# INTRODUCTION

WHAT STATISTICAL CONSULTANT NEEDS TO KNOW

EXAMPLE OF HOW SCIENTIST AND CONSULTANT COOPERATE

1st Scientist leads  2nd Equal  3rd Statistician leads.

TESTING HYPOTHESES AGAINST ERRORS OF 1st and 2nd KIND

AN EXAMPLE IN CHECKING ACCURACY and PRECISION

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# Discussion of Papers of

# B. L. Clarke and F. J. Power

by

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Bell Telephone Laboratories, Inc.

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Meeting of New York Section of The American Chemical Society at Hotel Pennsylvania, New York City, Friday evening, April 2, 1943, 8:15 P.M.

# INTRODUCTORY REMARKS

In last Tuesday's mail I received the following note from a friend of mine who has been trying to use statistics in his own industry:

"I shall forsake all others to attend the A.C.S. New York meeting this Friday next. We are still in the running and still find statistics an uphill job in an old company.

"The purpose of this note is simply to pass along a definition of a statistician from a recent issue of Chemical Industries:

"A statistician is a man who draws a mathematically precise line from an unwarranted assumption to a foregone conclusion.

I am sure that this definition will help you to appraise any comments that I as an industrial statistician may make. The worst part of it, however, is that this friend did not go far enough. He should also have called attention to the fact that a statistician can serve only as a consultant to the scientist, and some one has described a consultant as

"an ordinary guy a long way from home."

My comments tonight are those of a statistical consultant.

Perhaps you will refrain, however, from "throwing stones" because the chemist on this score lives in a kind of glass house. Many, if not all, of you use at some time or other the classical theory of errors. At least one of the world's great scientists, Lord Rayleigh, has said that the theory of errors is a good subject to read up on and then forget. Yet in the latest book that I have come across (1940) written by a chemist, the classical error theory as it stood at the turn of the century is carefully expounded and, lo and behold! in illustrating its usefulness, an example is taken from the work of Lord Rayleigh.

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In this same book we are told, as we are told in scores of others employing the error theory as it was known at the turn of the century, that if we take a series of observations,

$$ {}_1, X_2, \ldots X_i, \ldots X_n , $$

and compute

$$ \bar{X} \pm \text{p.e.} $$

50% of future observations may be expected to fall within these limits. Such statements have been made so often by high authorities that they are often taken more or less as gospel truth or as things that <u>we know for sure</u> from the theory of errors.

At this point I am reminded of the sound advice of C. F. Kettering:

"It ain't them things you don't know what gets you into trouble, it's them things you know for sure what ain't so."

Now it happens that the statement that 50% of the future observations will fall within

$$X \pm p.e.$$

although thought to be known for sure, just ain't so.

<u>Before you chemists</u> would want to <u>throw stones</u> at the statistician, I am sure that you will want to <u>find out what is so</u> in statistics.

There is a second reason for you as chemists to be lenient with the statistician. It was, in fact, a physical chemist, one of the most brilliant graduated from Oxford, who really started on a grand scale the applications of statistics in applied science - William S. Gossett of the Guiness Brewery, better known by his psuedonym "Student". He not only started the statistical ball rolling and made statistics pay in the biggest brewery in the world but he pointed out why certain conclusions based on the older theory weren't so.

