M.Tech. (Computer Science) Dissertation Series

Image Segmentation by Stochastic Active Contour

a dissertation submitted in partial fulfilment of the requirements for the M. Tech. (Computer Science) degree of the Indian Statistical Institute

by

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Certificate of Approval

This is to certify that the dissertation titled *Image segmentation by Stochastic Active Contour* submitted by Sudhansu Ranjan Dash, towards the partial fulfillment of the requirements for the Degree of M.Tech in Computer Science at Indian Statistical Institute, kolkata, embodies the work under my supervision. His work is satisfactory.

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Sudhansu Ranjan Dash Dt- 18/07/05

Abstract

Active Contours, or snakes are used extensively for image segmentation and in related applications of image processing. In the present work we discuss an active contour based image segmentation technique where points on the active contour are moved randomly following a stochastic scheme (Stochastic Active Contour). The evolution of active contour in an image is guided by an energy function. In this work, we derive an energy function suited for random movement of active contour points. Further the energy function is minimized using simulated annealing approach. The methodology is tested for a number of synthetic and real images that demonstrates the efficacy of the proposed scheme. Further the approach is extended for open curve evolution after designing energy functions suitable for the random moves of the points on the open curve. Open curve evolution has also shown promising results.

Contents

1 Introduction

Image segmentation is an essential step of many applications in image processing. Image segmentation entails the division or separation of image into regions such that any two regions differ from each other by some region attributes or features. These attributes may be gray-scale value or some other features. Though object boundary is perceived naturally to human observer it is difficult and complex to obtain the boundary automatically in an image space. The problem is more difficult when the image is degraded due to noise or for some other reasons (i.e. image filtering). There are various techniques present in the literature to obtain image segmentation. One of the popularly used image segmentation techniques to segment images is the *active contour* model.

Active contour model, first described by Kass, Witkin, Terzopoulus [1] and referred popularly as *snakes*, is used extensively in image processing applications. In absence of any prior knowledge of objects or complicated structures present in the image, the generic description of an object could be made using a closed contour formed by the image edges (image edge is the image gradient information and includes edges of the objects contained in the image). Snakes or parametric active contours are moved towards the object of interest present in the image through satisfying some image constraints expressed as an energy function. Snakes are parametric closed contours defined within an image domain and snakes move under the influence of external forces coming from the image edge and internal forces coming from the geometry of snake (curve) itself.

The basic principle of the snake type model evaluation is to solve an energy minimization (maximization) problem where the energy function contains both structural information of the curve or snake and image constraints. The snake stops evolution or movement when the energy function is locally minimized (maximized).

At first a contour is initialized near the region of interest inside the image; then over a series of iterations the forces (external and internal) drives the snake to the nearest region of interest. This is achieved through minimization (maximization) of the energy function containing internal and external forces. This is further explained in Figure 1.

In Figure 1 the outer oval shows the contour at time t and the dotted oval shows the contour at time $t+1$ and the arrows show the amount of movement the contour makes in one iteration. The bold line is the boundary of the region of interest (the edge of the object).

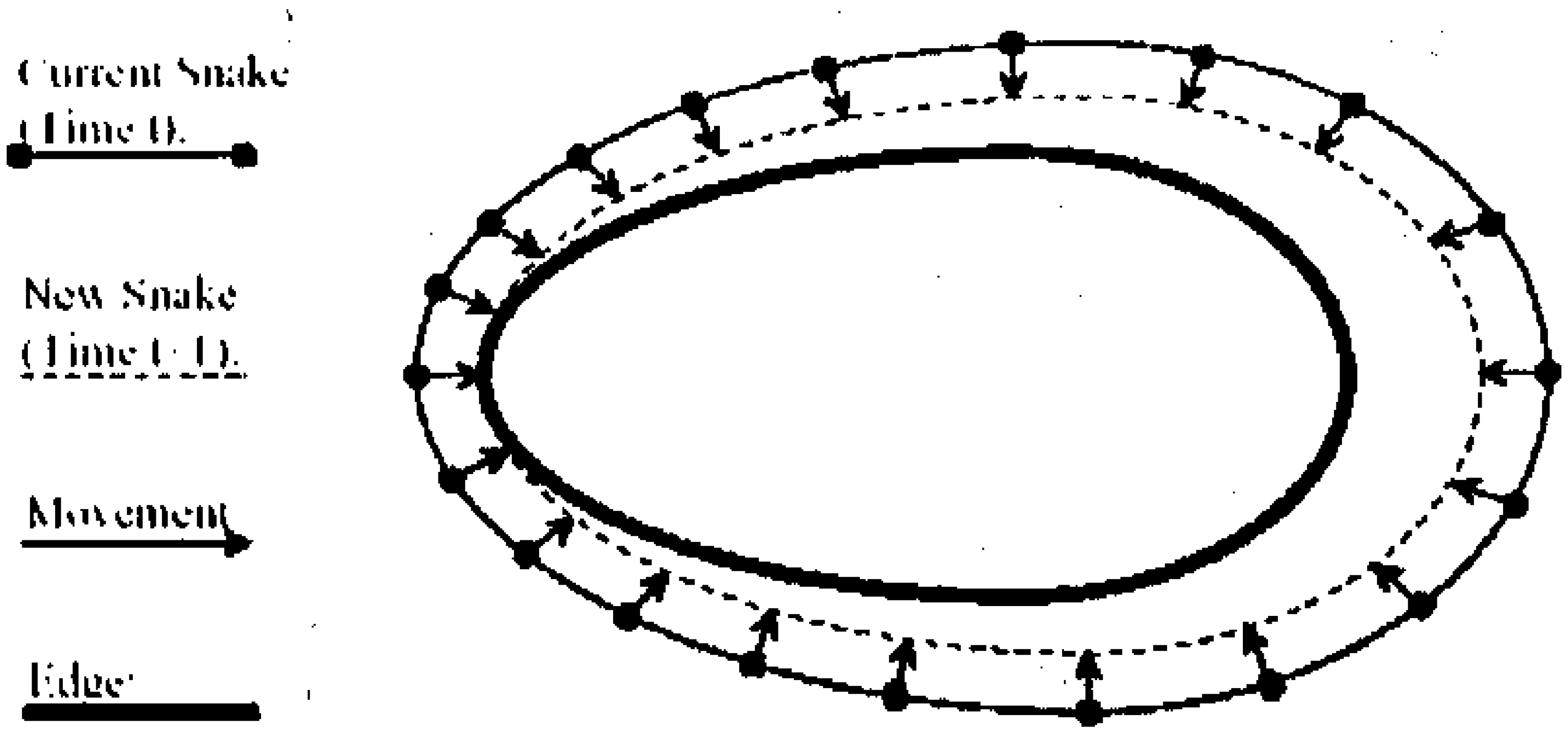


Figure 1: A Closed Active Contour Model.

