M. Tech. (C. S.) Dissertation Series

Angle Arrival Estimation In non Gaussian Noise

A dissertation submitted towards partial fulfillment of the requirements for the **M.Tech. (Computer Science)** degree of **Indian Statistical Institute**

By

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Certificate of Approval

This is to certify that the thesis entitled *Angle Arrival Estimation in non Guaussian noise* submitted by Subhendu Mandal, towards partial fulfillment of the requirement for M. Tech. in Computer Science degree of the *Indian Statistical Institute, Calcutta*, is an acceptable work for the award of the degree.

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(Supervisor)

(External Examiner)

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Subhendu Mandal

ARRIVAL ANGLE ESTIMATION

IN

NON-GAUSSIAN NOISE

INTRODUCTION

Several important problems in signal processing field,among them Direction finding with narrow-band sensor arrays,estimation of the Parameters of multiple superimposed exponential signals in noise And resolution of overlapping echoes can be reduced to parameters in the following model:

$$Y(t)=A(\theta)x(t) + e(t) \qquad t=1,2,\ldots N$$

Y(t) is a C^{m*1} is the noisy data vector

X(t) is a C^{n*1} is the vector of signal amplitudes

E(t) is a C^{m*1} is an additive noise vector

$A(\theta)$ is a C^{m*n} and has the following special structure

$$A(\theta)=[a(w_1),a(w_2),\ldots a(w_n)]$$

Where $\{w_I\}$ are real parameters and $a(w_I)$ which is C^{m*1} is a so Called transfer vector between ith signal and y(t) and $\theta=[w_{1,\ldots\ldots}w_n]^T$

There are three main problems associated with fitting models of the form as discussed to the data {y(1),y(2)....y(N)}

1.*Estimation of the number of signals n*
Method s for estimating n are well documented in the Literature and won't be discussed here.In this discussion We will assume that the number of signals is given.

2.*Estimation of the signal amplitudes*
Once an estimate of θ is available,the estimation of **x**(t) Reduces to a simple least square fit.We will omit any Explicit discussion on the problem of estimating x(t)

3.*Estimation of the parameter vector θ*
Methods for accomplishing this task and their performance Are the main topics of discussion.

A class of methods for estimating θ which has received significant Attention is based on the eigendecomposition of the sample Covariance matrix of y(t).A representative member of this class Is the MUSIC(Multiple Signal Characterization) algorithm. There has been considerable interest recently in the analyzing The statistical performance of the MUSIC.Some interesting and related studies of the resolvability of MUSIC have been reported However an expression for the covariance matrix of the MUSIC estimate of θ has not been derived in these papers.Here we discuss An explicit expression for the covariance matrix that holds for sufficiently large values of N.

Comparison with the performance corresponding to CRAMER RAO LOWER BOUND is of interest.An expression for the CRLB On the covariance matrix of any unbiased estimator of the parameters θ in the general model does not appear to be avaliable in the literature.Here we derive the CRLB on the covariance matrix

The classical maximum-likelihood(ML) method can also be used Under appropriate assumptions to estimate the parameter vector θ

Next, we introduce some basic assumptions on the model. The MUSIC and the ML methods are based on different sets of assumptions. However some assumptions are common to both methods. The common assumptions are listed first:

A1:m>n and the vectors a(w) corresponding to (n+1) different values of w are linearly independent.

A2:$E\{e(t)\}=0$ $E\{e(t)e'(t)\}=\sigma^2 I$ and $E\{e(t)e^T(t)\}=0$

This is a more restrictive assumption that is essential for the MUSIC algorithm; for the ML method, relaxation of A2 is possible in principle, but would lead to considerable complications.

The following additional assumption is needed for MUSIC.

AMU: The matrix

$$P=E\{x(t)x'(t)\}$$

Is non-singular (positive definite) and N>m:

The following one is needed for the MLE:

AML:

$E\{e(t)e'(s)\}=E\{e(t)e^T(s)\}=0$ for t not equal to s

And e(t) is Gaussian distributed.

Direction Finding with Uniform Linear Sensor Arrays

The problem of determining the direction of n plane waves impinging on a linear uniform narrow band array of m sensors Can be formulated as that of estimating the parameters θ of the model where x(t) is the vector of complex wave amplitudes, N is the number of "snapshots" and

And $\mathbf{a}(w)=[1 \ \exp(jw) \ \exp(2jw) \ldots\ldots\ldots \exp(j(m-1)w)]$

Note that in this case, A(θ) is a *Vandermonde matrix* and therefore assumption A1 is satisfied. Assumption A2 and AML mean that the

Noise is spatially and temporally uncorrelated, and the assumption AMU means that the plane waves are not fully coherent and the Number of snapshots is greater than the number of sensors in the array. All these assumption look reasonable and could be satisfied. Thus both the MUSIC and the MLE could be usable in this type of application.

