A STUDY ON SHUFFLE-EXCHANGE NETWORKS: DYNAMIC AND STATIC

a dissertation submitted in partial fulfillment of the requirements for the M.Tech(Computer Science)degree of The Indian Statistical Institute

by

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Abstract

It has been known that shuffle-exchange connection provides efficient interconnection scheme for parallel computation of many problems. In our work both dynamic and static Shuffle-Exchange networks have been studied.

Firstly we have considered dynamic shuffle-exchange network. A single stage Shuffle Exchange network is an interconnection between N inputs and N outputs with N/2 2*2 switching elements having a shuffle interconnection at the input.To realize any N*N permutation from N inputs to N outputs i passes may be needed through this single stage shuffle exchange network (**single stage multipass technique**) where $1 \leq i \leq 2n-1, n = \log_2 N$.In our work we have characterised the set of permutations realizable in i passes ($1 \leq i \leq 2n-1$) through a single stage multipass shuffle exchange network and given any permutation the above characteristics are made use of to find the minimum number of passes required to realize the permutation.

We have also studied **static shuffle network topology** where given any number of nodes N,we have tried to find out the most suitable arrangement of it as n * k Shuffle-Exchange network,where n=2^m is the number of nodes in a stage and k is the number of stages to give the lowest possible diameter.We have simulated the values of diameter and average diameter for a given value of N with different n.

Contents

Chapter 1

Introduction

Parallel processing is a technique used to achieve high speed computing.In this technique a number of processors and memory modules are linked together through an interconnection network.

Multistage Interconnection networks(MINs) are used to implement the required communication between processors and memory modules. A MIN is implemented by 2 * 2 switching elements with a number of stages. Each stage having N/2 switching elements with switches being set either in the cross state or in the straight state.

The desired communication between the resources is essentially a one -to -one mapping and can be represented by a permutation.If the N resources on either side are identified by integers from 0 to N-1,then an interconnection can be represented by a permutation[0,1,......,N-1].

Shuffle-Exhange networks provide an effective interconnection scheme for parallel computation of many problems.The Omega network by Lawrie[1] consists of $n = log_2N$ Shuffle-Exchange stages.It can realize only $(2^{N/2})^{log_2N}$ permutations of the possible N!.

So an economic way to realize the permutations would be to use single stage multipass technique.Parker[2] has shown that 3n passes are sufficient to realize all permutations and Wu and Feng [3] have shown an upper bound of 3n-1 passes. 2n-1 passes are necessary for $N \leq i \leq 8$.Akram Abdennadher and Tse-yun-Feng[4] have shown that a $2log_2N$-1 stage Shuffle-Exchange network is rearrangeable(All permutations are realizable).

The class of multistage cube-type networks(MTCN) refers to those MINs which are topologically equivalent to the Generalised Cube including Omega,Indirect Cube.A N*N permutation is said to be admissible to an MTCN if N conflict-free paths defined by the permutation can be established simultaneously.This is PA(permutation admissibility) problem.A closely related problem is:If a given permutation is not admissible ,how should N pairs be divided into a minimum no. of groups(passes) such that conflict-free paths for all pairs within a group can be established simultaneously.This is the MP(minimum pass) problem.

A k-Extra stage MTCN(K-EMTCN)is obtained by adding k more stages in front of an MTCN which provides 2^k disjoint paths between any pair of source and destination.

The admissibility of an arbitrary permutation to a N*N MTCN was studied by X.Shen,M.Xu and X.Wang [5].X.Shen in [6] has shown an admissibility algorithm for 1-Omega(1-Extra stage Omega Network) which can be extended to any 1-EMCTN's. (1 Extra stage MTCN) .The MP problem was first raised by Wu and Feng in [3].Raghevendra and Varma solved the MP problem for BPC(Bit Permute Complement) permutation on 0-Omega[7].Shen [8] extended the result to k-Omega network.Q.Hu ,X.Shen and W.Liang in [9] solved the MP problem for LC(Linear Combination And Complement permutation for k-Extra -Stage Cube-Type Networks.

A Shuffle Exchange network with $\log N$ stages(Omega network)can realize only $(2^{N/2})^{logN}$ of the total N! permutations.It is known that a $2n-1$ stage Shuffle-

Exchange network is rearrangeable,and not all permutations need $2n-1$ passes to realize it ,so an economic way to realize any permutation would be to take a single stage multipass Shuffle-Exchange network.A permutation realizable in k-Extra stage network may need less than k stages if we use single stage multipass Shuffle-Exchange network(1<=no of passes <=2n-1) thus reducing the delay.The earlier works have considered special classes of permutations[BPC and LC] but we have considered any permutation.We have studied the characteristics of permutations passable in i-stages of a Shuffle-Exchange network where $1 \leq i \leq 2n-1$. We have taken up the problem of ,given any permutation to find the minimum number of passes of the single stage multipass Shuffle-Exchange network required to route it.

We have also considered the static shuffle exchange network topology and studied the characteristics with varying number of stages.The classical definition of a Shuffle Network (p,k) has N nodes ($N = kp^k$) where p is the node degree of the network and k is the number of columns.The p^k nodes are linearly arranged in a column and two adjacent columns are connected in a perfect shuffle by unidirectional links.The last column is wrapped around to the first column in a cylindrical manner.In this arrangement of N($N = kp^k$)the realizable values of N are very sparse and many of the intermediate values of N are not realizable.

S.W.Seo,P.R.Prucnal and Hiasachi Kobayashi in [10] have used a new definition of Shuffle Network (N=nk) where n(n=2^m) is the number of nodes per stage and k is the number of stages.The nodes in adjacent stages are connected in perfect shuffle by unidirectional links.The last stage is wrapped around to the first in a cylindrical manner.This definition makes it possible to realize a shuffle network in a variety of different ways with a given N.In their paper they have calculated the expected number of hops for different n with a particular N.

In our work we have considered a static Shuffle-Exchange Network topology with N(N=nk) nodes with n (n=2^m) being the number of nodes in each stage

, the nodes between adjacent stages connected in a perfect shuffle by bidirectional links.The last stage is wrapped around to the first stage in a cylindrical manner.The network is degree bounded.We have considered the various values of Diameter and Average Diameter for various values of n with a given N and tried to find the most suitable arrangement for a given N. The results show that for most values of N ,it is possible to find a $n * k(n = 2^m)$configuration of N nodes where maximum diameter D $\leq \lceil \log N \rceil$.

Chapter 2

Preliminaries

As a way of achieving high computing power ,computer systems built upon a large number of processors and memory modules are becoming increasingly important.Interconnection networks provide the communication paths among processors and memories.If the N resources on either side are identified by integers [0,1,......,N-1],then a permutation is any one-to-one mapping of N resources on input side to N resources on the output side. If we consider the interconnection between N processors and N memory modules,thenfor N=4 the permutation [0,3,1,2] can be used to represent the interconnection of processor 0 to memory module 0, processor 1 to memory module 3,processor 2 to memory module 1, and processor 3 to memory module 2.

2.1 Interconnection Networks

The interconnection networks can be classified into the 2 categories static and dynamic based on network topologies.

Static Interconnection Networks: There are static (dedicated)communication links among PEs so that the interconnection has a distinct topology.Linear array, mesh,ring,star,systolic array ,hypercube,cube-connected cycles are popular static interconnection topologies.

FIGURE 1 shows some static network topologies.

Dynamic Interconnection Networks: Since it may be required that any processing element be able to access any MM ,the data paths need to be dynamic.This can be achieved by Interconnection Networks which have programmable datapaths so that a path can be dynamically established between a pair of input-output lines.The dynamism of datapaths is achieved by using programmable solid state switching elements (SE) grouped together in one or more stages.

*Single Stage Interconnection Network:*It is composed of a stage of switching elements cascaded to a link connection pattern. The Shuffle-Exchange network is a single-stage network based on perfect-shuffle interconnection.

*Multistage Interconnection Network(MIN's):*It consists of more than one stage of switching elements and is usually capable of connecting an arbitrary input terminal to an arbitrary output terminal.

2.1.1 Properties and Classification of MINs

The number of stages ,interstage topology and switching elements of SE characterise a MIN.The simplest switch is $2 * 2$ switch having 4 possible input-output routings.Most MINs use $2 * 2$ switching elements and restrict their valid states to straight (S-mode) and cross(X-mode) only.

Figure 2 shows a switching element with its 4 possible states.

MINs can be classified as

*Blocking networks:*Simultaneous connections of more than one terminal pair may result in conflicts in the use of network communication links. Example: Baseline,Omega.

*Rearrangeable nonblocking network:*If a network can perform all possible connections between inputs and outputs by rearranging its existing connections so that a connection path for a new input-output pair can always be established.Example Benes network.

Nonblocking network:A network which can handle all possible connections without blocking is called a nonblocking network.Example :The CLOS network.

Figure 3 shows some of the Multistage Interconnection Networks.

