Economic Quality Control of Manufactured Product[1]

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That we cannot make all pieces of a given kind of product identically alike is accepted as a general truth. It follows that the qualities of pieces of the same kind of product differ among themselves, or, in other words, the quality of product must be expected to vary. The causes of this variability are, in general, unknown.

The present paper presents a scientific basis for determining when we have gone as far as it is economically feasible to go in eliminating these unknown or chance causes of variability in the quality of a product. When this state has been reached, the product is said to be *controlled* because it is then possible to set up limits within which the quality may be expected to remain in the future. By securing control, we attain the five economic advantages discussed in Part III.

I Introduction

1. *What is the Problem of Control?*

WHAT is the problem involved in the control of quality of manufactured product? To answer this question, let us put ourselves in the position of a manufacturer turning out millions of the same kind of thing every year. Whether it be lead pencils, chewing gum, bars of soap, telephones or automobiles, the problem is much the same. He sets up a standard for the quality of his product and then tries to make all pieces of product conform with this standard. Here his troubles begin. For him standard quality is a bull's-eye, but like a marksman shooting at such a target, he often misses. As is the case in everything we do, unknown or chance causes exert their influence. The problem then is: how much may the quality of a product vary and yet be controlled? In other words, how much variation should we leave to chance?

To make a thing the way we want to make it is one popular conception of control. We have been trying to do this for a good many years and we see the fruition of this effort in the marvelous industrial development around us. We have accepted the idea of applying scientific principles but now a change is coming about in the principles themselves which necessitates a new concept of control.

A few years ago we were inclined to look forward to the time when a manufacturer would be able to do just what he wanted to do. We shared the enthusiasm of Pope when he said "All chance is but direction thou canst not see," and we looked forward to the time when we would see that direction. In other words, emphasis was laid on the *exactness*

[1] Paper presented before A. A. A. S. on December 28, 1929, at Des Moines, Iowa.

of physical laws. Today, however, the emphasis is placed elsewhere as is indicated by the following quotation from a recent issue, July, 1927, of the journal *Engineering:*

"Today the mathematical physicist seems more and more inclined to the opinion that each of the so-called laws of nature is essentially statistical, and that all our equations and theories can do, is to provide us with a series of orbits of varying probabilities."

The breakdown of the old orthodox scientific theory which formed the basis of applied science in the past necessitates the introduction of certain new concepts into industrial development. Along with this change must come a revision in our ideas of such things as a controlled product, an economic standard of quality and the method of detecting lack of control or those variations which should not be left to chance.

Realizing, then, the statistical nature of modern science, it is but logical for the manufacturer to turn his attention to the consideration of available ways and means of handling statistical problems. The necessity for doing this is pointed out in the recent book on the "Application of Statistics in Mass Production," by Becker, Plaut and Runge. They say:

"It is therefore important to every technician who is dealing with problems of manufacturing control to know the laws of statistics and to be able to apply them correctly to his problems."

Another German writer, K. H. Daeves, writing on somewhat the same subject says:

"Statistical research is a logical method for the control of operations, for the research engineer, the plant superintendent, and the production executive."

This statement is of particular interest because its author has for several years been associated with the application of statistical methods in the steel industry.

The problem of control viewed from this angle is a comparatively new one. In fact, very little has been written on the subject. Progress in modifying our concept of control has been and will be comparatively slow. In the first place, it requires the application of certain modern physical concepts and in the second place, it requires the application of statistical methods which up to the present time have been for the most part left undisturbed in the journals in which they appeared. This situation is admirably summed up by the magazine *Nature* of January, 1926, as follows:

"A large amount of work has been done in developing statistical methods on the scientific side, and it is natural for any one interested in science to hope that all this work may be utilized in commerce and industry. There are signs that such a movement has started, and it would be unfortunate indeed if those responsible in practical affairs fail to take advantage of the improved statistical machinery now available."

2. *Object*

The object of this paper is the presentation of a scientific basis for interpreting the significance of chance variations in quality of product and for eliminating causes of variability which need not be left to chance, making possible more uniform quality and thereby effecting certain economies.

3. *Nature of Control*

Let us consider a very simple example of our inability to do exactly what we want to do and thereby illustrate two characteristics of a controlled product.

Write the letter *a* on a piece of paper. Now make another *a* just like the first one; then another and another until you have a series of *a*'s, *a*, *a*, *a*, *a*, ... You try to make all the *a*'s alike but you don't; you can't. You are willing to accept this as an empirically established fact. But what of it? Let us see just what this means in respect to control. Why can we not do a simple thing like making all the *a*'s just alike? Your answer leads to a generalization which all of us are perhaps willing to accept. It is that there are many causes of variability among the *a*'s: the paper was not smooth, the lead in the pencil was not uniform and the unavoidable variability in your external surroundings reacted upon you to introduce variations in the *a*'s. But are these the only causes of variability in the *a*'s? Probably not.

We accept our human limitations and say that likely there are many other factors. If we could but name all the reasons why we cannot make the *a*'s alike, we would most assuredly have a better understanding of a certain part of nature than we now have. Of course this conception of what it means to be able to do what we want to do is not new; it does not belong exclusively to any one field of human thought; it is a commonly accepted conception.

The point to be made in this simple illustration is that we are limited in doing what we want to do; that to do what we set out to do, even in so simple a thing as making *a*'s that are alike requires almost infinite knowledge compared with that which we now possess. It follows, therefore, since we are thus willing to accept as axiomatic that we cannot do what we want to do and that we cannot hope to understand why we cannot, that we must also accept as axiomatic that a controlled quality will not be a constant quality. Instead a controlled quality must be a *variable* quality. This is the first characteristic.

But let us go back to the results of the experiment on the *a*'s and we shall find out something more about control. Your *a*'s are different from my *a*'s; there is something about your *a*'s which makes them yours and something about my a's that makes them mine. True, not all of your a's are alike. Neither are all of my a's alike. Each group of a's varies within a certain range and yet each group is distinguishable from the others. This distinguishable and, as it were, constant variability *within limits* is the second characteristic of control.

4. *Definition of Control*

For our present purpose a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon will be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within the given limits.

In this sense the time of the eclipse of the sun is a predictable phenomenon. So also is the distance covered in successive intervals of time by a freely falling body. In fact, the prediction in such cases is extremely precise. It is an entirely different matter, however, to predict the expected length of life of an individual at a given age; the velocity of a molecule at a given instant of time; the breaking strength of a steel wire of known cross section; or numerous other phenomena of like character. In fact, a prediction of the type illustrated by forecasting the time of an eclipse of the sun is almost the exception rather than the rule in scientific and industrial work.

In all forms of prediction an element of chance enters. The specific problem which concerns us at the present moment is the formulation of a scientific basis for prediction, taking into account the element of chance, where, for the purpose of our discussion, any unknown cause of a phenomenon will be termed a *chance* cause.

II. Scientific Basis for Control

1. *Three Important Postulates*

What can we say about the future behavior of a phenomenon acting under the influence of unknown or chance causes? I doubt that, in general, we can say anything. For example, let me ask: "What will be the price of your favorite stock thirty years from today?" Are you willing to gamble much on your powers of prediction in such a case? Probably not. However, if I ask: "Suppose you were to toss a penny one hundred times, thirty years from today, what proportion of heads would you expect to find?," your willingness to gamble on your powers of prediction would be of an entirely different order than in the previous case.

The recognized difference between these two situations leads us to make the following simple postulate:

Postulate 1. All chance systems of causes are not alike in the sense that they enable us to predict the future in terms of the past.

Hence, if we are to be able to predict the quality of product at least within limits, we must find some criterion to apply to observed variability in quality to determine whether or not the cause system producing it is such as to make possible future predictions.

