R E C O G N I T I O N   A N D   F I T T I N G   O F   C I R C L E S   A N D   E L L I P S E S   I N   D I G I T A L   I M A G E

A dissertaion submitted in partial fulfilment of the requirements for the M.Tech (Computer Science) degree of Indian Statistical Institute.

by

K. C. G U P T A

S U P E R V I S O R : B. B. C H A U D H U R I

Indian Statistical Institute

203 Barrackpore Trunk Road

Calcutta 700035

India

July 1993

Chapter 1

O P T I M U M   C I R C L E   F I T T I N G   A P P R O A C H

1. introduction There are two stages of parametric shape analysis in image processing & pattern recognition.

1) Detecting the points which belong to a particular shape.

2) Optimum curve fitting from the set of points.

For detection we will use Hough Transform (H T). Though Hough transform is a very useful method for shape detection & fitting but it does not optimize any criterion of fitting . For optimum fitting two methods can be mentioned.

1) Conic section fitting by regression [2].

2) Closed form expression [1]

Both optimizing some objective criterion.

In Chaudhuri[1] a generalized circle fitting on multidimensional weighted data is described.

It is shown that the method is effective even if the data set makes an arc of 90 degree. this method is used in the program to find parameter (centre & radius) for a single circle. When a set of points are given we assume in this case that they belong to the same circle. Next, we tested if multiple circles can be fitted in a single image space using the same technique. The problem involves both localization and fitting of circles. We have observed that for two circles localization and fitting can be effectively done when they are separated by an order of (r1+r2) where r1 and r2 are the radius of two circles.

In the same paper[1] it is noted that iterative circle fitting method [2] minimizes exactly same form of error function as in the method due to Thomas and Chan[3] and they should lead to the same value of circle parameters. It is proved that they are indeed the same.

An interesting problem is to find the location of centre of a circle from the supplied points when the radius is known. Although less number of parameters are involved, it is seen that no closed form expression for the parameters are found by optimizing the objective function. On the other hand if the centre is given, the radius of the optimum circle can be found in closed form.

2. Two methods and special cases.

2.1 Method proposed by Bookstein[2].

Any point (x,y) on a circle with centre (X_0,Y_0) and radius 'r' are related as $(x - X_0)^2 + (y - Y_0)^2 = r^2$

or $\quad x^2 + y^2 = \alpha_0 + \alpha_1 x + \alpha_2 y$

where $\quad \alpha_0 = r^2 - X_0^2 - Y_0^2$

$\alpha_1 = 2X_0$

$\alpha_2 = 2Y_0$

Now error for i_{th} point can be defined as

$$\delta_i = x_i^2 + y_i^2 - \alpha_0 - \alpha_1 x_i - \alpha_2 y_i$$

$$f(\alpha_0,\alpha_1,\alpha_2) = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} (x_i^2 + y_i^2 - \alpha_0 - \alpha_1 x_i - \alpha_2 y_i)^2$$

$$\delta f/\delta\alpha_0 = 0 \quad \text{or} \quad \sum_{i=1}^{n} (x_i^2 + y_i^2 - \alpha_0 - \alpha_1 x_i - \alpha_2 y_i) = 0$$

$$\delta f/\delta\alpha_1 = 0 \quad \text{or} \quad \sum_{i=1}^{n} (x_i^2 + y_i^2 - \alpha_0 - \alpha_1 x_i - \alpha_2 y_i)x_i = 0$$

$$\delta f/\delta\alpha_2 = 0 \quad \text{or} \quad \sum_{i=1}^{n} (x_i^2 + y_i^2 - \alpha_0 - \alpha_1 x_i - \alpha_2 y_i)y_i = 0$$

let $\quad \Sigma x_i = a_1$, $\Sigma x_i^2 = a_2$, $\Sigma x_i y_i = a_3$, $\Sigma x_i^2 y_i = a_4$, $\Sigma x_i^3 = a_5$

$\Sigma y_i = b_1$, $\Sigma y_i^2 = b_2$, $\Sigma x_i y_i^2 = b_4$, $\Sigma y_i^3 = b_5$

So $\alpha_0 = 1/n\ (a_2 + b_2 - a_1\alpha_1 - b_1\alpha_2)$...............(1)

$b_4 + a_5 - a_1\alpha_0 - a_2\alpha_1 - a_3\alpha_2 = 0$...............(2)

$b_5 + a_4 - b_1\alpha_0 - a_3\alpha_1 - b_2\alpha_2 = 0$...............(3)

From (1) & (2) we have

$\alpha_1(na_2 - a_1^{\ 2}) + \alpha_2(na_3 - a_1b_1) = (nb_4 + na_5 - a_1a_2 - a_1b_2)$.......(4)

From (1) & (3) we have

$\alpha_1(na_3 - a_1b_1) + \alpha_2(nb_2 - b_1^{\ 2}) = (nb_5 + na_4 - b_1a_2 - b_1b_2)$.......(5)

