results are available from authors on request.

[28] The sample period is Q2-1999 to Q2-2010

3.3.1 Parameter Values

We set the exogenous labor augmenting technological progress for India at $\gamma = 0.047$ as estimated by Bhattacharya et al. (2013). We fix the quarterly capital depreciation rate at $\delta = 0.025$ which approximately matches the annual depreciation rate in India of 0.1 (see Gabriel et al. (2012)). We choose $\alpha = 0.4$ from Ghate et al. (2012). The capital adjustment cost parameter, ϕ is fixed at 60.[29] We assume the bond holding cost parameter $\kappa = 0.0001$ as in Tiryaki (2012). We calculate the steady state bond holdings to output ratio, $\frac{b}{y}$ as in Neumeyer and Perri (2005), using the estimates for the net foreign assets to output ratio $\left(\frac{NFA}{Y}\right)$ for India from Lane and Ferretti (2007).[30] We arbitrarily the share of consumption in the utility function, i.e., $\mu = 0.75$. We fix the value of the discount rate at $\beta = 0.99$ and the value of coefficient of risk aversion parameter at $\sigma = 2.3$. The choice of μ, β and σ are such that the calibrated value of $\overline{R}^P$ is approximately consistent with average long run value of the Prime Lending Rates (PLR) of three major banks in India.[31] We fix the value of $\theta = 1$ as in Neumeyer and Perri (2005). We choose $\theta_G \leq \theta$ according to our choice of θ. In our baseline calibrations, we arbitrarily set $\theta_G = 1$[32], which means that the entire working capital constraint is subsidized. We arbitrarily choose a value of $\Theta > 1$.[33] We choose s such that

$$\overline{R}^G = \overline{R}^P(1 - s) > 1.$$

Given that India has a very narrow income tax base and depends more on generating revenue from indirect taxation, we allow for a high tax on consumption and a low income tax (see Poirson (2006)). In particular, the value of τ_c is fixed at 0.12 to match the VAT rate applicable in India. We fix the factor income tax rates low at $\tau_k = \tau_w = 0.01$ which follows the estimated average effective tax rates in Poirson (2006). Table 3 summarizes our choice

[29] The value of ϕ is higher than what is typically assumed in the literature. Neumeyer and Perri (2005) assume $\phi = 25$ and $\phi = 40$ in their numerical experiments. We choose a higher value of ϕ to capture the large adjustment costs of capital in low income EMEs.

[30] See Appendix for details.

[31] We consider the average nominal PLR of three major banks in India - the State Bank of India SBI, ICICI bank and IDBI bank. We construct the quarterly data from the daily data available for each bank in the CEIC database. For the CEIC database visit http://www.ceicdata.com/en/countries/india. Some of the missing datapoints on the PLR for SBI was obtained using the data published by Reuters India. For Reuters India visit http://in.reuters.com/article/2013/06/10/india-plr-idINL3N0EM1YU20130610?type=companyNews. We then deflate the quarterly interest rates using the quarterly inflation using the CPI data. See Table 170, Handbook of Statistics of the Indian Economy, RBI. http://www.rbi.org.in/scripts/PublicationsView.aspx?id=14528.

[32] Though we choose $\theta_G = 1$, the subsidy s is very small, which implies the effective amount of subsidized loan from the government is not very large.

[33] In our baseline calibrations, we arbitrarily fix $\Theta = 5$. Since the representative agent takes G as given in every time period, $\Theta > 1$ is feasible. A high value of Θ implies that government consumption is very efficient and the representative agent attaches high weightage to it. As we will show, this assumption is crucial for making consumption more volatile than output in our model.

of deep parameters in our model.[34]

Parameter Name	Symbol	Value
Coefficient of risk aversion (calibrated)	σ	2.3
Share of consumption in utility function (calibrated)	μ	0.75
Depreciation rate	δ	0.025
Rate of technical progress (Bhattacharya et al. (2013))	γ	0.047
Ratio of wage bill to be paid in advance	θ	1
Discount rate (calibrated)	β	0.99
Effective discount rate (calibrated)	$\widetilde{\beta}$	$\beta(1+\gamma)^{\mu(1-\sigma)}$
Real interest rate (calibrated)	$\overline{R}^P$	$\frac{(1+\gamma)}{\widetilde{\beta}}$
Share of capital in production (Ghate et al. (2012))	α	0.4
Bond holding costs (Tiryaki (2012))	κ	0.0001
Capital adjustment costs	ϕ	60
Subsidized portion of the advance wage bill ratio	θ_G	$\leq \theta$
Subsidy on working capital loans	s	0.1
Tax on consumption (VAT rate in India)	τ_c	0.12
Tax on labor income (Poirson (2006))	τ_w	0.01
Tax on capital income	τ_k	$= \tau_w$
Weightage of government consumption in c_t^*	Θ	$\geqslant 1$
Steady state TFP	$\overline{A}$	1

Table 3: Summary of Parameter Values

3.3.2 Estimation of the data generating processes

We calibrate the model using total factor productivity (TFP) shocks and interest rate shocks. We obtain annual data for total factor productivity for the period 1980-2008 from the Penn World Tables version 8.0 (2014).[35] We use the variable "rtfpna", a TFP index with base year 2005, as reported in the Tables.[36] The aggregate log-TFP data is then de-trended using

[34]The rest of our endogenous and exogenous variables are derived at the steady state based on these parameter values.
[35]See http://www.rug.nl/research/ggdc/data/penn-world-table
[36]For 2005, rtfpna = 1.

a HP-Filter using a standard annual smoothing parameter equal to 100 such that

$$\begin{aligned} \widehat{A}_t &= \rho_A \widehat{A}_{t-1} + \varepsilon_{tA}, \\ \text{where } \rho_A &= 0.42 \end{aligned} \tag{3.28}$$

with a standard error of regression $\sigma_A = 0.012$. $\widehat{A}_t$ is the de-trended log-TFP data.

To estimate the data generating process for real interest rates, we use the annual real interest rates data published by World Bank for the period 1980 - 2008. We choose annual data to maintain consistency in the frequency across all data generating processes.[37] The domestic interest rate on bonds is modelled as a mark-up on the world interest rate, i.e.,

$$R_t^P = R_t^* D_t \tag{3.29}$$

where R_t^P is the gross domestic real lending rate in India and R_t^* is the world interest rate which is assumed to be the US gross real lending rate. D_t is the country spread over R_t^*.[38] We de-trend the gross real interest rate data using a standard annual smoothing parameter equal to 100 such that

$$\widehat{R}_t^P = \widehat{R}_t^* + \widehat{D}_t \tag{3.30}$$

where a variable $\widehat{x}_t$ is the de-trended value of x_t from its steady state $\overline{x}$. We then estimate an AR(1) process on $\widehat{R}_t^*$ is an AR(1) process to be

$$\begin{aligned} \widehat{R}_t^* &= \rho_R \widehat{R}_{t-1}^* + \varepsilon_{tR} \\ \text{where } \rho_R &= 0.462 \end{aligned} \tag{3.31}$$

with standard error $\sigma_{R^*} = 0.004$.

As in Neumeyer and Perri (2005), we assume that the de-trended country spread component $\widehat{D}_t$ depends on future expected total factor productivity. In other words, with a higher future expected total factor productivity, the repayment capacity of borrowers increases, which causes a reduction in the country spread risk. Therefore $\widehat{D}_t$ contains two components - an idiosyncratic risk component (u_t) and second a term that depends upon the expected

[37]The real interest rate is calculated as the lending rate adjusted for inflation using the GDP deflator. See http://data.worldbank.org/indicator/FR.INR.RINR. These lending rates are rates at which short and medium term financing needs of the private sector are met. These lending rates are differentiated according to the credit-worthiness of borrowers.

[38]Neumeyer and Perri (2005) and Tiryaki assume 91-day US Treasury bill rate. Ghate et. al (2013) also report the second order moments for India using a 91-day Treasury bill rate.

future total factor productivity, i.e.,

$$\begin{aligned} \widehat{D}_t &= -\eta E_t \widehat{A}_{t+1} + u_t \qquad (3.32) \\ & u_t \text{ is a random shock} \\ \eta &= 0.4425 \end{aligned}$$

with a standard error of regression $\sigma_U = 0.006$. We find that the above relation between country spreads and expected TFP is statistically significant.[39] This means, a higher expected total factor productivity in time period $t+1$ indeed lowers the country spread in time period t. We assume that all shocks are uncorrelated.

3.3.3 Single Period Shocks

There are three shocks in our model – TFP shocks $\left(\widehat{A}_t\right)$, world interest rate shocks $\left(\widehat{R}_t^*\right)$, and shocks to the country spread risk (u_t). Since from equation (3.30)

$$\widehat{R}_t^P = \widehat{R}_t^* + \widehat{D}_t,$$

a TFP shock will lower $\widehat{R}_t^P$ through a reduction in D.

We will analyze the effect of single period shocks on $\widetilde{y}_t$, $\widetilde{c}_t$, $\widetilde{x}_t$, $\widetilde{G}_t$, $\widetilde{l}_t$, and net exports, $\widetilde{nx}_t$.[40] We will see how a single period 10% shock affects the deviations of these variables from their corresponding steady state values. In particular, for any variable z, we define variable lz as the deviation of the variable z from it's steady state value of $\overline{z}$ i.e.,

$$lz = z_t - \overline{z}.$$

[39] In Neumeyer and Perri (2005), this model is called *the induced country risk case.* They also estimate another case, *the independent country risk case,* where, $\widehat{D}_t$, is assumed evolve according to an exogenous process. This exogenous process is assumed to follow an AR(1) process. However, an AR(1) fit for $\widehat{D}_t$ in our model was not statistically significant given our choice of the interest rate series. We therefore report all our calibration results only for the induced country risk case. As an alternative exogenous process, we assume

$$\widehat{D}_t = \varepsilon_{tD},$$

results for which are available on request.

[40] We have chosen the value of $\tau_k = \tau_w = \tau = 0.01$ for generating the impulse responses in this section. While the impulse responses are on net exports, our calibrated second order moments are on net export to output ratio. This is similar to Neumeyer and Perri (2005), Ghate et al. (2013), and Tiryaki (2012).

TFP shock

Figure 3.7 plots the impulse response functions due to a single period shock in total factor productivity (A).

[INSERT FIGURE 3.7]

A one period positive total factor productivity shock, instantaneously causes output to increase. As a result, the deviation of output from its corresponding steady state value (ly) increases. This is because an increase in the firm's productivity causes an increase in labor demand and demand for private capital. An increase in the demand for private capital causes an increase in private investments (lx). While an increase in the firm's demand for labor causes an increase in equilibrium labor (ll) on impact and raises output, the positive income effect causes consumption (lc) to also increase in comparison to it's steady state value. Government consumption (lg) also increases on impact first due to a positive TFP shock. This is because an increase in output raises total tax revenue. R_t^P (shown as R in Figure 3.7) falls due to a reduction in the country spread risk. This occurs with a lag. Therefore interest incomes accruing to the government in time period $t+1$ falls. This causes a drop in lg in period $t+1$, and thereafter, converges very slowly to the steady state from above. This is because $\widehat{A}$ is more persistent than $\widehat{R}^P$, $|\eta| < 1$, and lc does not converge to the steady state even after 40 quarters. The savings-investment (S-I) gap (shown as si_gp) falls on impact because savings decrease and investment (lx) increases. The public revenue-expenditure gap (shown as tr_gp) increases in time period $t+1$ because

$$TR_t - G_t = -\left\{\left[R_{t-1}^G - 1\right]\theta_G\right\} w_t l_t. \tag{3.33}$$

This is because tr_gp in time period t depends on the interest rates of time period $t-1$ which falls because of a TFP shock. An instantaneous drop in net exports ($ln\ x$) therefore occurs due to a fall in the savings-investment gap. Net exports thereafter quickly converges to the corresponding steady state value because the public revenue-expenditure gap increases in period $t+1$.

Interest rate shock

Figure 3.8 shows the impact of a single period shock to the world interest rate R^*. The domestic interest rate (R) increases which causes an instantaneous drop in private consumption (lc) due to the inter-temporal substitution effect.

[INSERT FIGURE 3.8]

Therefore, equilibrium labor (ll) increases on impact due to a reduction in (lc). Output in time period t depends on l_t and k_{t-1}. Since k_{t-1} in time period t is given from time period $t-1$, ly increases on impact due to an increase in ll. Government consumption (lg) drops in time period t before it increases above the steady state level in $t+1$. This is because an initial reduction in tax revenue from lower consumption levels dominates the increase in tax revenue from the wage and rental income taxation.[41] A positive interest rate shock causes government consumption lg to then jump above the steady state in time period $t+1$ before it starts converging to the steady state from levels below zero. A combination of opposite movements in lc and lg causes ll to finally converge to zero. Investments (lx) falls because the private rate of return (lr) increases through the no arbitrage condition. An increase in savings due to postponement of consumption and reduction in investments causes the savings-investment gap (shown as si_gp) to increase on impact. The government revenue to expenditure gap denoted by tr_gp (see (3.33)) falls because labor (ll) increases. An increase in the savings-investment gap and a reduction in tr_gp therefore makes lnx increase on impact as shown in Figure 3.8.

The impact of a single period shock to idiosyncratic risk (u) is shown in Figure 3.9.

[INSERT FIGURE 3.9]

The intuition for a single period shock to u is identical to a single period shock to the world interest rate R^* at time period t. The impulse responses however seem to be converging to the steady state very quickly since u is not persistent.

3.3.4 Multi-period Shocks

Next, we calibrate our model with multi-period uncorrelated shocks to TFP $\left(\widehat{A}_t\right)$, world interest rates $\left(\widehat{R}^*_t\right)$ and idiosyncratic shocks to the country spread $(\widehat{u}_t)$ and compare the second order moments of our simulated data with the Indian quarterly data from 1999 Q2 to 2010 Q2.[42] Table 4 summarizes our calibration results. We calibrate the model in stages to assess goodness of fit. First, we estimate the second order moments of our model when there is no fiscal policy in the baseline model. The results from estimating this model are reported in the column "No Fiscal Policy". In this case, the fiscal policy wedge, $\Gamma_t = \overline{\Gamma} = 1$. Second,

[41] Clearly, the initial rise or fall in lg depends on the choice of fiscal policy parameters. As we discussed above, our choice of $\tau_c > \tau_w = \tau_k$ also puts higher weightage on the tax on consumption in comparison to tax on wage and capital income.

[42] See table (5) in Ghate et. al (2013) for the Indian data. The simulated series estimated by the model was generated for 500 time periods and was then hp-filtered with the value of the multiplier chosen to be 1600, a standard value used to hp-filter quarterly data.

we include only government consumption $\widetilde{G}_t$ financed by factor income taxes. The results from estimating this model are reported in the column "Only G". Third, we assume that in addition to $\widetilde{G}_t$ the government also subsidizes working capital loans, where $R_t^G = R_t^P(1-s)$, to firms on the fraction, $\theta_G w_t l_t$, of their wage payments. We report results obtained by estimating this model in the column "G and S". The column "G and S (with high Θ)" reports results for a high value of $\Theta = 75$. Finally, the column "Actual Data" reports the actual second order moments of the Indian data from Ghate et al. (2013).

Moments	No Fiscal Policy	Only G	G and S	G and S (with high Θ)	Actual Data
(1)	(2)	(3)	(4)	(5)	(6)
$\rho(C,Y)$	0.6033	0.4586	0.5126	0.5045	0.51
$\rho(X,Y)$	0.1330	0.1022	0.1103	0.0247	0.69
$\rho(R,Y)$	-0.0832	-0.0458	-0.0546	0.0754	0.38
$\rho(\frac{NX}{Y},Y)$	0.1912	0.2562	-0.1505	-0.1792	-0.15
$\rho(G,Y)$	–	0.6882	-0.32	-0.0229	-0.35
$\sigma(C)/\sigma(Y)$	0.3548	0.3236	1.20	1.69	1.31
$\sigma(X)/\sigma(Y)$	10.9	10.11	10.23	7.23	3.43
$\sigma(R)/\sigma(Y)$	0.48	0.439	0.44	0.28	1.77
$\sigma(NX)/\sigma(Y)$	11.13	10.57	10.64	7.82	1.04
$\sigma(G)/\sigma(Y)$	–	0.358	1.55	0.23	5.53

Table 4 : Comparison between the model and the data[43]

No fiscal policy

In this case, the fiscal policy wedge, $\Gamma_t = \overline{\Gamma} = 1$. This is because: $\tau_k = \tau_w = \tau_c = 0$, $s = 0$, and $\theta_G = \Theta = 0$. The labor demand equation (3.26) becomes

$$l_t^D = \left[\frac{(1-\alpha)A_t}{\widetilde{w}_t\left[(1-\theta)+\theta R_{t-1}^P\right]}\right]^{\frac{1}{\alpha}} \frac{\widetilde{k}_{t-1}}{(1+\gamma)}. \tag{3.34}$$

Unlike in equation (3.26), in equation (3.34) the effect of a positive shock to R_t^P on labor demand is fully transmitted through the labor market by lowering labor demand. The labor

[43] $\rho(Z,Y)$ is the correlation coefficient of variable Z with output Y. $\sigma(Z)/\sigma(Y)$ is the relative standard deviation of variable Z with output Y. Also, refer to table (2) for second order moments of the Indian data.

supply equation (3.24), becomes the standard labor supply equation in the absence of fiscal policy:

$$l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right). \tag{3.35}$$

As shown in column 2 in Table 4, we observe that consumption, investment, and the net-export to output ratio are pro-cyclical, whereas real interest rates are weakly counter-cyclical. While this model under-estimates the relative volatility of consumption and the real interest rate, it over-estimates the relative volatility of investment and the net-export to output ratio.

Government consumption

Suppose that the government imposes factor income taxes and spends it only on government consumption that affects utility. The government, however, does not subsidize working capital loans. Under this specification,

$$\begin{aligned}
\tau_k &= \tau_w = \tau > 0, \tau_c > 0 \\
s &= \theta_G = 0 \\
\Theta &> 0.
\end{aligned}$$

Labor supply in (3.24) is now given by

$$l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\Gamma_t^{'}. \tag{3.36}$$

The fiscal policy wedge, in this case, $\Gamma_t^{'}$, is given by

$$\Gamma_t^{'} = \left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\Psi_t^{'}}{D_{t-1}^{'}}$$

such that

$$\begin{aligned}
D_{t-1}^{'} &= 1 + \Theta\tau_w\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right) > 1 \text{ and} \\
\Psi_t^{'} &= \left[1 + \Theta\tau_c + \frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t} + \frac{\Theta\tau_w\widetilde{w}_t}{\widetilde{c}_t}\right] > 1.
\end{aligned}$$

As shown in column 3 government consumption is quite strongly pro-cyclical. This is expected because government expenditure given by

$$\widetilde{G}_t = \tau_c \widetilde{c}_t + \tau_w \widetilde{w}_t l_t + \tau_k r_t \frac{\widetilde{k}_{t-1}}{(1+\gamma)}.$$

which is a fraction of private income. Therefore any change in consumption and income directly affects government consumption. It is however estimated to be less volatile than output which is at odds with the actual data.

In this model real interest rates are even less counter-cyclical compared to the "No Fiscal Policy" case in column 2, and investment is less pro-cyclical. A positive interest rate shock does not have a direct effect on the fiscal policy wedge, although it does affect the wedge, and therefore l_t^S, indirectly through other endogenous variable such as $\widetilde{c}_t$, r_t, $\widetilde{k}_t$, and $\widetilde{w}_t$. In particular, a positive interest rate shock always increases the fiscal policy wedge, which tends to offset the outward movements in labor supply due to a reduction in consumption. Interest rates therefore tend to become less counter-cyclical.

The important difference however, is that the relative volatility of consumption, investments, and net exports have all fallen compared to the model with no fiscal policy. This suggests that $\widetilde{G}_t$ has a stabilizing effect. Consumption becomes less volatile because on the one hand households have to pay factor income taxes while on the other, a reduction in consumption due to taxes are returned to the agents through $\widetilde{G}_t$ which is more volatile than consumption.

Including $\widetilde{G}_t$ also makes the net-exports to output ratio more pro-cyclical. We can rewrite the net exports-to-output ratio as,

$$nx_t = \frac{(s_t - x_t) + (TR_t - G_t)}{y_t},$$

where s is savings and x is investments. Since in every time period,

$$TR_t = G_t,$$

we get

$$nx_t = \frac{s_t - x_t}{y_t}.$$

Since investments are weakly pro-cyclical with output, the cyclicality of savings with output dominates. Therefore the net-exports to output ratio will be pro-cyclical.

Government consumption and subsidy

Now suppose

$$\begin{aligned}\tau_k &= \tau_w = \tau > 0, \tau_c > 0\\ s &> 0,\ \theta_G > 0\\ \Theta &> 0.\end{aligned}$$

This is our model as discussed in Section 3.2 in which labor supply is now given by equation (3.24) and labor demand is given by equation (3.26). The moments are summarized in column 4 of Table 4. In contrast to the model with only government consumption, we now get counter-cyclical government consumption, counter-cyclical net-exports to output ratio, pro-cyclical consumption and investments, and weakly counter-cyclical real interest rates.[44] Our model qualitatively replicates the standard stylized facts that motivate the theoretical framework of Neumeyer and Perri (2005) through an alternate but compatible mechanism.

We find that contemporaneous correlation of consumption, the net exports-to-output ratio, and government expenditure with respect to output are very close to the Indian data. While private investments are less pro-cyclical as compared to the data (which is due to a highly over-estimated investment volatility), interest rates continue to be weakly counter-cyclical. Relative volatility of consumption, investments, net exports, and government expenditures are all closer to the Indian data and higher in the presence of subsidies to the working capital loans to firms.

Government expenditures are now significantly counter-cyclical due to high subsidies given to firms. The net-exports to output ratio, given by

$$nx_t = \frac{(s_t - x_t) + (TR_t - G_t)}{y_t},$$

are counter-cyclical because of a falling savings-investment gap and a negative public revenue-expenditures gap. Finally, strong income effects from TFP on consumption and the dampening effect of subsidies on the labor, and therefore output, increases the relative volatility of consumption with respect to output.

While the model with all three shocks and both roles for fiscal policy qualitatively match the stylized facts of EMEs, we find so far that the calibrated moments of this variant of the model (full fiscal policy) are also closer to the Indian data than columns 2 and 3. However, while predicted relative consumption volatility is approximately close to the Indian data,

[44] In this section, we choose $\Theta = 5$. In the section on counter-factuals, we discuss the effect of Θ and θ_G on the cyclical properties of interest rates, government expenditure, and the net exports-output ratio.

relative volatility of government consumption is under estimated and that of investment and net exports are over estimated. We also find that contemporaneous correlations of investment and real interest rates are much less as compared to the actual Indian data.

Government consumption and subsidy (under high Θ)

In our model, fiscal policy affects the economy by distorting the labor market equilibrium. On the one hand, a subsidy from the government affects the labor demand, whereas on the other, the utility enhancing government, which imposes a time-varying fiscal policy wedge, affects labor supply by influencing not just its inward-outward movements, but also its slope. As a result, fiscal policy affects the transmission of the interest rate shocks on the labor market, and therefore affects the fluctuations in other endogenous variables such as output, consumption, and net exports. In this section, we therefore analyze the effect of changes in Θ on some of our calibration results.