Perhaps the natural course to follow is to glean what we can about the workings of unknown chance causes which are generally acknowledged to be controlled in the sense that they permit of prediction within limits. Perhaps no better examples could be considered than those which influence length of human life and molecular motion, for it often appears that nothing is more uncertain than life itself, unless perhaps it be molecular motion. Yet there is something certain about these uncertainties. In the assumed laws of mortality and distribution of molecular displacement, we find some of the essential characteristics of control within limits.

A. Law of Mortality

The date of death always has seemed to be fixed by chance even though great human effort has been expended in trying to rob chance

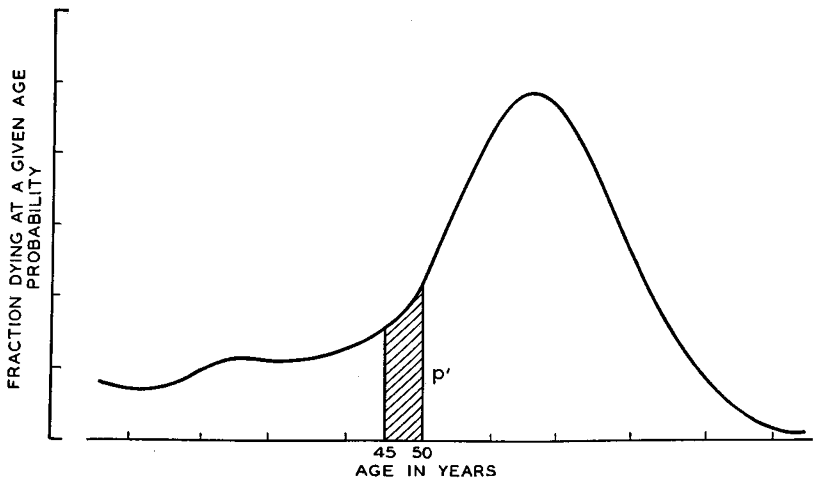

Fig. 1—Law of mortality—law of fluctuations controlled within limits.

of this prerogative. We come into this world and from that very instant on are surrounded by causes of death seeking our life. Who knows whether or not death will overtake us within the next year?

If so, what will be the cause? These questions we cannot answer. Some of us are to fall at one time from one cause, others at another time from another cause. In this fight for life we see then the element of uncertainty and the interplay of numerous unknown or chance causes.

However, when we study the effect of these chance causes in producing deaths in large groups of individuals, we find some indication of a controlled condition. We find that this hidden host of causes produce deaths at an average rate which does not differ much over long periods of time. From such observations we are led to believe that, as we approach the condition of homogeneity of population and surroundings, we approach what is customarily termed a "Law of mortality" such as indicated schematically in Fig. 1. In other words, we believe that in the limiting case of homogeneity the causes of death function so as to make the probability, let us call it dy, of dying within given age limits, such as forty-five to fifty, constant: That is, we believe these causes are controlled. In other words, we assume the existence of a kind of statistical equilibrium among the effects of such an unknown system of chance causes expressable in the assumption that the probability of dying within a given age limit, under the assumed conditions, is an *objective* and constant reality.

B. Molecular Motion

Just about a century ago, in 1827 to be exact, an English botanist, Brown, saw something through his microscope that caught his interest. It was motion going on among the suspended particles almost as though they were alive. In a way it resembled the dance of dust particles in sunlight, so familiar to us, but this dance differed from that of the dust particles in important respects—for example, adjacent particles seen under the microscope did not necessarily move in even approximately the same direction, as do adjacent dust particles suspended in the air.

Watch such motion for several minutes. So long as the temperature remains constant, there is no change. Watch it for hours, the motion remains characteristically the same. Watch it for days, we see no difference. Even particles suspended in liquids enclosed in quartz crystals for thousands of years show exactly the same kind of motion. Therefore, to the best of our knowledge there is remarkable permanence to this motion. Its characteristics remain constant. Here we certainly find a remarkable degree of constancy exhibited by a chance system of causes.

Suppose we follow the motion of one particle to get a better picture of this constancy. This has been done for us by several investigators, notably Perrin. In such an experiment he noted the position of a particle at the end of equal intervals of time, Fig. 2. He found that

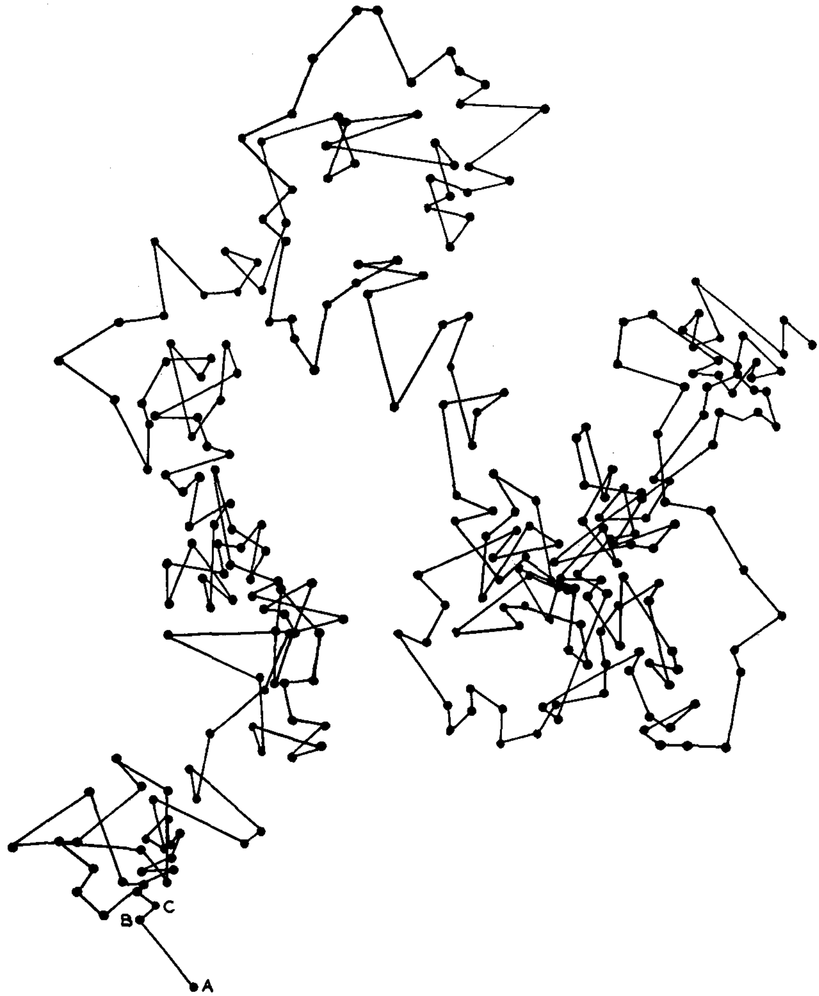

Fig. 2—A close-up of molecular motion appearing absolutely irregular, yet controlled within limits.

the direction of this motion observed in one interval differed, in general, from that in the next succeeding interval. He found that the direction of the motion presents what we instinctively call absolute irregularity. Let us ask ourselves certain questions about this motion.

Suppose we fix our attention on the particle at the point A. What made it move to B in the next interval of time? Of course we answer by saying that a particle moves at a given instant in a given direction, say AB, because the resultant force of the molecules hitting it in a plane perpendicular to this direction from the side away from B is greater than that on the side toward B; but at any given instant of time there is no way of telling what molecules are engaged in giving it such motion. We do not even know how many molecules are taking part. Do what we will, so long as the temperature is kept constant, we cannot change this motion in a given system. It cannot be said, for example, when the particle is at the point B that during the next interval of time it will move to C. We can do nothing to control the motion in the matter of displacement or in the matter of the direction of this displacement.

Let us consider either the x or y components of the segments of the paths. Within recent years we find abundant evidence indicating that these displacements appear to be distributed about zero in accord with what is called the normal law. That is to say, if x represents the deviation from the mean displacement, zero in this case, the probability dy of x lying within the range x to $x + dx$ is given by

$$dy = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x^2/2\sigma^2)} dx, \tag{1}$$

where σ is the root mean square deviation.