Estimation of Complex Sine Wave frequencies from Multiple-Experiment data

Consider the following signal model

$$\gamma_k(t) = \Sigma\, \gamma_p(t)\exp(j\omega_p k) + e_k(t) \quad k=1\ldots m, \quad t=1,\ldots N$$

The summation is over p=1…n

Where m denotes the number of samples in the experiment, N is the number of experiments, $\{\gamma_p(t)\}$ and $\{\omega_p\}$ are the amplitudes and the frequencies of the complex sine wave and $e_k(t)$ is an additive noise. The model can be rewritten as follows

$$Y(t)=[y_1(t)\ldots\ldots y_m(t)]$$
$$X(t)=[\exp(j\omega_1)\gamma_1(t) \ldots \ldots\ldots \exp(j\omega_n)\gamma_n(t)]$$
$$E(t)=[e_1(t)\ldots\ldots\ldots e_m(t)]$$

And $a(\omega)=[1 \ \exp(j\omega) \ldots\ldots\ldots \exp(j(m-1)\omega)]^T$

The conditions A2 and AML mean that the noise within the experiment is white and the noises of any two different experiments are uncorrelated, which is plausible.

Estimation of Complex Sine wave frequencies From single experiment data

And $y_k = \Sigma\gamma_p \exp(j\omega_p k) + e_k \quad k=1,2\ldots m$

The summation is over p=1..n

Assumption is satisfied if m>n,A2 means that the noise e_k is white And AML reduces to the requirement that noise is gaussian. Thus MLE may be usable.

To use MUSIC we write it in this form

And $y(t)=[y_t\ldots\ldots\ldots y_{t+m-1}]$

And $a(\omega)=[1 \quad \exp(j\omega) \ldots\ldots\ldots \exp(j(m-1)\omega)]$

And $x(t)=[\gamma_1 \exp(j\omega_1 t)\ldots\ldots\gamma_n(t)] \qquad t=1,\ldots\ldots,M-m+1$

And $e(t)=[e_t\ldots\ldots\ldots e_{t+m-1}]$

THE MUSIC ESTIMATOR

Here we briefly describe the MUSIC algorithm

The MUSIC Algorithm

In this subsection we assume that conditions A1,A2 and AMU hold. Under these assumptions the covariance matrix of the observation vector is given by

$$R=E\{y(t)y'(t)\}=A(\theta)PA'(\theta) \ + \sigma^2 I$$

For the notational convinience we simply write A instead of $A(\theta)$ Whenever there is no possibility of confusion.

Let $\lambda_1>=\lambda_2>=..........>=\lambda_m$ denote the eignvalues of R. Since Rank(APA')=n it follows that

$$\lambda_i>\sigma^2 \text{ for } I=1,2\ldots n \text{ and}$$
$$=\sigma^2 \text{ for } I=n+1,\ldots.m$$

It will be assumed throughout this paper that the eignvalues Are distinct.

Denote the unit norm eignvectors associated with $\lambda_1,...\lambda_n$ By $s_1,s_2\ldots.s_n$ and the corresponding with $\lambda_{n+1},\ldots\lambda_m$ By $g_1\ldots..g_{m-n}$. Also define

$S=[s_1\ldots\ldots s_n]$
$G=[\ g_1\ldots.g_{m-n}]$

Next, observe that

$$RG=APA'G + \ \sigma^2 G=\sigma^2 G$$

Which readily implies that

$$A'G=0$$

Or equivalently $\quad a'(\omega)GG'a(\omega)=0$

For $\omega= \omega_1,...\omega_n$

Since the eignvectors are orthonormal

$$SS'+GG'=I$$

So $a(\omega)[I-SS']a'(\omega)=0$ for $\omega=\omega_1,\ldots..\omega_n$

It is not difficult to see that the true values of $\{\omega_1\ldots.\omega_n\}$ are the only solutions to the above equation. The proof is by contradiction. Assume that there exists another solution ω_{n+1}>the matrix SS' is the orthonormal projection operator onto the subspace spanned by the columns of S. Thus it would follow that

The linearly independent vectors $a(\omega_I)$ for $I=1,2,..n+1$

Belong to the column space of S. However this is impossible since The space is of dimension

In practice true covariance matrix is unknown.