## IMPORTANT STEPS

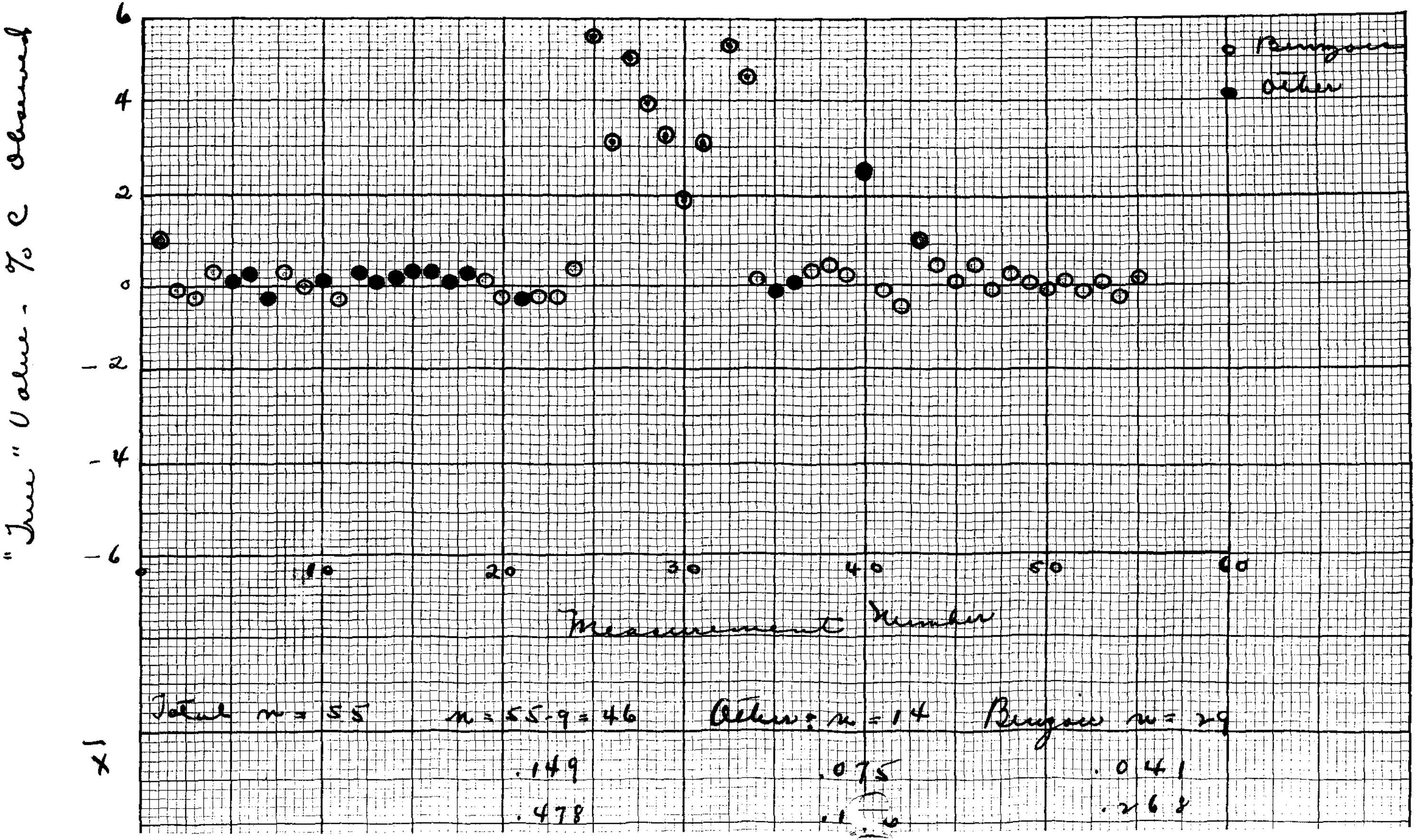
| | | |
|---|---|---|
| Student | 1908 | Confidence intervals |
| Control chart | 1924 | Z's of ASA |
| Errors 1st / 2nd kind | 1925 | |
| Valid prediction within tolerance limits | 1928<br>1941 | |
| Run chart | 1940 | |
| Analysis of variance | | |

## WHERE USED

Research

Production

Inspection


"True" Value - % C Observed
6
4
2
0
-2
-4
-6
10
20
30
40
50
60
Burgess
Other
Measurement Number
Total n = 55
n = 55-9 = 46
Other: n = 14
Burgess n = 29
x̄
.149
.478
.075
.1?6
.041
.268

## Slides

I - Birthplace of modern statistics.

Guinness Brewery

Slide # 22023

II - B. L. Clarke data

Slide # 23453

Sequence of 55 measurements of (% Carbon - True Carbon) for a number of different compounds.

1. Chemist would most likely reject nine points ⊙

   Run i of 11 above true value 0.