1.1 Active Contour Model

As mentioned in the last section, a snake is a parametric contour that deforms over a series of iterations. For a snake c each element of c (i.e. each snake point) depends on two parameters: the space parameter and the time parameter. So a snake c can be written as: $c(s,t)$ where s is the space (curve) parameter and t is the time (iteration) parameter.

Again, as mentioned earlier the snake is influenced by image forces and internal forces to move it towards the feature of interest. Briefly the forces can be described as:

- *Internal Forces*: Internal constraints are designed to hold the snake together (elastic forces) and to keep the contour from bending too much (bending forces).
- *Image Forces*: Image forces is used to drive the contour towards the salient features such as edge of the object present in the image.

Ignoring the time component, a contour can be defined as a mapping M from a parameter $s \in [0,1] \subset \Re$, into the $2D$ Euclidian space defined as:

$$M : s \rightarrow c(s) = (x(s), y(s)) \quad (1)$$

where $s \in [0,1]$ is the parameter and $c(s)$ represents the whole contour. The energy term involving internal and external forces mentioned above can be written as [1]:

$$E = \int_0^1 \{\frac{1}{2}\alpha[(\frac{\partial x}{\partial s})^2 + (\frac{\partial y}{\partial s})^2] + \frac{1}{2}\beta[(\frac{\partial^2 x}{\partial s^2})^2 + (\frac{\partial^2 y}{\partial s^2})^2]\}ds - \int_0^1 E_{ext}ds, \quad (2)$$

where α is the parameter for weighting elasticity of the contour and β is the parameter for weighting rigidity of the contour. In the above equation the first order derivative terms are strain energy terms and second order derivative terms are bending energy terms of the contour. E_{ext} is the external energy around the contour derived from the image data e.g. gradient magnitude of the image intensity values. A solution to (2) can be written as [1]:

$$\delta x = -\delta t(\frac{\partial E_{ext}}{\partial x} - \alpha x'' + \beta x''''), \quad (3)$$

and

$$\delta y = -\delta t(\frac{\partial E_{ext}}{\partial y} - \alpha y'' + \beta y''''), \quad (4)$$

where δt is the time step. The equations (3) and (4) give the amount of movement that is given to the snake point (x_i^k, y_i^k) in the kth iteration. So the new snake point (x_i^{k+1}, y_i^{k+1}) obtained at $(k+1)$th iteration from the following equations:

$$x_i^{k+1} = x_i^k + \delta x, \quad (5)$$

and

$$y_i^{k+1} = y_i^k + \delta y. \quad (6)$$

The gradient of the internal and external energy terms, which are the internal forces arising within the snake, and the external forces exerted by the image data respectively, are designed such that they try to balance each other while the contour is moving towards the object of interest in the image.

1.2 Evolution of Active Contour using Simulated Annealing

Annealing is a metallurgical process of heating up a solid and then cooling the solid slowly until it crystallizes. The atoms of this solid have high energies at very high temperatures. This gives the atoms a great deal of freedom in their ability to restructure themselves. As the temperature is reduced the energy of these atoms decreases. If this cooling process is carried out too quickly many irregularities and defects will be seen in the crystal structure. The process of rapid cooling is known as rapid *quenching*. Ideally the temperature should be decreased at a slower rate. A gradual fall to the lower energy states allows a more consistent crystal structure to form. This stable crystal form allows the metal to be much more durable.

Simulated annealing seeks to emulate this process to solve energy minimization problem. In the context of active contour evolution, simulated annealing encourages random move of snake points at high temperature and gradually restricts the randomness of the movement of snake point as the temperature is reduced. The energy function designed for capturing the object of interest is evaluated at every move of the snake points. If due to random moves of the snake points the energy is minimized the movements of the snake points are accepted. If the energy is not minimized due to random movement of snake point, the new position of the snake is accepted based on ceratin probability value [6]. The advantages of using such a random move for snake evolution is that:

- Snake can still move towards the object present in the image even if the snake is initialized badly.
- Poor convergence of active contour to object edges due to noise etc. can be overcome using random movement of snake points.

The primary objectives of the present work are to exploit the above advantages. In this context the specific contributions of this work are:

- Designing appropriate energy function for movement of snake points.
- Designing an implementation scheme for random movement of the snake points.
- Investigating technique to stop inconsistent random moves by the snake points.
- Extending the present work for movement of open curve.

1.3 Organization of the Report

In the next section, the background of the proposed scheme is discussed. The proposed energy function and the numerical implementation is described in Section 3. The proposed methodology is extended for evolving an open curve is discussed in Section 3.3. Results obtained after applying the proposed models to both synthetic and real images are given in Section 4 followed by conclusions and scope of future work.