We estimate R by

$$R= 1/N(\Sigma y(t)y'(t))$$

The summation is over $t=1,\ldots N$

Then we have to take the eignvectors of R estimated

And select eigenvectors with 2 greatest eignvalues.

That is how we form SS'

Then we minimize $a'(\omega)[I-SS']a(\omega)$

To pick n values of ω for which it is minimized.

THE CRAMER RAO LOWER BOUND

Here we assume that A1, A2, and AML hold. Under these Assumptions the CRLB of the covariance matrix Is given by

$$CRLB(\theta)=\sigma^2/2\{\ \Sigma real[X'(t)D'[I-A(A'A)^{-1}A']DX(t)]\ \}^{-1}$$

$X(t)$=matrix with diagonals given by $x_1(t), x_2(t), \ldots x_n(t)$

$D=[d(\omega_1), \ldots d(\omega_n)]$

Where $d(\omega)$=derivative of $a(\omega)$ with respect to ω

THE MAXIMUM LIKELIHOOD ESTIMATOR

Here, we assume that A1,A2 and AML hold. The likelihood Function of the observations y(t) is given by

$L=\text{const} - mN\ln\sigma^2 - 1/\sigma^2\Sigma[y(t)- Ax(t)]'[y(t)- Ax(t)]$

The summation is over t=1,2,3……N

Estimate of $\sigma^2=(1/m)\text{trace}[I - B(B'B)^{-1}B](\text{estimate of } R)$

Where B=estimate of A

$F(\theta)=\text{trace}[I - A(A'A)^{-1}A](\text{estimate of } R)$

ML estimates of ω are obtained by minimizing $F(\theta)$ in a fine grid.

NUMERICAL STUDY

We simulated music for n=2(two signals)
We varied σ^2 and ρ(the correlation between the two signals)
And we also varied $\Delta\omega(\omega 1 - \omega 2)$ and we also varied m=
No of sensors. The results obtained were the same as given in the book.

IMPROVEMENT
ON
MUSIC
FOR
NON-GAUSSIAN NOISE

CRAMER-RAO BOUND

Let the signal amplitudes be deterministic.
Suppose the 2m-variate distribution of e(t) have density f, which is known except for the scale parameters σ.The following assumptions are imposed on f:

1.f is differentiable with respect to each argument.
2.The partial derivative of f with respect to each argument vanishes as that argument goes to +infinity or –infinity.
3.f is symmetric about zero with respect to each argument for all values of other arguments.

We have $x(t)=\mu(\theta,\phi) + e(t) \quad t=1,2,3\ldots N$

Where the parameter ϕ consists of unknown signal amplitudes.

$I_\mu = E\{\delta \ln f(u)\, \delta \ln f(u)^T\}$

Here the real vector u is formed by stacking real and imaginary parts of e(t) one after another. After taking the inverse of I_μ

We get the cramer rao bound of $\omega 1$ and $\omega 2$.

MLE WITH MIXED-GAUSSIAN NOISE

Mixed Gaussian noise is when we take a uniform random
Distribution giving values between 0 and 1.
When the distribution gives a value < epsilon(say 0.1)
We take e(t) with variance say$\sigma_I^2 = 100\sigma_B^2$ and
Whenever it gives value greater than epsilon
We take e(t) with variance σ_B^2.

The density function for e(t) is
(1-epsilon)*product of normal$(0,\sigma_B^2)$ +(epsilon)*product of normal$(0,\sigma_I^2)$

The power of the background noise is σ_B^2 and the power of
The foreground noise is σ_I^2.The total noise is
(1-epsilon)*σ_B^2 + epsilon*σ_I^2

In the mle for mixed gaussian noise we take the weight
For each y(t) as (1/u1)*(derivative of f(u) with respect to u1)

MODIFICATION OF MUSIC

Step 1.Using music calculate $\omega 1$ and $\omega 2$.

Step2.Calculate estimate of total power using the estimate Of e(t)(from the values of y(t))

Step3.From the values of estimated e(t) we calculate the Weight function .

Step 3.Run music again after recalculating the covariance Matrix from the weighted observations.

WEIGHTED MUSIC WITH METHOD1

The role of the weight function is to reduce the contribution of Those y(t) which have high value of e'*e(that is whose sum of Squares of e in the vector e is high).To do this we take the value Of weight function as I $(u^T u < cm\sigma^2)$ where c is a positive constant
(we have taken c=2) and I is an indicator function.

ROBUST MUSIC

We repeatedly run the above method until we get a convergence. This we call robust music.