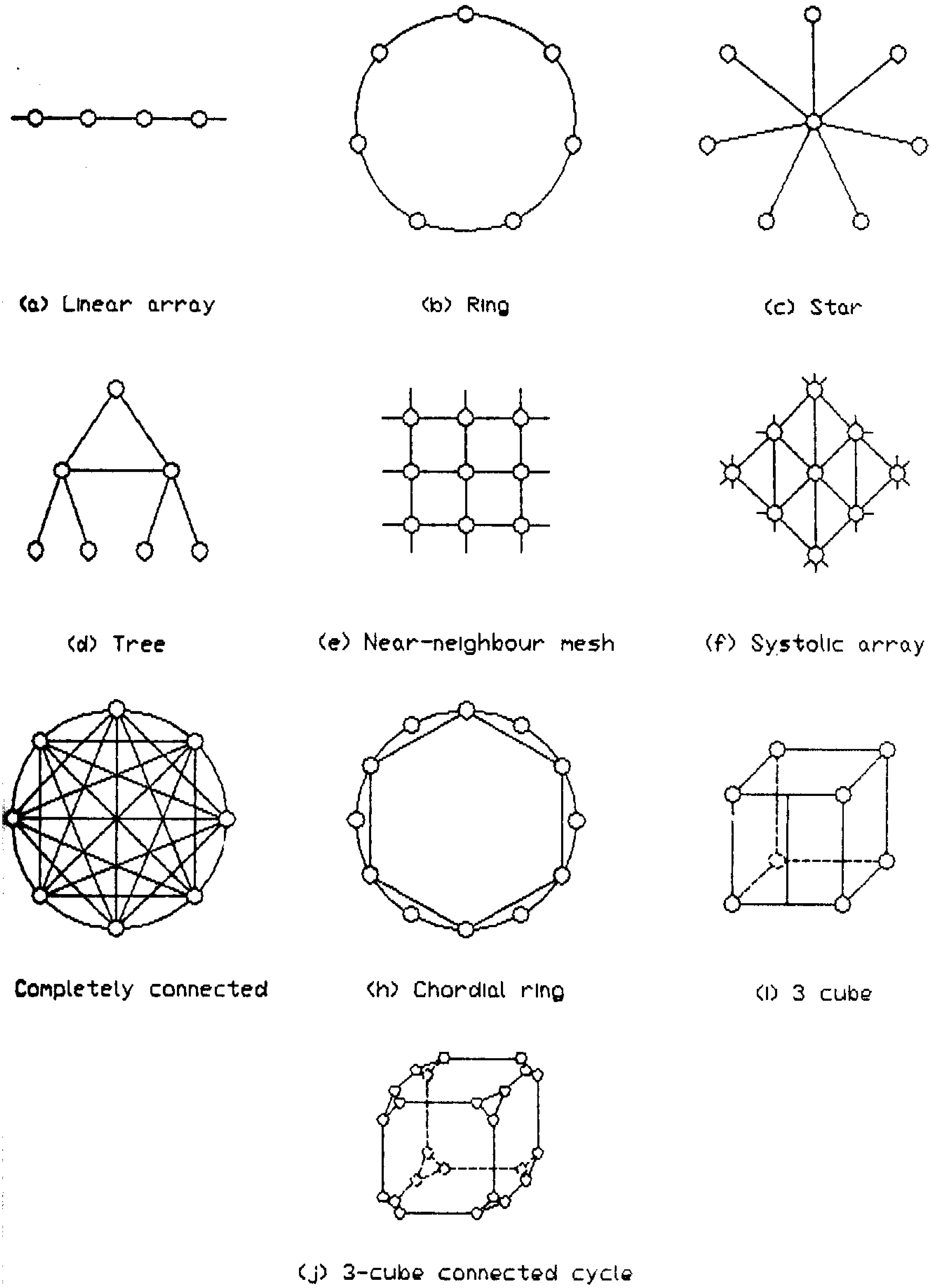
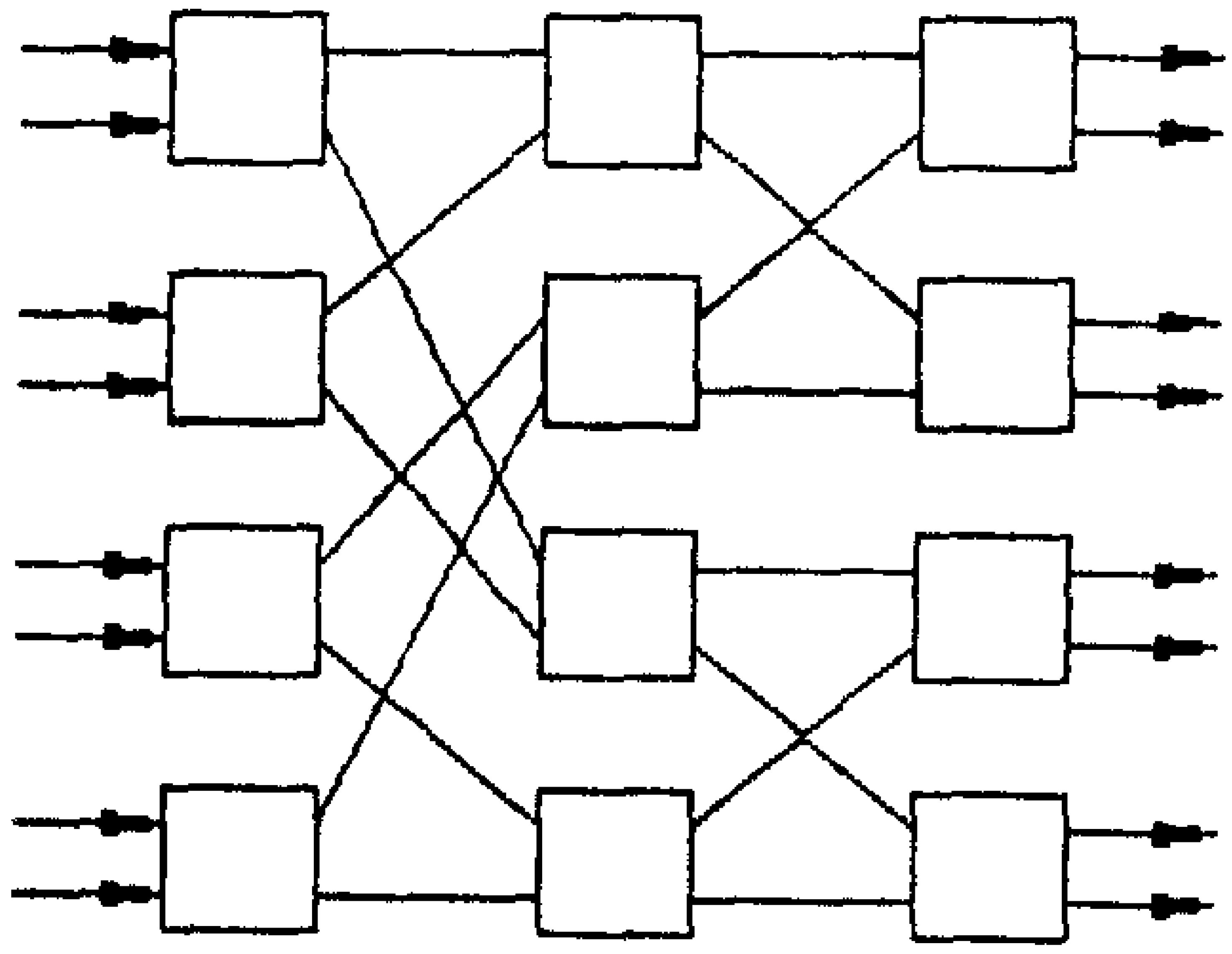
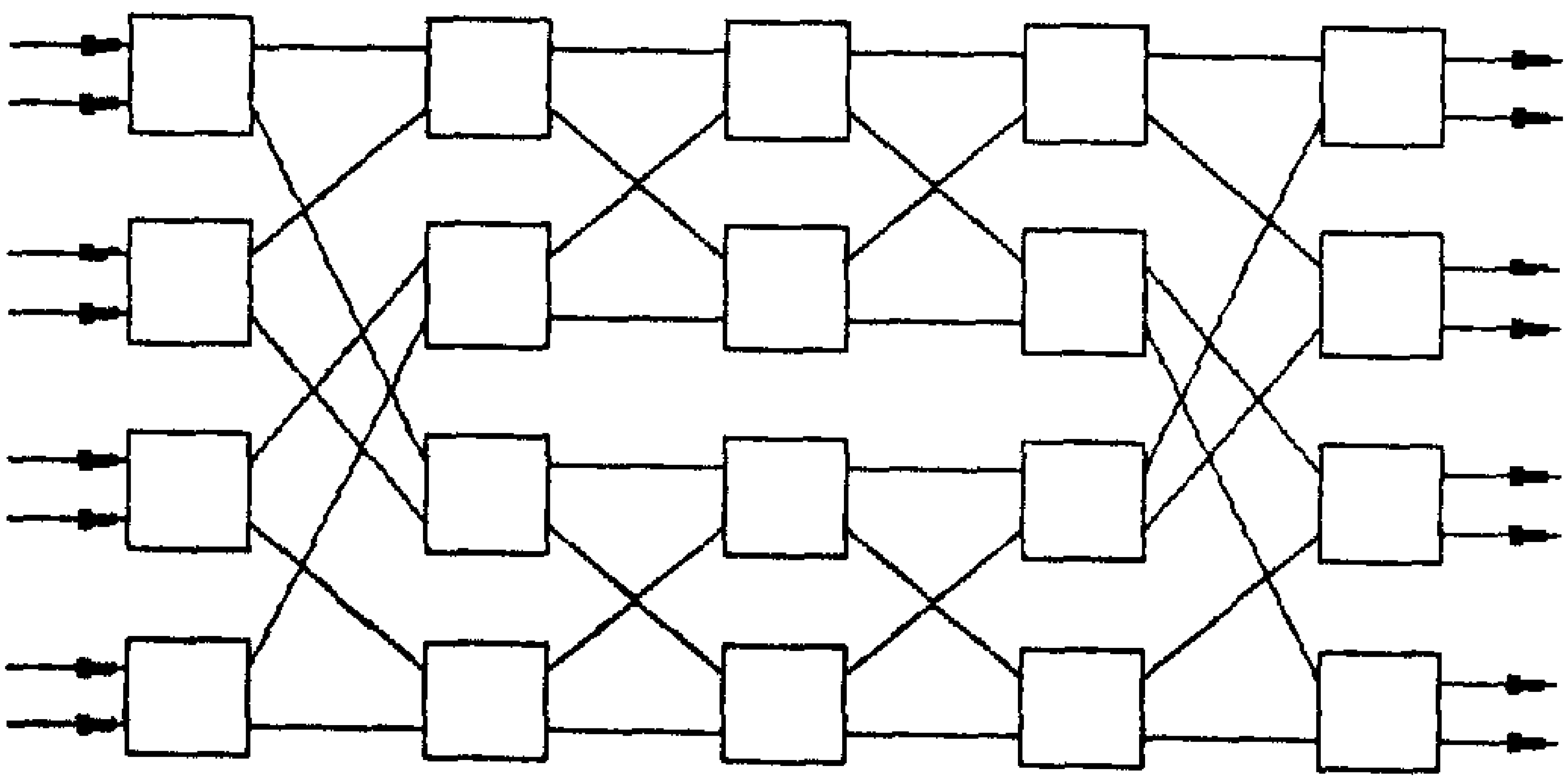