Such evidence as that provided by the law of mortality and the law of distribution of molecular displacements leads us to assume that there exist in nature phenomena controlled by systems of chance causes such that the probability dy of the magnitude X of a characteristic of some such phenomenon falling within the interval X to $X + dX$ is expressable as a function f of the quantity X and certain parameters represented symbolically in the equation

$$dy = f(X, \lambda_1, \lambda_2, \cdots, \lambda_m) dX, \tag{2}$$

where the λ's denote the parameters. Such a system of causes we shall term *constant* because the probability dy is independent of time. We shall take as our second postulate:

> *Postulate 2—Constant systems of chance causes do exist in nature.*

To say that such systems of causes exist in nature, however, is one thing; to say that such systems of causes exist in a production process is quite another thing. Less than ten years ago it seemed reasonable to assume that such systems of causes existed in the production of telephone equipment. Today we have abundant evidence of their existence. The practical situation, however, is that in the majority of cases there are unknown causes of variability in the quality of a product which do not belong to a constant system. This fact was discovered very early in the development of control methods, and these causes were called *assignable.* The question naturally arose as to whether it was possible, in general, to find and eliminate causes of variability which did not form a part of a constant system. Less than ten years ago it seemed reasonable to assume that this could be done. Today we have abundant evidence to justify this assumption. We shall, therefore, adopt as our third postulate:

Postulate 3—Assignable causes of variation may be found and eliminated.

Hence, to secure control, the manufacturer must seek to find and eliminate assignable causes. In practice, however, he has the difficulty of judging from an observed set of data, whether or not assignable causes are present. A simple illustration will make this point clear.

2. *When Do Fluctuations Indicate Trouble?*

In many instances the quality of the product is measured by the fraction non-conforming to engineering specifications or as we say the fraction defective. Table 1 gives for a period of 12 months the observed fluctuations in this fraction for two kinds of product designated here as Type A and Type B. For each month we have the sample size n, the number defective n_1 and the fraction $p = n_1/n$. We can

TABLE 1

Apparatus Type A				Apparatus Type B			
Month	n No. Insp.	n_1 No. Def.	$p = n_1/n$ Fraction Def.	Month	n No. Insp.	n_1 No. Def.	$p = n_1/n$ Fraction Def.
Jan.	527	4	.0076	Jan.	169	1	.0059
Feb.	610	5	.0082	Feb.	99	3	.0303
Mar.	428	5	.0017	Mar.	208	1	.0048
Apr.	400	2	.0050	Apr.	196	1	.0051
May	498	15	.0301	May	132	1	.0076
June	500	3	.0060	June	89	1	.0112
July	395	3	.0076	July	167	1	.0060
Aug.	393	2	.0051	Aug.	200	1	.0050
Sept.	625	3	.0058	Sept.	171	2	.0117
Oct.	465	13	.0280	Oct.	122	1	.0082
Nov.	446	5	.0112	Nov.	107	3	.0280
Dec.	510	3	.0059	Dec.	132	1	.0076
Average	483.08	5.25	.0109		149.33	1.42	.0095

better visualize the extent of these fluctuations in fraction defective by plotting the data as in Fig. 3-*a* and Fig. 3-*b*.

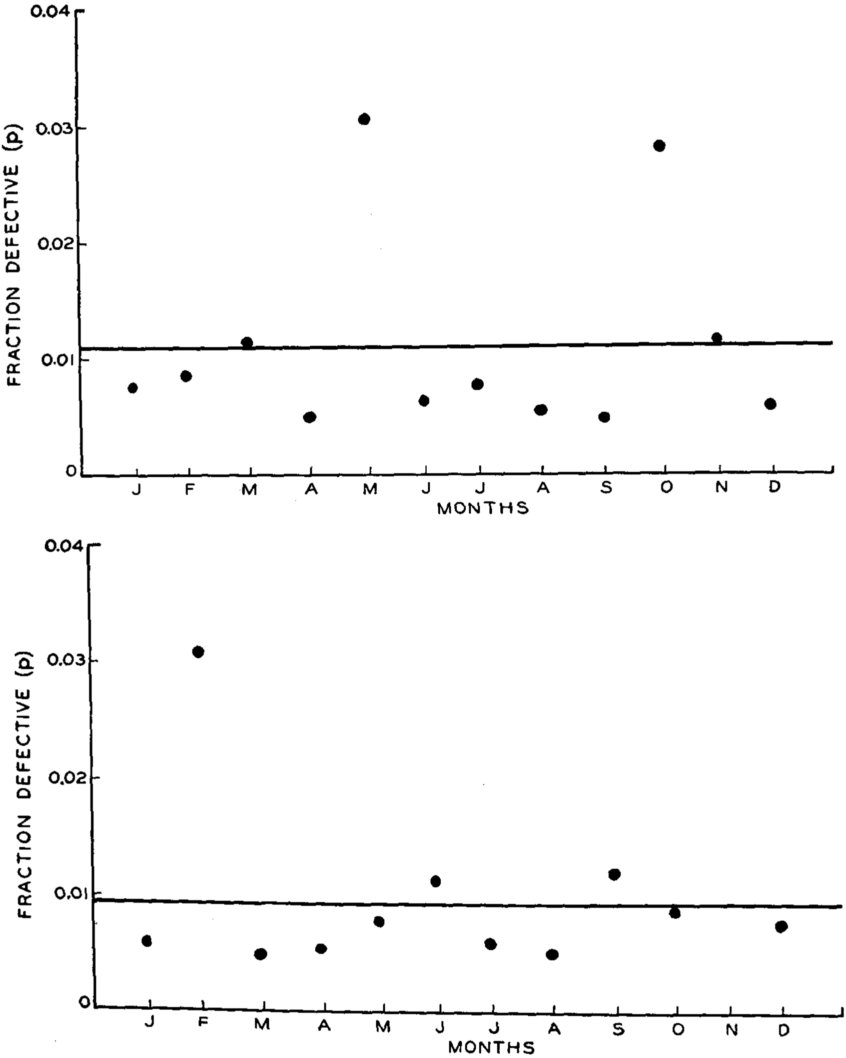

Fig. 3—Should these variations be left to chance?

a. Apparatus Type A.
b. Apparatus Type B.

What we need is some yardstick to detect in such variations any evidence of the presence of assignable causes. Can we find such a yardstick? Experience of the kind soon to be considered indicates an affirmative answer. It leads us to conclude that it is feasible to establish criteria useful in detecting the presence of assignable causes of