WEIGHTED MUSIC WITH METHOD2

The weight function in this method is

(1/u1)*(derivative of ln(u) with respect to u1)

This method gives low weight to y(t) with high error Terms and high weight to y(t) with high value of error.

MLE

By repeatedly running the above method till convergence is Obtained we get MLE.

SIMULATION

We simulated MUSIC, WEIGHTED MUSIC with both methods And ROBUST and MLE for the following data

Epsilon=0.1

Interfering noise=100*background noise

And $\omega 1=0.4\pi$

And $\omega 2=0.8\pi$

And m=8

And N=50(no of samples)

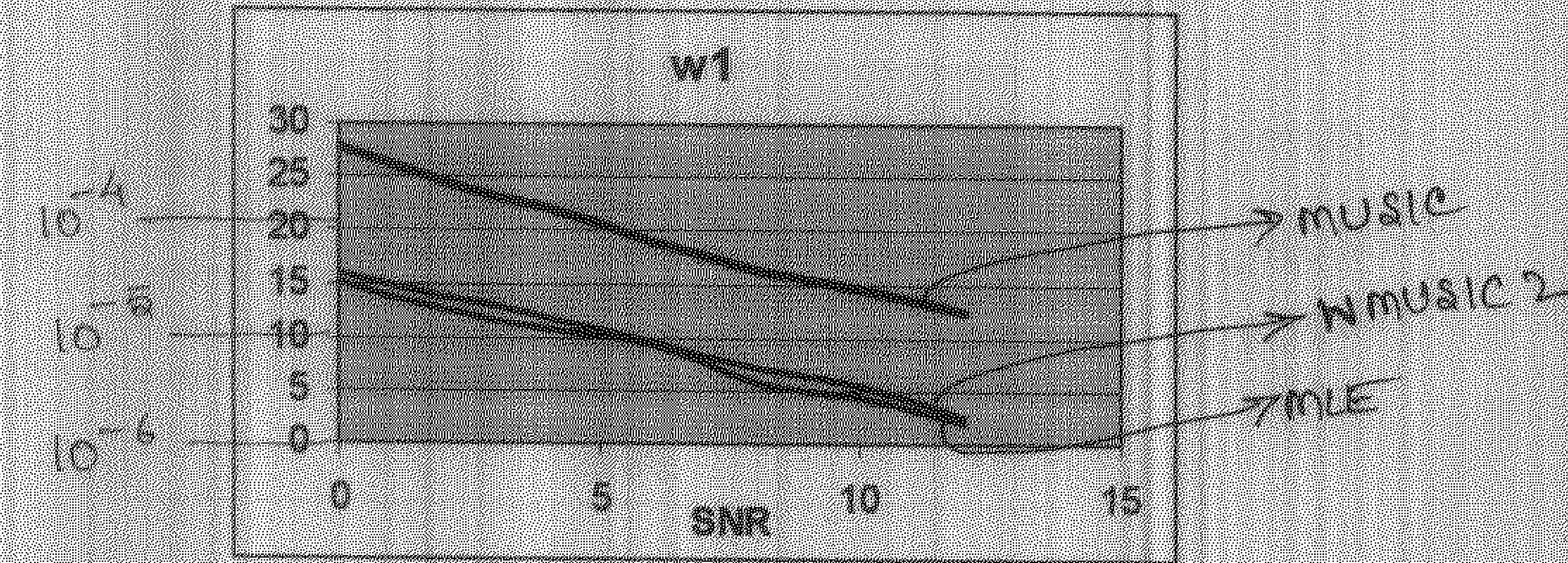
And the no of experiments=700

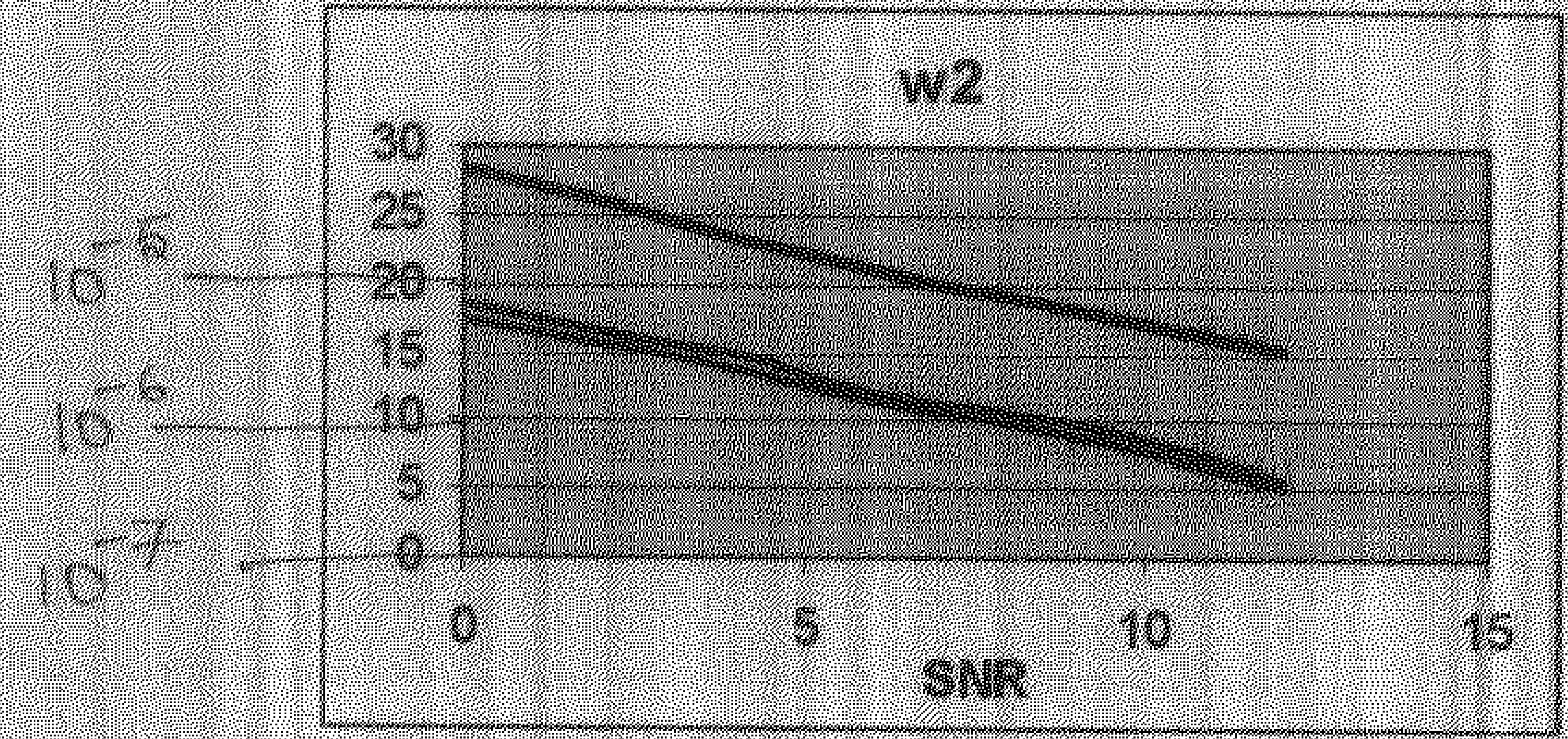
And c=2

FOR MIXED GAUSSIAN NOISE