In our simulations for arbitrary values of $\Theta = \{0.5, 5, 75\}$, we obtain the following observations.[45] While government expenditures are relatively unimportant in comparison to private consumption for $\Theta < 1$, an increase in Θ makes government consumption more important for the agent. Hence, an increase in Θ makes consumption more volatile, as can be seen in Column 5 of Table 4. The higher volatility causes a reduction in the contemporaneous correlation between private consumption and output. In particular, when $\Theta < 1$, private consumption is less volatile compared to output, which is not true for our sample of EMEs. Our model suggests that a higher Θ generates this higher consumption volatility.

As can be seen from Column 5, for very high values of Θ, $\rho(R, Y) > 0$, i.e., real interest rates, from being counter-cyclical, become pro-cyclical. As Θ increases, private consumption becomes more volatile. As a result, a positive interest rate shock causes a bigger reduction in current consumption. A big reduction in current consumption dominates the dampening effect of an increase in Γ on labor supply. The net effect is that equilibrium labor increases by more due to a positive interest rate shock. The real interest rate is however only mildly pro-cyclical because the productivity shock has also exerted a simultaneous contemporaneous positive income effect.

Government expenditures also become weakly counter-cyclical (with a higher Θ)$-$ as in Column 5 – which therefore makes the net exports-to-output ratio marginally more counter-cyclical. From equation (3.7), we know that government expenditure is financed by taxing consumption, household wage and rental income, and a net interest income from lending to firms. As Θ increases, with a positive interest rate shock, on the one hand, an increase in income and an increase in the interest rate increases revenue generated from wage, capital,

[45] These results are available from the authors on request.

and interest incomes. This makes government expenditures less counter-cyclical with a progression towards becoming pro-cyclical. On the other hand, a big reduction in consumption due to an interest rate shock causes a reduction in revenue from taxing consumption. This makes government expenditures fall for an increase in output, thereby making it counter-cyclical. Since taxing consumption is a bigger revenue source for the government, the net effect is that increases in revenue from taxing incomes is dampened by reductions in revenue from consumption. This makes government expenditures weakly counter-cyclical or almost a-cyclical. Being a residual, the net-export-to-output ratio becomes marginally more counter-cyclical.

3.4 The Role of Fiscal Policy as an Automatic Stabilizer

Our model outlines how fiscal policy can play the role of an automatic stabilizer when an economy is hit with interest rate shock which adversely affects labor market outcomes and also real output. Since government expenditure in our model is non-discretionary, it adjusts automatically with other endogenous variables. Table 4 shows how fiscal policy dampens overall volatility in the economy, but leads to a trade-off. A rise in Θ results in pro-cyclical interest rates *and* lesser relative volatility for X, R, NX, and G, even though these outcomes obtain at the expense of higher consumption volatility (see Column 5).[46] Higher consumption volatility happens because of a strong inter-temporal substitution effect driven by the private and public components of consumption beings perfect substitutes and Θ is high. This makes private consumption's response to a positive interest rate shock high. A higher τ_c would also make private consumption's response to a positive interest rate shock high making interest rates more pro-cyclical.

Table 4 also identifies the intuition behind why government spending volatility goes down. This happens because as long as τ_c is sufficiently large, a positive interest rate shock reduces $\tau_c \widetilde{c}_t$, which also reduces $\widetilde{G}_t$. A reduction in $\widetilde{c}_t$ leads to an increase in labor supply and output. Thus $\widetilde{y}_t$ increases and $\widetilde{G}_t$ falls. A rise in Θ causes a bigger increase in $\widetilde{l}_t$ which makes $\widetilde{y}_t$ more volatile. This causes a reduction in the relative volatility of $\widetilde{G}_t$.

As discussed in the previous section, since the impact of the fiscal policy wedge $\left(\frac{\partial \Gamma_t}{\partial R^P_{t-1}}\right)$ is increasing in Θ, a higher value of Θ reduces labor supply more when Θ is higher. However, a higher Θ also leads to a larger reduction in private consumption, $\widetilde{c}_t$, and this inter-temporal substitution effect leads to an increase in labor supply which off-sets the reduction in labor

[46] While we do not assess welfare in the model, Gali (1994) shows that large taxes and large government expenditures – while stabilizing in nature – have welfare reducing effects.

supply from the fiscal policy wedge. The net effect on labor supply is therefore positive. Therefore the strength of substitutability between, $\widetilde{c}_t$, and $\widetilde{G}_t$, captured by Θ is crucial for fiscal policy's role as an automatic stabilizer.

3.5 Conclusion

We build a tractable small open economy RBC model in which fiscal policy has a role in making *pro-cyclical* real interest rates consistent with counter-cyclical net exports and higher consumption volatility. Our theoretical model contributes to the growing literature on fiscal policy in small open economies. In particular, we show that by adding a role for fiscal policy in the Neumeyer and Perri (2005) setup, we are able to establish a causal link between the nature of fiscal policy (counter-cyclical or a-cyclical), real interest rates (pro-cyclical or counter-cyclical), counter-cyclical net exports, and higher relative consumption volatility. Our framework therefore can be seen as a more general framework to understand the effect of interest rate shocks on the real economy discussed in the empirical business cycle literature. We then calibrate the model to India to qualitatively match its business cycle properties. We also discuss the role that fiscal policy as an automatic stabilizer in the context of our model.

From a policy standpoint, our model suggests how the adverse effects of interest rate shocks on labor market outcomes can be mitigated by fiscal policy. For future work, we want to introduce sovereign debt and endogenize country spreads with sovereign default risks. We also want to undertake a welfare analysis of various types of fiscal policy in our model.

3.6 Technical Appendix

Steady state calculations

The following are the set of steady state conditions that will be used in the model

$$\overline{R}^P = \frac{(1+\gamma)}{\widetilde{\beta}} \tag{3.37}$$

$$\overline{r} = \frac{\overline{R} - (1-\delta)}{(1-\tau_k)} \tag{3.38}$$

From the firm's FOC,

$$\overline{y} = \frac{\overline{r}\overline{k}}{(1+\gamma)\alpha} \tag{3.39}$$

and

$$(1-\alpha)\overline{y} = \overline{w}\overline{l}\left[(1-\theta) + \overline{R}^P(\theta - s\theta_G)\right]. \tag{3.40}$$

Rearranging equation (3.21) we get

$$\overline{l} = \frac{1}{\left[1 + \frac{(1-\mu)}{\mu}\frac{(1+\tau_c)}{(1-\tau_w)}\frac{\frac{\overline{c}}{\overline{y}} + \Theta\frac{\overline{G}}{\overline{y}}}{\frac{\overline{w}\overline{l}}{\overline{y}}}\right]}. \tag{3.41}$$

From the output technology we know

$$\widetilde{y}_t = \frac{A_t}{(1+\gamma)^\alpha}\left[\widetilde{k}_{t-1}\right]^\alpha l_t^{1-\alpha}.$$

This implies, at steady state,

$$\overline{y} = \frac{\overline{A}}{(1+\gamma)^\alpha}\overline{k}^\alpha\overline{l}^{1-\alpha}.$$

Substituting from equation (3.39)

$$\frac{\overline{r}\overline{k}}{(1+\gamma)\alpha} = \frac{\overline{A}}{(1+\gamma)^\alpha}\overline{k}^\alpha\overline{l}^{1-\alpha},$$

which in rearranging we get

$$\overline{k} = \left[\frac{\overline{A}\alpha}{\overline{r}}\right]^{\frac{1}{1-\alpha}}(1+\gamma)\,\overline{l}. \tag{3.42}$$

From (3.40),

$$\overline{w} = \frac{(1-\alpha)\overline{y}}{\left[(1-\theta) + \overline{R}^P(\theta - s\theta_G)\right]\overline{l}}. \tag{3.43}$$

To calculate $\overline{k}$, $\overline{y}$, $\overline{w}$, and $\overline{r}$, we need $\overline{l}$. To obtain this, we proceed as follows. From the budget constraint

$$\frac{\overline{c}}{\overline{y}} = \frac{(1-\tau_w)\frac{\overline{w}\overline{l}}{\overline{y}}}{(1+\tau_c)} + \frac{(1-\tau_k)\frac{\overline{r}\overline{k}}{\overline{y}}}{(1+\tau_c)(1+\gamma)} + \frac{\frac{\overline{R}^P\overline{b}}{\overline{y}}}{(1+\tau_c)(1+\gamma)} - \frac{\frac{\overline{k}}{\overline{y}}}{(1+\tau_c)} + \frac{(1-\delta)\frac{\overline{k}}{\overline{y}}}{(1+\gamma)(1+\tau_c)} - \frac{\frac{\overline{b}}{\overline{y}}}{(1+\tau_c)}. \tag{3.44}$$

where, $\left(\frac{b}{y}\right)$ is known. From the government budget constraint

$$\frac{\overline{G}}{\overline{y}} = \left[\tau_c\frac{\overline{c}}{\overline{y}} + \left\{\left[\overline{R}^P(1-s)-1\right]\theta_G + \tau_w\right\}\frac{\overline{w}\overline{l}}{\overline{y}} + \frac{\tau_k}{(1+\gamma)}\frac{\overline{r}\overline{k}}{\overline{y}}\right]. \tag{3.45}$$

The above two expressions imply

$$\begin{aligned}
\frac{\overline{c}}{\overline{y}} + \Theta\frac{\overline{G}}{\overline{y}} &= \frac{\overline{w}\overline{l}}{\overline{y}}\left[\frac{(1-\tau_w)(1+\Theta\tau_c)}{(1+\tau_c)} + \Theta\left\{\left[\overline{R}^P(1-s)-1\right]\theta_G + \tau_w\right\}\right] \\
&+ \frac{\overline{r}\overline{k}}{\overline{y}(1+\gamma)}\left[\frac{(1-\tau_k)(1+\Theta\tau_c)}{(1+\tau_c)} + \Theta\tau_k\right] \\
&+ \frac{\overline{b}(1+\Theta\tau_c)}{\overline{y}(1+\tau_c)}\left[\frac{\overline{R}^P}{(1+\gamma)} - 1\right] - \frac{\overline{k}(1+\Theta\tau_c)}{\overline{y}(1+\tau_c)}\left[\frac{\gamma+\delta}{(1+\gamma)}\right]
\end{aligned}$$

Simplifying, we get,

$$\begin{aligned}
\frac{\overline{c}}{\overline{y}} + \Theta\frac{\overline{G}}{\overline{y}} &= \frac{(1-\alpha)}{\left[(1-\theta)+\overline{R}^P(\theta - s\theta_G)\right]}\left[\frac{(1-\tau_w)(1+\Theta\tau_c)}{(1+\tau_c)} + \Theta\left\{\left[\overline{R}^P(1-s)-1\right]\theta_G+\tau_w\right\}\right] \\
&+\alpha\left[\frac{(1-\tau_k)(1+\Theta\tau_c)+(1+\tau_c)\Theta\tau_k}{(1+\tau_c)}\right] \\
&+\frac{(1+\Theta\tau_c)\left[\overline{R}^P-(1+\gamma)\right]}{(1+\tau_c)(1+\gamma)}\left[\frac{(1-\alpha)\theta}{\left[(1-\theta)+\overline{R}^P(\theta-s\theta_G)\right]} + \frac{\overline{NFA}}{\overline{y}}\right] \\
&-\frac{(1-\tau_k)(1+\gamma)(1+\Theta\tau_c)\alpha(\gamma+\delta)}{\left[\overline{R}^P-(1-\delta)\right](1+\tau_c)(1+\gamma)} \\
&= \Upsilon, \text{ say.}
\end{aligned}$$

Recall, $\frac{NFA}{Y}$ is a parameter, calculated using the Lane and Ferretti (2007) data. Therefore, substituting for $\frac{\overline{c}}{\overline{y}} + \Theta\frac{\overline{G}}{\overline{y}}$ in equation (3.41), we get,

$$\overline{l} = \frac{1}{\left[1 + \frac{(1-\mu)}{\mu}\frac{(1+\tau_c)}{(1-\tau_w)}\frac{\Upsilon\left[(1-\theta)+\overline{R}^P(\theta-s\theta_G)\right]}{(1-\alpha)}\right]}.$$

We obtain all other parameters as follows

$$
\begin{aligned}
\frac{\overline{k}}{\overline{y}} &= \frac{(1-\tau_k)(1+\gamma)\alpha}{\overline{R}^P-(1-\delta)} \\
\frac{\overline{w}\overline{l}}{\overline{y}} &= \frac{(1-\alpha)}{\left[(1-\theta)+\overline{R}^P\left(\theta-s\theta_G\right)\right]} \\
\frac{\overline{r}\overline{k}}{\overline{y}} &= (1+\gamma)\alpha \\
\overline{r} &= \frac{\overline{R}^P-(1-\delta)}{(1-\tau_k)} \\
\frac{\overline{b}}{\overline{y}} &= (\theta-\theta_G)\frac{(1-\alpha)}{\left[(1-\theta)+\overline{R}^P\left(\theta-s\theta_G\right)\right]}+\frac{\overline{NFA}}{\overline{y}} \\
\frac{\overline{c}}{\overline{y}} &= \frac{(1-\alpha)}{\left[(1-\theta)+\overline{R}^P\left(\theta-s\theta_G\right)\right]}\frac{(1-\tau_w)}{(1+\tau_c)}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\alpha(1-\tau_k)}{(1+\tau_c)}-\frac{(1-\tau_k)\left(\gamma+\delta\right)\alpha}{\left[\overline{R}^P-(1-\delta)\right](1+\tau_c)} \\
&+\frac{1}{(1+\tau_c)}\left[\frac{\overline{R}^P}{(1+\gamma)}-1\right]\left[\frac{(1-\alpha)\theta}{\left[(1-\theta)+\overline{R}^P\left(\theta-s\theta_G\right)\right]}+\frac{\overline{NFA}}{\overline{y}}\right]
\end{aligned}
$$

$$
\frac{\overline{G}}{\overline{y}} = \left[\tau_c\frac{\overline{c}}{\overline{y}}+\left\{\left[\overline{R}^P(1-s)-1\right]\theta_G+\tau_w\right\}\frac{\overline{w}\overline{l}}{\overline{y}}+\frac{\tau_k}{(1+\gamma)}\frac{\overline{r}\overline{k}}{\overline{y}}\right], \text{ follows.}
$$

Finally for net exports,

$$
\begin{aligned}
\widetilde{NX}_t &= \widetilde{y}_t-\widetilde{c}_t-\widetilde{x}_t-\widetilde{G}_t \\
&\Rightarrow \frac{\widetilde{NX}_t}{\widetilde{y}_t}=1-\frac{\widetilde{c}_t}{\widetilde{y}_t}-\frac{\widetilde{x}_t}{\widetilde{y}_t}-\frac{\widetilde{G}_t}{\widetilde{y}_t} \\
&\Rightarrow \widetilde{nx}_t=1-\frac{\widetilde{c}_t}{\widetilde{y}_t}-\frac{\widetilde{x}_t}{\widetilde{y}_t}-\frac{\widetilde{G}_t}{\widetilde{y}_t} \\
&\Rightarrow \overline{nx}=1-\frac{\overline{c}}{\overline{y}}-\frac{\overline{x}}{\overline{y}}-\frac{\overline{G}}{\overline{y}}, \text{ clearly follows from above.}
\end{aligned}
$$

Proof of Proposition 3.1

We know from the agents problem, labor supply is given by

$$l_t^S = 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left(\frac{\widetilde{c}_t^*}{\widetilde{w}_t}\right)$$

which implies

$$\begin{aligned}
l_t^S &= 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left(\frac{\widetilde{c}_t + \Theta\widetilde{G}_t}{\widetilde{w}_t}\right) \\
&= 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\left\{\widetilde{c}_t + \Theta\left[\begin{array}{c}\tau_c\widetilde{c}_t \\ +\left[\left\{R_{t-1}^P(1-s)-1\right\}\theta_G + \tau_w\right]\widetilde{w}_t l_t \\ +\frac{\tau_k r_t \widetilde{k}_{t-1}}{1+\gamma}\end{array}\right]\right\}}{\widetilde{w}_t} \\
&= 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\left\{(1+\Theta\tau_c)\,\widetilde{c}_t + \Theta\left[\begin{array}{c}\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}\widetilde{w}_t l_t \\ +\frac{\tau_k r_t \widetilde{k}_{t-1}}{1+\gamma}\end{array}\right]\right\}}{\widetilde{w}_t}
\end{aligned}$$

This implies

$$\begin{aligned}
& l_t^S\left[1 + \Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}\right] \\
&= 1 - \left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\widetilde{c}_t}{\widetilde{w}_t}\left\{(1+\Theta\tau_c) + \frac{\Theta\tau_k r_t\widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t}\right\}
\end{aligned}$$

Define

$$D_{t-1} = \left[1 + \Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}\right].$$

Therefore,

$$\begin{aligned}
l_t^S &= \frac{1}{D_{t-1}} - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left[\frac{1+\Theta\tau_c}{D_{t-1}} + \frac{\Theta\tau_k r_t\widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t D_{t-1}}\right] \\
&= \frac{1 + \Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}}{D_{t-1}} \\
&\quad - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left[\frac{1+\Theta\tau_c}{D_{t-1}} + \frac{\Theta\tau_k r_t\widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t D_{t-1}}\right] \\
&\quad - \frac{\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}}{D_{t-1}}
\end{aligned}$$

$$\Rightarrow \;\; l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\left[\frac{1+\Theta\tau_c}{D_{t-1}} + \frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t D_{t-1}} + \frac{\Theta\left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\}\widetilde{w}_t}{D_{t-1}\widetilde{c}_t}\right]$$

$$\Rightarrow \;\; l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\Psi_t}{D_{t-1}}$$

$$\Rightarrow \;\; l_t^S = 1 - \frac{\widetilde{c}_t}{\widetilde{w}_t}\left(\frac{1-\mu}{\mu}\right)\Gamma_t.$$

Let's now define

$$\varrho_{t-1} = \left\{\left[R_{t-1}^P(1-s)-1\right]\theta_G + \tau_w\right\} > 0.$$

We can therefore re-write Γ_t

$$\Gamma_t = \left(\frac{1+\tau_c}{1-\tau_w}\right)\frac{\left[1+\Theta\tau_c + \frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\widetilde{c}_t} + \frac{\Theta\varrho_{t-1}\widetilde{w}_t}{\widetilde{c}_t}\right]}{\left[1+\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\varrho_{t-1}\right]}.$$

$\Gamma_t > 1$ if

$$(1+\tau_c)\left[1+\Theta\tau_c + \frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\,\widetilde{c}_t} + \frac{\Theta\varrho_{t-1}\widetilde{w}_t}{\widetilde{c}_t}\right] > (1-\tau_w)\left[1+\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\varrho_{t-1}\right].$$

Ignoring the positive term $\frac{\Theta\tau_k r_t \widetilde{k}_{t-1}}{(1+\gamma)\widetilde{c}_t} + \frac{\Theta\varrho_{t-1}\widetilde{w}_t}{\widetilde{c}_t}$, if

$$(1+\tau_c)(1+\Theta\tau_c) > (1-\tau_w)\left[1+\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\varrho_{t-1}\right],$$

we can be assured that Γ_t surely is greater than 1. We will look for sufficient conditions under which the above inequality holds.

$$\begin{aligned}(1+\tau_c)(1+\Theta\tau_c) \;\; &> \;\; (1-\tau_w)\left[1+\Theta\left(\frac{1-\mu}{\mu}\right)\left(\frac{1+\tau_c}{1-\tau_w}\right)\varrho_{t-1}\right] \\ &\Rightarrow \;\; \tau_c + \tau_w + \Theta\tau_c + \Theta\tau_c^2 > \Theta\left(\frac{1-\mu}{\mu}\right)(1+\tau_c)\,\varrho_{t-1}.\end{aligned}$$

Now if

$$\mu > 0.5,$$

clearly, for

$$\left[\frac{\tau_c + \tau_w + \Theta\tau_c + \Theta\tau_c^2}{\Theta(1+\tau_c)\,\varrho_{t-1}}\right] > 1 > \left(\frac{1-\mu}{\mu}\right).$$

to hold true, we need

$$\tau_c + \tau_w + \Theta\tau_c + \Theta\tau_c^2 > \Theta\left(1+\tau_c\right)\varrho_{t-1},$$

or,

$$\tau_c + \tau_w + \Theta\tau_c + \Theta\tau_c^2 > \Theta\left(1+\tau_c\right)\left[R_{t-1}^{P}(1-s)-1\right]\theta_G + \Theta\tau_w\left(1+\tau_c\right).$$

The LHS is given by

$$\tau_c\left(1+\Theta\right) + \tau_w + \Theta\tau_c^2$$

and the RHS is given by

$$\Theta\left(1+\tau_c\right)\left[R_{t-1}^{P}(1-s)-1\right]\theta_G + \Theta\tau_w + \Theta\tau_w\tau_c.$$

If we assume that

$$\begin{aligned} \tau_c &> \tau_w, \\ \text{and } 0 &< \Theta < 1, \end{aligned}$$

we get

$$\tau_w + \Theta\tau_c^2 > \Theta\tau_w + \Theta\tau_w\tau_c.$$

Hence now the comparison is between $\tau_c\left(1+\Theta\right)$ and $\Theta\left(1+\tau_c\right)\left[R_{t-1}^{P}(1-s)-1\right]\theta_G$. Define

$$\overline{x} = \left[R_{t-1}^{P}(1-s)-1\right]\theta_G.$$

Clearly

$$0 < \overline{x} < 1.$$

Since

$$\begin{aligned} 0 &< \Theta, \overline{x} < 1 \\ \text{and } 0 &< \overline{x} < 1, \end{aligned}$$

$$\tau_c\left(1+\Theta\right) > \Theta\overline{x}\left(1+\tau_c\right),$$

if

$$\begin{aligned} \tau_c &> \overline{x}, \\ \mu &> 0.5. \end{aligned}$$

Note, if

$$\tau_c > \tau_w,$$

we get

$$\begin{aligned} \left[\frac{\tau_c + \tau_w + \Theta\tau_c + \Theta\tau_c^2}{\Theta\left(1+\tau_c\right)\varrho_{t-1}}\right] &> 1 \\ &\Rightarrow \Gamma_t > 1. \end{aligned}$$

Proof of Proposition 3.3

In equation (3.27),

$$\frac{\partial l_{t+1}^D}{\partial R_t^P} = -\frac{l_{t+1}^D\left(\theta - s\theta_G\right)}{\alpha\left[\left(1-\theta\right) + R_t^P\left(\theta - s\theta_G\right)\right]},$$

and hence

$$\begin{aligned} \left|\frac{1}{\frac{\partial l_{t+1}^D}{\partial R_t^P}}\right| &= \frac{\alpha\left[\left(1-\theta\right) + R_t^P\left(\theta - s\theta_G\right)\right]}{l_{t+1}^D\left(\theta - s\theta_G\right)} \\ &= \frac{\alpha\left(1-\theta\right)}{l_{t+1}^D\left(\theta - s\theta_G\right)} + R_t^P. \end{aligned}$$

Therefore $\left|\frac{\partial l_{t+1}^D}{\partial R_t^P}\right|$ is decreasing in s and θ_G.