2 Stochastic Scheme for Curve Evolution

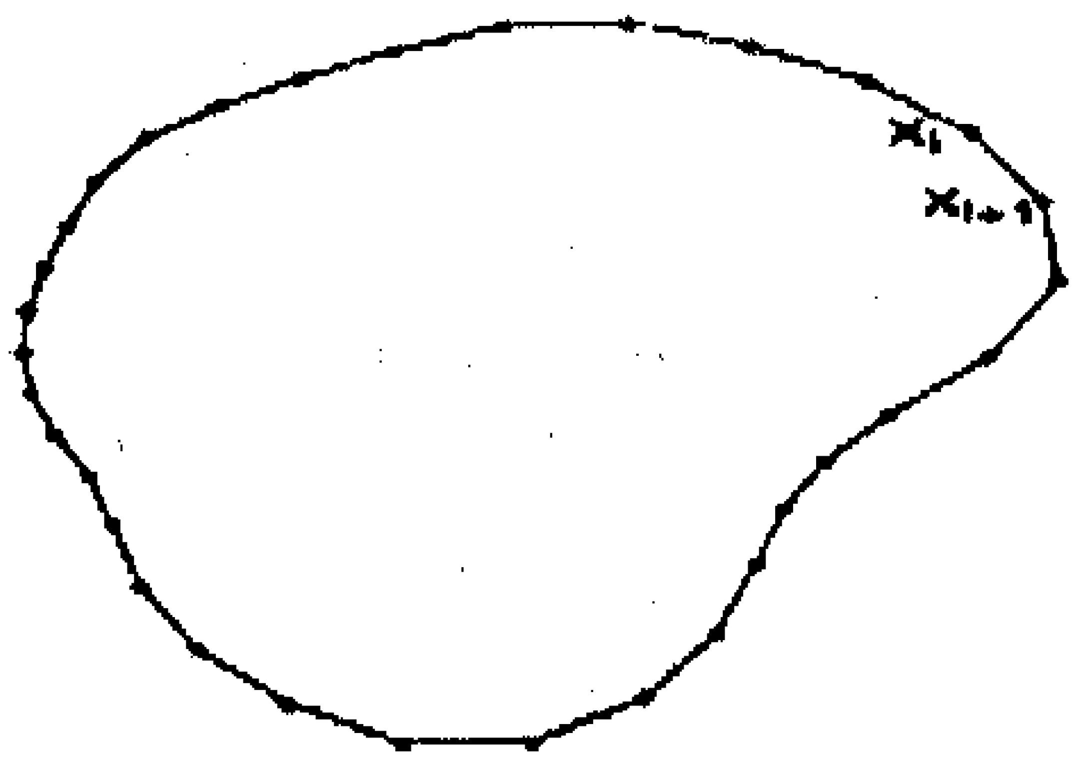


Figure 2: Polygon representation of the contour.

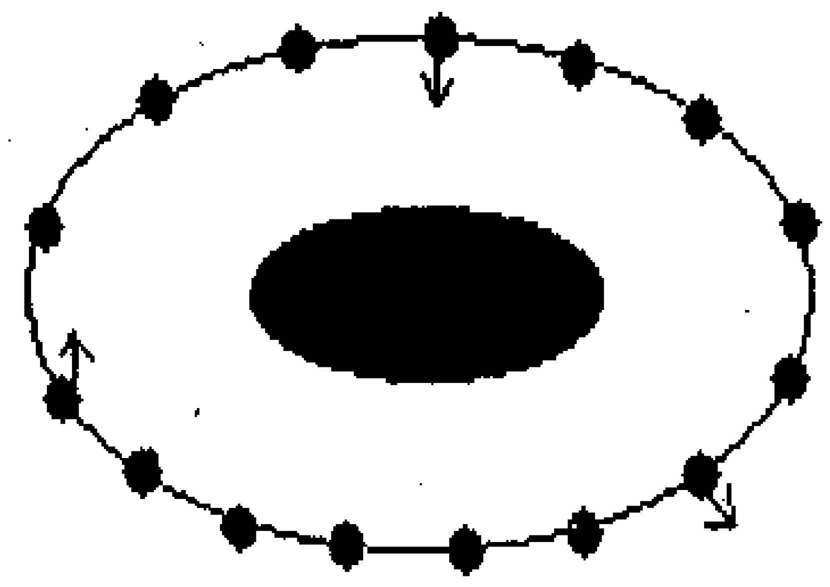


Figure 3: Nodes showing the direction of movement on the contour.

A polygon representation of a contour as shown in Figure 2 is defined by a set of nodes. These nodes on the contour are specified in a circular (either clockwise or anti-clockwise) fashion. Between two nodes the contour is represented by a straight line. For an active contour the number of nodes is dependent on the shape or length of the object that the contour is going to capture. Figure 3 demonstrates the type of random movement the nodes are given in each iteration. The black dots are some intermediate nodes and the nodes with arrows show the nodes chosen for giving random movement. Therefore, not all the nodes at a particular iteration of the contour are given random movement. The arrow shows the direction of move the node takes.

A way to simulate the random movement of the nodes of the active contour and acceptance or rejection of such random moves can be guided by Bayesian paradigm [5]. Given z as the observed image, the stochastic evolution of a curve in image domain is guided by following four principles:

1. Finding a prior probability distribution $\pi(c(s))$ where $c(s)$ is to be reconstructed.

2. Find the conditional probability density $f(z|c(s))$.

3. A posterior density $p(c(s)|z)$ is constructed from $f(z|c(s))$ and $\pi(c(s))$ by Bayes Theorem giving:

$$p(c(s)|z) \propto \pi(c(s))f(z|c(s)). \tag{7}$$

4. Base any inferences about $c(s)$ i.e. acceptance or rejection of the new position of $c(s)$ on the posterior distribution $p(c(s)|z)$.

In the active contour approach discussed in Section 1.1 the internal and external energy functions are associated with the contour. The internal energy function serves as a smoothness constraint and the external energy function guides the curve towards the image features [1]. Let the total energy across the contour $c(s)$ is $E(c(s); z)$ and as $E(c(s); z)$ consists of both internal and external energies, we can write:

$$E(c(s); z) = E_{int}(c(s)) + E_{ext}(c(s); z), \tag{8}$$

where E_{int} is the internal energy with respect to contour and E_{ext} is the external energy depending upon the observed image z and the position of the contour $c(s)$ in z. A prior model for $c(s)$ can be defined by:

$$\pi(c(s)) = \frac{1}{Z_{int}} e^{E_{int}(c(s))}, \tag{9}$$

where Z_{int} is a normalization constant [2]. Similarly the likelihood of the observed image z given the contour $c(s)$ is given by the equation [2]:

$$f(z|c(s)) = \frac{1}{Z_{ext}} e^{-E_{ext}(c(s);z)}, \tag{10}$$

where Z_{ext} is a normalization constant [2] and has the role similar to Z_{int}. Now the posterior probability for the contour conditioned on the image can be defined as:

$$p(c(s)|z) \; \alpha \; \pi(c(s)) f(z|c(s)) \; \alpha \; e^{-E_{int}(c(s))+E_{ext}(c(s);z)} \tag{11}$$

However optimizing (11) directly is nearly impossible. Therefore an iterative algorithm that move the contour dynamically towards the optimal solution has been proposed. Algorithms based on the dynamic programming can be designed to find optimal solution in the neighborhood of the contour. In many cases only a very closed neighborhood close to the object present in the image is used and the optimal solution is found by an iterative algorithm using the solution obtained from the previous iteration as the starting point.

