

*Dissertation Submitted to the Indian Statistical Institute in Partial Fulfilment of  
the Requirement for the Degree of Master of Technology in Computer Science*

# Super Resolution Imaging

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# Indian Statistical Institute

## *Certificate of Approval*

This is to certify that the thesis entitled “**Super Resolution Imaging**” by Pulak purkait towards partial fulfillment for degree of M.Tech in computer science at Indian Statistical Institute, Kolkata. It is fully adequate, in scope and quality as a dissertation for the required degree.

The thesis is a faithfully record of bona fide research work carried out under my supervision and guidance. It is further certified that no part this thesis has been submitted to any other university or institute for the award of any degree or diploma.

(Prof. Bhabatosh Chanda)

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## Abstract

This work focuses on the super resolution of images. The term *super resolution (SR)* is used to describe the process of obtaining a *high resolution (HR)* image, or a sequence of HR images, from a set of low resolution (LR) observations. This process has also been referred to in the literature as *resolution enhancement (RE)*. SR has been applied primarily to spatial and temporal RE. We only concentrate on motion based spatial RE, where every LR observations has sub-pixel different shifts.

In this project we would try to model a *iterative back propagation* algorithm for super resolution *image reconstruction*. This approach is an *ill-posed problem*. Hence the resultant image consisting of periodic noise and we analysis some *image restoration* technique in spatial as well as in frequency domain to remove this noise.

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# Chapter 1

## Introduction

In almost every application of digital image processing, it is desirable to generate an image that has a very *high resolution*(**HR**). Thus, a HR image could contribute to a better classification of regions in a multi-spectral image. The resolution of an image is dependent on the resolution of the image acquisition device. However, as the resolution of the image generated by a sensor increases, so does the cost of the sensor and hence it may not be an affordable solution. Hence the basic question is that given the resolution of an image sensor, is there any algorithmic way of enhancing the resolution of images without increasing sensor of the camera? The answer is definitely affirmative and there are various ways of enhancing the *image resolution*.

### 1.1 The Word Resolution

Resolution is perhaps a confusing term in describing the characteristics of a visual image since it has a large number of competing terms and definitions. In its simplest form, *image resolution* is defined as the smallest discernible or measurable detail in a visual presentation. Researchers in *digital image processing* and *computer vision* use the term resolution in three different ways.

- ***Spatial resolution*** refers to the spacing of pixels in an image and is measured in pixels per inch (ppi). The higher the spatial resolution, the greater the number of pixels in the image and correspondingly, the smaller the size of individual pixels will be. This allows for more detail and subtle color transitions in an image
- ***Brightness resolution*** refers to the number of brightness levels that can be recorded at any given pixel. This relates to the quantization of the light energy collected at

a charge-coupled device (CCD) element.

- **Temporal resolution** refers to the number of frames captured per second and is also commonly known as the frame rate. It is related to the amount of perceptible motion between the frames.

In this project the term resolution unequivocally refers to the spatial resolution, and the process of obtaining a high resolution image from a set of low resolution observations is called *super resolution imaging*.

## 1.2 Image Zooming

The quality of the interpolated image generated by any of the single input *image interpolation* algorithms is inherently limited by the amount of data available in the image. *Image zooming* cannot produce the high frequency components lost during the low resolution sampling process unless a suitable model for zooming can be established. Because of this reason *image zooming* methods are not considered as *super-resolution imaging* techniques.

## 1.3 Super Resolution Reconstruction

*Super-resolution (SR)* is a fusion process for reconstructing a *high resolution (HR)* image from several *low resolution (LR)* images covering the same region in the world. It extends classical single frame image reconstruction/restoration methods by simultaneously utilizing information from multiple observed images to achieve resolutions higher than that of the original data. These observations can be LR images captured simultaneously or at different times by a single or multiple imaging devices. This methodology, also known as *multi frame super-resolution reconstruction*, registers the observed images to a common HR reference frame in order to formulate the problem of fusion as one of constrained *image reconstruction* with missing data.

The general strategy that characterizes super-resolution comprises four major processing steps [1]:

1. **LR image acquisition:** Acquisition of a sequence of LR images from the same scene with non-integer (in terms of inter-pixel distances) geometric displacements between any two of the images.



2. **Image registration/motion compensation:** Estimation of the sub-pixel geometric transformation of each source image with respect to the reference HR desirable grid.
3. **HR image reconstruction:** Solution of the problem of reconstructing a HR image from the available data supplied by the source images.
4. **Image Restoration:** Motion blur and noise removal to get desired image.

Most of the multi-frame methods for SR *image reconstruction* proposed in the literature are in the form of a three-stage *registration, interpolation, and restoration* algorithm. They are based on the assumption that all pixels from available frames can be mapped back onto the reference frame, based on the motion vector information, to obtain an up-sampled frame. Next, in order to obtain a uniformly spaced up-sampled image, interpolation onto a uniform sampling grid is done. Finally, *image restoration* is applied to the up-sampled image to remove the effect of sensor PSF blur and noise. The block diagram of constructing

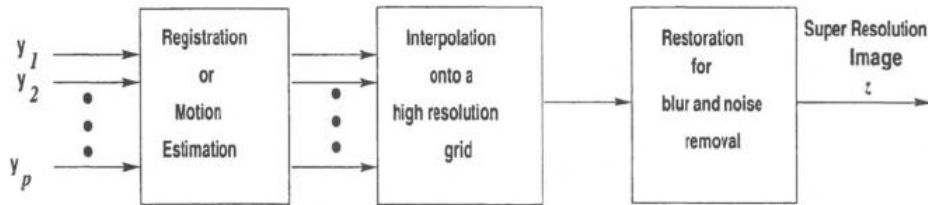


Figure 1.1: Scheme for *SR image reconstruction* from multi-frame shifted observations.

a high resolution frame from multiple LR frames is shown in Figure 1.1. Here, the LR frames  $y_1, y_2, \dots, y_p$  are input to the motion estimation or registration module, following which the registered image is interpolated onto a *high resolution* grid.

## 1.4 Review of Earlier work

The literature on SR can be broadly divided into methods employed for still images and those for video. Most of the research in still images involves an image sequence containing *sub-pixel* shifts among the images. Although some of the techniques for SR video are extensions of their still image counterpart, a few different approaches have also been proposed. In this project report, we briefly review the available literature for generation of SR images or frames from still images or a video sequence.

### 1.4.1 Super-resolution from still images

**Tsai and Huang** [1] were the first to address the problem of reconstructing a *high resolution* image from a sequence of *low resolution* under-sampled images. They assume a purely translational motion and solve the dual problem of registration and restoration. Later **Tom, Gatassanos and Katsaggelos** [2] model the former method in both spatial as well as frequency domain and regularize the iterative SR reconstruction algorithm which is an ill-posed problem. They used a *Maximum Likelihood(ML)* based *image registration* algorithm to evaluate sub-pixel shifts of LR images. **Rajan and S. Chaudhuri** [10] focused on some *generalized interpolation* for structure preserving super resolution and object based grouping. **Boult and Chiang** [11] worked on the issues related with supporting *image warping* algorithms for super resolution, examples of how *image wrapping* algorithms impact super resolution image quality, and the development of quantitative techniques for super resolution algorithm evaluation.