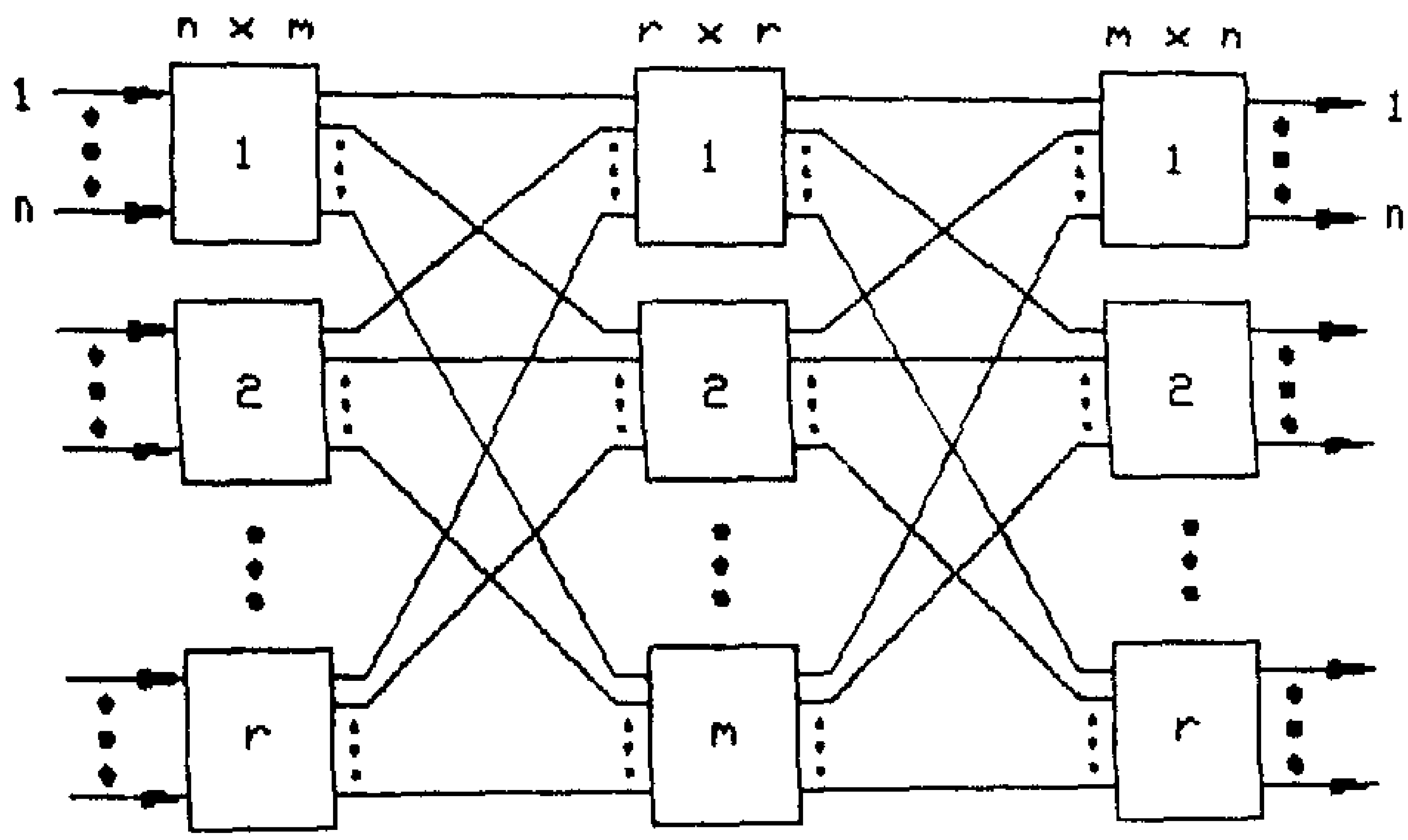
Fig.1. Static interconnection network topologies.



(a) 8 × 8 baseline network



(b) 8 × 8 Benes network



(c) Clos network

Fig 3. Multistage interconnection networks

Figure 5 shows a single stage Shuffle Exchange network and Omega network. The Omega network can realize only $(2^{N/2})^{log_2 N}$ permutations out of the total N! permutations due to conflicts in the communication paths.

Simultaneous connection of more than one input and output may result in conflicts in the communication paths and so not all permutations are realizable in a single pass. We have considered a single stage multipass shuffle exchange network and tried to find the restrictions on the permutations realizable in an i-pass single stage shuffle exchange network where $(1 \leq i \leq 2n-1)$.

