Generalized Lower Bound on Bandwidth for k-band Buffering in Hexagonal Cellular Networks

M.Tech dissertation submitted by
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Certificate of Approval

This is to certify that this thesis titled "**Generalized Lower Bound on Bandwidth for k-band buffering in Hexagonal Cellular Networks**", submitted by **Prashant Kumar Sinha (CS0316)** submitted towards partial fulfillment of requirements for the degree of M.Tech in Computer Science at the Indian Statistical Institute, Kolkata embodies the work done under my supervision.

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Contents

Abstract

A cellular network is often modeled as a graph and the channel assignment problem (CAP) is formulated as a coloring problem of the graph.

Existing studies on the channel assignment problem (CAP), focus on developing efficient heuristic algorithms, owing to its NP-completeness. The solutions thereby generated are often less than satisfactory, and what is worse, there is no way of checking how far away they are from the optimum.

Studies on deriving lower bounds on the number of frequencies required for CAP's have thus arisen out of their capacity of indirectly checking the quality of the assignment solutions at hand.

This project deals with proposing a lower bound on the number of frequency channels required for the CAP on a hexagonal graph, where the channel interference does not extend beyond k cells, i.e. the CAP with k-band buffering, where k is any non-negative integer.

1. Introduction

1.1 Background

In a cellular radio communication network, the service area is divided into hexagonal cells (geographical areas), each under the control of a given base station. Each base station is in-charge of communication with all mobile users currently present within its cell boundary. Communication between a mobile terminal and the base station is wireless in nature, while the base stations themselves are connected by a wired network in general. A number of mutually adjacent cells may be grouped together to form a location area (LA) which is controlled by a mobile switching centre (MSC).In a GSM cellular environment, communication is effected through a channel which may use the basic frequency division and/or time division multiplexing, while in a CDMA cellular environment code division multiplexing is used.

The total currently available bandwidth for cellular communication is 230 MHz, in the UHF part of the frequency spectrum. In 1995, it was decided that 170 MHz be allocated for terrestrial communication, and 80 MHz be allocated for satellite communication. However, currently 175 MHz is allocated for terrestrial communication and 75 MHz for satellite communication. We can see that, the total bandwidth which can be allocated is more or less constant, whereas the number of users is increasing rapidly. So, like all with all scarcely available resources, the cost of frequency-use provides the need for economic use of the available frequencies.

Reuse of frequencies within a wireless communication network can offer considerable economies. However, reuse of frequencies also leads to loss of quality of communication links. The use of (almost) the same frequency for multiple wireless connections can cause interference between the signals, which is unacceptable. So, we have to maintain certain minimum frequency spacing between frequencies assigned to two users, this minimum frequency spacing depends on the power of the signal that the base station uses to communicate with the users within it's cell boundary, and the geographical spacing between the base stations under whose control the two users, who have to be allocated frequency channels, are. If they are within control of the same base station, then the minimum frequency spacing to be maintained between the frequencies assigned to the two users, is called S_0, if the two users are within control of base stations that are adjacent, then the minimum frequency to be maintained is called S_1, if the two users are within control of base stations that are distance-k neighbors of each other, then the minimum frequency spacing to be maintained is called S_k.

For a network, the available radio spectrum is divided into non-overlapping frequency bands of equal length, each of these bands is called a channel, and these channels are numbered as 0, 1, 2 . . . from the lower end. In this context, the terms *channel assignment* and *frequency assignment* can be used interchangeably.

When the mobile cellular network is designed, each cell of the network is assigned a set of channels to provide services to the individual calls of the cells. The channel assignment problem (CAP) is that, given a set of cells, each with *a priori* known demand of a given number of users, we are to assign frequencies to each cell, so as to satisfy its demand and to minimize the total radio frequency bandwidth of the system, subject to some frequency separation constraints, so as to avoid channel interference. The highest numbered channel required in an assignment problem is called the required bandwidth.

Three types of interference that are taken into consideration in the form of constraints. They are,

(i) *co-channel constraint*, due to which the same channel is not allowed to be assigned to certain pairs of cells simultaneously.

(ii) *adjacent-channel constraint*, due to which adjacent channels are not allowed to be assigned to certain pair of cells simultaneously.

(iii) *Co-site constraint*, due to which any pair of channels assigned to the same cell must be separated by a certain number.

1.2 Motivation

A cellular network is often modeled as a graph and the channel assignment problem (CAP) is formulated as a coloring problem of the graph.

Existing studies on the channel assignment problem (CAP), focus on developing time-efficient approximate algorithms using simulated annealing, and genetic algorithms owing to its NP-completeness, which however cannot guarantee optimal solutions, thus the assignments generated by them may require more frequencies than an optimal assignment.

In heuristic approaches like genetic algorithm, the process terminates after a certain number of iterations, so a prior idea about lower bounds will help the procedure a lot. Also in the simulated annealing approach, the technique starts from a known lower bound and improves the results in each iteration. Therefore, in both approaches, the algorithms will give results in lesser time if we know a good lower bound on the minimum number of frequencies needed for a solution.

Studies on deriving lower bounds on the number of frequencies required for CAP's have thus arisen out of their capacity of indirectly checking the quality of the assignment solutions at hand. A tighter lower bound obviously gives a better judgment about how far away the solutions of the CAP that is under consideration, is from the optimum.

2. Generalized lower bound for k-band Buffering

2.1. Problem definition

This project deals with proposing a lower bound on the number of frequency channels required for the CAP on a hexagonal graph, where the channel interference does not extend beyond k cells, i.e. the CAP with k-band buffering.

The objective of this project is to tighten the lower bound for k-band buffering, where k is any non-negative integer.

2.2. Problem formulation

The Channel Assignment Problem (CAP) is represented by the model described by the following components [1]:

(i) A set **X,** of n distinct cells, with cell numbers as 0, 1, 2 . . . (n-1).

(ii) A demand vector **W** = (w_i), $(0 \le i \le n-1)$, where w_i represents the number of channels required for cell i.

(iii) A frequency separation matrix **C** = (c_{ij}), where c_{ij} represents the frequency separation requirement between a call in cell i and a call in cell j $(0 \le i, j \le n-1)$.

(iv) A frequency assignment matrix $\Phi = (\phi_{ij})$, where ϕ_{ij} represents the frequency assigned to call j in cell i $(0 \le i \le n-1, 0 \le j \le w_i - 1)$. The assigned frequencies ϕ_{ij}'s are assumed to be evenly spaced, and can be represented by integers ≥ 0.

(v) A set of frequency separation constraints specified by the frequency separation matrix:
$|\phi_{ik} - \phi_{jl}| \ge c_{ij}$ for all i, j, k, l (except when both $i = j$ and $k = l$).

Then, a triple **P** = (**X**, **W**, **C**) characterizes a CAP. A frequency assignment Φ for **P** is said to be *admissible* if ϕ_{ij}'s satisfy the component (v) above $\forall\ 0 \le i \le n-1, 0 \le j \le w_i - 1$. The span **S**($\Phi$) of a frequency assignment Φ is the maximum frequency assigned to the system, i.e. $S(\Phi) = \max_{i,j} (\phi_{ij})$.

Thus, the objective of the CAP is to find an admissible frequency assignment with the minimum span $\mathbf{S_0(P)}$, where $\mathbf{S_0(P)} = \min\{\mathbf{S}(\Phi) \mid \Phi \text{ is admissible for } \mathbf{P}\}$.

However, in order to judge the quality of the resulted solutions, it is essential to know the lower bound on the minimum bandwidth requirement for a given problem.