### 1.4.2 Super Resolution from video

Most of the super-resolution algorithms applicable to video are extensions of their single frame counterpart. **Irani and Peleg** [2] minimize the mean squared error between the observed and simulated images using the **back projection** method. **Schultz and Stevenson** use the modified *hierarchical block matching* algorithm to estimate the sub-pixel displacement vectors and then solve the problem of estimating the high resolution frame given a *low resolution* sequence by formulating it using the *MAP estimation*, resulting in a constrained optimization problem with unique minimum [3]. Patti propose a complete model of video acquisition with an arbitrary input sampling lattice and a non-zero aperture time [4]. They propose an algorithm based on this model using the theory of **POCS** to reconstruct a super-resolution video from a low resolution time sequence of images.

## 1.5 Why and when Super Resolution is possible ?

A fundamental question is what makes SR possible. Intuitively, each LR-observed image represents a sub-sampled (i.e., *aliased*) version of the original scene. The *aliasing* could not be removed if we were to process one image only. Due to the sub-pixel shifts, however, each observed image contains complementary information. With exact knowledge of the shifts, the observed images can be combined to remove the *aliasing* and generate a higher resolution image. If we assume that the resolution of this HR image is such that

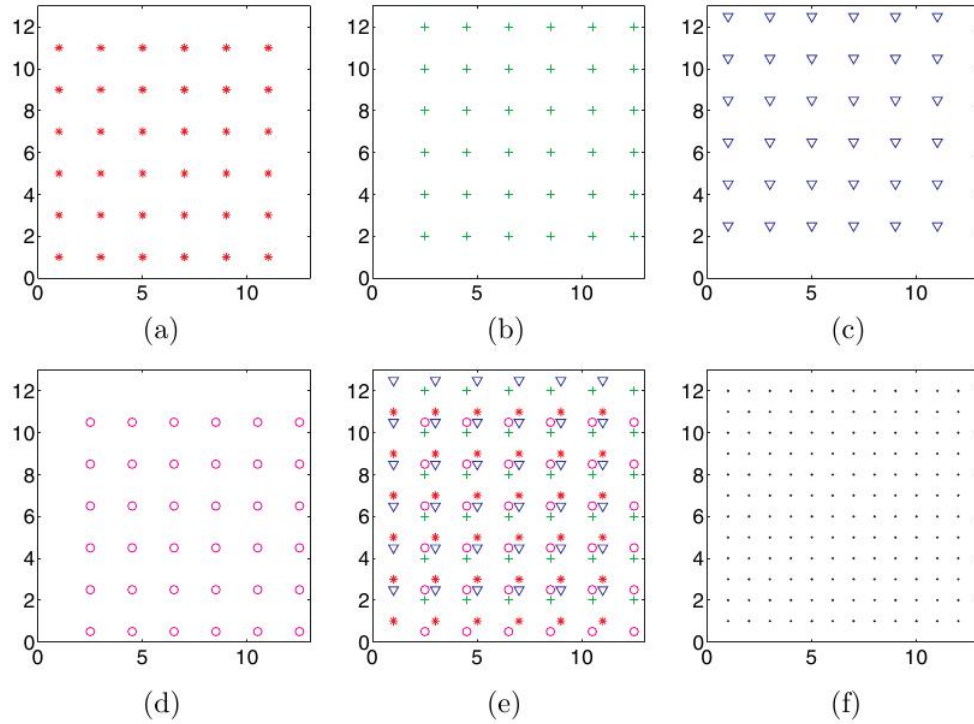


Figure 1.2: (a)-(d) Relationship between **LR** images, (e) the irregularly sampled **HR** image, and (f) the regularly sampled **HR** image

the Nyquist sampling criterion is satisfied, this **HR** image then represents an accurate representation of the original (continuous) scene. Aliased information can therefore be combined to obtain an *alias-free* image.

Figure 1.1 shows the relationship among the sub-sampled **LR** images and the **HR** image in this *degradation-free* example. Assuming that the image in Fig. 1.1(e) is available, that is, assuming that the sub-pixel shifts are known, the **SR** problem becomes a re-sampling problem, that of converting an arbitrarily sampled image (Fig. 1.1(e)) to a uniformly sampled one (Fig. 1.1(f)). If sub-pixel shifts are not known we can model (for example **ML**) it to calculate sub-pixel shifts. So only requirement is that every **LR** images of a same scene should have different sub-pixel shifts.

## 1.6 Organization of the Report

The concept of *super resolution* and it's need in digital image processing is now established. In the next chapter we would model multiple low resolution images to reconstruct

a *high resolution* image and associated *additive noise* in the output images for develop an *image restoration* technique. Here we would concentrate mainly on some *iterative back-propagation* algorithms for *image reconstruction* technique. It is an *ill-posed* problem so we would regularize it make limited number of solution and model an *adaptive image restoration* technique to approximate original high resolution image. We would also try to medal a mask in *frequency domain* for removal of *periodic noise*.

## Chapter 2

# Super Resolution Reconstruction

In this chapter the problem of reconstructing a *high resolution* image from multiple aliased and shifted by sub-pixel shifts *low resolution* images is considered. As discussed in the previous chapter *Super Resolution image reconstruction* consists of three basic steps (a)Image Registration (b)Image Interpolation into high resolution grid and (c)Image Restoration.

### 2.1 Image Registration

*Low resolution* images are possibly degraded by unknown blurs and their sub-pixel shifts are not known. These *low-resolution* images can be either obtained as a sequence taken over time, or taken at the same time with different sensors. A pictorial example of the overlay of three misregistered images is shown in Figure 2.1, where the sub-pixel shifts for each frame,  $\delta_x, \delta_y$  are also shown.

Here the first image taken as referenced image and rest has non-zero sub-pixel shifts respect to first image. Establishing a correspondence between low resolution images requires matching of identical shapes in the related image pairs. There are various technique for registering images. They can be categorized in two types, area based techniques(viz. Maximum Likelihood based Restoration-Registration) and feature-based techniques. Barbara Zitova and Jan Flusser[14] has given some novel method for image registration. Area-based techniques are preferably applied when local shapes or structures vary over the images due to blurring and noise and the distinctive information is provided by graylevels/colors rather than by local shapes and structures. In this project we have used area based registration technique. We divide the reference low resolution images into small block of same size. Then find the best matching into small neighborhood for each

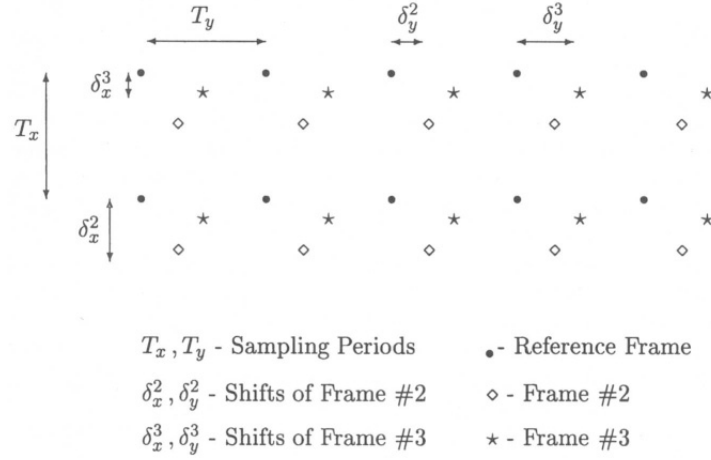


Figure 2.1: Overlay of three misregistered images ( $\delta_x^1 = 0, \delta_y^1 = 0$ )

block on the other low resolution images. Here best matching is done depend upon the covariance of blocks.

