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Some Problems on Guarding Monotone Polygons

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CERTIFICATE

This is to certify that the dissertation entitled **"Some Problems on Guarding Monotone Polygons"** submitted by **Ratan Lal** to Indian Statistical Institute, Kolkata, in partial fulfillment for the award of the degree of **Master of Technology in Computer Science** is a bonafide record of work carried out by him under my supervision and guidance. The dissertation has fulfilled all the requirements as per the regulations of this institute and, in my opinion, has reached the standard needed for submission.

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Abstract

The art gallery problem is a well-studied visibility problem in computational geometry. It originates from a real-world problem of guarding an art gallery with the minimum number of guards who together can observe the whole gallery. In the computational geometry version of the problem the layout of the art gallery is represented by a simple polygon and each guard is represented by a point in the polygon. A set S of points is said to guard a polygon if, for every point p in the polygon, there is some q $\in$ S such that the line segment between p and q does not leave the polygon. Finding the smallest cardinality of guarding set of simple polygon is known to be NP-hard. Many researcher approached for an approximation algorithm. Subhir K. Ghosh [reference 1] proposed $log(n)$-factor approximation algorithm for simple polygon in $O(n^4)$ in 2010. It is also known that There exist a constant $\varepsilon \geq 0$ such that an approximation ratio of $1 + \varepsilon$ can not guaranteed by any polynomial time approximation algorithm unless P = NP. In a recent paper, B. J. Nilsson [2013] proposed a 30-factor approximation algorithm for monotone polygon. L. Gewali [1992] proposed an $O(n)$ time algorithm for covering a horizontally convex orthogonal polygon with minimum number of orthogonal star-shaped polygons. In this thesis, we are dealing with the art gallery problem for uni-monotone and special case of monotone polygon. For simple uni-monotone, we are assuming that there is some fixed guards G already placed inside the polygon P. If G covers the whole polygon then can we partition it to k-sets such that each set individually covers P. If we get such a partition, G is said to be fault tolerant at level k. In case the preplaced guard can not partition in to k-sets, find the smallest number of extra guards that is to be added to G such that it can be partitioned into k-sets which individually covers P. For orthogonal uni-monotone polygon, we are showing $O(nlogn)$ time algorithm for optimal guarding. For monotone polygon, If we are considering only upper chain(lower chain) to form a graph based on visibility region of convex pieces inside the polygon then this graph is not chordal. We are also showing sub case of monotone polygon for optimal guarding. Our result can be used in the geometrical application where there is requirement to maintaining fault tolerant. For example, In ad-hoc network there is need to maintain fault tolerant at some level so by modifying the definition of coverage of point, our result may be used.

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Chapter 1

Introduction

The original art gallery problem was introduced by Victor Klee in 1973 in a discussion with Vasek Chvatal. This first problem asked for the following question: How many guards situated in the vertices of a gallery with n walls are always enough and sometimes necessary to see all points inside this gallery restricted to the shape of a simple polygon?. Two years later, Chvatal [reference 7] solved it, demonstrating that $\lfloor \frac{n}{3} \rfloor$ guards cover all possible galleries. This was the beginning of art gallery problem variation studies; changing gallery shape, changing guard situation and mobility, etc.. A set S of points is said to guard a polygon if, for every point p in the polygon, there is some q $\in$ S such that the line segment between p and q does not leave the polygon. We call this set S a guarding set. The optimization problem is thus defined as finding a Guard set of smallest possible cardinality. Many variation of this problem is studied but the most popular variations are namely vertex guarding, edge guarding and point guarding. Vertex guarding deals with the case when guards are restricted to place on vertices of the polygon. Edge guarding deals with the case when guards are restricted to place on boundary of polygon. Point guarding deals with the case when guards can be placed any where inside the polygon. Point guarding of simple polygon can be formulated in to Set Cover problem which is known to be NP-complete and can not be approximated to a constant approximation factor unless P = NP [reference 1]. Subir K. Ghosh proposed $O(logn)$-approximation algorithms with running time $O(n^4)$ for simple polygons. Later Justin Iwerk and Joseph S.B. Mitchell [reference 8] in 2012 proposed sufficient guard in term of reflex and convex vertices. Let $G(r, c)$ be the function for the guard number in terms of $r \geq 0$, of reflex vertices and $c \geq 3$, of convex vertices of simple polygon P $(n = r + c)$,

$$G(r,c) = \begin{cases} 1,\ if\ r = 0 \\ r,\ if\ r \leq \left\lfloor \frac{c}{2} \right\rfloor \\ \left\lfloor \frac{n}{3} \right\rfloor,\ if\ \left\lfloor \frac{c}{2} \right\rfloor < r < 5c - 12 \\ 2c - r,\ if\ r \geq 5c - 12 \end{cases} \tag{1.1}$$

The terrain guarding problem can be divided in two problems, a discrete version and a continuous version. The discrete version focuses on guarding only the vertices of the terrain (or some discrete set of chosen points). Wherever the continuous version focuses guarding on entire terrain. Guarding a terrain is also NP-hard. Erik Krohn and James King [reference 2] gave a proof of that. Ben-Moshe [reference 3] proposed the first constant factor approximation. Later J. King's paper [reference 4] provides a 5-approximation to the terrain guarding problem. Erik Krohn and B. J. Nillson has proved its vertex guarding of monotone polygon to be NP-Hard in [reference 5]. But its point guarding does not immediately follow from that claim. The same authors Erik Krohn and B. J. Nillson has proved its point guarding to be NP-Hard [reference 6]. Erik Krohn and B. J. Nillson also gave constant factor 30-approximation algorithm for point guarding monotone polygons. Gewali et al.[reference 9] and Lingas et al.[reference 10] separately proposed linear time algorithm for guarding monotone orthogonal polygons with star shaped polygons. In this thesis, our primary focus on guarding different variation of uni-monotone polygon and sub case of monotone polygon. We also taking care of guarding with preplaced guard set and partitioning the guard set in to k-sets such that each set individually covers whole polygon. We have shown this approach may be useful in Ad-hoc networking. In Chapter 2, we have shown preliminaries things which is required to work on guarding problem. In chapter 3, we have shown different variation of guarding for uni-monotone polygon. In chapter 4, we have shown different variation of guarding for monotone polygon. In chapter 5, we have concluded the paper and proposed a few further direction of research.

Chapter 2

Preliminaries

2.1 Monotone Polygons and Terrains

Below we introduce some common definition on polygon and Terrain.