From (4) we have

$\hat{B}\alpha_1 + \hat{A}\alpha_2 = \hat{D}$ and $\hat{A}\alpha_1 + \hat{C}\alpha_2 = \hat{E}$

Where $\hat{A} = (na_3 - a_1b_1)$, $\hat{B} = (na_2 - a_1^{\ 2})$, $\hat{C} = (nb_2 - b_1^{\ 2})$

$\hat{D} = (nb_4 + na_5 + a_1a_2 - a_1b_2)$, $\hat{E} = (nb_5 + na_4 - b_1a_2 - b_1b_2)$

Hence $\alpha_1 = \dfrac{\hat{A}\,\hat{E} - \hat{C}\,\hat{D}}{\hat{A}^2 - \hat{B}\,\hat{C}}$, $\alpha_2 = \dfrac{\hat{A}\,\hat{d} - \hat{E}\,\hat{B}}{\hat{A}^2 - \hat{B}\,\hat{C}}$

$\alpha_0 = 1/n\ (a_2 + b_2 - b_1 \dfrac{\hat{A}\,\hat{d} - \hat{E}\,\hat{B}}{\hat{A}^2 - \hat{B}\,\hat{C}} - a_1 \dfrac{\hat{A}\,\hat{E} - \hat{C}\,\hat{D}}{\hat{A}^2 - \hat{B}\,\hat{C}})$

2.2 Closed form expression Method [3].

The centre (c_1, c_2) and radius r_0 are given by

$$c_1 = (B'C - BC')/(AB' - A'B) \;,\; c_2 = (A'C - AC')/(A'B - AB')$$

$$r_0^2 = \sum_{i=1}^{n} \left[\frac{(x_i - c_1)^2 - (y_i - c_2)^2}{n} \right]$$

Where $A = \Sigma (x_i - \bar{x})x_i \;,\quad A' = \Sigma (y_i - \bar{y})x_i$

$B = \Sigma (x_i - \bar{x})y_i \;,\quad B' = \Sigma (y_i - \bar{y})y_i$

$C = \frac{1}{2} \Sigma (x_i - \bar{x})(x_i^2 + y_i^2) \;,\quad C' = \frac{1}{2} \Sigma (y_i - \bar{y})(x_i^2 + y_i^2)$

2.2.1 Special case 1 : Radius r_0 given .

No closed form expression when radius is given

Let (a,b) be centre

$$f(a,b) = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} (x_i^2 - 2ax_i + y_i^2 - 2by_i + a^2 + b^2 - r^2)^2$$

$\delta f/\delta a = 0$ or

$$\sum_{i=1}^{n} (x_i^2 - 2ax_i + y_i^2 - 2by_i + a^2 + b^2 - r^2)^2 (a - x_i) = 0 \;\ldots.(1)$$

$\delta f/\delta b = 0$ or

$$\sum_{i=1}^{n} (x_i^2 - 2ax_i + y_i^2 - 2by_i + a^2 + b^2 - r^2)^2 (b - y_i) = 0 \;\ldots.(2)$$

Equation (1) & (2) are quadratic in a , b but for which we have no closed form expression.

2.2.2 Special case 2 : Centre (a,b) given

$\delta f/\delta r = 0$ or $\sum_{i=1}^{n} (x_i^2 - 2ax_i + y_i^2 - 2by_i + a^2 + b^2 - r^2)^2 = 0$ or

$r = SQRT\ [\ (\Sigma x_i^2 - 2a\,\Sigma x_i + \Sigma y_i^2 - 2b\,\Sigma y_i + na^2 + nb^2)/n\ ]$

i,e when (a , b) is given the soln for r is trivial

3. Equivalence of the two methods

We have to show that $\alpha_1/2 = c_1$, $\alpha_2/2 = c_2$ & $r = r_0$

Now $\hat{A} = n\,\Sigma x_i y_i - n\,\bar{x}\,\Sigma y_i = n\,(\Sigma x_i y_i - \bar{x}\,\Sigma y_i) = n\,B'$ ..(7)

Similarly $\hat{B} = n\,A$.....(8) , $\hat{C} = n\,B'$ ....(9)

$\hat{E} = n\,\Sigma y_i^3 + n\,\Sigma x_i^2 y_i - \Sigma y_i\,\Sigma x_i^2 - \Sigma y_i\,\Sigma y_i^2$

$= n\,\Sigma (y_i - \bar{y})(y_i^2 + x_i^2)$

i.e. $\bar{E} = 2\,n\,C'$........(10) similarly $\bar{D} = 2\,n\,C$........(11)

Now $\alpha_1/2 = \dfrac{\hat{A}\,\hat{E} - \hat{C}\,\hat{D}}{2(\hat{A}^2 - \hat{B}\,\hat{C})} = \dfrac{nB2nC' - nB'\,2nC}{2(\,nA' - nB - nAnB')}$