## 2.2 Image Interpolation into High Resolution Grid

After registration is done we need to interpolate low resolution images into high resolution grid.

Figure 2.2 shows the relationship between the sub-sampled, multiple *low-resolution* images and the *high-resolution* image. The main reason that a single *high-resolution* frame can be constructed from *low-resolution* frames is that the *low-resolution* images are sub-sampled (aliased) as well as misregistered with sub-pixel shifts. If the images are shifted by integer amounts, then each image contains the same information (intensity values at the same spatial location), and thus there is no new information that can be used. In this case, a simple interpolation scheme (*bilinear, cubic spline, etc.*) can be used to increase the resolution. However, if the images have sub-pixel shifts, and if aliasing is present, then each image cannot be obtained from the others, assuming each image has different shifts.

### 2.2.1 Mathematical Model

Let us denote by  $f(x, y)$  the continuous two-dimensional (2D) image, by  $f_{HR}(m, n)$  the *high-resolution* discrete *high resolution* image of size  $2L_x P_1 \times 2L_y P_2$  and by  $f_{LR}^k(m, n)$  the

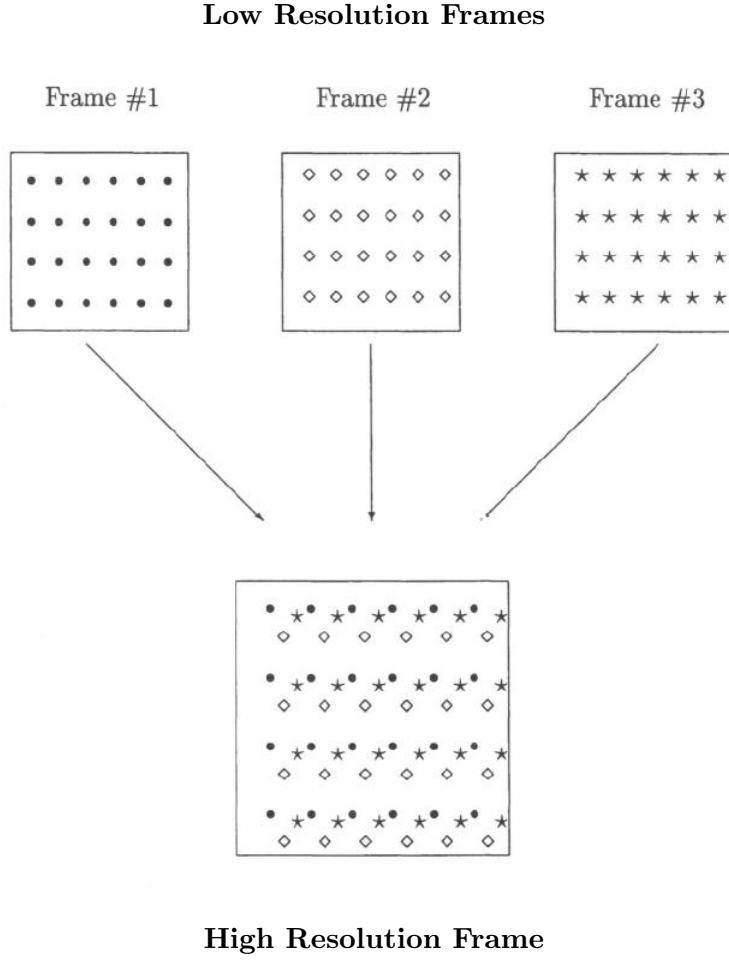


Figure 2.2: Relationship between *low-resolution* images and the *high-resolution* image

$k$ -th *low-resolution* discrete image of size  $P_1 \times P_2$ . They are related by

$$f_{HR}(m, n) = f(mT_x, nT_y) \quad (2.1)$$

Where  $T_x$  and  $T_y$  are sampling periods in x and y directions and also

$$f_{LR}^k(m, n) = f(mT'_x + \delta_x^k, nT'_y + \delta_y^k), \quad \text{for } k = 1, 2, \dots, K \quad (2.2)$$

with the sampling periods  $T'_x$  and  $T'_y$  are given by  $T'_x = 2L_x T_x$ ,  $T'_y = 2L_y T_y$  and  $\delta_x^k$  and  $\delta_y^k$  represent the shifts in the x and y direction of the  $k^{\text{th}}$  *low resolution* image with respect to a referenced image, and  $K$  is the total number of *low resolution* image.

*Super-resolution* reconstruction can be presented as a sparse linear optimization problem. The implementation is operated by standard operations such as convolution, warping, sampling, and etc. The relationship between the *low resolution* images and the *high-resolution* image can be formulated as [5]. *Super-resolution* reconstruction can be presented as a sparse linear optimization problem. The implementation is operated by standard operations such as convolution, warping, sampling, and etc. The relationship between the LR images and the *high-resolution* image can be formulated as [5]

$$g_k = DH_k^2 F_k H_k^1 f_{hr} + e_k \quad (2.3)$$

Considering only atmospheric blurring alternate matrix formulation of the super-resolution problem is presented as

$$g_k = DF_k H_k^1 f_{hr} + e_k \quad (2.4)$$

Considering only camera lens/CCD blur, another matrix formulation of the super-resolution problem is presented as

$$g_k = DH_k^2 F_k f_{hr} + e_k \quad (2.5)$$

Where  $g_k$  is a lexicographically ordered vector of the  $k^{th}$  *low resolution* image  $f_{LR}^k$  with size  $M_1 \times M_2$ ;  $f_{hr}$  is a lexicographically ordered vector of HR image  $f_{HR}(m, n)$  with size  $N_1 \times N_2$ ;  $e_k$  is a lexicographically ordered vector of noise with size  $M_1 \times M_2$  which is generally seen to be the normally distributed additive noise;  $F_k$  is a geometric warp matrix of size  $N_1 N_2 \times N_1 N_2$ ;  $D$  is the decimation matrix of size  $M_1 M_2 \times N_1 N_2$ ;  $H_k^1$  is atmospheric blurring matrix of size  $N_1 N_2 \times N_1 N_2$ ;  $H_k^2$  is camera lens/CCD blurring matrix of size  $N_1 N_2 \times N_1 N_2$ ,  $1 \leq k \leq K$  and  $K$  is the number of low-resolution images.

In conventional imaging systems (such as video cameras), camera lens/CCD (Charge Coupled Device) blur has more important effect than the atmospheric blur (which is very important for astronomical images). In this project we use the model (2.5).