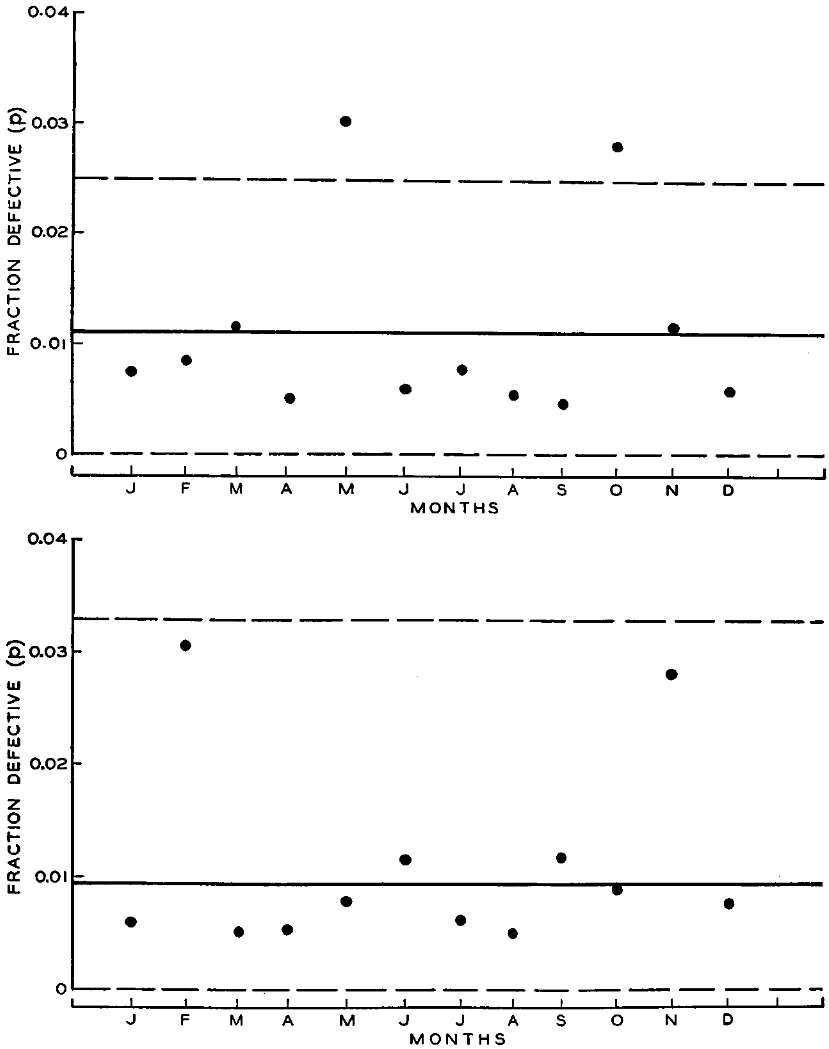

Fig. 4—Should these variations be left to chance?

a. No.

b. Yes.

variation or, in other words, criteria which when applied to a set of observed values will indicate whether or not it is reasonable to believe that the causes of variability should be left to chance. Such criteria are basic to any method of securing control within limits. Let us, therefore, consider them critically. It is too much to expect that the criteria will be infallible. We are amply rewarded if they appear to to work in the majority of cases.

Generally speaking, the criteria are of the nature of limits derived from past experience showing within what range the fluctuations in quality should remain, provided they are to be left to chance. For example, when such limits are placed on the fluctuations in the qualities shown in Fig. 3, we find (see Fig. 4) that in one case two points fall outside the limits and in the other case no point falls outside the limits. Upon the basis of the use of such limits, we look for trouble in the form

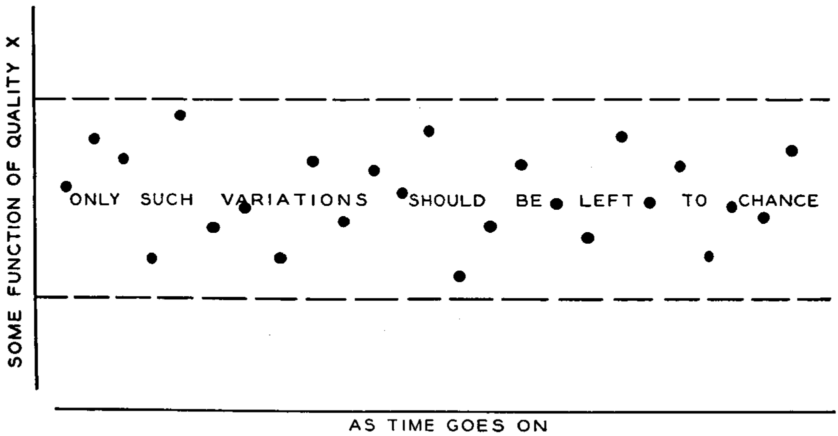

Fig. 5—Art plus modern statistical machinery makes possible the establishment of such limits.

of assignable causes in one case but not in the other. However, to be of really practical interest, we should be able to answer the following question: Can we expect to be able to find and eliminate causes of variability *only* when deviations fall outside the limits? First, let us see what statistical theory has to say in answer to this question.

Upon the basis of postulate 3, it follows that we can find and remove causes of variability until the remaining system of causes is constant or until we reach that state where the probability that the deviations in quality remain within any two fixed limits (Fig. 5) is constant. However, this assumption alone does not tell us that there are certain limits within which all observed values of quality should remain provided the causes cannot be found and eliminated. In fact so long as the limits are set so that the probability of falling within the limits is less than unity, we may always expect a certain percentage of observations to fall outside the limits even though the system of causes be constant. In other words, the acceptance of this assumption gives us a right to believe that there is an objective state of control within limits but in itself it does not furnish the practical criterion for determining when variations in quality, such as given in Fig. 3, should be left to chance.

Furthermore, we may say that mathematical statistics as such does not give us the desired criterion. What does this situation mean in plain every day engineering English? Simply this: such criteria, if they exist, cannot be shown to exist by any theorizing alone, no matter how well equipped the theorist is in respect to probability or statistical theory. We see in this situation the long recognized dividing line

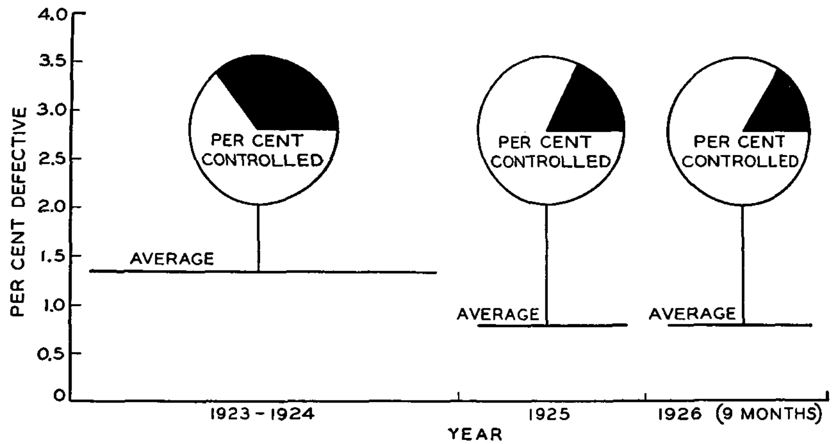

Fig. 6—Evidence of improvement in quality with approach to control.

between theory and practice. The available statistical machinery referred to by the magazine *Nature* is, as we might expect, not an end in itself but merely a means to an end. In other words, the fact that the criterion which we happen to use has a fine ancestry of high-brow statistical theorems does not justify its use. Such justification must come from empirical evidence that it works. As the practical engineer might say, the proof of the pudding is in the eating. Let us therefore look for the proof.

3. *Evidence that Criteria Exist for Detecting Assignable Causes*

A. Fig. 6 shows the results of one of the first large scale experiments to determine whether or not indications given by such a criterion applied to quality measured in terms of fraction defective were justified by experience. About thirty typical items used in the telephone plant and produced in lots running into the millions per year were made the basis for this study. As shown in this figure during 1923–24, these items showed 68 per cent control about a relatively low average of 1.4 per cent defective.[1] However, as the assignable causes indicated by deviations in the observed monthly fraction defective falling outside of control limits were found and eliminated, the quality of product approached the state of control as indicated by an increase of from 68 per cent to 84 per cent control by the latter part of 1926. At the same time the quality improved; in 1923–24 the average per cent defective was 1.4 per cent whereas by 1926 this had been reduced to .8 per cent. Here we get some typical evidence that, in general, as the assignable causes are removed, the variations tend to fall more nearly within the limits as indicated by an increase from 68 per cent to 84 per cent. Such evidence is, of course, one sided. It shows that when points fall outside the limits, experience indicates that we can find assignable causes, but it does not indicate that when points fall within such limits, we cannot find causes of variability. However, this kind of evidence is provided by the following two typical illustrations.

TABLE 2

Electrical Resistance of Insulations in Megohms.
Should Such Variations be Left to Chance?