Definition *(Simple Monotone Polygon) A polygon P in the plane is called monotone with respect to a straight line l, if every line orthogonal to l intersects P at most twice.*

Definition *(X-Monotone Polygon) A monotone polygon with respect to x-axis is called x-monotone polygon.*

Definition *(Y-Monotone Polygon) A monotone polygon with respect to y-axis is called y-monotone polygon.*

Definition *(Orthogonal Polygon) A orthogonal polygon is a polygon all of whose edges meet at right angles. Thus the interior angle at each vertex is either* 90° *or* 270°.

Definition *(Orthogonal Monotone Polygon) A orthogonal polygon P is said to be orthogonal monotone with respect to given line l if any line perpendicular to l makes either empty or single line segment inside P.*

Definition *(1.5D Terrain) Usually when we use terrain we generally use a special version of it which is 1.5D terrain. It is an x-monotone chain T consisting of a set of points* $p_1(x_1, y_1)$, $p_2(x_2, y_2)$,$\cdots$, p_n (x_n, y_n) *where* (p_i, p_{i+1}) *are connected by a line segment, i = 1, 2,,*$\cdots$*, n-1 and the line joining* p_1 *and* p_n *does not intersect the chain.*

Definition *(Visibility Polygon) Given a polygon P and an interior point q, the visibility polygon of a point q is said to be the area inside the polygon P such that for any point p in this area, there is line segment joining p and q does not intersect the boundary of polygon P.*

2.2 Some definition and results on Graph & Approximation Algorithm

Definition *(Graph) A graph is an ordered pair $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines, which are 2-element subsets of V.*

Definition *The complement graph $\overline{G} = (V, \overline{E})$ of a graph $G = (V, E)$ is defined by $\overline{E} = \{xy : x, y \in V \text{ and } x \neq y \text{ and } xy \notin E\}$.*

Definition *Let $G = (V, E)$ be a graph.*

- *A graph $G' = (V', E')$ is a sub graph of G if $V' \subseteq V$ and $E' \subseteq E$.*
- *A sub graph $G' = (V', E')$ is an induced sub graph of G if $E' = \{uv : uv \in E \text{ and } u, v \in V'\}$. We also say that G' is induced by V' and usually write $G(V')$ for G'.*
- *A graph property P is hereditary if the property P holds for every induced sub graph of G whenever it holds for G.*

Definition *Let $G = (V, E)$ be a graph.*

- *$V' \subseteq V$ is an independent set or stable set in G (or empty sub graph of G) if for all $u, v \in V', uv \notin E$.*
- *$V' \subseteq V$ is a clique in G (or complete sub graph) if for all $u, v \in V', u \neq v, uv \in E$.*
- *A stable set (clique) S in G is maximal if there is no stable set (clique) $S' \neq S$ in G with $S \subset S'$.*
- *A stable set (clique) S is maximum if $|S|$ is the maximum possible size of a stable set (clique) in G.*

Definition *Let $G = (V, E)$ be a graph.*

- $\alpha(G) = max\{|V'| : V' \subseteq V \text{and } V' \text{ is an independent set in } G\}$
- $\omega(G) = max\{|V'| : V' \subseteq V \text{and } V' \text{ is a clique in } G\}$
- $\chi(G) = min\{k : \exists \text{ a partition of } V \text{ in to } k \text{ disjoint independent sets}\}$

- $\kappa(G) = min\{k : \exists$ *a partition of V in to k disjoint cliques*$\}$

  Note 1: *for every graph G,* $\omega(G) \leq \chi(G)$ *and* $\alpha(G) \leq \kappa(G)$. $\chi(G)$ *often called the chromatic number of G, since a partition* $V_1, V_2, ..., V_k$ *of V in to independent sets* V_i*, i = 1, 2, ..., k, is exactly a coloring of G such that no two adjacent vertices have the same color.*
  Note 2: *for every graph G,* $\alpha(\overline{G}) = \omega(G)$ *and* $\chi(\overline{G}) = \kappa(G)$. *It is well known that determining each of the parameters* $\alpha(G), \omega(G), \chi(G), \kappa(G)$ *is an NP-complete problem.*

There is some classes of graph for which above parameters can be found optimally.

1. *Chordal Graph*
2. *Interval Graph*
3. *Perfect Graph*

Definition *(Perfect Elimination Order) A perfect elimination order* v_1, v_2,$\cdots$, v_n *such that Pred(*v_i*) is a clique for all i = 1, 2,*$\cdots$*, n.*

There is algorithm for finding perfect elimination order of graph G if one exist.

Algorithm 1: PerfectEliminationOrder

Input: Graph G.
Output: Perfect elimination order of vertices of G if one exists.

begin
    for i = n,$\cdots$, *1* **do**
        Let G_i be graph induced by V - $\{v_{i+1}, \cdots, v_n\}$.
        Test whether G_i has simplicial vertex v.
        if *NO* **then**
            stop. // G_i has no perfect elimination order.
        else
            v_i = v;
    $v_1, v_2, \cdots, v_n$ is a perfect elimination order.
end

Lemma 2.2.1 *Let $v_1, v_2, \cdots, v_n$ be a perfect elimination order. Then $C =$ Pred(v_i) $\cup \{v_i\}$ is not a maximal clique if and only if there exists a successor v_j of v_i such that v_i is the last predecessor of v_j and indeg(v_j) = indeg(v_i) + 1.*

Proof Assume there exists such a successor v_j. Since v_i is v_j's last predecessor, all predecessors of v_j are either v_i or a predecessor of v_i,so *Pred(v_i)* $\subseteq$ *Pred(v_i)* $\cup \{v_i\}$ = C. By *indeg(v_j) = indeg(v_i) + 1*, equality holds, so v_j is adjacent to all vertices in C, and C $\cup \{v_j\}$ is a bigger clique.
For the other direction, assume C is not maximal. Let j be the minimal such that $v_j \notin C$ and C $\cup \{v_j\}$ is a clique.Vertex v_j is adjacent to v_i, but it is not a predecessor, otherwise it would be in C. So v_j is a successor of v_i, which implies C $\subseteq$ *pred(v_j)* and *indeg(v_j)* $\geq$ *indeg(v_i) + 1.*
We claim that v_i is the last predecessor of v_j. Assume it is not, so v_j has a predecessor v_k with $i < k < j$. Then v_k is adjacent to all of C, and C U $\{v_k\}$ is a clique, contradicting the minimality of j. Therefore, any predecessor of v_j is either v_i or a predecessor of v_i, so *pred(v_j)* $\subseteq$ C and *indeg(v_j)* $\leq$ *indeg(v_i) + 1.* This proves the claim. $\square$

There is linear time algorithm for finding all maximal cliques of chordal graph.