2.3 Preliminaries

Definition 1. *The cellular graph is a graph where each cell of the cellular network is represented by a node and two nodes have an edge between them if the corresponding cells are adjacent to each other (i.e. when the two cell boundaries share a common segment).*

Note: The cellular graph represents the topology of the cellular structure, so, since the topology of the cellular structure is hexagonal, so the cellular graph too is hexagonal.

Definition 2. *The cellular network is said to belong to a k-band buffering system if it is assumed that the interference does not extend beyond k cells away from the call originating cell.*

Note: The minimum frequency spacing to be maintained between channels assigned to two cells will be denoted by S_i, if the two cells are distance i apart, where $0 \le i \le k$. Since we are assuming here only single demand per cell, so S_0 does not play any role.

Definition 3. *Suppose* $G = (V, E)$ *is a cellular graph. A subgraph* $G_0 = (V_0, E_0)$ *of the graph* $G = (V, E)$ *is defined to be a distance-k clique, if every pair of nodes in* G_0 *is connected in* G *by a path of length at most k.*

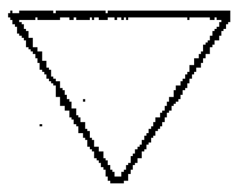
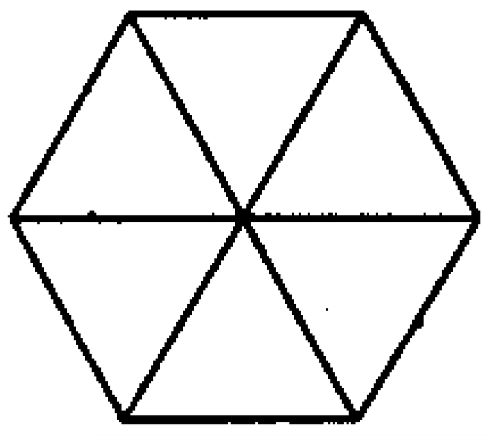
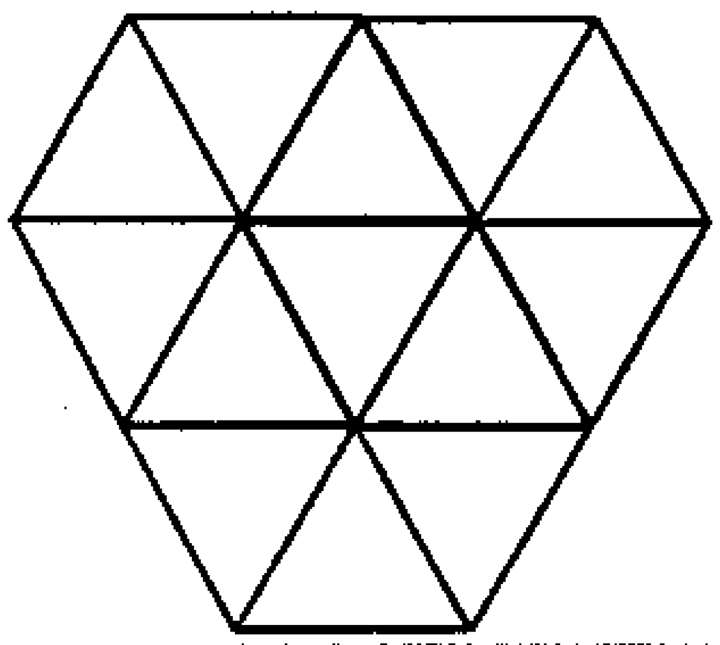
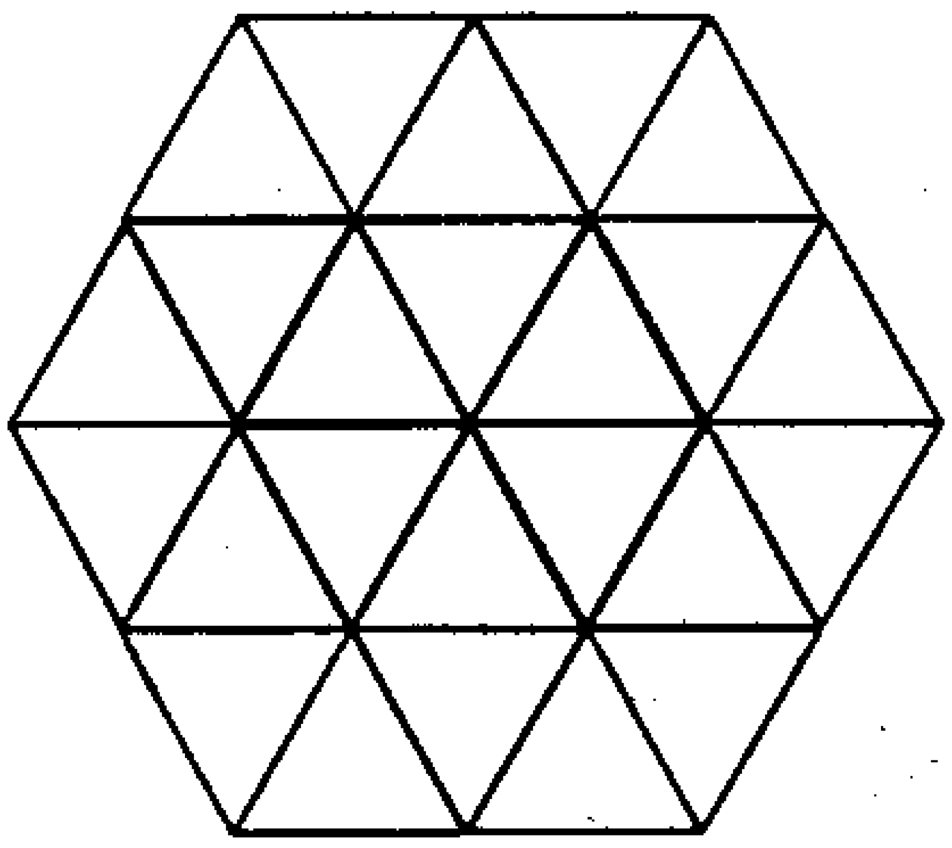
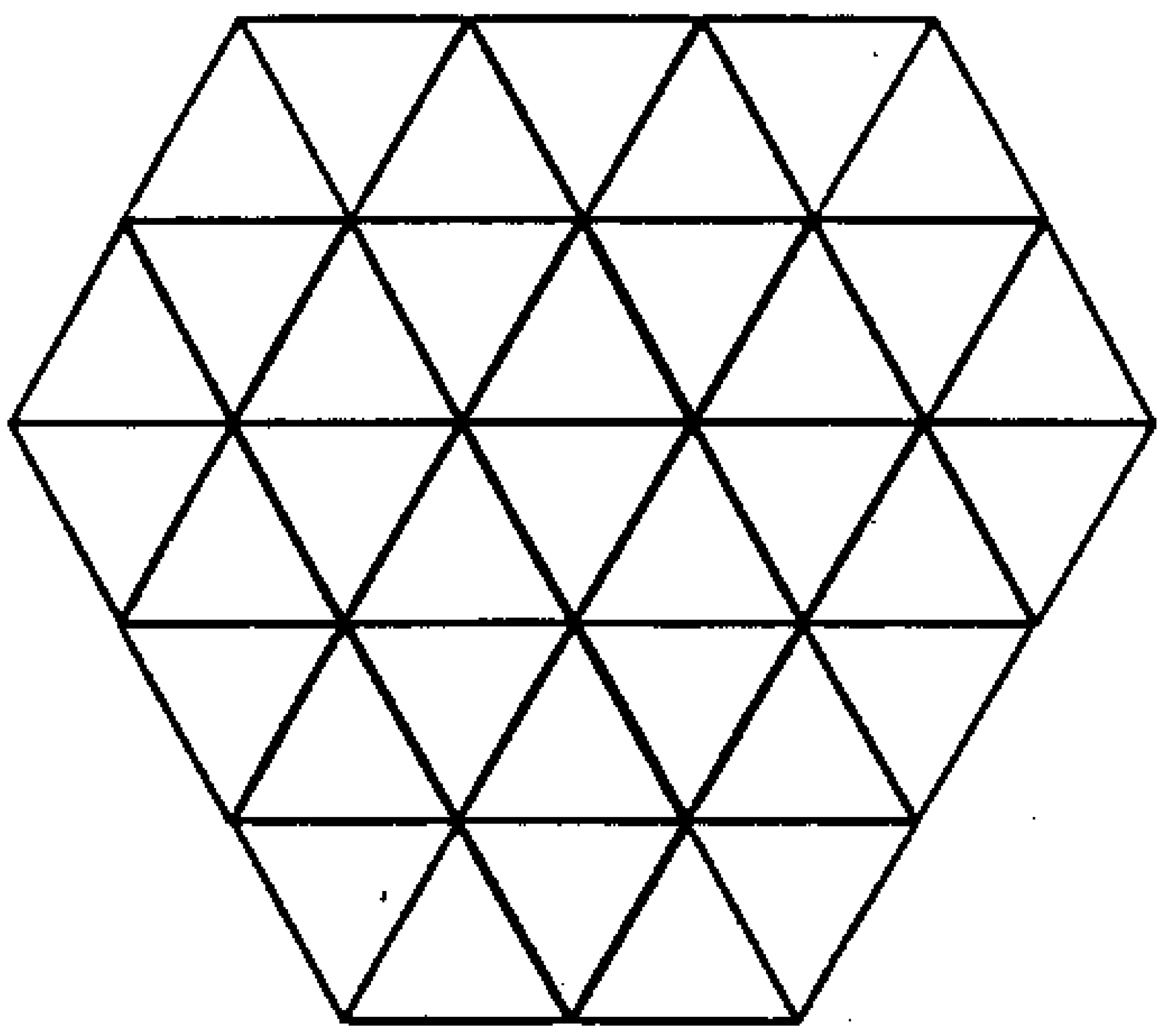
Definition 4. *Suppose* $G = (V, E)$ *is a cellular graph. A subgraph* $G_0 = (V_0, E_0)$ *of the graph* $G = (V, E)$ *is defined to be a complete distance-k clique, if every pair of nodes in* G_0 *is connected in* G *by a path of length at most k and* G_0 *is a maximal subgraph with this property.*