Stacking K vector equations from the different LR images into a single matrix-vector:

$$\begin{bmatrix} g_1 \\ \cdot \\ \cdot \\ \cdot \\ g_k \end{bmatrix} = \begin{bmatrix} DH_1^2 F_1 \\ \cdot \\ \cdot \\ \cdot \\ DH_k^2 F_k \end{bmatrix} f_{hr} + \begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_k \end{bmatrix} = \begin{bmatrix} A_1 \\ \cdot \\ \cdot \\ \cdot \\ A_k \end{bmatrix} f_{hr} + \begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_k \end{bmatrix} \Leftrightarrow g = Af_{hr} + e \quad (2.6)$$

Now assuming images are taken from same camera lens/CCD (Charge Coupled Device), then lens blur is same for all the LR images. Moreover assuming that the picture is taken on same atmospheric and lighting condition  $e_k$  is also same. Hence equation (2.5) can be written as

$$g_k = A_k f_{hr} + e, \quad \text{for, } k = 1, 2, \dots, K \quad (2.7)$$

The computation of high resolution image from given low resolution image is a reverse process, where  $g_k$  is known to us however  $A_k$  is still unknown. To compute  $A_k$ , we will estimate the sub-pixel shifts  $\delta^k$  taking one of the image as referred image, for which *image registration* is required. There is lot of algorithm (*viz, Maximum likelihood based registration*) available in literature for image registration. So we are left with only one unknown  $f_{hr}$ .

### 2.2.2 Some Iterative Algorithms

In the above section we end up with solving M linear equation, where M is the number of low resolution images. So we can write above image model as

$$A\bar{x} = b. \quad (2.8)$$

Where the observed data  $b = (b^1, b^2, \dots, b^M) \in \mathbb{R}^M$ , the approximated value  $\bar{x} = (x_1, x_2, \dots, x_N) \in \mathbb{R}^N$  of high resolution image  $f_{hr}$ , and  $A = (A_{ij})$  is a non-zero  $M \times N$  matrix. The problem is to reconstruct the image  $\bar{x}$  from the data b.

A direct solution is not feasible with conventional direct methods because of the ill-posedness of the problem, the noisy corrupt data b and huge data dimension in practice.

The iterative approach has been important because of their superior performance

in the above context. With the rapid development of computer technology, iterative algorithms receive increasingly more attention

The algebraic reconstruction technique (ART) [13] and expectation maximization (EM) algorithm are the primary algorithms widely used in the community, due to their simplicity, efficiency and performance. The algorithms what we have used for image reconstruction are following

- **ART like algorithm** : The ART is the first iterative algorithm used in CT

$$x_j^{(n+1)} = x_j^n + \lambda_n \frac{A_{ij}}{\|A^i\|^2} (b^i - A^i x^{(n)}) \quad (2.9)$$

where  $i = n \bmod(M) + 1$ ,  $\|A^i\|^2 = \sum_{j=1}^N A_{ij}^2$  is the Euclidean norm of the  $i$ -th row of  $A$ . This method was originally discovered by Kaczmarz.

- **SART like algorithm** : In 1984, the SART was proposed as a major refinement of the ART. Let

$$A_{i,+} = \sum_{j=1}^N |A_{ij}|, \text{ for } i = 1, 2, \dots, M \quad (2.10)$$

$$A_{+,j} = \sum_{i=1}^M |A_{ij}|, \text{ for } j = 1, 2, \dots, N \quad (2.11)$$

Then the **SART** is

$$x_j^{(n+1)} = x_j^n + \lambda_n \frac{1}{A_{+,j}} \sum_{i=1}^M \frac{A_{ij}}{A_{i,+}} (b^i - A^i x^{(n)}) \quad (2.12)$$

where  $i = n \bmod(M) + 1$ , and Cimmino modify this algorithm as following.

- **Cimminos Algorithm** : The Cimminos simultaneous projection method is

$$x_j^{(n+1)} = x_j^n + \lambda_n \frac{1}{M} \sum_{i=1}^M \frac{A_{ij}}{\|A^i\|^2} (b^i - A^i x^{(n)}) \quad (2.13)$$

where  $i = n \bmod(M) + 1$ , and  $\lambda_n$  is the parameter that control the convergence of the above iterative methods.

However the **high resolution** image construction is an ill-posed problem and the

solution obtained using those iterative method is not unique. Suppose  $\bar{e}$  be a noise term such that

$$A\bar{e} = 0 \Rightarrow A(\bar{x} + \bar{e}) = A\bar{x} + 0 \Rightarrow A(\bar{x} + \bar{e}) = b$$

which shows that if  $\bar{x}$  is a solution then  $\bar{x} + \bar{e}$  also be a solution. So we need to regularize the algorithm.

### 2.2.3 Regularization

Regularization has been used in conjunction with iterative techniques for restoration of noisy and degraded images. Therefore using an iterative regularize approach, Eq. (2.7) can be written as

$$\bar{f}_{hr}^{(n+1)} = \bar{f}_{hr}^{(n)} + \lambda_n(A_k^T(\delta)f_{lr} - (A_k^T(\delta)A_k(\delta) + \alpha_n Q^T Q))\bar{f}_{hr}^{(n)}, k = n \text{ mod}(M) \quad (2.14)$$

Where  $\lambda_n$  is the parameter controlling the convergence and the speed of the convergence,  $Q$  is the regularizing operator and  $\alpha_n$  is the regularizing parameter, usually a five point Laplacian. The choice of  $\alpha_n$  is a very important issue, and has been studied extensively [6]. A good initial choice is

$$\alpha_0 = \frac{\min_k \{A_k^T A_k\}}{Q^T Q} \quad (2.15)$$

## 2.3 Image Restoration

After project the low resolution images into a high resolution grid, we apply some Image Restoration technique to restore the images. We would model some image restoration technique in both spatial as well as frequency domain.

### 2.3.1 Spatial Domain Filtering

We take a look at some adaptive filters whose behavior changes based on the statistical characteristics of the output image obtained from applying the above iterative method. Here adaptive filter is chosen as because we have seen that amount of noise is depends on the image characteristics. As shown in the fig (2.3), we see that some periodic noise appear with period equal to the resolution factor(no of times we want to increase size the of image) appear parallelly to the abrupt changes in the image. So adaptive filter doing

well to remove the noise, however the detail of the output lost little bit.



Figure 2.3: (a) Scaled and cropped SR output of iterative algorithm. (b) After smoothing by some adaptive spatial domain filtering technique

Here we look at the behavior changes based on the statistical characteristic of the image inside the filter region defined by the  $m \times n$  rectangular window  $S_{xy}$ , where  $m$  and  $n$  are the resolution factor along  $x$  and  $y$  direction respectively. The response of the filter at any point  $(x, y)$  on which the region is centered is to be based on four quantities : (a)  $\bar{f}_{hr}(x, y)$  the value of the noisy image at  $(x, y)$ ; (b)  $\sigma_{\eta}^2$ , the variance of the noise corrupting  $f_{hr}(x, y)$  to form  $\bar{f}_{hr}(x, y)$ ; (c)  $m_L$ , the local mean of the pixels in  $S_{xy}$ ; and (d)  $\sigma_L^2$  the local variance of the pixels in  $S_{xy}$ . We want to behavior of the filter as follows :

1. If  $\sigma_{\eta}^2$  is zero, the filter should return simply the value of  $\bar{f}_{hr}(x, y)$ . This is the trivial, zero-noise case in which  $\bar{f}_{hr}(x, y)$  is equal to  $f_{hr}(x, y)$ .
2. If the local variance  $\sigma_L^2$  is high relative to  $\sigma_{\eta}^2$ , the filter should return a value close to  $\bar{f}_{hr}(x, y)$ . A high local variance typically is associated with edges and these should be preserved.
3. If the two variances are equal, we want the filter to return the arithmetic mean value of the pixels in  $S_{xy}$ . This condition occurs when the local area has the same properties as overall image, and local noise is to be reduced simply by averaging.