Algorithm 2: AllMaximalClique

Input: Chordal Graph G.
Output: All maximal cliques.

begin
  for j = *1, 2,*$\cdots$*,* n **do**
    find all predecessors of v_j
    store *indeg(*v_j*)* and which vertex is the last predecessor of v_j
  for i = *1, 2,*$\cdots$*,* n **do**
    find all successors of v_i
    for *each* *successor* v_j *of* v_i **do**
      if *if (*v_i *is the last predecessor of* v_j *and indeg(*v_j*)=indeg(*v_i*)+1)* **then**
        discard *pred(*v_i*)* $\cup$ $\{v_i\}$ //it is not a maximum clique
    if *pred(*v_i*)* $\cup$ $\{v_i\}$ *has not been discarded* **then**
      output it as one maximal clique of the graph.
end

Note: This algorithm takes O(*deg(v) + 1*) time per vertex, and hence has linear time.

Definition *(simplicial vertex) A simplicial vertex of a graph G is a vertex such that the neighbours of v form a clique in G.*

Definition *(Chordal Graph) A graph G is a chordal graph if it does not contain an induced k-cycle for* $k \geq 4$*.*

Definition *(Comparability Graph) A graph that has an acyclic transitive orientation is called a comparability graph.*

Definition *(Interval Graph) A graph G is an interval graph if and only if G is a chordal graph and* $\overline{G}$ *is a comparability graph.*

Definition *(Alternative)(Interval Graph) A graph G is an interval graph iff there exists a linear order of its maximal cliques such that for each vertex v, all maximal cliques containing v are consecutive.*

Remark Some result about Interval graph G

- Interval graph can be recognized in O(n) time.

- Largest clique can be computed in O(nlog(n)) time.
- Minimum clique cover can be computed in O(nlog(n)) time.
- Maximum Independent set can be computed in O(nlog(n)) time.

Definition *(Perfect Graph) A graph G is perfect if* $\omega(G) = \chi(G)$ *and* $\omega(H) = \chi(H)$ *for every induced sub graph H of G.*

Theorem 2.2.2 *The graph G has perfect elimination order if and only if graph G is chordal.*

Definition *(Approximation Algorithm) An* α*-approximation algorithm is a polynomial-time algorithm which always produces a solution of value within* α *times the value of an optimal solution.*
That is, for any instance of the problem,
Zalgo / Zopt $\leq \alpha$*, (for a minimization problem)*
where Zalgo is the cost of the algorithm output,
Zopt is the cost of an optimal solution.
α *is called the approximation guarantee (or factor) of the algorithm.*

Chapter 3

Guarding Uni-Monotone Polygon

3.1 Simple Uni-Monotone Polygon

Definition *(Simple Uni-Monotone Polygon) A simple monotone polygon with respect to line l is called uni-monotone if one of its chain is l-monotone chain and other is the line l.*

Lemma 3.1.1 *If we rotate two lines with respect to a point in same direction then the angle of intersection between lines remain same.*

Proof Let us assume two intersecting line l_1 and l_2 have slope θ_1 and θ_2 respectively and without loss of generality $\theta_1 < \theta_2$. Now after rotating θ angle both lines in anti clock wise direction, slope becomes θ_1' and θ_2' respectively, so $\theta_1' < \theta_2'$ and $\theta_1' = \theta_1 + \theta$, $\theta_2' = \theta_2 + \theta$.
Let us take after rotation angle between line l_1 and l_2 be θ'. so $\theta' = \theta_2'$ - θ_1' = θ_2 - θ_1. □

Lemma 3.1.2 *Every uni-monotone polygon with respect to line l can be converted in to uni-monotone polygon with respect to x-axis.*

Proof Since rotation of lines in same direction does not change the angle of intersection among all rotating lines. Rotate whole polygon with angle θ, where θ is the angle of intersection between line l and x-axis. After rotation, all the internal angles are same, so structure of polygon does not change. Thus rotating polygon again uni-monotone polygon with respect to x-axis. □

Theorem 3.1.3 *Minimum number of guards required to cover all the edges of uni-monotone polygon is equal to minimum number of guards required to cover whole uni-monotone polygon.*

Proof Let us take uni-monotone polygon P, G be the guard set with smallest possible cardinality to cover all the edges and G' be another guard set with smallest possible cardinality to cover whole polygon P. Now we have to show that $|G| = |G'|$.
(i) $|G| \leq |G'|$
Since G' cover whole polygon P, so G' also cover all the edges of P ($\because$ P is bounded by edges).
(ii) $|G'| \leq |G|$
Let us assume that guard set G are not sufficient to cover whole polygon P, then $\exists$ non empty region R inside the polygon P which is not seen by guard set G.
Let us take a point p $\in$ R and make a vertical line l passing through p. Since P is uni-monotone, so line l must intersect the polygon P exactly two point (say A and B). Since $\exists$ guard g $\in$ G can see both point A and B, so g can see any point on line l inside the P i.e. guard g can see p.
Similarly $\forall$ p $\in$ R, $\exists$ guard g $\in$ G such that g can see point p. Since all point p $\in$ R can be seen by guard set, so R is empty. This is contradiction to our assumption.
From (i) and (ii),
$|G| = |G'|$. □

Observation Minimum number of guards required to see the edges of P is not equal to minimum number of guards required to see the vertices of P.

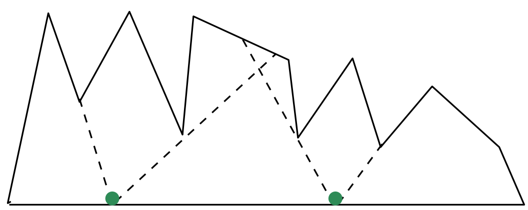

Figure 3.1: Counter Example For Vertex and Edge Guarding

3.1.1 Transformation of Guarding problem in to Graph

We need to define projection of edge in uni-monotone polygon P with respect to x-axis.

Definition *Projection of edge e on x-axis is the intersection of V(e) and the line x-axis. where V(e) is the set of points of P such that each point $p \in$ V(e) can see edge e.*

Lemma 3.1.4 *Projection of each edge e form single interval on x-axis.*

Proof Let us assume projection of edge e does not form single interval on x-axis then there are two cases:
case (i): Projection of edge e does not form interval on x-axis i.e. there is no point p on x-axis such that p can see edge e. Since P is uni-monotone so $\exists$ vertical line l of x-axis which intersect edge e at point A and x-axis at point p. p can see point A. Let us take C and D are end points of edge e. Now $\triangle$pCA and $\triangle$pAD are formed, so p can see CA and AD. Thus p can see whole edge e. This is contradiction to our assumption.
case (ii): Let us assume projection of edge e form more than one interval on x-axis i.e. $\exists$ two interval (a_1, b_1), (a_2, b_2) on x-axis this implies that no point p between b_1 and a_2 can see whole edge e. Let us take a point p between b_1 and a_2. Since b_1 and a_2 both can see C so b_1, a_2 and C form a triangle, so p can see C. Similarly p can see D. since pCD form a triangle so p can see whole edge e. This is contradiction.
In both cases we found contradiction. Thus projection of each edge e form single interval on x-axis. □

Transformation:

Let us take simple uni-monotone polygon P with (V, E) where V is the set of vertices and E is the set of edges in P with respect to x-axis. Consider the sequence of convex pieces of upper chain C_1, C_2, $\cdots$, C_k arranged in order, where each piece consist of at least two vertices among which first and last vertices are reflex except C_1 and C_k. Last vertex of first convex piece and first vertex of last convex piece must be reflex. $\cup_{i=1}^{i=k} C_i$ consist of all the vertices of the P. For each convex piece we define interval by common portion of intervals of projection of all edges of convex piece on x-axis. Thus there are k number of intervals $[a_i, b_i]$ where i = 1, 2,$\cdots$, k.
Now define graph G = (V', E') where V' contains vertices corresponding to each interval and edge between two vertices u, v $\in V'$ if corresponding intervals overlaps each other.

