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GRAY LEVEL IMAGE THINNING – A FUZY SET THEORETIC APPROACH

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ABSTRACT

A new approach to the problem of Thinning gray level images is proposed whereby it is possible to obtain the skeleton of the image using fuzzy theory. The object pixels are assigned membership values indicating the degree of 'belongingness' to the object and suitable 'skeleton membership functions' are proposed. The skeleton is obtained from this version of the object based on the histogram measure and compactness measure. The effectiveness of the algorithms is demonstrated for images of a specimen handwriting, chromosome and biplane.

FUZZY SET THEORETIC APPROACH TO IMAGE PROCESSING

The theory of fuzzy sets as proposed by Zadeh and its subsequent developments provides suitable mathematical tools and techniques in analysing complex systems and decision processes where the pattern indeterminancy is due to inherent varieability and /or vaguness (fuzziness) rather than randomness. Fuzzy set theory has proved itself to be of significant importance in image processing problems, both using decision theoretic and syntactic approaches. On the one hand fuzzy algorithms for different image processing problems are being developed, and on the other hand fuzzy languages, fuzzy grammars and fuzzy automata theories have been developed and applied. For example in the problem of automatic speech recognition, since speech is a pattern of biological origin and carries information regarding the message, the speakers sex, age, health and other factors, it is to a large extent fuzzy in nature. Similarly, during the gray tone image processing, because the gray tone picture possesses ambiguity withn the pixels due to possible multi-valued level of brightness, it is justified to apply the concept and logic of fuzzy set rather than ordinary set theory to image processing problems. Keeping this in mind, an image can be considered as an array of fuzzy singletone, each with a value of membership function denoting the degree of having same brightness level.

The standard approach to image analysis and recognition begins by segmenting the image into regions and computing various properties of and relationships among these regions. However, the regions are not always 'crisply' defined (as mentioned earlier), it is sometimes more appropriate to regard them as fuzzy subsets of the image. For example, when we segment an image by classifying its pixels, we may want to estimate their probabilities of belonging to the classes but not omit ourselves to the final classification step.

Fuzzy equivalent definitions for a variety of properties which exist in gray level image processing like connectedness, surroundedness, convexity, area, perimeter, compactness, extent, diameter, elongatedness, etc. are also given. And all the definitions given for the fuzzy set approach hold good even for the two level processing, i.e., for crisp sets. More important, the fuzzy properties are approximations of the crisp properties if the fuzzy set is approximately crisp, thus they allow an image to be meaningfully described in terms of its parts and relationships without the need to commit oneself to crisp segmentation.

The main drawbacks of fuzzy subsets are (i) they are more cumbersome to represent and (ii) their properties are somewhat more expensive to compute.

But as computer memory and computing power continue to became cheaper, the fuzzy approach to image description is well justified.

Fuzzy Subsets :

A 'fuzzy' subset of a set S is a mapping μ from S into $0,1$. For any $P \in S$, $\mu(P)$ is called a 'degree of membership' of P in μ. A crisp (i.e. ordinary or nonfuzzy) subset can be regarded as a special case of a fuzzy subset in which the mapping μ is into $\{0,1\}$, in fact, such a μ is the characteristic function of a subset, namely $\{P \in S \mid \mu(P)=1\}$.

The standard set theoretic relations and operations have natural generalizations to fuzzy subsets.

Digital fuzzy sets are also referred to as 'piecewise constant' fuzzy sets, i.e., having constant value on each of a finite set of bounded regions that meet pairwise along rectifiable arcs. And in digital fuzzy sets these regions are unit squares or pixels.

THINNING

In the field of image processing, thinning techniques have always had a great importance due to simplicity of object representation they allow. Thinning algorithms have been used widely for the analysis of thresholded image of written characters (signature analysis) and even the 'skeletonization' of chromosomes and a number of other types of images.

Two-tone images can be thinned to arcs and curves, without changing the connectedness, by repeatedly deleting the border ppints whose deletion does not disconnect the object. A number of such thinning algorithms (for two tone images) exist. Any graylevel image can be converted to a binary image by thresholding it and the two-tone thinning algorithms can be applied. But the selection of the threshold has to be done judiciously.

Gray-level thinning has the advantage that it does not require commitment to a particular threshold. In an image, with the pixels having gray-level values, the regions are not well-defined, there is an inherent amount of vagueness in it. So to get a natural feeling of the image and also to avoid committing to a particular thresholded version a fuzzy set representation of the image has to be considered.

For pattern recognition, before the actual recognition of the various objects (in the image) can be done, the image should be 'pre-processed' to quite an extent to enhance the features necessary for the object recognition. For example, in character recognition, before identifying the character from the image, the skeleton should be extracted so that syntactic pattern recognition techniques can be applied.

OBJECT EXTRACTION

The object can be extracted from the given image using any technique and it isn't necessary that the object extracted definite or be exactly as the original, because the data itself might be vague or fuzzy to quite an extent.

The method used during the course of this work is given below.

1. Edge Detection

The Robert's Gradient Operator is used for this purpose. Difference operators yield high values at places where the gray level is rapidly changing. At seperation of the object from the background, since the change in the gray level is appreciable, the application of the hence the edge detection is carried out. Consider the following case, where we have to compute the Robert's gradient for the pixel with gray level a 'a'

c	b
d	a

The gradient at the 'pixel a' is

$$\sqrt{\frac{(a-c)^2 + (b-d)^2}{2}}$$

After the gradient is computed for all the pixels in the image, the tracking of the edge pixels begins. Starting with a pixel having a high gradint value, the other edge points are detected by choosing a pixel with a maximum gradient value in the 8-neighbourhood of the pixel. And this process is continued, till a closed loop is detected. After this, if there are any other objects present, they are also detected in the same manner. In case of more than one pixel having the maxima value in the 8-neighbourhood then the ambiguity is resolved by adoptiɹg a policy of selecting the pixel which has more edge points along its direction of traversal.