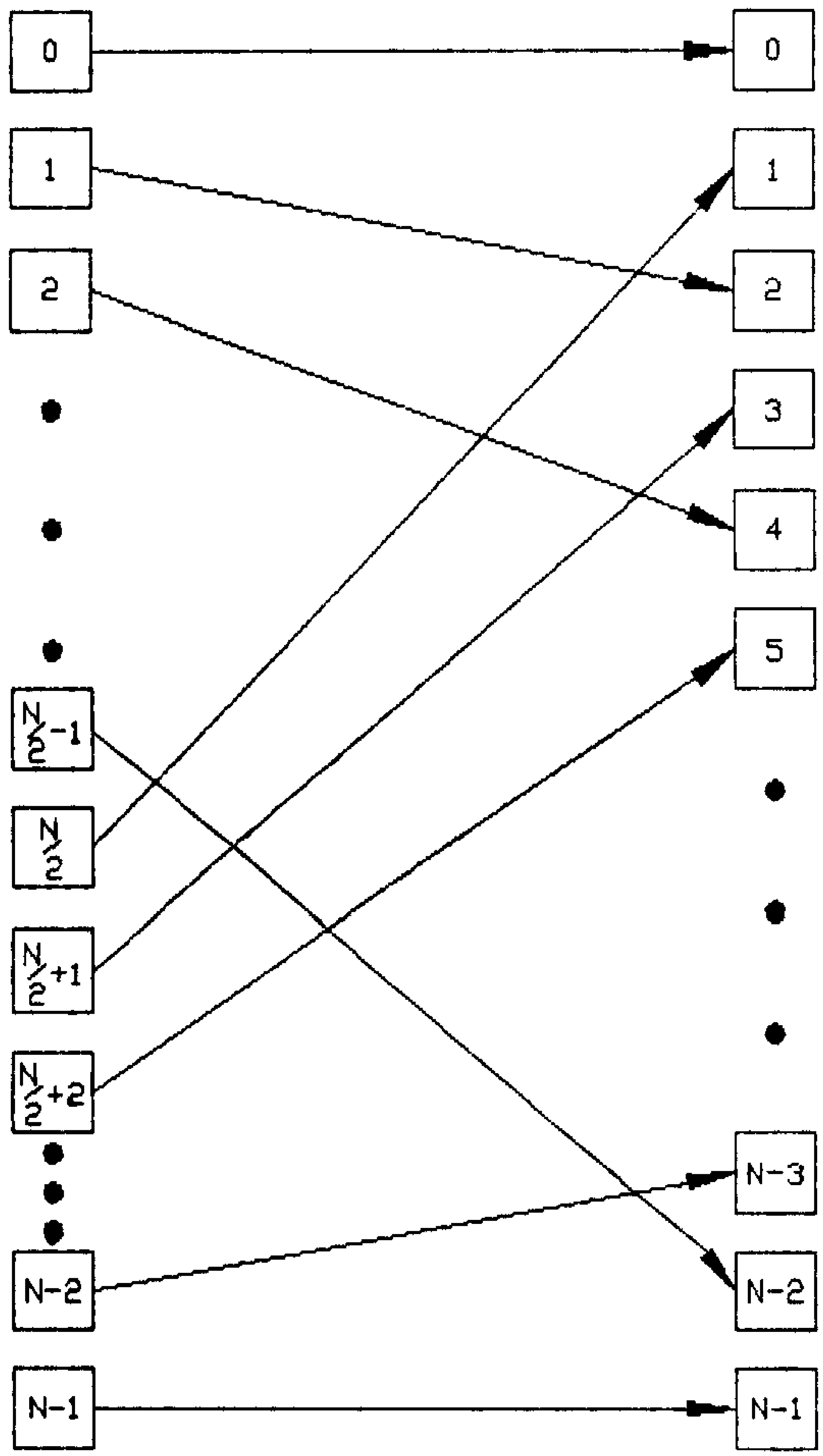


Fig 4. The perfect shuffle of an N element vector.

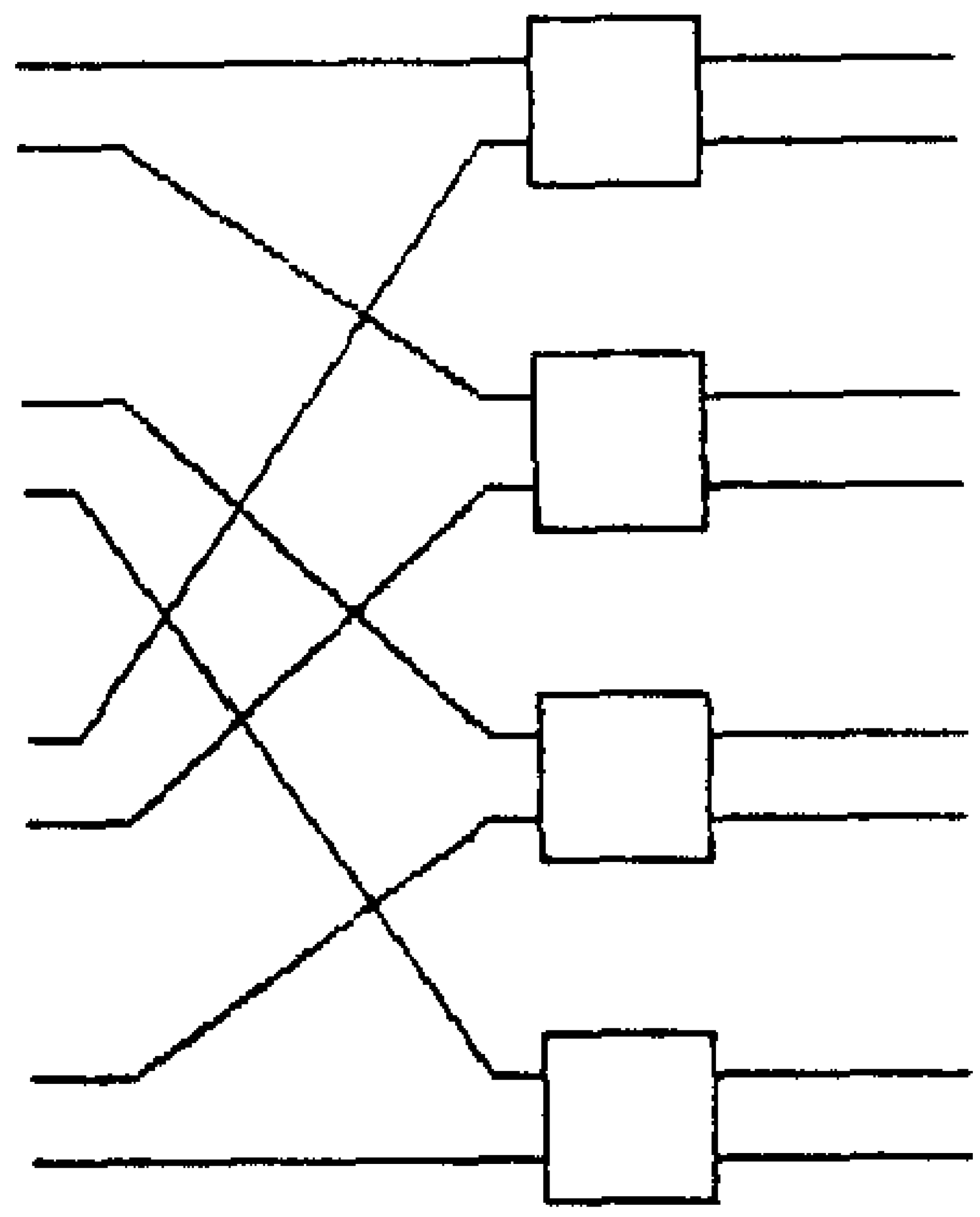


Fig 5a. A single stage shuffle-exchange network

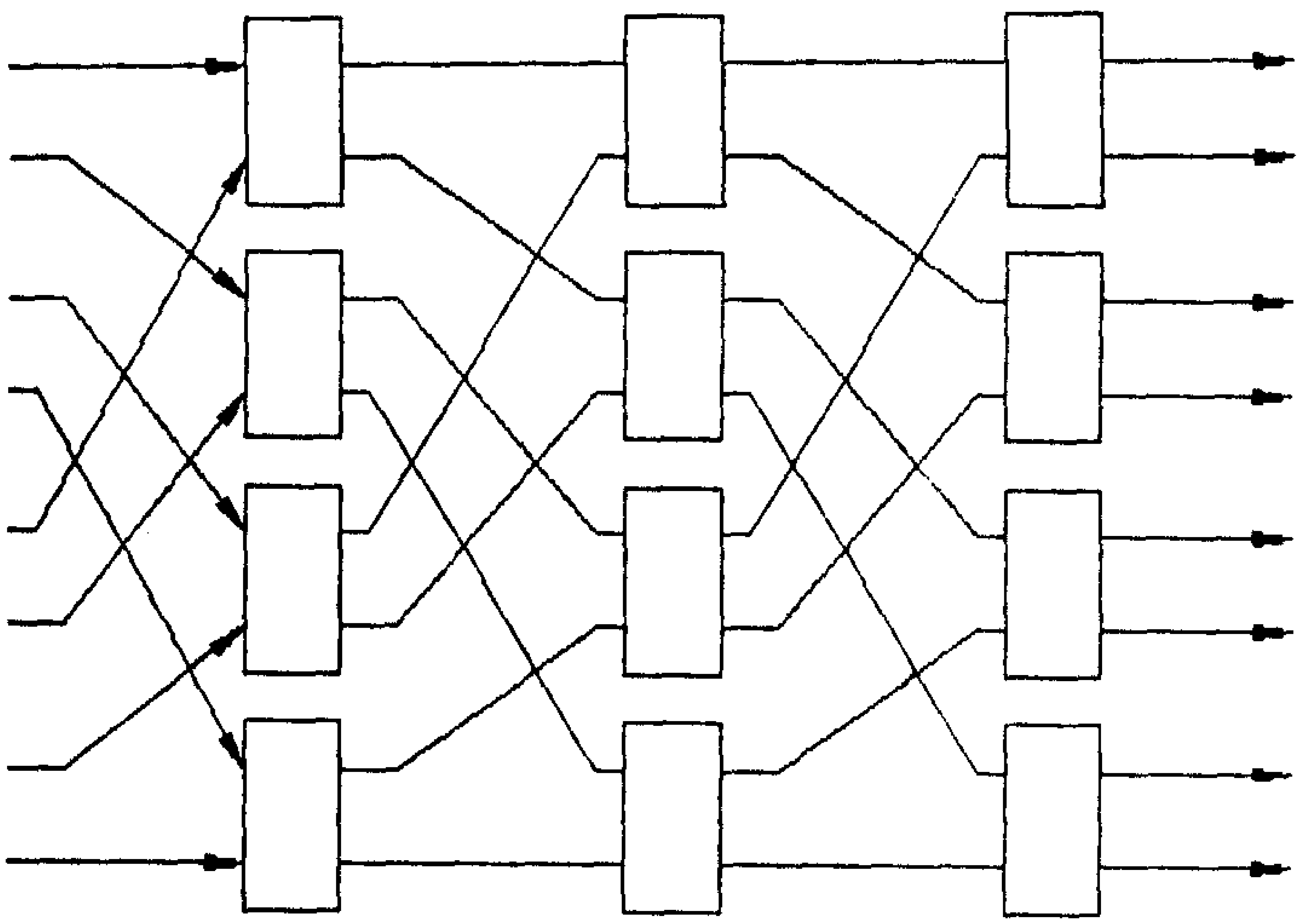


Fig 5b. A 8 x 8 Omega network.