   $P_{i=11,n=55} \pm \underline{.011}$

   Assignable cause found and removed.

2. Chemist questions 3 points ⊛

   $n = 46 \quad s = \sqrt{\Sigma v^2/n-1}$

   $t = \frac{1.01-\bar{X}}{s} = \frac{1.01-.1494}{.4781} = 1.80$

   $P_{t=1.80} = \underline{.036}$

3. 43 points left.

Run of i = 8 above zero.

$$P_{i=8,n=43} = \underline{.071}$$

7 of these points other than benzoic
6 of these "Victron".

4. 14 points other than benzoic.

Run of i = 8 above 0.

$$P_{i=8,n=14} = \underline{.0156}$$

5. Benzoic n = 29.

| i | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f_i$ | 5 | 5 | 3 | 1 |

Total

| Obs. | Theory |
|---|---|
| 14 | 15 |

6. Benzoic o

**B. L. Clarke**

$n_1 = 29$

$\overline{X}_1 = 68.7593$

$s_1 = .2677$

**Father Power**

$n_2 = 34$

$\overline{X}_2 = 68.9868$

$s_2 = .1639$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s\sqrt{\frac{n_1+n_2}{n_1 n_2}}} = \frac{68.7593 - 68.9868}{.0550} = 4.13$$

$P_{t=4.13} < .001$

$F = s_1^2/s_2^2 = 2.65 \qquad P_F < .01$

# ROUTINE ANALYSES

A. Testing hypothesis against errors of 1st and 2nd kind.

Example

$H_o$ - Material under test is pure benzoic acid.

Alternative Hypotheses
- $H_1$ - Impurity Type A present
- $H_2$ - Impurity Type B present

Slide #19000

- o Density of points $H_o$
- o Density of points $H_1$
- o Density of points $H_2$

## Rule of Action

Must adopt some rule of action in which you choose a region $\omega$ in the $X_1X_2$ plane and reject if in that region but accept if not in that region.

Any such rule is subject to two kinds of errors:

$e_1$ - Reject when in fact $H_o$ is true.
$e_2$ - Accept when in fact $H_o$ is not true.

Problem is one of finding region $\omega$ that will maximize the probabilities of rejection of $H_1$ and $H_2$ for a given $P_{e_1}$.

# ROUTINE ANALYSIS (Cont'd)

B. Check for accuracy and precision.

Assumption

Take Power's 34 observations as setting standard

$$\bar{X}' = 68.9868$$

$$\sigma' = .1639$$

Radius of 99% circle = $3.0349\,\sigma'$

Distance between dotted lines = $2(2.58)\sigma'$

Shift in accuracy - along 45° axis.