An adaptive expression for obtaining  $\hat{f}_{hr}(x, y)$  based on the above assumption may be written as

$$\hat{f}_{hr}(x, y) = \bar{f}_{hr}(x, y) - \frac{\sigma_{\eta}^2}{\sigma_L^2} [\bar{f}_{hr}(x, y) - m_L] \quad (2.16)$$

The only quantity that needs to be known or estimated is the variance of the overall noise,  $\sigma_{\eta}^2$ . The other parameters are computed from the pixels in  $S_{xy}$  at each location

$(x, y)$  on which filter window is centered. Here we have assumed that noise variance is less than the variance of the window.

### 2.3.2 Frequency Domain Filtering

Since the property of the noise introduced during the iterative process is like a *periodic noise*, *Frequency domain Filtering* works well to remove those noise.

Figure (2.4)(b) shows the occurrence of periodic noise where we have taken resolution factor four to reconstruct super resolution. The Fourier transform of original high resolution image and the output super resolution image are exactly same except for four black horizontal and vertical lines.

Now for removal of noise in frequency domain we use following two methods

- **Using mean/median difference :** For a fixed resolution factor to reconstruct high resolution image, the effect of noise appears at the same position in the Fourier spectrum. We apply this algorithm for a dataset consisting of images of the same size. Take the mean difference of the Fourier transform of the original and reconstructed images to get the Fourier mask. Instead of mean we could take the median of the difference, which will give a different Fourier mask.
- **Using Selective Gaussian filtering (Notch Filter) :** A selective notch filter in general rejects (or passes) frequencies in a predefined neighborhood about the center of the frequency rectangle. Here instead of rejecting (or passing) frequencies we modify the predefined neighborhood (the black spot shown in fig:2.4) by a Gaussian function as shown below

$$H(u, v) = 1 + \alpha e^{-[(u-u_0)^2 + (v-v_0)^2]/2\sigma^2} \quad (2.17)$$

where  $(u_0, v_0)$  are the centers of the black spots in the Fourier spectrum,  $\alpha$  is a positive constant,  $H(u, v)$  is the convolution operator in frequency domain.

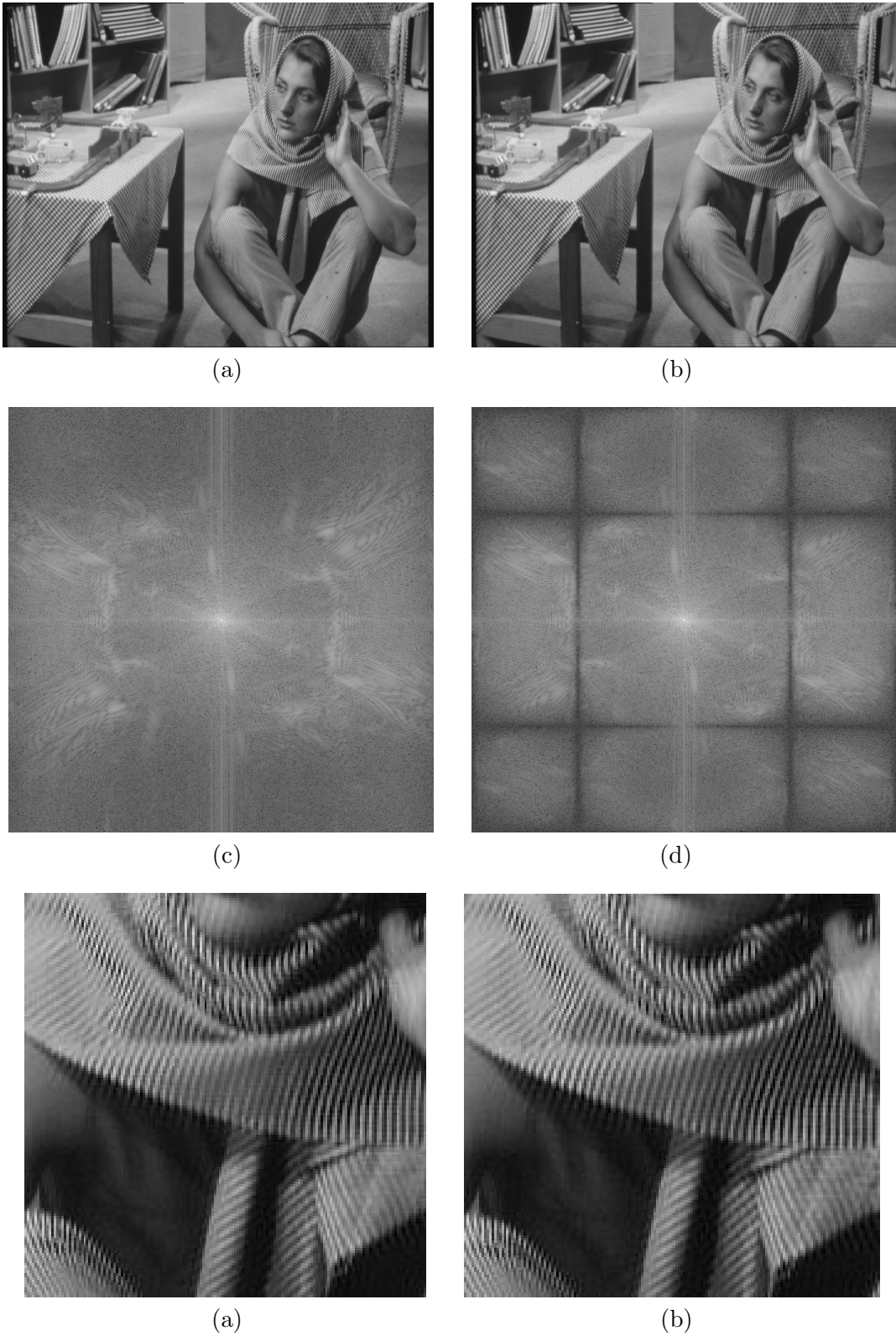


Figure 2.4: (a) **Fourier spectrum** of original *High Resolution* Barbara image. (b) **Fourier spectrum** of output image applying iterative *Super Resolution* algorithm (c) Zoomed and cropped Barbara image obtained from SR iterative algorithm. (d) same portion of the image after applying frequency domain filtering.

## Chapter 3

# Experimental Results

### 3.1 Super Resolution Image construction

Here we have shown some results obtained by applying ART and SART like *Iterative algorithm* which we have discussed detail in the previous chapter. In fig.3.1 we have shown some *low resolution* images of boat with different sub-pixel shifts with referred to first image as referenced image.