$= (BC' - B'C)/(A'B - AB') = c_1 = X_0$

similarly $\alpha_2/2 = c_2 = y_0$

now $r^2 = \alpha_0 + X_0^2 + Y_0^2$

$= 1/n(\,a_2 + b_2 - a_1\alpha_1 - b_1\alpha_2\,) + X_0^2 + Y_0^2$

$= 1/n\,(\,\Sigma x_i^2 + \Sigma y_i^2 - 2X_0\,\Sigma x_i - 2Y_0\,\Sigma y_i\,) + X_0^2 + Y_0^2$

$= 1/n\,(\,\Sigma x_i^2 + \Sigma y_i^2 - 2c_1\,\Sigma x_i - 2Y_0\,\Sigma y_i + n\,c_1^2 + n\,c_2^2\,)$

$= 1/n(\sum_{i=1}^{n} [\,(x_i - c_1)^2 + (y_i - c_2)^2\,] = r_0^2$

4. Experimental results and discussions.

If multiple circles are present in a single image space then problem involves both localization and fitting of circles.Here concept of window is introduced.We have taken an input image of size (128*128) as in fig-1. It is assumed that ratio of largest radius to smallest radius is less than two.In the table shown below we have outputs for different window size.Window size is needed as we require that at least in one window an arc greater than 90 degree of a circle is included.We have taken care of those windows which correspond to the same circle to get more accurate result.

window size	centre-1 (x, y)	centre-2 (x , y)	radius-1	radius-2
25	48.03,49.96	69.88,68.11	18.09	14.53
20	49.40,49.49	69.17,69.56	19.91	15.74
15	46.77,46.26	68.74,71.60	16.67	12.91

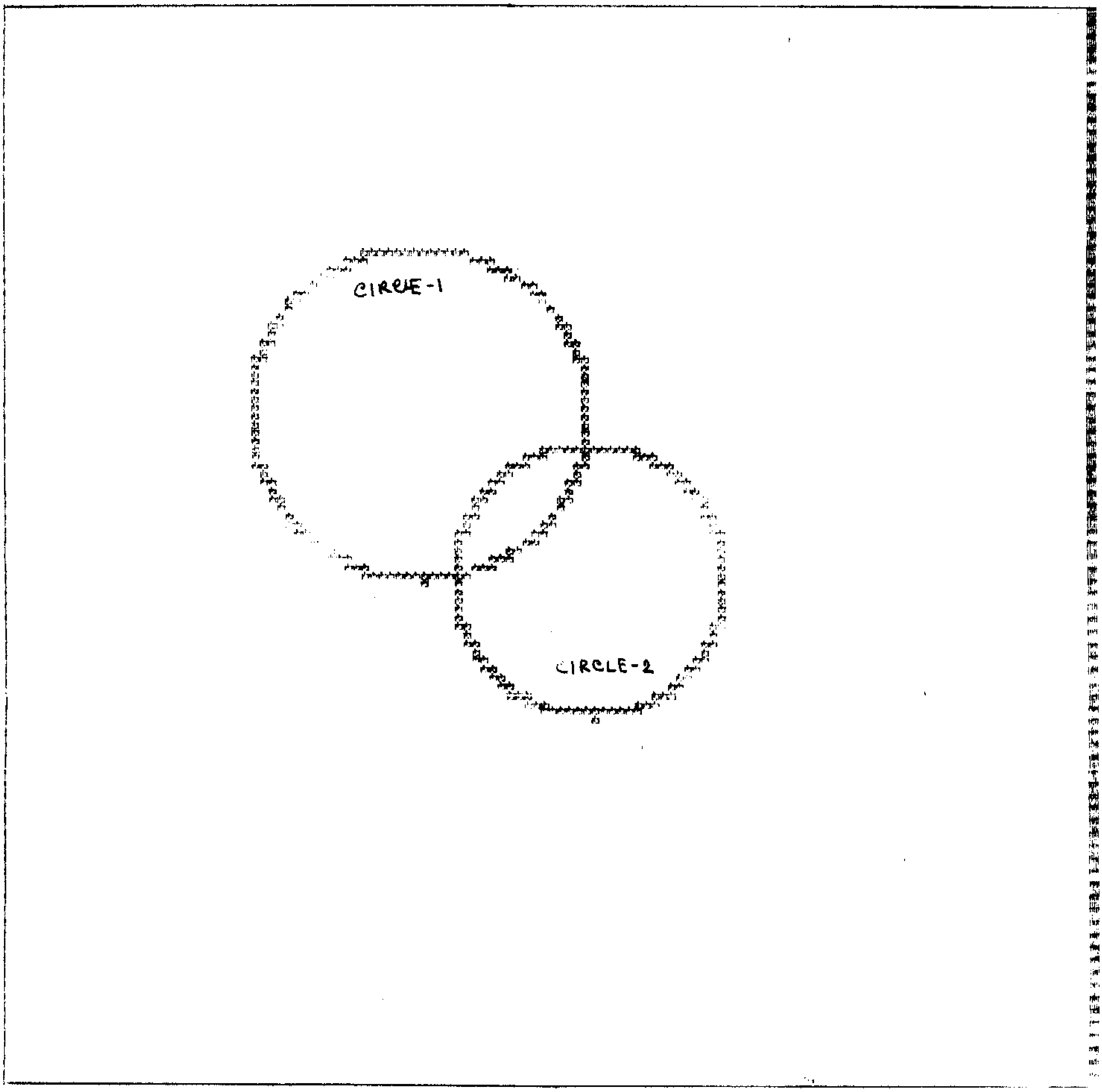

Fig-1

5. Acknowledgements.

We wish to acknowledge the help from Punam Kumar Saha during various stages of this work.

