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THE CONTROL OF QUALITY
OF
MANUFACTURED PRODUCT

BY

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PREFACE, TABLE OF CONTENTS AND PART I

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PREFACE

The present book is a natural outgrowth of an investigation started about five years ago to develop a theoretical basis for the solution of problems involved in the control of quality of manufactured product. As such, the book does not pretend to be other than a record of progress and an indication of the way in which future developments may be expected to take place.

This work was started in the field of inspection engineering. It was realized in the beginning that inspection engineering involves not only measurement and the sorting out of the good from the bad but the measurement of the right thing, in the right way, the right number of times. The object of inspection engineering, however, does not stop here,- it is necessary to use methods of analysis of data which will yield all of the essential information contained in inspection data in a form to be of greatest service to the research, development, design and purchasing organizations in the better control of quality of product through the weeding out of those causes of variation which should not be left to chance.

The book starts in Part I with the broad definition of quality of product and a consideration of some of the problems of quality control as they arise in a modern engineering organization. Quality itself is naturally expressed in the form of certain measurements, which must be interpreted in a physical and engineering way. In general, the first step in any such interpretation is the reduction of masses of data to a few statistics which contain the essential information and the methods for doing this are outlined in Part II. The next step involves the interpretation of the results in terms of these statistics or simple functionsof the original data, and the basis for thus interpreting the data is provided in Part III.

As is to be expected, it turns out that there is more than one logical framework which may be erected to form the basis for the interpretation of measurements of quality. Modern statistical theory becomes an exceedingly serviceable tool in the formulation of a basis for quality control. It indicates quite clearly the nature of variations in quality that it is useless to try to modify without changing the whole manufacturing process. It provides criteria by which to judge when variations in quality are such that they indicate lack of control; it affords a basis for setting economic standards of quality for raw materials by means of making the best use of information already available and it furnishes criteria for determining the most economical method of measuring quality and for interpreting the results of sampling inspection. These phases of the subject are treated in Parts IV to VIII.

Statistical theory, however, serves merely as a tool. The logical basis for the theory is for the most part that of the theory of probability. The mathematical formulas used naturally are based upon certain sets of assumptions which may or may not hold good in the field of application. To a certain extent, therefore, in the use of statistical theory, as in that of any other, we can never be quite sure that the underlying assumptions are those which fit the practical problem in hand. There is, therefore, always a question in the application of any theory as to whether or not it will work, that is to say, there is an experimental side.

In fact, it turns out, as we shall see, that the process of reasoning involved in the application of statistics is to some extent an individual matter and therefore it follows that the methods proposed for solving the problems of quality control as given in this book may not be the best in all cases. It was early recognized that the real test of the theory depended upon whether or not it worked in practice. It is of particular interest, therefore, to know that at least the methods proposed have been tested experimentally. Therefore, it can be said that although future work will doubtless reveal methods of quality control far better than those under present discussion, experience has shown that the methods provided herein are at least experimentally sound and represent a real step forward.

The work reported upon is naturally the result of the cooperation of a considerable group of individuals. Some of the experimental data reported upon could have been accumulated only in a large industry. Detailed acknowledgements to the literature in the field are presented as occasion arises. On the theoretical side, the author wishes to acknowledge the very helpful and suggestive criticism given by his colleagues in the Bell System, Dr. T. C. Fry and Mr. E. C. Molina. On the practical side he owes a great debt to another colleague, Mr. H. F. Dodge, for helpful criticisms at almost every step in the development of the subject in its present form.

The arduous task of accumulating and analyzing the large amount of data presented has been borne largely by Miss Marion B. Cater and Miss Miriam S. Harold and the task of getting all of the material into its final form has been shared by Mr. F. W. Winters. To each of these the author is deeply indebted.

Naturally, in such a new field involving the application of parts of modern statistical theory which are only in the formative state, it is reasonable to expect that many helpful suggestions and criticisms may arise in the minds of those who try to make use of the material presented in this book. It is hoped that wherever possible these criticisms may be brought to the attention of the author.

The author is particularly indebted to Dr. R. L. Jones and Mr. G. D. Edwards for their constant inspiration, helpful guidance and sympathetic understanding of the many practical as well as theoretical problems that have arisen in the course of this work.

W. A. SHEWHART

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PART I

Introduction

In which we clearly define the various practical concepts of quality and indicate the ways in which the problem of control of quality arises in research, production, development, design, inspection and distribution.

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CHAPTER I

Definition of Quality

1. Introductory Note

When we begin to analyze our conception of quality, we find that this term is used in several different ways. Hence, in the consideration of quality control, it is essential that we decide, first of all, whether the discussion is to be limited to a particular concept of quality or to be so framed as to include the essential element in each of the numerous conceptions of quality. Our purpose in considering the various definitions of quality is merely to show that in any case the measure of quality is a quantity which may take on different values. In other words, the measure of quality, no matter what the definition of quality, is a variable, which we shall represent by the symbol X. In future chapters when we are discussing quality control, we shall treat of the control of the measurable part of quality as defined in any one of the different ways indicated below.

2. Popular Conception of Quality

Dating at least from the time of Aristotle, there has been some tendency to conceive of quality as indicating the goodness of an object. In this sense Shakespeare says, "The Quality of Mercy is not Strained" and even now this same significance of the term illustrates the popular conception of the difference between the quality and quantity of production. The majority of advertisers of the present day appeal to the public upon the basis of the quality of product. In doing this, they implicitly assume that there is a measure of goodness which can be applied to all kinds of product whether it be vacuum tubes, sewing machines, automobiles, grape nuts, books, cypress flooring, Indiana limestone or correspondence school courses.

The need for such a measure is obvious. How wonderful it is to visualize a universal yardstick for the measurement of anything and everything! The search for such a measure is one form of a fundamental human endeavor, viz., the search for permanence and invariance in this changing world.

What universal yardstick exists whereby we may measure the goodness of everything and compare the magnitudes of goodness or quality? To begin with we need a more concrete conception of quality.

3. Conception of the Quality of a Thing as a Variable

Quality, in Latin qualitas, comes from qualis, meaning "how constituted" and signifies such as the thing really is. In general, the quality of a thing is that which is inherent in it and we cannot alter the quality without altering the thing. It is that from which anything can be said to be such and such and may, for example, be a characteristic explainable by an adjective admitting degrees of comparison.