3.1.2 Guarding The Region of Polygon With Minimum Guards

Lemma 3.1.5 *If the vertices of graph G are arranged in order to left end of corresponding interval on x-axis then this order of vertices are perfect elimination order.*

Proof Let us assume vertices order based on left end point of corresponding interval does not follow perfect elimination order. This means that there exist vertex v_i such that $pred(v_i) \cup \{v_i\}$ does not form clique. i.e. there exist vertex v_j and $v_k \in pred(v_i)$ such that $(v_j,\, v_k) \notin E'$. Since $(v_j,\, v_i)$ and $(v_k,\, v_i) \in E'$ so interval corresponding v_j and v_k must overlap on the vertical line l passing through the left end point of interval corresponding to v_i. Thus v_j and v_k must overlap on line l i.e. there is an edge $(v_j,\, v_k) \in E'$. This is contradiction to our assumption so vertices order based on left end point of corresponding interval are the perfect elimination order. □

Note: Above graph G = $(V',\, E')$ is chordal.

Lemma 3.1.6 *Graph $\overline{G}$ is comparability graph.*

Proof Let us assume u and v are two vertices that are not adjacent in G i.e. adjacent in $\overline{G}$. As each vertex represent interval so two intervals I_u, I_v of these vertices do not intersect. There are now two possibilities Either I_u is to the left of I_v, or I_u is to the right of I_v. This naturally imposes edge direction for $\overline{G}$. For a pair u, v $\notin$ E(G), direct the edge as u→v if I_u is to the left of I_v and as v→u otherwise.
(Transitive Orientation) Let us assume graph $\overline{G}$ does not have transitive orientation then there exist three vertices u, v, w such that (u,v) and (v, w) are edges of $\overline{G}$ but (u,w) is not an edge of $\overline{G}$. This implies edge (u,w) belong from edge set in G. since interval corresponding v in G in the middle of interval corresponding u and w so there must be overlap between interval corresponding either u and v (or) v and w. This is contradiction to our assumption. Thus $\overline{G}$ have transitive orientation.
(Acyclic Orientation) Let us assume graph $\overline{G}$ has cycle then there exist some vertices of $\overline{G}$ are in cycle order v_1, v_2,$\cdots$, v_n where $v_1 = v_n$. since there is edge (v_{n-1}, v_1) so interval corresponding v_{n-1} is to left of v_1 and by transitive property, there should be edge $(v_1,\, v_{n-1})$ i.e. v_1 is to left of v_{n-1}. This is contradiction, Hence graph $\overline{G}$ have acyclic orientation. Thus $\overline{G}$ have acyclic and transitive orientation so this is comparability graph. □

Theorem 3.1.7 *Graph G is interval graph.*

Proof Since graph G is chordal and $\overline{G}$ is comparability graph so this is interval graph. □

Theorem 3.1.8 *Finding the minimum number of guard required to cover uni-monotone polygon P have taken $O(n + klog(k))$ time, where n is the number of vertices of P and k is the number of intervals.*

Proof Since transformation of polygon in to interval graph will take $O(n)$ time where n is number of vertices. After transformation, finding the minimum clique cover of interval graph will take $O(klog(k))$ time, where k is number of intervals. so finding the minimum number of guards have taken $O(n + klog(k))$ time. □

3.1.3 Guarding With Some Fixed Preplaced Guards

Let us take simple uni-monotone polygon P and some fixed guard locations L where guards are already placed inside P. Now following operation can be done in $O(n + klogk)$ time, where k is number of intervals of convex pieces on x-axis. Whether r fixed preplaced guards are sufficient for point guarding of P. Can we partition of guarding set in to m sets such that each set individually cover P. In case, if there is no partition of guarding set in to m sets then find the minimum number of extra guards.

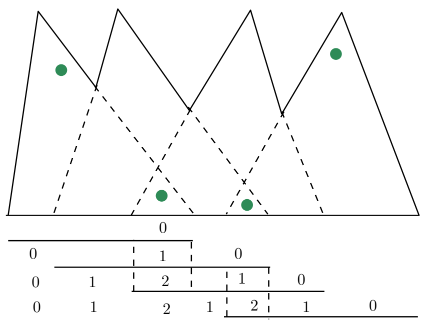

Figure 3.2: Two Partition Guarding

In following algorithm, clique region are the intersection visibility region of convex pieces associated with maximal clique interval.

Algorithm 3: Guard Uni-monotone Polygon

Input: Uni-monotone Polygon P and fixed pre-placed guard locations L, partition size m

Output: Yes/No

Initialization: $C_R = \phi$, I = ϕ, C = ϕ, $L_R = \phi$, $C_R = \phi$, $G_R = \phi$
I = Find intervals of convex pieces on x-axis of polygon P
C = Find smallest number of maximal clique intervals which covers I
for ***each*** *c in C* **do**
    Find clique region R for maximal clique interval c
    $C_R = C_R \cup \{R\}$
for ***each*** *R in C_R* **do**
    L_R = List of visibility region of convex pieces associated with clique region R
Return Yes, if each clique region R in C_R have atleast m guard location from L.
Find the clique regions having less than m guard location from L (say G_R).
for ***each*** *R in G_R* **do**
    L_R = List of visibility regions of convex pieces associated with clique region R
    if *each region in L_R have atleast m guard location from L* **then**
        continue;
    else
        Return No
Return Yes

Algorithm 4: Find Extra Guard

```
Input: Uni-monotone Polygon P, Guard Locations L, G_R, partition
       size m
Output: Minimum number of extra guard
Initialization: MinExtraGuard = 0
for each R in G_R do
    L' = φ, minmax = 0;
    for each R_1 in L_R do
        if at least m guard location in R_1 then
            continue;
        else
            temp = min guard location required in region R_1 to be m
            guards.
            if minmax ≤ temp then
                minmax = temp;
        L' = Add minmax number of guard locations in R
        L = L ∪ L'
        MinExtraGuard += minmax
    Return MinExtraGuard
```

Algorithm 5: Guard Partition

Input: Uni-monotone Polygon P, Guard Locations L, C_R, partition size m

Output: Partition of size m

Initialization: $\mathcal{P} = \{S_1, S2, \cdots, S_m\}$ where $S_i = \phi$

begin

  for ***each*** R *in* C_R **do**

    for ***each*** R_1 *in* L_R **do**

      M_i = unmarked $\forall i = 1, 2, \cdots, m$

      G_m = guards in region R_1 which have already assign to some S_i.