6. References.

1. Chaudhuary, B. B (1990), Optimal circular fit to objects in two and three dimensions. Pattern Recog. Letters, 11, 571-574.

2. Bookstein, F. L. (1979), Fitting conic sections to scattered data. Computer Vision, Graphics and Image Proces., 9, 56-71.

3. Thomas, S. M. and Chan Y. T. (1989), A simple approach for the estimation of circular arc center and its radius. Computer Vision, Graphic and Image Proces., 45, 362-370.

Chapter 2

ELLIPSE AND CIRCLE DETECTION BY HOUGH TRANSFORMATION

1. Introduction.

In an image the pertinent information about an object is very often contained in the shape of its boundary. So first step in image processing is to find gradient edge image. Which can be found by Sobel's operator. Our primary goal is to extract ellipse and circles. Circular objects appear as ellipses when viewed from on oblique angle. The rapid and accurate extraction of ellipses from images can, therefore, be highly important for any model based vision system. Fast extraction of ellipse contour is possible using methods such as Freeman chain coding and polyg onal approximation, however, the reliability of these technique are likely to suffer if the edge data is noisy or if the image feature space is cluttered. Direct application of Hough technique requires five dimensional accumulator array for ellipse extraction.

The task of extracting ellipses from images can be made practical by making use of the gradient direction of contour points, by breaking down the problem into one that can be solved in two, or more, sequentially executed stages [iii] and exploiting neighbourhood information.

2. Existing approaches.

Tsuji and Matsumato[v] have presented a scheme for ellipse recognition whereby the centers of candidate ellipse are extracted during the initial pass.The technique by which ellipse centers are found is based on a simple geometric property of ellipses. If a pair of points with equal tangents lie on contour of the same ellipse then their midpoint is the centre of the ellipse. In the first stages of the centre finding proce dure, pairs of image points with equal, or near equal, tangents are extracted. A two dimensional histogram of the midpoints of each extracted pair of points is then constructed. Peaks in this histogram define cen tres of candidate ellipses in the image. The set of points which contrib uted to the extracted centre are then identified and a least mean square technique is applied to these to obtain estimates of all five parameters.This method however suffer if outliers or irrelevant data is present in the image or gradient information is not very accurate. Sec ondly, the straight lines in the image must be located and removed before ellipse are searched far.

A new centre finding technique has recently been presented by Illing warth and Kittler as part of a scheme for ellipse detection. The adap tive Hough transform [iv] is used to extract the remaining 3 param eters.The AHT implementation uses a small(9x9x9) accumulator array and iteratively focuses in on parameter regions of highest accumulator densi ty. It does this by dynamically and independently increasing the resolu tion of each parameter in the array.

3. A tristage approach[3]

3.1 Centre estimation.

The first stage in most Hough-based ellipse detection schemes is that of centre-finding. It is perhaps the most important, since the accuracy with which the remaining three parameters can be determined frequently depends on the ellipse centre measurements. Successful extraction of the ellipse boundary is made possible if good centre-finding techniques are used. One centre-finding method (used in References 5) evaluated the mid-points of pairs of contour points with approximately equal tangents. If the contour points were both located on the boundary of an ellipse then their midpoint defined the ellipse centre. A two-dimensional histogram was used to accumulate the midpoint locations. Peaks in the histogram defined the locations of possible ellipse centres. However, partial occlusion of the ellipse, often resulted in a significant reduction in the number of points symmetrical about the ellipse centre. In these situations, the technique was liable to fail.

Because of its ability to handle partial occlusion, we have used the ellipse centre-finding method of Yuen et al[6].This procedure is based on a simple geometric property of ellipses. Consider the two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, lying on the boundary of an ellipse as shown in Fig. 1.