Figure 3.1: (a) A *low resolution* boat image of size  $128 \times 128$  taking as referenced image. (b) *low resolution* boat image with same size having sub-pixel shift  $(.64, -.32)$ . (c) *low resolution* boat image with same size having sub-pixel shift  $(.78, .16)$ . (d) *low resolution* boat image with same size having sub-pixel shift  $(-.32, .75)$ .

The fig.3.2(a)-(d) are the boat images obtained after applying ART like iterative algorithm with different no of iterations. In fig.3.2(e)-(f) are the boat image after converging iterative ART and SART like algorithms as discussed in previous chapter taking 64 low resolution boat image with four resolution factor along x and y direction.

We can apply the above *Super resolution* algorithms on color images also. For a color image we apply SR algorithm on every component (*viz.* **RGB**,  $YC_bC_r$ , **HSI** etc) sep-

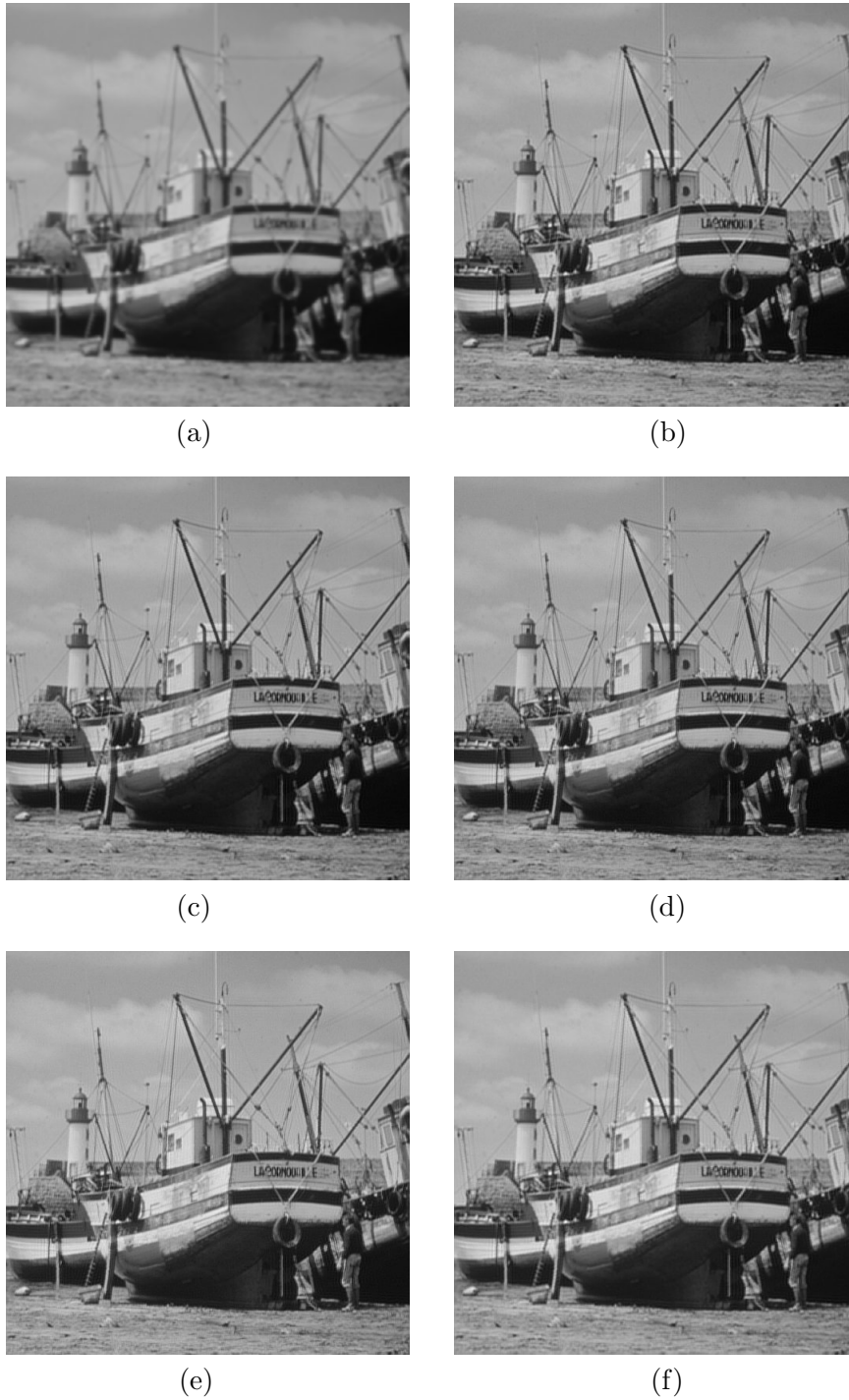


Figure 3.2: (a) Output of ART like algorithm after 1st iteration taking 64 low resolution image of size 128. (b)After 3th iteration. (c)After 5th iteration. (d) After 10 iteration. (e)-(f) After the iterative method converge (ART-SART)



arately and then combining we get SR image. In fig.3.3 we have shown results of Iterative algorithms on color images.

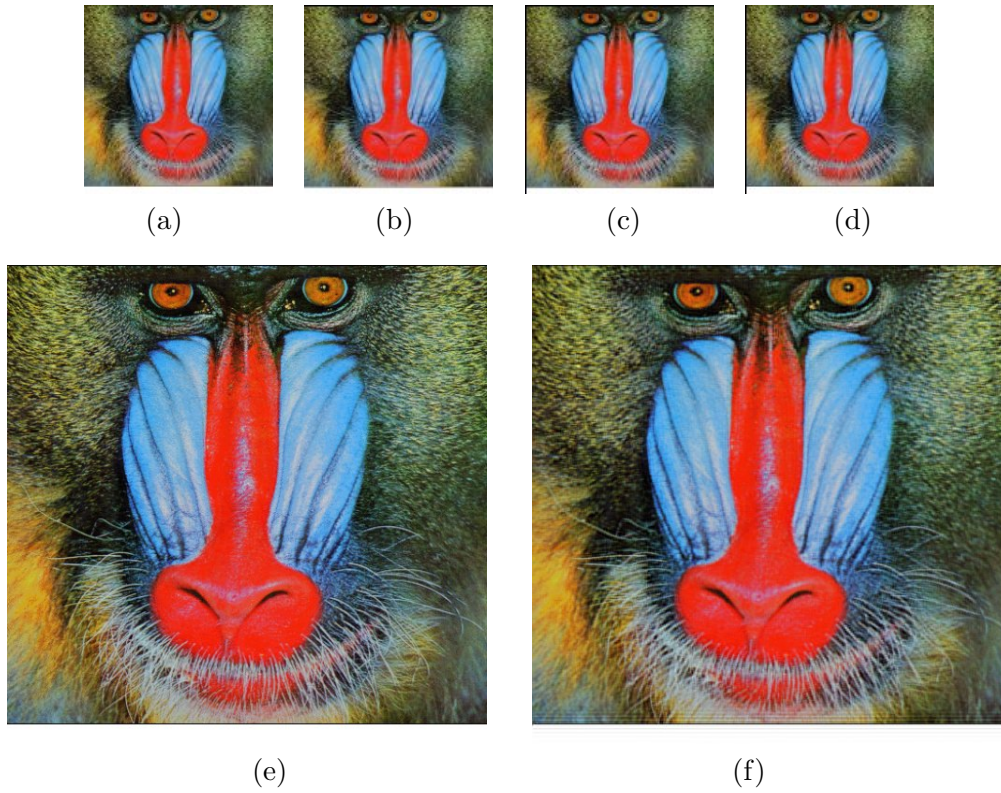


Figure 3.3: (a)-(d) low resolution color baboon images of size  $128 \times 128$  with different sub-pixel shifts (e) Original baboon image of size  $512 \times 512$ . (f) Output of *Iterative ART* algorithm taking input images as in RGB with resolution factor four.