Chapter 3

Permutations and Number of stages in a Shuffle Exchange Network

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Here we study the characteristics of permutations realizable in an i $(1 \leq i \leq 2n-1)$ stage shuffle-exchange network ,i.e the permutation capabilities of different stages.

Definition 1:

Input Groups:For a N*N Shuffle Exchange Network the inputs are grouped into different levels.Each input is represented by an n-bit binary number s,
$s = \{s_0 s_1 .... \ s_{n-2} s_{n-1}\}$ where s_0 is the MSB ,$n = \log N$,
any input group at level $i(1 \leq i < n)$ consists of 2^i elements, those inputs form a group which have fixed values for positions $\{s_i, s_{i+1} .... \ s_{n-1}\}$ and with all possible combinations for $\{s_0, s_1, ... \ s_{i-1}\}$. The input groups at level 0 consists of a single element ,their being N such groups and a single group at level n consists of all elements (0,1,2,.........,N-1).
Figure 6 shows the input groups for N=8.

Let $g_i(x, i)$ denote a group at level i ,x being the number obtained by taking the decimal value of the the binary number $\{s_i s_{i+1} ...... s_{n-1}\}$ ($\{s_i s_{i+1} ... s_{n-1}\}$ being the fixed bits in the binary representation of the inputs in the group.)

Example 1:

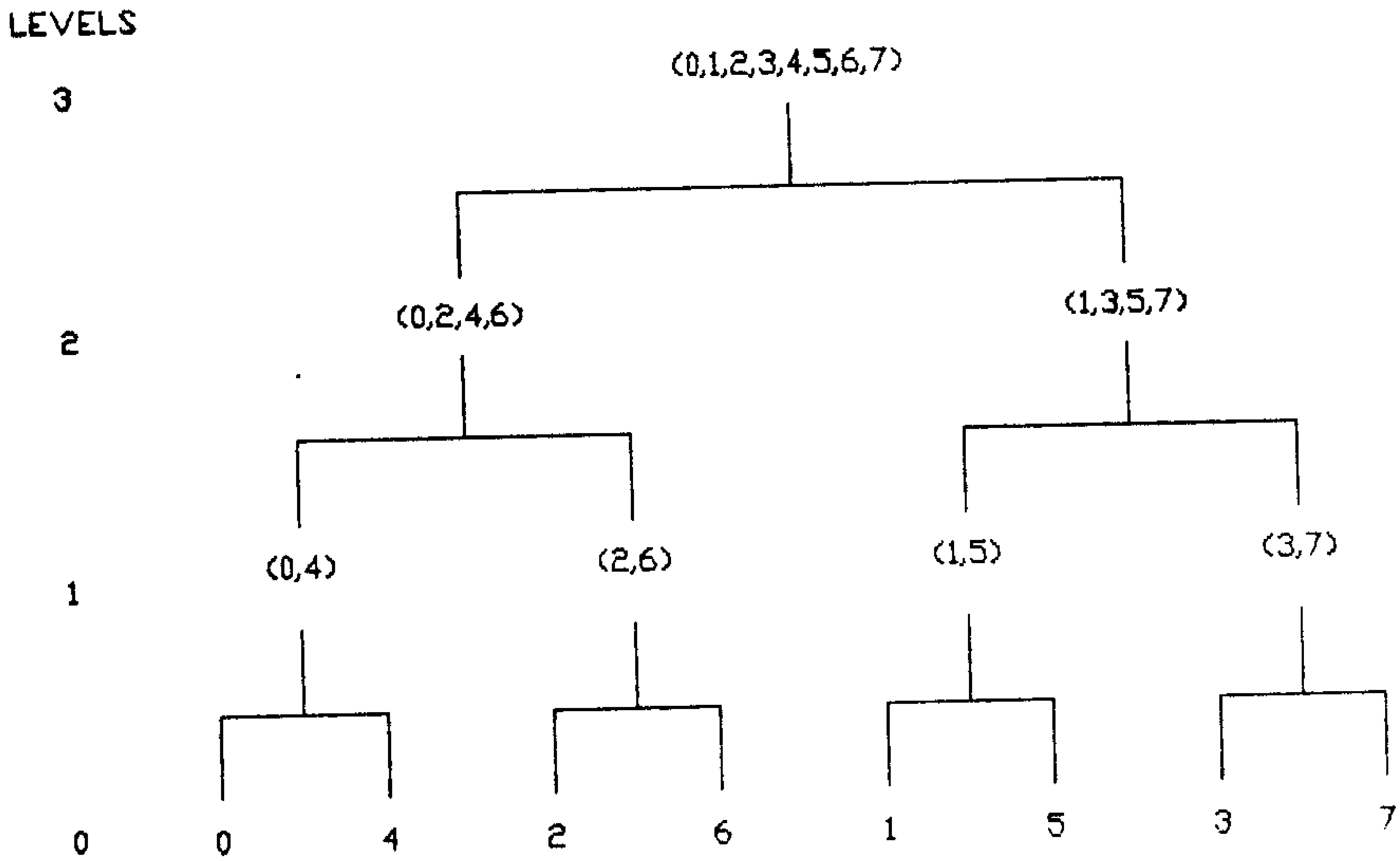


Fig 6. The input groups of a 8 x 8 shuffle exchange network.

For $N = 8$:

g_i(0,0)=0 g_i(2,0)=2

g_i(4,0)=4

g_i(0,1)=(0,4) g_i(2,1) = (2,6)

g_i(1,1)=(1,5) g_i(3,1)=(3,7)

g_i(0,2)=(0,2,4,6) g_i(1,2)=(1,3,5,7)

g_i(0,3)=(0,1,2,3,4,5,6,7).

Definition 2:

Output Groups:For a N*N Shuffle Exchange network,the outputs can be grouped into different levels.Each output is represented by an n-bit binary number d, $d = \{d_0 d_1, \ldots\ldots\ldots\ldots d_{n-2} d_{n-1}\}$where $d_0$is the MSB, an output group at level i $(1 \leq i < n)$ contains 2^i elements ,those outputs form a group which have fixed values for bit positions $\{d_0, d_1, \ldots\ldots\ldots d_{n-i-1}\}$ and with all possible combinations for $\{d_{n-i}, \ldots\ldots, d_{n-1}\}$. The output groups at level 0 consists of a single output their being N such groups and there is a single output group at level n which comprises of all outputs (0,1,......N-1).

Figure 7 shows the output groups at different levels for N =8.

Let $g_0(x, i)$ denote an output group at level i, x being the number obtained by taking the decimal value of the binary number $\{d_0 d_1 \ldots\ldots\ldots d_{n-i-1}\}$ $(\{d_0, d_1 .. d_{n-i-1}\}$ being fixed bits in the binary representation of the outputs in the group)

Example 2:For N=8:

