INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2016-17

Course Name: M. Tech. (CS) I Year

Subject Name: Introduction To Programming

Date: 03/09/2016 **Maximum Marks: 60** **Duration: 3 hours**

Note: Default Standard is ANSI C. Whenever you write C code, your program should check all possible errors during runtime (syntactically correct) and should be neatly written. Otherwise justify your answers.

Answer any three questions (3x20=60).

1. Prime Factors of a positive integer are the prime numbers that divide that integer exactly. Write a program which takes a positive integer (command line) and prints its distinct prime factors. eg: if your input is 24 your program should print

$$24 = 2, 3.$$

20

2. Write a program to count the frequency of words from a text file given as command line input. You may assume words are separated by blank, comma, Semicolon and full-stop only. Print your result in ascending order of frequency.
   i. the source file name (valid C source code)
   ii. the destination file name (the same C code without any comments)

20

3. Write a program to remove all comments from a C program. Your program should take two command line arguments –
   i. the source file name (valid C source code)
   ii. the destination file name (the same C code without any comments)

20

4. Answer any eight from the following:
   I. Answer/justify the following question w.r.t. C programming.
      i. What are the various methods of communicating data between functions?
      ii. How to declare and define:
         a. Functions
         b. Variables
      iii. Pointers provide machine dependent or independent address arithmetic?
      iv. Does C provide operations to deal directly character strings?
      v. Is there any difference between **'C'** and **"C"** ?

P.T.O

II. What may be the effect of the following C codes?

i.
```
#include <stdio.h>
main()
{
int iVar = 10;
iVar = !iVar>14;
printf ("iVar = %d", iVar);
}
```

ii.
```
#include <stdio.h>
int aiArray[2][3] = {1,2,3,4,5,6};
main()
{
  printf("%d, aiarray[1][1]);
}
```

iii.
```
#include <stdio.h>
myFunction();
main()
{
  myFunction();
}
  myFunction()
{
  static int siVar = 5;
  printf("%d ", siVar--);
  if (siVar)
    myFunction();
}
```

iv.
```
#include <stdio.h>
main()
{
  float fValue = 1.1;
  double dValue = 1.1;
  if (fValue == dValue)
    printf("Opt One");
  else
    printf("Opt Two");
}
```

v.
```
#include <stdio.h>
#define int char
main()
{
  int iVar = 65;
  printf("sizeof(iVar) = %d", sizeof(iVar));
}
```

8x2.5=20

INDIAN STATISTICAL INSTITUTE

PERIODICAL EXAMINATION
M.TECH.(CS) I YEAR

PROBABILITY AND STOCHASTIC PROCESSES

Date: 05.09.2016 Maximum marks: 90 Duration: 3 hours

The paper contains 110 marks. Answer as much as you can, the maximum you can score is 90.

1. (a) Define a probability space and a random variable using the measure-theoretic framework. Define all basic notions required to define these concepts.
   (b) If A_1 and A_2 are independent events, show that A_1 and $\overline{A}_2$ are also independent events.
   (c) Define the Borel σ-algebra $\mathcal{B}(\mathbb{R})$ of $\mathbb{R}$. Let $\pi(\mathbb{R}) = \{(-\infty, x] : x \in \mathbb{R}\}$. Show that $\sigma(\pi(\mathbb{R})) \subseteq \mathcal{B}(\mathbb{R})$.
   (d) Define a distribution function $F : [-\infty, \infty] \to [0, 1]$ such that F has exactly two atoms.
   (e) Let $A_1, A_2, \ldots$ be a sequence of events. Define $\limsup_{n\to\infty} A_n$. If $A_1, A_2, \ldots$ are independent events such that $\sum_{i\geq 1} \Pr[A_i] \to \infty$, show that $\Pr[\limsup_{n\to\infty} A_n] = 1$.
   (f) Define the statistical distance $d(p, q)$ between two discrete distributions p and q defined over a finite non-empty set $\mathcal{Y}$. Show that $d(p, q) = \max\{|p(\mathcal{Y}_0) - q(\mathcal{Y}_0)| : \mathcal{Y}_0 \subseteq \mathcal{Y}\}$.

   $(6 + 5 + 7 + 5 + 10 + 7 = 40)$

2. (a) Let $\{a_n\}_{n\geq 1}$ be a sequence of real numbers such that for each n, a_n is in the interval $[0, 1]$. Suppose a real number X is picked uniformly at random from the interval $[0, 1]$. What is the probability that X is equal to a_n for some n?
   (b) On rolling a die 18 times, what is the probability of getting each face three times?
   (c) Ten dice are thrown. Given that at least one six turns up, what is the probability of obtaining two or more sixes.
   (d) Two persons toss a fair coin n times each. What is the probability that the number of heads seen by the two persons will be equal.
   (e) A biometric identification system accepts a geninue person with probability $p < 1$. Find an expression for the minimum n such that the probability of having at least one rejection in n independent trials is at least α.
   (f) Let $A_1, \ldots, A_n$ be events and such that $\Pr[A_i | \overline{A_1} \wedge \cdots \wedge \overline{A_{i-1}}] = p_i$, for $i = 1, \ldots, n$; $q_1 = 0$ and for $i = 2, \ldots, n$, $\Pr[A_1 \vee \cdots \vee A_{i-1}] = q_i$. Show that $\Pr[A_1 \vee \cdots \vee A_n] = \sum_{i=1}^{n} p_i(1 - q_i)$. Here $\wedge$ denotes 'AND' and $\vee$ denotes 'OR'.

   $(5 \times 6 = 30)$

3. (a) There are two processes, named A and B, for manufacturing computer chips. Process A has 99% reliability while Process B is cheaper, but, has 90% reliability. Factories I and II produce chips using these processes; Factory-I uses Process A to produce 75% of its output, while Factory-II uses Process-A to produce 90% of its output. The two factories have the same production capacity. What is the probability that a randomly chosen chip is reliable?