3.7 Figures

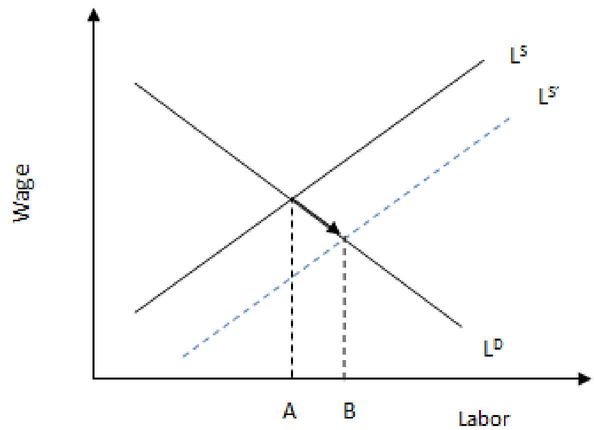

Figure 3.2: The effect of an increase in R_t^P on l_t^S

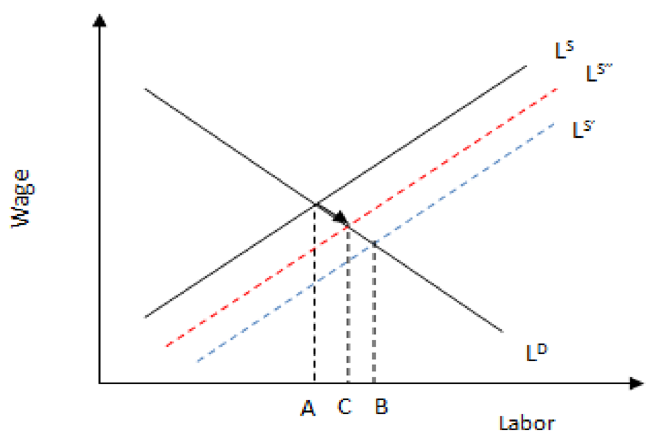

Figure 3.3: The effect of the wedge Γ_t on l_t^S

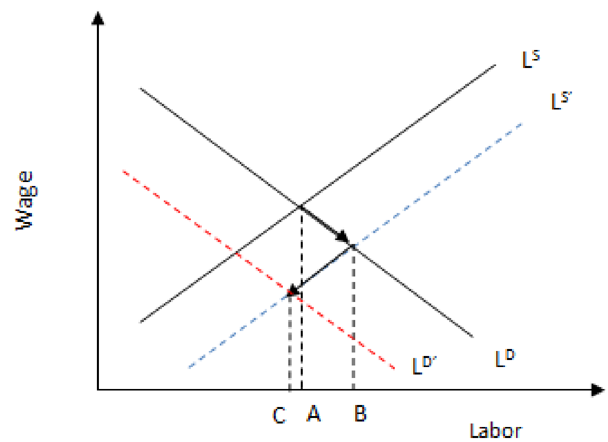

Figure 3.4: The effect of an increase in R_t^P on l_{t+1}^D

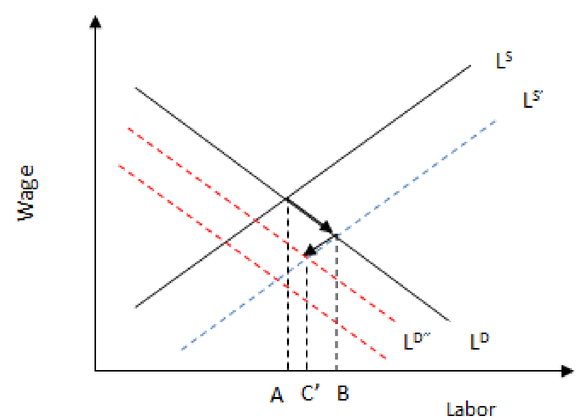

Figure 3.5: The effect of a subsidy θ_G and s on l_{t+1}^D

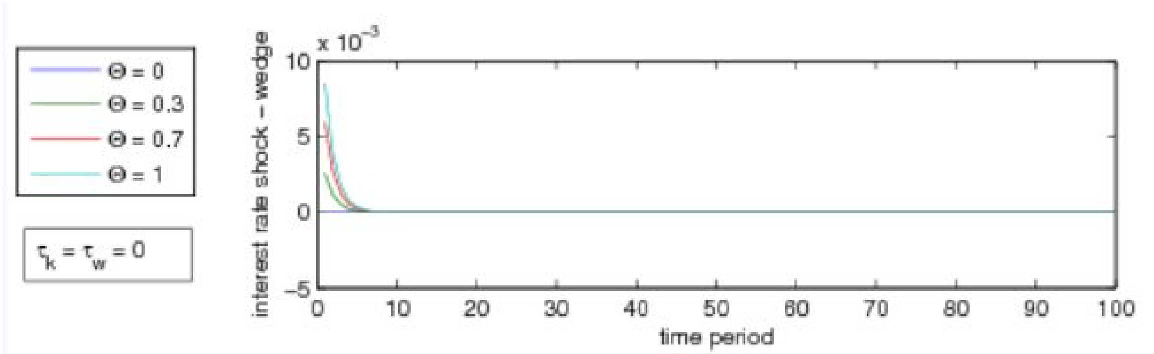

Figure 3.6: The effect of a single period interest rate shock on Γ_t

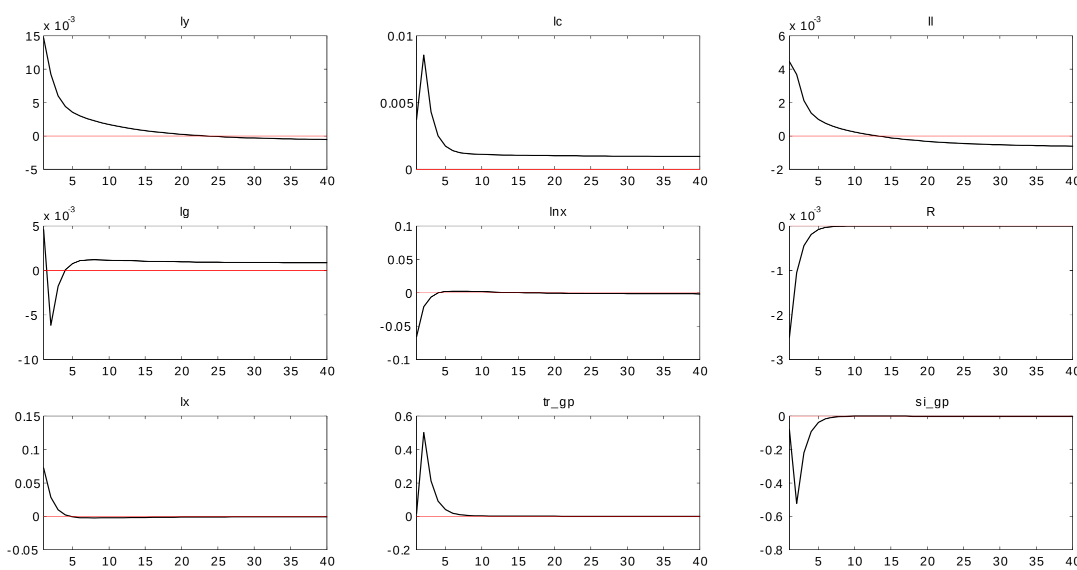

Figure 3.7: Impact of a single period TFP $\left(\widehat{A}\right)$ shock

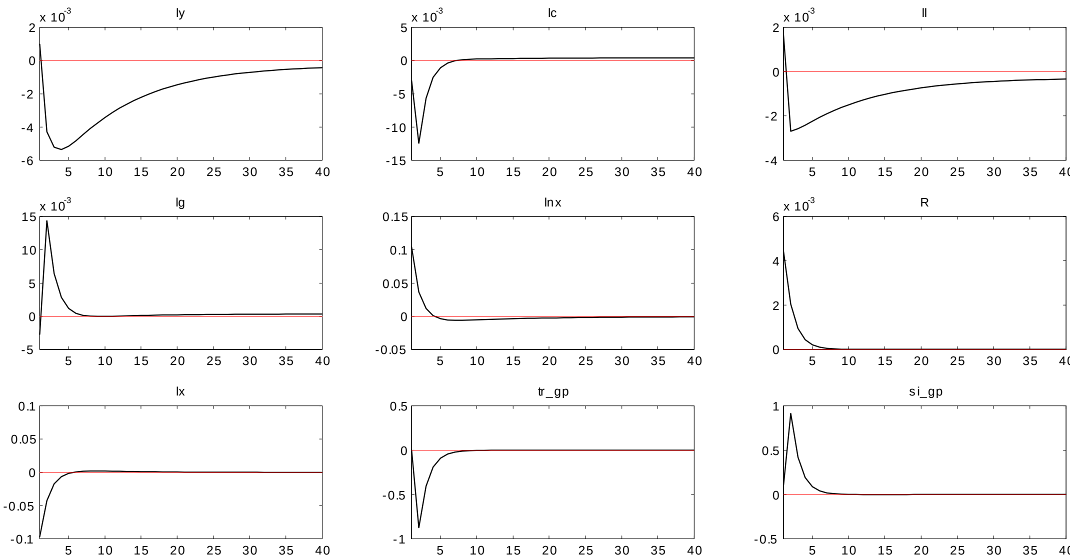

Figure 3.8: Impact of a single period international interest rate $\left(\widehat{R}^{*}\right)$ shock

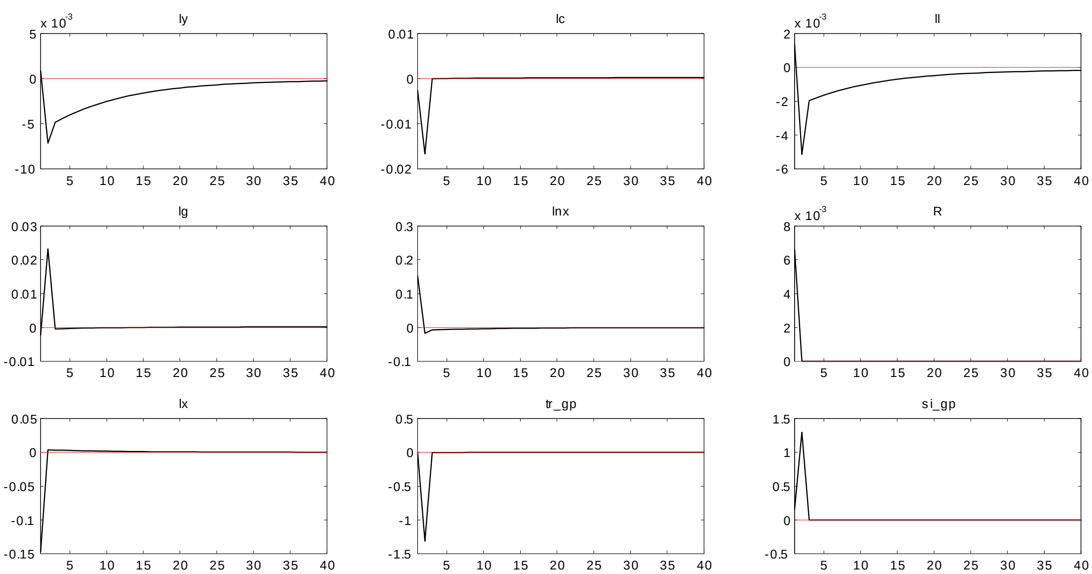

Figure 3.9: Impact of a single period spread $\left(\widehat{D}\right)$ shock

Chapter 4

Tax Policy and Food Security

4.1 Introduction

Post 2008 world food price shocks, food security concerns have come to the forefront of developmental policy. In the period 2000–12, even though the world food production outpaced the world population growth (wheat grew at an annualized rate of 1.05%, rice by 1.45%, and meat by 2.12% while the world population grew by 1.11%), in 2013 about 12% of the world population was undernourished (FAO, IFAD, and WFP (2013)). Despite the fact that food is available, it seems that either food is not available in a consistent manner or some people lack access to food.[1]

The FAO 2013 report (FAO, IFAD, and WFP (2013)) highlights problems in all three aspects of food security – food affordability, access to food and its nutritional content. In the period 2000-12, world real food prices rose by 4.37% annually (FAOSTAT database), which means that food was not affordable by the marginalized sections of the society. Other factors like decline in agricultural investment, higher volatilities in short-run supply and demand, rapidly increasing oil prices, diversions of maize to ethanol production, and middlemen hoarding have contributed to people's lack of access to food. Even when there is access to food, the nutritional content of food is a worrisome issue. On the one hand the developing world is facing widely prevalent undernourishment and on the other hand the developed nations are fighting obesity problems.

Policy makers across the world have taken concrete measures to combat food insecurity. The United Nations World Food Programme has several projects aimed at improving ac-

[1] In 2011-13, around one in eight people in the world are likely to have suffered from chronic hunger, not having enough food for an active and healthy life. The vast majority of these hungry people – 827 million of them – live in developing regions, where the prevalence of undernourishment is now estimated at 14.3%.(See FAO, IFAD, and WFP (2013))

cess to nutritious food for people living in developing countries, like food e-vouchers and vouchers, cash and food for work, improving food logistics, connecting farmers to market, to name a few (World Food Programme (2013)). Several countries have laid constitutional and legal protection to the human right to food (Knuth and Vidar (2011)). Recently, India joined the existing group of nine countries[2] to provide its citizens the right to food by law. Knuth and Vidar (2011) note that while legal protection of right to food is an important step towards ensuring food security, it needs to be accompanied by dedicated government efforts in implementing it. Countries which have made significant progress in improving their food security status have done so primarily through social programs like food subsidies, employment schemes, support to agricultural production, school meals, etc. (FAO, IFAD, and WFP (2014)). To consider an example, India implemented the seminal Right to Food Act in 2013, where this law aims to provide 'food and nutritional security [...], by ensuring access to adequate quantity of quality food at affordable prices to people to live a life with dignity' (The Gazette of India, September, 2013). The law plans to achieve its goals by providing subsidized food grains to approximately two-thirds of India's 1.2 billion people in the hope that it would significantly improve their nutritional status. Pregnant women, lactating mothers, and certain categories of children are eligible for daily free meals. In a country where 40% of children below 5 years of age are undernourished, the intent of this law is to 'meet the domestic demand as well as access, at the individual level, to adequate quantities of food at affordable prices' (see The Gazette of India, September, 2013).

These food subsidy programs across the globe, on the one hand will provide nutrition to the poorer sections of the society who in turn can work more efficiently and contribute positively to the country's GDP. On the other hand, the wealthier sections of the society would be taxed to finance these social programs, which may curb investment and long run growth of the economy. Who gains, who loses and in what conditions – are some questions that need to be answered. Further, the food subsidy program has a differential impact on the output of the agricultural sector as compared to (say) the manufacturing sector. It is an interesting question to ask how do the sectoral outputs change and what are the effects on relative prices. In this paper, we isolate the 'increased nutrition' channel through which the food subsidy is intended to affect the economy, and try to understand the effects of food subsidy program on sectoral outputs, their prices and welfare of agents.

We model a developing economy, where the agents are heterogeneous in their assets ownership.[3] One agent, entrepreneur, is endowed with capital while the other agent, farmer,

[2] In 2013, the nine countries that recognized the right to food as a separate and stand-alone right were Bolivia, Brazil, Ecuador, Guyana, Haiti, Kenya, South Africa, Nepal and Nicaragua. (Knuth and Vidar (2011))

[3] Antoci et al (2010) assert that in developing economies asset ownership is highly concentrated. Like our

is not. The agents are consumer-producers, whose objectives are to maximize their individual utilities subject to their respective budgets. The farmer uses his labor to produce food crop and cash crop, where the former is a final good and the latter is an intermediate good. The entrepreneur employs cash crop, his labor, and capital to produce the manufacturing output, which is another final good.

While the consumption of both final goods provides utility to the two agents, these goods have other additional purposes. Consumption of food provides energy to the agents and is the source for their labor abilities. Agents need to consume a minimum quantity of food to survive. After this subsistence need is met, food consumption increases labor capacity. The relation is increasing and concave. This is a novel feature of this paper where we use a metabolism function to capture the food to labor supply conversion. It is this route through which the food subsidy affects the well-being of the agents and hence affects the other macroeconomic variables like output and prices. The manufacturing good has the additional role of being the capital good. The entrepreneur invests a part of his income in augmenting his next period's capital stock – this means that the entrepreneur participates in a saving technology to which the farmer has no access. This is a typical feature of developing countries (as noted in Conning and Udry (2007)) and later forms the basis for the tax structures that are imposed on the agents.

To begin with, we assume that the economy is food secure, i.e. the productivity of food production is sufficiently high so that the subsistence needs of both agents are met. The government introduces a social program where it provides subsidy on the food consumption to both agents at an exogenous rate. The two agents may get different subsidies. The government may finance this program by either levying a direct tax or an indirect tax. Under the direct tax regime, the entrepreneur has to pay taxes proportional to his income,[4] while in the indirect tax regime, a consumption tax is imposed on both the farmer and the entrepreneur on their manufacturing goods consumption. The tax rates are fixed so that the government balances its budget. The model is fairly complex. We examine the effect of farmer's subsidy and the entrepreneur's subsidy on the different variables through numerical simulations.

In this economy, we find that in both tax regimes, the subsidy program increases the steady state agriculture output but lowers the steady state manufacturing output. The

economy, in their model of a developing economy, they assume heterogenous agents where one agent owns capital while the other does not.

[4] In India agricultural income is exempted from taxation. China also abolished agricultural taxes in 2006. Other developing countries like South Africa, Brazil, etc. farmers are subjected to proportional income taxes. However, in these countries taxation of entrepreneurs is a larger and a more significant source of the government's income (see China Internet Information Centre (2005)).

two taxes are levied either on manufacturing income or manufacturing consumption, which negatively affects either the supply or the demand of manufacturing good. In both cases, the net effect is that the subsidies adversely affect the manufacturing output. At the same time, by providing food subsidy the government makes the consumption of food cheaper, which in turn boosts the demand for food. Thus, the food subsidy program increases food output at the expense of manufacturing output.

The effects on relative prices are different in the two tax regimes. In the income tax regime, the long run price of the food crop relative to the price of the manufacturing good declines with subsidies, while in the consumption tax regime it increases with subsidies. In both tax regimes, the subsidy program raises the demand for food, which increases the nominal price of food. However, the tax regime has different effects in the manufacturing sector. In the income tax regime the subsidies lower the supply of manufacturing output, which increases the nominal price of manufacturing good – so much so that price of food relative to that of manufacturing falls. In the consumption tax regime the subsidies lower the demand for manufacturing consumption, hence lowering the nominal price of food which implies that relative price of food relative increases. The differential effect of the tax regimes on manufacturing demand and supply explains the subsidy effect on relative prices.

We also determine the program's welfare effects on the farmer and the entrepreneur. Compared to the no subsidy regime, the agents' steady state welfare improves only for a certain range of subsidies. To understand this, let us consider the farmer's welfare. The farmer's food consumption increases with his own subsidies. This translates into higher labor units and hence increased leisure. At the same time, an increase in the farmer's subsidy decreases the supply of manufacturing output and hence the farmer's consumption of manufacturing good falls with his subsidy. It is only for medium levels of farmer's subsidy, which boosts the farmer's consumption of food and leisure, and does not have a large adverse effect on his manufacturing goods consumption, that the farmer's welfare is higher in the subsidy program. The entrepreneur's subsidy has an unambiguously negative effect on farmer' welfare. The entrepreneur's subsidy does not boost farmer's food consumption or leisure, and further it adversely affects his manufacturing consumption. Thus, it is the combination of low subsidies to the entrepreneur and medium subsidies to the farmer that improves farmer's welfare. Analogous reasoning holds for the entrepreneur's welfare – medium entrepreneurial subsidies and low farmer's subsidies yield higher entrepreneur's welfare. In fact, our simulations suggest that there may be no subsidy combination in which both the agents are better off in the subsidy program as compared to the no subsidy program. One agent's welfare improvement may be accompanied with a loss in the other agent's welfare. This highlights that government needs to be prudent in choosing the level of subsidies to the agents as different subsidy combinations benefit different categories of people.

Comparing between the income tax regime and the consumption tax regime, we find that financing this program using an indirect consumption tax regime compared to a direct income tax regime gives higher welfare to both agents. On normative grounds, our paper therefore suggests that while such a subsidy program may only have limited gains in a heterogeneous agent economy, it is best to implement this program by sharing the tax burden between the two agents – through an indirect tax – to finance the food subsidy program. The subsidy program will unequivocally improve the health status of all the beneficiaries, but this by itself does not yield any significant welfare improvements. In this economy, though subsidy increases the labor capacity of both agents, but due to capital market frictions, it comes at the cost of capital deaccumulation. The subsidy program increases only the farmer's income, but he can not invest his income in any saving technology which implies that health improvements do not translate into higher growth of the economy. The paper outlines that there are limits to the benefits of a food subsidy program. Other complementary policy interventions are needed which enable better health to yield increase in output, welfare and possibly growth.

In the next section, we model the income tax regime. We build the model and present the simulation results. In Section 4.3 we analyze the economy with an alternative manufacturing consumption tax. Finally we conclude in Section 4.4 with policy recommendations.

4.2 Income Tax Regime

In this section we present the model economy with the government financing the food subsidy program using a distortionary income tax. This is a heterogenous agent economy, where the two infinitely lived agents are – a farmer and an entrepreneur. The entrepreneur is born with capital, while the farmer is not. This difference in ownership of asset also dictates the choice of the agents occupation. Further, there is occupational immobility – the farmer cannot participate in entrepreneurial activities and the entrepreneur does not want to do the labor-intensive farming work. The entrepreneur does not prefer to do agricultural work over manufacturing jobs because the former is more labor-intensive and hence harder. Further, working in the capital sector may be considered more modern and hence is looked up to, which tilts the entrepreneur's occupational choice towards manufacturing production. We capture this occupational immobility in the model by assuming that agents prefer to work in the sector where they can make use of their resources. Thus, the farmer uses his labor to produce two agricultural goods – a food crop and a cash crop. The cash crop is used only as an intermediate input. The entrepreneur uses cash crop along with his labor and capital to produce manufacturing goods only. Introduction of cash crops enable us to analyze the effects of the subsidy program within the agriculture sector, in particular, to compare subsidized food crop production with other agricultural products.

As in Jiny and Zengz (2007) these agents are household producers. Consumption of the manufacturing good, food, and leisure provides utility to the agents. We now present the model economy in greater detail.

4.2.1 The Representative Farmer

The farmer produces – a food crop, Q_{at}, and a cash crop, Q_{ct}. The two crops are produced using fully labor intensive CRS technologies, such that

$$\begin{aligned} Q_{at} &= AL_{at} \\ Q_{ct} &= CL_{ct} \end{aligned} \tag{4.1}$$

where L_{at} is labor employed in food production and L_{ct} is labor employed in cash crop production. A and C are total factor productivities (TFPs) that augment the production of the two crops. A and C are assumed to be constants.

Labor capacity is endogenous. We assume the following simple function, which is termed metabolism function as it captures the conversion of food to labor units,

$$L_t^F = \begin{cases} 0 & \text{for} \quad X_{at} < 1 \\ 1 - \frac{1}{X_{at}} & \text{for} \quad X_{at} \geq 1 \end{cases} \tag{4.2}$$

where X_{at} denotes farmer's consumption of food crop. The metabolism function, L_t^F, is plotted in Figure 4.1. L_t^F is a continuous function in X_{at}. For $X_{at} > 1$, farmer's labor capacity is strictly increasing and concave in food consumption. $X_{at} = 1$ captures subsistence consumption, below which the farmer has no energy to supply labor.