5045	4635	4700	4650	4640	3940	4570	4560	4450	4500	5075	4500
4350	5100	4600	4170	4335	3700	4570	3075	4450	4770	4925	4850
4350	5450	4110	4255	5000	3650	4855	2965	4850	5150	5075	4930
3975	4635	4410	4170	4615	4445	4160	4080	4450	4850	4925	4700
4290	4720	4180	4375	4215	4000	4325	4080	3635	4700	5250	4890
4430	4810	4790	4175	4275	4845	4125	4425	3635	5000	4915	4625
4485	4565	4790	4550	4275	5000	4100	4300	3635	5000	5600	4425
4285	4410	4340	4450	5000	4560	4340	4430	3900	5000	5075	4135
3980	4065	4895	2855	4615	4700	4575	4840	4340	4700	4450	4190
3925	4565	5750	2920	4735	4310	3875	4840	4340	4500	4215	4080
3645	4190	4740	4375	4215	4310	4050	4310	3665	4840	4325	3690
3760	4725	5000	4375	4700	5000	4050	4185	3775	5075	4665	5050
3300	4640	4895	4355	4700	4575	4685	4570	5000	5000	4615	4625
3685	4640	4255	4090	4700	4700	4685	4700	4850	4770	4615	5150
3463	4895	4170	5000	4700	4430	4430	4440	4775	4570	4500	5250
5200	4790	3850	4335	4095	4850	4300	4850	4500	4925	4765	5000
5100	4845	4445	5000	4095	4850	4690	4125	4770	4775	4500	5000

B. In the production of a certain kind of equipment, considerable cost was involved in securing the necessary electrical insulation by means of materials previously used for that purpose. A research program was started to secure a cheaper material. After a long series of preliminary experiments, a tentative substitute was chosen and an

[1] Jones, R. L., "Quality of Telephone Materials," *Bell Telephone Quarterly*, June, 1927.

extensive series of tests of insulation resistance were made on this material, care being taken to eliminate all known causes of variability. Table 2 gives the results of 204 observations of resistance in megohms taken on as many samples of the proposed substitute material. Reading from top to bottom beginning at the left column and continuing throughout the table gives the order in which the observations were made. The question is: "Should such variations be left to chance?"

No *a priori* reason existed for believing that the measurements forming one portion of this series should be different from those in any other portion. In other words, there was no rational basis for dividing the

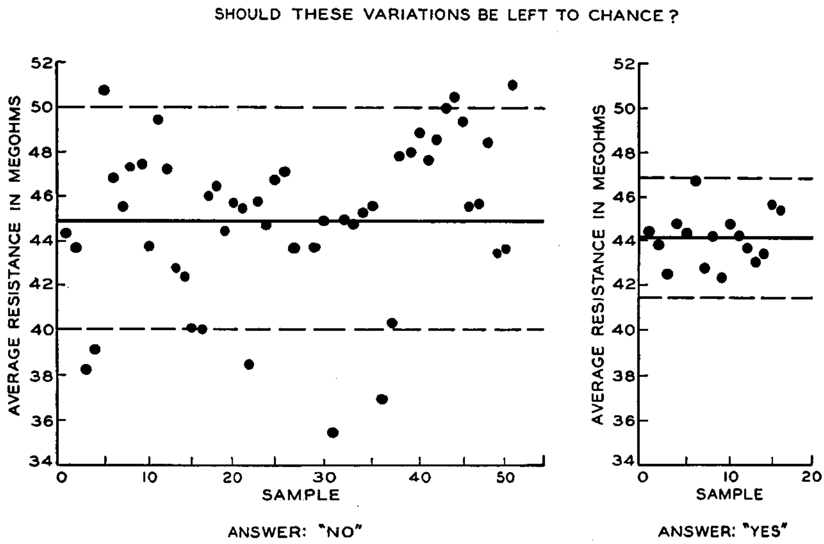

Fig. 7.

total set of data into groups of a given number of observations except that it was reasonable to believe that the system of causes might have changed from day to day as a result of changes in such things as atmospheric conditions, observers, and materials. In general, if such a change is to take place, we may readily detect its effect provided we divide the total number of observations into comparatively small sub-groups. In this particular instance, the size of the sub-group was taken as four and the black dots in Fig. 7-*a* show the successive averages of four observations in the order in which they were taken. The dotted lines are the limits within which experience has shown that these observations should fall, taking into account the size of the sample, provided the variability should be left to chance. Several of the observed values lie outside these limits. This was taken as an indication of the existence of causes of variability which could be found and eliminated.

Further research was instituted at this point to find these causes of variability. Several were found and after these had been eliminated, another series of observed values gave the results indicated in Fig. 7-*b*. Here we see that all of the points lie within the limits. We assumed, therefore, upon the basis of this test, that it was not feasible for research to go much further in eliminating causes of variability. Because of the importance of this particular experiment, however,

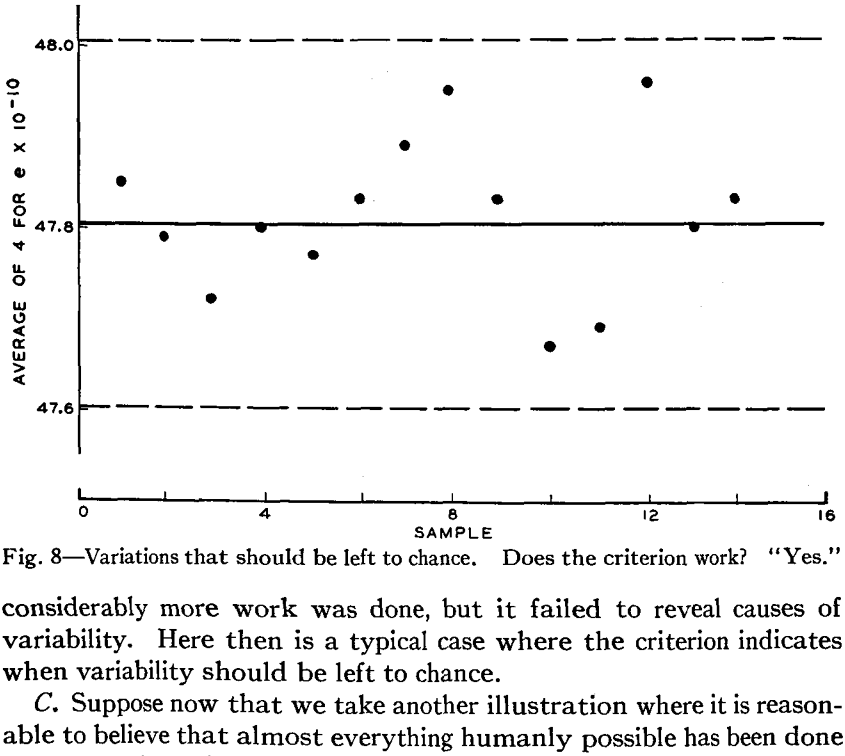

Fig. 8—Variations that should be left to chance. Does the criterion work? "Yes."

considerably more work was done, but it failed to reveal causes of variability. Here then is a typical case where the criterion indicates when variability should be left to chance.

C. Suppose now that we take another illustration where it is reasonable to believe that almost everything humanly possible has been done to remove the assignable causes of variation in a set of data. Perhaps the outstanding series of observations of this type is that given by Millikan in his famous measurement of the charge on an electron. Treating his data in a manner similar to that indicated above, we get the results shown in Fig. 8. All of the points are within the dotted limits. Hence the indication of the test is consistent with the accepted conclusion that those factors which need not be left to chance had been eliminated before this particular set of data were taken.