   (b) A lake contains N fishes. A first catch of n fishes are made, these are marked and then released into the lake. A second catch of r fishes is made. Let M be the number of marked fishes in the second catch. Find the mean and variance of M.

   (c) Given a biased coin whose probability of turning up head is p, show how to select one out of three persons uniformly at random.

   (d) Suppose your waiting time in a queue is given by a random variable X_0. Suppose the waiting times of your friends in independent queues are $X_1, X_2, \ldots$ (assuming you have an infinite number of friends). Let N be the integer valued random variable which takes the value n if your n-th friend is the first one who has to wait for a longer time than you (i.e., $X_n > X_0$). Find $\Pr[N > n-1]$ assuming that $X_0, \ldots, X_n$ are all distinct values. Hence, or otherwise, find $\Pr[N = n]$. What can you say about the expectation of N?

$(10 + 10 + 10 + 10 = 40)$

INDIAN STATISTICAL INSTITUTE
M. Tech. (CS) - I year : 2016-2017
Computer Organization
Mid Semester Examination

Date: 06. 09. 2015 Marks: 50 Time: 3 Hours

Answer any part of any question. The question is of 55 marks. The maximum marks you can get is 50. Please write all the part answers of a question at the same place.

1. (a) Take your date of birth in DDMMYYYY format and consider that as an 8-digit Decimal number. Write the number in Binary, Octal and Hexadecimal format.
   (b) Explain r's and $(r-1)$'s complements in detail with examples of 8-bit binary numbers.

   $3+7=10$

2. (a) Can you implement a 4-input OR gate with 2-input XOR gates (as many as you require)? Justify.
   (b) Simplify the Boolean function $F(x,y,z)=\sum(0,2,4,5,6)$ using Karnaugh map.
   (c) Prove that all n-input 1-output Boolean functions can be implemented with two input NAND gates.

   $2+3+5=10$

3. (a) Explain the complete circuit diagram of a 4×1 MUX using combinational logic gates of your choice.
   (b) Use two 4×1 MUXes with some additional logic gates to implement an 8×1 MUX.
   (c) Implement $F(x,y,z,w)=\sum(0,1,3,4,8,9,15)$ with a suitable MUX.

   $5+2+3=10$

4. (a) Describe relevant tables to characterize D and T Flip-Flops.
   (b) Use these tables to implement a 4-bit binary counter with only D Flip-Flops and only T Flip-Flops. You may use additional combinational logic gates.
   (c) Which implementation is efficient? Give reasons.

   $4+4+2=10$

5. Consider that we need to study the evolution of the recurrence relation $x_{n+4}=x_{n+1}\oplus x_n$, where $x_i\in\{0,1\}$.
   (a) How will you implement a combinational/sequential circuit to study this?
   (b) Consider that you start with $x_0=1, x_1=x_2=x_3=0$. Clearly explain how your circuit evolves.
   (c) What will be the evolution in case of the recurrence relation $x_{n+4}=x_{n+2}\oplus x_n$?
   (d) Differentiate the characteristics of the evolutions corresponding to the two different recurrence relations.

   $4+4+4+3=15$

3. (a) There are two processes, named A and B, for manufacturing computer chips. Process A has 99% reliability while Process B is cheaper, but, has 90% reliability. Factories I and II produce chips using these processes; Factory-I uses Process A to produce 75% of its output, while Factory-II uses Process-A to produce 90% of its output. The two factories have the same production capacity. What is the probability that a randomly chosen chip is reliable?

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$(10 + 10 + 10 + 10 = 40)$

INDIAN STATISTICAL INSTITUTE
M. Tech. (CS) - I year : 2016-2017
Computer Organization
Mid Semester Examination

Date: 06. 09. 2015 Marks: 50 Time: 3 Hours

Answer any part of any question. The question is of 55 marks. The maximum marks you can get is 50. Please write all the part answers of a question at the same place.

1. (a) Take your date of birth in DDMMYYYY format and consider that as an 8-digit Decimal number. Write the number in Binary, Octal and Hexadecimal format.
   (b) Explain r's and $(r-1)$'s complements in detail with examples of 8-bit binary numbers.

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   $2+3+5=10$

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   (c) Implement $F(x,y,z,w) = \sum(0,1,3,4,8,9,15)$ with a suitable MUX.

   $5+2+3=10$

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   (b) Use these tables to implement a 4-bit binary counter with only D Flip-Flops and only T Flip-Flops. You may use additional combinational logic gates.
   (c) Which implementation is efficient? Give reasons.

   $4+4+2=10$

5. Consider that we need to study the evolution of the recurrence relation $x_{n+4} = x_{n+1} \oplus x_n$, where $x_i \in \{0,1\}$.
   (a) How will you implement a combinational/sequential circuit to study this?
   (b) Consider that you start with $x_0 = 1, x_1 = x_2 = x_3 = 0$. Clearly explain how your circuit evolves.
   (c) What will be the evolution in case of the recurrence relation $x_{n+4} = x_{n+2} \oplus x_n$?
   (d) Differentiate the characteristics of the evolutions corresponding to the two different recurrence relations.

   $4+4+4+3=15$

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination : 2016

M. Tech.(C.S.) I year

Elements of Algebraic Structures

Date :07.09.2016 Maximum Marks :80 Duration :2.5 hrs

Note: Answer as many as you can. The maximum you can score is 80. Notation is as used in the class.

1. (a) Let H be a subset of a multiplicative group G. Show that H is a subgroup of G iff for all $a, b \in H, ab^{-1} \in H$.