[INSERT FIGURE 4.1]

The parametrization of endogenous labor capacity in our model is technically similar to the functional relationship between food consumption and labor productivity as in Bliss and Stern (1978). A similar functional relationship between labor productivity and food consumption is also assumed in Dasgupta and Ray (1986) and Dasgupta (1997). In these papers, the authors assume that all households are endowed with a fixed number of labor hours, however the productivity of these labor hours depends on food consumption. Unlike in this literature however, we do not differentiate between labor hours and labor productivity.

In this paper the metabolism function is the 'effective' labor hours. An analogous way of interpreting this is as if the agent (in this economy) is endowed with one unit of labor hours and the labor productivity function is of the form L_t^F.

As mentioned before, food consumption has dual purposes, as an input in the labor capacity function and as a utility providing good. In all, the farmer derives utility from three goods: consumption of food, consumption of manufacturing good, and leisure. His utility function is

$$U_t^F = \phi_1 \ln X_{mt} + \phi_2 \ln X_{lt} + (1 - \phi_1 - \phi_2) \ln X_{at}, \quad 0 < \phi_1, \phi_2 < 1 \tag{4.3}$$

where X_{mt} is his manufacturing good consumption and X_{lt} is units of the labor endowment spent in leisure. The farmer has two sources of income: revenues from sale of food crop and cash crop. He spends this income in purchasing food and the manufacturing good. His budget is

$$(1 - f_1) p_{at} X_{at} + X_{mt} = p_{at} A L_{at} + p_{ct} C \left(1 - \frac{1}{X_{at}} - L_{at} - X_{lt}\right), \tag{4.4}$$

where we have assumed that the manufacturing good is the numeraire, and p_{at} and p_{ct} denote the relative price of the food crop and the cash crop respectively. Note that we have already used the farmer's full employment condition in the budget constraint by substituting it for employment in cash crop production (L_{ct}) as $L_{ct} = L_t^F - L_{at} - X_{lt}$.The government extends a per-unit subsidy of f_1 on the farmer's consumption of the food crop, so effectively the farmer has to spend $(1 - f_1) p_{at}$ for purchasing one unit of food. The farmer maximizes his utility (4.3) subject to its budget (4.4) by choosing X_{mt}, X_{at}, X_{lt}, and L_{at}. The optimization yields

$$\begin{aligned}
1 - \frac{2}{X_{at}} &= \frac{1}{\phi_3} \left[\frac{X_{at}(1 - f_1)}{A} - \frac{1}{X_{at}}\right], && (4.5)\\
X_{mt} &= \left(\frac{\phi_1}{1 - \phi_1 - \phi_2}\right) p_{at} A \left[\frac{X_{at}(1 - f_1)}{A} - \frac{1}{X_{at}}\right], && (4.6)\\
X_{lt} &= \left(\frac{\phi_2}{1 - \phi_1 - \phi_2}\right) \left[\frac{X_{at}(1 - f_1)}{A} - \frac{1}{X_{at}}\right], && (4.7)\\
\frac{p_{at}}{p_{ct}} &= \frac{C}{A}. && (4.8)
\end{aligned}$$

Eq. (4.5), can be rewritten as

$$X_{at} = \frac{(1 - \phi_1 - \phi_2) A \pm \sqrt{(1 - \phi_1 - \phi_2)^2 A^2 + 4(1 - f_1)(2\phi_1 + 2\phi_2 - 1) A}}{2(1 - f_1)}, \tag{4.9}$$

and hence for any positive A, i.e., $A > 0$, the sufficient condition for a real solution of X_{at} is

$$\phi_1 + \phi_2 > \frac{1}{2}. \tag{4.10}$$

Further, this condition also ensures that there is only one positive solution of X_{at} and hence ensures a unique feasible solution of X_{at}. Henceforward, we assume condition (4.10) always holds true. With this condition we find that the consumption of manufacturing good and leisure are strictly positive (from (4.6) and (4.7)).

Proposition 4.1 *The farmer's food consumption does not change over time. Further, it is positively related with his entitled food subsidy. These properties also hold for the farmer's labor capacity.*

Proof. Condition (4.10) along with equation (4.9) gives this. ■

We can easily see from (4.5) that higher the farmer's subsidy, higher would be his food consumption. A greater subsidy provided to the farmer increases his food consumption and hence his labor capacity. Thus, by improving the farmer's health, the per-unit subsidy of f_1 on food consumption also acts as 'food security'. To understand this, suppose $f_1 = 0$ and $A = 1/(\phi_1 + \phi_2)$. For these values, $X_{at} = 1$ which implies $L^F_{at} = 0$. Thus at this level of productivity, the farmer is not eating sufficiently to have any labor capacity. Now suppose the government provides the farmer a per-unit food subsidy, i.e. $f_1 > 0$, then his food consumption increases to $X_{at} > 1$. By providing subsidy, the farmer can now work as opposed to in the case of no-subsidy when the farmer would not even have existed at $A = 1/(\phi_1 + \phi_2)$. Through this logic we say that food subsidy provides food security as the marginalized farmer can now meet his subsistence food requirements to live and work. In a similar manner, we shall see that food subsidy to the entrepreneur also provides him food security.

4.2.2 The Representative Entrepreneur

The entrepreneur has an identical labor capacity function as the farmer, which is denoted by L^E_t. He employs labor L_{mt}, capital K_t, and the cash crop q_{ct} to produce manufactures using a CRS Cobb-Douglas production function

$$Q_{mt} = M L^{\alpha}_{mt} q^{\beta}_{ct} K^{1-\alpha-\beta}_t \tag{4.11}$$

where Q_{mt} is manufacturing output and M is TFP of the manufacturing production. Note, the manufacturing good is the numeraire.

Like the farmer, the entrepreneur is also assumed to be self employed. His felicity function is same as that of the farmer

$$U_t^E = \phi_1 \ln Y_{mt} + \phi_2 \ln Y_{lt}, + (1 - \phi_1 - \phi_2) \ln Y_{at}, \quad 0 < \phi_1, \phi_2 < 1$$

where Y_{mt} is his manufacturing goods consumption, Y_{lt} denotes the entrepreneur's leisure units, and Y_{at} is the entrepreneur's consumption of the food crop. The entrepreneur spends his after-tax income from sale of the manufacturing good on consumption of goods, purchase of cash crops, and capital investment. Thus, his budget constraint is

$$(1 - f_2)p_{at}Y_{at} + Y_{mt} + p_{ct}q_{ct} + K_{t+1} - (1 - \delta) K_t = (1 - \tau_t) M \left(1 - \frac{1}{Y_{at}} - Y_{lt}\right)^{\alpha} q_{ct}^{\beta} K_t^{1-\alpha-\beta}, \tag{4.12}$$

where f_2 is the food subsidy given by the government to the entrepreneur and we have substituted the entrepreneur's full employment condition, i.e. $L_{mt} = L_t^E - Y_{lt}$, for manufacturing employment in the manufacturing production function. In the income tax regime the tax burden falls on the capital owning agent and here the entrepreneur pays a proportional tax of τ_t on his income from selling manufactures. The assumed structure of taxation mimics the developing economies. As noted in Gordon and Li (2009), in developing countries, personal income tax rates are differentiated across different income groups, where usually the capital owning agents pay higher taxes. Further, corporate income tax is one of the most important sources of revenue for these countries. In this sense, by taxing the entrepreneur's income, we capture both these features of developing economies in our model.

Conditional on his budget and given initial capital stock K_0, the entrepreneur maximizes the following discounted lifetime utility

$$\sum_{t=0}^{\infty} \rho^t \left[\phi_1 \ln Y_{mt} + \phi_2 \ln Y_{lt}, + (1 - \phi_1 - \phi_2) \ln Y_{at}\right],$$

subject to his budget constraint (4.12), by choosing $\{Y_{at}, Y_{mt}, Y_{lt}, q_{ct}\}_{t=0}^{\infty}$ and $\{K_t\}_{t=1}^{\infty}$. The following is the Lagrangian corresponding to the entrepreneur's discounted lifetime problem

$$\max_{Y_{at}, Y_{mt}, Y_{lt}, q_{ct}, K_{t+1}} \sum_{t=0}^{\infty} \rho^t \left[\phi_1 \ln Y_{mt} + \phi_2 \ln Y_{lt}, + (1 - \phi_1 - \phi_2) \ln Y_{at}\right] + \sum_{t=0}^{\infty} \rho^t \left[\begin{array}{c} (1 - \tau_t) M \left(1 - \frac{1}{Y_{at}} - Y_{lt}\right)^{\alpha} q_{ct}^{\beta} K_t^{1-\alpha-\beta} - (1 - f_2)p_{at}Y_{at} - Y_{mt} \\ -p_{ct}q_{ct} - K_{t+1} + (1 - \delta) K_t \end{array} \right]$$

The first order conditions yield

$$
\begin{aligned}
Y_{lt} &= \frac{1-\frac{1}{Y_{at}}}{1+\frac{\phi_1 \alpha}{\phi_2}\frac{(1-\tau_t)Q_{mt}}{Y_{mt}}}, && (4.13)\\
(Y_{at}-1)\left[Y_{at}-\frac{(1-\phi_1-\phi_2)}{\phi_1(1-f_2)}\frac{Y_{mt}}{p_{at}}\right] &= \frac{1}{1-f_2}\left[\frac{\phi_2}{\phi_1}\frac{Y_{mt}}{p_{at}}+\alpha\frac{(1-\tau_t)Q_{mt}}{p_{at}}\right], && (4.14)\\
q_{ct} &= \frac{\beta(1-\tau_t)Q_{mt}}{p_{ct}}, && (4.15)
\end{aligned}
$$

and the Euler equation,

$$
\frac{Y_{mt+1}}{Y_{mt}} = \rho\left[1-\delta+(1-\alpha-\beta)\frac{(1-\tau_{t+1})Q_{mt+1}}{K_{t+1}}\right], \qquad (4.16)
$$

where ρ is the discount factor.

4.2.3 Market clearing conditions

The manufacturing and agricultural (i.e., food crop and cash crop) goods market clearing conditions respectively are

$$
\begin{aligned}
Q_{mt} &= K_{t+1}-(1-\delta)K_t+X_{mt}+Y_{mt} && (4.17)\\
AL_{at} &= X_{at}+Y_{at} && (4.18)\\
C\left(1-\frac{1}{X_{at}}-X_{lt}-L_{at}\right) &= q_{ct}. && (4.19)
\end{aligned}
$$

Finally, the government balances budget in every time period

$$
f_1 p_{at} X_{at} + f_2 p_{at} Y_{at} = \tau_t Q_{mt}. \qquad (4.20)
$$

We assume that the subsidies to the beneficiaries are fixed. So f_1 and f_2 are given and the government fixes taxes τ_t to balance its budget.

4.2.4 Static System

The static system is reduced to the following four equations.

$$
\beta(1-\tau_t)\frac{Q_{mt}}{p_{at}} = A\left[\left\{1-\frac{1}{X_{at}}\left(\frac{1-2\phi_2-\phi_1}{1-\phi_1-\phi_2}\right)\right\}-\frac{X_{at}}{A}\left(\frac{1-\phi_1-\phi_2 f_1}{1-\phi_1-\phi_2}\right)\right]-Y_{at} \qquad (4.21)
$$

$$Q_{mt} = M\left[\left(1-\frac{1}{Y_{at}}\right)\frac{\frac{\phi_1\alpha}{\phi_2}\frac{(1-\tau_t)Q_{mt}}{Y_{mt}}}{1+\frac{\phi_1\alpha}{\phi_2}\frac{(1-\tau_t)Q_{mt}}{Y_{mt}}}\right]^{\alpha}\left[\frac{\beta C}{A}\frac{(1-\tau_t)Q_{mt}}{p_{at}}\right]^{\beta}K_t^{1-\alpha-\beta} \tag{4.22}$$

$$(Y_{at}-1)\left[Y_{at}-\frac{(1-\phi_1-\phi_2)}{\phi_1(1-f_2)}\frac{Y_{mt}}{p_{at}}\right] = \frac{1}{1-f_2}\left[\frac{\phi_2}{\phi_1}\frac{Y_{mt}}{p_{at}}+\alpha\frac{(1-\tau_t)Q_{mt}}{p_{at}}\right]$$

$$\tau_t = \frac{1}{Q_{mt}/p_{at}}\cdot(f_1X_a+f_2Y_{at}).$$

We get the first equation from (4.7), (4.8),(4.15), (4.18) and (4.19). It is the reduced form of agents food consumption optimization condition and the agricultural goods market clearing conditions. The next equation is derived on substituting the entrepreneur's optimization conditions (4.13)-(4.15) into manufacturing production function (4.11). The last two equations are from entrepreneur's optimization (4.14) and from government budget (4.20) respectively. The static system yields

$$Q_{mt} = Q_m(Y_{mt}, K_t), \quad Y_{at} = Y_a(Y_{mt}, K_t), \quad p_{at} = p_a(Y_{mt}, K_t), \quad \tau_t = \tau(Y_{mt}, K_t).$$

There are a few of points to note here.

1. The explicit form of the aforementioned functions can not be determined.

2. The $[\cdot]$ term in eq. (4.21) captures the farmer's residual labor units after deducting leisure and labor required to produce own food consumption from the farmer's total labor units i.e. $(1-1/X_{at}) - X_{lt} - X_{at}/A$. In the absence of subsidy, $f_1 = 0$, we get from (4.5) that the farmer's residual labor is positive. However, in the subsidy program the farmer's leisure and food consumption increase with his subsidy and we can show that his residual labor decreases with increase in f_1. This implies that there is an upper-limit to the food subsidy offered to the farmer, beyond which the farmer's residual labor is negative. Now we know from (4.21) that for positive after-tax income from manufacturing production, i.e. $(1-\tau_t)Q_{mt} > 0$, it is necessary for the $[\cdot]$ term to be positive. Thus there is an upper-limit to the food subsidy that can be feasibly offered to the farmer.

3. Even though subsidies are fixed in the economy, taxes vary over time.

4.2.5 Dynamic System

The dynamics of the economy is spelled by Euler equation (4.16) and the capital accumulation equation (4.17). It is determined by the growth of two variables Y_{mt} and K_t. In this economy, there is no long run growth. At steady state,

$$Y_{mt} = Y_m^*, \quad K_t = K^*$$

Using this in the dynamic equations (4.16) and (4.17), we get

$$Q_m^* = Y_m^* + \delta K^* + X_m^* \tag{4.23a}$$

$$\frac{(1-\tau^*)Q_m^*}{K^*} = \frac{{}^1/_\rho - 1 + \delta}{1-\alpha-\beta}. \tag{4.23b}$$

The above equations with the static system solves for the steady state. Closed form solution does not exist. We therefore simulate the model for analyzing the change in macroeconomic variables with change in agents' subsidies.

4.2.6 Simulation

The complexity of the model makes it difficult to analytically solve the model. Hence, we resort to numerical simulations to characterize the effect of subsidy program on output and welfare. For this purpose, India is an ideal economy to model as it is a developing country which has recently implemented a food security act. The effects of the food subsidy program, which are calculated in this simulation exercise, would also be relevant for other developing countries.

The structural parameters for India are fixed in accordance with the existing literature, discount factor is 0.98 and the annual depreciation rate is 0.1 (as in Gabriel et al. (2012)). We calculate the preference parameters using data from the RBI handbook of statistics and CSO database. The preference parameter, ϕ_2, is taken to be the share of total output which is not consumed,

$$\phi_2 = 1 - \frac{C}{Y}$$

where C/Y is the average aggregate consumption to output ratio. The ratio of private final consumption expenditure (PFCE) to GDP, averaged over the years 1999-2007, yields $\phi_2 = 0.4$. Further, as the agents consume two goods – food and manufacturing – their respective weights are

$$\phi_1 = \left(\frac{V_M}{V_M + V_A}\right) \times \frac{C}{Y}, \quad \phi_3 = \left(\frac{V_A}{V_M + V_A}\right) \times \frac{C}{Y},$$

where V_M is the average manufacturing value added, and V_A is the average agricultural value added for the period 1999-2007. We get $\phi_1 = 0.24$ and $\phi_3 = 0.36$.[5].

The manufacturing production requires three inputs, namely capital, labor, and cash crop. Thus, the value of manufacturing output Q_{mt} is the sum of capital payments, wage payments, and the spending on cash crop intermediates. Similar to the methodology in Verma (2012), wage payments is estimated by compensation of employees, and the capital payments by the sum of consumption of fixed capital and operating surplus. The estimation of expenditure on cash crops inputs is a more involved process. Dholakia (2009) tabulates the input-output (I-O) tables for India in which he reports the cash crop intermediate inputs in manufacturing production. While Dholakia (2009) reports the I-O table for the years 1968, 1973, 1978, 1983, 1989, 1993, 1999, and 2003, we consider only the last two reported years. Our choice of this time period is to maintain consistency with the time period for the other aggregate and sectoral variables. We calculate the average share of cash crop intermediates of the total intermediates inputs used in producing manufacturing good for the years 1998 and 2003. This gives that cash crop input accounts for about 8.7% of total intermediate goods consumption in the manufacturing sector. Considering this cash crop input usage constant over time, we capture the expenditure on cash crops equal to 8.7% of the intermediate consumption in manufacturing goods production. Thus, the manufacturing sector's labor income share equals compensation of employees/(compensation of employees + operating surplus + consumption of fixed capital + 8.7% of intermediates consumption), which gives $\alpha = 0.19$. Similarly, we calculate the capital shares, $\beta = 0.25$ and $1 - \alpha - \beta = 0.56$.

Finally, the productivity parameters are arbitrarily fixed at $A = 100$, $C = 100$, and $M = 100$. Since we are interested in analyzing and comparing the effect of the subsidies f_1 and f_2 with the no food subsidy case, we conduct our numerical experiments in steady state for different values of f_1 and $f_2 \in [0, 1)$.

4.2.7 Subsidy Program Effects

The tax revenues finance the food subsidy, therefore, it follows that the steady state income tax increases with the subsidies, $\tau^* = \tau^*(\underset{+}{f_1}, \underset{+}{f_2})$. We plot the steady state tax rates for different subsidy combinations in Figure 4.2. The x-axis denotes the farmer's subsidy and the y-axis captures the tax rates. For different entrepreneur's subsidies we plot different curves. As one moves along the x-axis the farmer's subsidy increases and as one moves from black solid line to purple dotted line the entrepreneur's subsidy increases. The figures shows that from zero taxes in no subsidy program (shown in green line), the taxes increase with

[5]See Table 3A, Handbook of Statistics on Indian Economy, RBI.

both farmer's subsidy as well as entrepreneur's subsidy.

[INSERT FIGURE 4.2]

On Food Consumption

The subsidy program is intended to primarily affect the agents food consumption. As noted in Proposition 4.1, the farmer's consumption of food is higher in the food subsidy program. X_{at} increases in f_1 and is independent of f_2.

The subsidy effects on entrepreneur's food consumption is more involved. The entrepreneur's subsidy has a direct effect on his food consumption. In addition, as his food consumption linked with farmer's production, it is also affected by the farmer's subsidy. Our simulations show that in steady state, the amount of food consumed by the entrepreneur is positively related to the subsidy he himself gets and negatively related to the farmer's subsidy. In particular, the entrepreneur's food consumption is affected by the subsidy program through two channels – through income and through prices. On the one hand, an increase in f_1 and f_2 implies that the entrepreneur has to pay higher taxes. This reduces his after-tax income and hence lowers his consumption of food. On the other hand, an increase in f_2 lowers the effective price the entrepreneur has to pay for consuming food. Our simulations suggest that in the steady state, for the entrepreneur, the latter effect of f_2 dominates the former effect, i.e., $Y_a^* = Y_a^*(\underset{-}{f_1}, \underset{+}{f_2})$. This is shown in Figure 4.3. It is therefore possible that for a low f_1 and a high f_2, the entrepreneur's food consumption is higher in the subsidy program.[6]

[INSERT FIGURE 4.3]

The trends in food consumption also determine how the subsidy program influences the agents' work capacity. The subsidies unequivocally increase the work capacity of the farmer, but the effect on the entrepreneur's work capacity depends on the subsidy combination. The low f_1 and high f_2 combination – at which the direct benefits of a higher f_2 dominates the indirect detriments of higher taxation – increases the work capacity of the entrepreneur.

[6] In the case of equal subsidies, i.e. $f_1 = f_2$, the the entrepreneur's food consumption is decreasing in the food subsidy. So it is the negative effect of higher taxes which dominates the positive food price effect and the net result is that this subsidy program adversely effects Y_a^*. It is important to highlight that if equal subsidies are offered to both agents, the farmer's food consumption increases but it reduces the entrepreneur's food consumption, in which case, the program provides additional food security only to the farmer.

On Farmer's Production

The food subsidy program has opposite effects on the farmer's production of the food crop and the cash crop. Simulations show that the food crop output increases in both subsidies while the cash crop output decreases in both subsidies. We have shown that the farmer's subsidy boosts his food consumption, but not the entrepreneur's food consumption. In contrast, the entrepreneur's food subsidy increases the entrepreneur's food consumption and has no effect on the farmer's food consumption. The net effect is that both subsidies raise demand for food and yield $Q_a^* = Q_a^*(\underset{+}{f_1}, \underset{+}{f_2})$. As a result, the food output is always higher in the presence of the food subsidy program. This is shown in Figure 4.4. This implies employment in the food production increases with subsidies.

The effect of subsidies on the production of the cash crop is exactly opposite, as illustrated in Figure 4.5. To comprehend this, let us rewrite eq. (4.21) at steady state as

$$Q_c^* = C\left[\left\{1 - \frac{1}{X_a^*}\left(\frac{1-2\phi_2-\phi_1}{1-\phi_1-\phi_2}\right)\right\} - \frac{X_a^*}{A}\left(\frac{1-\phi_1-\phi_2 f_1}{1-\phi_1-\phi_2}\right)\right] - \frac{C}{A}Y_a^*. \tag{4.24}$$

We have already noted that the farmer's residual labor, [.] term above, is decreasing in f_1. In addition, our simulations show that the entrepreneur's food consumption, Y_a^*, decreases in f_1 and increases in f_2. These two findings together indicate that with increase in both f_1 and f_2 the farmer shifts his labor units involved in production (total labor minus leisure) towards food production and away from cash crop production. As a result, $L_c^* = L_c^*(\underset{-}{f_1}, \underset{-}{f_2})$ and $Q_c^* = Q_c^*(\underset{-}{f_1}, \underset{-}{f_2})$. Thus, the food subsidy program, by increasing the demand for food production, adversely affects the cash crop output, as shown in Figure 4.5.

[INSERT FIGURE 4.4]
[INSERT FIGURE 4.5]

On Entrepreneur's Production

To understand the effects of subsides on the manufacturing output, we rewrite the steady state manufacturing production function as

$$(Q_m^*)^{\alpha+\beta} = M\left(\frac{1/\rho - 1 + \delta}{1-\alpha-\beta}\right)^{1-\alpha-\beta}(1-\tau^*)^{1-\alpha-\beta}(L_m^*)^{\alpha}(q_c^*)^{\beta}$$

where we have used (4.23b) to substitute for K^*.As already discussed, subsidies unequivocally increase taxes and reduce the supply of cash crop. So the effect of subsidies through $\tau^*$and q_c^* is to reduce manufacturing output. At the same time, the subsidies *may* increase the labor capacity of the entrepreneur which implies that subsidies may possibly increase the manufacturing employment. Our simulations suggest that the subsidies affect the manufacturing employment in the same way as entrepreneur's work capacity, i.e., $L_m^* = L_m^*(\underset{-}{f_1}, \underset{+}{f_2})$. As shown in Figure 4.6, compared to the economy without the food subsidy program, a higher subsidy to the entrepreneur along with a low subsidy to the farmer increase L_m^*.