Definition 5. *Suppose* $G = (V, E)$ *is a cellular graph and* $G_0 = (V_0, E_0)$ *is a distance-k clique of the graph* $G = (V, E)$*, A node* $v \in V_0$ *is said to be a central node of the distance-k clique* G_0*, if it's maximum distance from any other node in* G_0 *is minimum, or in other words, it's eccentricity is minimum, i.e. in our case* $\lceil k/2 \rceil$.

Definition 6. *Suppose* $G = (V, E)$ *is a cellular graph and* $G_0 = (V_0, E_0)$ *is a distance-k clique of the graph* $G = (V, E)$*, A node* $v \in V_0$ *is said to be a peripheral node of the distance-k clique* G_0*, if it's maximum distance from any other node in* G_0 *is maximum, or in other words, it's eccentricity is maximum, i.e. in our case* k.

Definition 7. *Suppose* $G = (V, E)$ *is a cellular graph and* $G_0 = (V_0, E_0)$ *is a distance-k clique of the graph* $G = (V, E)$*. A seed subgraph* G^s *is that subgraph of the distance-k clique* G_0 *whose vertices are the central nodes of* G_0*.*

Note: Consider the distance-k cliques in a hexagonal cellular graph, for different k values, as shown in table 1. We can see that there are two types of seed subgraphs, one for even k, and another for odd k, we will call them G^s_{even} and G^s_{odd}, as we can see G^s_{even} has one node and G^s_{odd} has three nodes.

k	No of nodes in the distance-k clique in a cellular graph	Distance-k clique in a hexagonal cellular graph	No of nodes having max distance k from any other node	No of nodes having max distance k-1 from any other node	No of nodes having max distance k-2 from any other node	No of nodes having max distance k-3 from any other node	No of central nodes
0	1	•	1	0	0	0	0
1	3		3	0	0	0	0
2	7		6	1	0	0	1
3	12		9	3	0	0	3
4	19		12	6	1	0	1
5	27		15	9	3	0	3

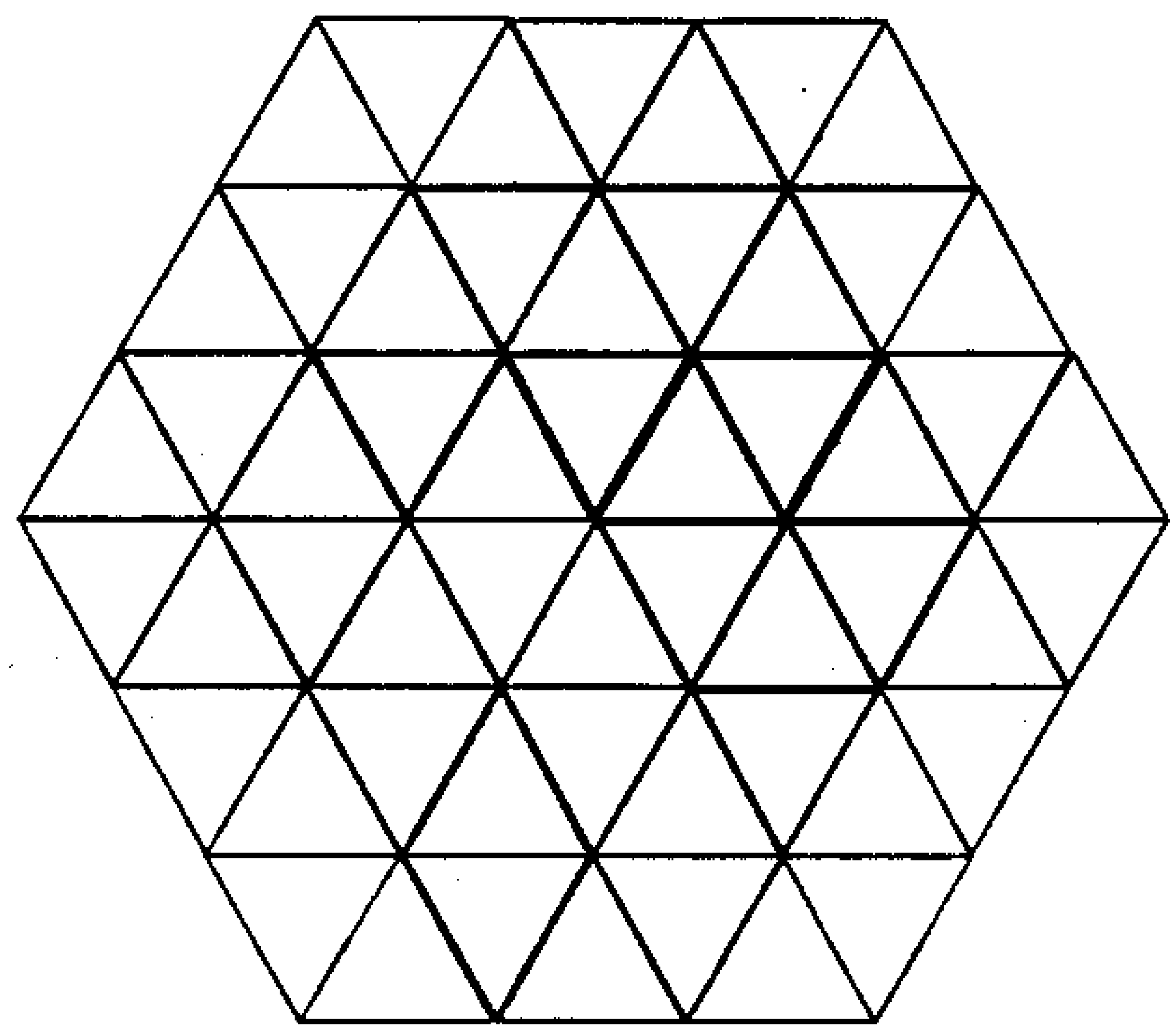
k	No of nodes in the distance-k clique in a cellular graph	distance-k clique in a hexagonal cellular graph	No of nodes having max distance k from any other node	No of nodes having max distance k-1 from any other node	No of nodes having max distance k-2 from any other node	No of nodes having max distance k-3 from any other node	No of central nodes
6	37		18	12	6	1	1