   (b) Let H be a subgroup of a group $< G, . >$. Show that for $a, b \in G; Ha = Hb$ iff $ab^{-1} \in H$. [5+6]

2. Let n be a positive integer. Let

$$G(n) = \{a \in Z_n : gcd(a, n) = 1\}.$$

   Show that $G(n)$ is a group under multiplication modulo n
   What is the order of this group?
   Hence deduce Euler's Theorem.
   Hence deduce Fermat's Little theorem [6+1+3+3]

3. Prove that if G is finite and has no non-trivial subgroups, then G must be cyclic of prime order [6]

4. Show that the right cosets of a group G form a partition of G. [8]

5. Show that a subgroup of a cyclic group is cyclic. [6]

6. - When is a subgroup of a group said to be normal? Let N be normal subgroup of a group G. Define clearly the quotient group G/N
   - State the Isomorphism Theorem for groups. [7+5]

7. • Show that every permuattion can be written as product of disjoint cycles.
Write the following permutation as product of disjoint cycles.

$$\pi(1) = 3, \pi(2) = 5, \pi(3) = 1, \pi(4) = 2, \pi(5) = 4.$$

Find the order of each cycle

• Show that the product of 2 tranpositions can be written as a product of 3-cycles. Hence prove that for $n \geq 3$, the subgroup generated by 3-cycles is A_n.

[10+6]

8. • For finite groups, write down the *class equation* or *class formula.* Explain all notation.

• State Sylow's Theorem [6+5]

9. Define a ring. Let R be a ring and $I \subseteq R$. When is I said to be an ideal?
Let I be an ideal in the ring of integers. Show that there is a positive integer n_0 such that $I = (n_0) = \{n_0 k : k \in \mathbb{Z}\}$. [12]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination

M. Tech.(CS) - I Year (Semester - I)

Data and File Structures

Date : 8 Sept. 2016 Maximum Marks : 60 Duration : 3:00 Hours

Note : You may answer any part of any question, but maximum you can score is 60.

1. Answer whether the following statements are True or False. Give a small justification in support of your answer.

   (a) A sorted array A of n integers contains an integer $A[i]$ that is the sum of two integers that appear in A such that $A[i] = A[j] + A[k]$. The value of i, j and k can be determined in $O(n^2)$ time.

   (b) Suppose a binary tree T has distinct set of values in each node and all the non-leaf nodes of T have two children. Suppose we know the preorder and postorder traversal sequences of a binary tree T. Then we can uniquely determine the binary tree.

   (c) Any n-node unbalanced tree can be balanced using $O(\log n)$ rotations.

   (d) Suppose that a search for key k in a binary search tree ends up in a leaf. Consider three sets: A, the keys to the left of the search path; B, the keys on the search path; and C, the keys to the right of the search path. Any three keys $a \in A$, $b \in B$ and $c \in C$ must satisfy $a \leq b \leq c$. [4*7=28]

2. Implement a data structure that supports the following data operations as efficiently as possible.

   I. **INIT(N):** Initialize the data structure for N empty rooms numbered $1, 2, \ldots N$, in polynomial time.

   II. **COUNT(l; h):** Return the number of available rooms in $[l, h]$, in $O(\log N)$ time.

   III. **CHECKIN(l; h):** In $O(\log N)$ time, return the first empty room in $[l, h]$ and mark it occupied, or return NIL if all the rooms in $[l, h]$ are occupied.

   I. **CHECKOUT(x):** Mark room x as not occupied, in $O(\log N)$ time. [10]

3. Suppose T is a height balanced tree having height h. What is the minimum possible length of a path from root of tree T to a leaf? [7]

4. Given an array of numbers (which could be positive or negative) indexed from 0 to $n-1$. Write an efficient algorithm for finding integer i and j such that $a[i] + a[i+1] + \ldots + a[j]$ is maximized. [14]

5. Given an $n \times n$ 0-1 matrix. A sink is defined as the integer $i \leq n$ such that for each $j \neq i$, $A[i, j] = 0$ and $A[j, i] = 1$. Compute the bound on the number of sinks in a matrix. Design an $O(n)$ time algorithm to compute a sink (if any) in the given matrix. [12]

Indian Statistical Institute

Mid-Semester Examination (2016-2017)

M.Tech. (CS) First Year

Discrete Mathematics

Date: September 9, 2016 Maximum Marks: 60 Time: 2.5 hours

Answer as much as you can. This question has two pages and carries 75 marks. The maximum you can score is 60 marks. Marks alloted to each question are indicated within square brackets near the right margin.

1. In each of the following cases, justify whether or not the set S of integers has a lower bound, and if it does, the find its least member:
   (a) $S = \{s \in \mathcal{Z} | s^2 \leq 16\}$,
   (b) $S = \{s \in \mathcal{Z} | s = 2t \text{ for some } t \in \mathcal{Z}\}$. [4]

2. Show by combinatorial argument that $\binom{2n}{n} = \Sigma_{i=0}^{n} \binom{n}{i}^2$. [5]

3. Let F_n be the n^{th} Fibonacci number. Show that $\sum_{i=0}^{n} F_i = F_{n+2} - 1$. Mention the method of proof. [5]

4. Count the number of distinct ways in which:
   (i) n pencils of same kind can be distributed among k children;
   (ii) subsets of k elements can be formed from the set $\{1, 2, \ldots, n\}$ such that in each subset there are no consecutive integers;
   (iii) n couples can sit in a row such that no couple sits next to each other. [4+5+6=15]