Going a little deeper we see that possibly without exception every conceptual "something" is really a group of conceptions more elementary in form. The minimum number of conceptions required to define an object may be called the qualities thereof and this definition is consistent with that given by Jevons, "The mind learns to regard each object as an aggregate of qualities and acquires the power of dwelling at will upon one or other of those qualities at the exclusion of the rest".[1]

The same conception underlies the definition of quality of manufactured product as given by a prominent author on this subject. Thus he says "The term quality as applied to the products turned out by industry, means a characteristic or group or combination of characteristics which distinguishes one article from another or the goods of one manufacturer from those of his competitors, or grades of product from a certain factory from another grade turned out by the same factory."[2] In this sense a thing has qualities and not a quality. For example a piece of material has weight, density, dimensions and so on indefinitely.

For our purpose we shall assume that, had we but the ability to see, we would find a very large number m' of different characteristics required to define what even the simplest thing really is. Let us represent the magnitudes of these characteristics by the symbols Q_1', Q_2', Q_3' ....$Q_{m'}'$ where it is assumed to be impossible to proceed further in breaking up or dividing the thing into its component parts. That is to say, the Q's, represent Elementary charac-

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[1] Jevons, W. S., "The Principles of Science", 2nd Edition, page 25.

[2] Radford, G.S., "Control of Quality and Manufacture", published by Ronald Press Company, 1922, page 4.

teristics. A thing is therefore formally defined in this sense, if the specific magnitudes of the m' characteristics are known.

Admittedly we do not know a single one of these nor even the number of possible ones in any given case. Those that we take as elementary, we believe to be but a combination of several truly elementary ones, so that the nearest we can approach to the description of any physical thing is to say that it has a finite number of measurable characteristics Q_1, Q_2, ... Q_m where, of course, m' is presumably greater than m.

Thus we might take the characteristics of capacity, inductance and resistance as defining the quality of a relay. Geometrically speaking the quality of a relay in the above sense could be thought of as a point $(P \equiv Q_{11}, Q_{21}, Q_{31})$ in three dimensional space with coordinate axes Q_1, Q_2 and Q_3. Of course, to define quality of the relay in terms of those characteristics which make it what it is would require a space of m' dimensions where m' is the number of independent characteristics required to define a relay.

To characterize a group of n monatomic gas molecules we need a space of 6 n dimensions, one dimension being required for each molecule for each of three space coordinates and for each of three velocity components.

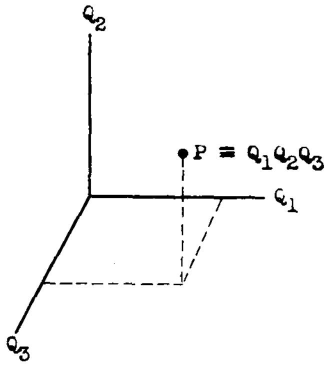

Fig. 1 - Quality as a Point in Space

Quality then as we shall use it may be a quantity having known physical dimensions such as length, velocity, resistance; a quantity representing the magnitude of any entity in units of the same kind or merely a number such as a rate or number defective, and so on.

4. Conception of the Quality of a Thing as an Attribute

Customary engineering practice specifies the limits or tolerances within which the different quality characteristics are supposed to lie provided the single piece of apparatus or thing under study is to be considered as satisfactory or conforming to specifications. Geometrically this can be represented for the previous example involving three quantities by Fig. 2. A piece of apparatus or thing having a quality falling within the rectangular element of volume[1] would be said to possess the positive attribute of conformance to

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[1] Obviously this element of volume may be large because only a lower bound is often given to some one or more of the quality characteristics.

specified standards. If the quality falls outside this volume the piece of apparatus or thing is said to possess the negative <u>attribute</u> of non-conformance. The property of positive attribute is variously characterized as good, satisfactory, conforming, standard, and of negative attribute, as unsatisfactory, non-conforming and so on.

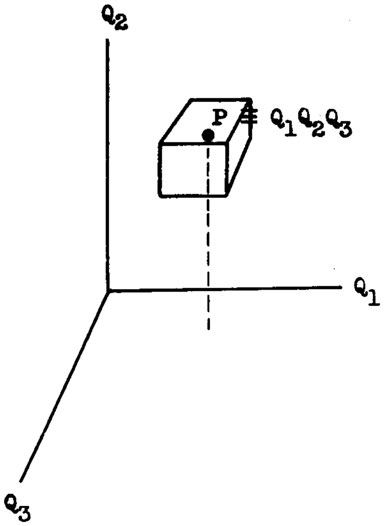

FIG. 2 - QUALITY CONFORMS IF WITHIN VOLUME

5. Quality of a Number of the Same Kind of Things - Attributes

Suppose that we wish to express the quality of a lot of, say 10, relays in terms of resistance only. Actually the 10 observed resistances might be given in the following tabular form.

TABLE 1

Number of Relay	Resistance in Ohms
1	100.9
2	100.5
3	99.9
4	101.0
5	101.4
6	102.7
7	100.5
8	99.0
9	99.9
10	99.6

Such a table gives the complete picture insofar as the single characteristic, resistance, is concerned. In practice, however, we need some short cut method of picturing the results to give us the greater part of the pertinent information contained within the ten measurements. This is particularly true when instead of 10 in the lot we may have several thousand.

Possibly the method most frequently used is to give the observed fraction p of the lot having a value of resistance within the specified limits, in this case let us say not less than 99.5 ohms nor more than 100.5 ohms. This information is expressed by saying that the fraction p non-conforming in the lot of 10 is $\frac{2}{10}$.

In the inspection of product being manufactured in quantities running into the thousands or even millions per year, it would be a very laborious task to measure and record as a variable the quality characteristic on each piece of apparatus or piece-part. Instead the practice is usually followed of recording only the fraction non-conforming in each lot of size N. In the course of a year, then, we might have a record such as shown graphically in Fig. 3 representing the quality of a given kind of apparatus measured in terms of fraction defective.

In the general case each piece of apparatus is supposed to possess several quality characteristics and the results of an inspection of a lot of size N on the basis of, say m, quality characteristics Q_1, Q_2, ... Q_m could be reported either as the fractions p_1, p_2, ... p_m within limits for the respective characteristics or the fraction p within all the limits. Obviously the fraction p alone does not give as much information about the product as the set of m such fractions.