Table 1

Definition 8. *We define a growth operation on a distance-(k-2) clique as the step of adding to it all nodes in the cellular graph* **G**, *which are at distance one from it's peripheral nodes.*

Note: The distance-k clique can be derived from the distance-(k-2) clique in one growth operation.

2.4 Results

We assume that the cellular graph is hexagonal in nature with a 3-band buffering restriction. Let us now consider the complete distance-3 clique as shown in table 1. Every pair of nodes in this graph is within distance three from each other, and therefore, they are going to interfere with each other in their frequency assignment. So, no frequency reuse is possible within this subgraph, hence the bandwidth requirement of this subgraph will give a lower bound on the bandwidth requirement of the whole cellular network.

We now have the following results on the lower bound on frequency requirement of the distance-3 clique. A lower bound is achievable if there exists an admissible frequency assignment of the distance-3 clique with span equal to the lower bound. Suppose frequencies in the interval $[0; p]$ are sufficient for assigning single channel to each of the nodes of the distance-3 clique. Therefore, our objective is to find minimum p such that there exists an admissible frequency assignment of the distance-3 clique with span equal to p.

We now have the following results on the lower bounds on bandwidth requirement for frequency assignment of distance-3 clique.

Theorem 1: *The minimum bandwidth required for assigning channels to the distance-3 clique, each cell of which has homogenous demand of single channel and 3-band buffering restriction with frequency separation* S_1, S_2, S_3, *is,* $11S_3$ *and this bound is achievable when* $S_2 \leq S_1 \leq 3S_3$ *and* $S_2 = S_3$.

Proof: Consider the twelve-node distance-3 clique as shown in figure 1. Since any two nodes of the subgraph shown are at most distance three apart from each other, the frequencies assigned on any two nodes of this subgraph must be separated by at least S_3.

Consider that a frequency channel number α has been assigned to a node, and frequency channel numberβ, has been assigned to another node of the subgraph, so the minimum gap that has to be maintained between α and β, is S_3.

Since there are twelve nodes, so considering the lowest frequency being assigned to a node of the subgraph to be 0, at least $11S_3$ frequencies will be required to satisfy the demand for this subgraph.

Here, and in all future discussions, a frequency $(iS_1 + jS_2 + kS_3)$ assigned to a node will be denoted by a 3-tuple (i, j, k).

When $S_2 \leq S_1 \leq 3S_3$, and $S_2 = S_3$, an admissible frequency assignment of the distance-3 clique using $11S_3$ number of channels has been shown if figure 1. However, this assignment is conflict free only when $S_2 \leq S_1 \leq 3S_3$, and $S_2 = S_3$. Hence the proof.

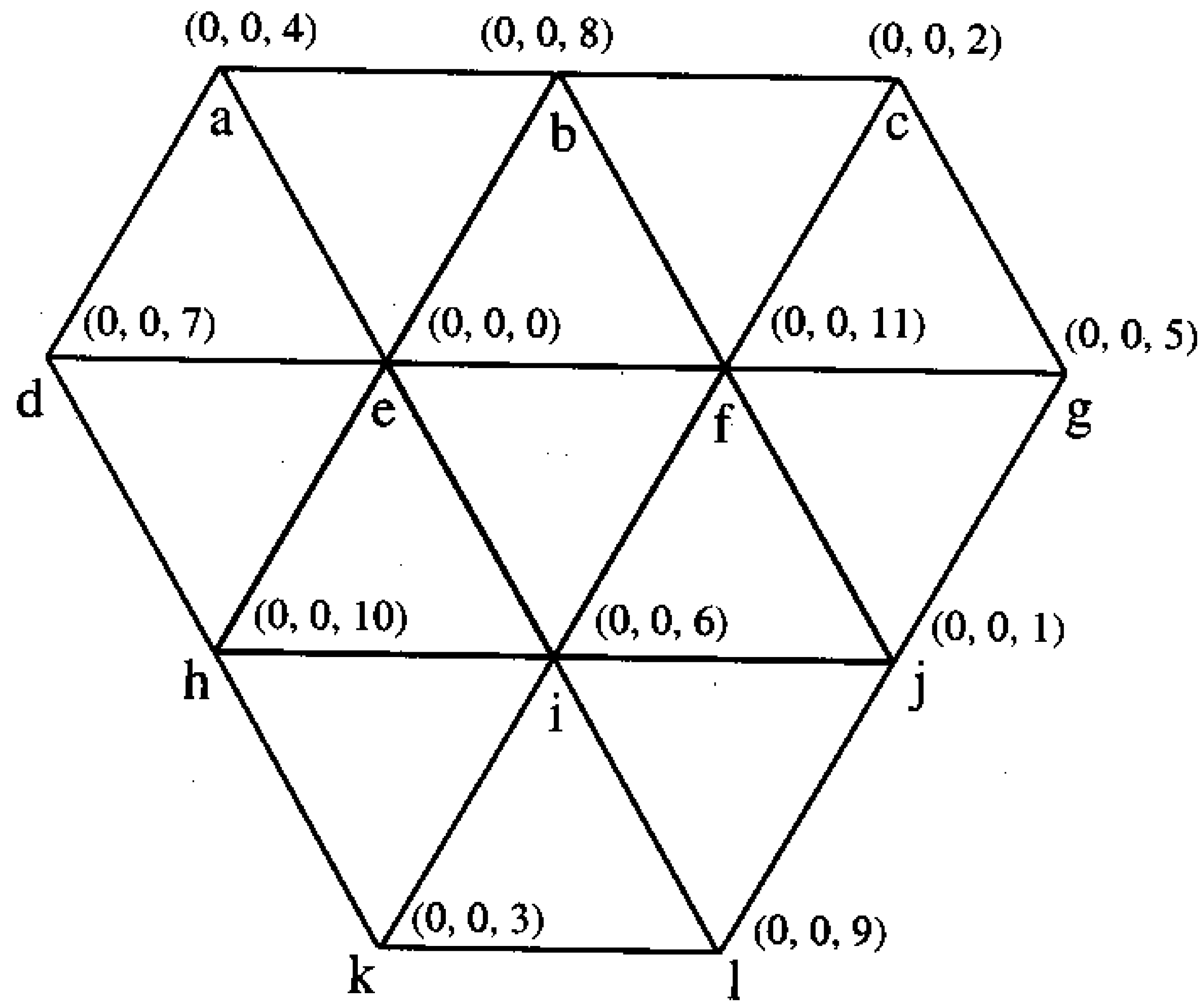


Figure 2.1 A conflict-free frequency assignment of distance-3 clique when $S_2 \leq S_1 \leq 3S_3$, and $S_2 = S_3$

Theorem 2: *The minimum bandwidth required for assigning channels to the distance-3 clique having homogenous demand of single channel per cell and 3-band buffering restriction with frequency separation S_1, S_2, S_3, is $(4S_2 + 7S_3)$ and this bound is achievable when $S_2 \leq S_1 \leq 3S_3$ and $S_3 \leq S_2 \leq 2S_3$.*

Proof: Frequencies assigned to any two nodes of the distance-3 clique must be separated by at least S_3. In addition there is no node which is distance-3 apart from the central nodes *e*, *f*, and *i*.