5. Suppose that B is a $n \times m$ board (like a chessboard) with certain forbidden squares; the rest are acceptable ones. A *rook* is a piece that moves either horzontally or vertically on the board. So, one rook is said to take another rook if both are in the same row or the same column. Let $r_k(B)$ be the number of ways to choose k acceptable squares such that no two lie in the same row or in the same column. In other words, $r_k(B)$ is the number of ways in which k rooks can be placed on the acceptable squares of board B.
   (a) Write an epression for the ordinary generating function $R(x, B)$ for $r_k(B)$.
   (b) Let the accceptable squares of B be partitioned into two sets I and J such that no acceptable sqaure in I lies in the same row or column as any acceptable sqaure in J. B_I and B_J) are the respective $n \times m$ boards with only the squares in I and J as acceptable. B_I and B_J are said to decompose the board B. Show that $R(x, B) = R(x, B_I)R(x, B_J)$. [3+6=9]

P.T.O

6. Let $P_m(n) = \Sigma_{k=1}^{n-1} k^m$ denote the sum of the m^{th} powers of the integers from 0 to $n-1$. Find the exponential generating function $P(x,n)$ for $P_m(n)$. Verify your answer for $m = 2$ and $m = 3$. [10]

7. Solve or give tight bounds for the following recurrences:

   (a) $a_n = 7a_{n-1} - 12a_{n-2} + 3n4^n$, and $a_0 = 0$, $a_1 = 2$.

   (b) $T(n) = 2T(n/4) + \sqrt{n}$, and $T(1) = 1$

   [7+5=12]

8. (a) Given any 9 integers whose prime factors lie in the set 3,7,11, prove that there must be two whose product is a square.
   (b) For any $n^2 + 1$ closed intervals of $\mathcal{R}$, prove that either $n+1$ of the intervals share a point, or $+1$ intervals are disjoint. [5+6=11]

9. Derive the number of derangements of a set of n distinct objects by using the Principle of Inclusion and Exclusion. [4]

10. (a) Write the generating function for the number of partitions of a positive integer n in which each part is at most 5.

    (b) Calculate $p(9,4)$, which is the number of distinct partitions of the integer 9 into exactly 4 parts.

    [5+5=10]

INDIAN STATISTICAL INSTITUTE

Semester Examination : 2016-17(First Semester)

M.Tech(CS) I yr

Elements of Algebraic Structures

Date :18.11.16 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. The total score of the question paper is 113. The maximun you can score is 100. Notation is as used in the class.

1. (a) Write down the Euclidean Algorithm for finding the GCD of two integers a, b. Prove its correctness.

   (b) State the Chinese Remainder Theorem (for 2 conguence relations). Hence, or otherwise, show that for positive integers a, b with $GCD(a, b) = 1$

   $$\phi(ab) = \phi(a)\phi(b).$$

   [8+8=16]

2. (a) When is a polynomial $p(x)$ in the ring $F[x]$, where F is a field, said to be irreducible?
   Show that the ideal $I =< p(x) >$ is maximal iff $p(x)$ is irreducible.

   (b) Show that $x^2 + x + 1$ is irreducible over $\mathbf{Z_2}$.
   Hence, or otherwise, construct a field F of order 4 and write down the multiplication table for F [(2+8)+(3+7)=20]

3. Define characteristic of a field. Show that if a field has finite characteristic, then its characteristic is prime. [2+5=7]

4. - Define a vector space over a field $\mathbf{F}$
   - Show that any vector space over $\mathbf{F}$ of dimension n is isomorphic to $\mathbf{F^n}$.
   - Find a basis of the following subspace of $I\!R^3$

   $$S = \{(x, y, z) \in I\!R^3 : 2x - 5y + z = 0\}.$$

   [4+6+6=16]

5. Suppose U is a subspace of a finite-dimensional vector space V. Prove that U has a complement. [7]

6. • Given a linear map $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$, obtain an $m \times n$ matrix A such that
$$f(\mathbf{x}^T) = A.\mathbf{x}^T; \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$
• Find the inverse of the following real matrix.
$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}.$$
[7+7=14]

7. Show that if a square matrix of order n has rank n, then it is non-singular. [7]

8. Let A be an $m \times n$ matrix. Show that the system of equations
$$A\mathbf{x} = \mathbf{b}$$
is consistent iff $\rho(A) = \rho([A : \mathbf{b}])$. [8]

9. Find a general solution of the following
$$x_1 + x_2 + 2x_3 + 3x_4 = 0$$
$$2x_1 + 4x_3 + 4x_4 = 0$$
$$x_1 - x_2 + 2x_3 + x_4 = 0$$
$$x_1 - 2x_2 + 2x_3 = 0.$$
[8]

10. Find the determinant of the following
$$\begin{pmatrix} a_1^3 & a_1^2 & a_1 & 1 \\ a_2^3 & a_2^2 & a_2 & 1 \\ a_3^3 & a_3^2 & a_3 & 1 \\ a_4^3 & a_4^2 & a_4 & 1 \end{pmatrix},$$
where a_1, a_2, a_3, a_4 are reals. [10]

Indian Statistical Institute

First Semester Examination (2016-2017)

M.Tech. (CS) First Year

Discrete Mathematics

Date: November 22, 2016 Maximum Marks: 100 Time: 3.5 hours

This question paper has two pages and carries 120 marks. Answer as many questions as you can. The maximum credit you may get is 100 marks.

Marks allotted to each question are indicated within square brackets near the right margin.

You are allowed to carry with you TWO A4-sized reference sheets with your name and roll no. written on them. These are not to be shared, and to be submitted with your answer-scripts at the end of this exam.

1. (a) Write the (i) converse, (ii) inverse, and (iii) contrapositive of the following statement: *If Kolkata is in Karnataka, then Bengaluru is in Bengal.*

   (b) Let x, y, and z be any real numbers. Let $p : x < y, q : y < z, r : x < z$ be three statements. Represent the following sentence symbolically: $x \geq y$ *and* $y \geq z$ *if and only if* $x \geq z$.