2. Object Extraction

As the edge points are already available, the object can easily be obtained by superimposing the edge enhanced image on the original image and filling the region between any 2 edge points by either the corresponding object pixels or background pixels. The pixels of the object are retained alongwith their corresponding gray level values, but those of the background are erased out i.e., all the background pixels are assigned a single gray level, say 0.

SKELETON MEMBERSHIP FUNCTIONS

Once the object has been extracted from the background, all the the object pixels have to be assigned a membership value, indicating the degree of 'belongingness' to the object.

The property of any gray level image, is that there is no abrupt change in the pixel intensity as we move from the core or the centre of the object towards the edge. In other words there is a gradual variation in the pixel intensity, decreasing as the object pixel moves away from the centre. The core on the central pixels have high intensities and this property is made use in assigning the primary membership value to the pixels depending upon their gray level intensities. The membership assigning functions should be devised taking these features into account. And depending upon the function quality, the skeleton extracted could be termed as acceptable or unacceptable. The membership functions can be defined in a number of ways. A few of them are discussed here with there merits and demerits.

Let the maximum gray level of the pixel in the object be g_{max}, let g be the gray level of the pixel under consideration and let p_o be the function which assigns the membership value to the pixel for its gray level.

So we define

$$p_o(g) = \frac{g}{g_{max}}$$

The $p_o(g)$ assigns a membership value from 0 to 1. When $g = 0$, i.e., the pixel under consideration is a background pixel then its primary membership value based on pixel intensity is 0 and when g has a value around g_{max} then

$p(g)$ is nearly 1, thereby exhibiting the correctness of the function $p(g)$. This is the simplest possible function, for the images where the object pixels have a higher gray level than the background pixels. For images with light objects and dark background (astronomical data or chromosome data) the function has to be modified to something like

$$p(g) = 1 - g/g_{max} .$$

Another factor which determines the core point is the distance of the pixel from the left edge and right edge for the horizontal distance and from the top edge and bottom edge for the vertical distance. Consider a pixel g which is at a distance x_1 from the left edge and x_2 from the right edge. (The distance being measured as the number of units seperating the pixel under consideration from the first background pixel along that direction). Let the horizontal distance function, $h(g)$, which will be the second primary membership function, be defined as

$$h(g) = h(x_1 , x_2) = \frac{x_1}{x_2} \quad \text{if} \quad x_1 <= x_2$$

$$= \frac{x_2}{x_1} \quad \text{if} \quad x_1 > x_2$$

Similarly the vertical membership function, making up the third primary membership function, will be defined as

$$v(g) = v(y_1,y_2) = \frac{y_1}{y_2} \quad \text{if} \quad y_1 <= y_2$$
$$= \frac{y_2}{y_1} \quad \text{if} \quad y_1 > y_2$$

where y_1 is the distance of the pixel under consideration from the first background pixel in the upward direction and y_2 is the destance from the first background pixel encountered in the downward direction.

So now we have the three primary membership functions -- $p(g)$, $v(g)$ and $h(g)$. A single membership function is then defined based on these primary membership functions.

The first function proposed is

$$C_1(g) = \max\ (\min\ (p(g),\ h(g),\ \min\ (h(g),\ v(g)),\ \min\ (p(g), v(g)).$$

A detailed analysis of this function will show that, a final value of 1 is assigned when atleast two of the three primary membership values are 1, i.e., a pixel in the object is defined as belonging to the core or the skeleton, if it is at the geometric centre of that region (when $x_1=x_2$, $h(g) = 1$ and $y_1=y_2$, $v(g) = 1$, so $C_1(g) = 1$) or if the pixel intensity is nearly equal to the g_{max} of the object and it is either at the horizontal centre on the vertical centre (in this case $p(g) \simeq 1$ and either $v(g)$ or $h(g) \approx 1$

in which case $C_1(g) \approx 1)$. This proposed function has a drawback that it holds very well only for well defined geometric objects, and it does not take shape of the region, i.e., the extent, into consideration. So even pixels which belong to the core have considerably low membership values assigned. Secondly, all the three primary membership values have been attributed equal weight for computing the membership $C_1(g)$.

So the second membership function, proposed, $C_2(g)$ combining the three is defined as

$$C_2(g) = w_1 p(g) + w_2 h(g) + w_3 v(g).$$

where $w_1 + w_2 + w_3 = 1.0$

And because there might be core pixels which do not belong to the fuzzy geometrical centre of the object, the weight attributed to the $p(g)$ - the primary membership function, concerning pixel intensity, -- is greater than the weights attributed to the other two. Usually $w_2 = w_3$.

The composite membership function in this case $C_2(g)$, is better than the previous proposed one, $C_1(g)$ but this too has a drawback that for a relatively broader region, there is not much variation in the membership value $C_2(g)$ for the pixels in that region. So instead of having a 'streak' of high membership values we have a 'patch'. For example consider 2 pixels 'a' and 'b', such that their pixel intensities are same, and $y_{1a} = y_{2a}$ and $y_{1b} = y_{2b}$, i.e., both the pixels are at the centres as far as their vertical distances are considered, so $v(a) = v(b)$ and $p(a) = p(b)$. But let $x_{1a} = 15$ and $x_{2a} = 15$, so that $h(a) = 1$ and let $x_{1b} = 12$ and $x_{2b} = 18$, so that $h(b) = 2/3$. So given the above data, we can infer that 'a' belongs to the skeleton much more than 'b'. But if we consider $C_2(a)$ and $C_2(b)$ for $w_1 = 0.6$, $w_2 = w_3 = 0.2$, we have

$$C_2(a) = 1.0 \quad \text{and} \quad C_2(b) = 0.94$$

We observe that the change in the membership value is not appreciable and so a third membership function is proposed.