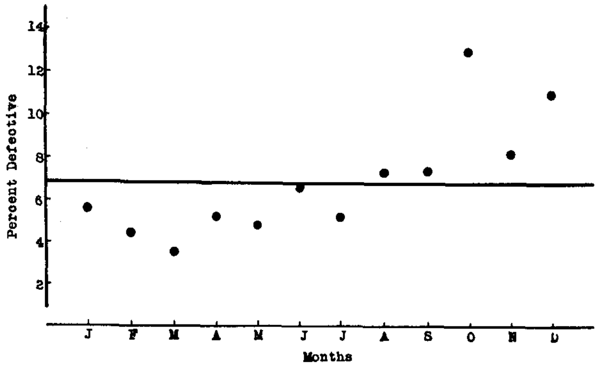

FIG. 3 - RECORD OF QUALITY IN TERMS OF FRACTION DEFECTIVE

6. Quality as a Rate

Starting with the very simple case already considered, viz., the quality of a single kind of apparatus expressed in terms of fraction defective, let us consider the problem of interpreting the results of monthly inspections throughout a year when the numbers N_1, N_2, ... N_{12} of pieces manufactured each month are all different. In tabular form the information would appear as in Table II.

TABLE 2

Month	Number Manufactured	Fraction Non-Conforming		
		Q_1	Q_2	Q_m
Jan.	N_1	p_{11}	p_{21}	$p_{m,1}$
Feb.	N_2	p_{12}	p_{22}	$p_{m,2}$
Dec.	N_{12}	$p_{1,12}$	$p_{2,12}$	$p_{m,12}$

To comprehend either the physical or practical significance of such a table is difficult. Naturally we need some quantity or rate X expressible as a function f of the quantities p_1, p_2, ... p_m and readily interpretable so that the qualities for the different months could be compared among themselves. Formally we would have

$$X = N\,f\,(p_1, p_2, \ldots\ldots p_m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

Another requirement placed upon this rate X is that it be inherently the same kind of quantity for all kinds of apparatus so that a particular value of rate X, for one kind of apparatus would have identical meaning with the same magnitude of rate for any other kind of apparatus.

Right here we find ourselves being forced back to the first of the above mentioned conceptions of quality, viz., something common to everything. However, we can make it take on significance by defining specifically what we mean by X. Since the p's and N are all numbers in Eq. 1, X must also be a number.

One of the simplest forms of f is the following linear relationship with coefficient $a_1, a_2, \ldots\ldots a_m$.

$$X = (a_1\,p_1 + a_2\,p_2 + \ldots + a_i + \ldots + a_m\,p_m)N \quad \ldots\ldots\ldots\ldots \quad 1'$$

As an example of the way a_i might be interpreted, we could let

$$a_i = \frac{c_i}{c_1\,p_1' + c_2\,p_2' + \ldots\ldots + c_m\,p_m'}$$

where c_i is the cost of a single non-conformance in the ith quality characteristic Q_i and p_i' is the accepted economic standard[1] fraction non-conforming in the ith quality characteristic.

Naturally the equation would still be true, if a_i were interpreted directly as cost per piece of apparatus of non-conformance in the ith characteristic, provided, of course, X was then interpreted as total cost of non-conformance in the observed lot of size N.

In equation (1') we have defined the rate X for a given piece of apparatus as proportional to the ratio of the cost of the observed fractions non-conforming in the m quality characteristics to the cost of the economic fractions non-conforming in these same characteristics.

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1. Every piece of apparatus possessing a non-conforming quality characteristic must either be junked or modified and the chosen procedure costs an amount c_i per modification of the ith characteristic on a single piece of apparatus. To try to control product so that no piece would ever be non-conforming in this respect would add to the cost of production. That fraction p_i' for which the net cost of production is a minimum is defined as the economic standard.

Now, instead of using the economic fractions defective, we may wish to use the observed average fraction non-conforming in each of the m qualities over a certain period of years known as the <u>base period.</u> The quantity within the parentheses in equation (1') would then become a sort of index I of the quality of the given piece of material over the past month as compared to base period quality. The rate X might then be defined by the equation

$$X = n\ (1 - I)$$

We see from the form of X that, if the quality of the apparatus for the past month has been better than, equal to, or worse than the quality of the same piece for the <u>base period</u>, then X will be correspondingly positive, zero, or negative. Also, if we have reason to believe that the quality of the product has been satisfactorily controlled over the base period, then the fluctuations in X from month to month will give an indication of the nature of the control of the quality of current product.

Graphically the month to month picture of this rate, as we have just defined it, is illustrated by Fig. 4.

7. <u>Quality of a Product - Attributes</u>

Let us now consider the case of measuring the quality of output of a manufacturing concern making many kinds of apparatus[1]. For any given period, say a month, there would probably be as many different rates as there are different kinds of apparatus. That is we would have a series of X_1, X_2, ... X_M, if there were M kinds. This situation introduces a new problem of major importance provided M is large.

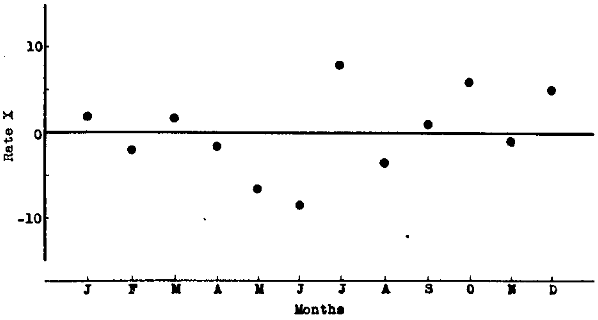

FIG. 4 - TYPICAL FLUCTUATION IN A QUALITY RATE

Of course these X's may be tabulated month after month, year in and year out, but when presented in this detailed form it is very difficult to grasp

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[1] The same reasoning obviously applies in getting an over-all rate for many different piece-parts of a single kind of apparatus.

the significance of the rates at least so that they contrast the present with the past. In other words, no two monthly sets of numbers are likely to be the same. The differences between sets may be partially attributed to that somewhat hazy element, chance, about which we will hear more, but now we must confine ourselves to a consideration of ways and means for measuring these differences in an adequate manner. Specifically our problem is one of finding certain functions of the M values of X which will contain the essential information in the complete set.