$g_o\ (0,0) = 0$ $g_o(1,0)=1$

$g_o(0,1) = (0,1)$ $g_o(1,1) = (2,3)$

$g_o(2,1) = (4,5)$ $g_o(3,1) = (6,7)$

$g_o(0,2) = (0,1,2,3)$ $g_o(1,2) = (4,5,6,7)$

$g_o(0,3) = (0,1,2,3,4,5,6,7)$

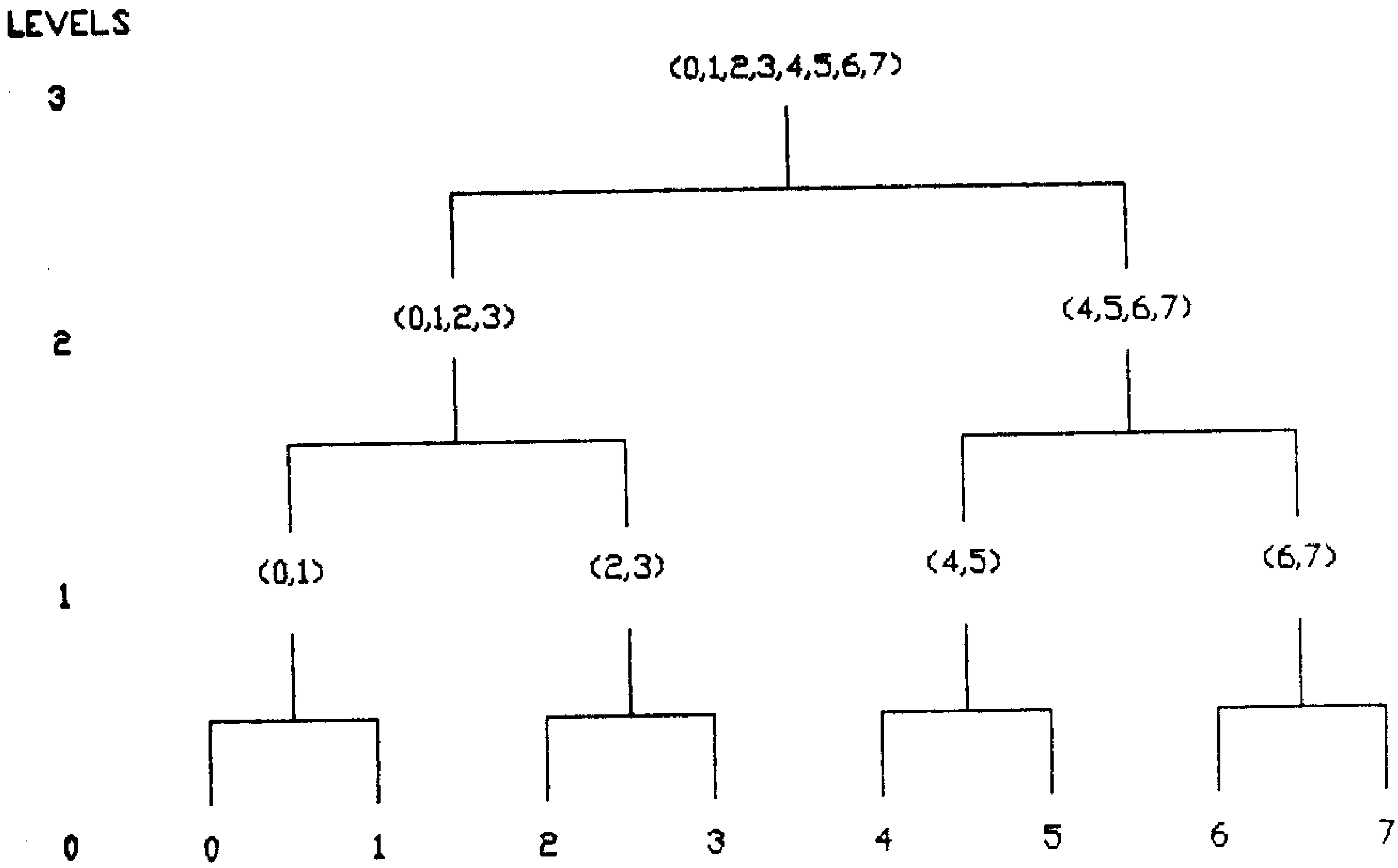


Fig 7. The output groups of a 8 X 8 shuffle exchange network.

3.1 Permutation Capabilities for various stages of a shuffle exchange network.

In a single stage multipass shuffle exchange network,for passes i($1 \leq i \leq n$) there exists a unique path in the network between every pair of input and output. For $i = 1$, each input can access either of 2 possible outputs.For $i = 2$ each input can access any of 4 possible outputs.At the n th pass the network possesses the property of Full Access (An input can be connected to any output).At pass i($1 \leq i \leq n$) $(2^{N/2})^i$ permutations are realizable.

For $i > n$,the network is Full Access network .At pass i ($i = n + k$), there exists 2^kdisjoint paths between a input and a output.Also different switch settings may realize the same permutation.

So we are to consider the two cases differently.Permutations requiring passes i:

1. $1 \leq i \leq n$:

   For these values of i the network will be referred to as reduced stage shuffle exchange network.

2. $n < i \leq 2n - 1$:

   For these values of i ,we refer to the networks as extra stage shuffle exchange network.

3.1.1 Permutations realizable in reduced stage shuffle exchange networks

Here we consider i-stage Shuffle Exchange network where $1 \leq i \leq \log N$. There are two types of restrictions to be considered.

1. The possible outputs which an input can access.

2. There are restrictions among the outputs to which a group of inputs can be connected (without conflict).

a) The possible outputs which an input can access

The possible outputs D(i) to which an input i at pass j can access ($j = 1$ to n)are 2^j in number and the possible outputs are :

D(i) = $(i * 2^j)$mod N to $((i * 2^j)$mod N + $2^j - 1)$ for $0 \leq i < N/2$

=$((2i - N) * 2^{j-1})$ mod N to $((2i - N) * 2^{j-1}$mod N+$2^j - 1)$

for $N/2 \leq i < N$ where mod denotes remainder after division.

At stage k,($1 \leq k \leq n$) the input groups at level k, $g_i(x, k)$ can access possible outputs from the output group $g_o(x, k)$,($0 \leq x < N/2^k$) ,their being $N/2^k$ such input and output groups at level k.

Let the set of realizable outputs at stage i ($0 \leq i \leq n$)for an input group $g_i(x, i)$ be R(x,i)

The elements of R(x,i) are the elements of $g_o(x, i)$

Example 3:For N=8.

For i=1(pass 1)

R(0,1)(Realizable set for 0,4) = $g_o(0, 1)$ = (0,1)

R(1,1)(Realizable set for 1,5) = $g_o(1, 1)$ = (2,3)

R(2,1)(Realizable set for 2,6) = $g_o(2, 1)$ = (4,5)

R(3,1)(Realizable set for 3,7) = $g_o(3, 1)$ = (6,7)

For i=2(pass 2)

R(0,2)(Realizable set for 0,2,4,6) = $g_o(0, 2)$ = (0,1,2,3)

R(1,2)(Realizable set for 1,3,5,7) = $g_o(1, 2)$ = (4,5,6,7)

For i=3(pass 3)

R(0,3)=Realizable set for 0,1,2,3,4,5,6,7) =$g_o(0, 3)$=(0,1,2,3,4,5,6,7)

b)Restrictions between input output pairs

For the i th pass: $1 \leq i \leq \log N$

if i=1

Consider the restrictions on the possible outputs only

else

For pass-no =i

Consider restrictions between outputs of elements in the

input groups from levels 1 to i-1

Let number of input groups at level i be No-Group = $N/2^i$

For x = 0 to (No-Group)-1

For j =1 to i -1

For k= 0 to $2^{i-j} - 1$

{For inputs in the group} -> {Their outputs

g_i(x+No-Group*k,j)} should belong to distinct

$g_o(x * 2^j + m, i - j)$

where $0 \leq m < 2^j$}

(i.e outputs belong to 2^j

distinct output groups}

Example 4:For N=8:

Restrictions on permutations realizable in i $(1 \leq i \leq 3)$passes

i=1(1st pass)

The realizable set of outputs for inputs in the group

$g_i(x, 1)$ is $g_o(x, 1)$ where $0 \leq x < 4$.

i=2(2nd pass)

The realizable set of outputs for inputs in the group

$g_i(x, 2)$ is $g_o(x, 2)$ where $0 \leq x < 1$

For each k(k=0,2)inputs in $g_i(k,1)$ should have outputs belonging to distinct g_o(x,1) where $0 \le x < 1$

For each k(k=1,3)inputs in $g_i(k,1)$ should have outputs belonging to distinct g_o(x,1) where $2 \le x < 4$

i=3(3 rd Pass):

The realizable set of outputs for inputs in the group $g_i(0,3)$ is g_o(0,3).

For each k($0 \le k < 4$),inputs in g_i(k,1)should have outputs belonging to distinct g_o(x,2) where $0 \le x < 2$

For each k($0 \le k < 2$),inputs in g_i(k,2)should have outputs belonging to distinct $g_o(x,1)$ where $0 \le x < 4$

So for reduced stage Shuffle Exchange Network, at the ith pass ,first the outputs are to be checked to see if they are Realizable outputs and then the restrictions between the input output pairs in various groups are to be checked.

3.1.2 Permutations realizable in extra stage shuffle exchange networks

Here we consider the permutations realizable in i passes ($n < i \le 2n-1$). An input can individually access all outputs(Full Access) ,so the Realizable set consists of all outputs.

For pass-no =i ($n < i \le 2n-1$) :

Let i=n+k

For j=k+m $1 \le m \le n-k$

For x = 0 to ($N/2^j - 1$)

{ For inputs in the group $g_i(x,j)$} —>{Each g_o(y,n-m) should

have 2^k outputs

where $0 \leq y < N/2^{n-m}$}

So for the $(2n-1)$th pass :

{ For inputs in the group g_i(0,n)}->{$N/2$ outputs belong to
g_o(0,n-1) and N/2
outputs to g_o(1,n-1)}

This confirms the fact that a 2n-1 stage shuffle exchange network is rearrangeable.