For the third membership function using the three primary membership functions, slightly different horizontal and vertical primary membership functions are used. Let $h'(g)$ be the modified horizontal primary membership function. Then $h'(g)$ is defined as

$$h'(g) = h'(x_1, x_2) = \frac{x_1}{x_2} \quad \text{if } d(x_1, x_2) <= 1 \text{ and } x_1 <= x_2$$

$$= \frac{x_2}{x_1} \quad \text{if } d(x_1, x_2) \ngtr 1 \text{ and } x_2 < x_1$$

$$= \frac{2x_1}{x_2(x_1 + x_2)} \quad \text{if } d(x_1, x_2) > 1 \text{ and } x_1 <= x_2$$

$$= \frac{2x_2}{x_2(x_1+x_2)} \quad \text{if} \quad d(x_1,x_2) > 1 \text{ and } x_2 < x_1$$

where $d(x_1,x_2) = | x_1 - x_2 |$

This definition of the horozontal membership function assigns high values (of nearly 1.0 for pixels near the core) and low values to pixels away from the core. For the pixels not belonging to the core, the factor/term (x_1+x_2) in the denominator of $h'(g)$ takes into consideration extent of the object segment and hence there is an appreciable amount of change in the membership value. Similarly the vertical primary membership function is also modified and is given by

$$v'(g) = v'(y_1,y_2) = \frac{y_1}{y_2} \quad \text{if} \quad d(y_1,y_2) <= 1 \text{ and } y_1 < y_2$$

$$= \frac{y_2}{y_1} \quad \text{if} \quad d(y_1,y_2) <= 1 \text{ and } y_2 < y_1$$

$$= \frac{2y_1}{y_2(y_1+y_2)} \quad \text{if} \quad d(y_1,y_2) > 1 \text{ and } y_1 < y_2$$

$$= \frac{2y_2}{y_1(y_1+y_2)} \quad \text{if} \quad d(y_1,y_2) > 1 \text{ and } y_2 < y_1$$

The single membership function combining the changed functions is given as

$$C_3(g) = w_1 p(g) + w_2 h'(g) + w_3 v'(g)$$

where $w_1 + w_2 + w_3 = 1$, and $w_2 = w_3$.

Of the three membership functions proposed the third one is found to be the best, whereas the other two are good for special images like those which are highly symmetrical, etc.

In this case, for ease of storage and computation the membership values have been rounded upto 2 decimal places' and discrete values 0.05, 0.10, 0.15, 0.20, ..., 0.90, 0.95, 1.00 have been assigned.

SKELETONISATION

Given the image, developed in the previous stage, with the object pixels having been assigned values indicating their degree of membership to the object, the skeleton of the object can be achieved by considering the 'level sets' at a particular value t.

The 'level sets' of a fuzzy set μ are the sets

$$\mu_t = \{P \in S \mid \mu(P) \geq t\}$$

where μ is the fuzzy subset of a set S, and $0 <= t \leq 1$. So for higher values of t, μ_t will have pixels with high membership values, the other pixels are marked.

The methods proposed here, do not ensure that the resulting skeleton would be connected. So if a connected skeleton is needed, then this version of the skeleton, a contour tracking algorithm can be applied to resolve the disconnectivity problem appropriately.

Energy Method

An energy measure of a fuzzy set is the 'power' 'fuzzy cardinality' defined as

$$P(\mu) = \sum_{x \in S} \mu(x)$$

So with this definition, the total energy of the whole fuzzy subset (i.e., the image with fuzzy membership values) P_t is calculated. As mentioned earlier, the membership values are discrete - 0.05, 0.10, 0.15, 0.20, ..., 1.0. So for each membership value r, the pixels having this value are counted. And so for each r, the percentage of the number of pixels falling in the range and the percentage of total energy contained in this range is computed. The membership value which has the highest percentage of pixels and energy, m is chosen as a sort of threshold and in the skeleton only those pixels having membership greater than m are retained.

So if a histogram is drawn, membership value versus percentage of pixels and/or percentage of energy contained, then the threshold corresponds to the peak of the histogram.

Compactness Method

The area of μ (a fuzzy subset) is defined as

$$a(\mu) \triangleq \int \mu$$

where the integral is taken even any region outside which $\mu=0$.

If μ : piecewise constant (for example, in a digital image) $a(\mu)$ is the weighted sum of the areas of the regions an which μ has constant values, by these values.

$$\text{i.e.,} \quad a(\mu) = \sum_{x \in S} \mu(x).$$

For a piecewise constant case, the perimeter is defined as

$$p(\mu) = \sum_{m=1}^{M} \sum_{n=1}^{N-1} | \mu(x_{m,n}) - \mu(x_{m,n+1}) | + \sum_{n=1}^{N} \sum_{m=1}^{M-1} | \mu(x_{mn}) - \mu(x_{m+1,n}) |$$

where the image is of size MXN.

The compactness is defined as

$$\text{Comp}(\mu) \triangleq a(\mu)/p^2(\mu)$$

So in this case, we consider the fuzzy subset obtained by deleting the pixels which have a membership value less than or equal to the 'r' under consideration (r = 0.05, 0.10, 0.15,.. 0.95, 1.0) and compute the compactness according to the above definition, for every membership value. It is observed that as r increases from 0.05 (in steps of 0.05), the compactness decreases to a certain minimum and later for a further increase in r, the compactness increases. The membership value, 'n', for which the compactness is minimum can be used as a threshold and the corresponding fuzzy subset can be used as the skeleton.