8. Quality of a Number of the Same Kind of Things - Variables

Suppose we have one thousand measurements before us representing the measured resistances of as many relays. What do we mean by the quality of this set of relays as determined by the single characteristic, resistance? One answer would be the thousand observed values but the significance of so many different values can not be easily grasped by the human mind. We must go a step further and take some single or at least only a few functions of this group of one thousand data as representing the quality. For example we often use only the average and some measure of dispersion.

In the more general case, we might wish to express the quality of this same set of relays as determined by more than one characteristic on each, such, for example, as the resistance, inductance and capacity. The problem may appear to become more complicated but in the last analysis so far as we are concerned, no matter what function of an observed set of data we take as representing the quality, it is, for our purposes, merely a definable quantity controlled by a system of physical causes.

CHAPTER II

Problems of Quality Control

1. Introductory Statement

The qualities of a group of similar things, will, in general, differ one from the other. No matter where we look, whether at the raw material, the finished product, or even some stage in between, we shall find quality differences. We assume that, if one tries to make two things alike in every respect, he is doomed to failure. Even with all of the developments in the field of applied science we are unable to control the elements of nature in such a way as to produce identity between two things. In the field of mass production, where millions of parts of the same kind are produced each year, we must, therefore, expect certain differences in the quality of these parts. What then do we mean by quality control?

We shall get in this chapter concrete illustrations of the nature of variations which exist in the quality of material at every stage in its transformation into finished equipment. Furthermore we shall see how even under the best controlled laboratory conditions it appears impossible to measure a simple property or quality of the material even though we have reason to believe that it is a constant of nature. Hence, we see that, before an engineer can give an actual picture of the quality of the finished product, he must be in a position to correct his results for errors of measurement so that variations introduced in measurement will not appear as variations in quality. Three important types of problems arise in this particular phase of the work and are illustrated below.

Similarly, variations in quality of raw material must be taken into account in the setting of standards and in the use of these standards in the design of equipment. At every step, from raw material to finished product we find an element of chance entering the picture and we come to see that a controlled product means at the best a product controlled within certain limits. Naturally the research and development engineer takes as his ideal the elimination of chance from the picture insofar as that is feasible. He wishes to find and apply scientific laws in the control of product.

Specific typical problems are introduced to show the nature of the variations in physical quantities with which the research and development engineer must work in his efforts to control the quality of the finished product.

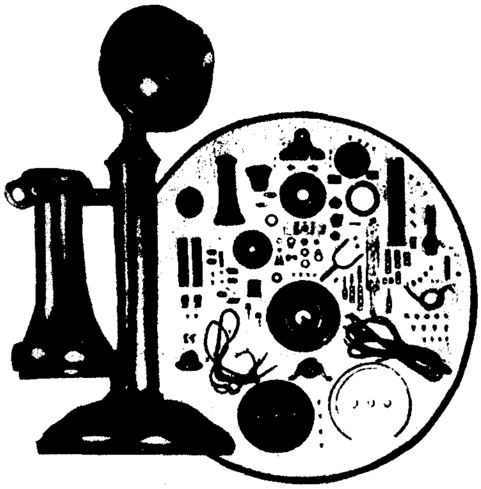
We get a picture of the design engineer's problem of fitting together the piece-parts into the finished product to attain the desired quality within the required range allowing for the chance variations in the piece-parts themselves. Finally we review the methods whereby the inspection engineer can insure with a known degree of probability that the quality of product as turned out does fall within the requisite limits. In each of the foregoing types of problems, we meet specific questions of a general nature which will be answered in later chapters.

In our discussion at this stage it is logical to proceed from errors of measurement to variations in raw materials and then on through the steps of development, design and inspection to the finished product. In our treatment in future chapters, however, it will not be feasible to carry through exactly this same order of presentation. In fact, we shall have occasion practically to reverse the order, leaving to later chapters the discussion of more difficult problems involving the correction of data for errors of measurement and the setting of engineering standards.

2. Steps in Production

In general, a manufacturer starts with the raw material as found in nature. Through research and development he refines this into desired materials of construction; through the use of known physical principles he brings together certain materials according to some well thought out design; he lays down specifications for production processes and after these steps have been taken and the product is turned out, he inspects it sufficiently to insure that established standards of quality have been maintained.

That we may visualize this process somewhat better, let us consider the telephone instrument with which you are so familiar but which is not so simple as it looks. To make it requires 201 parts as pictured in Fig. 5 and to connect it with another instrument requires approximately 110,000 more parts. The annual production of most of these parts runs into the millions and the total annual production of parts runs into the billions. Picture, if you will, the twenty or more raw materials, such as gold, silver, platinum, copper, tin, lead, wool, rubber, silk, etc., literally collected from the four corners of the earth and finally transformed into the telephone communication system of the present day. In this you have a bird's-eye view of what mass production may mean. At every step in this process, we find variations in that which is supposed to be the same and yet in the end we must come out with a product economically controlled within well-defined limits.



Not so simple as it looks

FIG. 5

3. The Problem of Maintaining Quality

If it were feasible or even possible for a manufacturer to go through all these steps and turn out a product in which every piece was identically the same as every other, we would have no problem of quality control in the sense to be considered. But he cannot attain this ideal. Instead every effort to do so is met with failure. No two pieces of raw material are identical and even though they were, measurements of their physical properties would most likely not be identical. Even the so-called principles or so-called physical laws of nature which enter into the steps of development and design are only approximations valid within certain limits. Uncontrolled variations in physical conditions such as temperature, humidity and similar factors introduce variations in the production process which in turn give rise to variations in the final product. In the face of these conditions, it is not economically feasible, even if it were physically possible, to insure identity of all pieces of product.

Since we cannot make all things alike, what then do we understand by maintenance of quality? What do we mean by such terms as standard quality or uniform quality? This brings us to a consideration of the idea of economic quality.

4. Economic Quality

As engineers and scientists we like to think that, if we had infinite knowledge of the principles or laws of nature, we could attain the ideal of making all pieces of product identical; to think that the more we know the closer we can come to this ideal.

But immediately the question arises: Would we be better off if we were able to achieve this goal? I think the answer is no. There is a point in the control of the elements of nature through a knowledge of the physical laws, beyond which the cost of reducing the variation between pieces of product would outweigh the economic value of such reduction. Hence there is an economic quality or one such that to eliminate causes of variation therein would cost more than the resulting advantages are worth.