      G'_m = guards in region R_1 which are not assigned to any S_i.

      for ***each*** g *in* G_m **do**

        mark M_i if g in S_i

      for i = *1 to* m **do**

        if M_i *is unmarked* **then**

          pick a guard from G'_m and assign to set S_i.

  Return $\mathcal{P}$

end

Lemma 3.1.9 *Visibility region of each convex piece have at least m guards iff $\exists$ a partition of guarding set in to m sets such that each set individually covers whole uni-monotone polygon P.*

Proof Let R = visibility region of each convex piece have at least m guards and S = $\exists$ a partition of guarding set in to m sets such that each set individually covers whole uni-monotone polygon P.

(i) R $\rightarrow$ S

Let us assume there is no partition of guarding set in to m sets such that each set individually covers P i.e. $\forall$ partition $\mathcal{P} = \{S_1, S_2, \cdots, S_m\}$. without loss of generality, Let $\mathcal{P}_1 = \{S_1, S_2, \cdots, S_n\}$ where S_j, $j = 1, 2, \cdots, n$ does not cover P.

$\mathcal{P}_2 = \{S_{n+1}, S_{n+2}, \cdots, S_m\}$ where S_k, $k = n+1, n+2, \cdots, m$ cover P.

$\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2$ and $\mathcal{P}_1 \neq \phi$

If S_j does not cover P then $\exists$ convex pieces $C^j = \{C_1, C_2, \cdots, C_l\}$ where $V(C_i)$ $i = 1, 2, \cdots, l$ does not cover by S_j.

Now, claim that $\exists$ r in $\{1, 2, 3, \cdots, n\}$ such that $\forall$ $t \neq r$, S_t has at most one guard from atleast one $V(C_i)$ where $C_i \in C^r$.

Let us assume our above claim is false i.e. $\forall$ r in $\{1, 2, 3, \cdots, n\}$ such that for each $C_i \in C^r$ $\exists$ $t \neq r$, S_t has more than one guard from $V(C_i)$. For each $C_i \in C^r$ pick one guard from S_t which belongs from $V(C_i)$ and put in S_r. So $\exists$ partition of guarding set in to m set such that each set individually covers P. This is contradiction to our assumption. By claim, $\exists$ r in $\{1, 2, 3, \cdots, n\}$ such that S_r does not cover whole P and $\exists$ convex piece $C_i \in C^r$ whose visibility region $V(C_i)$ does not cover by S_r. Since there is no S_t, $t \neq r$ has more than one guard from $V(C_i)$, so $V(C_i)$ contains at most $m-1$ guards. Thus for each partition of guarding set in to m set $\exists$ convex piece C_i such that $V(C_i)$ contains at most $m-1$ guards. By contrapositive, R $\rightarrow$ S.
(ii) S $\rightarrow$ R
Let us assume $\exists$ convex piece C_l such that $V(C_l)$ have at most $m-1$ guards. Since any guarding set must have atleast one guard from $V(C_l)$, so there are $m-1$ guarding set are possible i.e. there is no partition of guarding set in to m sets such that each set individually cover P. By contrapositive, S $\rightarrow$ R.
From **(i)** and **(ii)**,
Visibility region of each convex piece have at least m guards iff $\exists$ a partition of guarding set in to m sets such that each set individually covers whole uni-monotone polygon P.

3.2 Orthogonal Uni-Monotone Polygon

Definition *(Orthogonal Uni-Monotone Polygon) A orthogonal monotone polygon with respect to line l is called uni-monotone if one of its chain is l-orthogonal monotone chain and other is the line l.*

Definition *(Top edge) A horizontal edge in upper chain such that both end point are convex vertices of P. It is denoted as h_t.*

Definition *(Bottom edge) A horizontal edge in upper chain such that both end point are reflex vertices of P. It is denoted as h_b.*

3.2.1 Guarding The Region of Polygon With Minimum Star Shape Polygon

Definition *(Star shaped polygon) A polygon P is star shaped if it contains an interior point r such that for all point p of P $\exists$ a rectangle R aligned with edges of P such that R contains line segment $\overline{rp}$.*

Definition *(Orthogonally visible point) A point p is orthogonally visible from a guard g if and only if both p and g are contained in within a rectangle (with* *a non zero area whose sides are aligned with edges of the polygon) that lies entirely within the polygon.*

Lemma 3.2.1 *Each maximal star shaped polygon S of orthogonal uni-monotone polygon P contains exactly one top edge h_t of P.*

Proof Let us assume $\exists$ a maximal star shape polygon S of P does not have exactly one h_t. There are two cases arises:
case (i): S has no h_t i.e. S must be in form of either up stair or down stair star shape polygon. In case of up stair star shape polygon, there must be atleast one h_t which will be after last vertex in S. Let us take $S' =$ S $\cup$ include part of P from last vertex of S to h_t. Since S' form star shape polygon. This is contradiction to maximality of S, Similarly for down stair star shape polygon.
case (ii): S has more than one h_t. let us take h_{t_1}, h_{t_2} are top edges of P inside S, so $\exists$ point p $\in$ S such that there is possible to make rectangles R_1 with h_{t_1} and R_2 with h_{t_2} having point p i.e. $R_1 \cap R_2 \neq \phi$. Since h_{t_1} and h_{t_2} are not adjacent so any rectangles form with h_{t_1} and h_{t_2} do not intersect each other i.e. $R_1 \cap R_2 = \phi$. This is contradiction.
In both cases, we found contradiction. Thus Each maximal star shape polygon S of orthogonal uni-monotone polygon P contains exactly one top edge h_t. $\square$

Theorem 3.2.2 *Finding the minimum number of guards to cover orthogonal uni-monotone polygon P such that each point $p \in P$ is orthogonally visible point from at least one guard can be computed in $O(n)$ time.*

Proof Let us compute minimum number of star shape polygon which cover whole P. Since each maximal star shape polygon of P contains exactly one h_t, so minimum number of star shape polygon of P is equal to number of h_t in P. This can be computed in $O(n)$ time. For each star shape polygon S, one guard is enough for all p $\in$ S to orthogonal visibility so number of h_t are minimum number of guards required to cover whole P with orthogonal visibility. $\square$