The increase in the value of the compactness, for values of 'r' higher than n, is because for these values (of r) majority of the core line or skeleton pixels (of the original object) are not considered, and so the skeletorized version consists of a number of disconnected objects, therefore the value of compactness, also increases. And the initial decrease in the value of compactness can be explained, by observing that for every value of r, the border pixels having membership values less than r, are not taken into consideration. So both the area and perimeter are less than those for the previous value of r. But the decrease in area is more than the decrease in the perimeter and hence the compactness decreases (initially) to a certain minimum.

The skeleton obtained at this stage (by either of the two methods) is disconnected. A connected skeleton can be obtained by checking that a pixel is deleted from the object only if it has a neighbour in the background and its deletion does not change the connectedness of the pixels in the neighbourhood of the pixel under consideration.

If a deterministic skeleton is desired then it can be obtained by applying a suitable contour tracking algorithm.

Elongatedness

For any gray level image, two important operations are shrinking and expanding. For a binary image, the shrinking process is carried out by deleting pixels of the image if they are within a given distance 'd' of the background and the expanding process is carried out by adding pixels to the object if they are with d of the object. The fuzzy generalizations of shrinking and expanding are local minimum and local maximum operations -- i.e., the object pixel is replaced by the min. (or max.) of all object pixels within a distance d of the pixel under consideration.

The fuzzy elengatedness is defined as

$$\bar{e}(\mu) = \max_{d > 0} \frac{a(\mu - \mu_{-d})}{(2d)^2}$$

where μ_{-d} is the result of the shrinking operation on μ.

As d increases, μ_{-d} gets smaller, so $\mu - \mu_{-d}$ gets larger and hence $a(\mu - \mu_{-d})$ reaches its maximum when $\mu_{-d} = 0$. Therefore the ratio $a(\mu - \mu_{-d})/(2d)^2$ cannot have its max. for d larger than this (the numerater remains constant while the denominator keeps increasing, but it may have its max. for same smaller d, (for e.g., if the object consists of a large amount of area disappears even when the shrinking operation is applied a small number of times).

So, for every value of the membership ranging from 0.05 to 1.00 (increments of 0.05), the elongatedness has been calculated for every d, $(d \geq 1)$ till $\mu_{-d} = 0$. The μ for the computation of the elongatedness is constant for all d, for a given r. And this μ is the level set of the object (with membership values) for that r.

Fuzzy shrinking operations, modified suitably to preserve fuzzy connectedness, can be applied to obtain a connected skeleton. The elongatedness has been calculated, but there is no fixed pattern recarding the manner in which the elongatedness is changing for the various level sets of the object. But the result of the computation has been included.

RESULTS

The block diagram is given in Fig.1. The algorithm has been applied to three image data -- a handwriting (SHU-DATA), chromosome data and the biplane data.

The histogram of the original image, and the histogram of the membership value (assigned) image with percentage of energy contained in every range have been included in the tables. The compactness measure tables have also been included. For the compactness, the rate of change of compactness has also been computed. By the histogram method, the 'optimum' skeleton is obtained at the membership value r for which the peak of the histogram (regarding percent of energy corresp ..uing to that membership value or the percent of number of object pixels falling in that range) is obtained. And by the compactness measure, the optimum skeleton is obtained at the value of r, for which the compactness is minimum or the rate of change in compactness is maximum. For each image, it can be observed that the optimum skeleton is obtained at almost the same membership value whether it is by the histogram measure or the compactness measure.

The values obtained for the elongatedness have also been included, for further study.

CONCLUSION

The skeleton membership functions, used in this work are simple and defined taking into account only the fixel intensity and distance from the background. And it can be expected that if the skeleton membership function is good, taking into account the geometry of the object and gray level distribution, etc. then a much better skeleton can be obtained. And if such a function can be defined, then this approach will yield a skeleton much faster than the other algorithms which are very sequential and thereby slow.

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FIG. 1. ORIGINAL IMAGE —— SHU DATA