We come face to face with the situation: To eliminate all variation in the quality of a manufactured product is neither possible nor economically desirable. Hence we must fix some criteria for determining how far we may and should go toward discovering and eliminating causes of variation so as to secure a controlled quality about an economic standard.

5. Quality Control

Already we have gained the helpful conception of quality control within certain limits. We have seen that a manufacturer should not strive blindly in development, design and production processes to secure identity of all manufactured parts but that instead he should strive to keep the variations within certain limits. He cannot make certain, however, that the variation will always be within certain limits. To do even this inherently demands on the part of the producer an infinite knowledge of natural physical principles or laws. How otherwise would one be able to insure with certainty that no two pieces of product would vary by more than a fixed amount? In our state of limited knowledge some of the causes of variation must remain unknown and hence the best that we can hope to do is to set up an ideal of control wherein we specify that the probability of the unknown causes producing a variation beyond certain limits must remain less than some fixed amount. Such a product will be said to be controlled.

So far we have been dealing somewhat in the abstract, depending upon the reader's experience to justify the assertion that variations in quality do exist. We must get a little closer to the nature and magnitude of such variations; we must see first hand some of the problems which arise.

In some way or other either directly or indirectly production depends upon measurements and we cannot discuss the quantitative aspect of quality unless we have measures of quality. As already noted it is, therefore, fitting to start with the consideration of the errors of measurement or as it were the effects of that nucleus of uncontrolled causes always present even after we have taken all human precautions to eliminate them.

6. Errors of Measurement

Problem 1 - Let us picture some physical quantity such as mass, resistance, and so on, the magnitude of which we have reason to believe is constant and see what we get when we make measurements of this quantity, remembering, of course, that such measurements are in our sense of the term measurements of quality. Possibly few so-called physical constants are of greater industrial or scientific interest and importance than the charge on an electron. Possibly for this reason as much as any other this quantity has been subjected perhaps to more refined measurement than any other. Hence in the measurements thereof we should expect to get an indication of the nature of the results which we may expect to get when we have gone nearly to the practical limit in trying to eliminate sources of error in the measurement of some quality.

Fig. 6 shows the best available distribution of the observed values of charge on an electron[1]. There can be little question even in this remarkably controlled experiment that causes of variation enter, and what happens here can, in general, be expected to exist to an even greater degree under less carefully conducted experiments.

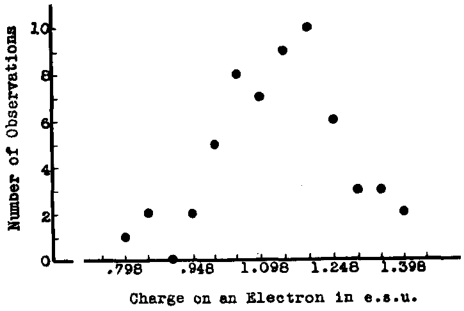


FIG. 6 - TYPICAL EVIDENCE OF UNCONTROLLED CAUSES OR SOURCES OF VARIATION IN THE MOST REFINED MEASUREMENTS

But how do the presence of such variations affect our every day use of the observed results? The answer is obvious. Engineering formula involve the use of the charge on an electron. Not knowing the true value of this charge, we must use some estimate based upon the n observed values. In this interesting case we believe that there is a true value of quality, say e', repre-

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[1] Millikan, R. A., "The Electron", Chicago University Press.

senting the true charge on an electron and from the observed set of values of this charge we must estimate the true quality. But how? We realize full well that the estimate so derived will most likely not be equal to the true value and hence we must have some way of obtaining an indication of the error of our estimate, that is to say, if we use a formula supposed to represent a law of nature and involving the charge on an electron we shall most likely introduce an error through the use of any estimate of this charge derived from observed data.

Hence even where we have reason to believe that a measurable thing is constant, we find that it is not possible to measure this thing with certainty. There always remains a nucleus of chance or unknown causes of variation in the most refined measurements and two specific questions must be answered:

1. Given a set of observed values X_1, X_2, ... X_i, .. X_n, of some quality supposed to have a true value X', what function of the observed set of values shall we take as being the best estimate of X'?

2. What is the probability that this best estimate will differ from the true value X' by less than some fixed amount?

The engineer may raise the objection that the problem of measurement generally lies in the field of physics where the errors of measurement are too small to introduce serious difficulty when the results are used in engineering calculations. What difference does it make that we do not know the charge on an electron more precisely than we do? His point is granted. We are only considering at this point the nature of the questions which arise even under ideal conditions. Let us hasten on, therefore, to two other typical problems of measurement which are indeed very bothersome to an engineer engaged in the measurement of quality.

Problem 2 - Let us take the case where the thing to be measured, instead of behaving itself nicely by remaining constant as does the charge on an electron varies under the influence of unknown causes. Suppose for example that you wish to measure the quality of each of a series of N pieces of a given kind of material in respect to some characteristic X. We might take, for example, the measurement of the viscosity of each of one hundred samples of lubricating oil under supposedly the same essential conditions. Each sample might have been taken from a different barrel as is often the case in practice. Naturally, the practical problem is to compare one barrel with another upon the basis of the measurement of the viscosity of n samples from each barrel. Here we have the error of measurement in the laboratory sense of the term, meaning thereby the errors involved in the determination of the viscosity from n measurements. From a practical viewpoint, we also must take account of the fact that the oil within the barrel is not uniform, homogeneous and isotropic throughout. It is quite possible that the differences in the viscosity of specimens drawn from the same barrel may be comparable to the differences in a set of specimens taken from different barrels. Quite naturally, if we measure the viscosities of more than one specimen from each barrel and use the average of a set of measurements for a given barrel, the resultant set of N such averages will, in general, show less variation than the corresponding set of N values in which only one observation was made on each barrel. In other words, the averages give us a better measure of the inherent differences between the viscosities of the oils in the different barrels than does a set of N single observations one on a sample from each barrel.

But the engineer must answer the question:- How many tests per barrel shall he take so as to reduce the effects of errors of measurement beyond the stage where it will give misleading results in the estimate of differences of quality?

We may state the general problem as follows:- Given a series of observed values consisting of n_1 observations of the magnitude of a physical quality X made on one of N things, n_2 observations on the same quality X of another of the N things and so on until we have finally n_N observations of this quality on the last of the N things, how shall we modify this distribution to correct for the errors of measurement involved?