MUSIC<WEIGHTED MUSIC WITH SIMPLE WEIGHT
<WEIGHTED MUSIC WITH COMPLEX WEIGHT
<ROBUST MUSIC<MLE

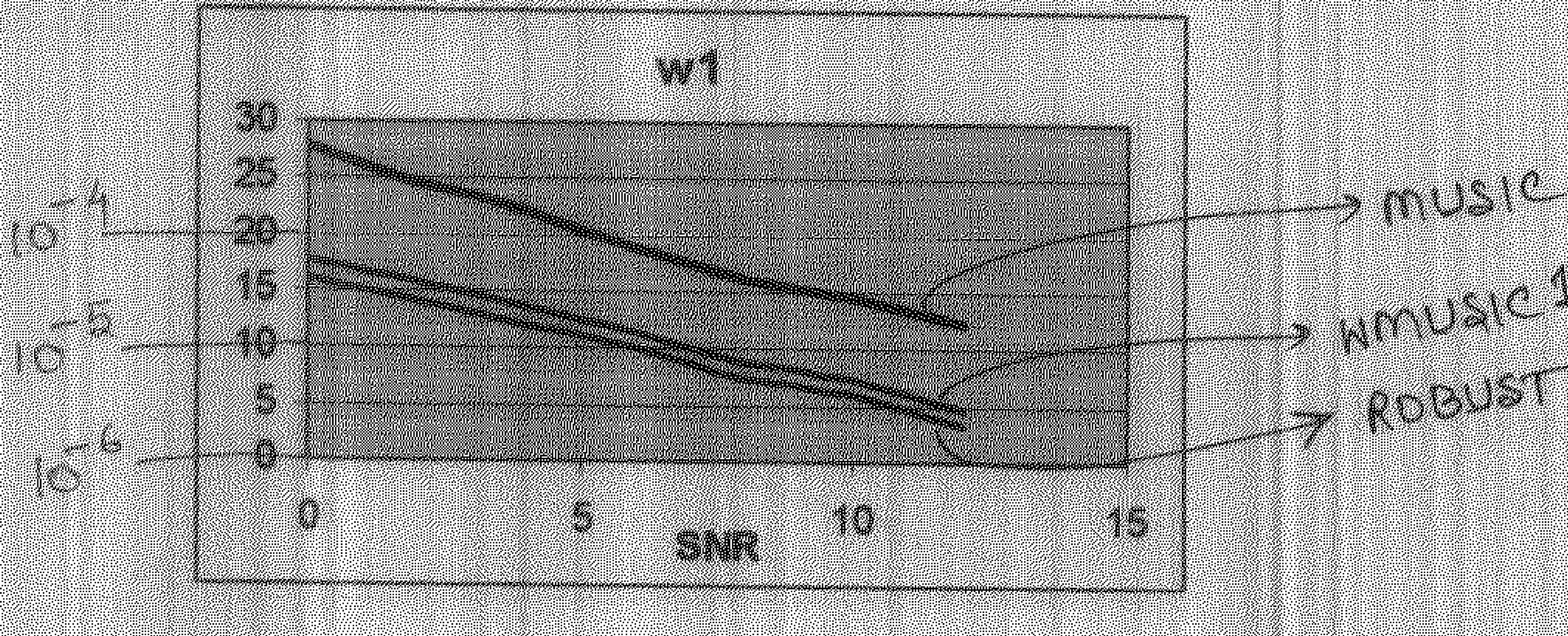
music < wmusic (simple) < robust < wmusic (complex) < mle.

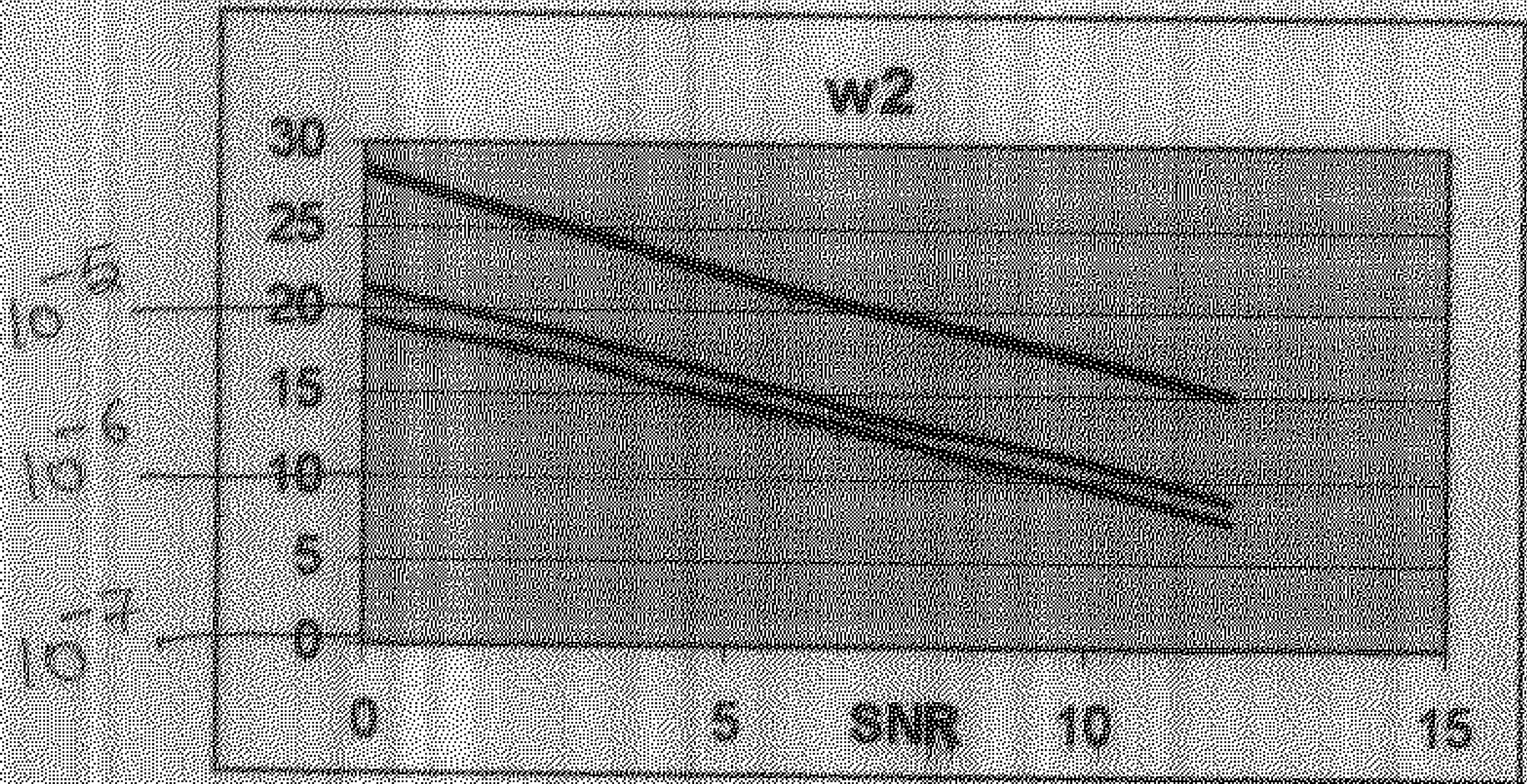
FOLLOWING IS THE PLOT OF MSE VERSUS SNR FOR MUSIC,WEIGHTED MUSIC WITH COMPLICATED WEIGHT AND MLE