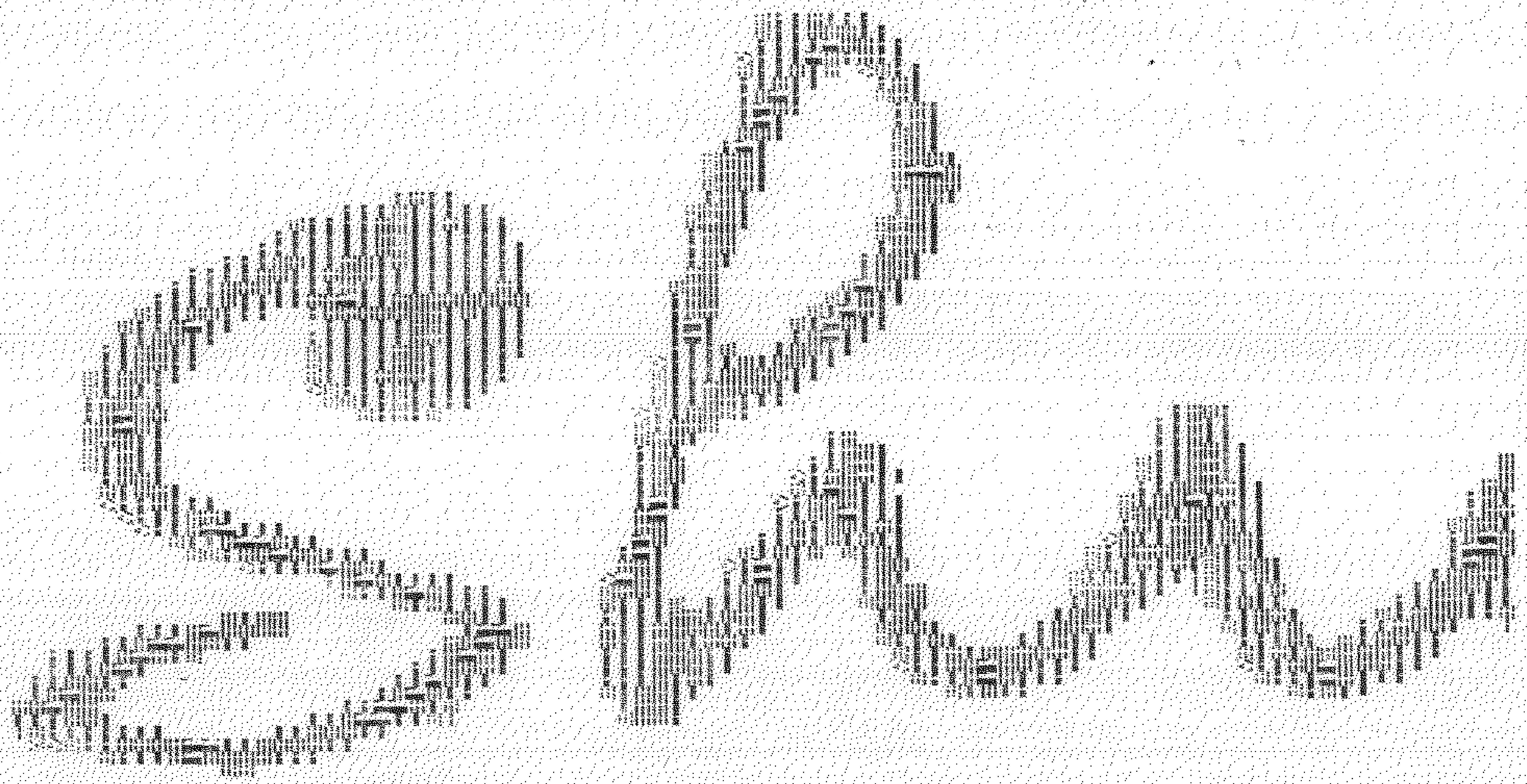

FIG. 2a. MEMBERSHIP PLANE —— SHU DATA

(Pixels with membership values 0.05, 0.10, [illegible] 0.95, 1.00 have been coded as [illegible] respectively)