4. *Rôle Played by Statistical Theory*

It may appear thus far that mathematical statistics plays a relatively minor rôle in laying a basis for economic control of quality. Such, however, is not the case. In fact, a central concept in engineering work of today is that almost every physical property is a *statistical distribution.* In other words, an observed set of data constitutes a sample of the effects of unknown chance causes. It is at once apparent, therefore, that sampling theory should prove a valuable tool in testing engineering

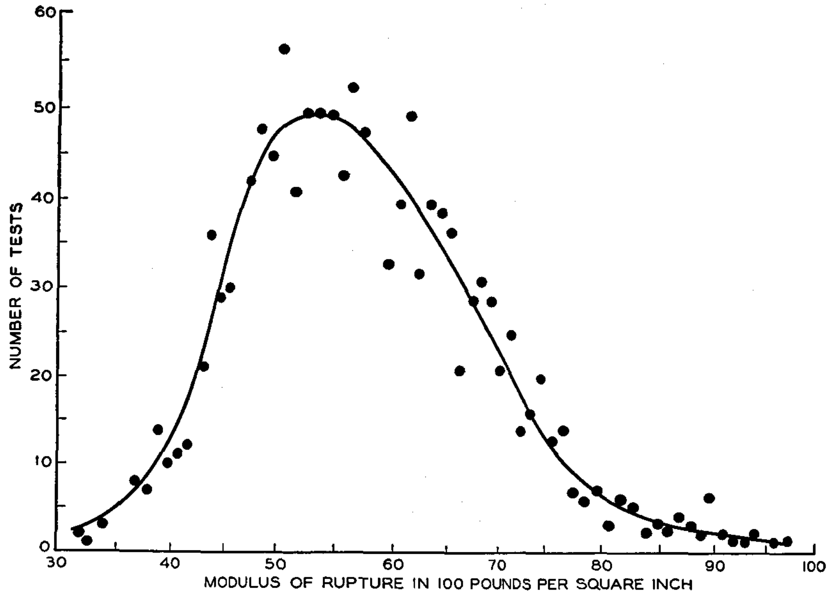

Fig. 9—Variability in modulus of rupture of clear specimens of green sitka spruce typical of the statistical nature of physical properties

hypotheses. Here it is that much of the most recent mathematical theory becomes of value particularly in analysis involving the use of comparatively small numbers of observations.

Let us consider, for example, some property such as the tensile strength of a material. Provided our previous assumptions are justified, it follows that after we have done everything we can to eliminate assignable causes of variation, there will still remain a certain amount of variability exhibiting the state of control. Let us consider an extensive series of data recently published by a member of the Forest Products Laboratories[2] (Fig. 9). Here we have the results of tests for tensile strength on 1304 small test specimens of sitka spruce, the kind

[2] Newlin, J. A., *Proceedings of the American Society of Civil Engineers,* September, 1926, pp, 1436–1443.

of material used in aeroplane propellers during the war. The wide variability is certainly striking. The smooth solid curve is an approximation to the distribution function for this particular property representing at least approximately a state of control. The importance of going from the sample to the smooth distribution is at once apparent and in this case a comparatively small amount of refinement in statistical machinery is required.

Suppose, however, that instead of more than a thousand measurements we had only a very small number, such as is so often the case in engineering work. Our estimation of the variability of the distribution function, representing the state of control, upon the basis of the information given by the sample would necessarily be quite different from that ordinarily used by engineers (see Fig. 10). This is true even though

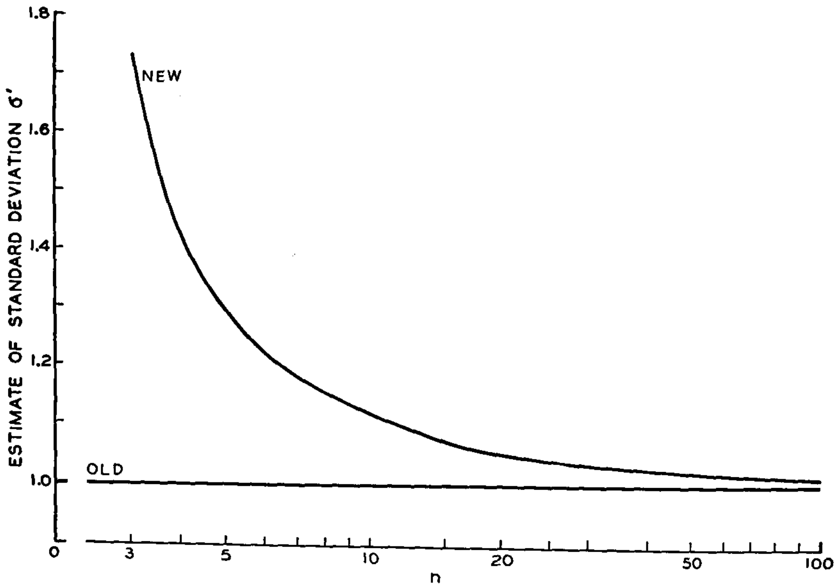

Fig. 10—Correction factors made possible by modern statistical theory are often large.—Typical Illustration.

we make the same kind of assumption to begin with as engineers have been accustomed to do in the past. This we may take as a typical example of the fact that the production engineer finds it to his advantage to keep abreast of the developments in statistical theory. Here we use new in the sense that much of modern statistical machinery is new to most engineers.

5. *Conclusion*

Based upon evidence such as already presented, it appears to be practicable to set up criteria by which to determine when assignable causes of variations in quality have been eliminated so that the product may then be considered to be controlled within limits. This state of control appears to be, in general, a kind of limit to which we may expect to go economically in finding and removing causes of variability without changing a major portion of the manufacturing process as, for example, would be involved in the substitution of new materials or designs.

III. Advantages Secured through Control

1. *Reduction in the Cost of Inspection*

If we can be assured that something we use is produced under controlled conditions, we do not feel the need for inspecting it as much as

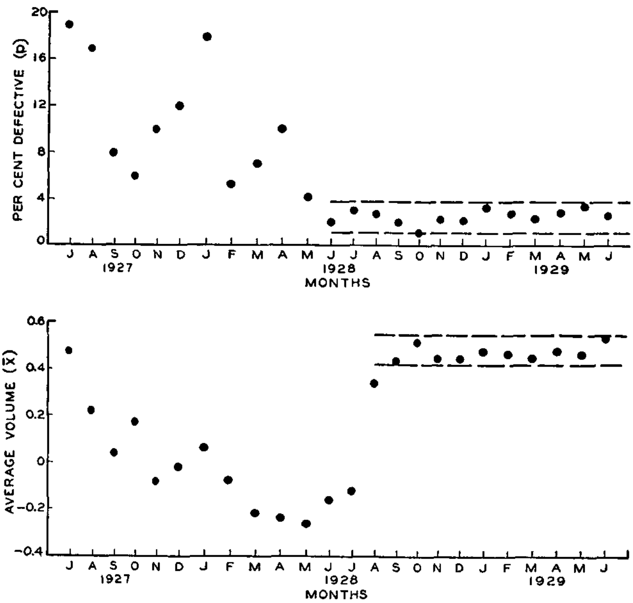

Fig. 11—Approach to stable equilibrium or control as assignable causes are weeded out, thus reducing the need for inspection.

we would if we did not have this assurance. For example, we do not waste our money on doctors' bills so long as we are willing to attribute the variability in our health to the effects of what in our present terminology corresponds to a constant system of chance causes.

In the early stages of production there are usually causes of variability which must be weeded out through the process of inspection. As we proceed to eliminate assignable causes, the quality of product usually approaches a state of stable equilibrium somewhat after the manner of the two specific illustrations presented in Fig. 11. In both instances, the record goes back for more than two years and the process of elimination in each case covers a period of more than a year.

It is evident that as the quality approaches what appears to be a comparatively stable state, the need for inspection is reduced.

2. *Reduction in the Cost of Rejections*

That we may better visualize the economic significance of control, we shall now view the production process as a whole. We take as a specific illustration the manufacture of telephone equipment. Picture, if you will, the twenty or more raw materials such as gold, platinum, silver, copper, tin, lead, wool, rubber, silk, and so forth, literally collected from the four corners of the earth and poured into the manufacturing process. The telephone instrument as it emerges at the end of the production process is not so simple as it looks. In it there are 201 parts, and in the line and equipment making possible the connection of one telephone to another, there are approximately 110,000 more parts. The annual production of most of these parts runs into the millions so that the total annual production of parts runs into the billions.