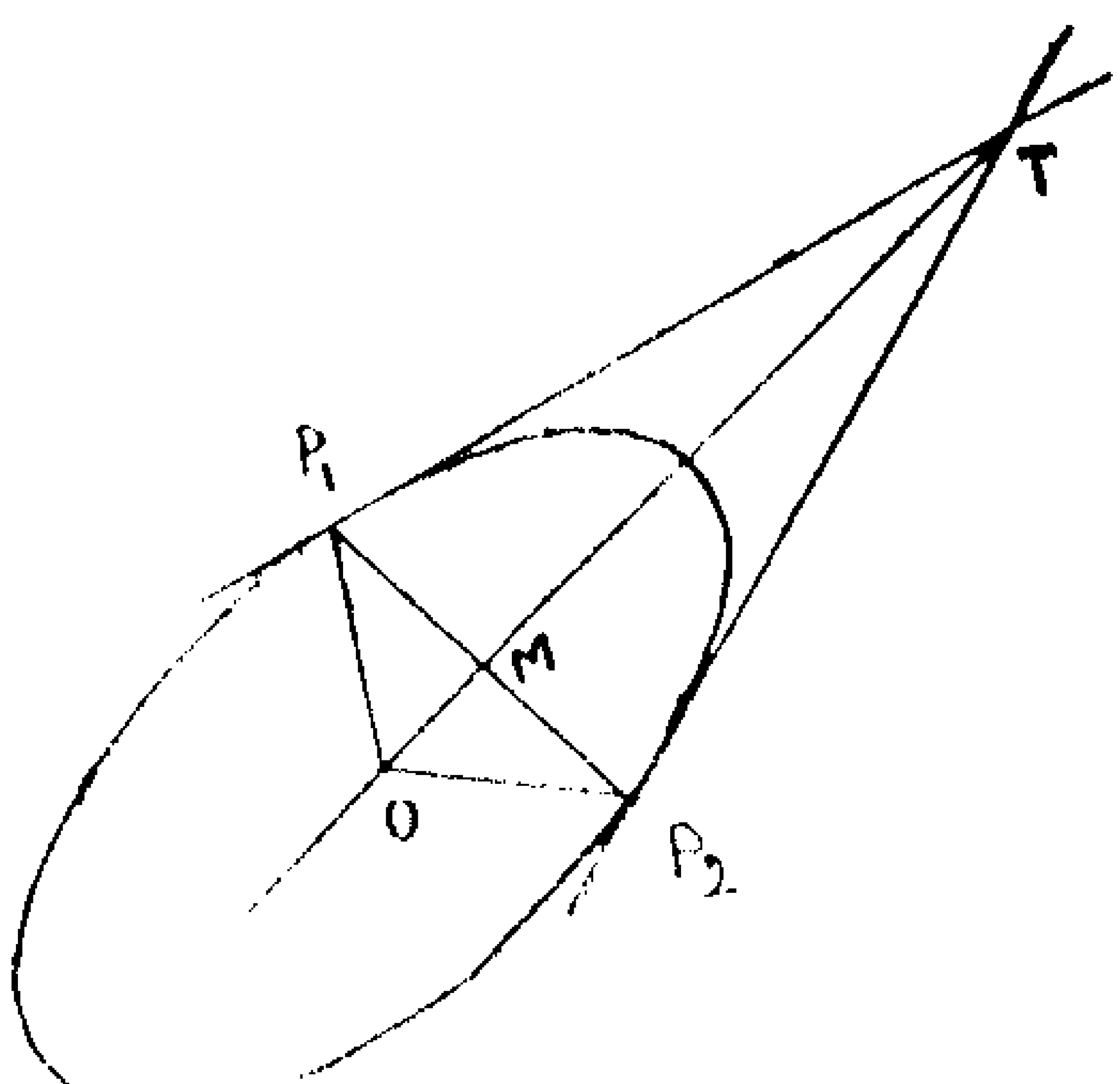

Fig-1

Given that P_1 and P_2 have intersecting tangents, then the line through the intersection point $T(t_x, t_y)$, and the midpoint $M(m_x, m_y)$, of P_1 and P_2 will pass through the centre of that ellipse. The equation of line TM can be expressed as:

$Y = cx + d$

Where x and y are the co-ordinates of points on the line. The gradient of the line c, and the offset d are given by:

$c = (t_y - m_y)/(t_x - m_x)$ & $d = m_y - c\, m_x$

The coordinate of T are given as

$t_x = y_1 - y_2 - g_1 x_1 + g_2 x_2$

$t_y = y_1 g_2 - y_2 g_1 + g_1 g_2 (x_2 - x_1)$

and for the point M:

$m_x = (x_1 + x_2)/2$ & $m_y = (y_1 + y_2)/2$

Where g_1, g_2 are the slopes of the tangents of P_1 and P_2 respectively. Lines constructed from all pairs of points lying on the same ellipse will intersect at one point, $O(x_0, y_0)$, the centre of the ellipse. In a practical implementation of this scheme, P_1 and P_2 cast their votes in a two-dimensional accumulator array called the centre array. Locations in this array which intersect the line TM are incremented. Local peaks in the array give evidence for possible ellipses and their centres in the images. Votes casted by pairs of points resulting from random noise or other shapes in the image will, in most cases, give to background accumu lation in the centre array . In certain situations however, the accumula tion of votes caused by pairing points on different ellipses or shapes can result in false peaks in the accumulator array. These peaks may disrupt the interpretation of the parameter space by masking out significant peaks relating to the centres of smaller or partially occluded ellipses.

Hence, we need only include a pair of image points, $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, in the voting process if they satisfy the following criteria.

$$d^2_{min} < (x_1 - x_2)^2 + (Y_1 - Y_2)^2 < d^2_{max}$$

where d_{max} and d_{min} are some preset values.