In the proposed approach, which is discussed in the next section, active contour can be driven towards near optimal solution when the initial contour is not present int the close neighborhood of the object.

3 Proposed Methodology

In Section 3.1 we discuss energy functions that are suitable for the random movement of the snake points of a closed curve and maximization of the total energy such that the active contour approaches the region of interest in the image. This is followed by the numerical implementation in Section 3.2. In Section 3.3 model for open curve evolution is discussed.

3.1 Energy Functions of Snake Points

Let the prior distribution $\pi(c(s))$ has of the following form:

$$\pi(c(s)) = \frac{1}{Z_\alpha} e^{-\alpha' E(c(s))}, \tag{12}$$

where α' is the weight vector that are given to different components of $E(c(s))$, Z_α is a normalization constant usually unknown due to large number of possible configuration of $c(s)$. $E(c(s))$ in (12) is the internal energy and its evaluation is discussed below.

The contour has a polygon representation i.e. $c(s) = (c(s_1), c(s_2), c(s_3), \ldots c(s_n))$ where each $c(s_i)$ gives the coordinate of the contour at node i, and n is the total number of nodes present on the contour $c(s)$. Generally the energy functions is defined using the local characteristics of the contour. Hence the energy function can be written as

$$E(c(s)) = \sum_i V(c(s_i)), \tag{13}$$

where $V(c(s_i))$ is some measure of potential around the node i and the total energy is the sum of potential along all the nodes of the contour. If at each iteration a random number of nodes are moved to new positions then a potential measure can be specified as follows:

$$E(c(s)) = \frac{1}{|c(s)|} \sum_i V(c(s_i)), \tag{14}$$

where $|c(s)|$ is the total number of nodes present in the contour. The total energy may consists of some other functions as well. Some of these are discussed below.

Smoothness of the contour can be enforced in the prior distribution. For active contour approaches first and second derivatives of active contour points are used as smoothness measures. These derivatives can be approximated by

$$V_i^1 = \|c(s_i) - c(s_{i-1})\|^2, \tag{15}$$

and

$$V_i^2 = \|c(s_{i+1}) - 2c(s_i) + c(s_{i-1})\|^2, \tag{16}$$

where V_i^1 is the approximation of the first derivative and V_i^2 is the approximation of the second derivative. As stated in [4] minimizing the potential corresponding to (15) causes the contour to shrink. This is because first derivative is the discrete version of the contour length. The first and second derivatives are scale dependent. Hence to make the energy function scale independent an alternative measure is to estimate the curvature of the contour.

William and Shah [5] proposed a method to measure the curvature of the contour, which can be formulated as,

$$V_i(c(s)) = cos^{-1}\frac{(c(s_i) - c(s_{i-1})) \cdot (c(s_{i+1}) - c(s_i))}{\|c(s_i) - c(s_{i-1})\| \cdot \|c(s_{i+1}) - c(s_i)\|}. \tag{17}$$

The above is the measure of angle between the vectors $c(s_i) - c(s_{i-1})$ and $c(s_{i+1}) - c(s_i)$.

In some cases energy-functions cannot be split into potentials of interest. One such type of function is fractal dimension of the object defined as:

$$E(c(s)) = \frac{(contour\ length)^2}{(area\ of\ the\ object)} \tag{18}$$

An approximation of the contour length can be found out by counting the number of pixels (in the discrete version) on the contour boundary and area can be approximated by counting the total number of pixels lies inside the contour.

Let us define the mean radius of the active contour as:

$$r = \frac{1}{|c(s)|}\sum_i(\|c(s_i) - c_{mean}\|^2), \tag{19}$$

which is to be minimized where mean radius c_{mean} is defined as:

$$c_{mean} = \frac{1}{|c(s)|}\sum_i c(s_i). \tag{20}$$

The number of nodes present on the contour is $|c(s)|$ and $c(s_i)$ is the ith node on the contour. Hence it is possible to formulate an internal energy E_{int} which is guided by rate of change of mean radius r.

The probability density $f(z|c(s))$ is dependent upon the observed image data z and the position of the contour $c(s)$ inside the image. In [1] energy functions connecting the contour to the image are defined through potentials along the contour, that is

$$E(c(s), z) = -\sum_i h(c(s_i); z), \tag{21}$$

where $h(c(s_i), z)$ is some local measure of the image at the contour point $c(s_i)$. Some examples of $h(c(s_i))$ are the gray levels measure or gradient measures of the image.. Such energy functions can be approximated for z given $c(s)$ by:

$$f(z|c(s)) = \frac{1}{Z}\prod_i e^{-h(c(s_i);z)}. \tag{22}$$

This is a measurement in the neighborhood of contour $c(s)$.

One measure of (22) can be defined as follows. Let $f(x, y)$ be an edge map function derived from the image $z(x, y)$. Therefore the external energy is given by,

$$E_{ext}(z(x,y)) = |f(z(x,y))| = |\nabla z(x,y)| \tag{23}$$

Note that f gives a high value near the edge and small value in homogeneous regions within the image.

Another measure can be the absolute difference of the mean gray values inside and outside the contour. That is

$$f(z|c(s)) = |\mu_{inside} - \mu_{outside}|, \tag{24}$$

where μ_{inside} and $\mu_{outside}$ are the mean gray values inside and outside of the contour $c(s)$ (on the observed image z) respectively. The mean gray value inside the contour can be found out by summing all gray values of the pixels on the observed image z that lies inside the contour and then dividing by the total number of pixels lies inside the contour. Similarly $\mu_{outside}$ can be computed.