## 3.2 Image Restoration

We applied image restoration technique on both spatial and frequency domain as discussed in the last chapter. Here we have shown some results of applying image restoration technique.

### 3.2.1 Spatial domain filtering

In spatial domain we used some adaptive filtering technique to remove periodic noise occurred during iterative process. In fig.3.4 we have shown the images before and after applying adaptive filter with the parameter (b)  $\sigma_\eta = 1$ , (c)  $\sigma_\eta = 3$ , (d)  $\sigma_\eta = 6$ .

### 3.2.2 Frequency domain filtering

As discussed on the section (2.4.2) we apply some frequency domain filtering technique. In Selective Gaussian Filtering (Notch Filter) on Fourier Transform, we have chosen parameter  $\sigma=4.2$  and  $t=1.4$  as shown in fig. 3.5(e). In fig.3.5(f) we convoluted Fourier Transform by mean of Fourier Transform difference of super resolved image and original image from a standard set of images of same size.

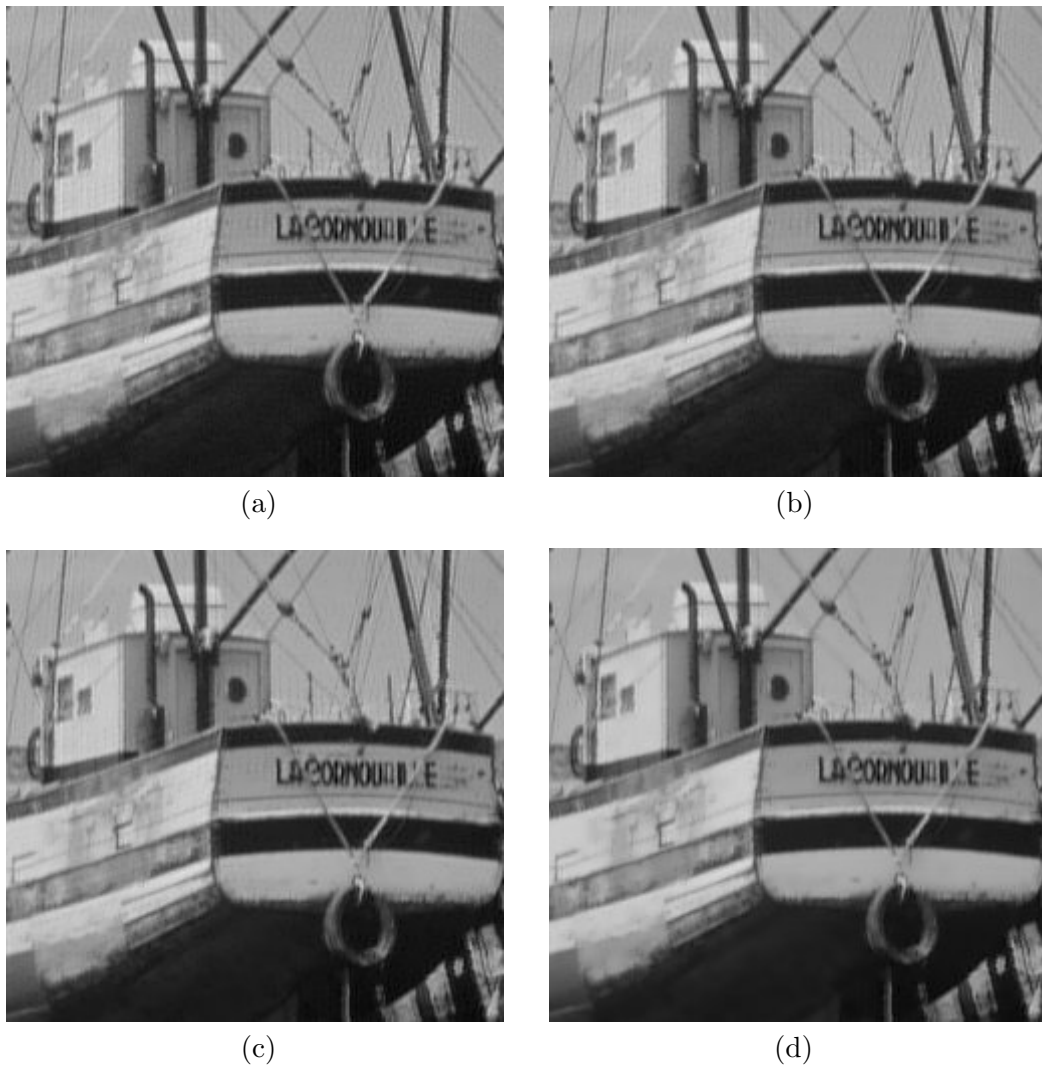


Figure 3.4: (a) zoomed and cropped boat image obtained from applying iterative algorithm for super resolution on low resolution images. (b)-(d) same images obtained after spatial domain adaptive filtering technique on (a) with different value of parameter  $\sigma_\eta$ .