   (c) Using quantifiers and predicates, express the following sentence symbolically: *Every outerplanar graph with at least seven vertices has a non-outerplanar complement.* [2*3+6+6=18]

2. Use the tree method to

   (a) show that $A \Rightarrow (B \Rightarrow C)$ is logically equivalent to $(A \wedge B) \Rightarrow C$.

   (b) to obtain a model for *Someone is talking and walking, and someone is talking and texting, so someone is walking and texting.* [6+8=14]

3. Give an example of a sentence which cannot be expressed in first-order logic. [6]

4. (a) Consider G, the join of two graphs G_1 and G_2. Prove or disprove that $\overline{G} = \overline{G}_1 + \overline{G}_2$.

   (b) Compute the diameter and radius of the graph $K_{m,n}$. [6+6 = 12]

5. (a) Let G be a maximal planar graph, and n_i be the number of vertices of G having degree i. Prove that $\sum_i (6-i)n_i = 12$.

   (b) Show that every planar graph is 6-colorable.

   (c) Argue whether $x^4 - 4x^3 + 3x^2$ is a chromatic polynomial of a graph. [5+6+5=16]

P.T.O

6. (a) Show that for any graph G, $\beta_1(G) \leq \alpha_0(G)$ and $\beta_0(G) \leq \alpha_1(G)$, where $\alpha_0(G)$, $\alpha_1(G)$, $\beta_0(G)$, and $\beta_1(G)$ denote respectively the cardinalities of a minimum vertex cover, a minimum edge cover, a maximum independent set and a maximum matching of G.

   (b) Prove that the graph G_1 shown in Figure 1 does not have a Hamiltonian cycle.

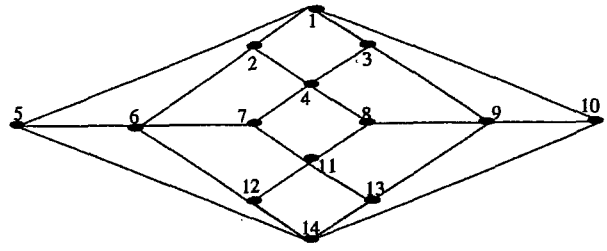


Figure 1: G_1

[8+6=14]

7. (a) Prove or disprove: For each $n \geq 1$, every simple digraph with n vertices has two vertices with the same outdegree or two vertices with the same indegree.

   (b) Let G be a graph of 11 vertices and chromatic number 3. Does G necessarily have a clique of 3 vertices or an independent set of 3 vertices? Justify your answer.

   (c) Show that the Ramsey number $R(5,5) \leq 70$. [5+5+5 =15]

8. The Bell number $B(n)$ is defined as $\Sigma_{k=1}^{n} S(n,k)$ where $S(n,k)$ is the Stirling's number of the second kind denoting the number of distinct ways in which n labeled marbles can be distributed in k unlabeled boxes. Show that
   (i) $B(n) = \frac{1}{e} \sum_{j=0}^{\infty} \frac{j^n}{j!}$, and then
   (ii) the exponential generating function for $B(n)$ is $e^{e^x - 1}$. [6+4=10]

9. Show that the number of partitions of an integer n into k or fewer parts is equal to the number of partitions of n into parts such that each part $\leq k$. [5]

10. Suppose 4-digit numbers are printed on a set of identical rectangular tags with one 4-digit number per tag. Find the smallest number of distinctly printed tags needed to cover all the 10^4 numbers? For example, the tag with 0066 may also be read as 9900. [10]

INDIAN STATISTICAL INSTITUTE
M. Tech. (CS) – I year : 2016–2017
Computer Organization
Semestral Examination

Date: 24. 11. 2016 Marks: 100 Time: 3 Hours

Answer any five questions. Each question is of 20 marks. Please write all the part answers of a question at the same place.

1. (a) Explain 1's and 2's complements in detail with examples of 8-bit binary numbers.
   (b) How is 'int' represented in C programming language on a 32-bit environment?
   (c) Write a C program to justify your claim above and provide necessary examples with proper input and output to your program.
   (d) How a floating point decimal number is implemented in a binary computer?

   $5 + 2 + 7 + 6 = 20$

2. (a) How can you implement a 4-input OR gate with 2-input NAND gates? Try to minimize the number of NAND gates.
   (b) Simplify the Boolean function $F(x, y, z) = \sum(0, 2, 3, 5, 6)$ using Karnaugh map.
   (c) Prove that all n-input 1-output Boolean functions can be implemented with two input NOR gates.

   $5 + 5 + 10 = 20$

3. (a) Consider an n-input decoder circuit. For an n of your choice ($n \geq 3$), implement the circuit with suitable logic gates.
   (b) Is there any relationship between a decoder and demultiplexer? Explain with examples.
   (c) Describe, with proper example, in which part of a computer circuit we can use a decoder?
   (d) Implement the function $F(x, y, z) = \sum(0, 1, 3, 4, 6, 7)$ with a decoder of your choice and a multi-input OR gate.

   $5 + 5 + 5 + 5 = 20$

4. (a) What is a J-K Flip-Flop?
   (b) Describe relevant tables to characterize the J-K Flip-Flop.
   (c) What is Race-Around Condition?
   (d) How one can solve the above issue with Master-Slave configuration?

   $3 + 4 + 3 + 10 = 20$

P.T.O

5. Consider that we need to study the evolution of the recurrence relation

$$x_{n+4} = x_{n+1} \oplus x_n, \text{ where } x_i \in \{0, 1\}.$$

(a) How will you implement a combinational/sequential circuit to study this?

(b) Consider that you start with $x_0 = 1, x_1 = x_2 = x_3 = 0$. Clearly explain how your circuit evolves.