The external energy function should also attract the deformable contour to the region of interest, such as object boundaries in an image. The external energy function used in the proposed model is the energy due to image gradient.

The internal and external energy functions used in the proposed model uses (19) and (23) respectively. Therefore the total energy for the model under consideration can be written as:

$$E_{total} = -E_{int} + \zeta E_{ext}, \tag{25}$$

where ζ is the weighting parameter. Note that $E(c(s))$ is same as E_{int}. Maximization of (25) causes the term $-E_{int}$ to take a high value which shrinks the contour. Since $-E_{int}$ is implemented through minimization of mean radius, active contour shrinks due to iterative maximization of (25). Similarly maximization of (25) causes the term E_{ext} to drive the active contour towards the edge of the object present in the image where E_{ext} gets a high value. Next we discuss the implementation issue and the details of the algorithm.

3.2 Numerical Implementation

The overall implementation steps are:

1. Provide certain random movement to a preset number of nodes of $c(s)$ to get the deformed contour $c'(s)$. The length of each random movement is one pixel is any one of the 8 neighborhood directions. The number of nodes moved in each iteration is fixed as 10% of the total active contour points.

2. Compute the energy of the deformed contour using (25). The weighting parameter ζ is set experimentally.

3. If the energy is maximized accept the move. If the energy is not maximized accept the move probabilistically. This acceptance rejection scheme is implemented using simulated annealing [6]. Let the initial temperature is $T(0)$, $T = T(0)$ and for each iteration k carry out the following steps:

   (a) Compute Acceptance Acc defined by the following equation:

   $$Acc = \frac{1}{1 + e^{(E^k - E^{k-1})/T}} \tag{26}$$

   where E^{k-1} and E^k the energies at $(k-1)$th and kth iteration respectively. E^{k-1} and E^k are E_{total} computed using (25).

   (b) Generate a random number $rand$ uniformly between 0 and 1 and change to contour $c'(s)$ if $Acc < rand$, otherwise retain the contour $c(s)$. Change the current temperature as described below.

For the convergence of the algorithm choice of $T(0)$ and λ are important factors. $T(k+1)$ can be obtained from $T(k)$ by any of the following equations:

$$T(k+1) = \lambda T(k), \qquad 0 < \lambda < 1 \tag{27}$$

or

$$T(k+1) = P/\log(k+1) \tag{28}$$

where P is a constant and k is the current iteration number. The use of equation (27) makes the temperature change in a G.P. series. If λ value is chosen such that λ is close to 0.5 then temperature at each iteration becomes almost half. This may cause the contour to converge to a local optimum solution rather than finding a global one. But if λ is chosen very near to 1 then temperature decreases slowly and a global optimum solution may be possible with significant number of iterations [2]. Use of (28) makes the rate of change of temperature decrease very slowly

3.3 Open Curve Evolution

Evolution of open curve is of interest in capturing long thin filament like structures which are essentially one dimensional. Very few works are reported for parametric open ended curve evolution. In the proposed methodology we have used the combination of stochastic move of snake points and simulated annealing for evolving open curve. To do so the magnitude of gradient of the image or the intensity of the image is distributed through out the image domain to form an energy distribution called *potential field*. The internal energies discussed in Section 3.1 cannot be used directly for the open curve as there cannot be any concept of mean radius for an open curve. A variation of internal energy that is used by the proposed model is described next after we specify the open curve.

Define the open curve as $c_{op} = (c_1, c_2 ... c_n)$, $c_1 \neq c_n$, and $c_i \in \Re^2$ and n is the total number of nodes in the contour. Let the internal energy of open curve be denoted as E^{op}_{int} and defined as:

$$E^{op}_{int} = \begin{cases} k_1 \|c_1 - c_n\|^2, & p_f(c(s_1)), p_f(c(s_n)) > Th; \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

where $p_f(c_i)$ is the potential field associated with the ith node on the open curve and Th is some threshold value depending upon the problem at hand. Maximizing E^{op}_{int} ensures that when the two end points (i.e. c_1 and c_n) of the curve lies in a region of potential value larger than Th they try to move away from each other.

The potential at a point in the image domain can be computed as:

$$p_f(x, y) = \sum_{j}^{r} \sum_{k}^{c} \frac{q(j, k)}{d^2} \tag{30}$$

where $q(j, k)$ is the charge associated with the point (i, j) in the image domain. The number of rows and columns of the image is given by r and c respectively. The charge at an image point can be the intensity associated with the corresponding point in the image or magnitude of gradient of the image depending upon the problem at hand. The Euclidian distance between the point (x, y) in the image domain and (j, k) is given by d. The potential field is computed by (30) for all the points in image domain where the open curve is to be evolved. Defining such type of potential field ensures that near the region of interest there is a high value of potential. As the distance of an image point becomes more from the region of interest potential filed becomes weaker.

Now we define the external energy E_{ext}^{op} associated with the curve and the image under consideration as:

$$E_{ext}^{op} = \sum_i p_f(c_i). \tag{31}$$

The total energy for the model can be computed as:

$$E_{tot}^{op} = E_{int}^{op} + \xi E_{ext}^{op} \tag{32}$$

where ξ is weighting parameter determined experimentally. Maximizing (32) causes (31) to take a high value, so the curve moves towards the region of interest. Note that the internal energy defined in (29) comes into action if both end points of the open curve lie on the region where the potential field value is more than the threshold value Th. Maximization of (29) expands the end points so that the curve can catch the object of interest. The algorithm for random movement of curve points and consequent acceptance or rejection is the same as discussed in Sections 3.2 and only the energy functions are different. The result obtained after applying the proposed model is presented in Section 4.3.

4 Results

In this section some results are shown that are obtained after applying the proposed active contour model of Section 3. In Section 4.1 there are two results on synthetic images. Section 4.2 shows three results on real images and Section 4.3 shows the results obtained after evolving open curve on a synthetic and a real image.

4.1 On Synthetic Images

(a) (b)

Figure 4: (a) Initialized contour and the object of interest (b) Final contour after grasping the object perfectly.