Shift in precision - if outside dotted lines

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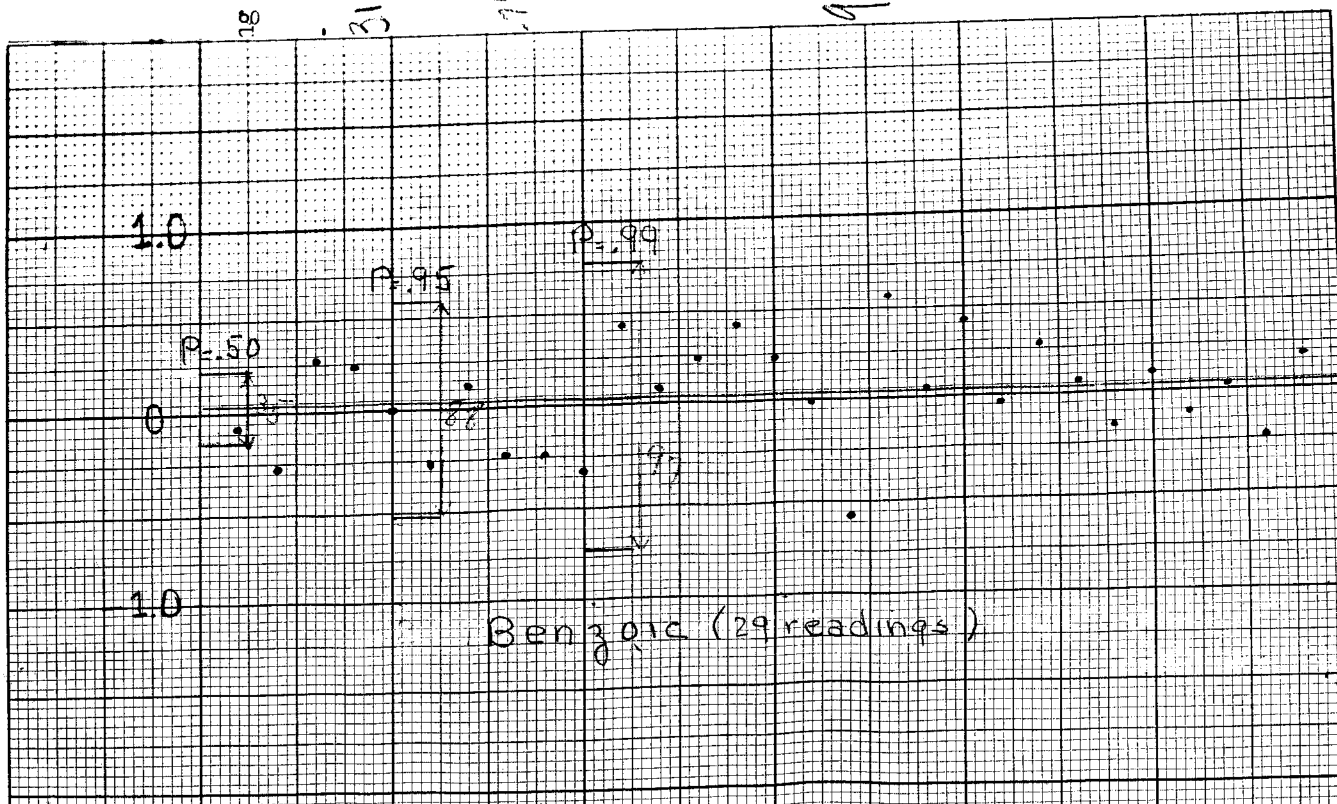



* At least 35, 88, and 97 in tolerance limits .50, .95, and .99 respectively

Benzoic (n = 29)