Obviously this problem is important from a commercial viewpoint because the manufacturer is never anxious to have his quality of product appear more variable than it really is and hence he always must correct for errors of measurement since, when present, they contribute to the observed dispersion between the measurements of quality thus making the quality appear more variable than it really is.

Problem 3 - Suppose you wanted to test the tensile strength of the steering rod on your automobile, what would you do? Of course you could break it and obtain the desired information, but then you wouldn't have any steering rod. This is just the dilemma in which the manufacturer finds himself particularly when the tests are destructive as is so often the case.

If he were lucky enough to know of a non-destructive test which would give him indirectly a measure functionally related to that of tensile strength, he would be very happy indeed. But in the measurement of tensile strength, as in the measurement of many other qualities of material, luck does not favor him in this way.

In this situation one quite common practice is to substitute a hardness test for that of tensile strength. As already stated, this would be highly satisfactory if it were possible to express a measure of hardness as a function of the measure of tensile strength. But this condition is not fulfilled as indicated by the typical set of data shown in Fig. 7. Here each test represents an observed pair of values of hardness and tensile strength for 59 pieces of aluminum alloy die castings.

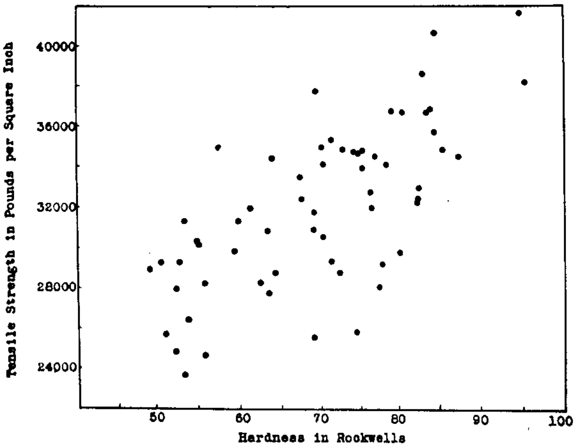

FIG. 7 - HOW SHALL WE USE SUCH RESULTS IN ESTABLISHING A SUBSTITUTE TEST FOR TENSILE STRENGTH?

Two typical questions confront us:

1. For a given value of hardness, what shall we take as a corresponding tensile strength?

2. If it is desired to control tensile strength within certain limits and in the sense introduced above, how can we use the measurements of hardness to establish the limits for such control?

And so we see that before we complete our discussion of quality control there will likely arise many cases where we must give due consideration to the correction of observed quality to eliminate insofar as possible the effects of errors of measurement. Furthermore, we shall find occasion to discuss methods of calibrating machines which give measures of certain kinds of quality which measures are only correlated with and not mathematical functions of the quality to be measured. Let us now pass on to a consideration of the variations in the quality of raw material.

Naturally, when an engineer starts to build something he thinks about the raw material which he must use. He compares the physical properties of raw materials to find the one best suited to his needs. Since the quality of a material in respect to a certain characteristic, such as density, tensile strength, hardness, and so on, varies, in general, from piece to piece, he must adopt some average figure. Furthermore he must take into account the nature of the variations that may be expected to occur in the quality of the material.

Let us become specific and consider the production of aeroplane propellers. Here, as in practically every other instance, the strength of the timber is an essential quality characteristic. Sitka spruce is the material most extensively used in the construction of aeroplane propellers because, among other things, it has a comparatively high modulus of rupture, which is an important strength characteristic. But pieces of Sitka spruce supposed to be the same may differ over a wide range in respect to modulus of rupture.

In fact one not familiar with the nature of such variations in the properties of raw materials may be interested in observing the wide variation in the modulus of rupture of Sitka spruce as reported by the Forest Products Laboratory.[1] The frequency distribution for 1304 tests is shown in Fig. 8. We see that some pieces give a modulus of rupture as low as 3100 pounds per square inch whereas others give a modulus of rupture more than three times as great, actually 9700 pounds per square inch.

Now consider the specification for a propeller. You cannot measure the modulus of rupture of a given propeller without destroying it. In this situation you would allow for the variability of the material. But how? Of course you could choose a safety factor large enough practically to insure that you would never have a condition where the propeller would be overloaded. But how large would this factor have to be to give a specified assurance? The answer to that

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[1] Newlin, J. A., Uni-Stresses in Timber, Transactions of the American Society of Civil Engineers, Part 1, September 1926, pages 1436-43.

question takes us far into the modern theory of sampling. Here we are justified in making the assurance very high indeed because of the possibility of loss of life accompanying failure of a propeller.

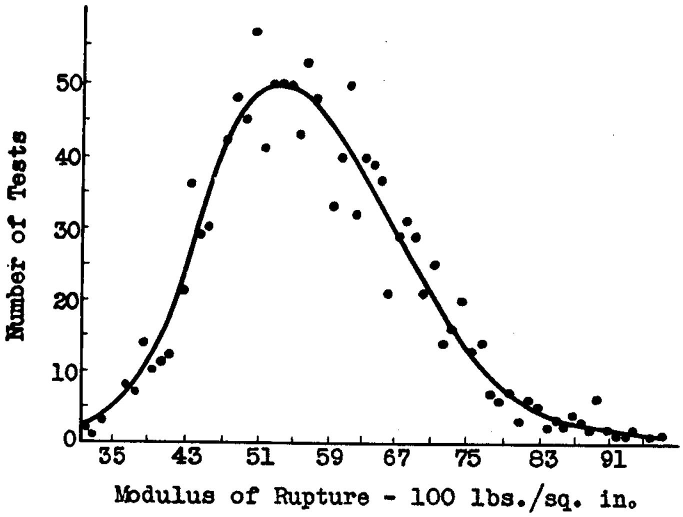

FIG. 8 - TYPICAL VARIATION IN PHYSICAL PROPERTY OR QUALITY OF THE SAME KIND OF MATERIAL. SITKA SPRUCE FOR AEROPLANE PROPELLERS.

In most instances however, material is used in a design where actual failure of a piece would not introduce a life hazard but instead would merely involve the added cost due to replacement of the broken piece. Here we are interested in knowing what would be the economic standard of modulus of rupture to be used. Obviously it would be cheaper to replace some pieces of broken material than it would be to use a modulus of rupture figure which would insure that no piece would break. There is some figure of modulus of rupture which, for the case in hand, would be the most economical. From the observed data we must make the most efficient estimate of the probability associated with a given range because this figure is required in the estimate of the number of failures that may be expected assuming a given value of modulus of rupture.