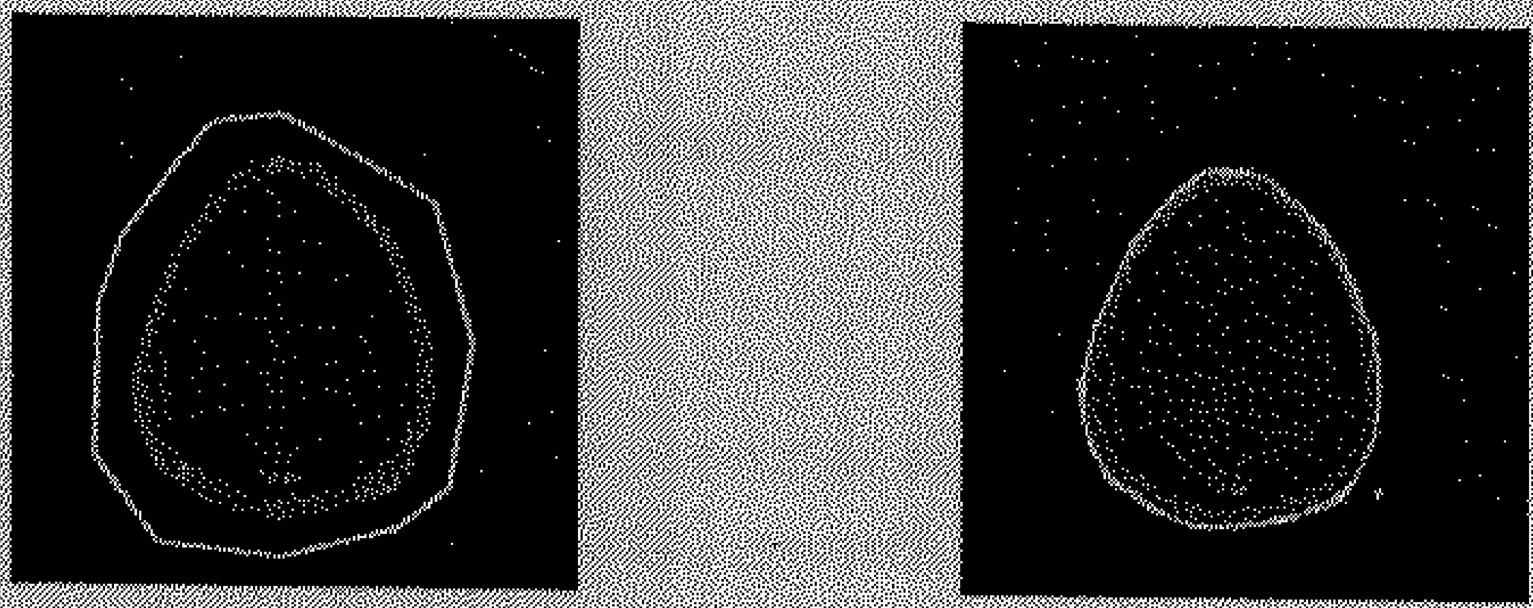
Figure 4(a) shows a synthetic image of a circle against a black background. We have defined our initial contour as the closed thin red curve around the circle as shown in Figure 4(a). Figure 4(b) shows the boundary of the synthetic circle has been captured after 20,000 iterations. The final position is shown in thick red.

(a) (b)

Figure 5: (a) A Synthetic image with initial contour (b) Synthetic object been fully captured by the contour.

Figure 5(a) shows a synthetic black shape against white background. The initial contour is defined around the synthetic object. After 40,000 iterations the contour catches the boundary of the object. The final contour is shown in thick red in Figure 5(b).

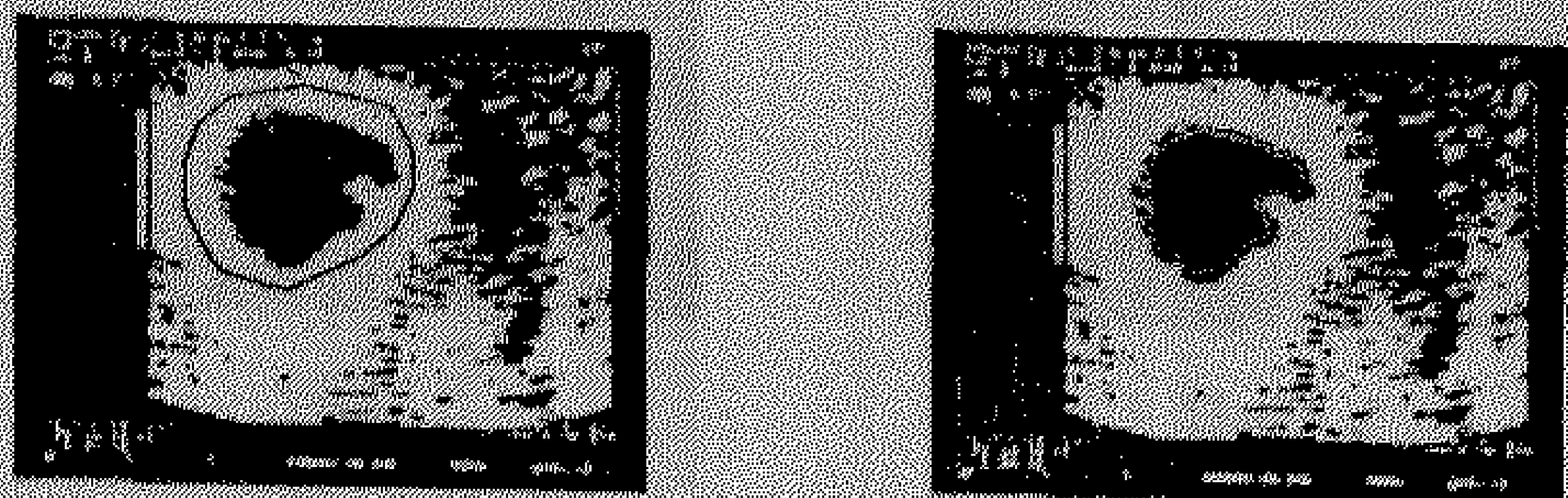
4.2 On Real Images



(a) (b)

Figure 6: (a) MRI of a brain transverse section and initial contour (b) Final contour captured the object.