Example 5:For N=8:

The permutations realizable in i pass($4 \leq i \leq 5$)extra stage shuffle exchange network

i=4($=n+1$)

{For each k(k=0,1) inputs in g_i(k,2)}—>{2 outputs belong to each of
g_o(x,2) where $0 \leq x < 2$}

i=5($=n+2$)

{For inputs in g_i(0,3)}—>{4 outputs belong to each of
g_o(x,2) where $0 \leq x < 2$}

This is always true for any permutation.

3.1.3 Algorithm

Input:The permutation.

Output:The minimum number of passes required to route the permutation

pass-no=1;

Check the permutation against the restrictions given for
pass-no($1 \leq pass-no \leq 2n-1$),rules formulated earlier)
if any restriction is violated

pass-no =pass-no+1.

check for next pass.

else

permutations require pass-no to route it.

Complexity

For pass-no =i

At level 0

Outputs of N inputs are checked to see if they are elements of the realizable set for that pass.This involves checking an output against the lower and upper limits of the Realizable set.

Considering input groups from level 1 to i-1

At level 1:N/2 pairs of inputs in groups at level 1 are checked to see if their outputs lie in two different output groups at level i-1.This requires N/2 comparisons.

N/2 comparisons and atmost N/2 exchanges required so pairs of outputs for inputs in the group being checked are kept in increasing sorted order.

For example checking for i=3 (pass-no),N=8,at level 1 inputs in the group $g_i(0,1)$ and $g_i(2,0)$ should have outputs in distinct $g_o(x,2)$ $0 \leq x < 2$.The lower numbered of the outputs being compared in a group is kept stored in a lower location in the array storing the permutations.

For input groups at level 2: For groups at level 2 :N/2 pairs of input are to be checked to see if in the input group at level 2 , the 4 outputs belong to 4 different output groups.For this only comparisons between those pairs of outputs which were in the same output group at level i-1 need be made. This involves N/2 comparisons.

Again N/2 comparisons and at most N/2 exchanges required to check so that pairs of outputs being compared are kept in increasing sorted order.

For example for i=3,N=8,the inputs in the input group $g_i(0,2)$ should have outputs belonging to different $g_o(x,1)(0 \leq x < 4)$,for this only those outputs need be compared among themselves which were in the same output group at level 2 .For example those in either $g_o(0,2)$ need be compared among themselves and likewise those in $g_o(1,2)$ need be compared among themselves and the lower numbered output is stored in a lower location in the array storing the permutations.
Similarly till level i-1 checking is carried on
Complexity for the ith pass:(O(N * i))
Complexity for $O(\log N)$ passes :$O(N(\log N)^2)$

Scope for parallel algorithms
$N/2$ comparisons in a level can be done in parallel but checking for different levels must be done sequentially and each pass must also be checked sequentially. Hence a it is possible to have a parallel algorithm with a complexity of $O((logN)^2)$ with $N/2$ processors.

Chapter 4

Generalized Shuffle Exchange Topology

We have previously considered the characterisations of permutations realizable in i passes ($1 \leq i \leq 2n-1$) of a single stage dynamic shuffle exchange network. Next we consider the static shuffle network topology.

A N($=kp^k$) node shuffle-exchange network is characterised by two parameters p and k and is represented as a (p,k) shuffle -exchange network,where p is the node degree of the network and k is the number of columns.In a (p,k) shuffle network p^k nodes are linearly arranged in a column and two adjacent columns are connected in perfect shuffle by unidirectional links.The column is wrapped around to its first column in a cylindrical manner .Having all the nodes arranged in a single column is known as the single stage shuffle-exchange network.The shuffle network is a regular network.The realizable values of N here are sparse.

4.1 Earlier Works

S.W.Seo,P.R.Prucnal and Hiasachi Kobayashi in [10] have used a new definition of shuffle network N=nk. The tight relationship ($N = kp^k$)between the number of stages (k) and the number of nodes per stage(n=2^m) is removed.The network is more flexible allowing two independent parameters n and k whose product is a constant.This allows larger number of values of N to be realized.In their paper they have calculated the expected number of hops for different n with a particular N.

4.2 Configurations for minimum Diameter

Here given any value of N,we have considered different n * k shuffle exchange configurations where n is the number of nodes in a stage and k is the number of stages. If the nodes in a stage are numbered 0...... n-1 in a stage,2 nodes u and v in adjacent stages have a link between them if

a)v is obtained by the left cyclic shift of the bits in the binary representation of u(shuffle operation).

b)v is obtained by the left cyclic rotation of the bits in the binary representation of of u followed by the complementation of the LSB of the resultant binary number. (shuffle-exchange operation).

The last stage is wrapped around to the first stage in a cylindrical manner.

Fig 8 shows the arrangement for N=16,n=8,k=2.

We have assumed the links to be bidirectional and used shortest path routing to find the Diameter and the average Diameter.,the algorithm for which is discussed below.

Algorithm 2.

Input:The total number of nodes N and the number of nodes n in a stage.

Output:The Diameter and average diameter.

List<– keeps a list of nodes whose neighbours (neighbour of a node :those nodes in adjacent stages which have a link between them) have not yet been visited.

Each node has a variable m to show whether it has been visited and marked

Each node has also a variable distance associated with it which is the number of hops of the node from the source node.

Let i be any node in the network,the distance of all the nodes from it are found.

i <- start node

Repeat steps 1-3 until all the nodes in the network are visited and marked.

Step 1 :From i visit all its neighbours and mark them as visited,

their distance as (i->distance)+1.

Step 2:Put the neighbours at the end of the List.

Step 3:i<-The node at the head of the list

Go to step 1. After all the nodes are marked and their distances found,the Diameter and average Diameter are calculated.

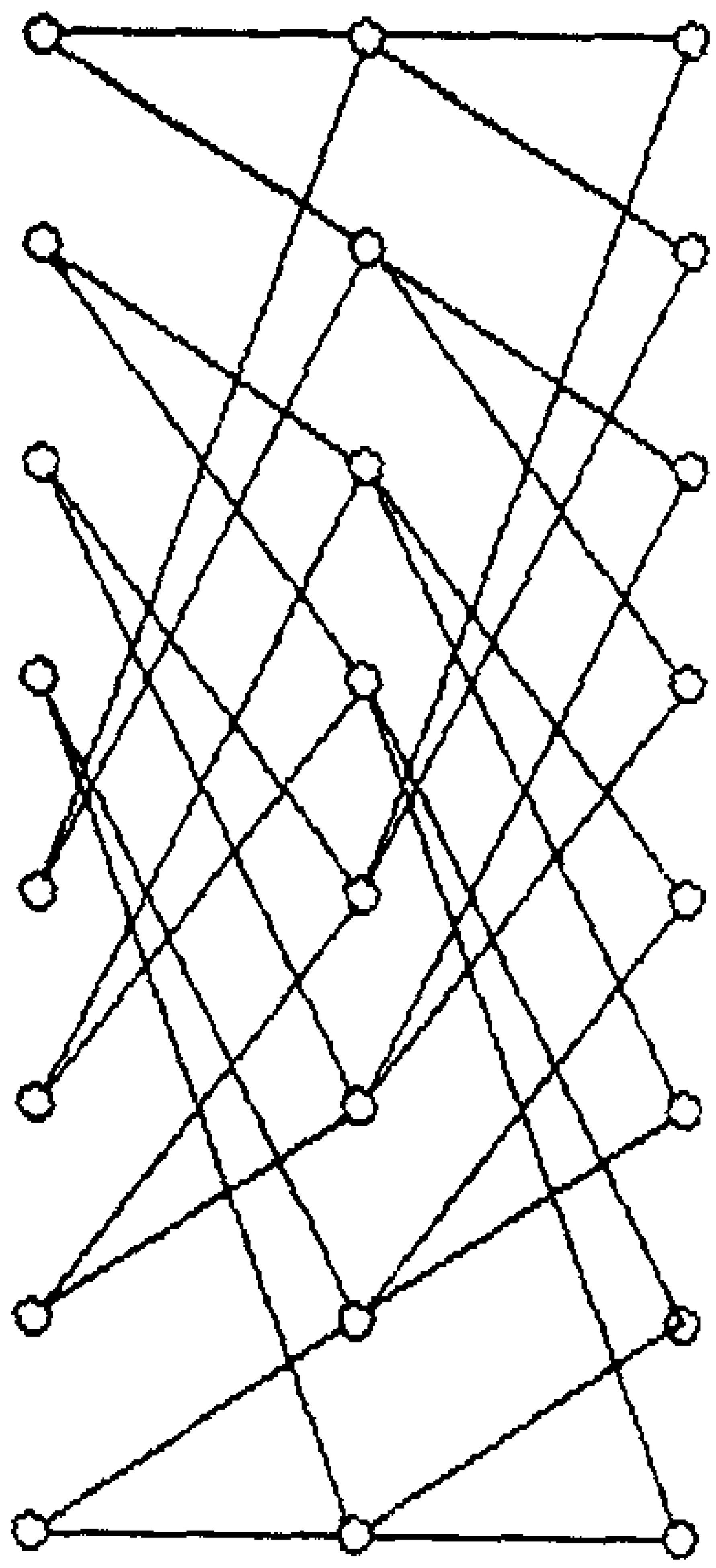
Lemma: For a particular arrangement $n * k$ of N nodes ,the diameter(maximum) has a range from $\log n + \lfloor k/4 \rfloor \leq D \leq \log n + \lfloor k/2 \rfloor$.