(c) What will be the evolution in case of the recurrence relation $x_{n+4} = x_{n+2} \oplus x_n$?

(d) Differentiate the periods of the evolutions corresponding to the two different recurrence relations.

$5 + 5 + 5 + 5 = 20$

6. (a) How can you implement a Full Adder?

(b) How this circuit can be used to construct a 4-bit adder?

(c) How can you modify the above circuit for both addition and subtraction of 4-bit numbers?

(d) What kind of properties of signed binary integers do you exploit in designing the above circuit?

$5 + 5 + 5 + 5 = 20$

7. (a) Explain different kinds of memory elements used in a computer.

(b) What is the importance of Cache memory?

(c) Consider the following address map:

000 to 07F	RAM 1
080 to 0FF	RAM 2
100 to 17F	ROM 3 (Lower Half)
180 to 1FF	ROM 3 (Upper Half)

Provide a circuit outline for this configuration showing the CPU and Memory elements.

$5 + 5 + 10 = 20$

8. (a) List various addressing modes.

(b) Describe three of such modes with examples.

(c) Consider a 16-bit CPU, with 4-bit opcode, and two operands, 6-bit each. Explain, with detailed examples, how will you design arithmetic operations in this context.

$4 + 6 + 10 = 20$

INDIAN STATISTICAL INSTITUTE

First Semester Examination: 2016-17

Course Name: M. Tech. (CS) I Year

Subject Name: Introduction to Programming

Date: 25/11/2016 **Maximum Marks: 100** **Duration: 3 hours**

Note: Your programs should be written in ANSI C and should check for all possible errors during runtime (e.g. invalid inputs, return values from system call, etc.).

Answer any four questions (4x25=100).

1. Write a C program for the following:

   Given an array of integers Print the sums of **all** subsets in it. Output sums may be printed in any order. E.g.: if your input is 2, 4, 5; your output sums will be 0 2 4 5 6 7 9 11.

2. A user needs the following basic set operations: Union, Intersection, Set Difference, and Subset (checking whether one set is a subset of another). Assume that the universal set U = {0, 1, …, MAX_INT}. Design an efficient data structure and functions so that a user can use them to declare such set data and perform such operations.

3. Explain with examples the following concepts in logic programming:
   a) Recursive Queries
   b) Constraint and Concurrent Logic Programming
   c) Negation As Failure

4. Explain with examples the following concepts of object-oriented programming:
   a) Data abstraction and encapsulation
   b) Inheritance
   c) Polymorphism

5. Answer the following:
   a) Prime Factors of a positive integer are the prime numbers that divide that integer exactly. Write an efficient program (using only a few variables and loops) which takes a positive integer (command line) and prints its distinct prime factors. E.g.: if your input is 900 your program should print

      900 = 2, 3 and 5.

b) Explain what the following C program does:

```
#include <stdio.h>
void aa(void *x, void *y)
{
  int *a, *b, c;
  a = (int *)x;
  b = (int *)y;
  c = *a + *b;
  printf("%d\n",c);
  return;
}
void bb(void *x, void *y)
{
  float *a, *b, c;
  a = (float *)x;
  b = (float *)y;
  c = *a + *b;
  printf("%f\n",c);
  return;
}
void cc(void *x, void *y, void(*dd)(void*,void*))
{
   (*dd)(x,y);
}
int main()
{
  float a,b;
  int m,n,k = 0;
  void *x,*y;
  while((k<0)||(k > 1))
  {
     scanf("%d",&k);
  }
  (k?scanf("%f %f",&a,&b):scanf("%d %d",&m,&n));
  cc((k)?((void *)&a):((void*)&m), (k)?((void *)&b):((void*)&n), (void (*)(void*, void*))(k?bb:aa));
  return(0);
}
```

INDIAN STATISTICAL INSTITUTE

Semestral Examination

M. Tech.(CS) - I Year (Semester - I)

Data and File Structures

Date : 28 Nov. 2016 Maximum Marks : 100 Duration : 4:00 Hours

Note : You may answer any part of any question, but maximum you can score is 100.

1. Answer whether the following statements are True or False. Give a small justification in support of your answer.

   (a) For sufficiently large n (say greater than 1 billion), and for any k (less than $\sqrt{n}$), largest, 2nd largest, upto kth largest elements of an array of n elements can be computed in $O(n)$ time.

   (b) For n points in two dimensional plane, a 2-dimensional range tree can be build in $O(n \log n)$ time.

   (c) Given an array A storing n elements. To identify whether there exist an element that appears in the array A more than $n/3$ times cannot be determined in O(n) time in worst case.

   (d) Consider the following algorithm $H(A)$ for building a heap of array A.

   $H(A)$
   Heapsize[A] $\leftarrow 1$
   for $i \leftarrow 2$ to *length[A] do Heap-Insert*$(A, A[i])$.

   The procedure $H(A)$ and standard *Build-Heap*(A) creates the same heap.

   (e) For sorting n numbers, only two stack data structure is not sufficient where stack data structure supports only push, pop and Is-Stack-empty.

   (f) You are given a red-black tree T storing a set S of n elements. You are also given an element $x \in S$. An algorithm can be designed to split T into two trees $T_<$ consisting of elements less than x and tree $T_>$ consisting of element greater than x in $O(log^2 n)$ time.

   (g) Given an array A of size n storing a sequence consisting of a series of 1s followed by a series of 0s and followed by 1s. In addition you are given an i such that $A[i] = 0$. An $O(\log n)$ time algorithm can be designed to compute the number of 0s in the array.

   (h) For the binomial heap of 2^k nodes, there are exactly $\binom{k}{i+1}$ nodes at depth i for $i = 0, 1, \ldots, k$.