[INSERT FIGURE 4.6]

Summing up, the farmer's subsidy increases taxes, reduces the cash crop output, and reduces manufacturing employment. It is evident that f_1 unambiguously reduces the manufacturing output. However, the net effect of f_2 on the manufacturing output is not obvious. We look at the simulation results in Figure 4.7 and find that the manufacturing output decreases with increases in entrepreneur's subsidy, f_2. It appears that the effect of f_2 on lowering the cash crop and raising taxes dominates the positive effect it has on the manufacturing employment. Hence, $Q_m^* = Q_m^*(\underset{-}{f_1}, \underset{-}{f_2})$.

[INSERT FIGURE 4.7]

Further, as the subsidy program lowers the manufacturing output, from (4.23b), it follows that subsidies also lower steady state capital stock. Increase in f_1 and f_2 implies a higher tax and a lower manufacturing output, which reduces the entrepreneur's after-tax income and hence adversely affects capital accumulation. This is depicted in Figure 4.8, $K^* = K^*(\underset{-}{f_1}, \underset{-}{f_2})$.

[INSERT FIGURE 4.8]

On Prices

The simulations yield that the relative prices of food and cash crops are negatively related to the two subsidies. As shown in Figures 4.9 and 4.10, $p_a^* = p_a^*(\underset{-}{f_1}, \underset{-}{f_2})$ and $p_c^* = p_c^*(\underset{-}{f_1}, \underset{-}{f_2})$.

[INSERT FIGURE 4.9]
[INSERT FIGURE 4.10]

To understand this, recall that the subsides increase the demand of the food crop and reduces the supply of the manufacturing good. This increases the nominal price of both the food crop and the manufacturing good. The increase in the nominal price of the manufacturing good is however higher than that of the food crop, which implies that the price of the food crop relative to the manufacturing good falls with subsidies. Thus, both subsidies lower p_a^*. Further, from equation (4.8), we know that p_a^* and p_c^* are one-to-one linked. As a result, the relative price of the cash crop also falls in steady state.

On Welfare

The representative farmer and the entrepreneur derive utility from consuming the manufacturing good, leisure, and food. In steady state, the representative farmer's per-period utility is given by

$$\Gamma^F = \phi_1 \ln X_m^*(\underset{-}{f_1}, \underset{-}{f_2}) + \phi_2 \ln X_l^*(\underset{+}{f_1}, \underset{0}{f_2}) + (1-\phi_1-\phi_2)\ln X_a^*(\underset{+}{f_1}, \underset{0}{f_2}).$$

Our simulations suggest that the subsidy program lowers X_m^*, as depicted in Figure 4.11. Intuitively, both subsidies make manufacturing consumption more expensive as compared to food consumption (as p_a^* falls), which lowers the demand for the manufacturing good.

It is easy to see that f_1 has two opposing effects on the farmer's welfare. On the one hand, it reduces the consumption of the manufacturing good and on the other hand it increases the consumption of leisure and agricultural good. We therefore find that for any given f_2, there exists an interior value of f_1 where the farmer's welfare is maximum. Further, the farmer's welfare is strictly decreasing in f_2. The farmer's per-period welfare is shown in Figure 4.12.

[INSERT FIGURE 4.11]
[INSERT FIGURE 4.12]

Our simulations suggest that for low levels of f_2 and medium levels of f_1 the farmer's welfare may be higher in the subsidy program.

The entrepreneur's steady state per-period utility is given by

$$\Gamma^E = \phi_1 \ln Y_m^*(\underset{-}{f_1}, \underset{-}{f_2}) + \phi_2 \ln Y_l^*(\underset{-}{f_1}, \underset{+}{f_2}) + (1-\phi_1-\phi_2)\ln Y_a^*(\underset{-}{f_1}, \underset{+}{f_2}).$$

The effect of the subsidy program on Y_m^* and Y_l^* are plotted in Figures 4.13 and 4.14 respectively. As in the farmer's case, due to an increase in the relative price of the manufacturing good as compared to the food crop, the entrepreneur reduces manufacturing consumption, which explains $Y_m^*(\underset{-}{f_1}, \underset{-}{f_2})$. Further, the entrepreneur's leisure follows the same trend as his work capacity – it increases with f_2 and decreases with f_1. It is clear that f_1 has an overall negative effect on the entrepreneur's welfare. The entrepreneur's food subsidy f_2, though negatively affects the consumption of manufacturing good, it increases leisure and consumption of the food crop. The entrepreneur's welfare effects in Figure 4.15 suggest that for any given f_1, there exists an interior value of f_2 where the entrepreneur's welfare is at its highest.

[INSERT FIGURE 4.13]
[INSERT FIGURE 4.14]
[INSERT FIGURE 4.15]

Our simulations depict that for low levels of f_1 and medium levels of f_2 the entrepreneur's welfare may be higher in the subsidy program. Our simulations also show that improvement in welfare of one agent comes at the expense of the other agent. We do not find any subsidy combination at which both agents gain from the subsidy program. However, if we look at the sum of welfare of the two agents, there are some combinations of subsidies at which the aggregate welfare of the economy is higher in the subsidy program (see Figure 4.16).

[INSERT FIGURE 4.16]

4.3 Consumption Tax Regime

In this section, we investigate an alternate form of financing the food subsidy program, i.e., imposing a tax on manufacturing consumption on the farmer and the entrepreneur. The idea is to see if a change in the method of financing the subsidy program has any differential effects on the economy. Importantly, unlike in the income tax regime where the entrepreneur solely bears the burden of taxation, in the consumption tax regime, the government taxes the farmer's and the entrepreneurs's consumption of the manufacturing good at a uniform rate τ_{st}. Except for the budget constraint, the two agents' optimization problem is unchanged. The farmer's new budget is

$$(1-f_1)p^s_{at}X^s_{at} + (1+\tau_{st})X^s_{mt} = p^s_{at}AL^s_{at} + p^s_{ct}C\left(1 - \frac{1}{X^s_{at}} - L^s_{at} - X^s_{lt}\right). \tag{4.25}$$

It is intuitive that the farmer's optimization conditions with respect to manufacturing consumption changes

$$X^s_{mt} = \left(\frac{\phi_1}{1-\phi_1-\phi_2}\right)\frac{p^s_{at}A}{1+\tau_{st}}\left[\frac{X^s_{at}(1-f_1)}{A} - \frac{1}{X^s_{at}}\right]. \tag{4.26}$$

whereas other conditions remain as in the previous income tax regime, i.e., (4.5), (4.7) and (4.8). Therefore,

$$X^s_{at} = X_{at}, \quad X^s_{lt} = X_{lt} \tag{4.27}$$

i.e., the farmer's food consumption and leisure are unchanged in the consumption tax regime. As a result, the farmer's total labor endowment L^{Fs}_t also remains unchanged, i.e.,

$$L^{Fs}_t = 1 - \frac{1}{X_{at}} = 1 - \frac{1}{X^s_{at}} = L^F_t. \tag{4.28}$$

We summarize this as follows:

Proposition 4.2 *The farmer's food consumption, his total labor endowment and his leisure are unchanged in the income tax regime and manufacturing consumption tax regime.*

The new tax regime similarly alters the entrepreneur's problem. His utility function is same as in the previous regime but now his manufacturing consumption, instead of income, is taxed. The entrepreneur's new budget constraint is

$$(1-f_2)p_{at}^s Y_{at}^s + (1+\tau_{st})\,Y_{mt}^s + p_{ct}^s q_{ct}^s + K_{t+1}^s - (1-\delta)\,K_t^s = M\left(1-\frac{1}{Y_{at}^s}-Y_{lt}^s\right)^{\alpha}(q_{ct}^s)^{\beta}\,(K_t^s)^{1-\alpha-\beta}.$$

The first order conditions are

$$Y_{lt}^s = \frac{1-\frac{1}{Y_{at}^s}}{1+\frac{\phi_1\alpha}{\phi_2}\frac{Q_{mt}^s}{(1+\tau_{st})Y_{mt}^s}}, \tag{4.29}$$

$$(Y_{at}^s-1)\left[Y_{at}^s-\frac{(1-\phi_1-\phi_2)}{\phi_1(1-f_2)}\frac{(1+\tau_{st})Y_{mt}^s}{p_{at}^s}\right] = \frac{1}{1-f_2}\left[\frac{\phi_2}{\phi_1}\frac{(1+\tau_{st})Y_{mt}^s}{p_{at}^s}+\alpha\frac{Q_{mt}^s}{p_{at}^s}\right], \tag{4.30}$$

$$q_{ct}^s = \frac{\beta Q_{mt}^s}{p_{ct}^s}, \tag{4.31}$$

and the Euler equation is

$$\frac{(1+\tau_{st+1})Y_{mt+1}^s}{(1+\tau_{st})Y_{mt}^s} = \rho\left[1-\delta+(1-\alpha-\beta)\frac{Q_{mt+1}^s}{K_{t+1}^s}\right]. \tag{4.32}$$

The goods market clearing conditions are unchanged as in (4.17), (4.18) and (4.19). As before, for any given f_1 and f_2,the government fixes taxes to balance its budget, which now is

$$f_1 p_{at}^s X_{at}^s + f_2 p_{at}^s Y_{at}^s = \tau_{st}(X_{mt}^s + Y_{mt}^s). \tag{4.33}$$

4.3.1 Static System

The economy can be expressed in four equations, which constitute the static system

$$\beta\frac{Q_{mt}^s}{p_{at}^s} = \left[A\left\{1-\frac{1}{X_{at}^s}\left(\frac{1-2\phi_2-\phi_1}{1-\phi_1-\phi_2}\right)\right\}-X_{at}^s\left(\frac{1-\phi_1-\phi_2 f_1}{1-\phi_1-\phi_2}\right)\right]-Y_{at}^s \tag{4.34}$$

$$Q_{mt}^s = M\left[\left(1-\frac{1}{Y_{at}^s}\right)\frac{\frac{\phi_1\alpha}{\phi_2}\frac{Q_{mt}^s}{(1+\tau_{st})Y_{mt}^s}}{1+\frac{\phi_1\alpha}{\phi_2}\frac{Q_{mt}^s}{(1+\tau_{st})Y_{mt}^s}}\right]^{\alpha}\left[\frac{\beta C}{A}\frac{Q_{mt}^s}{p_{at}^s}\right]^{\beta}[K_t^s]^{1-\alpha-\beta} \tag{4.35}$$

$$(Y_{at}^s - 1)\left[Y_{at}^s - \frac{(1-\phi_1-\phi_2)}{\phi_1(1-f_2)}\frac{(1+\tau_{st})Y_{mt}^s}{p_{at}^s}\right] = \frac{1}{1-f_2}\left[\frac{\phi_2}{\phi_1}\frac{(1+\tau_{st})Y_{mt}^s}{p_{at}^s} + \alpha\frac{Q_{mt}^s}{p_{at}^s}\right]$$

$$\tau_{st}\left[\left(\frac{\phi_1}{1-\phi_1-\phi_2}\right)\frac{A}{1+\tau_{st}}\left\{\frac{X_{at}^s(1-f_1)}{A} - \frac{1}{X_{at}^s}\right\} + \frac{Y_{mt}^s}{p_{at}^s}\right] = f_1 X_{at}^s + f_2 Y_{at}^s. \qquad (4.36)$$

The first equation is the reduced form of the food and cash crop optimization, and market clearing conditions. The next equation is derived on substituting the entrepreneur's optimization conditions (4.29)-(4.31) into manufacturing production function (4.11). The third equation is the entrepreneur's optimization condition (4.30) and the last is the government budget constraint, where we have substituted for X_{mt}/p_{at} from (4.26) into (4.33) to get (4.36). Note, we already know the value of X_{at}^s from (4.5), hence the static system yields

$$Q_{mt}^s = Q_m^s(Y_{mt}^s, K_t^s), \quad Y_{at}^s = Y_a^s(Y_{mt}^s, K_t^s), \quad p_{at}^s = p_a^s(Y_{mt}^s, K_t^s), \quad \tau_{st} = \tau_s(Y_{mt}^s, K_t^s).$$

4.3.2 Steady State

The capital accumulation equation (4.17) and the Euler equation (4.32) constitute the dynamic equations of the economy. In steady state, the dynamic variables are constant so

$$Y_{mt}^s = Y_m^{s*}, \quad K_t^s = K^{s*}$$

and from the dynamic equations we get

$$Q_m^{s*} = \delta K^{s*} + Y_m^{s*} + X_m^{s*} \qquad (4.37a)$$

$$\frac{Q_m^{s*}}{K^{s*}} = \frac{{}^1/_\rho - 1 + \delta}{1-\alpha-\beta}. \qquad (4.37b)$$

The above equations along with the static system solves for the steady state. In this regime, as was in the previous case, closed form steady state solutions do not exist. However, it can be shown,

Proposition 4.3 *In steady state, the entrepreneur's consumption of the food crop is same in the consumption tax regime as in the income tax regime, i.e.,* $Y_a^{s*} = Y_a^*$.

Proof. See Appendix ■

The intuition lies in the fact that the two methods of financing do not alter the behavior of the economy in steady state. In the income tax regime, depending on entrepreneur's food consumption, the cash crop employment is determined which in turn determines the entrepreneur's disposable income in terms of food prices (eq. (4.21)). This yields $(1 - \tau^*)(Q_m^*/p_a^*)$ as a function of Y_a^*. This relation together with the steady state relation (4.23b) and the steady state entrepreneur's budget, $(1-f_2)Y_a^* + (Y_{mt}/p_a^*) = (1-\beta)\,(1-\tau^*)\,(Q_m^*/p_a^*) - \delta(K^*/p_a^*)$, determines the budget-wise link between (Y_m^*/p_a^*) and Y_a^*. Finally all these links are brought together in optimization condition (4.14) which solves for Y_a^*. A change in the tax regime affects the variables but not the linkages. As compared to the income tax regime, in the presence of consumption tax, the entrepreneur's disposable income is Q_m^*/p_a^* and his expenditure on manufacturing good consumption, in terms of food prices, is $(1+\tau_s^*)\,(Y_m^*/p_a^*)$. Apart from this the chain of how demand for the entrepreneur's food determines the supply of cash crop, which in turn is linked with the entrepreneur's disposable income in terms of food prices, which finally determines the entrepreneur's food consumption, is exactly the same in both tax regimes. This explains $Y_a^{s*} = Y_a^*$.

Proposition 4.3, together with eqs. (4.13) and (4.29) yields that the entrepreneur's steady state total labor units, manufacturing employment and leisure remain unchanged in the two tax regimes. That is,

$$L^{Es*} = L^{E*}, \quad L_m^{s*} = L_m^*, \quad Y_l^{s*} = Y_l^*.$$

Further, Proposition 4.3 along with (4.27) implies that in steady state the farmer's allocation of labor for food production and production of cash crops also remain unchanged in the two tax regimes, i.e.,

$$L_a^{s*} = L_a^*, \quad L_c^{s*} = L_c^*.$$

We summarize these findings as follows.

Proposition 4.4 *In steady state, the sectoral employments (in food crop, cash crop and manufacturing output production) are unchanged in the two tax regimes. Further, in steady state, the entrepreneur's leisure is unaffected by the tax structures.*

Proof. Discussed above. ■

The unchanged employment in food and cash crops sectors imply that food and cash crop outputs are same in the two tax regimes. However, this equality does not hold for steady state manufacturing output:

Proposition 4.5 *The steady state capital and the steady state manufacturing output is higher in the consumption tax regime compared to the income tax regime, i.e., $K^{s*} > K^*$ and $Q_m^{s*} > Q_m^*$. Therefore the steady state relative price of the food crop is higher in the consumption tax regime, i.e., $p_a^{s*} > p_a^*$.*

Proof. Substituting the steady state eqs (4.23b) and (4.37b) into their respective manufacturing production functions (4.22) and (4.35), we get

$$\frac{Q_m^{s*}}{Q_m^*} = (1-\tau^*)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} > 1.$$

In both regimes, the steady marginal product of capital is the same (eqs. (4.23b) and (4.37b)). However, in the income tax regime, the after-tax value of manufacturing output is lower, hence capital stock is lower in this regime,

$$\frac{K^{s*}}{K^*} = \frac{Q_m^{s*}}{(1-\tau^*)\,Q_m^*} > 1.$$

Further, as the food consumptions are equal in the two tax regimes, the cash crop market clearing conditions (4.21) and (4.34) yield,

$$\frac{p_a^{s*}}{p_a^*} = \frac{Q_m^{s*}}{(1-\tau^*)\,Q_m^*} > 1.$$

■

The higher food prices, with no change in cash crop and food crop output, implies that farmer's income is higher in the consumption tax regime. As his food consumption is unaffected by the tax structure, the increase in his income is spent on increasing his manufacturing goods consumption. Similar increase in entrepreneur's income translates into higher manufacturing consumption by the entrepreneur. We summarize this as follows

Proposition 4.6 *The steady state consumption of the manufacturing output for the farmer and the entrepreneur is higher in the consumption tax regime compared to the income tax regime, i.e.,* $X_m^{s*} > X_m^*$ *and* $Y_m^{s*} > Y_m^*$.

Proof. From steady state eqs. (4.23a) and (4.37a) we get,

$$\frac{X_m^{s*}+Y_m^{s*}}{X_m^*+Y_m^*} = \frac{1-\delta\Psi}{1-(1-\tau^*)\delta\Psi}\frac{Q_m^{s*}}{Q_m^*} = \frac{1-\delta\Psi}{1-(1-\tau^*)\delta\Psi}\frac{1}{(1-\tau^*)^{\frac{1-\alpha-\beta}{\alpha+\beta}}} \equiv \Omega(\tau^*), \tag{4.38}$$

where $\Psi = (1-\alpha-\beta)/({}^1/_\rho - 1 + \delta)$. It is easy to see that $\Omega(0) = 1$ and $\Omega(1) = \infty$. Further $\Omega'(\tau^*) > 0$. Thus for $1 > \tau^* > 0$ it is evident that $\Omega(\tau^*) > 1$. In other words, the total manufacturing consumption by the two agents in the food subsidy program is higher in the presence of consumption tax as compared to income tax. Now, as $Y_a^{s*} = Y_a^*$ and $Q_m^{s*}/p_a^{s*} = (1-\tau^*)(Q_m^*/p_a^*)$, we get from (4.14) and (4.30) that

$$\frac{(1+\tau_s^*)Y_m^{s*}}{p_a^{s*}} = \frac{Y_m^*}{p_a^*}. \tag{4.39}$$

The above expression together with (4.6) and (4.26) yields

$$\frac{(1+\tau_s^*)}{p_a^{s*}}\left(X_m^{s*}+Y_m^{s*}\right) = \frac{1}{p_a^*}\left(X_m^*+Y_m^*\right). \tag{4.40}$$

We know $X_m^{s*} + Y_m^{s*} > X_m^* + Y_m^*$ and with the aforementioned relation, we get $(p_a^{s*}/p_a^*) \cdot (1 + \tau_s^*)^{-1} > 1$. This further with (4.39) and (4.40) gives $X_m^{s*} > X_m^*$ and $Y_m^{s*} > Y_m^*$. ■

The higher manufacturing consumption in the consumption tax regime also implies that the utility of both agents is now higher. That is,

Proposition 4.7 *The steady state per-period utilities of the farmer and the entrepreneur is higher in the consumption tax regime as compared to the income tax regime, i.e.,* $\Gamma^{sF} > \Gamma^F$ *and* $\Gamma^{sE} > \Gamma^E$.

Proof. As $X_a^{s*} = X_a^*$, $X_l^{s*} = X_l^*$ and $X_m^{s*} > X_m^*$, it gives that utility of the farmer is higher in the consumption tax regime as compared to income tax regime, $\Gamma^{sF} > \Gamma^F$. Similarly, as $Y_a^{s*} = Y_a^*$, $Y_l^{s*} = Y_l^*$ and $Y_m^{s*} > Y_m^*$, the utility of the entrepreneur is higher in the consumption tax regime as compared to income tax regime, $\Gamma^{sE} > \Gamma^E$. ■

Thus, financing this program using an indirect consumption tax regime compared to a direct income tax regime is Pareto improving. This is because in the steady state, moving from the income tax regime to the consumption tax regime causes an increase in the consumption of the manufacturing output by both agents. As a result, sharing the tax burden, by imposing an indirect tax, is Pareto superior. An interesting normative insight we get is that sharing the tax burden – between the farmer and the entrepreneur – via manufacturing consumption tax is beneficial in terms of aggregate welfare.

4.3.3 Simulation

For the same parameter values used in the income tax regime, we simulate the model to determine long run effects of the subsidy program on the economy in the consumption tax regime. As shown in Figure 4.17, compared to the no-subsidy case, the consumption tax is positive. Further, since the government fixes the tax rate for a given pair of farmer's and entrepreneur's subsidies, higher the subsidies, the government would have to set a higher tax rate $\tau_s^* = \tau_s^*(\underset{+}{f_1}, \underset{+}{f_2})$.

[INSERT FIGURE 4.17]

On Outputs

We have already shown that $X_a^{s*} = X_a^*$ and $Y_a^* = Y_a^{s*}$. Hence the food consumption plot for the entrepreneur is the same as in Figure 4.3. Further, employment in the food crop and cash crop production are same as were in the income tax regime. Thus, the farmer's production of the food crop and cash crop are exactly the same as in the previous regime (shown in Figures 4.4 and 4.5). Simulations show that in this regime too, subsidies reduce steady the state manufacturing output as well as capital. The subsidy program reduces the cash crop production and this adversely affect manufacturing production, which in turn also lowers the steady state capital stock. These effects are shown in Figures 4.18 and 4.19.

[INSERT FIGURE 4.18]
[INSERT FIGURE 4.19]

On Price

The subsidy effect on the relative prices differs in the income tax regime and consumption tax regime. In the consumption tax regime, the relative price of the food crop increases with higher f_1 and f_2, i.e., $p_a^{s*} = p_a^{s*}(\underset{+}{f_1}, \underset{+}{f_2})$. An increase in f_1 and f_2, increases the demand for the food crop and therefore increases the nominal price of food. The consumption tax reduces the demand for manufacturing consumption good which reduces the nominal price of manufacturing good. The joint effect is an increase in the relative price of the food crop. Since p_c^{s*} is proportional to p_a^{s*}, $p_c^{s*} = p_c^{s*}(\underset{+}{f_1}, \underset{+}{f_2})$. The effect of subsidies on the food crop and cash crop relative prices is shown in Figures 4.20 and 4.21.