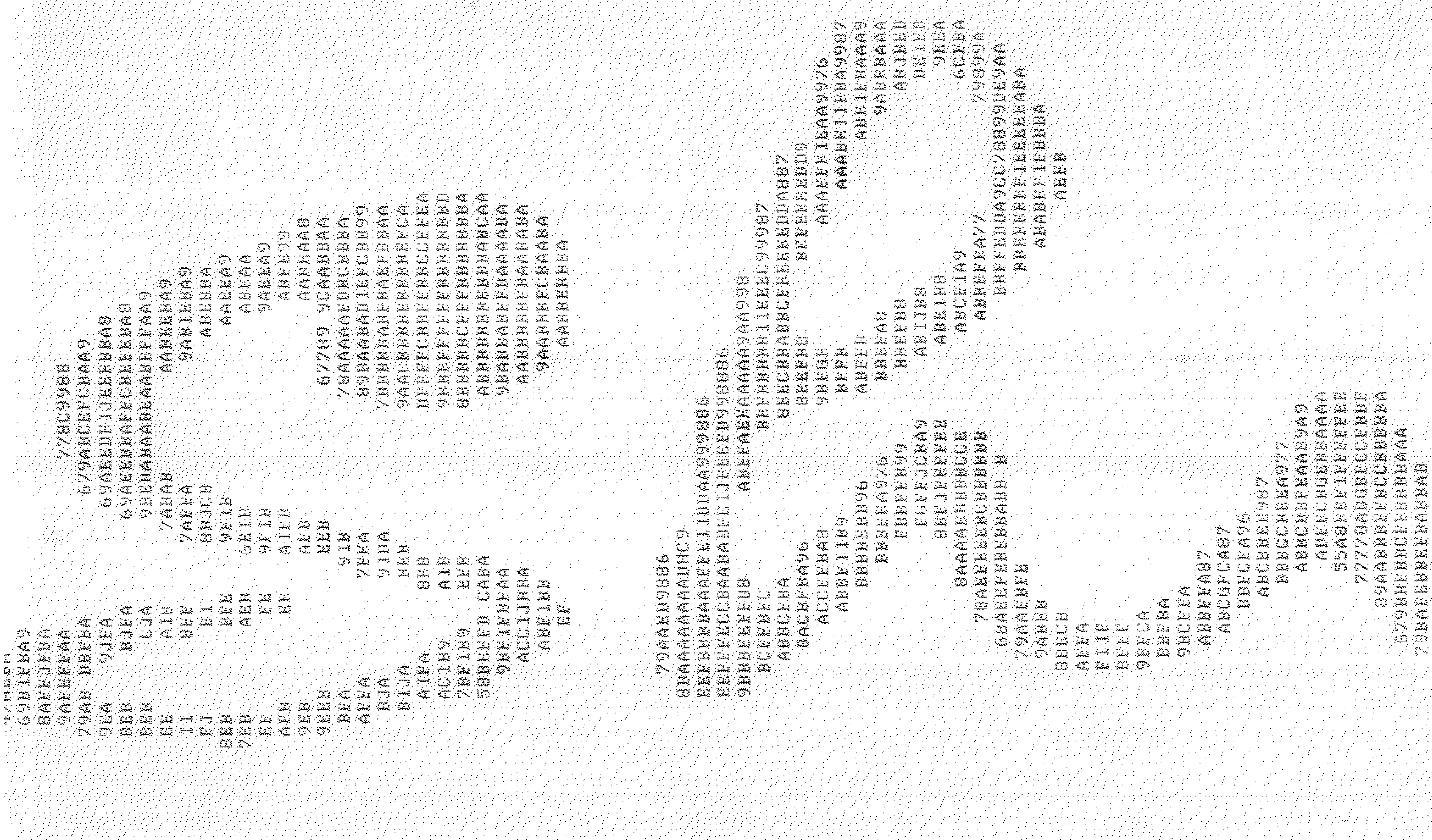

Fig. 2b CODED MEMBERSHIP PLANE — SHU-DATA
(Pixels with membership values 0.05, 0.10, ..., 0.45, 0.50, 0.55, ..., 0.90, 0.95, 1.00 have been coded as 1, 2, ..., 9, A, B, ..., I, J, K respectively)

FIG. 3 SKELETON SHU DATA
(CUT OFF 0.60)