Hence, if frequency $\alpha \in [0; p]$ be assigned to any node of the subgraph. Then to satisfy the interference criteria, no frequency channel within the open interval $(\alpha - S_3; \alpha)$ and $(\alpha; \alpha + S_3)$ can be assigned to any node of the subgraph. That is, there would be an unusable gap of $(\alpha - S_3; \alpha)$ before α, and also another such gap of $(\alpha; \alpha + S_3)$ after α, on the frequency spectrum line, where gap $(x ; y)$ implies that the integers within the open interval $(x ; y)$ cannot be assigned to any node of the subgraph. We refer $(y - x)$ as the length of gap $(x ; y)$. So, for $\alpha = 0$ (or, p), one of these gaps of unusable frequencies, e.g., $(\alpha - S_3; \alpha)$ (or, $(\alpha; \alpha + S_3)$) will be beyond the interval $[0; p]$.

Now, consider the three central nodes in figure 1, i.e. nodes *e*, *f* and *i*. These three nodes are at maximum distance two apart from any other node in the subgraph, so, the frequency gaps to be maintained between the frequency assigned to these nodes and any other node has to be at least S_2, instead of S_3.

Suppose, during the assignment, a frequency channel $\chi \in [0; p]$ be assigned to any of these three central nodes. Then, to satisfy the interference criteria, no frequency channel within the open interval $(\chi - S_2; \chi)$ and $(\chi; \chi + S_2)$ can be assigned to any node of the subgraph. That is, there must be unusable gaps $(\chi - S2; \chi)$ and $(\chi; \chi + S_2)$ before and after χ, respectively. If $\chi = 0$ (or p), one of these gaps of unusable frequencies, e.g., $(\chi - S_2; \chi)$ (or, $(\chi; \chi + S_2)$) falls beyond the interval $[0; p]$.Since $S_2 \geq S_3$, it follows from above that we can have more number of usable frequency channels if the minimum and maximum frequencies are assigned to the central nodes, rather than to any other nodes.

Now, it is possible that one of the central nodes be assigned the minimum frequency channel, call it frequency channel χ_1, and another one the maximum frequency channel, call it frequency channel χ_3, but the third central node will have to be assigned a frequency channel which lies inside the open interval $(0; p)$, i.e. strictly between 0 and p but neither 0 nor p, call it frequency channel χ_2.

So, in view of this, there will be one unusable gap, $(\chi_1; \chi_1 + S_2)$, of length S_2 within $[0; p]$ due to the lowest frequency χ_1 being assigned to a central node, another unusable gap, $(\chi_3 - S_2; \chi_3)$, of length S_2 within $[0; p]$ due to the maximum frequency χ_3 being assigned to a central node, and two more unusable gaps $(\chi_2 - S_2; \chi_2)$, and $(\chi_2; \chi_2 + S_2)$ each of length S_2 due to the frequency χ_2 being assigned to a central node. That makes 4 unusable gaps each of length S_2.

So, in the earlier lower bound of $11S_3$ we must add $4(S_2 - S_3)$, because there are 4 nodes that contribute to unusable gaps each of length S_2 instead of S_3. So, $(4S_2 + 7S_3)$ is a lower bound for assigning channels to the distance-3 clique of a hexagonal cellular network, having homogenous demand of single channel and 3-band buffering.

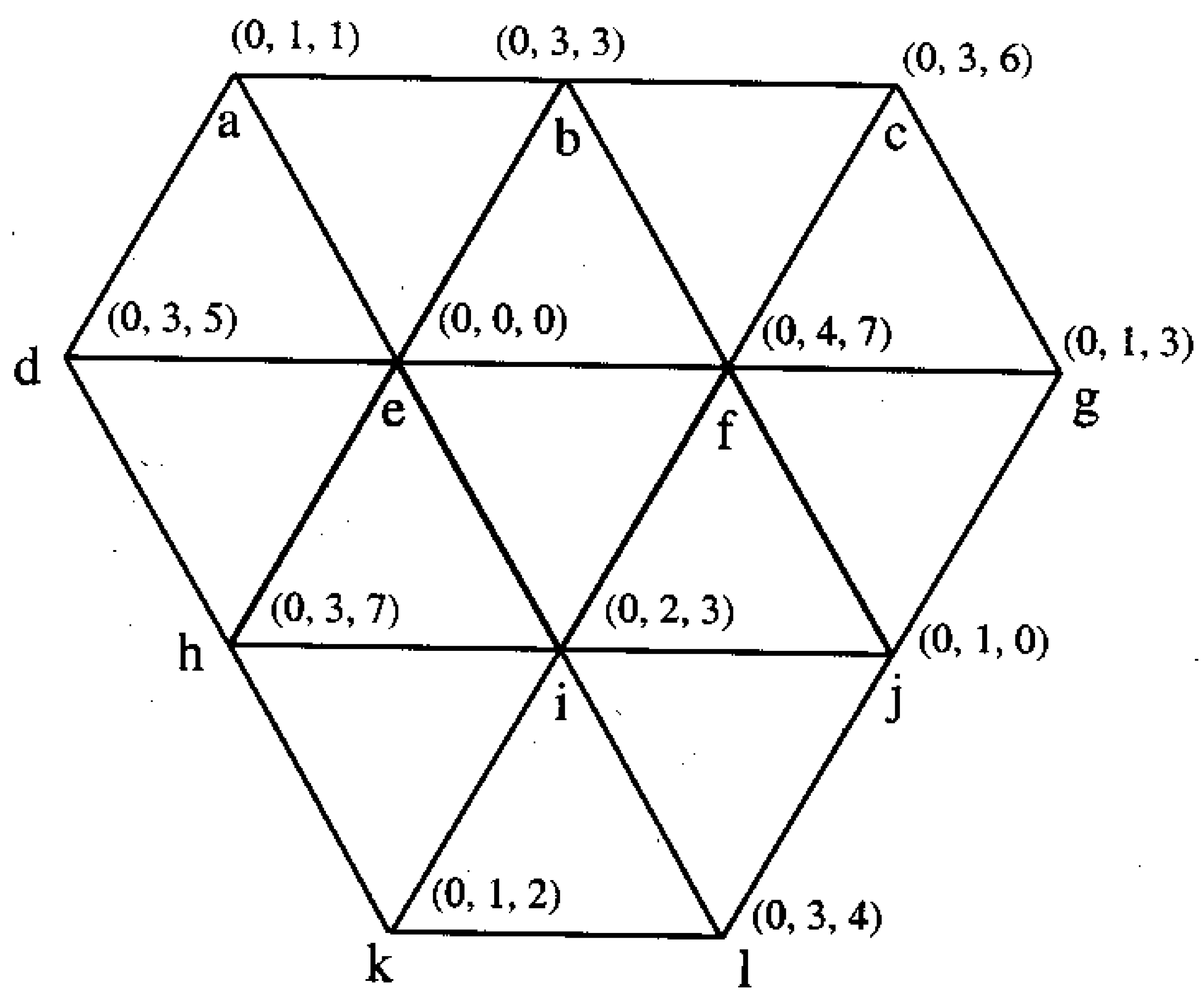


Figure 2.2 A conflict-free assignment of the distance-3 clique when $S_2 \leq S_1 \leq 3S_3$ and $S_3 \leq S_2 \leq 2S_3$

When $S_2 \leq S_1 \leq 3S_3$ *and* $S_3 \leq S_2 \leq 2S_3$, an admissible frequency assignment of the distance-3 clique using $4S_2 + 7S_3$ number of channels has been shown in figure 2. However, this assignment is conflict free only when $S_2 \leq S_1 \leq 3S_3$ *and* $S_3 \leq S_2 \leq 2S_3$. Hence the proof.