How shall the production process for such a complicated mechanism be engineered so as to secure the economies of quantity production and at the same time a finished product with quality characteristics lying within specified tolerances? One such scheme is illustrated in Fig. 12. Here the manufacturing process is indicated schematically as a funnel, at the small end of which we have the 100 per cent inspection screen to protect the consumer by assuring that the quality of the finished product is satisfactory. Obviously, however, it is often more economical to throw out defective material at some of the initial stages in production rather than to let it pass on to the final stage where it would likely cause the rejection of a finished unit of product. For example, we see to the right of the funnel, piles of defectives, which must be junked or reclaimed at considerable cost.

It may be shown theoretically that, by eliminating assignable causes of variability, we arrive at a limit to which it is feasible to go in reducing the fraction defective. It must suffice here to call attention to the kind of evidence indicating that this limiting situation is actually approached in practice as we remove the assignable causes of variability.

Let us refer to the information given in Fig. 6 which is particularly significant because it represents the results of a large scale experiment carried on under commercial conditions. As the black sectors in the pie charts decrease in size, indicating progress in the removal of assignable causes, we find simultaneously a decrease in the average frac-

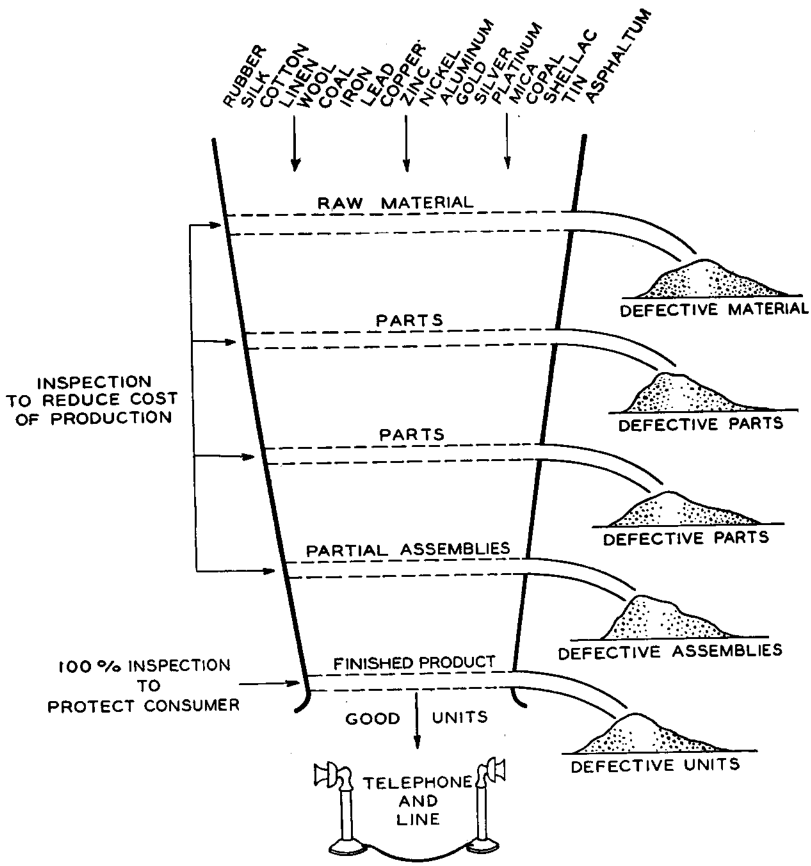

Fig. 12—An economic production scheme.

tion defective from .014 to .008. Here we see how control works to reduce the amount of defective material. However, this is such an important point that it is perhaps interesting to consider an illustration from outside the telephone field.

Recent work of the Food Research Institute of Stanford University shows that the loss from stale bread constitutes an important item of cost for a great number of wholesale as well as some retail bakeries. They estimate that this factor alone costs people of the United States millions of dollars per year. The sales manager of every baking corporation is interested, therefore, in detecting and finding assignable causes of variation in the returns of stale bread provided that by so doing he may reduce to a minimum the loss arising in this way.

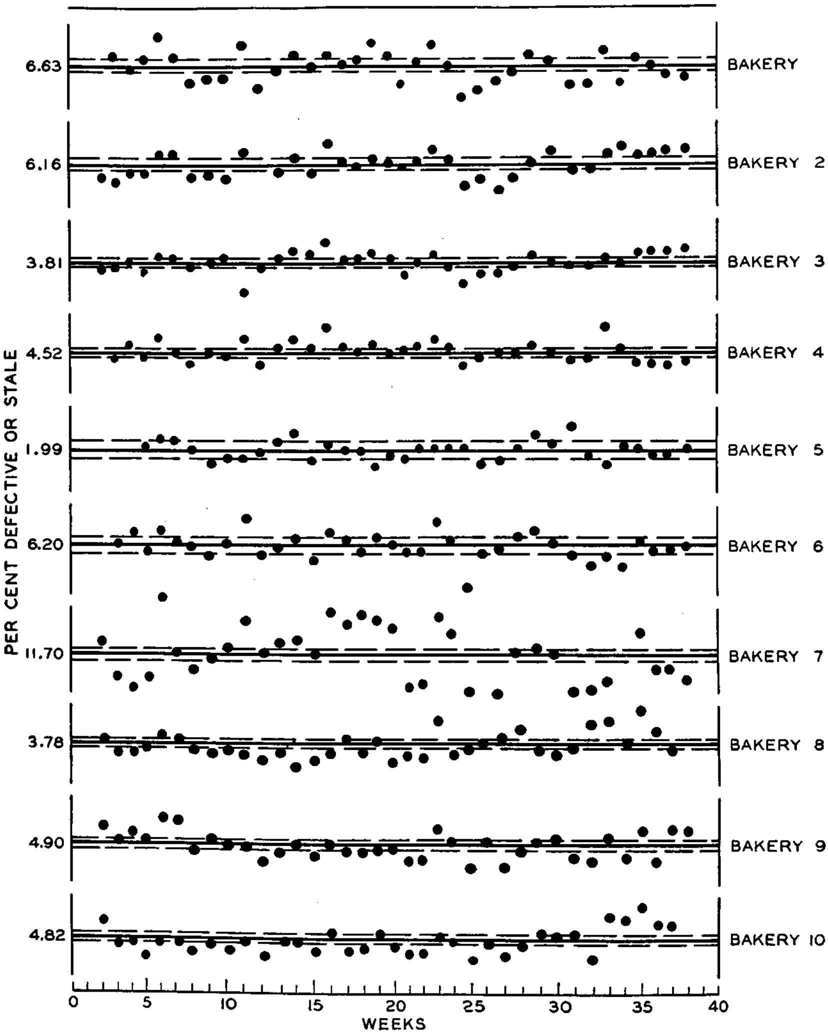

Fig. 13—Results showing how control effects a reduction in the cost of rejections.

Some time ago it became possible to secure the weekly record of return of stale bread for ten different bakeries operating in a certain metropolitan district. These observed results are shown graphically in Fig. 13. At once we see that there is a definite lack of control on the part of each bakery. The important thing for us to note, however, is that the bakery having the lowest percentage return, 1.99 per cent, also shows better control than the other bakeries as judged by the number of points falling outside the control limits in the period of 36 weeks.

3. *Attainment of Maximum Benefits from Quantity Production*

The quality of the finished product depends upon the qualities of raw materials, piece parts and the assembling process. It follows from simple theory that so long as such quality characteristics are controlled, the quality of the finished unit will be controlled, and will therefore exhibit *minimum variability*. Other advantages also result. For example, by gaining control, it is as we have already seen, possible to establish standard statistical distributions for the many quality characteristics involved in design. Very briefly, let us see just how these statistical distributions, representing states of control, become useful in securing an economic design and production scheme.