Instead of having a very large number of observations, (1340 in this case) from which to determine the figure to be used in design, we customarily have but a very few observations. For example, the most comprehensive and valuable source of information on the strengths of timber is perhaps the series of bulletins published by our own government laboratories. As a case in point, Table 1 of Bulletin 556 of United States Agricultural Department gives the results of modulus of rupture tests on 126 species of wood. The number of trees tested per species, however, varies from only two to sixty and the number of trees most frequently tested is only five.

The way one uses such information is in answering questions such as the following: Suppose you are to use 1000 poles of a given species, what percentage will have a modulus rupture within a given range? Customary error theory will not give the best available answer to this question. The nearest approach to a satisfactory solution comes through the use of some of the modern results in sampling which will later be set forth.

7. Development and Research

Industrial progress and development have taken great strides within recent years and yet the changes brought about have more the characteristics of steady growth than of revolutionary modifications. The automobile of this year is better than that of last and the automobile of tomorrow will no doubt be better than the one of today. We firmly believe that the more we know about the laws of nature, the more nearly are we able to do what we want to do; the more nearly are we able to make advancement in a given direction. But every step we take is fraught with difficulties because there is always the element of chance arising in that we do not always know as a certainty that what appears to be an improvement is an actual improvement at least in the initial stages of development work. We must allow for variations usually attributed to unknown or uncontrolled causes of variation.

The very spirit of research is to discover these causes and through their control make possible the development of a high standard of quality. Certain typical problems arise.

Problem 1 - The manager of a gas plant observes the record shown in Fig. 9 giving the number of empirical thermal units per cubic foot of gas produced from cracking of the oil over a period of thirty-one days. Should he attribute the variations to chance? Would you consider it likely that research work would reveal the causes of these fluctuations?

Such a series of observations all of which are assumed to have been taken under the same essential conditions but which differ widely among themselves, is a challenge to the man of research. He wants to know: Why do they vary and how can

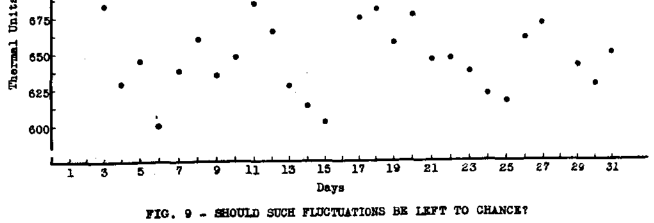

FIG. 9 - SHOULD SUCH FLUCTUATIONS BE LEFT TO CHANCE?

the variation be reduced. These and other similar questions take on even greater meaning as we ponder more over the fundamental research problems as we shall have occasion to do in considering other problems of quality control. Let us bring ourselves back, however, to a consideration of a somewhat more common problem.

Problem 2 - We are to find a contact material to replace expensive platinum contacts; to develop a new alloy having certain desirable properties including high tensile strength and high resistance to corrosion; to develop a piece of apparatus to function in a certain way and so on. In all such cases we follow the general procedure of modifying certain conditions and then observing the results of such changes. For example, in the development of contact material we make different alloys and try a few specimens of each alloy under different circuit conditions. Then we compare the measurements of the quality characteristics of the new contact material with that of the old. We are forced to decide whether or not the differences are significant. Sometimes such work is entered into by different scientific laboratories or by a group of both industrial and scientific laboratories. As an illustration certain investigations of the latter kind are being carried on by committees of the American Society for Testing Materials in experiments such as the following: A few samples of a given alloy supposed to be weather resisting are exposed on a roof in Havana, others in Seattle and so on in various scattered places. Later, the results are to be brought together and compared. It will then be necessary to determine whether or not the observed differences are really significant in the sense that they indicate real differences in the effects of atmospheric conditions upon corrosion of the materials.

In many such cases it is necessary to compare sets of observations representing not only one but several quality characteristics. We may illustrate this by a very simple case. Fig. 10 shows the results of measurements of two characteristics of leather from different sources. In the particular case for which the leather was to be used it was desirable to control within certain limits both the thickness and the tensile strength of the material. From a comparison of these two tests of data and the requirements to be met by the material the development engineer was called on to decide between the two sources of supply.

Problems involving more than two characteristics could be represented in a similar way by geometrical constructions in as many dimensions as there are characteristics to be compared. Enough has been said, however, to give a hint as to the nature of this type of problem and so we pass on to another.

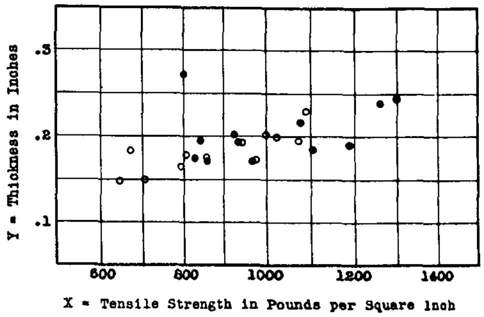

FIG. 10 - WHICH SOURCE OF SUPPLY SHOULD BE CHOSEN?

○ Leather from Source A
● Leather from Source B

Problem 3 - Look at Fig. 11 and decide whether or not temperature should be controlled in the production of this material. Here we have pictured graphically a series of observations of the temperature of roast and the resulting resistance of the material which was being produced. It was very desirable indeed to confine the variations of the resistance within narrow limits. Each point in the diagram represents the temperature-resistance condition of one lot of material. What process should we use in determining whether or not variation in temperature is a marked contributing factor to the variation in the resistance of the material?

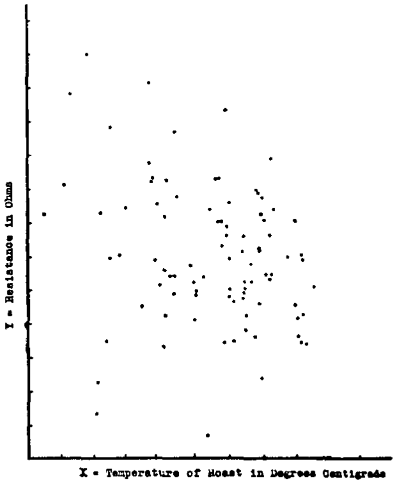

FIG. 11 - SHOULD TEMPERATURE BE CONTROLLED IN THIS PRODUCTION PROCESS?