This criteria was applied by Yuen et al. [6] in their implementation of the ellipse centre-finding scheme. Careful choice of d_{max} and d_{min} provided a degree of control over the particular size of ellipse that was extracted. In general, d max was set to a value equal to twice the expected maximum of the major radius.

The process of pairing up a set of points results in a line along TM that passes at some point through the centre of the ellipse. For practical purposes, lines of length L, starting from M, in the direction of the expected ellipse centre were generated. In order that sufficient votes were generated to allow accurate estimates of ellipse centre, the length L was set in the region of the expected maximum of the major ellipse radius.

Since the Hough transform is an incremental evidence gathering procedure, shape extraction is possible even if part of the data is only processed. Point hopping was introduced to speed extraction, whereby qualifying feature points were paired with every nth point in the feature list rather than with every point.

3.2 Determining ellipse orientation

Once an estimate of the location of an ellipse centre has been made, the process of determining ellipse orientation can be carried out. If a shape is known to have an axis of symmetry with slope θ, then the angular slopes (in radians) of its k axes of symmetry are given by :

$$\theta_i = \theta_0 + (i-1)\,\Pi/k \quad i=1,2,\ldots k$$

An ellipse posses two axes of symmetry, θ_1 and θ_2 whose angular slopes are given by :

$\theta_1 = \theta_0$ and $\theta_2 = \theta_0 + \pi/2$

The procedure for estimating ellipse orientation involves identifying the set of midpoints M. and tangent intersections T, which lie on either axis of symmetry of the ellipse. An important assumption is that the intersection point of the axes of symmetry of the ellipse lies on the estimated ellipse centre. If the distances OP_1 and OP_2, from any pair of points, P_1 & P_2, respectively, to the ellipse centre are equal, then the points M and T are assumed to lie on either axis of symmetry of the ellipse. Points M and T which lie on the axes of symmetry are then rotated by a factor of pi/2 about the estimated ellipse centre, so that they lie in the first quadrant of a co ordinate frame whose origin lies on that centre.

The angles subtended by OM and OT increment a one-dismensional orientation histogram. The mode in the histogram defines the angular slope of either the major or the minor axis (θ_1 or θ_2) of the ellipse. Which axis the angular slope relates to is unimportant at this stage and can be resolved after the major and minor radii have been determined. The procedure for determining ellipse orientation is out lined below .

```
DO  =1,M    !Point Pi
  DO j = j + 1,M   !Point Pj
     d1 = OPi
     d2 = OPj
    IF  ((d1 .EQ. d2) .AND. (Pi .NE. Pj)) THEN
     { Work out M and T }
     { Rotate M and T about 0 by a factor of   pi/2 so that they lie
     in the first quadrant of a co-ordinate frame centered at 0 }
      theta1 = arctan ((my - yo) / (mx - xo))
     theta2 = arctan ((ty - yo) / (tx - xo))
     acc (theta1) = acc (theta1) +1
     acc (theta2) = acc (theta2) +1
    ENDIF
   ENDDO
ENDDO
```

{The location with maximum votes in array 'acc' is obtained }

3.3 Estimating the major and minor ellipse radii

Once a maximum, θ_{opt}, has been extracted from the orientation histogram the estimation of ellipse radii can proceed. Pairs of points, P_1 & P_2 in a third pass through the image are rotated to

$P_1'(x_1', y_1')$ and

$P_2'(x_2', y_2')$

through an angle θ about the estimated ellipse centre, according to the transformation :

$$x' = (x - x_0) \cos \theta_{opt} - (y - y_0) \sin \theta_{opt} + x_0$$

$$y' = (x - x_0) \cos \theta_{opt} - (y - y_0) \sin \theta_{opt} + y_0$$

This results in a subset of transformed points situated on the boundary of an unrotated ellipse. For a pair of points

$P_1'(x_1', y_1')$, and $P_2'(x_2', y_2')$

on the contour of an unrotated ellipse, with centre $O(X_0, Y_0)$.

We have

$$(x_1' - x_0)^2 / a^2 + (y_1' - y_0)^2 / b^2 = 1$$

$$(x_2' - x_0)^2 / a^2 + (y_2' - y_0)^2 / b^2 = 1$$

These can be combined by eliminating b to give

$$(x_1' - x_0)^2 (y_2' - y_0)^2 / a^2 (y_1' - y_0)^2 - (x_2' - x_0)^2 / a^2$$

$$= (y_2' - y_0)^2 / (y_1' - y_0)^2 - 1$$

Solving for a,

$$a = c/d \quad ---- (1)$$

$$c = ((x_1' - x_0)^2 (y_2' - y_0)^2 - (x_2' - x_0)^2 (y_1' - y_0)^2)^{1/2}$$

$$d = ((y_2' - y_0)^2 - (y_1' - y_0)^2)^{1/2}$$

We can also combine the two equations so as to eliminate a and solve for b to give