3.2.2 Guarding The Vertices of Polygon With Minimum Guards

Lemma 3.2.3 *Visibility region of Any guard $g(g_x, g_y)$ inside orthogonal uni-monotone polygon P is subset of visibility region of guard g' at g_x on x-axis.*

Proof Let us assume V(g) $\nsubseteq$ V(g') i.e. $\exists$ point p $\in$ V(g) and p $\notin$ V(g'). Since g' below g so there must be an edge e of P below $\overline{gp}$ which intersect line segment $\overline{g'p}$ at point q. Now draw vertical line l passing through q and it intersect at s on x-axis. Since $\overline{gp}$ is completely inside the P so there must be boundary of P above $\overline{gp}$ so line l intersect on boundary at point r. Thus line l make two line segment $\overline{qs}$ and $\overline{qr}$. This is contradiction of P being orthogonal uni-monotone polygon. $\square$

Theorem 3.2.4 *Finding the minimum number of guards required to cover all the vertices of orthogonal uni-monotone polygon P can be computed in $O(nlogn)$ time.*

Proof Let us first compute projection of each vertices on x-axis that form intervals. Now we generate interval graph of these intervals and compute minimum clique cover in $O(nlogn)$ time. Since generating graph will take $O(n)$ time, so overall computation time $O(nlogn)$.

3.2.3 Guarding The Region of Polygon With Minimum Guards

Lemma 3.2.5 *Let V(g) and V(g') be the visibility region of guards g(g_x, g_y) and g'(g'_x, g'_y) in orthogonal uni- monotone polygon P respectively. If $g_x \geq g'_x$ then $V_l^L(g) \subseteq V_l^L(g')$ where l is the vertical line passing through g'_x and $V_l^L(g)$ is the part of V(g) which is left of line l.*

Proof Let us assume $V_l^L(g) \nsubseteq V_l^L(g')$ i.e. there exist point p $\in V_l^L(g)$ which can not be seen by guard g'. So line segment $\overline{gp}$ is possible and $\overline{g'p}$ is not possible. This implies there is edge of P below $\overline{gp}$ such that $\overline{g'p}$ is not possible. There exist a point q below $\overline{gp}$ such that vertical line passing through q divides in to two line segment inside the polygon. This is the contradiction of orthogonal uni-monotone polygon.

Approach for Placing Guards

Let us take orthogonal uni-monotone polygon P where lower chain is x-axis. We are placing guards on x-axis such that each guard must hold the following property:
P_1**:** Placement of i^{th} guard $g_i(g_{i_x}, g_{i_y})$ ensure that left part of P from line l must be visible by guard set $\{g_{1,2}, \cdots, g_i\}$ where l is vertical line passing through g_{i_x}.
P_2**:** $\forall\, \varepsilon > 0$ if g_i is shifted right with ε-distance then property P_1 should not

hold.

There are some notation which are used in following algorithm.
g_l : Latest placed guard location
p_g : The point where next guard is expected to be placed up to current visiting vertex.
e_l : Latest edge either top edge or bottom edge.
θ : Either this angle measure minimum angle from g_l to all reflex vertices up to next top edge with x-axis if e_l is bottom edge or measure maximum angle from p to all reflex vertices up to next bottom edge with x-axis if e_l is top edge.
p : Intersection point of top edge and line passing through g_l with angle θ.

Following algorithm holds above both property.

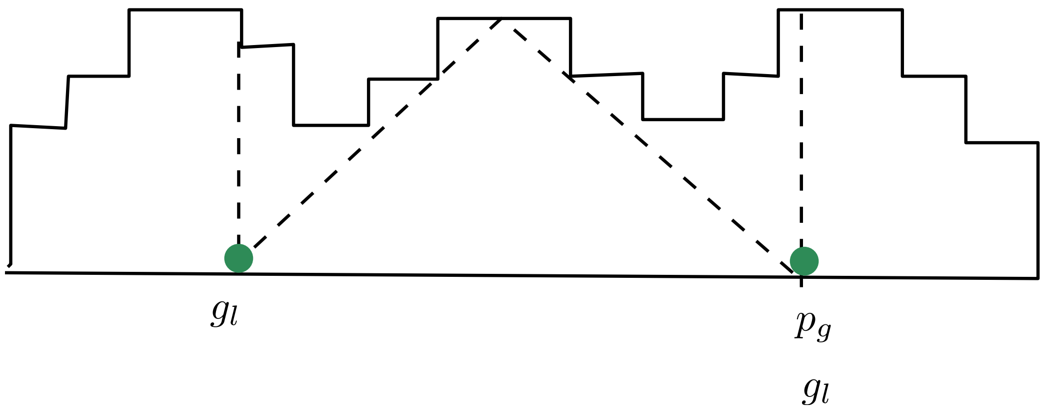

Figure 3.3: Guarding Orthogonal Uni-Monotone Polygon

Algorithm 6: *GuardingOrthogonalUnimonotonePolygon*

Input: A orthogonal uni-monotone polygon P.
Output: Minimum Guarding set G.

Initialization: $g_l = \phi$, $p_g = \phi$, $p = \phi$, $e_l = \phi$, $\theta = 90°$
begin
    Add each vertex in queue Q and u = delete(Q);
    // it return vertex which has lowest x-coordinates. In case of tie return lowest x-coordinates having lowest(highest) y-coordinates depends on e_l.
    while *Q is not empty* **do**
        v = delete(Q);
        if $p_g == \phi$ **then**
            ProcX(u, v);
        else
            ProcY(u,v);
        u = v;
    if $p_g \neq \phi$ **then**
        HandleGuard(v, e_l);
end