[8*9=72]

P.T.O

2. Indicate whether you use an Array, Linked List or Hash Table to store data in each of the following cases. Justify your answer.

   (i) A set of employee records need to be maintained dynamically in a manner that is easy to find max or min in the set.

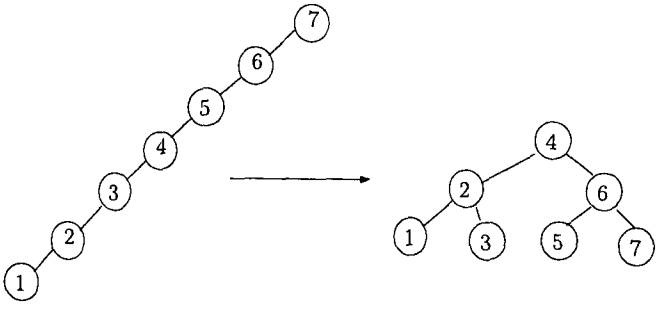
   (ii) A data set contains many records with duplicate keys. Only thing needed is to keep the list in sorted order.

   (iii) A library needs to maintain books by their ISBN number. Only thing important is to find them as soon as possible.

   (iv) A dynamic data set needs to be maintained in order to find the median of the set quickly.

   [12]

3. Suppose that you are given a binary tree which consists of a linear chain, where each entry except the leaf, has its left child. Suppose further that the tree has exactly $n = 2^k - 1$ nodes, for some integer $k \geq 1$. Derive an algorithm using only single-left and single-right-rotations of AVL tree, that maps this tree into a perfectly balanced complete tree. Explain how your algorithm works on the given example. [15]



4. Suppose that we insert n keys into a hash table of size m using open addressing and uniform hashing. Let $p(n, m)$ be the probability that no collisions occur. Show that $p(n, m) \leq e^{-n(n-1)/2m}$. Argue that when n exceeds $\sqrt{m}$, the probability of avoiding collisions goes rapidly to zero. [Hint: $e^x = 1 + x + x^2/2 + \ldots$.] [12]

Examination: Back Paper Exam (2014-15)
M.Tech I Year
Automata, Languages and Computations.

Date: 22.12.2016 Maximum Marks:100 Duration:3hours

Note: Answer as many as you can.
Unless otherwise stated, notation used is as defined in the class.

1. State Kleene's Theorem.
   Define regular expressions over an alphabet Σ.
   Let $\mathbf{r}$ and $\mathbf{s}$ be two regular expsessions. Prove the following identity

$$(\mathbf{r}+\mathbf{s})^* = (\mathbf{r}^* + \mathbf{s}^*)^*.$$

2. Write an algorithm to test, given a DFA $\mathcal{M}$, whether $\mathcal{L}(\mathcal{M}) = \phi$. Prove its correctness. [9]

3. Find a CFG that generates the set of all palindromes over the alphabet $\{a, b\}$. [6].

4. When is a grammar said to be in Chomsky Normal Form(CNF)? Convert the following grammar into an equivalent grammar in CNF.

$$S \to aB/Ab;\ A \to aAB/a$$
$$B \to b/ABb$$

[2+6=8]

5. State Bar-Hillel's Pumping Lemma.
   Use it to show that $\{a^p : p \text{ is prime}\}$ is not context-free. [3+7=10]

6. (a) Explain in detail when a pushdown automata (PDA) accepts a string by empty stack.
   (b) Show that if $\mathcal{L}$ is a context-free language then there is a PDA that accepts $\mathcal{L}$.
   (c) When is a PDA said to be deterministic? Show that if $\mathcal{L}$ is a language accepted by a deterministic PDA by empty stack, then $\mathcal{L}$ has the prefix property *i.e.* no proper prefix of a string in $\mathcal{L}$ is in $\mathcal{L}$.

(d) Show that if $\mathcal{L}$ is a context-free language and $\mathcal{R}$ is regular, then $\mathcal{L} \bigcap \mathcal{R}$ is context-free by constructing a suitable PDA.(Show only the major steps.)

Hence, show that

$$\mathcal{L} = \{ww : w \in (0+1)^*\}$$

is not context-free

[5+7+7+(6+7)=32]

7. State the Bar-Hillel's Pumping Lemma.
Show that the language
$\{w : w \in \{a,b,c\}^*$, w contains equal numbers of a's, b's and c's $\}$
is not context-free.

Construct a Turing Machine which accepts this language.

[(4+7+5)=16]

8. (a) Design (i) single-tape (ii) multitape Turing machines(TMs) that accept the set of all strings over $\{a,b\}$ containing an equal numbers of a's and b's. Compare the number of moves made by the respective TMs on accepting a string of length n.

Also, construct a TM that uses only $\lceil \log n \rceil$ cells (not counting cells on the input tape)

(b) A language $\mathcal{L}$ over Σ is said to be recursive if both $\mathcal{L}$ and $\Sigma^* - \mathcal{L}$ are recursively enumerable. Show that $\mathcal{L}$ is recursive iff there is a TM accepting $\mathcal{L}$ that halts on all inputs. [(5+5+2+5)+8=25]

9. (a) When is a function $f : \mathbb{N}^k \rightarrow \mathbb{N}$ said to be (Turing) computable?
Show that the following functions are computable.

i. $f(n,m) = \lfloor n/m \rfloor = \begin{cases} 0 & \text{if } m = 0 \\ \text{the largest integer} \leq n/m & \text{if } m > 0 \end{cases}$

ii. $p(n) =$ the nth prime number.

iii. $lcm(n,m) = \begin{cases} 0 & \text{if } n = 0 \text{ or } m = 0 \\ \text{the least common multiple of } n, m & \text{otherwise} \end{cases}$

iv. $gcd(n,m)=$ the greatest common divisor of n and m.

v. $f(n) = \lceil \log n \rceil$. [$(2 + 5 \times 4) = 22$]

10. (a) Define the classes $\mathcal{P}$ and $\mathcal{NP}$. When is a language said to be $\mathcal{NP}$-complete?