$$b = e/f \quad ---- (2)$$

$$e = ((x_2' - x_0)^2 (y_1' - y_0)^2 - (x_1' - x_0)^2 (y_2' - y_0)^2)^{1/2}$$

$$f = ((x_2' - x_0)^2 - (x_1' - x_0)^2)^{1/2}$$

For a pair of points attributed to lie on the ellipse, eqns. 1 and 2 can be used to calculate values for the major and minor radii, a and b. Points P_1 and P_2 votes for major and minor radii in a quantized two dimensional radius accumulator array. The location of a peak in this array defines the a and b radii of the detected ellipse. Ellipse orien tation is generally defined as the angle subtended by the major axis with the horizontal co-ordinate of the image frame. Symmetry of the ellipse about $\Pi/2$ means that the ambiguity arising in the measurement of orientation can be resolved by inspection of the measured a and b radii. If b is greater than a, the ellipse orientation becomes

$$\theta_0 = \theta_{opt} + \Pi/2$$

and a and b are exchanged. Otherwise, the measured parameters remain valid. In a situation where contours of concentric ellipses with equal orientation exist in the feature map, these will map to separate peaks in the radius array.

3.4 Feature point labeling and accumulator array simplification.

Applying the orientation and radius finding procedures (stages 2 and 3) directly to a feature map in which candidate ellipse centres have been extracted is likely to be computationally expensive. Also, it may yield inaccurate results if the feature space is complex and multiple el lipses exist. To reduce the complexity of the image feature space, it was necessary to identify the points from which the significant accumu lator counts originated. That is, a subset of points lying on the contour of the ellipse whose centre has been found was required. Appli cation of stages 2 and 3 to the subset of contour points representing the ellipse boundary meant that procedures implemented for extracting the remaining three parameters were more robust and less computational ly intensive. Extraction of the relevant boundary points was achieved using a method derived from a backmapping technique introduced by Gerig and Klein [7]. The backmapping technique is based on the assumption that a given boundary point would be a member of only one curves in image space. Membership to that curve is defined by an accompanying accumulator cell with the highest count and hence, an optimum parameter set a_{opt}

To identify the optimum parameter set for a given boundary point, the Hough procedure is repeated a second time. But instead of incre menting cells intersected by each voting surface, the location of the cell with the highest count a_{opt} is found. The boundary point is then labelled with this location. The identification of boundary points that contributed to a peak (optimum parameter set) in the accumulator array becomes straightforward. Incorporating the Gerig and Klein backmapping procedure can also result in a simplification of the accumulator array. During the backmapping procedure, if all cells except for a_{opt} that intersect the voting surface are cancelled, then the distribution of votes in the accumulator array is concentrated at peak locations

4 Extending the technique to detect multiple ellipses

(Two extensions of the tristage Hough transform to detect instances of multiple ellipses are outlined).

4.1 Ellipse contour removal (ECR)

in the first extension, ellipses extracted using the tristage technique are removed from the binary contour .The centre-finding procedure is reapplied and further peaks searched for until the extracted peak is below a predetermined threshold. At this point, no further ellipses are assumed to be present in the image and the search ends. Peak threshold selection is arbitrary but is largely governed by the expected ellipse size and the extent to which it may be obscured.

4.2 Accumulator peak removal (APR)

Rather than delete the detected ellipse from the binary edge map, we can instead delete the associated peak in the accumulator array. This forms the basis of the second extension. Centre-finding and feature point labelling are applied only once to points in the binary edge map. Peaks associated with the extracted ellipse are removed by setting accumulator cells in a 5*5 neighbourhood centred at the extracted peak location to zero. Remaining peaks are subsequently searched for until the extracted peak is below a predetermined threshold, at which point the search ends.

5 Experimental results

We have taken an 128*128 image of coffee-beans as input image (Fig-2) Fig-3 is the edge image of fig-2. In 1st stage we get the centres.
In 2nd stage we get the orientation in degree.
In the final stage we get major and minor axis .

ellipse	Centre (x,y)	Orientation (in degree)	Major axis	Minor axis
1	(41 , 20)	0	42	35
2	(97 , 32)	48	51	32
3	(31 , 61)	134	49	31
4	(68 , 83)	136	48	30
5	(107 , 83)	104	42	30
6	(28 , 104)	33	50	28