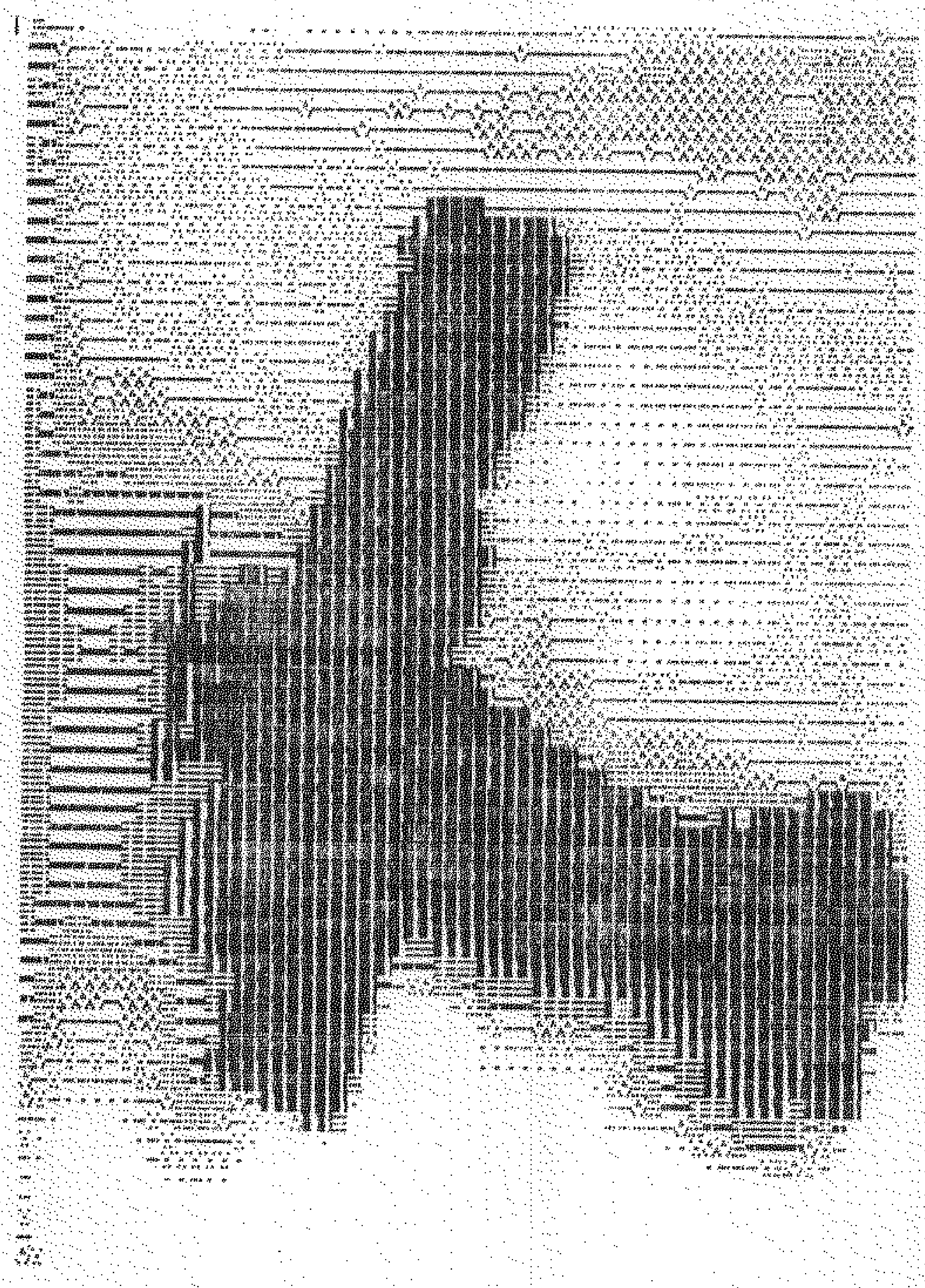

FIG. 4 ORIGINAL IMAGE — BIPLANE DATA

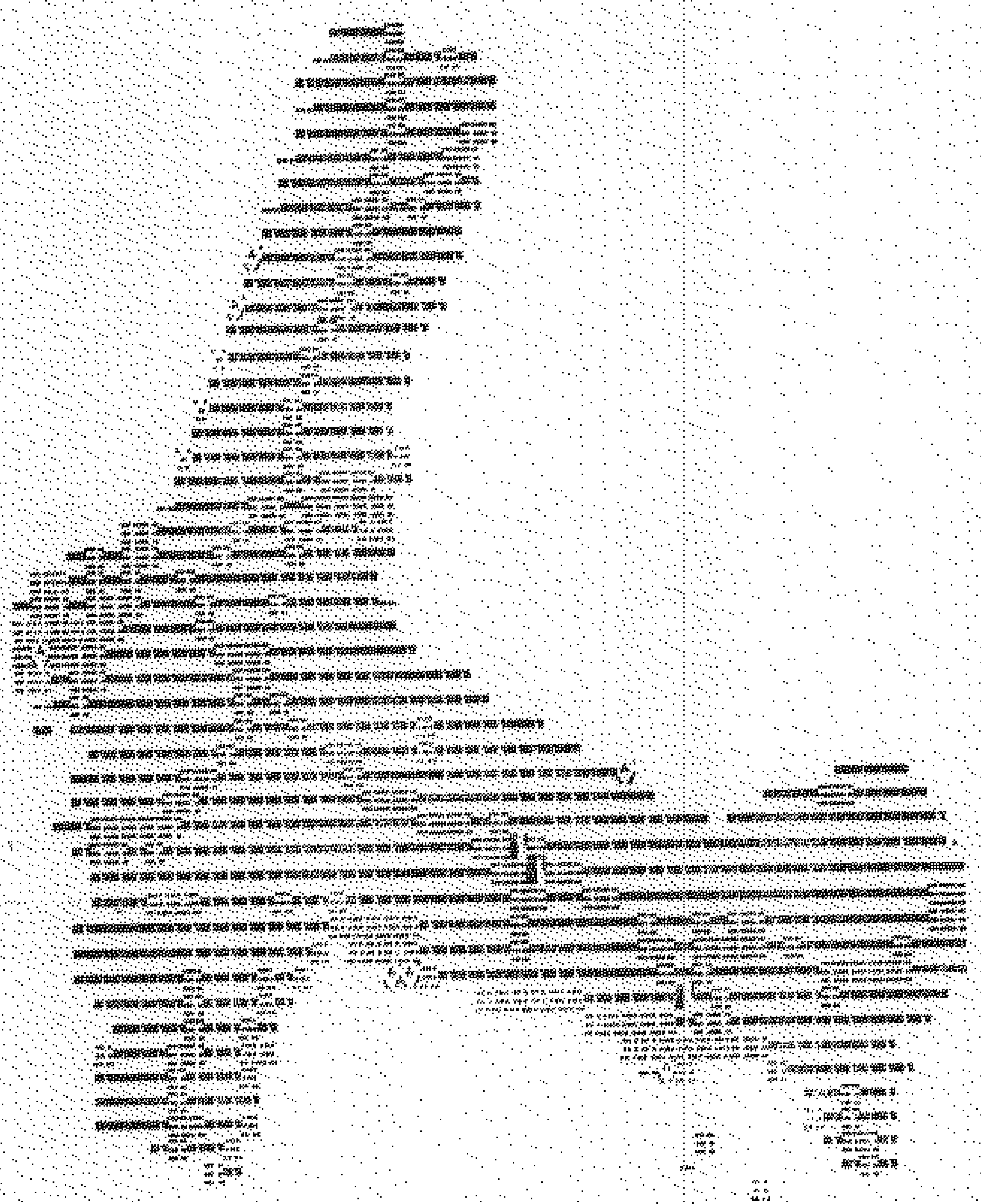

FIG. 5 MEMBERSHIP PLANE
BIPLANE DATA

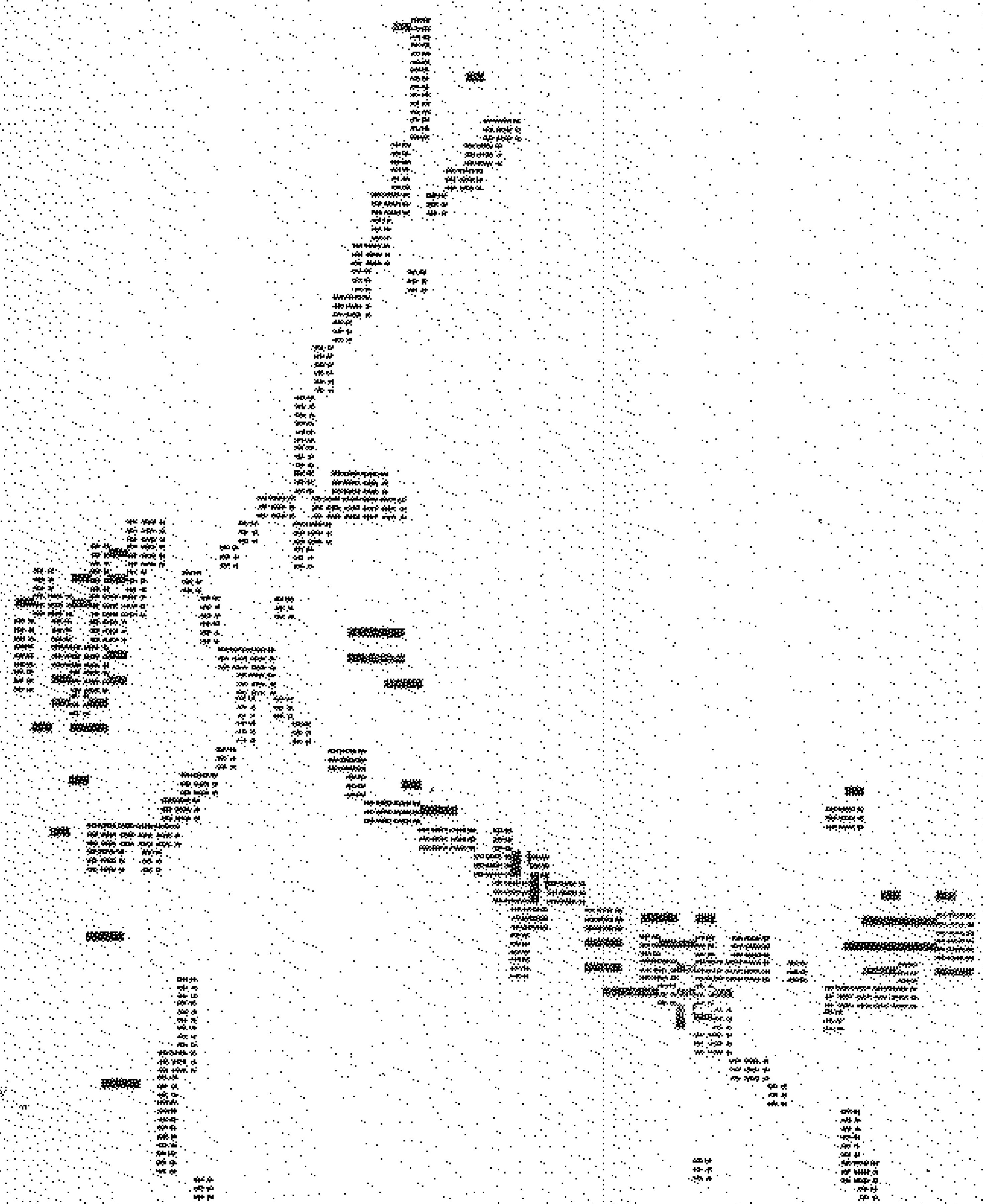

FIG 6. SKELETON - BIPLANE DATA
(CUTOFF — 0.55)

TABLE 1

ENERGY DISTRIBUTION OF MEMBERSHIP PLANE FOR PLANE DATA

MEMBERSHIP VALUE	PERCENT OF PIXELS	PERCENT OF ENERGY
0.05	0.0992	0.0098
0.10	0.1984	0.0394
0.15	0.0992	0.0295
0.20	0.2976	0.1181
0.25	0.7937	0.3939
0.30	0.6944	0.4135
0.35	1.6865	1.1715
0.40	3.9683	3.1502
0.45	30.5556	27.2828
0.50	38.2929	38.0980
0.55	6.3492	6.9305
0.60	3.0754	3.6621
0.65	6.7460	8.7025
0.70	5.8532	8.1315
0.75	0.8929	1.3290
0.80	0.0	0.0
0.85	0.0	0.0
0.90	0.2976	0.5316
0.95	0.0	0.0
1.00	0.0	0.0

TABLE 2

ENERGY DISTRIBUTION OF MEMBERSHIP PLANE FOR SHU DATA

MEMBERSHIP VALUE	PERCENT OF PIXELS	PERCENT OF ENERGY
0.05	0.0	0.0
0.10	0.0	0.0
0.15	0.0	0.0
0.20	0.0746	0.0254
0.25	0.3731	0.1590
0.30	1.2687	0.6489
0.35	2.9104	1.7366
0.40	4.0299	2.7481
0.45	7.4627	5.7252
0.50	19.0299	16.2214
0.55	26.2687	24.6310
0.60	4.9254	5.0382
0.65	2.0149	2.2328
0.70	11.9403	14.2494
0.75	14.2537	18.2252
0.80	0.6716	0.9160
0.85	0.0746	0.1081
0.90	3.2836	5.0382
0.95	1.4179	2.2964
1.00	0.0	0.0

TABLE 3

COMPACTNESS VALUES FOR THE BIPLANE DATA