Analytically we have the same type of problem in many fields of research where we must investigate the nature of the relationship between the variations in two or more physical quantities which are not related functionally in the mathematical sense. Specific illustrations of such relationships would be the depth of pitting of contacts versus percent of a given constituent material; the density versus tensile strength versus elongation of materials and so on indefinitely.

8. Design

To secure the economies of quantity production we must insure to a marked degree the interchangeability of piece-parts. But in so doing we introduce the problem of determining the influence of possible variations in the quality of each piece-part upon the quality of the assembled unit. This problem has already been hinted at in the discussion of standards for raw materials. Let us consider a simple example.

Suppose a circuit is composed of n different elements such as relays, induction coils, line sections, etc., Such a circuit naturally may be thought of as being assembled at random by taking one of each of the n elements from the store house where the piece-parts are kept. Now, it is obvious that in the production of each kind of element it will not be feasible to make them all alike. Suppose we knew the distribution of resistances in each of the different elements as produced. The problem becomes one of determining the probability that the resistance of n different elements constituting a given circuit will lie within a previously specified range.

It may be necessary that a relay be designed so as to function in this circuit at least 99% of the time. What would be the necessary limits on the variation of the different elements so that this condition would be met?

Of course, the practical problem is much more complicated in its details but from the theoretical viewpoint its nature is clearly set forth in this simple illustration. In general, the design engineer must be able to allow for the variations which may occur in the piece-parts which go into the design.

9. Production

After everything has been said and done the manufacturer will still find variations in the quality of his product. He may experience, for example, a condition such as that illustrated in Fig. 12 which shows the twelve monthly frequency distributions of the efficiency of a given kind of instrument.

Obviously the producer is interested in doing two things:- (1) Securing some simple method of quantitatively expressing the quality of product from month to month. (2) Knowing whether or not the observed variation in product is such that it should not be left to chance. Therefore, let us consider another typical problem which touches every one of us.

Almost everyone is interested in the cost of bread and hence may be interested in the following statement referring to the annual loss due to the return of stale bread[1]: "The loss to bakers which is largely passed on to con-

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[1] "Stale Bread Loss as a Problem of the Baking Industry", published by the Food Research Institute of Stanford University, California.

sumers, almost certainly exceeds five million dollars and it may amount to as much as ten million dollars per year". Fig. 13 shows the actual record of the percentage of stale bread returned by ten bakers in a certain metropolitan district over a period of 37 weeks.

How shall we proceed to analyze these data to determine whether or not the managers of the different bakeries should have reason to believe that they can reduce the net returns of stale bread? Is it reasonable to believe that such variations must continue to exist and be excused upon the basis of chance?

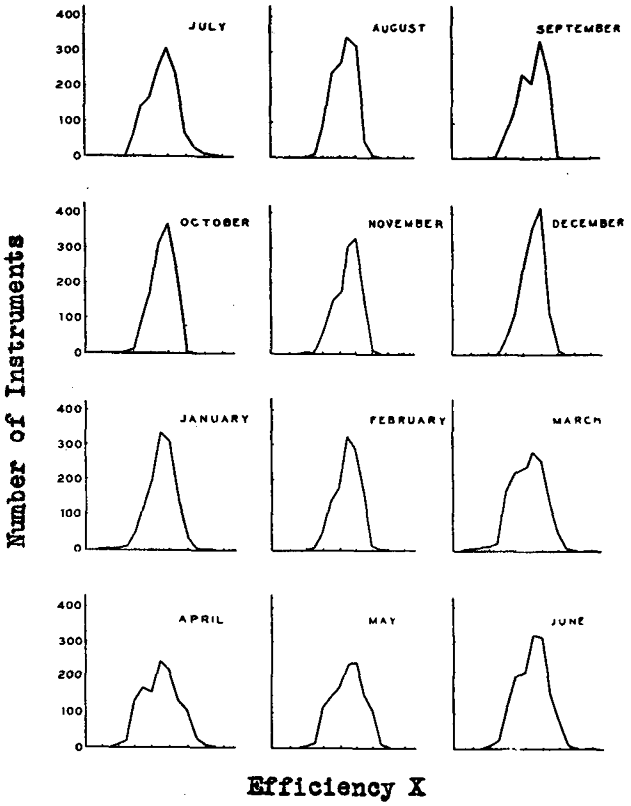

FIG. 12 - DISTRIBUTIONS SHOWING THE VARIATION IN EFFICIENCY IN A GIVEN KIND OF TELEPHONE INSTRUMENT

Thus we get a glimpse of some of the problems which trouble the manufacturer. Those confronting the engineer of distribution are closely similar, except that instead of working with physical qualities of product, he works with such things or qualities as number of sales, net returns, etc. Analytically, however, the problems are identical.

10. Inspection

In many instances and in particular when the tests are destructive it is not economically feasible to inspect the quality of every piece-part in the course of production. Instead we must rely upon sampling inspection to give us an indication of the quality. Under these conditions we must develop sampling theory to give us the requisite assurance that quality standards are being met at a minimum of inspection costs. We must therefore present the theoretical basis for establishing the requisite

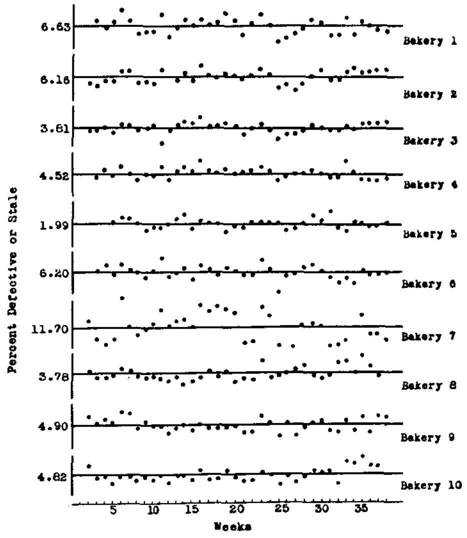

FIG. 13 - MUST SUCH DIFFERENCES IN THE RETURNS BY DIFFERENT FACTORIES BE LEFT TO CHANCE?

inspection methods.

Having seen how certain typical problems of quality control arise in every step of production from raw material to finished product, we pass on to a consideration of the ways of analyzing data to reduce the essential information contained in a set of observed data to a few simple functions with which we will have to deal in quality control.