Suppose we consider a simple problem in which we assume that the quality characteristic Y in the finished product is a function f of m different quality characteristics, $X_1, X_2, \cdots, X_m$, representable symbolically by Equation (3).

$$Y = f(X_1, X_2, \cdots, X_m). \tag{3}$$

For example, one of the X's might be a modulus of rupture, another a diameter of cross section, and Y a breaking load. Engineering requirements generally place certain tolerances on the variability in the resultant quality characteristic Y, which variability is in turn a function of the variabilities in each of the m different quality characteristics.

As already stated, the quality characteristic Y will be controlled provided the m independent characteristics are controlled. Knowing the distribution functions for each of the m different independent variables, it is possible to approximate very closely the per cent of the finished product which may be expected to have a quality characteristic Y within the specified tolerances. If it is desirable to minimize the variability in the resultant quality Y by proper choice of materials, for example, and, if standard distribution functions for the given quality characteristics are available for each of several materials, it is possible to choose that particular material which will minimize the variability of the resultant quality at a minimum of cost.

4. *Attainment of Uniform Quality Even Though Inspection Test Is Destructive*

So often the quality of a material of the greatest importance to the individual is one which cannot be measured directly without destroying the material itself. So it is with the fuse that protects your home; with the steering rod on your car; with the rails that hold the locomotive in its course; with the propeller of an aeroplane, and so on indefinitely. How are we to know that a product which cannot be tested in respect to a given quality is satisfactory in respect to this same quality? How are we to know that the fuse will blow at a given current; that the steering rod of your car will not break under maximum load placed upon it? To answer such questions, we must rely upon previous experience. In such a case, causes of variation in quality are unknown and yet we are concerned in assuring ourselves that the quality is satisfactory.

Enough has been said to show that here is one of the very important applications of the theory of control. By weeding out assignable causes of variability, the manufacturer goes to the feasible limit in assuring uniform quality.

5. *Reduction in Tolerance Limits*

By securing control and by making use of modern statistical tools, the manufacturer not only is able to assure quality, even though it cannot be measured directly, but is also often able to reduce the tolerance limits in that quality as one very simple illustration will serve to indicate.

Let us again consider tensile strength of material. Here the measure of either hardness or density is often used to indicate tensile strength. In such cases, it is customary practice to use calibration curves based upon the concept of functional relationship between such characteristics. If instead of basing our use of these tests upon the concept of functional relationship, we base it upon the concept of statistical relationship, we can make use of planes and surfaces of regression as a means of calibration, thus in general making possible a reduction in the error of measurement of the tensile strength and hence the establishment of closer tolerances. It follows that this is true because, when quality can be measured directly and accurately, we can separate those samples of a material for which the quality lies within given tolerance limits from all others. Now, when the method of measurement is indirect and also subject to error, this separation can only be carried on in the probability sense assuming the errors of measurement are controlled by a constant system of chance causes. It is obvious that, corresponding to a given probability, the tolerance limits may be reduced as we reduce the error of measurement.

Fig. 14 gives a simple illustration. Here the comparative magnitudes of the standard deviations of strength about the two lines of regression and the plane [3] of regression are shown schematically by the

[3] For definition of these terms see any elementary text book on statistics.

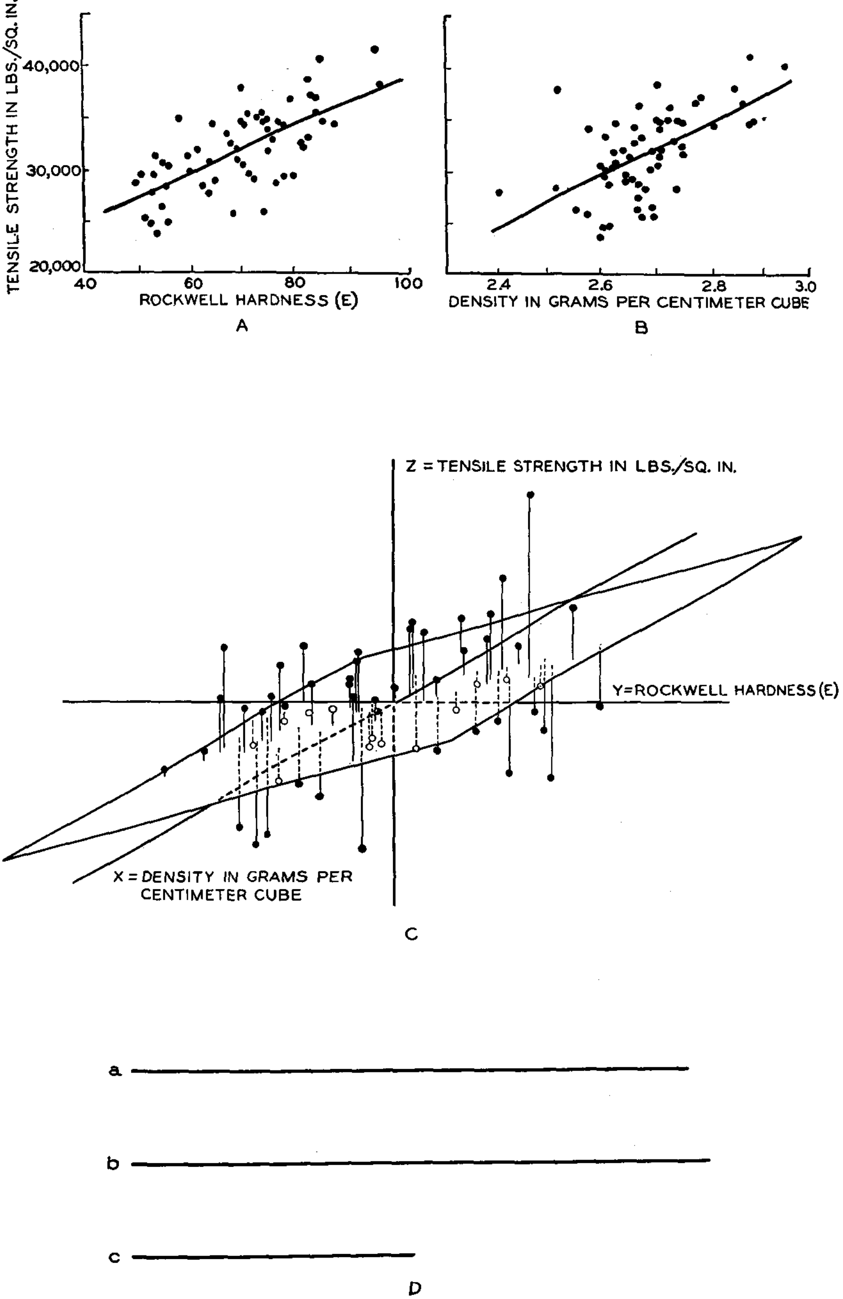

Fig. 14—How control makes possible improved quality through reduction in tolerance limits.

lines in Fig. 14-*d*. The lengths of these are proportional to the allowable tolerance limits corresponding to a given probability. Customary practice is to use the line of regression between tensile strength and hardness. Note the improvement effected by using the plane of regression. By using the hardness and density together as a measure of tensile strength in this case, the tolerance limits on tensile strength corresponding to a given probability can be reduced to approximately one-half what they would be if either of these measures were used alone.

IV. Conclusion

It seems reasonable to believe that there is an *objective state of control*, making possible the prediction of quality within limits even though the causes of variability are unknown. Evidence has been given to indicate that through the use of statistical machinery in the hands of an engineer artful in making the right kind of hypothesis, it appears possible to establish criteria which indicate when the state of control has been reached. It has been shown that by securing this state of control, we can secure the following advantages:

1. Reduction in the cost of inspection.
2. Reduction in the cost of rejections.
3. Attainment of maximum benefits from quantity production.
4. Attainment of uniform quality even though inspection test is destructive.
5. Reduction in tolerance limits where quality measurement is indirect.