ProcX(u, v) **begin**

    if *edge uv is* h_t *and* $p == \phi$ **then**
        HandleGuard(v, e_l);
    else
        if *edge uv is* h_b *and* $p \neq \phi$ **then**
            q = intersection point of x-axis and line passing through p with angle θ;
            if $q_x \leq v_x$ **then**
                HandleGuard(v, e_l);
            else
                p_g = q;
        else
            if *v is reflex* **then**
                if e_l *is* h_t *and* $p \neq \phi$ **then**
                    θ_1 = angle between line pv and x-axis.
                    if $\theta \leq \theta_1$ **then**
                        $\theta = \theta_1$;
                if e_l *is* h_b *and* $g \neq \phi$ **then**
                    θ_1 = angle between line g_lv and x-axis.
                    if *(*$\theta \geq \theta_1$ **then**
                        $\theta = \theta_1$;

end

```
ProcY(u, v) begin
    if edge uv is $h_t$ then
        if $u_x \leq p_{g_x} \leq v_x$ then
            HandleGuard($p_g$, $e_l$);
        if $p_{g_x} \geq v_x$ and $p == \phi$ then
            HandleGuard(v, $e_l$);
    else
        if uv is $h_b$ then
            if $p \neq \phi$ then
                q = intersection point of x-axis and line passing through
                p with angle $\theta$
                if $u_x \leq p_{g_x} \leq v_x$ then
                    if $q_x \leq p_{g_x}$ then
                        HandleGuard(q, $e_l$);
                    else
                        HandleGuard($p_g$, $e_l$);
                else
                    if $q_x \leq p_{g_x}$ then
                        if $q_x \leq v_x$ then
                            HandleGuard(q, $e_l$);
                        else
                            $p_g$ = q;
end
```

```
HandleGuard(r, e) begin
    Place guard on x-axis at $r_x$.
    $g_l$ = r, $p_g = \phi$, $p = \phi$
    if e is $h_t$ then
        $\theta = 0°$
    if e is $h_b$ then
        $\theta = 90°$
end
```

Note: This algorithm will take $O(nlogn)$ time.

Lemma 3.2.6 *Guarding set G on x-axis of orthogonal uni-monotone polygon which satisfy properties P_1 and P_2 is unique.*

Proof Let us assume there are two guarding set G and G' which are not equal i.e. $\exists$ g $\in$ G and $g' \in G'$ be the first i^{th} guard on x-axis in G and G' which are not equal. Without loss of generality, let us take $g_x < g'_x$, By property P_2 if g is shifted right then it will loss property P_1. Since g' also hold property P_1 so right shifting of guard g must hold property P_1. This is contradiction to our assumption. □

Lemma 3.2.7 *If guarding set G and G' on x-axis of orthogonal uni-monotone polygon where G satisfy property P_1 but not P_2 and G' satisfy both properties then $|G| \geq |G'|$.*

Proof Let us take guard set $G' = \{g_1, g_2, \cdots, g_n\}$ such that $g_{1_x} < g_{2_x} < \cdots < g_{n_x}$. Since G' holds both properties so atleast one guard from G before g_1 otherwise G violate the property P_1.
Now my claim is that at least one guard g $\in$ G must be between g_i to g_{i+1} where i = 1, 2,$\cdots$, n-1. Let us assume this is not true i.e. $\exists$ i such that there is no guard g $\in$ G in between g_i to g_{i+1}. Let $g_r \in$ G be the right most guard in G before g_i and $g_l \in$ G be the left most guard in G after g_{i+1}. let us take vertical line l_1 passing through g_i and l_2 passing through g_l. Now, shifting guard g_{i+1} towards right at g_l. All point of p $\in$ P between l_1 and l_2 either seen by g_r or g_l. If p is seen by g_r then it must be seen by g_i. If p is seen by g_l then it must be seen by g_{i+1}. Thus all left part of polygon from line l_2 is seen by guards $\{g_1, g_2, \cdots, g_{i+1}\}$ i.e. g_{i+1} does not hold P_2. This is contradiction to our assumption. Thus there are atleast one guard g $\in$ G must be between g_i to g_{i+1} where i = 1, 2,$\cdots$, n-1 i.e. $|G| \geq$ n this implies $|G| \geq |G'|$. □

Theorem 3.2.8 *Cardinality of any guarding set on x-axis of orthogonal uni-monotone polygon P which satisfy properties P_1 and P_2 is equal to cardinality of optimal guarding of P.*

Proof Let us assume G be optimal guarding of P and G' be guarding on x-axis of P which satisfy both properties. we have to show that $|G| = |G'|$.
case (i): $|G| \leq |G'|$.
Since G is optimal guarding set so any other guarding set must have atleast $|G|$ number of guards. Thus $|G| \leq |G'|$.
case (ii): $|G| \geq |G'|$.

Let us first vertical shifting of each guard of G on x-axis. This shifting does not reduce the visibility of any guard (By lemma 3.2.3).
Now claim is that Guard set G must hold property P_1.
Let us assume G = $\{g_1, g_2, \cdots, g_n\}$ such that $g_{1_x} < g_{2_x} < \cdots < g_{n_x}$ does not hold property P_1 i.e. $\exists$ i such that $\{g_1, g_2, \cdots, g_i\}$ can not see $P_l^L(g_i)$ where $P_l^L(g_i)$ is left part of polygon P from vertical line l passing through g_i. The unseen part of $P_l^L(g_i)$ by guards $\{g_1, g_2, \cdots, g_i\}$ can not be seen by $\{g_{i+1}, g_{i+2}, \cdots, g_n\}$ (By lemma 3.2.5). This is contradiction of guarding set G. Thus G hold the property P_1.
If G hold property P_2 then G = G'(By lemma 3.2.6) i.e. $|G| = |G'|$.
If G does not hold property P_2 then $|G| \geq |G'|$ (By lemma 3.2.7).
Thus $|G| \geq |G'|$.
From case(i) and case (ii),
$|G| = |G|'$. $\square$

Chapter 4

Guarding Monotone Polygon

4.1 Guarding Simple Monotone Polygon

Art Gallery problem was for vertex guarding was not known to be NP Hard for Monotone polygon until E.Krohn and B.J. Nillson[2] proved it to be NP Hard. NP hardness of the interior guarding does not immediately follow from that claim. However, Erik Krohn and B.J. Nillson[7] gave the NP Hardness proof of its interior guarding.

Definition *(Visibility region of convex piece) Set of point $p \in P$ such that p can see whole convex piece.*

4.1.1 Transformation of Guarding Problem in to Graph

Let us take simple monotone polygon P contains upper chain U(V, E) where V is finite set of vertices and E is finite set of edges in upper chain. Consider the sequence of convex pieces of upper chain C_1, C_2,$\cdots$, C_k arranged in order, where each piece consist of at least two vertices among which first and last vertices are reflex except C_1 and C_k. Last vertex of first convex piece C_1 and first vertex of last convex piece C_k must be reflex. $\cup_{i=1}^{i=k} C_i$ consist of all the vertices of the U. For each convex piece we would find visibility region V(C_i) where i = 1, 2,$\cdots$, k inside P.
Now define graph G = (V', E') where V' contains vertices corresponding to each convex piece and edge between two vertices u, v $\in V'$ if intersection of visibility region of corresponding convex pieces are non empty.

Approach

Since interior guarding of monotone polygon is NP hard, so our intention is to reduce approximation factor. If graph G belongs to chordal graph then we would able to say 2-approximation factor for interior guarding. This will be huge reduction from 30-approximation factor. My main focus is to identify whether G belongs to such a graph class in which minimum clique cover can be found in polynomial time.