Theorem 3: *The minimum bandwidth required for assigning channels to the distance-3 clique having homogenous demand of single channel per cell and 3-band buffering restriction with frequency separations* S_1, S_2, S_3 *is* $(2S_1 + 4S_2)$ *when* $S_2 \leq S_1 \leq 2S_2$, *and* $(3S_1 + 2S_2)$ *when* $S_1 \geq 2S_2$.

Proof: Since any two nodes of the distance-3 clique is either distance-1, or distance-2 or distance-3 apart from each other, so accordingly the frequencies assigned on any two nodes of this subgraph must be separated by at least S_1, S_2 or S_3 apart.

Now, let us consider a complete distance-2 clique induced by the node set $\{a, b, d, e, f, h, i\}$ as shown in figure 3. Any pair of nodes is within distance two of each other, and node e is the central node.

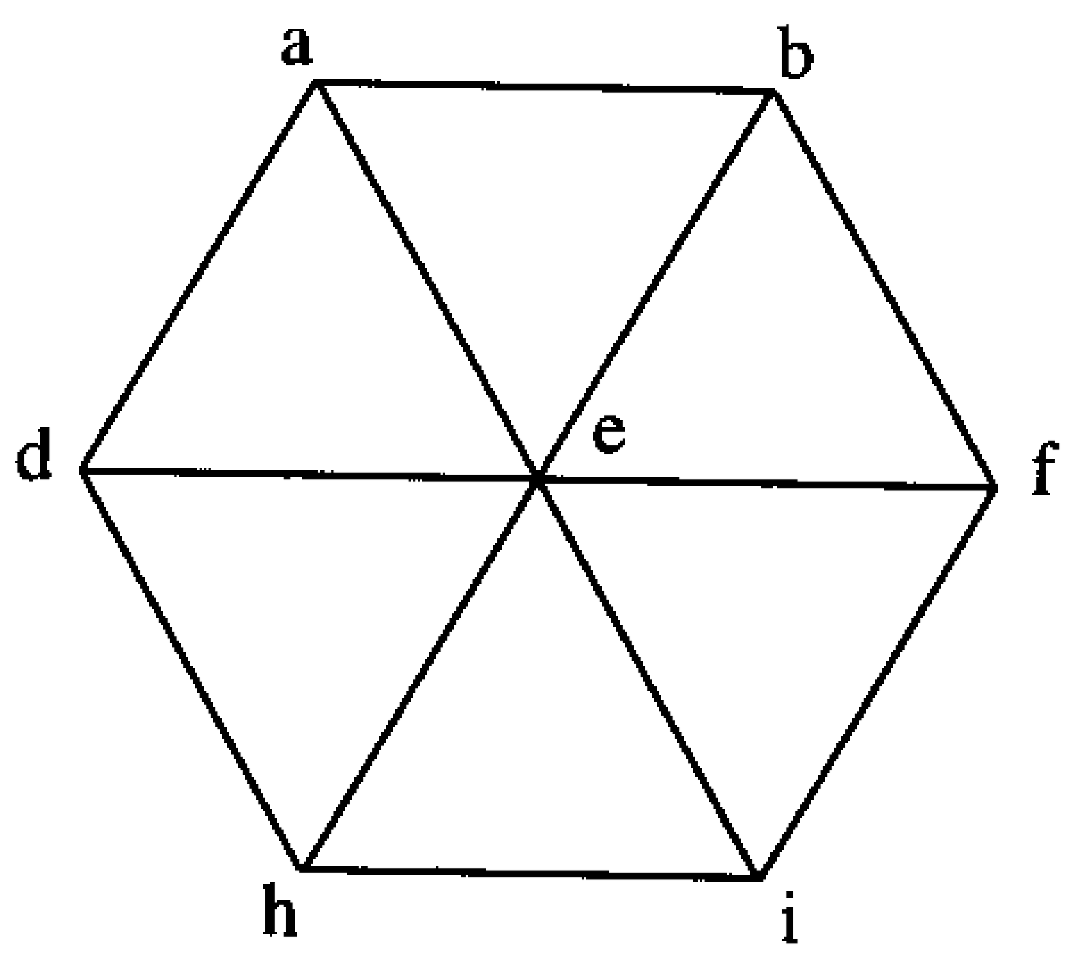


Figure 2.3 complete distance-2 clique induced by the node set $\{a, b, d, e, f, h, i\}$

The following result on minimum bandwidth requirement for assigning channels to this complete distance-2 clique has been reported in [1].

Result: *The minimum bandwidth required for assigning channels to the complete distance-2 clique of a hexagonal cellular network having homogenous demand of single channel and 2-band buffering restriction with frequency separation* S_1, S_2 *is* $(S1 + 5S2)$, *when* $S_2 \leq S_1 \leq 2S_2$, *and* $(2S_1 + 3S_2)$ *when* $S_1 \geq S_2$.

However, this result is only achievable when either the minimum or the maximum frequency can be assigned to the central node. Note that in the distance-3 clique there are three complete distance-3 cliques induced by the node sets $\{a, b, d, e, f, h, i\}$, $\{b, c, e, f, g, i, j\}$ and $\{e, f, h, i, j, k, l\}$ with central nodes e, f and i respectively. In addition, the central nodes e, f and i are distance-1 apart from each other. Therefore, at least one central node must be assigned an intermediate frequency, i.e. a frequency which is neither the minimum nor the maximum. So following the same approach as reported in [1], the minimum bandwidth requirement is given by $(S_1 + 5S_2) + (S_1 - S_2) = (2S_1 + 4S_2)$, when $S_2 \leq S_1 \leq 2S_2$ and $(2S_1 + 3S_2) + (S_1 - S_2) = (3S_1 + 2S_2)$, when $S_1 \geq 2S_2$. Hence the proof

Lemma 1: *A lower bound on the number of channels required to assign channels to the complete distance-2 clique of a hexagonal cellular network having homogenous demand of single channel and 3-band buffering restriction with frequency separation S_1, S_2, S_3 is $max((4S_2 + 7S_3), (2S_1 + 4S_2))$, when $S_2 \leq S_1 \leq 2S_2$ and $max((4S_2 + 7S_3), (3S_1 + 2S_2))$, when $S_1 \geq 2S_2$.*

Proof: From theorem 2 gives a lower bound on the distance-3 clique, which is $(4S_2 + 7S_3)$. and theorem 3 gives us a lower bound depending on the relative values of S_1 and S_2, i.e. either $S_2 \leq S_1 \leq 2S_2$ or $S_1 \geq 2S_2$.So, $\max((4S_2 + 7S_3), (2S_1 + 4S_2))$ is a lower bound when $S_2 \leq S_1 \leq 2S_2$, and $\max((4S_2 + 7S_3), (3S_1 + 2S_2))$ is a lower bound when $S_1 \geq 2S_2$. Hence the proof.