[INSERT FIGURE 4.20]
[INSERT FIGURE 4.21]

On Welfare

As in the income tax regime, the representative farmer's per-period steady state utility is given by

$$\Gamma^{Fs*} = \phi_1 \ln X_m^{s*}(\underset{-}{f_1}, \underset{-}{f_2}) + \phi_2 \ln X_l^{s*}(\underset{+}{f_1}, \underset{0}{f_2}) + (1 - \phi_1 - \phi_2) \ln X_a^{s*}(\underset{+}{f_1}, \underset{0}{f_2}),$$

and similarly, the representative entrepreneur's steady state per-period utility is given by

$$\Gamma^{Es*} = \phi_1 \ln Y_m^{s*}(\underset{-}{f_1}, \underset{-}{f_2}) + \phi_2 \ln Y_l^{s*}(\underset{+}{f_1}, \underset{0}{f_2}) + (1 - \phi_1 - \phi_2) \ln Y_a^{s*}(\underset{+}{f_1}, \underset{0}{f_2}).$$

Financing the subsidy program using tax on manufacturing consumption does not qualitatively change agents' welfare effects. The effects of subsidies are still the same, except that the magnitude of the effects have altered. We present in Figures 4.22 and 4.23 that the two subsidies have a negative effect on the manufacturing consumption of both agents.

[INSERT FIGURE 4.22]
[INSERT FIGURE 4.23]

The welfare of the farmer and the entrepreneur for different subsidies is shown in Figures 4.24 and 4.25 respectively. We present the aggregate welfare in Figure 4.26.

[INSERT FIGURE 4.24]
[INSERT FIGURE 4.25]
[INSERT FIGURE 4.26]

As discussed before, simulations also show that welfare gains are higher in consumption tax regime as compared to income tax regime. An increase in consumption of the manufacturing good and unaltered consumptions of the food crop and leisure, by both farmer and entrepreneur, explains higher welfare gains in the consumption tax regime. Further, the entrepreneur witnesses larger welfare gains than the farmer as a result of moving from the income tax to the consumption tax regime. The simulation results for welfare gains from the subsidy program in the two tax regimes, for the case of $f_2 = 0.81$ and different levels of f_1, are shown in Figure 4.27. The pattern does not change for different subsidy combinations. Intuitively, switching from the income tax regime to the consumption tax regime has resulted in an increase in incomes for the farmer and the entrepreneur, which results in an increase in the consumption of the manufacturing output. In addition, higher gains for the entrepreneur are on the account of sharing the tax burden with the farmer. On normative grounds therefore, our model suggests that despite there being marginal gains from introducing the subsidy program, it is better to finance such a scheme using a uniform distortionary consumption tax compared to a discriminatory income tax regime.

[INSERT FIGURE (4.27)]

4.4 Conclusion

Our work is motivated by the recent food security schemes announced across several developing and middle income economies to fulfill their millennium developmental goals. Several economies like India and South Africa have made "Right to Food" as a constitutional act. The objective of our paper was to analyze the effects of a food subsidy program on output and employment. To do this, we build a two sector heterogenous agent model of a farmer and an entrepreneur, both of whom are eligible for a subsidy on food consumption. The novelty of our paper is that food consumption augments the labor capacity of a representative agent who then decides how to allocate this capacity towards work and leisure. This ensures "food security" even with low levels of agricultural productivity.

We then assume two different tax regimes. The government may finance this subsidy by levying a distortionary income tax or through a tax on manufacturing consumption. In the long run, the subsidy program increases the output of the food sector but lowers the manufacturing output, independent of the method of its financing. While the price of food crop relative to the price of manufacturing good falls under an income tax regime, it increases under the consumption tax regime.

We also determine the welfare effects of the food subsidy program on the farmer and the entrepreneur under both tax regimes. The program may have long-run welfare gains for the two agents only for a certain range of subsidies. However, financing this program using an indirect consumption tax regime is Pareto superior to a direct income tax regime.

This exercise also suggests that introducing a universal food subsidy program may not necessarily have large benefits for an economy in the long run. Introducing other welfare measures to enhance labor productivity, for instance, may complement a subsidy program which has partial coverage. This will also enable us to analyze the effectiveness of introducing such welfare schemes in highly debt driven economies. Future work can therefore extend this framework by adding public debt as an alternative source of financing the subsidy program. We may also extend our model by allowing for international trade.

Appendix

Proof of Proposition 3

As the tax regimes does not differentially affect the farmer's optimization conditions, so $X_{at} = X^s_{at}$. Further from eqs. (4.21) and (4.34) we get,

$$(1-\tau_t)\frac{Q_{mt}}{p_{at}} = \Upsilon_1(Y_{at}),\ \frac{Q^s_{mt}}{p^s_{at}} = \Upsilon_1(Y^s_{at}). \tag{4.41}$$

The two functional forms are same. forms of This implies that the respective implicit functions are equal. In steady state of the income tax regime, using (4.20), (4.23a) and (4.23b) we get

$$\begin{aligned}\frac{Y^*_m}{p^*_a} &= f_1 X^*_a + f_2 Y^*_a + \left(\frac{{}^1/_\rho - 1 + \delta}{1-\alpha-\beta} - \delta\right)\left(\frac{1-\alpha-\beta}{{}^1/_\rho - 1 + \delta}\right)\frac{(1-\tau^*)\,Q^{s*}_m}{p^{s*}_a} \\ &\quad - \left(\frac{\phi_1}{1-\phi_1-\phi_2}\right)A\left\{\frac{X^{s*}_a\,(1-f_1)}{A} - \frac{1}{X^{s*}_a}\right\},\end{aligned} \tag{4.42}$$

and similarly in the consumption tax regime using (4.36) , (4.37b), and (4.37a), we get

$$\begin{aligned}(1+\tau^*_s)\,\frac{Y^{s*}_m}{p^{s*}_a} &= f_1 X^{s*}_a + f_2 Y^{s*}_a + \left(\frac{{}^1/_\rho - 1 + \delta}{1-\alpha-\beta} - \delta\right)\left(\frac{1-\alpha-\beta}{{}^1/_\rho - 1 + \delta}\right)\frac{Q^{s*}_m}{p^{s*}_a} \\ &\quad - \left(\frac{\phi_1}{1-\phi_1-\phi_2}\right)A\left\{\frac{X^{s*}_a\,(1-f_1)}{A} - \frac{1}{X^{s*}_a}\right\}.\end{aligned} \tag{4.43}$$

As $X^*_a = X^{s*}_a$ and together with (4.41), (4.42) and (4.43) we get

$$\frac{Y^*_m}{p^*_a} = \Upsilon_2(Y^*_a),\ (1+\tau^*_s)\,\frac{Y^{s*}_m}{p^{s*}_a} = \Upsilon_2(Y^{s*}_a). \tag{4.44}$$

Substituting (4.41), (4.44) in the entrepreneur's food optimization condition (4.14) and (4.30) we get that in steady state