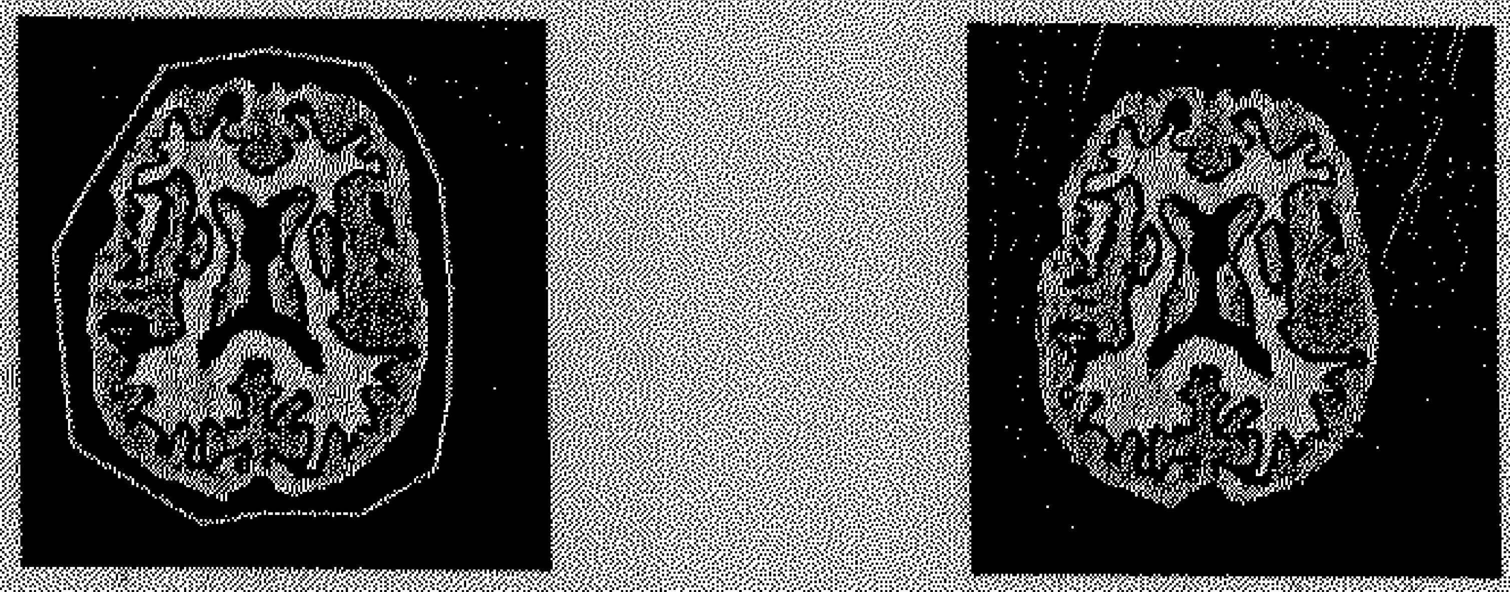
Figure 6(a) shows the MRI of the brain and an active contour is initialized around the brain. Figure 6(b) shows the segmentation obtained after 70,000 iterations. Final contour is shown in thick red.



(a) (b)

Figure 7: (a) An Ultrasound image and the initial contour (b) Segmentation obtained on the Ultrasound image.

Figure 7(a) shows an Ultrasound image of stomach and the initialization of the contour. Figure 7(b) shows the segmentation obtained after 50,000 iterations.



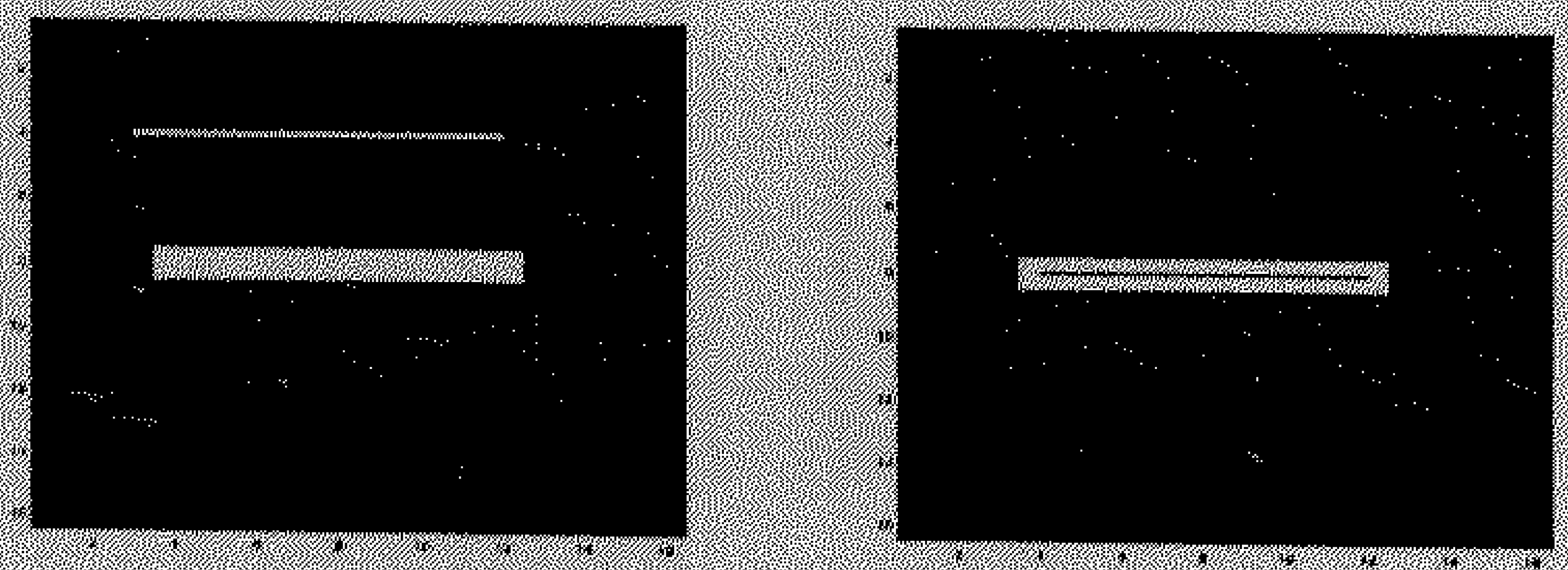
(a) (b)

Figure 8: (a) Another transverse section of brain and the initial contour (b) Segmentation of the brain image.