VALUE	AREA	PERIMETER	COMPACTNESS	RATE OF CHANGE
0.05	507.85	220.70	0.01042632	
0.10	507.65	220.50	0.01044113	-0.0014204
0.15	507.50	219.90	0.01049508	-0.00516
0.20	506.90	219.90	0.01048267	-0.0011824
0.25	504.90	222.90	0.01034699	-0.0129433
0.30	502.80	222.10	0.01019292	-0.0148903
0.35	496.85	223.50	0.00994652	-0.0241736
0.40	480.85	240.30	0.00832730	-0.162790
0.45	342.25	332.10	0.00310317	-0.627349
0.50	148.75	297.10	0.00168521	-0.456939
0.55	113.55	267.40	0.00158806	-0.05764
0.60	94.95	230.20	0.00179179	-0.128288
0.65	9.45	34.80	0.00780321	3.354998
0.70	2.70	10.80	0.02314815	1.97

TABLE 4

COMPACTNESS VALUES FOR THE SHU DATA

VALUE	AREA	PERIMETER	COMPACTNESS	RATE OF CHANGE
0.15	785.99	622.40	0.00202901	
0.20	785.80	622.40	0.00202849	-0.000256
0.25	784.55	622.40	0.00202527	-0.0001587
0.30	779.45	622.39	0.00201210	-0.0006545
0.35	765.79	623.80	0.00196800	-0.021917
0.40	744.20	626.20	0.00189786	-0.035640
0.45	699.20	630.70	0.00175775	-0.073825
0.50	571.70	674.70	0.00125598	-0.285518
0.55	378.10	693.40	0.00077473	-0.3738
0.60	388.50	661.00	0.00077473	-0.01483
0.65	320.95	649.30	0.00076128	-0.01736
0.70	208.95	513.50	0.00079243	0.040917
0.75	65.70	216.50	0.00140169	0.76885
0.80	58.50	192.50	0.00157869	0.126276