Proof: Let the variable k' denote the number of stages in a conventional shuffle network with n nodes/stage (k'=$\log n$).The source can access every other node in two stages ($s_{k'}$,$s_{k''}$) reached after $\log n$ hops The two stages $s_{k'}$ and $s_{k''}$ are reached after traversing $\log n$ hops through the stages from the source node in the forward and backward directions.

If the two stages $s_{k'}$ and $s_{k''}$ reached after log n hops are such that they divide the total number of stages k into equal partitions($k/4$) then the maximum distance of a node from another node in the $s_{k'}$,$s_{k''}$ stages would be $\lfloor k/4 \rfloor$ .Thus the maximum distance of the source node to any other node can be $\log n + \lfloor k/4 \rfloor$ hops. The upper bound is attained when the nodes are so arranged that there exists a node that is at a distance of $\lfloor k/2 \rfloor$ hops from a node in the stage $s_{k'}$,$s_{k''}$.

4.3 Simulation Results

We have simulated the values of average Diameter and Diameter using algorithm 2. Table 1 shows the average number of hops for various values of n for a particular N using unidirectional links [10] and also the average diameter from our simulated



n = 8, k = 2

N = 16

Fig 8. The generalised shuffle exchange topology.

results. It is seen that the average diameter is less considering the links to be bidirectional.

TABLE 1

* Expected Number of Hops for different $n's$considering unidirectional links.

Average diameter for various $n's$ considering bidirectional links.

	n=2	4	8	16	32	64	128	256	512
N=8 *	2.286	2.000	2.107						
#	1.25	1.5	1.75						
N=16 *	4.267	2.933	2.730	2.833					
#	2.125	2	2.25	2.5					
N=24 *	6.261	3.913	3.261						
#	3.083	2.1667	2.2917						
N=32*	8.258	4.903	3.742	3.565	3.693				
#	4.0625	2.5	2.625	3.00	3.3475				
N=64*	16.254	8.889	5.714	4.635	4.511	4.595			
#	8.0312	4.25	3.156	3.375	3.844	4.203			
N=128*	32.252	16.882	9.701	6.614	5.635	5.380	5.512		
#	16.015	8.125	4.528	4.141	4.203	4.703	5.125		
N=160*	40.252	20.881	11.698	7.610	6.069				
#	30.0125	10.100	5.463	4.212	4.312				
N=256*	64.251	32.878	17.694	10.604	7.561	6.923	6.708	6.834	
#	32.001	16.0625	8.289	5.070	5.070	5.125	5.625	6.047	
N=384*	96.251	48.877	25.691	14.601	9.556	7.536	7.892		
#	48.005	24.05	12.193	6.713	5.33	5.442	5.690		
N=512*	128.250	64.877	33.691	18.599	11.554	8.532	8.031	7.648	7.965
#	64	32.03	16.414	8.535	5.742	5.8125	6.0313	6.547	6.99

Table 2. shows the Diameter for various n for a particular N.It is seen the Diameter has a large range of values.For $n = N$ it becomes a single stage shuffle network with a diameter D $= \log N$.

TABLE 2

Diameter for different $n's$

* Those values of n for which |N - nlogN | is least.

	n=2	4	8	16	32	64	128	256	512	1024	2048
N=8	2	3*	3								
N=16	4	4	4*	4							
N=24	6	4	4*								
N=32	8	4	4*	5	5						
N=64	16	8	6	6*	6	6					
N=128	32	16	8	8	6*	7	7				
N=160	40	20	10	8	7*						
N=256	64	32	16	8	8	8*	8	8			
N=384	96	48	24	12	10	9*	8				
N=512	128	64	32	16	10	8*	8	9	9		
N=640	160	80	40	20	10	10	9*				
N=768	192	96	48	24	12	12	9*	9			
N=896	224	112	56	28	14	12	10*				
N=1024	256	128	64	32	16	12	10*	10	10	10	
N=1280	320	160	80	40	20	12	10*	10			
N=1536	384	192	96	48	24	12	12	10*	10		
N=1792	448	224	112	56	28	14	14	10*			
N=2048	512	256	128	64	32	16	14	12*	10	11	11
N=4608	1152	576	288	144	72	36	18	16	13*		
N=10240	2560	1280	640	320	160	80	40	20	18	15*	12
N=22528	5632	2810	1408	704	352	176	88	44	22	20	16*

Some experimental results:

case 1: $D > \log N$

$N = 32$, $n = 2$,$k = 16$

$D = 8$, $\log N = 5$

$n = 2$, $\log n = 1$

After 1(=logn) hop we reach stages $s_{k'}$ and $s_{k''}$ .All the nodes in these 2 stages can be accessed from the source node.The total number of stages is 16.The maximum number of hops between nodes in any other stage and in the the stages $s_{k'}$ and $s_{k''}$ is 7.So to reach a node in the stage which is 7 hops from $s_{k'}$ or $s_{k''}$ would require a maximum of 7+1 =8 hops from the source. This confirms the experimental result.

case 2:D=log N

a>$N = 32$,$n = 16$,$k = 2$

$D = 5$, $\log N = 5$

$n = 16$, $\log n = 4$

The total number of stages is 2.The stages $s_{k'}$and $s_{k''}$reached after 4(= $\log n$) hops from the source are the same. So to access a node in the remaining stage would require another hop since it is known that the source can access all the nodes in the stages $s_{k'}$,$s_{k''}$ in maximum k' hops.Hence the result.

b> $N = 64$,$n = 8$,$k = 8$

D=6, $\log N = 6$

n=8, $\log n$=3

The stages reached after 3 (=$\log n$) hops from the source $s_{k'}$ and $s_{k''}$ are separated by a distance of one stage but there exists stages which are 3 hops from $s_{k'}$or $s_{k''}$.To access nodes in those stages 3 hops from $s_{k'}$and $s_{k''}$ would be required ,thus a total of 3+3 hops will be required from the source.

case 3:$D < \log N$ $N = 32$,$n = 8$,$k = 4$

D=4, $\log N$=5

n=8, $\log n$ =3

The stages $s_{k'}$ and $s_{k''}$ reached after 3(=$\log n$) hops are separated by one stage and the maximum number of hops of any other stage from fro $s_{k'}$ or $s_{k''}$ is 1.So D = 3

+ 1 =4.

As seen from the tabular values a wide range of Diameters are got for different arrangements.The arrangement should be so chosen that D is less.For higher values of k in a n*k arrangement of N nodes , higher values of diameter are got,so the arrangement should be such that values of k are less.

From the results a conjecture can be made that if the arrangement n*k is so chosen that |N-n logn | is low for a given N ,then a low value of diameter can be got.The values are shown in Table 2.In almost all such cases it is seen that D$\leq \lceil \log N \rceil$.As N becomes larger this bound is exceeded.

Chapter 5

CONCLUSIONS

It is known that a 2n-1 stage shuffle exchange network is rearrangeable, not all permutations need 2n-1 stages for realization. The permutation realizable by an k-extra stage Omega network may need less number of passes if single stage multipass technique is used.We have considered the case of single stage multipass Shuffle Exchange networks and characterised the permutations realizable in i-pass Shuffle Exchange network $(1 \leq i \leq 2n-1)$. An 0(N $(\log N)^2$) algorithm has been used to find the minimum number of passes needed to realize any arbitrary permutation. For passes i, $1 \leq i \leq n$,a unique path exists between every source destination pair and the routing can be done by destination tag routing but for passes >n the path is not unique,its routing needs to be considered.Also here only realizability of permutations(one-to-one-mapping) using 2 states of the switch straight and cross have been considered,the restrictions existing in case of broadcasts also needs to be considered.

We have also considered a shuffle topology with N nodes (N=nk,n $= 2^m$).The realizable values of N are larger than when N=kp^k.We have studied the change in characteristics of this topology with variations in the arrangement.An arrangement with Diameter $\leq \lceil logN \rceil$ can be obtained for almost all N. Also it is observed that as the values of N become larger the bound on Diameter $\leq \lceil \log N \rceil$ is not always valid.This topology needs to be studied in more detail and its routing for the optimal arrangement needs to be examined.

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