Figure 8(a) shows another transverse cross section of the brain and the initial contour position. After 90,000 iterations the contour catches the object perfectly which is shown in Figure 8(b).

4.3 On Open Curve

This section shows results obtained after applying the proposed model of open curve evolution of Section 3.3 and using the energy function of (32).

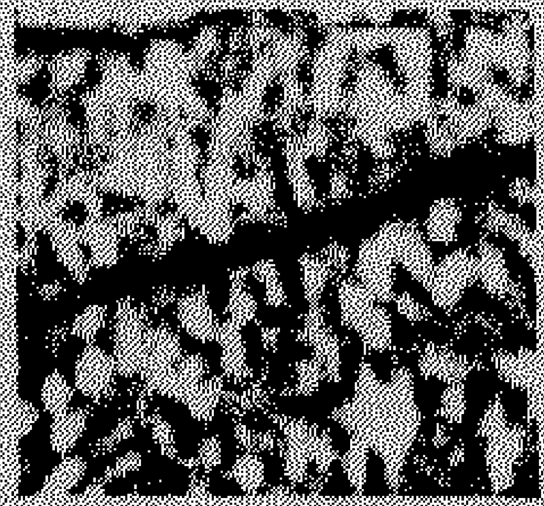


(a) (b)

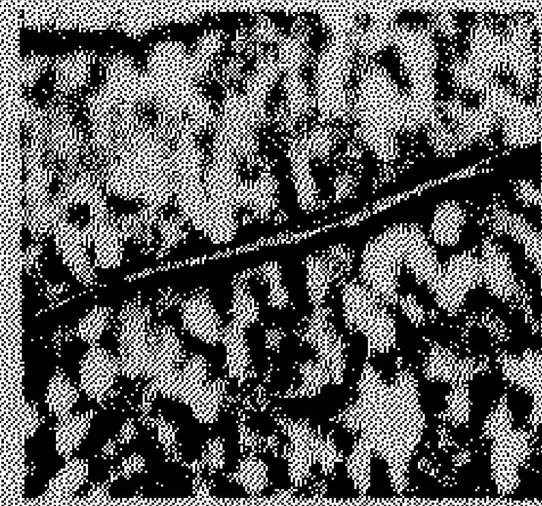
Figure 9: (a) The object and the initialized open curve (b)After the open curve catched the object

Figure 9(a) shows the position of the initialized open curve and the position of the object. Figure 9(b) shows the final curve after 5,000 iterations. The curve catches the middle of the synthetic object as expected. In this experiment the center of the object is of interest not the edge.

Figure 10(a) shows a road and the initialized curve. Figure 10(b) shows the final curve after catching the object of interest. The potential field is computed after taking negative of the image and then thresholding. In this



(a)



(b)

Figure 10: (a) The Open Curve initialization. (b)The open Curve catches the Object (road).

example the region of interest is the center of object and not the edge (gradient of the image).

For all results shown in Section 4, change of temperature for simulated annealing is computed by (27) and λ lies between 0.999 and 0.9999. At each iteration one node is chosen for random move on the contour. The direction of move for a node on the contour at one iteration is any one of all the 8 neighborhood directions. Allowing all possible 8 directions give enough freedom to the snake points so that the active contour can reach the object boundary.

5 Conclusion and Scope of future work

The proposed methodology is used for segmenting two dimensional images (both synthetic and real). The contour is assumed to be simple closed curve and a prior probability distribution is defined for the contour. The observed image is combined with the contour through a conditional probability density.

An iterative algorithm is developed for solving the segmentation problem and the concept of simulated annealing is used to obtain the global optimum solution. Throughout the iterations the contour moves dynamically towards the feature of interest (the edges of the object present in the image).

The main drawback of the proposed methodology for closed contour evolution is time complexity (very large). The internal energy used is guided by the rate of change of mean radius. So if the gradient of the image is not smooth the resultant contour is not smooth. To overcome this some other internal energy function should combine with the proposed internal energy so that the resultant contour becomes smooth even if the gradient of the image is not smooth. In designing the external energy for the proposed model only the magnitude of gradient energy is considered. It may be possible to design an external force so that it can only guide the contour toward the object of interest. If more number of points can be randomly moved in each iteration, the time complexity can be reduced significantly.

In the proposed model we could not avoid all possible inconsistent random moves (loop formation or very sharp edges) made by the snake. Removing such type of random moves may improve the time of convergence of the contour towards the region of interest.

In the Section 3.3 we have discussed a methodology for open curve evolution. The main drawback is that the open curve makes many inconsistent move which should be avoided. The proposed methodology works if noise is uniform and if noise is not uniform then the curve may stuck to a local minimum. These issues need further investigation.

6 Bibliography

References

[1] M.Kass,A. Witkin, and D. Terzopoulos, "Snakes: Active contour models," *Int. J. Comuter Vision*, Vol. 1, no.4, pp.321-331, 1988.

[2] Geir Storvic, "A Bayesian Approach to Dynamic Contours through Stochastic sampling and Simulated Annealing," *IEEE Transactions on PAMI,* Vol. 16, N0. 10, October 1994.

[3] V.Caselles, F. Catte, T. Coll, and F. Dibos, "A Geometric model for active contours," *Numer. Math.*,vol.66, pp.1-31, 1993.

[4] D.J. Williams and M.Shah, "A fast algorithm for active contours and curvature estimation," *CVGIP:Image understanding*, vol.55. no.1, pp.14-26, 1992.

[5] J. Besag, "Towards Bayesian image analysis, " *J. Appl. Statist.*, vol.16, no. 3, pp.395-407, 1989.

[6] William H. Press, Saul A. Teukolsky, William T. Vellerling, Brain P. Flannery, "*Numerical Recipes in C - The Art of scientific Computing,*" Cambridge University Press, pp.444-455, 1993.