AN EXTENSION OF HALD'S TABLE FOR THE ONE-SIDED CENSORED NORMAL DISTRIBUTION

By NIKHILESH BHATTACHARYA

Indian Statistical Institute, Calcutta

SUMMARY. Hald (1949) outlined a very convenient method of maximum likelihood estimation of the parameters of a one-sided censored normal distribution and gave tables for facilitating the process. Table 1 below is an extension of Hald's main table (Table III). Hald's table gave the values of a certain function $z = f(h, y)$, for values of $h = 0.05, 0.10, \ldots, 0.80$, and for some appropriate values of y, h being the fraction of censored observations in the sample. The present extension gives the values of z for some values of h below 0.05, for use in situations where the censored observations cannot be ignored for purposes of estimation, even though they form less than 5% of the total sample.

Hald's method of estimation is briefly as follows :

Suppose there are n observations from a normal distribution with mean ξ and variance σ^2, and it is known that a number, say a of these observations are less than or equal to a known point of truncation. The values of these a observations are not further specified, unlike the values above the truncation point, which may be denoted by $x_1, x_2, \ldots, x_{n-a}$.

The point of truncation is taken as the origin. Let now

$$\xi = -\frac{\zeta}{\sigma},$$

$$\phi(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}, \quad \Phi(u) = \int_{-\infty}^{u} \phi(x)dx, \quad \psi(u) = \log_e \Phi(u),$$

and $\psi'(u)$ the first derivative of $\psi(u)$.

Let $h = \frac{a}{n}$ denote the observed degree of truncation in the sample.

Hald defines

$$g(h, z) = \frac{1}{\frac{h}{1-h}\psi'(z) - z},$$

and

$$F(h, z) = \tfrac{1}{2}\, g(h, z)\left[g(h, z) - z\right].$$

Let now the inverse function to $y = F(h, z)$ with respect to z be denoted by $z = f(h, y)$. This function was tabulated in Table III of Hald (1949) for $h = 0.05, 0.10, \ldots, 0.80$ and for some appropriate values of y.

The estimate ζ of $\hat{\zeta}$ is then obtained by calculating

$$y = \frac{(n-a)\sum_{i=1}^{n-a} x_i^2}{2\left(\sum_{i=1}^{n-a} x_i\right)^2}$$

and reading $\hat{\zeta} = f(h, y)$ from the Table.

The next step is to calculate

$$\hat{\sigma} = g(h, \hat{\zeta})\frac{\sum_{i=1}^{n-a} x_i}{n-a}$$

and finally $$\hat{\xi} = -\hat{\zeta}\hat{\sigma}.$$

The function $g(h, z)$ can be easily calculated. Table IV of Hald's paper may be used for this purpose, but direct calculation is not difficult.

The need of the present extension was felt in certain cases of fitting one-sided censored normal distributions to grouped data. The values of $h = \frac{a}{n}$ were found to be often below 0.05, and sometimes of the order of 0.01. Although the censored part could be ignored without much loss of information, it would be desirable to make use of it, especially because for examining goodness of fit the tails are valuable. Values of $z = f(h, y)$ in Hald's table change sharply with h, as h approaches small values. Graphical extrapolation was out of question.

The present extension intends to facilitate interpolation for values of h below 0.05. The column for $h = 0.001$ is particularly in point. This value of h is clearly outside the range of practical interest. However, cases with $h = 0.005$ or 0.008 are not uncommon and the column for $h = 0.001$ will enable one to interpolate for such values.

The calculations were based on the Table of Normal Probability Functions, published by the National Bureau of Standards. The figures tabulated are correct to the third place of decimals.

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REFERENCES

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Tables of Normal Probability Functions, National Bureau of Standards, Applied Mathematics Series, 23.

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TABLE 1. VALUES OF FUNCTION $z = f(h, y)$ FOR FITTING ONE-SIDED CENSORED NORMAL DISTRIBUTIONS

y	h				
	0.001	0.010	0.020	0.035	0.050
(1)	(2)	(3)	(4)	(5)	(6)
0.500					−4.135
0.505				−4.465	−3.774
0.510				−4.030	−3.494
0.515			−4.337	−3.715	−3.268
0.520			−3.959	−3.468	−3.080
0.525	−4.421	−4.021	−3.665	−3.248	
0.530	−4.043	−3.723	−3.429	−3.072	
0.535	−3.747	−3.482	−3.232	−2.922	
0.540	−3.508	−3.283	−3.066	−2.792	
0.545	−3.310	−3.114	−2.923	−2.677	
0.550	−3.142	−2.969	−2.799	−2.576	
0.555	−2.997	−2.843	−2.680	−2.485	
0.560	−2.870	−2.731	−2.591	−2.404	
0.565	−2.759	−2.632	−2.503	−2.329	
0.570	−2.659	−2.542	−2.423	−2.262	
0.575	−2.569	−2.463	−2.351	−2.199	
0.580	−2.488	−2.388	−2.284	−2.142	
0.585	−2.416	−2.321	−2.223	−2.089	
0.590	−2.347	−2.259	−2.167	−2.040	
0.595	−2.285	−2.202	−2.115	−1.993	
0.600	−2.227	−2.148	−2.066	−1.950	
0.610	−2.124	−2.053	−1.978	−1.872	
0.620	−2.034	−1.969	−1.900	−1.802	
0.630	−1.954	−1.894	−1.830	−1.739	
0.640	−1.883	−1.828	−1.768	−1.683	
0.650	−1.820	−1.768	−1.712	−1.632	
0.660	−1.762	−1.713	−1.660	−1.585	
0.670	−1.710	−1.663	−1.613	−1.541	
0.680	−1.662	−1.618	−1.570	−1.501	
0.690	−1.617	−1.576	−1.530	−1.464	
0.700	−1.577	−1.537	−1.493	−1.430	
0.710	−1.539	−1.500	−1.450	−1.398	
0.720	−1.503	−1.466	−1.426	−1.368	
0.730	−1.470	−1.435	−1.396	−1.340	
0.740	−1.440	−1.405	−1.368	−1.313	
0.750	−1.410	−1.377	−1.341	−1.288	
0.760	−1.383	−1.351	−1.316	−1.264	
0.770	−1.357	−1.326	−1.292	−1.242	
0.780	−1.333	−1.303	−1.269	−1.220	
0.790	−1.310	−1.280	−1.248	−1.200	
0.800	−1.288	−1.259	−1.227	−1.181	
0.850	−1.192	−1.167	−1.138	−1.096	
0.900	−1.115	−1.092	−1.066	−1.027	
0.950	−1.052	−1.030	−1.006	−0.970	
1.000	−0.998	−0.977	−0.955	−0.921	
1.050	−0.951	−0.932	−0.910	−0.878	
1.100	−0.911	−0.892	−0.872	−0.841	
1.150	−0.875	−0.857	−0.838	−0.808	
1.200	−0.843	−0.826	−0.807	−0.779	
1.250	−0.816	−0.798	−0.780	−0.752	
1.300	−0.789	−0.773	−0.755	−0.728	
1.350	−0.765	−0.750	−0.732	−0.706	
1.400	−0.744	−0.728	−0.712	−0.686	
1.450	−0.724	−0.709	−0.692	−0.668	
1.500	−0.705	−0.691	−0.675	−0.650	