$\bar{x} = .04069$

$\sigma = .2630$

$s = 1.0177\,\sigma$

$= .26766$

$\sqrt{\frac{30}{29}}\,s = 1.0171\,s$

$= .27224$

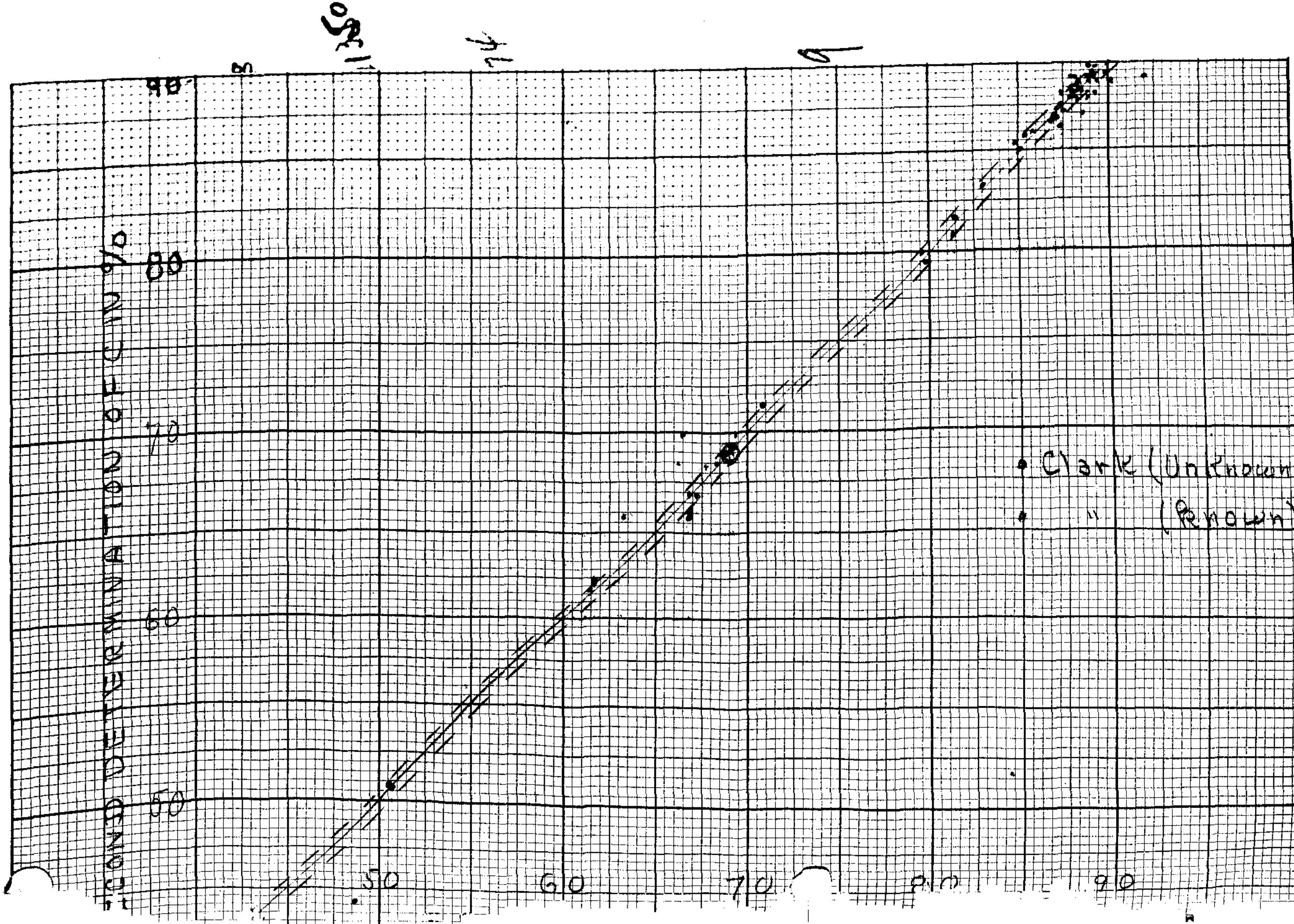
| P | $t_a$ | $t_a\sqrt{\frac{30}{29}}\,s$ |
|---|---|---|
| .50 | .684 | .1862 |
| .95 | 2.052 | .5586 |
| .99 | 2.772 | .7546 |

| | $\bar{x} \pm t_a\sqrt{\frac{30}{29}}\,s$ | tolerance limits $\bar{x} \pm t_a\sqrt{\frac{30}{29}}\,s$ |
|---|---|---|
| .50 | $\bar{x} \pm .6957\,s$ | +.2269<br>−.1455 |
| .95 | $\bar{x} \pm 2.0871\,s$ | +.5993<br>−.5179 |
| .99 | $\bar{x} \pm 2.8194\,s$ | +.7951<br>−.7137 |

| $p$ | $\sigma$ |
|---|---|
| .50 | .05772 |
| .95 | .02574 |
| .99 | .006355 |

| $\bar{p}$ | $3\sigma$ | $\bar{p}+3\sigma$ | $\bar{p}-3\sigma$ |
|---|---|---|---|
| .50 | .1732 | .6732 | .3268 |
| .95 | .07722 | 1.0272 | .8722 |
| .99 | .01906 | 1.0091 | .9709 |

| $\bar{p}$ | $2.58\sigma$ | $\bar{p}+2.58\sigma$ | $\bar{p}-2.58\sigma$ |
|---|---|---|---|
| .50 | .1489 | .6489 | .3511 |
| .95 | .06641 | 1.0164 | .8836 |
| .99 | .01640 | 1.0064 | .9736 |


COND DETERMINATION OF C IN %
50
60
70
80
90
• Clark (Unknown)
" (Known)
1350
74
9

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