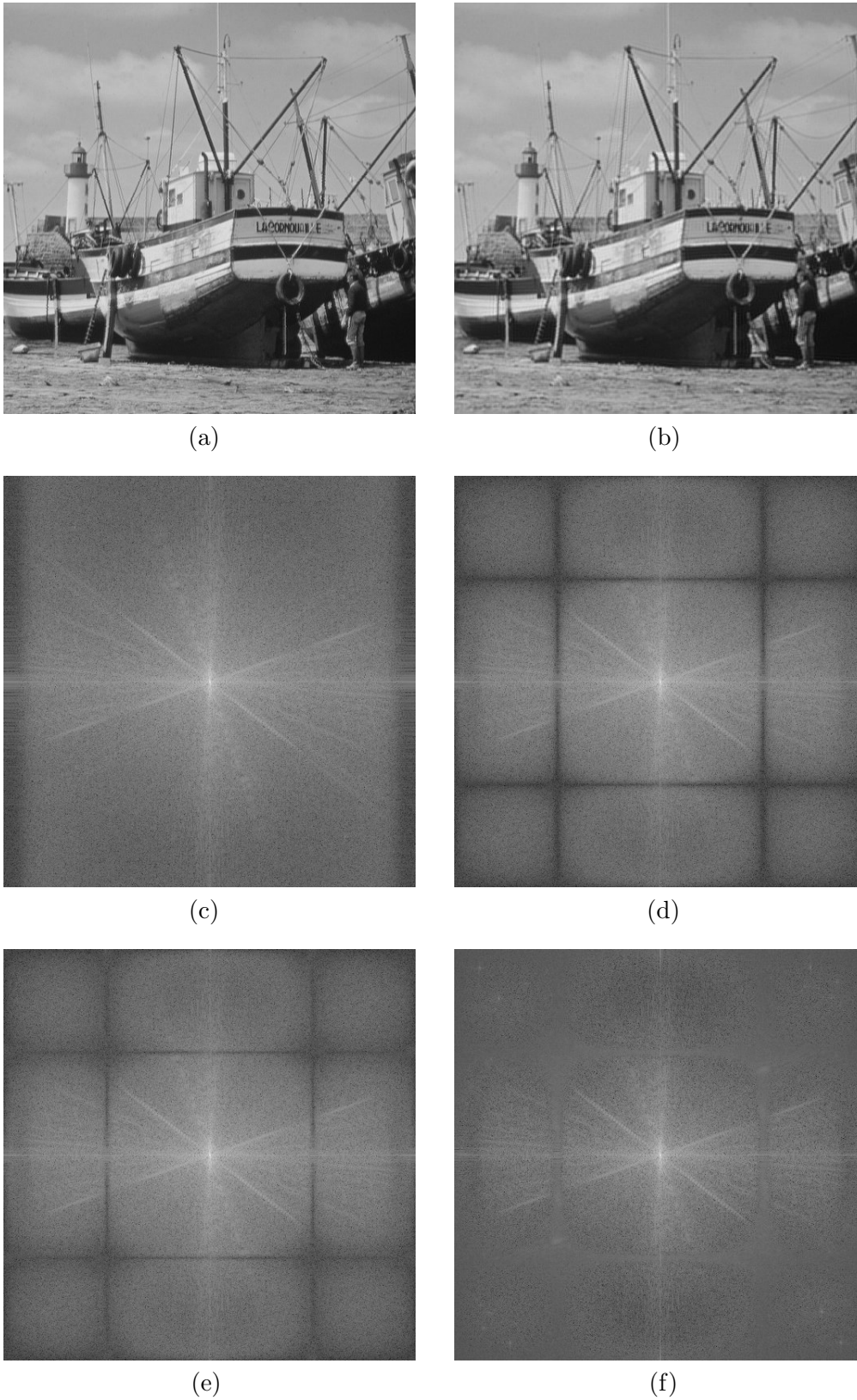


Figure 3.5: (a) Original boat image. (b) Super Resolved boat image (c) Fourier Spectrum of original image. (d) Fourier Spectrum of Super Resolved boat image. (e) Using Selective Gaussian Filtering on Fourier Transform of Super Resolved image. (f) Using mean difference of Fourier Transform.

## Chapter 4

# Conclusions and Future Work

In this paper the problem of reconstructing a high-resolution image from multiple degraded low-resolution images is addressed. In the first chapter we discussed some basic model of Super Resolution framework and why do we need super resolution in real-life application. Then we mathematically model for three main steps and corresponding results. Although the framework is presented for Image Registration and Reconstruction in the discrete spatial domain, modeling in discrete frequency domain may work faster and efficiently. Since each iterative algorithm discussed in second chapter is time consuming, however after regularization and with efficient data structure we could solve this problem with some extend, but evolving some more efficient algorithm for image reconstruction is still possible. Applying image restoration in frequency domain whatever we discussed in this report increases PSNR by some amount but it is really hard to recognize the difference in spatial domain. Hence modeling more efficient mask for image restoration technique in frequency domain is seems to be a good future work.

# Bibliography

- [1] **R. Y. Tsai, T. S. Huang**, “Multiframe image restoration and registration”, in *Advances in Computer Vision and Image Processing*, JAI Press Inc., pp. 317-339, 1984.
- [2] **Brian C. Tom, Aggelos K. Katsaggelos**, “Reconstruction of a high-resolution image by simultaneous registration, restoration and interpolation of low-resolution images”, in *Proc. of Int Conf. Image Processing*, Washington D.C., pp. 539-542, 1995.
- [2] **Michal Irani, Shmuel Peleg**, “Motion analysis for image enhancement resolution, occlusion and transparency”, *Journal of Visual Communication and Image Representation*, vol. 4, no. 4, pp.324-335, December 1993.
- [3] **R. R. Schultz and R. L. Stevenson**, “Extraction of high-resolution frames from video sequences”, *IEEE Trans. on Image Processing*, vol. 5, pp. 996-1011, June 1996.
- [4] **Andrew J. Patti, M. Ibrahim Sezan, and A. Murat Tekalp**, “Superresolution video reconstruction with arbitrary sampling lattices and nonzero aperture time”, *IEEE Trans. on Image Processing*, vol. 6, no. 8, pp. 1064-1076, August 1997.
- [5] **Farsiu, S., Robinson, M.D., Elad M., and Milanfar, P.**, “Fast and robust multi-frame super resolution”, *IEEE Transaction on Image Processing*, (2004) vol.13, 1327-1344
- [6] **Subhasis Chaudhuri**, “Super Resolution Imaging ”, pp. 73-105, 2003.
- [7] **Rafael C. Gonzalez, Richard E. Woods**, “Digital Image Processing ”, *Pearson Prentice Hall, Third Edition*, pp. 199-388, 2008.

- [8] **Bhabatosh Chanda, Dwijesh Dutta Majumder**, “Digital Image Processing and analysis ”, *Prentice Hall of India*, 2000.
- [9] **Ishita De, Bhabatosh Chanda, Buddhajyoti Chattopadhyay** “Enhancing effective depth-of-field by image fusion using mathematical morphology ”, *Image and Vision Computing*, pp. 1278-1287, 2006
- [10] **Deepu Rajan, Subhasis Chaudhuri**, “A physics-based approach to generation of super-resolution images,”in *Proc. Indian Conf. on Comp. Vis., Graphics and Image Proces*, New Delhi, India, pp. 250-254, Dec. 1998.
- [11] **Terrance E. Boulton, Ming-Chao Chiang** “Super Resolution via imaging warping ”, *Image and Vision Computing Journal Special Issue*, 2000.
- [12] **Aggelos K. Katsaggelos, Rafael Molina and Javier Mateos** “Super Resolution of Images and Video ”, *Synthesis Lectures on Image Video and, Multimedia Processing 7*
- [13] **R. Gordon, R. Bender, G.T. Herman**, “Algebraic reconstruction techniques (ART) for three-dimensional electron microscopy and x-ray photography” , *Journal of Theoretical Biology*, 471-482, 1970.
- [14] **B. Zitova, J. Flusser**, “Image registration methods”, a survey, *Image and Vision Computing*, vol. 21, pp. 977-1000, November 2003.
- [15] **N. P. Galatsanos, M. Wernick, and A. K. Katsaggelos**, “Multichannel Image Recovery ”, in *Handbook of Image and Video Processing*, A. Bovik, editor, ch. 3.7, pp. 161-174, Academic Press, 2000.
- [16] **B. C. Tom and A. K. Katsaggelos**, “Resolution Enhancement of Monochrome and Color Video Using Motion Compensation” , *Trans Image Proc.*, vol. 10, no. 2, pp. 278-287, Feb. 2001.
- [17] **Ming Jiang, Ge Wang**, “Development of iterative algorithms for image reconstruction” , *Journal of X-Ray Science and Technology* pp. 77-86, 2002