$$Y^*_a = Y^{s*}_a.$$

4.5 Figures

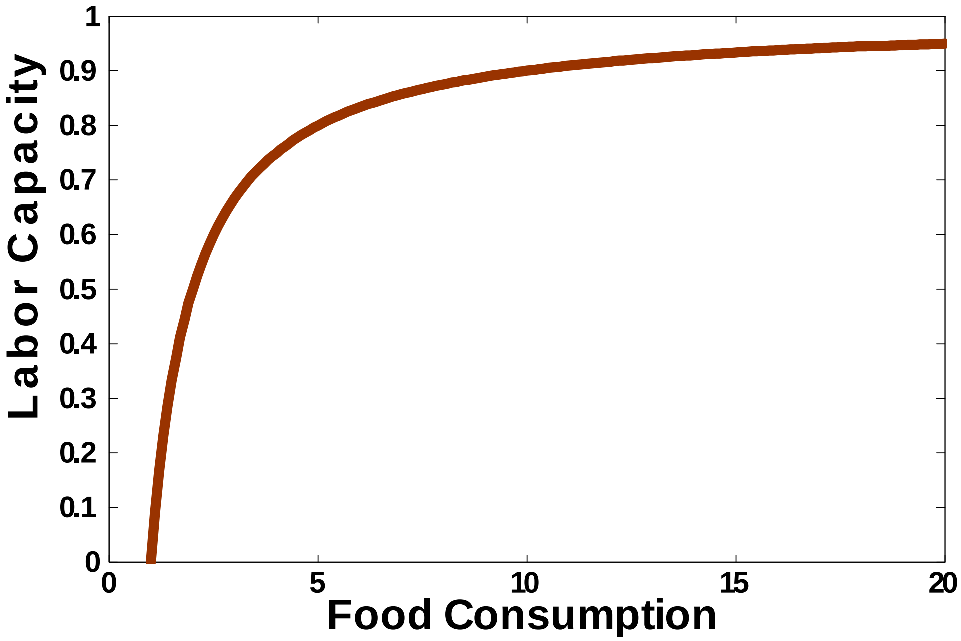

Figure 4.1: Metabolism Function

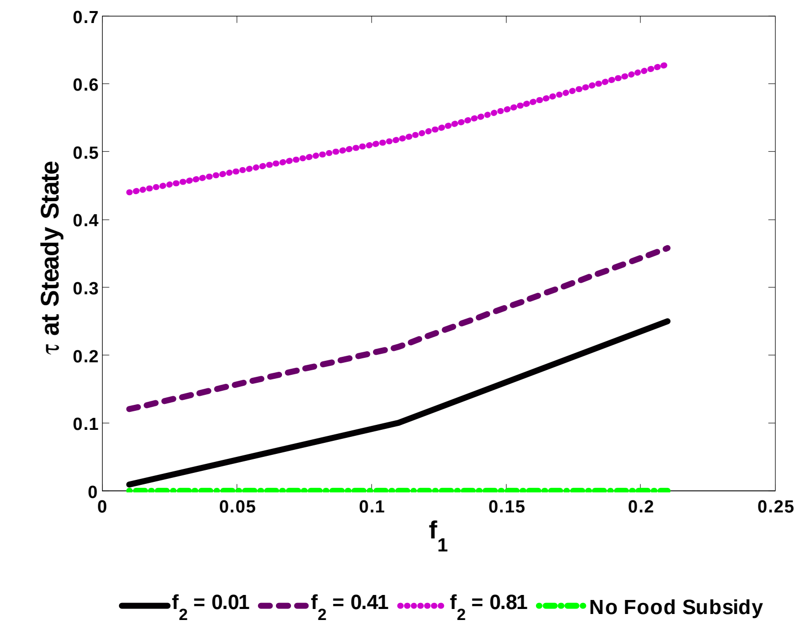

Figure 4.2: The effect of the food subsidy program on τ^*

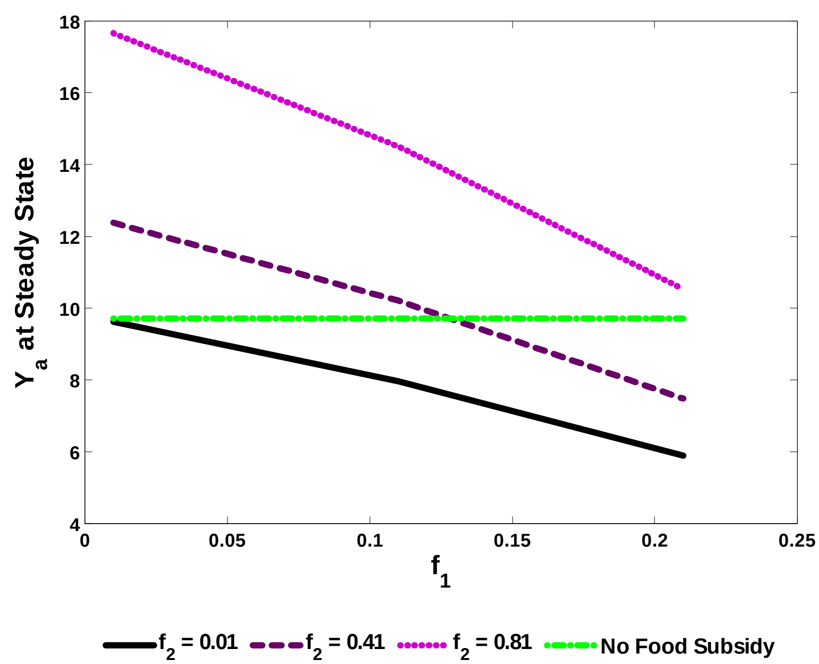

Figure 4.3: The effect of the food subsidy program on Y_a^*

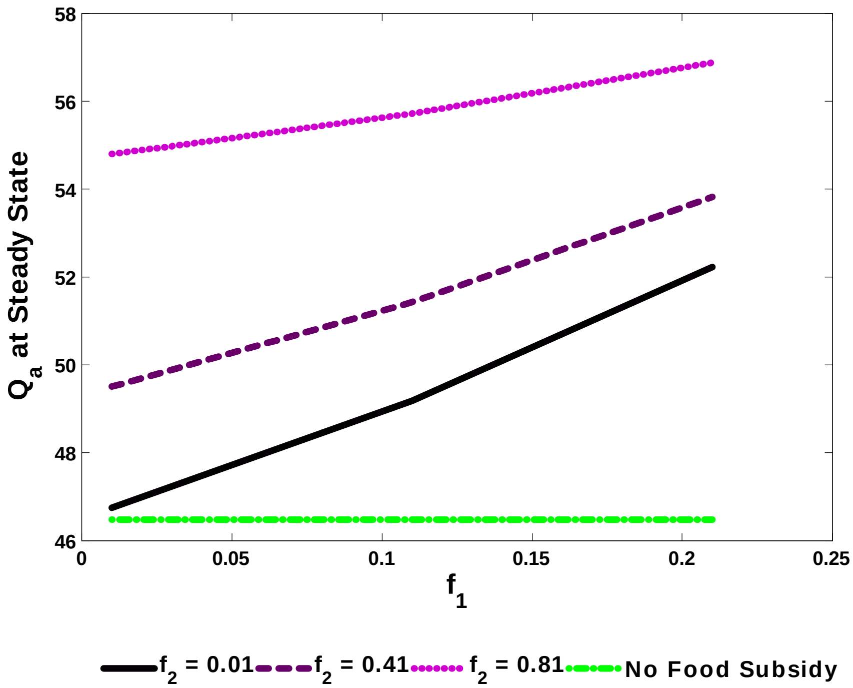

Figure 4.4: The effect of the food subsidy program on Q_a^*

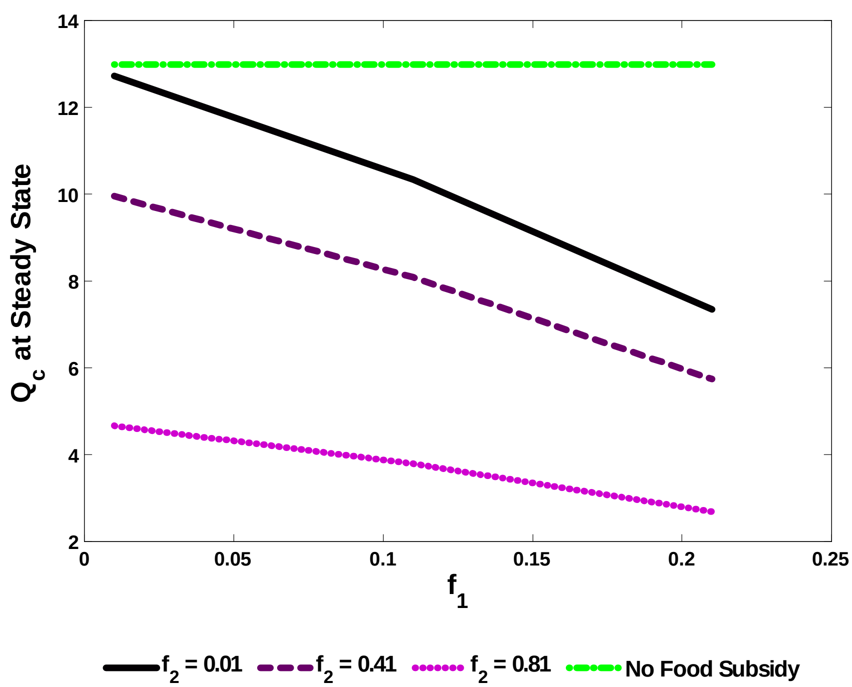

Figure 4.5: The effect of the food subsidy program on Q_c^*

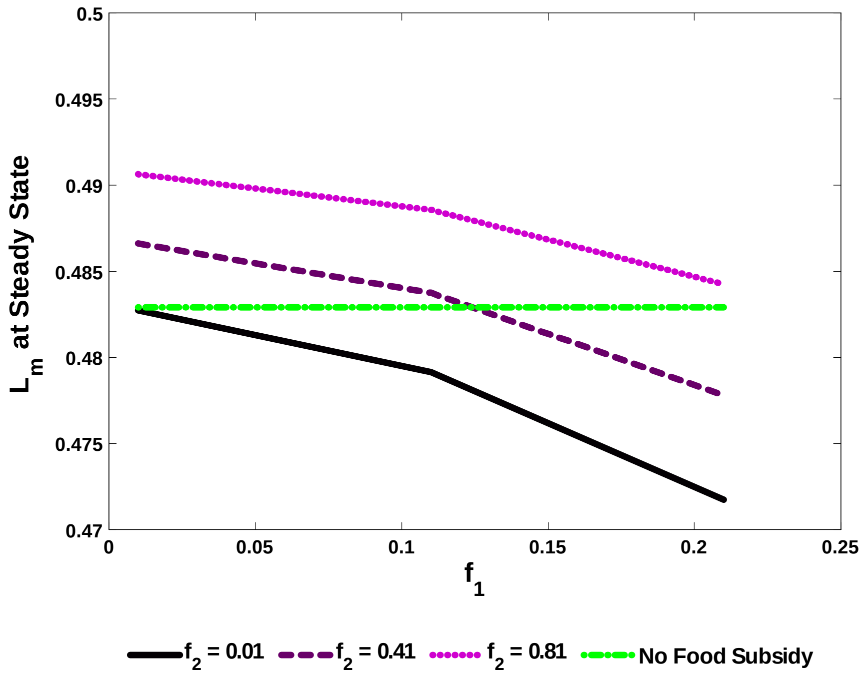

Figure 4.6: The effect of the food subsidy program on L_m^*

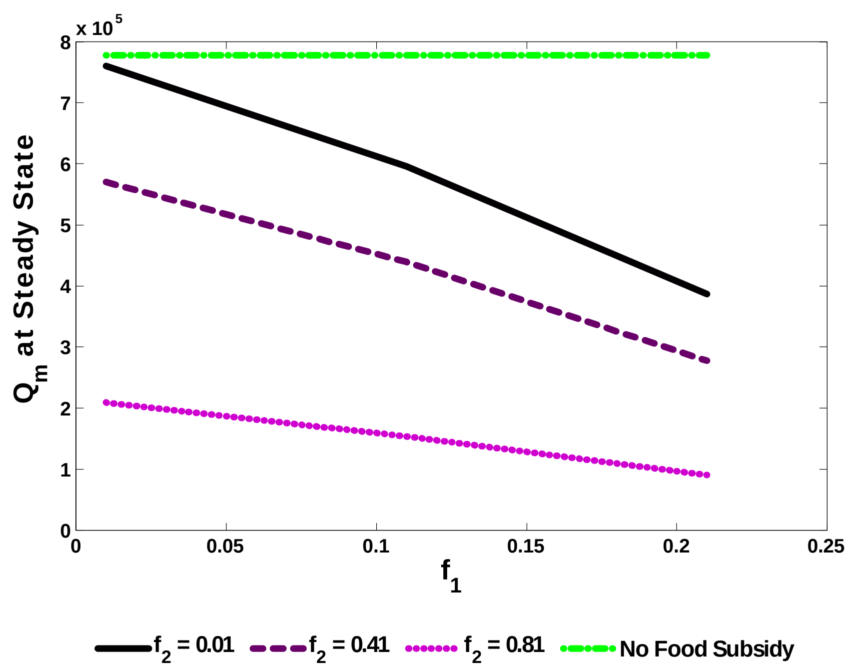

Figure 4.7: The effect of the food subsidy program on Q_m^*

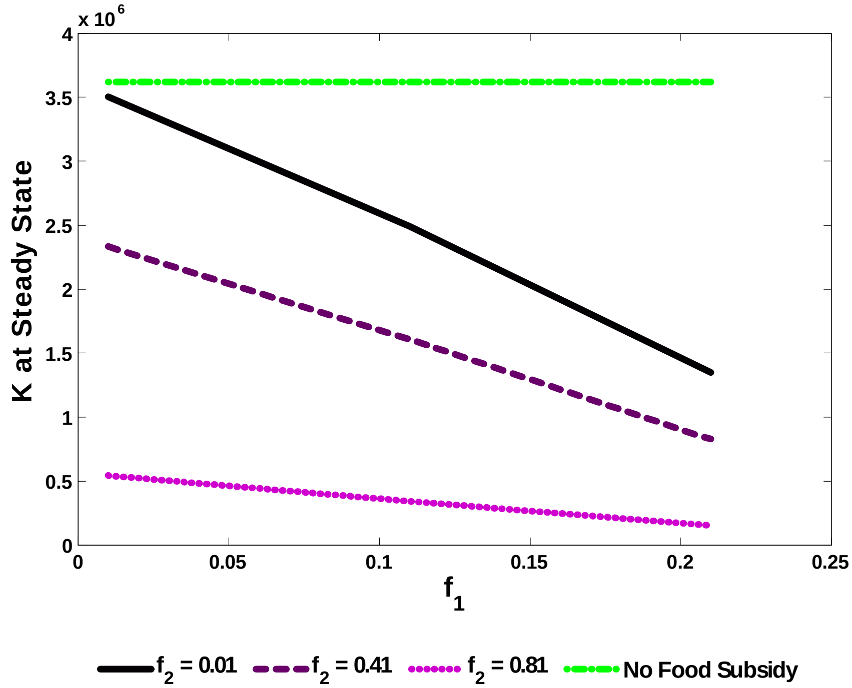

Figure 4.8: The effect of the food subsidy program on K^*

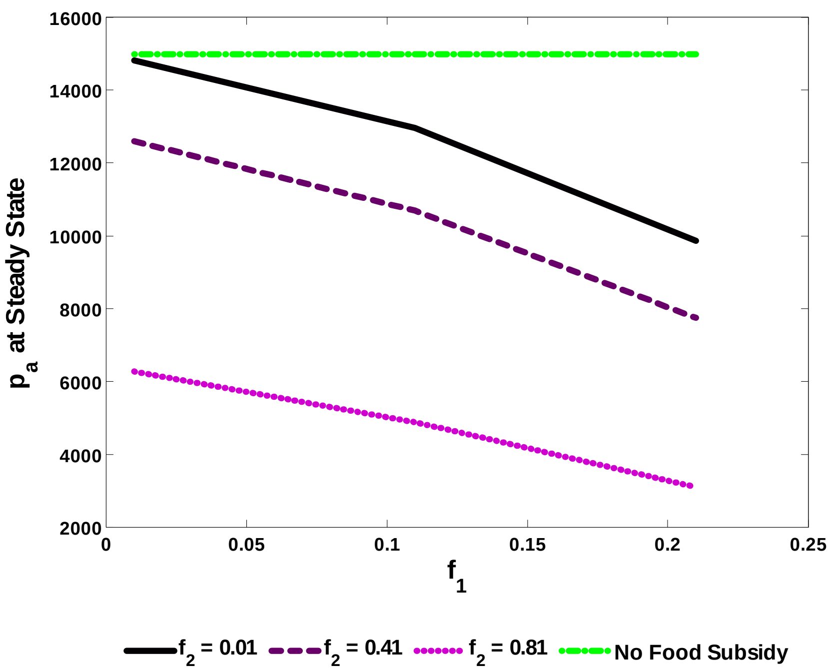

Figure 4.9: The effect of the food subsidy program on p_a^*

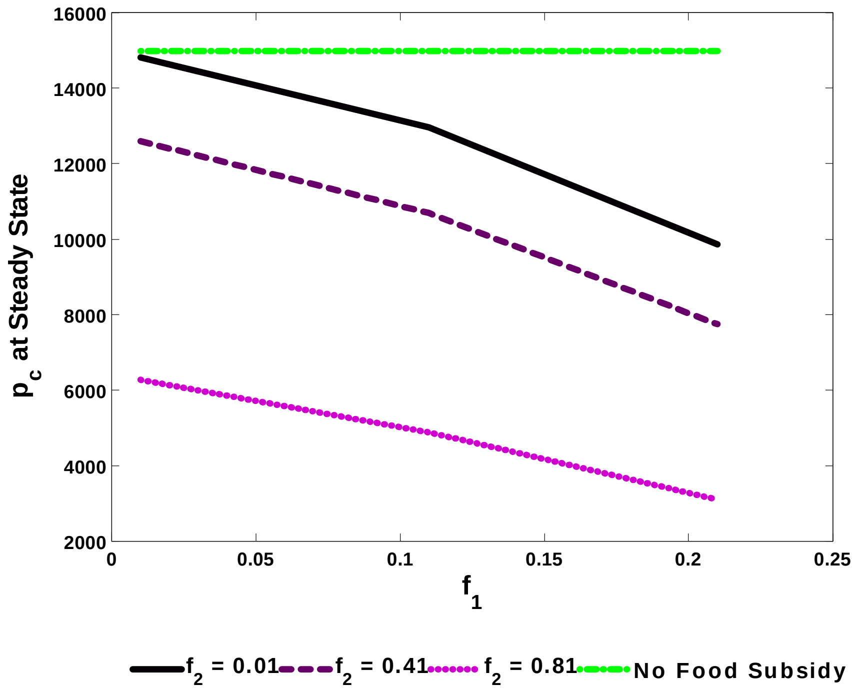

Figure 4.10: The effect of the food subsidy program on p_c^*

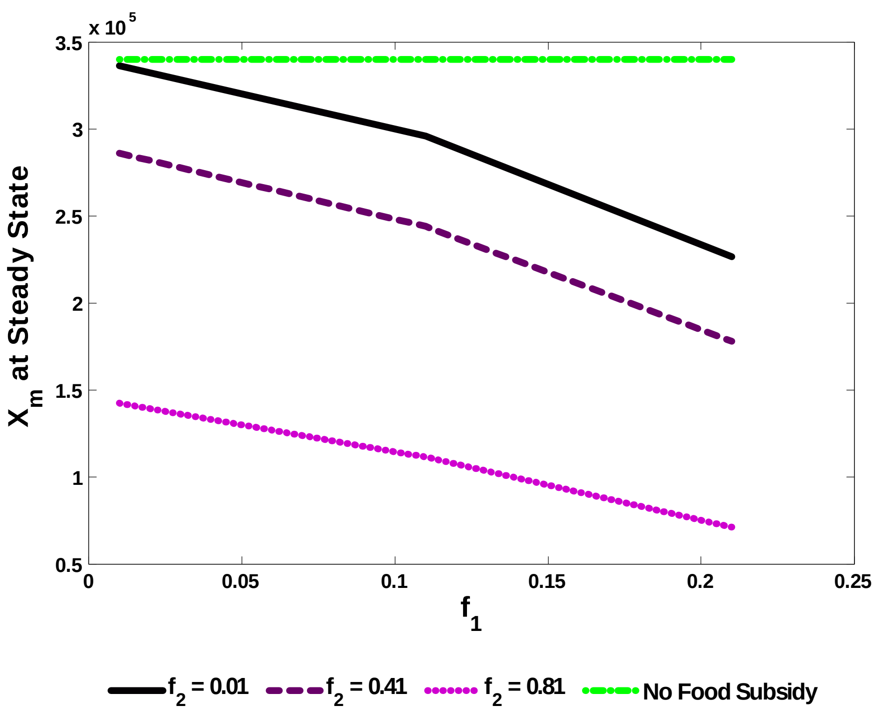

Figure 4.11: The effect of the food subsidy program on X_m^*

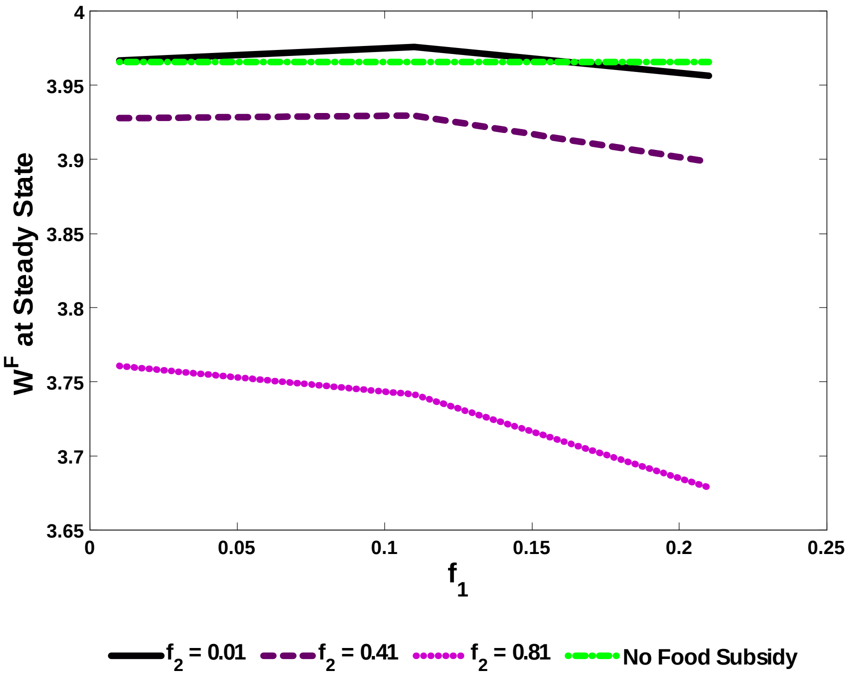

Figure 4.12: The effect changing f_1 for a given f_2 on W^{F*}

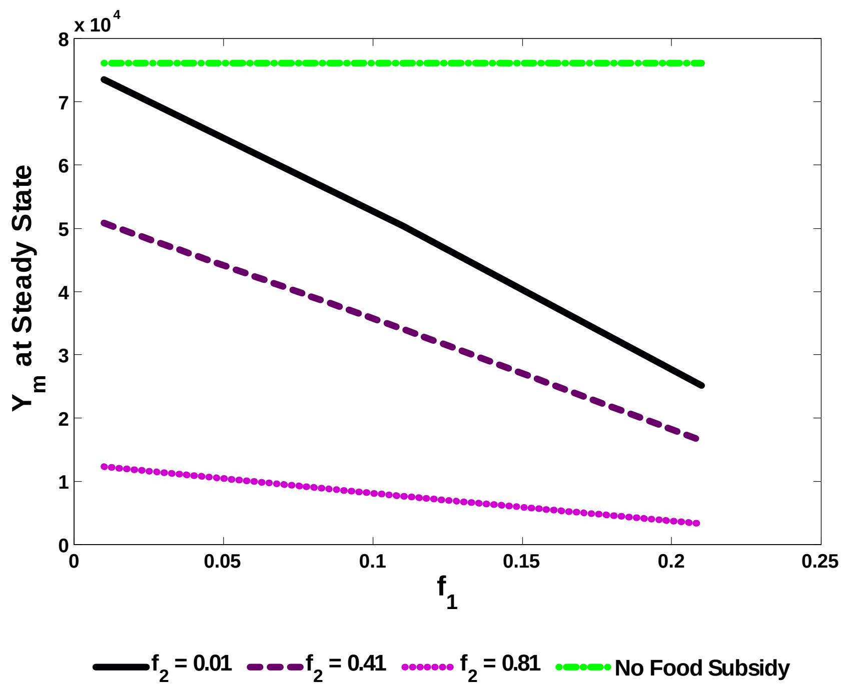

Figure 4.13: The effect of the food subsidy program on Y_m^*

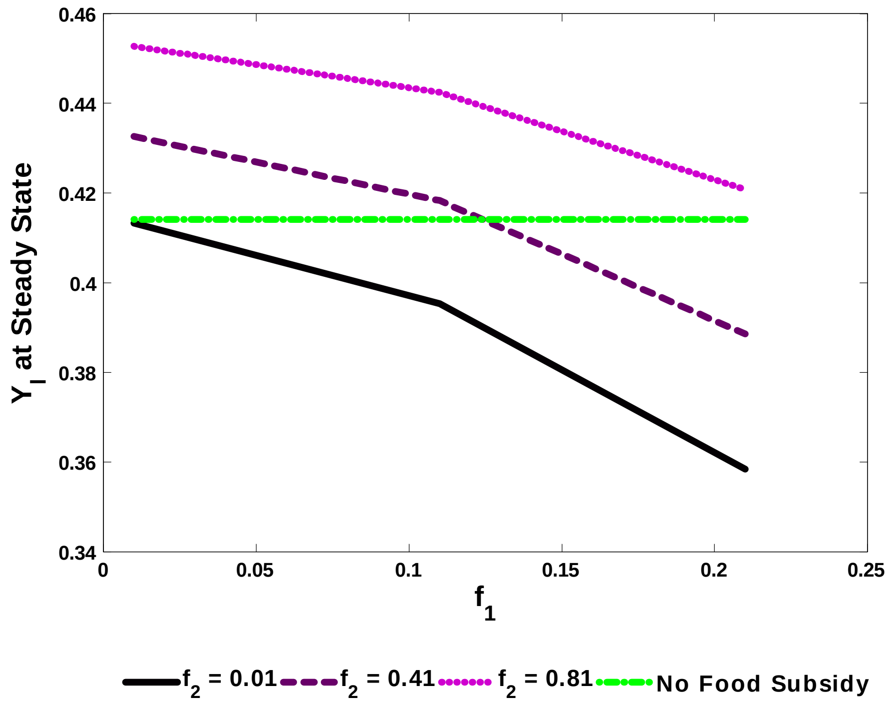

Figure 4.14: The effect of the food subsidy program on Y_l^*

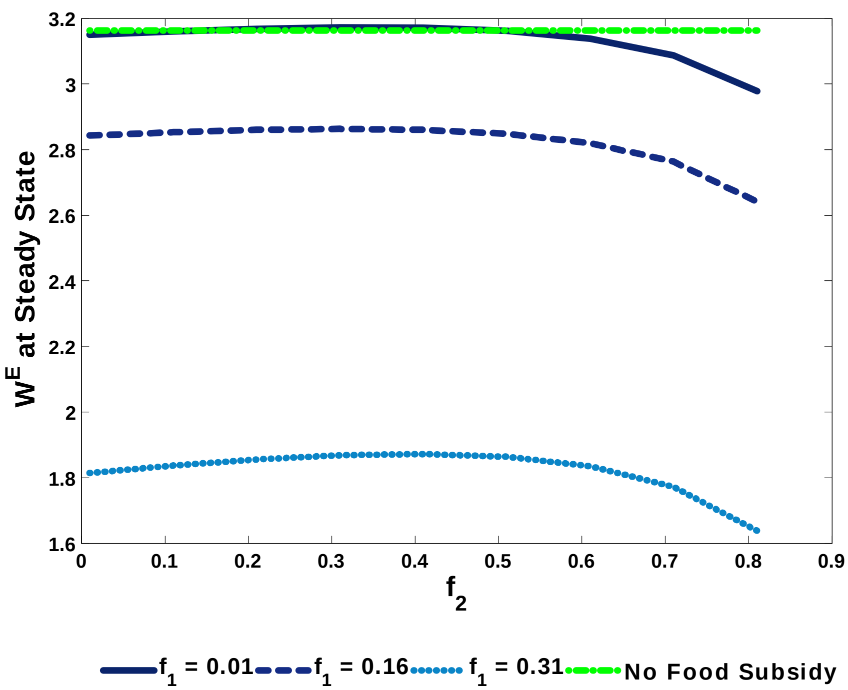

Figure 4.15: The effect of a change in f_2 for a given f_1 on W^{E*}

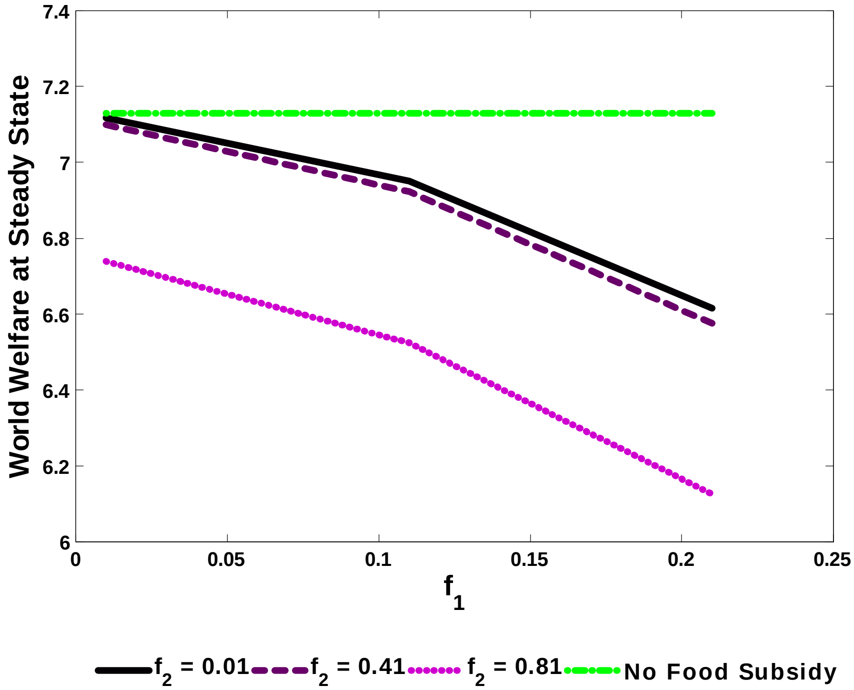

Figure 4.16: The effect of the subsidy program on W^{O*}

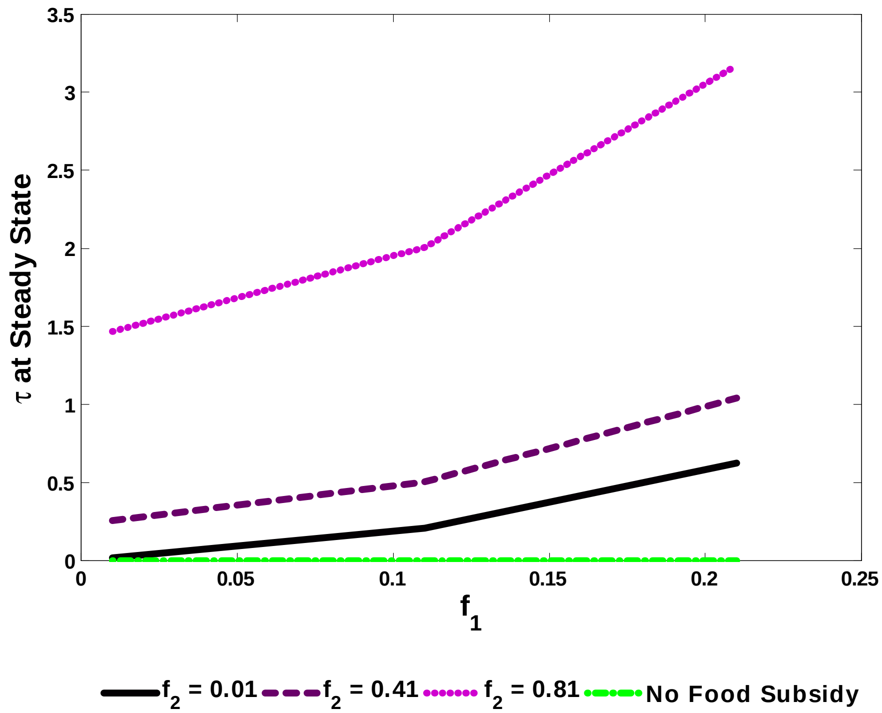

Figure 4.17: The effect of the subsidy program on τ_s^*

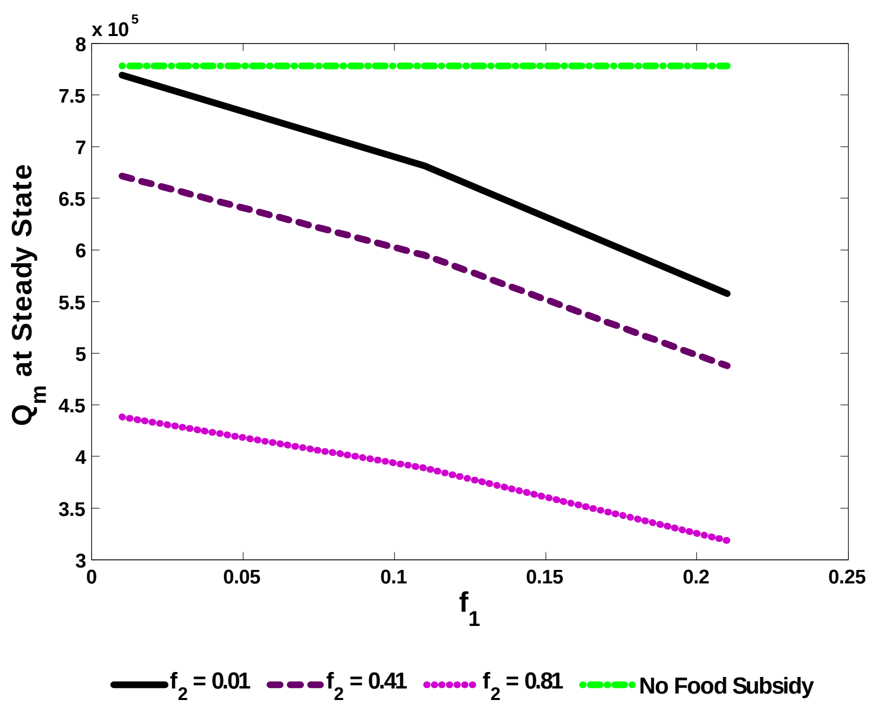

Figure 4.18: The effect of the subsidy program on Q_m^{s*}

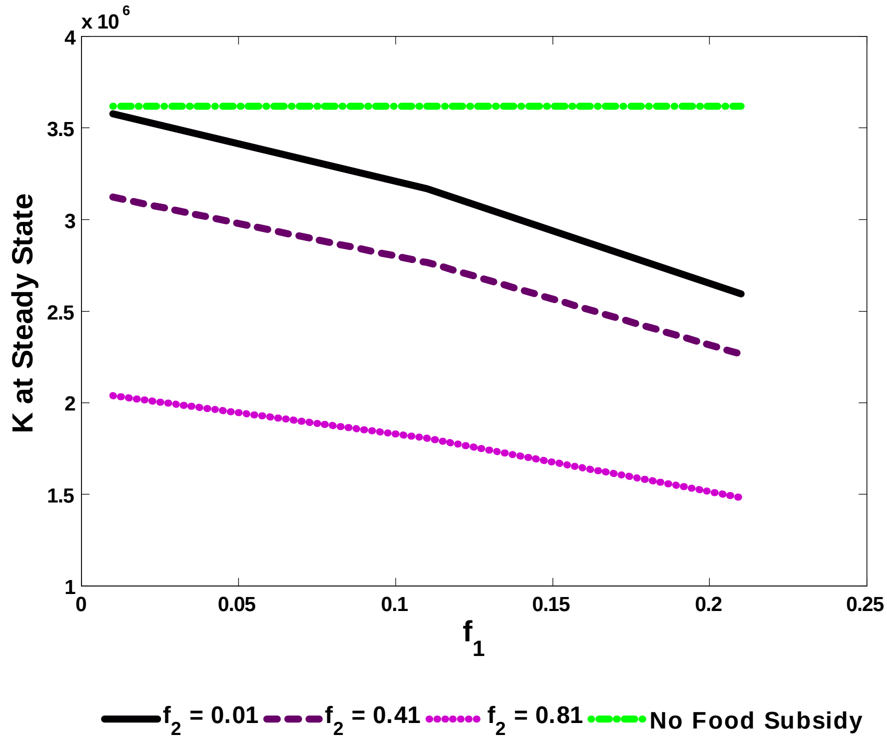

Figure 4.19: The effect of the subsidy program on K^{s*}

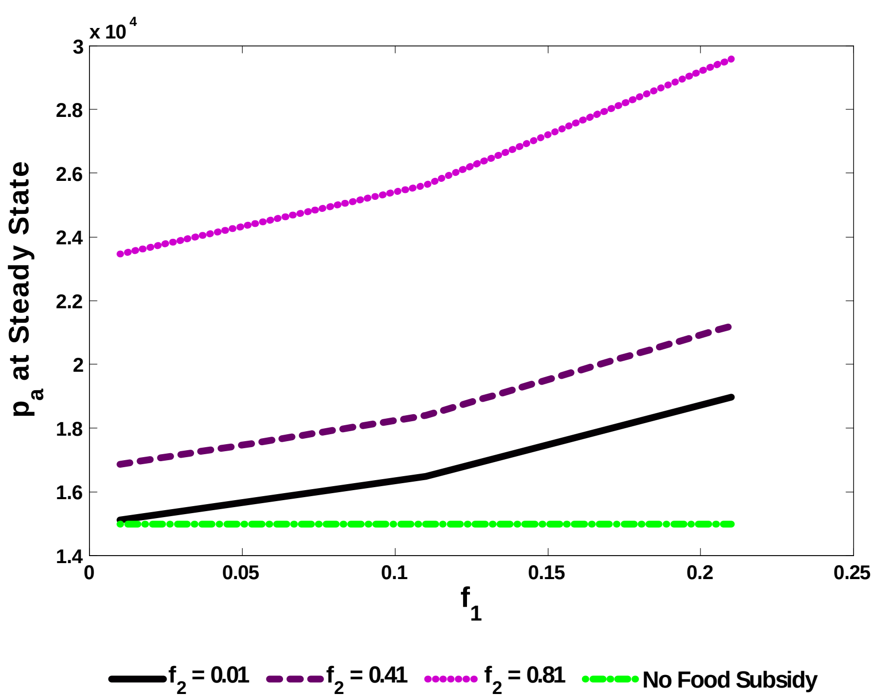

Figure 4.20: The effect of the subsidy program on p_a^{s*}

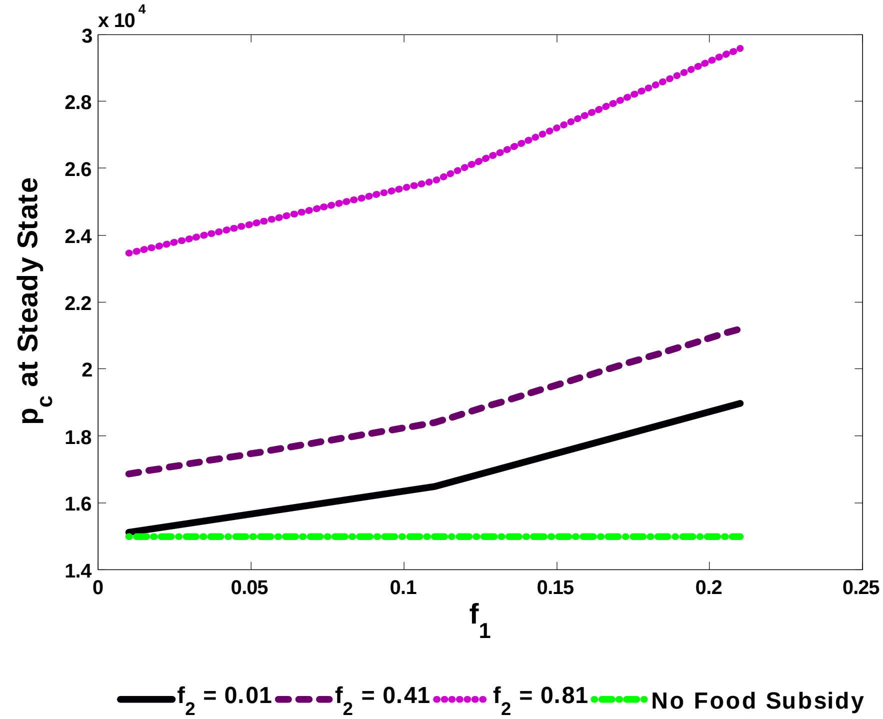

Figure 4.21: The effect of the subsidy program on p_c^{s*}

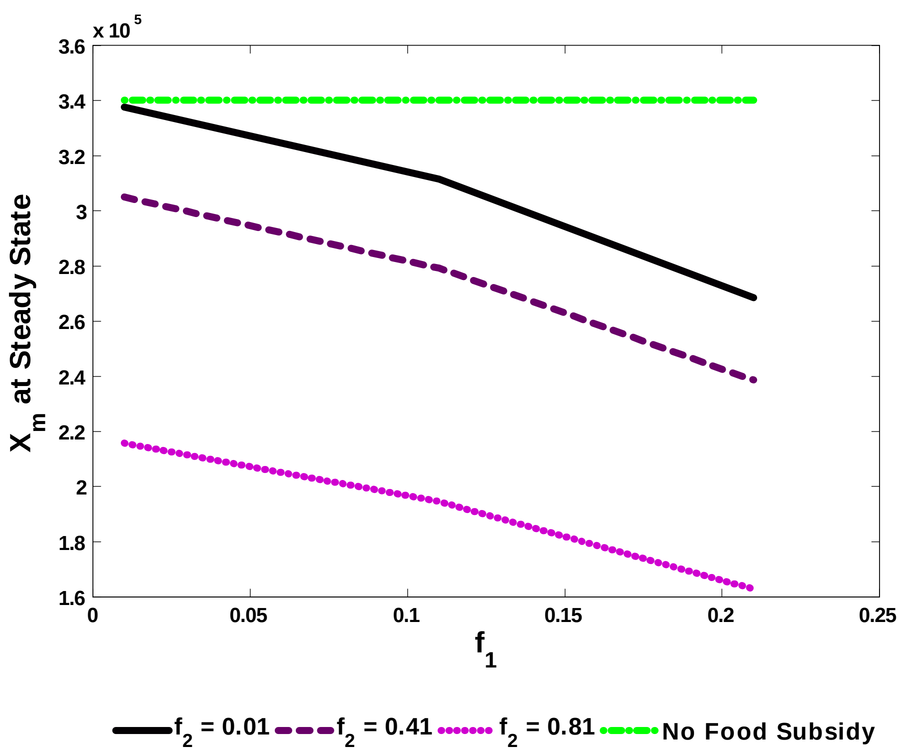

Figure 4.22: The effect of the subsidy program on X_m^{s*}

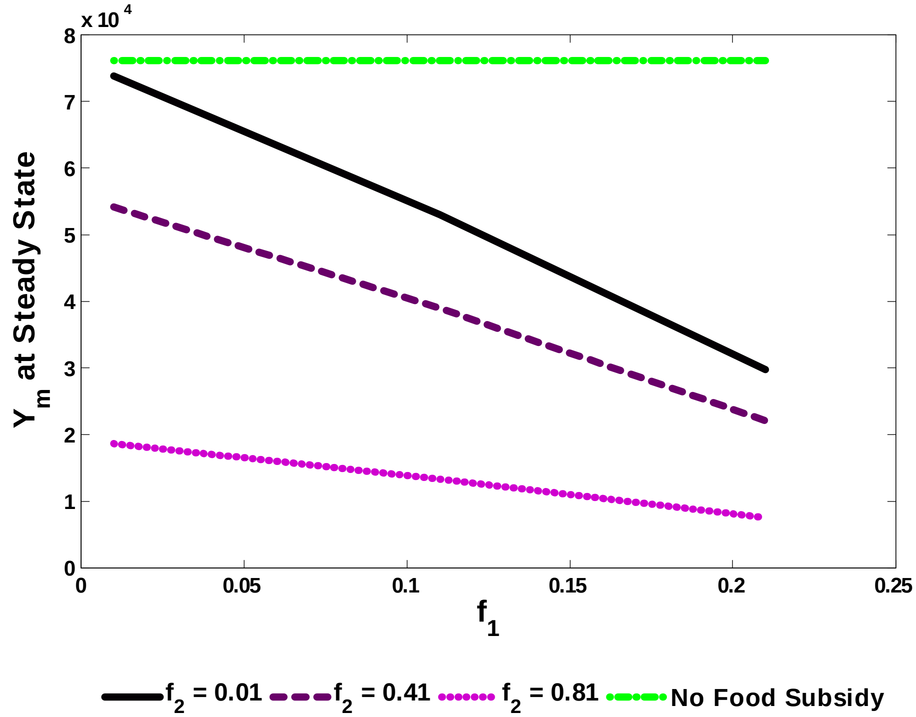

Figure 4.23: The effect of the subsidy program on Y_m^{s*}

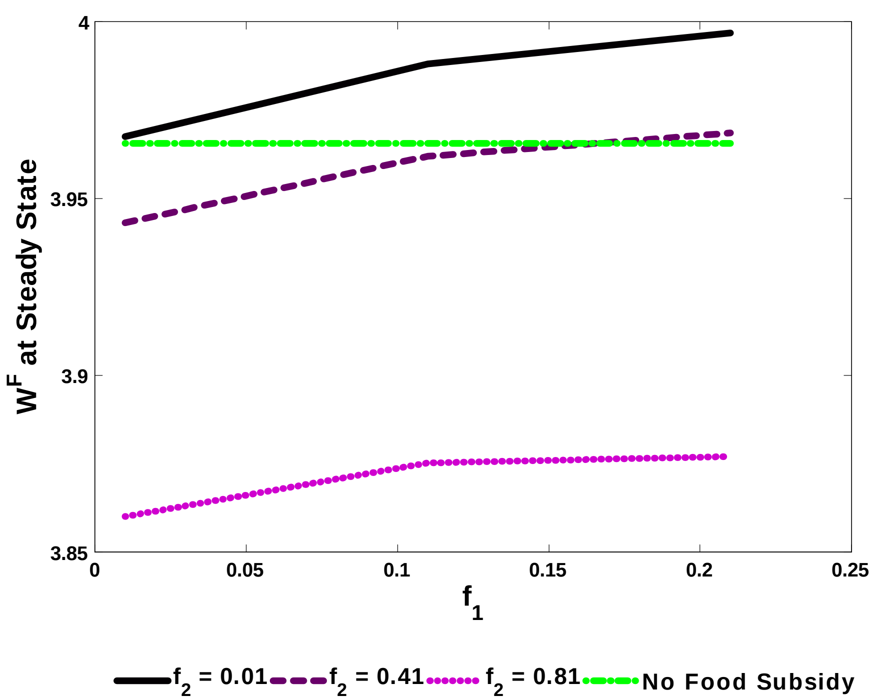

Figure 4.24: The effect of the subsidy program on W^{sF*}

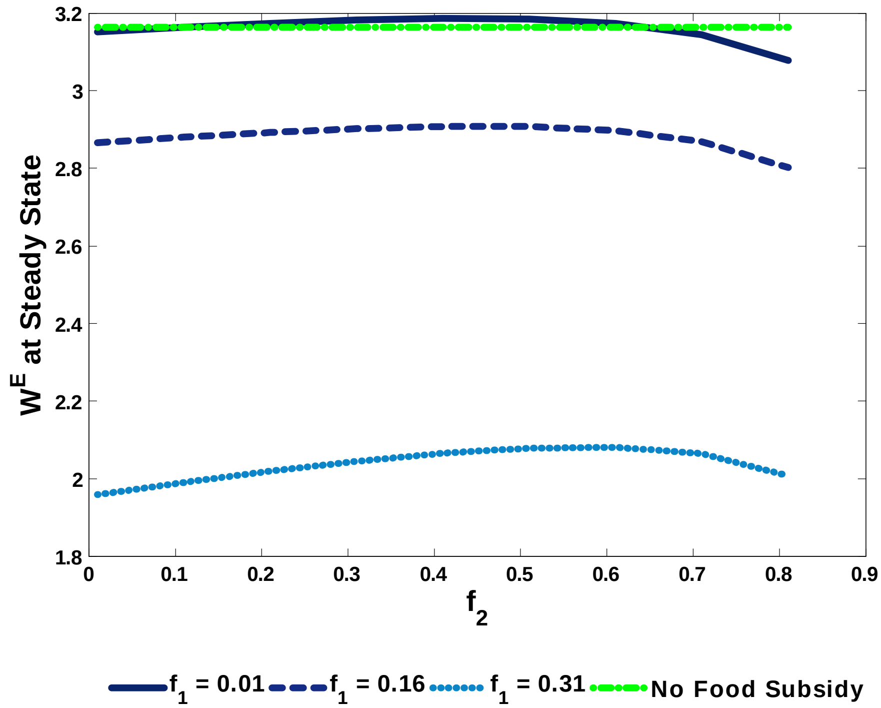

Figure 4.25: The effect of a change on f_2 for a given f_1 on W^{sE*}

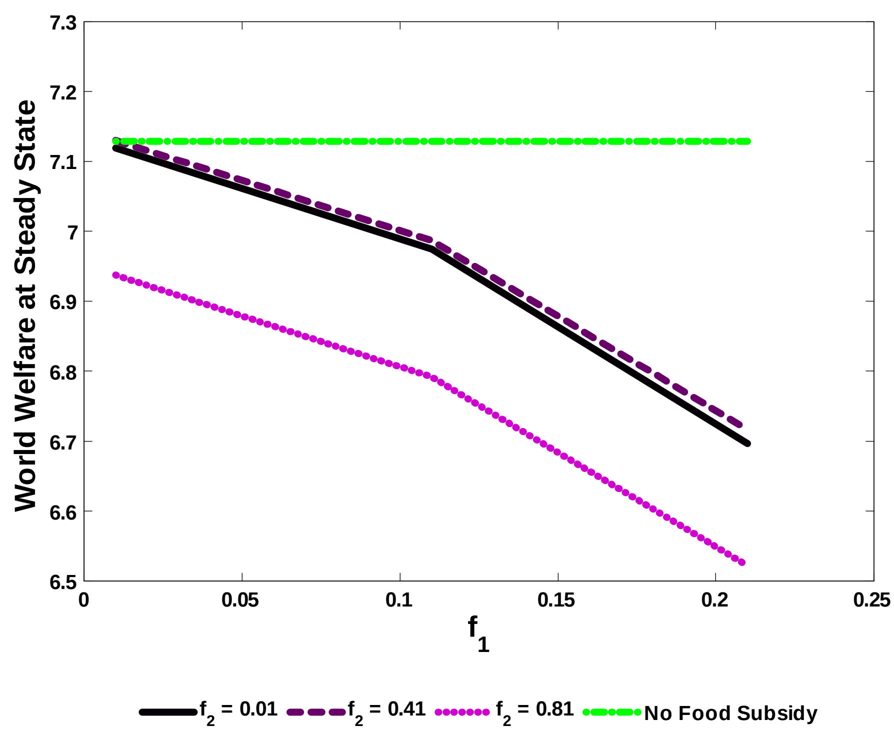

Figure 4.26: The effect of the subsidy program on W^{sO*}

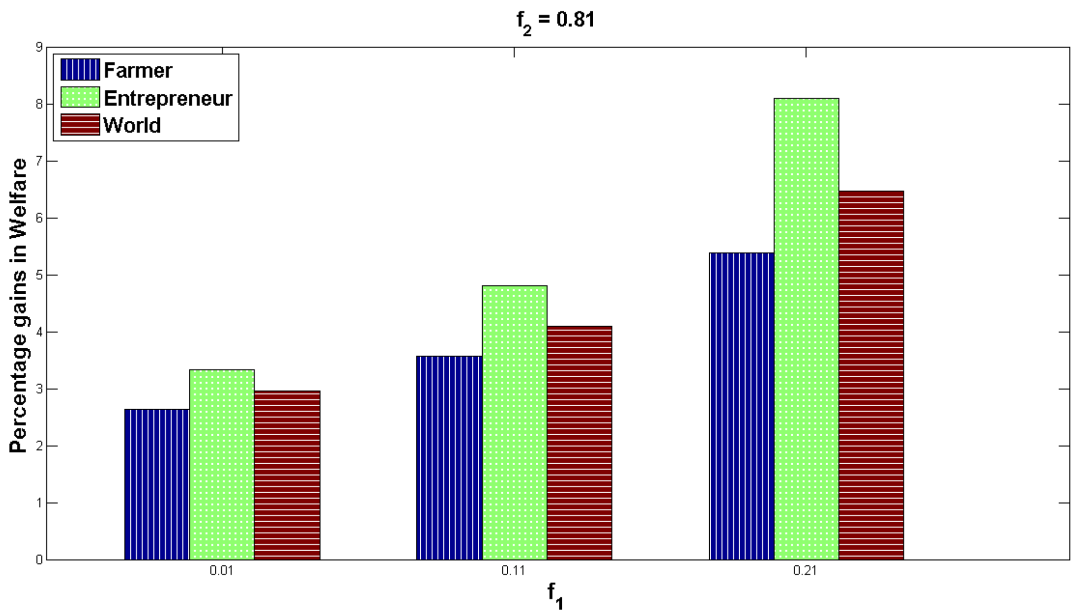

Figure 4.27: Percentage gains in welfare for $f_2 = 0.81$

Chapter 5

Ongoing and Future Work

The focus of this thesis was to analyze the effect of fiscal policy on three different aspects – the effect of taxation and government spending on long-run growth in advanced economies, the stabilization effects of government expenditures in emerging economies business cycles, and the costs and benefits of implementing a welfare measure so as to meet developmental goals in a heterogeneous agent economy. Chapter 2 of this thesis was motivated towards explaining why several OECD economies exhibiting different factor income tax combinations are observed have identical output growth rates. To explain this, a model of endogenous investment specific technological change (ISTC) with fiscal policy, building on Huffman (2008) was used. ISTC is augmented by public and private capital stock externalities, and a specialized labor. Spillovers from this specialized labor also augments the final goods production. The framework developed in Chapter 2 therefore provided a tractable channel to explain how the presence of these externalities affect the magnitude of the factor income tax gap and generates distinct rankings across different factor income tax rates, as observed in the OECD data.

Chapter 3 was motivated towards building an emerging market economy (EME) real business cycle model with fiscal policy, to explain the wide EME business cycle experience. In particular, the model provided an explanation as to why could some EMEs like India, have pro-cyclical real interest rates with counter-cyclical government expenditures, contrary to what was documented in previous literature. Building on Neumeyer and Perri (2005), the model proposed in Chapter 3 incorporated fiscal policy into a standard EME real business cycle model as a stabilization tool which makes real interest rates a-cyclical or pro-cyclical. The model was then used to replicate qualitatively some of the key features of the Indian business cycle.

Chapter 4 provided a framework to analyze the effect of introducing a food subsidy program in a heterogenous economy consisting of two type of agents, i.e., a farmer and an entrepreneur. The effects of introducing the subsidy program on sectoral outputs, prices, and welfare, using two different tax regimes – a distortionary income tax regime where the entrepreneur's output is taxed and a consumption tax regime where the consumption of the manufacturing output by both agents are taxed – are analyzed. The main normative insight from the model was that welfare gains from such a subsidy program are limited. However, financing the subsidy program using a consumption tax regime, where the tax burden is shared between both agents, is Pareto superior compared to financing it using a distortionary income tax regime.

The following sections discuss some possible extensions, and scope for future work, building on some of the key results obtained in the previous chapters.

5.1 Investment Specific Technological Change

A fundamental empirical observation over a long time period in advanced economies was that the share of labor income in total income was observed to be constant over time (see Kaldor (1957)). This stylized fact, among others, motivated economists to build long-run balanced growth models to understand economies that experience steady state balanced growth. Karabarbounis and Neiman (2014) have recently documented that the labor income share in total income has fallen since 1980s in a large majority of advanced and labor abundant emerging economies like India and China. Investment Specific Technological Change, due to technological advancements, is attributed as one of the key reasons for this stylized observation across countries.

A possible outcome of falling labor income shares in total income is rising wage inequalities due to skill accumulation. He and Liu (2008) provide a framework of ISTC to explain the increase in the ratio of skilled to non-skilled workers and therefore rising skill accumulation, along with rising wage inequality in the US since 1980s. To understand this, they build a model of skill heterogeneity, endogenous skill accumulation, and ISTC, where equipment capital is more complementary to skilled workers than unskilled workers. The model is able to qualitatively replicate most of the increase in wage inequality in the US post 1980s. The government imposes taxes on labor and capital income, which it then transfers back to households. In their counter-factual experiments, a decrease in the tax on capital income has strong welfare gains but with some marginal increase in inequality due to high capital-skill complementarity. Further, a progressive increase in the labor income tax does not reduce wage inequality and at the same time has a negative effect on overall welfare and productivity.