Observation *Graph G is not chordal graph.*

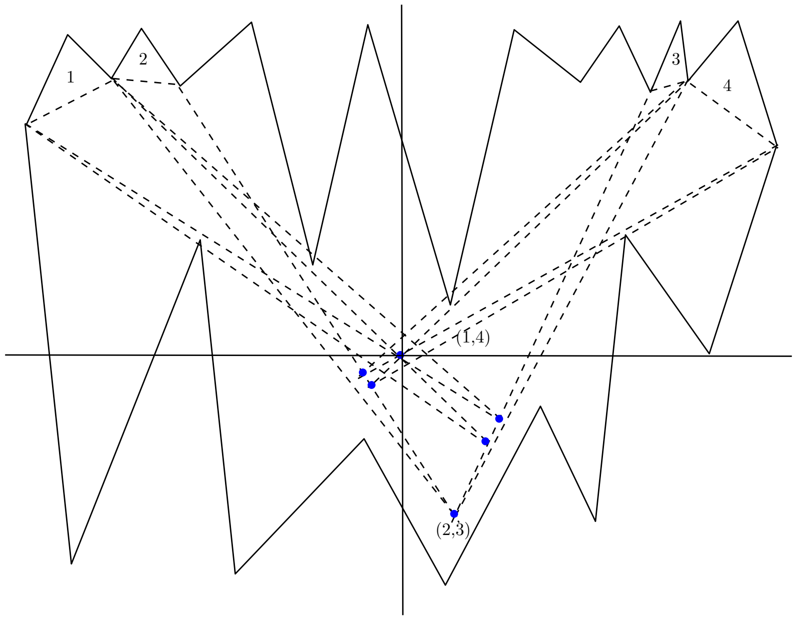

Figure 4.1: Counter Example of Chordal Graph

4.1.2 Guarding Restricted Simple Monotone Polygon

Let us consider simple monotone polygon P such that it holds following two properties:
P_1: for each pair of continuous convex pieces of upper chain (lower chain) $\exists$ a point p $\in$ P from where both convex pieces can be seen.
P_2: for each pair of non continuous convex pieces of same upper chain (lower chain) $\nexists$ a point p $\in$ P from where both convex pieces can be seen.

Lemma 4.1.1 *Any guard inside restricted simple monotone polygon P can see at most two continuous convex piece of the same chain.*

Proof Let us assume $\exists$ guard g inside P can see more than 2 continuous convex pieces of same chain i.e. $\exists$ a point p from where two non continuous pieces can be seen. This is contradiction to hold property P_2 by P. $\square$

Approach

Let us take restricted simple monotone polygon P and assume there are m convex pieces C_1, C_2,$\cdots$, C_m in upper chain and n convex pieces C'_1, C'_2,$\cdots$, C'_n in lower chain. Now we will find visibility regions $V(C_1)$, $V(C_2)$,$\cdots$, $V(C_m)$ and $V(C_1) \cap V(C_2)$, $V(C_2) \cap V(C_3)$,$\cdots$, $V(C_{m-1}) \cap V(C_m)$ for upper chain and $V(C'_1)$, $V(C'_2)$,$\cdots$, $V(C'_n)$ and $V(C'_1) \cap V(C'_2)$, $V(C'_2) \cap V(C'_3)$,$\cdots$, $V(C'_{n-1}) \cap V(C'_n)$ for lower chain. we will make linked list where nodes contains $V(C_i)$ named as left and $V(C_i) \cap V(C_{i+1})$ named as common where i = 1, 2,$\cdots$, m-1 for upper chain. Similarly for lower chain.

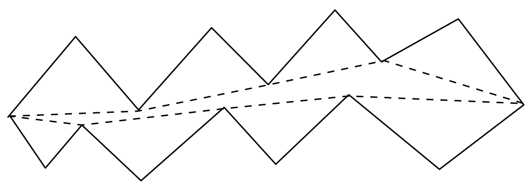

Figure 4.2: Restricted Monotone Polygon

Algorithm 7: Minimum Guarding Set

Input: Linked list of upper chain and lower chain.
Output: Guarding Set G
Initialization: G = ϕ
begin
  while $L_1 \neq \phi$ *and* $L_2 \neq \phi$ **do**
    if $L_1 \rightarrow common \cap L_2 \rightarrow common \neq \phi$ **then**
      Place a guard g in this non empty region
      G = G $\cup \{g\}$
      $L_1 = L_1 \rightarrow next \rightarrow next$
      $L_2 = L_2 \rightarrow next \rightarrow next$
    else
      if $L_1 \rightarrow common$ *is left of* $L_2 \rightarrow common$ **then**
        Place a guard g in $L_1 \rightarrow common$ region
        G = G $\cup \{g\}$
        $L_1 = L_1 \rightarrow next \rightarrow next$
        if $L_1 \rightarrow common \cap L_2 \rightarrow left \neq \phi$ **then**
          $L_2 = L_2 \rightarrow next$
      else
        if $L_2 \rightarrow common$ *is left of* $L_1 \rightarrow common$ **then**
          Place a guard g in $L_2 \rightarrow common$ region
          G = G $\cup \{g\}$
          $L_2 = L_2 \rightarrow next \rightarrow next$
          if $L_2 \rightarrow common \cap L_1 \rightarrow left \neq \phi$ **then**
            $L_1 = L_1 \rightarrow next$
  while $L_1 \neq \phi$ **do**
    Place a guard g in $L_1 \rightarrow common$
    G = G $\cup \{g\}$
    if $L_1 \rightarrow next \rightarrow next == \phi$ **then**
      $L_1 = L_1 \rightarrow next$
    else
      $L_1 = L_1 \rightarrow next \rightarrow next$
  while $L_2 \neq \phi$ **do**
    Place a guard g in $L_2 \rightarrow common$
    G = G $\cup \{g\}$
    if $L_2 \rightarrow next \rightarrow next == \phi$ **then**
      $L_2 = L_2 \rightarrow next$
    else
      $L_2 = L_2 \rightarrow next \rightarrow next$
end

Chapter 5

Conclusion and Further Work

In this thesis, some special cases of art gallery problem for uni-monotone and monotone polygon have been studied. We have given the algorithm for finding the partition of guarding set of size m such that each set in partition cover whole simple uni-monotone polygon if exists. For orthogonal uni-monotone polygon, an $O(nlogn)$ time algorithm for finding optimal guarding. We have given optimal guarding algorithm for special case of monotone polygon. In case of simple monotone polygon, if upper chain (lower chain) transform in to graph based on visibility region of convex pieces then it is not chordal graph. Here there is challenge to find such graph classes in which it belongs and polynomial time minimum clique cover algorithm exist. As we found that graph theoretic orientation of art gallery problem may give better result.

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