In view of the above results, a generalized lower bound, for any non-negative k, both even as well as odd. has been proposed below.

Lemma 2: *The number of central nodes in a distance-k clique in a hexagonal cellular graph is one, if k is even, and three, if k is odd.*

Proof: By starting with the seed subgraph G^s_{even}, for even k, or G^s_{odd} for odd k, we can derive the distance-k clique from the corresponding seed subgraph in $\lfloor k/2 \rfloor$ growth operations. The nodes of the seed subgraphs will have minimum eccentricity in the distance-k clique, therefore, their nodes will be the central nodes of the distance-k clique. Since there are only two possible seed subgraphs, one corresponding to even k and one corresponding to odd k, so the number of central nodes will be one if k is even and three if k is odd. Hence the proof.

Lemma 3: *In the ω^{th} growth operation on the seed subgraph, the number of nodes being added to the graph will be 6ω* for even k, and $(6\omega + 3)$ for odd k, where $\omega \in [1, 2, 3, \ldots, k/2]$, for even k, and $\omega \in [1, 2, \ldots, \lfloor k/2 \rfloor]$, for odd k.

Proof: The number of nodes in a distance-k clique in a cellular graph is given by, [2]

(i) $3/4\,(k+1)^2$, *if k is odd, i.e. k = 1, 3, 5 . .*

(ii) $3/4\,(k+1)^2 + 1/4$, *if k is even, i.e. k = 0, 2, 4 . .*

So, we can see that the distance-k clique in a cellular graph will have 3k number of nodes more than the distance-(k-2) clique in a cellular graph, for both cases i.e. k is even or k is odd. Now, if k is even, i.e. $k = 2\omega$, then, $3k = 3(2\omega) = 6\omega$, and if k is odd, i.e. $k = 2\omega + 1$, then $3k = 3(2\omega + 1) = 6\omega + 3$.Hence the proof.

Lemma 4: *A lower bound on the number of channels required for assigning channels to the N-node, distance-k clique in a hexagonal cellular network having homogeneous demand of single channel and k-band buffering is $(N\text{-}1)S_k$. Where $N = 3/4\ (k+1)^2$, if k is odd, and $N= 3/4\ (k+1)^2 + 1/4$, if k is even.*

Proof : Since any two nodes of the subgraph shown are at most distance k apart from each other, the frequencies assigned on any two nodes of this subgraph must be separated by at least S_k.

Consider that a frequency channel number α has been assigned to a node, and frequency channel number β, has been assigned to another node of the subgraph, so the minimum gap that has to be maintained between α and β, is S_k.

Since there are N nodes, so considering the lowest frequency being assigned to any node of the subgraph to be 0, at least $(N\text{-}1)S_k$ frequencies will be required to satisfy the demand for this subgraph. Hence the proof.

Theorem 4: *A lower bound on the bandwidth required for assigning channels to the N-node, distance-k clique in a hexagonal cellular network, having homogenous demand of single channel and k-band buffering for even k is,*

$$(N\text{-}1)S_k + (S_\Psi - S_k) + 11(S_{\Omega 1} - S_k) + \sum_{\omega=2}^{k/2} 12\omega\,(S_{\Omega\omega} - S_k), \text{ where } N = 3/4\ (k+1)^2 + 1/4$$

$$\Psi = \lceil k/2 \rceil$$

$$\Omega 1 = (\lceil k/2 \rceil + 1)$$

$$\Omega\omega = (\lceil k/2 \rceil + \omega).$$

And for odd k is,

$$(N\text{-}1)S_k + 4(S_\Psi - S_k) + \sum_{\omega=1}^{\lfloor k/2 \rfloor} 2\,(6\omega + 3)\,(S_{\Omega\omega} - S_k), \text{ where } N = 3/4\ (k+1)^2$$

$$\Psi = \lceil k/2 \rceil$$

$$\Omega 1 = (\lceil k/2 \rceil + 1)$$

$$\Omega\omega = (\lceil k/2 \rceil + \omega).$$

Proof: In lemma 4 we have shown that $(N\text{-}1)S_k$ is a lower bound, now we shall make it tighter.

We know that the number of nodes in a distance-k clique subgraph in a hexagonal cellular network, is

$N = 3/4\ (k+1)^2$, if k is odd, i.e. $k = 1, 3, 5$ . ., and
$N = 3/4\ (k+1)^2 + 1/4$, if k is even, i.e. $k = 0, 2, 4$ . .

Consider the N-node, distance-k clique in a hexagonal cellular network. Suppose, using the frequency channels within the closed interval *[0; p]*, it is possible to assign frequency channels to each of the nodes of the subgraph satisfying all interference constraints. Therefore, our objective is to find *minimum p*.

Since, any two nodes of the subgraph are within distance k from each other. Therefore, any two frequencies assigned to two nodes of the subgraph must be separated by at least S_k, suppose, the frequency channel $\alpha \in [0; p]$ be assigned to any node of the subgraph. Then to satisfy the interference criteria, no frequency channel within the open interval $(\alpha - S_k; \alpha)$ and $(\alpha; \alpha + S_k)$ can be assigned to any node of the subgraph. That is, there would be an unusable gap of $(\alpha - S_k; \alpha)$ beforeα, and also another such gap of $(\alpha; \alpha + S_k)$ after α, on the frequency spectrum line, where gap $(x ; y)$ implies that the integers within the open interval $(x ; y)$ cannot be assigned to any node of the subgraph. We refer $(y - x)$ as the length of gap $(x; y)$. So, for $\alpha = 0$ (or, p), one of these gaps of unusable frequencies, e.g., $(\alpha - S_k; \alpha)$ (or, $(\alpha; \alpha + S_k)$) will be beyond the interval $[0; p]$.

Now, consider the central nodes in the subgraph, according to lemma 2, if k is even, then there is only one central node, and if k is odd, then there are three central nodes,. These nodes are at maximum distance $\lceil k/2 \rceil$ apart from any other node in the subgraph, i.e., ceiling of k/2, so, the frequency gaps to be maintained between the frequency assigned to these nodes and any other node has to be at least S_Ψ, instead of S_k, where $\Psi = \lceil k/2 \rceil$.Suppose, during the assignment, a frequency channel $\chi \in [0; p]$ be assigned to any of these central nodes. Then, to satisfy the interference criteria, no frequency channel within the open interval $(\chi - S_\Psi; \chi)$ and $(\chi; \chi + S_\Psi)$ can be assigned to any node of the subgraph. That is, there must be unusable gaps $(\chi - S_\Psi; \chi)$ and $(\chi; \chi + S_\Psi)$ before and after χ, respectively. If $\chi = 0$ (or p), one of these gaps of unusable frequencies, e.g., $(\chi - S_\Psi; \chi)$ (or, $(\chi; \chi + S_\Psi)$) falls beyond the interval $[0; p]$.Since $S_n \geq S_m, \forall n \leq m$, it follows from above that we will have more number of usable frequency channels if the minimum and maximum frequencies are assigned to the central nodes, rather than to any other nodes.

Case 1: k is even.

Now, consider the case when k = even, then, according to lemma 2, the seed subgraph will have only one node, i.e., there will be only one central node, and, we can assign to it, either the lowest or the highest frequency channel. In view of this, there will be only one unusable gap of length S_Ψ ,within $[0; p]$, either $(\chi_1; \chi_1 + S_\Psi)$, due to the lowest frequency χ_1 being assigned to the central node, or unusable gap, $(\chi_N - S_\Psi; \chi_N)$, due to the maximum frequency χ_N being assigned to the central node. So, in view of this, to the earlier lower bound, $(N - 1)S_k$, where $N = 3/4\ (k+1)^2 + 1/4$, (since k is even), we must add $(S_\Psi - S_k)$.