Whereas in the above context there is no productive role for government spending, a possible extension, building on He and Liu (2008), could be one where productive government spending, which augments human capital accumulation, can be chosen so as to lower long-run wage inequality. For instance, the effect of public spending on education and health outcomes, and therefore output growth, has also been widely analyzed in many theoretical and empirical papers (see Agénor (2008, 2008a, and 2010), Wang (2003), Bloom et al. (2004), and Wagstaff and Claeson (2004)). Building on the model proposed in Chapter 2 this extension could therefore introduce skill differences between labor employed in producing the final output and ISTC.

In addition to augmenting ISTC directly through a productive public capital stock, government spending towards health and education can augment skill accumulation which in turn enhances ISTC. Public spending may therefore be optimally allocated towards accumulating a public capital stock that directly augments ISTC as in Chapter 2, and public spending on health and education that augments human capital accumulation. The model can then be used to identify the right combination of government spending allocations, and factor income taxes, so as to lower wage inequalities in the long run.

5.2 Emerging Market Economy Business Cycles

An important aspect of emerging market business cycles that has received little attention in this thesis is the importance of financial development in smoothening business cycle fluctuations. In a cross-country empirical study, da Silva (2002) shows that after controlling for all other factors that affect business cycle fluctuations, countries with more developed financial systems, where more credit is given to the private sector relative to the public sector, have less volatile economic fluctuations. This is because stronger financial systems are characterized by lower credit market imperfections.[1] This enables a better screening of borrowers who are less credit-worthy. Aghion et al. (1999) show that following a productivity shock, countries with more developed financial markets, will converge faster to their new steady state compared to countries with less developed financial markets (also see Aghion et al. (2000, 2001, and 2004), and Eichengree et al. (2006)).[2]

Fiscal policy, through public debt, can also potentially affect financial development. For

[1] Theoretically, financial development in a model can captured by introducing risky borrowers as in Kiyotaki and Moore (1997), Scharler (2008), and Mitra (2013). In Scharler (2008) and Mitra (2013), firms face two kinds of shocks, a liquidity shock through working capital requirements, and an indiosyncratic productivity shock which determines their ability to repay, or alternatively, the probability with which they default.

[2] In more recent literature on small and emerging open economies, news on expected future TFP, and financial news is also a source of business cycle fluctuations (see Jaimovich and Rebelo (2008) and Gunn and Johri (2011 and 2013)).

instance, Ismihan and Ozkan (2012) show that public debt can harm financial development especially in countries where a major portion of total bank lending goes as credit to the government. Fiscal expansion is therefore more contractionary, i.e., public debt crowds out credit to the private sector more, particularly in emerging market economies or underdeveloped countries with limited financial depth. The model built in Chapter 3 can therefore be extended by introducing public debt to address this question.

Development of financial markets also affects the asset portfolio of households. In the absence of profitable lending or investment options, or because of poor financial coverage, households may prefer to invest in 'safer options' such as gold and silver. In addition, a household may also attach high utility from the pruchase of precious metals particularly in the form of ornaments (see Patrick (1966)).[3] To meet its high domestic demand mainly in the form of ornaments, India, for instance, relies heavily on importing gold and silver, the share of which in total imports, are very high at around 12%, next only to crude oil at 34%.[4] Therefore, while on the one hand, under-developed financial markets may cause crowding out of credit available to firms, on the other hand, the extent of the development of financial markets also has an effect on the economy's current account. In addition, higher inflationary expectations can trigger an increase in the current demand for gold and silver, which may in turn put pressure on the local currency to depreciate. This may further cause higher future realized inflation because of spillovers from a higher import bill, through the import of factor inputs such as petroleum and coal. This may in turn raise the volatility of the economy's business cycle. This is an ongoing project.

5.3 Food Security

As discussed in Chapter 4, availability of food is one factor that crucially affects food security. A combination of macroeconomic imbalances and changes in policy via trade restrictions, affect the short run agricultural prices via the futures market and speculative trading (see Headey (2011) and Giordani et al. (2014)). The availability of food in the short run mainly depends on inventory levels and how food stocks accumulate. If prices of food grains in the futures market are higher than current food prices, this causes an increase in the accumulation of food stocks. On the contrary, asymmetric information may induce speculative trading . This results in trade shocks which distort the domestic availability of food. In some countries like Bangladesh, however, trade liberalization has in the past improved food secu-

[3] This is analogous to Hansen and Imrohoroglu (2013) who assume that in Japan, households derive utility from holding public debt.

[4] See http://www.rbi.org.in/scripts/PublicationsView.aspx?id=15921 for the break-up on India's imports of principal commodities.

rity (see Dorosh (2001)). Previous evidences therefore suggest that trade liberalization may improve food security, but asymmetries in information which encourages speculative trading may create shortage of food. Hence the effect of free trade with frictions due to asymmetric information can be analyzed in the context of food availability and food security.

The model introduced in Chapter 4 assumes an environment where there is no occupational mobility. In another extension, occupational mobility may be assumed, by introducing a third representative agent who optimally allocates his total labor supply towards production of the agricultural output and the manufacturing output. The agent solves a dynamic problem in which he consumes the food crop and the manufacturing output and saves for the future. In addition to introducing a food subsidy program in this framework, the effect of introducing a policy reservation wage for supplying labor to the manufacturing sector can also be analyzed. While the model in Chapter 4 predicts that the food subsidy program negatively affects the steady state manufacturing output since labor supplied to the manufacturing sector falls, introducing a reservation wage may encourage migration of surplus labor from the agricultural sector to the manufacturing sector. This may offset the negative effect of the food subsidy program on the manufacturing sector.

Another feature that may be introduced in the manufacturing sector is friction in the form of labor adjustment costs (see Li (2011)). Due to strong labor laws, hiring and firing of employees in the manufacturing sector is costly. Introducing labor adjustment costs would therefore offset the negative effect of the food subsidy program on labor demand. Coupled with a high reservation wage, the subsidy program on the one hand may increase total labor supply, whereas on the other hand it may not lower the manufacturing production. This may have an overall positive effect aggregate income and welfare.

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