fig-2

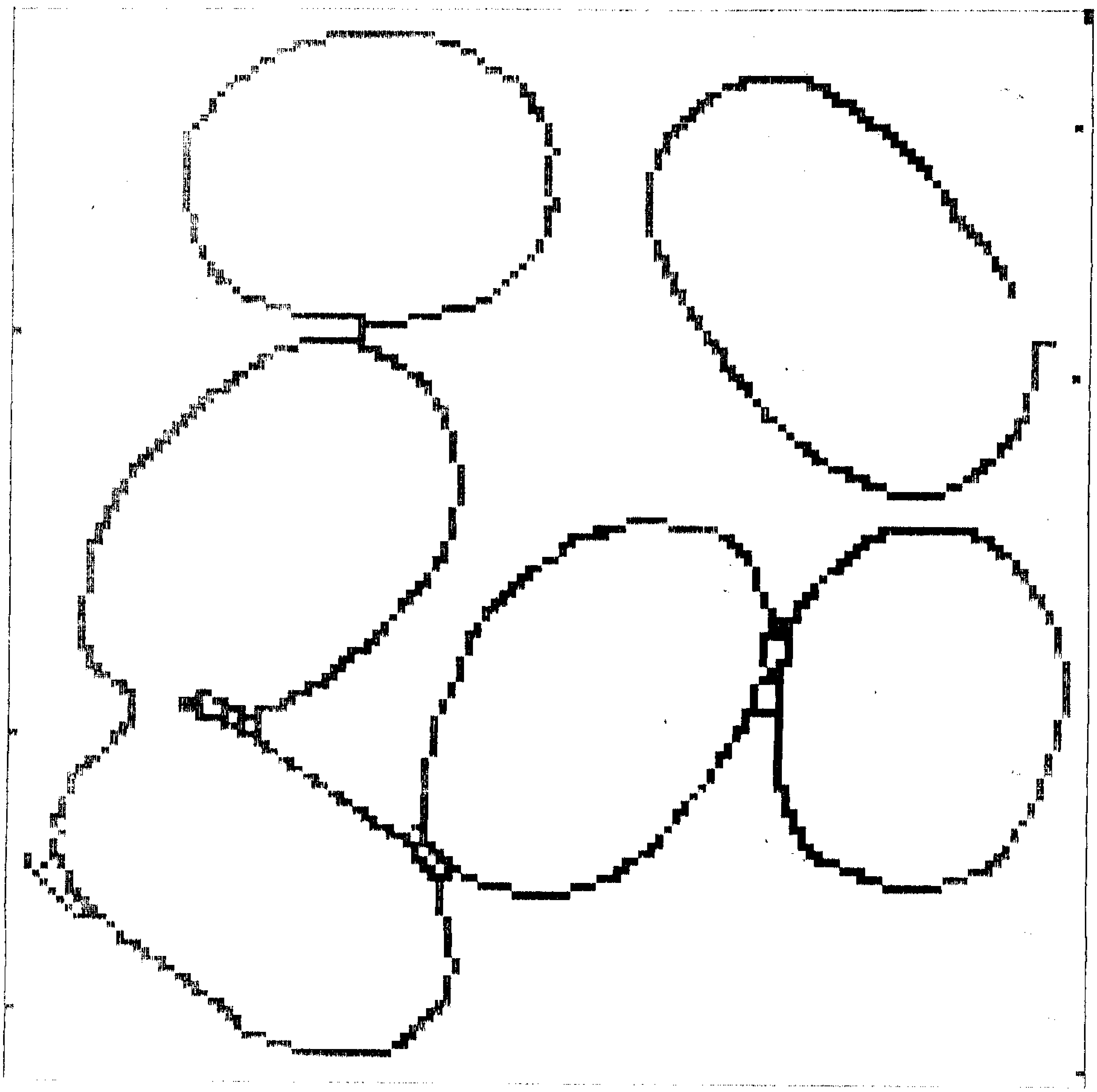

fig 8

6. Conclusion

We have demonstrated the feasibility of circle and ellipse detection using Hough transformation. The method is robust and able to detect parameter from partial data. The amount of storage and the time required is much less than that used in the standard H T implementation and this greatly increases the range of application which can be implemented on small memory computers. Exploiting neighbourhood information the system is made more robust. If points are connected or if they are neighbouring edge pixel with the same edge direction then more incrementing factor.

7. Acknowledgements

We wish to acknowledge the help from Punam Kumar Saha and Nirupam Sarkar during various stages of this work.

8. REFERENCES

1. R.O. DUDA and P.E. HART, "Use of the Hough transform to detect lines and curves in picture", communication of the ACM, vol 15, pp 11-15,1972.

2. D.H. BALLARD, "Generalizing the Hough transform to detect arbitrary shapes", pattern recognition, vol 13, no 2, pp 111-122, 1981.

3. H.K. MUAMMAR and M.NIXON, "Tristage Hough transform for multi ple ellipse extraction", IEE proceedings-E, vol 138,No 1,pp 27-35, Janu ary 1991.

4. J. ILLINGWORTH and J.KITTLER, "Adaptive Hough transform", IEEE Transac tions on pattern analysis and machine intelligence vol PAMI- 9, No-5,pp 690-697, September 1987.

5. TSUJI and MATSUMOTO, "Detection of ellipses by modified Hough trans formation", IEEE Trans, vol c-27, no 8, pp 777-781, 2978.

6. H.K.YUEN.,J.ILLINGWORTH and J.KITTLER, "Detecting partially occluded ellipses using the Hough transform", Image and vision computing, vol 7, no 1, pp 31-37, 1989.

7. G.GERIG and F.KLEIN , "Fast contour identification through efficient Hough transformand simplified interpretation strategy", proc 8th int. conf. on pattern recognition, Paris, pp 498-500,Oct. 1986.