Now, we will construct the distance-k clique in a cellular graph, layer by layer, by adding $\lfloor k/2 \rfloor$ layers over the seed subgraph, where $\lfloor k/2 \rfloor$ denotes the floor of k/2.According to lemma 2, the ω^{th} layer will contribute $3(2\omega)$ nodes, i.e. 6ω nodes when k is even, where $\omega = 1, 2, 3, \ldots \lfloor k/2 \rfloor$.

Now, we grow the first layer over the seed subgraph, so the number of nodes which are added is 6ω, where $\omega = 1$, so the number of nodes which are added in the first layer is 6. Now, of the lowest and the highest frequency channels, only one was assigned to the central node, the other one is available for assignment, so since $S_n \geq S_m, \forall\ n \leq m$, we will have more number of usable channels if we assign the other of these, to one of the nodes belonging to the first layer. In view of this, there will be one unusable gap of length $S_{\Omega 1}$ within *[0; p]*, due to the highest (or lowest) frequency channel being assigned to one of these nodes, where $\Omega 1 = (\lceil k/2 \rceil + 1)$. To the other 5 nodes whatever frequency α we assign, the unusable gap in the frequency spectrum due to each node each node will be $2S_{\Omega 1}$, since this frequency α will lie strictly inside *[0; p]*, so both the gaps $(\alpha - S_{\Omega 1}; \alpha)$ and $(\alpha; \alpha + S_{\Omega 1})$ will also lie strictly inside *[0; p]*, for each of these 5 nodes. So, to the lower bound of lemma 1, we must add $11(S_{\Omega 1} - S_k)$.

Now, when the second layer is added, the graph obtained will have 6ω more nodes, where $\omega = 2$, so 12 more nodes will be added due to the second layer. These nodes can only be assigned frequency channels which lie strictly inside *[0; p]*, so whatever frequency α we assign to any one of these nodes, the unusable gap in the frequency spectrum due to each node will be $2S_{\Omega 2}$, one gap of size $S_{\Omega 2}$ before α, i.e. $(\alpha - S_{\Omega 2}; \alpha)$, and another of same size $(\alpha; \alpha + S_{\Omega 2})$, where $\Omega 2 = (\lceil k/2 \rceil + 2)$.So, to the lower bound of lemma 3, we must add $24(S_{\Omega 2} - S_k)$.

Similarly, the ω^{th} layer will contribute 6ω nodes, and each node will contribute two unusable gaps each of size, $S_{\Omega\omega}$, where $\Omega\omega = (\lceil k/2 \rceil + \omega)$.So, to the lower bound of lemma we must add $12\omega\ (S_{\Omega\omega} - S_k)$. So, a lower bound on the number of channels required for assigning channels to the N- node, distance-k clique in a hexagonal cellular network, having homogenous demand of single channel and k-band buffering, when k is even, is

$$(N-1)S_k + (S_\Psi - S_k) + 11(S_{\Omega 1} - S_k) + \sum_{\omega=2}^{\lfloor k/2 \rfloor} 12\omega\ (S_{\Omega\omega} - S_k), \text{ where } N = 3/4\ (k+1)^2 + 1/4$$

$$\Psi = \lceil k/2 \rceil$$
$$\Omega 1 = (\lceil k/2 \rceil + 1)$$
$$\Omega\omega = (\lceil k/2 \rceil + \omega).$$

Case 2: k is odd

Now, according to lemma 1, for odd k, the seed subgraph has three nodes, i.e. there are three central nodes in the distance-k clique in the hexagonal cellular network,. These nodes are at maximum distance $\lceil k/2 \rceil$ apart from any other node in the subgraph, so, the frequency gaps that have to be maintained between the frequencies assigned to these nodes and any other node has to be at least S_Ψ, instead of S_k, where $\Psi = \lceil k/2 \rceil$.As mentioned earlier, we will have more number of usable frequency channels if the minimum and maximum frequencies are assigned to the central nodes, rather than to any other nodes.

Now, we can always assign the lowest and highest frequency channels to any two of these central nodes, but the third node has to be assigned a frequency channel which lies strictly inside *[0;p]* in view of this, there will be four unusable gaps each of length S_Ψ ,within *[0; p]*, they are $(\chi_1; \chi_1 + S_\Psi)$, due to the lowest frequency χ_1 being assigned to one central node, $(\chi_N - S_\Psi; \chi_N)$, due to the maximum frequency χ_N being assigned to another central node, and two more $(\chi - S_\Psi; \chi)$ and $(\chi; \chi + S_\Psi)$, due to frequency χ being assigned to the third central node because χ lies strictly inside *[0; p]*.So, in view of this, to the earlier lower bound, $(N-1)S_k$, where $N = 3/4\ (k+1)^2$, (since k is odd), we must add $4(S_\Psi - S_k)$.

Now, we will construct the distance-k clique in a cellular graph, layer by layer, by adding $\lfloor k/2 \rfloor$ layers over the seed subgraph, where $\lfloor k/2 \rfloor$ denotes the floor of k/2.According to lemma 2, the ω^{th} layer will contribute $3(2\omega+1)$ nodes, when k is odd, where $\omega = 1, 2, 3, \ldots \lfloor k/2 \rfloor$,.

Now, we grow the first layer over the seed subgraph, the graph obtained will have $6\omega+3$ more nodes, where $\omega = 1$, so the number of nodes which are added due to the first layer is 9.These nodes can only be assigned frequency channels which lie strictly inside *[0; p]*, so whatever frequency α we assign to any one of these nodes, the unusable gap in the frequency spectrum due to each node will be $2S_{\Omega 1}$, one gap of size $S_{\Omega 1}$ before α, i.e. $(\alpha - S_{\Omega 1}; \alpha)$, and another of same size $(\alpha;\alpha + S_{\Omega 1})$, where $\Omega 1 = (\lceil k/2 \rceil + 1)$.So, to the lower bound of lemma 3, we must add $18(S_{\Omega 1} - S_k)$.

Similarly, the ω^{th} layer will contribute $6\omega+3$ nodes, and each node will contribute two unusable gaps each of size, $S_{\Omega\omega}$, where $\Omega\omega = (\lceil k/2 \rceil + \omega)$.So, to the lower bound of lemma we must add $2(6\omega + 3)(S_{\Omega\omega} - S_k)$.

So, a lower bound on the number of channels required for assigning channels to the N-node, distance-k clique in a hexagonal cellular network, having homogenous demand of single channel and k-band buffering, when k is odd, is

$$(N-1)S_k + 4(S_\Psi - S_k) + \sum_{\omega=1}^{\lfloor k/2 \rfloor} 2(6\omega + 3)(S_{\Omega\omega} - S_k), \text{ where } N = 3/4\,(k+1)^2$$

$$\Psi = \lceil k/2 \rceil$$

$$\Omega 1 = (\lceil k/2 \rceil + 1)$$

$$\Omega\omega = (\lceil k/2 \rceil + \omega).$$

Hence the proof.

3. Concluding Remarks

Theorem 4 can be used to find a lower bound on the number of channels required for assigning channels to the N-node, distance-k clique in a hexagonal cellular network, having homogenous demand of single channel and k-band buffering and the bandwidth requirement of this subgraph will give a lower bound on the bandwidth requirement of the whole hexagonal cellular network with k-band buffering restriction. These lower bounds enable us to compare the results obtained by heuristic approaches in a better way. Moreover, these bounds can be used in either terminating heuristic algorithms following a genetic algorithm approach, or starting with a better initial solution in simulated annealing approaches.

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