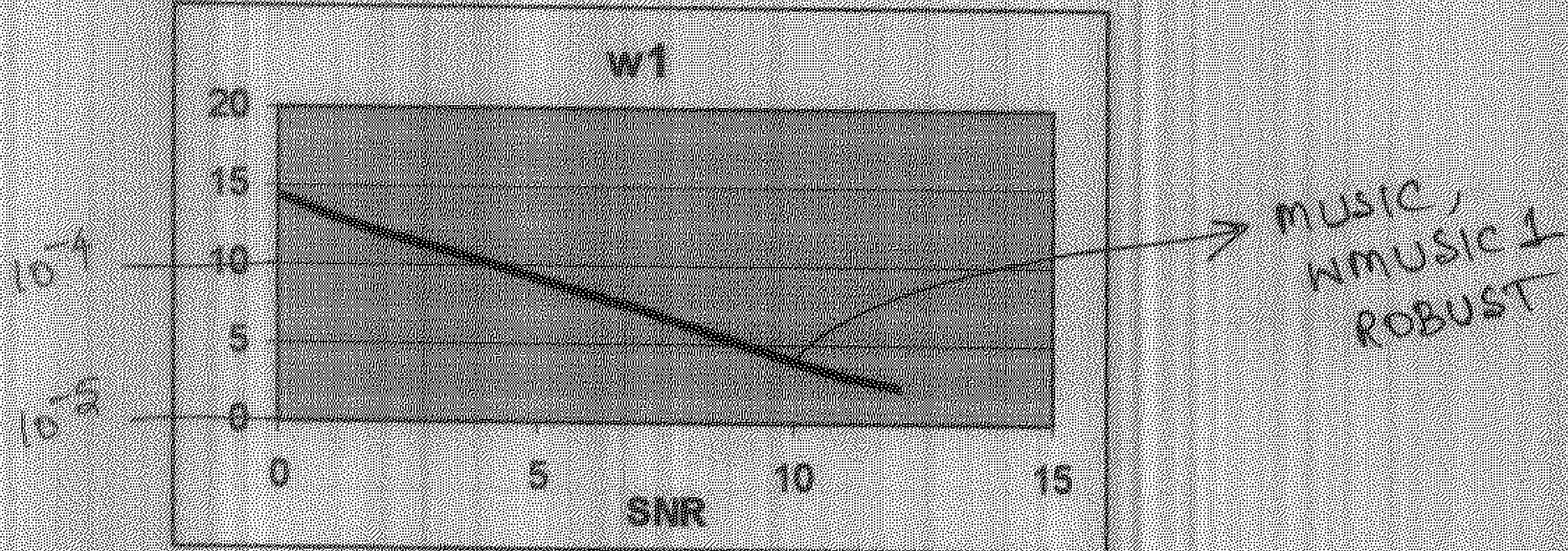


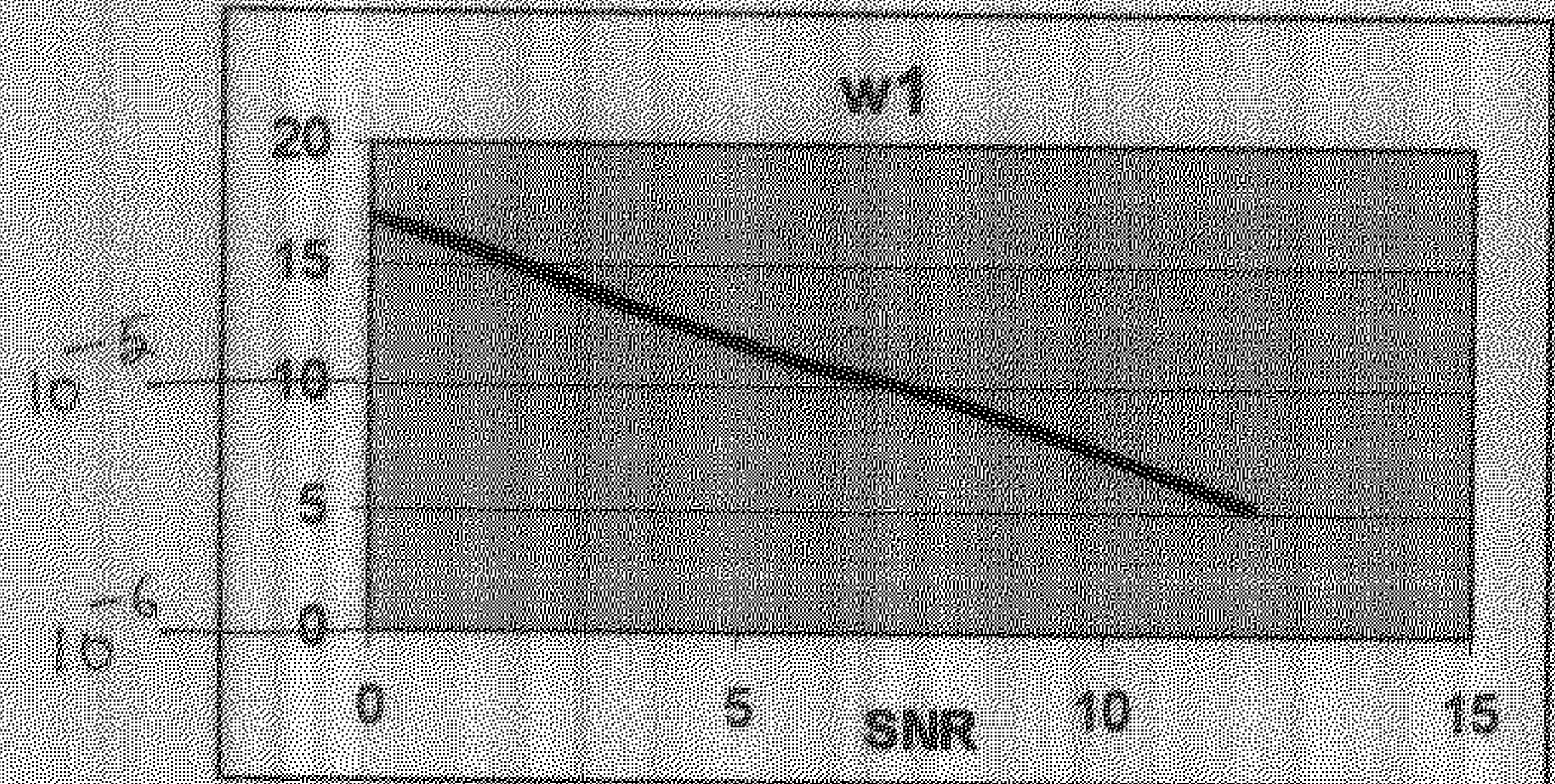
FOLLOWING IS THE PLOT FOR MSE VERSUS FOR MUSIC,WMUSIC WITH SIMPLE WEIGHT AND ROBUST MUSIC





FOR GAUSSIAN NOISE THE FOLLOWING IS THE PLOT FOR MSE VERSUS SNR BEST RESULT IS FOR MUSIC THE WMUSIC WITH SIMPLE WEIGHT THEN ROBUST MUSIC.ROBUST MUSIC AND WEIGHTED MUSIC WITH SIMPLE WEIGHT ARE ALMOST EQUAL





THE FOLLOWING IS THE PLOT OF MSE VERSUS SNR FOR GAUSSIAN NOISE FOR MUSIC, WMUSIC WITH COMPLEX WEIGHT AND MLE
MUSIC IS BEST FOLLOWED BY WMUSIC. MLE IS ALMOST EQUAL TO WMUSIC