    (b) Show that SAT is in $\mathcal{NP}$.

    (c) Describe the COMPLETE SUBGRAPH problem. Assuming that SAT is $\mathcal{NP}$-complete, prove that COMPLETE SUBGRAPH is also $\mathcal{NP}$-complete. [7+5+8=20]

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Back Paper Examination

M. Tech (CS) - I Year (Semester - II)

Design and Analysis of Algorithms

Date : 26.12.2016 Maximum Marks : 100 Duration : 3.5 Hours

1. Write a sorting algorithm of time complexity $O(n \log k)$ for an unsorted array S containing n integer values, where k is the number of distinct values in the array S. You must justify the time and space complexities of your proposed algorithm. Is your proposed algorithm in-place? [8+4+3=15]

2. Write a method to check whether a given string of characters $T = a_1 a_2 \ldots a_n$ is the circular rotation of another string of characters $T' = b_1 b_2 \ldots b_n$. For example, *arc* and *car* are cyclic rotation of each other. Analyze the time complexity of your proposed method. [10]

3. Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of n points distributed on a 2D plane. The objective is to test whether the convex hull of P is a hexagon or not. Write a linear time implementable algorithm for solving this problem. Justify the time complexity of your algorithm. [10]

4. (a) When is a graph said to be bi-connected ?

   (b) Write an algorithm to identify the bi-connected components in a graph. Mention the time complexity of your algorithm.

   [5+10 = 15]

5. You are given a connected undirected graph $G = (V, E)$ in which the weight of each edge is either 1 or 2. Present an $O(V + E)$ time algorithm to compute a minimum spanning tree for G. Explain the correctness of your algorithm, and analyze its time complexity. [7+4+4=15]

6.(a) Show that if the Ford-Fulkersons' algorithm for computing maximum flow between a pair of vertices s and t in a flow network terminates, it produces the maximum flow value from s to t.

(b) Decide whether the following statement is *true* or *false*. If it is true, give proper justification, and if it is false, give a counterexample.

> Let $G = (V, E)$ be an arbitrary flow network, with a source s and a sink t. Each edge $e \in E$ is attached with a positive integer capacity c_e. Let (A, B) be the minimum $s - t$ cut in graph G with respect to the edge capacities $\{c_e, e \in E\}$. Now, if we add 1 to every edge capacity, then (A, B) still remains the minimum $s - t$ cut in the revised flow network.

[7+8 = 15]

7.(a) When is a problem said to be (i) in NP, and (ii) NP-complete?

(b) Let $G = (V, E)$ be a weighted directed graph. Show that, the problem of getting a traveling salesman tour in G of cost $\rho \times opt$ is NP-complete, where *opt* indicates the cost of the optimum traveling salesman tour in G, and ρ is a given constant.

(c) Also show that if the edge costs of the above graph G satisfy triangle inequality, then a traveling salesman tour of cost $2 \times opt$ can be found in polynomial time.

$[6 + 6 + 8 = 20]$

INDIAN STATISTICAL INSTITUTE

Backpaper Examination : 2016-17(First Semester)

M.Tech(CS) I yr

Elements of Algebraic Structures

Date :30.12.16 Maximum Marks :100 Duration :3 hrs

Note: Answer as many as you can. Notation is as used in the class.

1. (a) Show that if G is a finite group and H a subgroup of G, then $|G/H| = |G|/|H|$

   (b) Let $f : G \to G'$ be a homomorphism of the group G *onto* G'. Show that the kernel $ker(f)$ is normal. Prove that $G/ker(f)$ is isomorphic to G'.

   (c) Consider the unit circle C of the complex plane as a multiplicative group. Apply the above result to show that the additive group $\mathbb{R}/\mathbf{Z}$ is isomorphic to C [5+10+7=22]

2. (a) For a positive integer n, let

$$\mathbb{Z}_n^* = \{a \in \mathbb{Z} : gcd(a, n) = 1\}.$$

   Show that $\mathbb{Z}_n^*$ is a multiplicative group where multiplication is defined modulo n.

   Hence, or otherwise, show that for any integer a with $gcd(a, n) = 1$,

$$a^{\phi(n)} \equiv 1 \bmod n.$$

   (b) Show that for integers a, b with $GCD(a, b) = d$, there exist integers λ and μ such that

$$a\lambda + b\mu = d.$$

[10+10=20]

3. Show that every ideal I in the ring $F[x]$, where F is a field is generated by a single polynomial i.e. $I =< p(x) >$ for some $p(x) \in F[x]$.

   Show that if $p(x)$ is irreducible, then I is maximal.
   Prove that $x^2 + 1$ is irreducible over $\mathbb{Z}_7$ [7+8+8=23]

Rao Bose

4.
   - Define a vector space over a field $\mathbf{F}$.
   - Show that any two vector spaces of the same dimension are isomorphic.
   - Find a basis of the following subspace of $\mathbb{R}^3$

$$\mathcal{S} = \{(x, y, z) \in \mathbb{R}^3 : 2x - 3y + z = 0\}.$$

[4+6+6=16]

5. Suppose U, W are subspaces of a finite-dimensional vector space V. Define the space $U + W$. Prove that

$$dim(U + W) = dim(U) + dim(W) - dim(U \bigcap W).$$

[10]

6. Let A, B be 2 square matrices of order n. Show that

$$\rho(AB) \leq min\{\rho(A), \rho(B)\}.$$

[10]

INDIAN STATISTICAL INSTITUTE

SEMESTRAL-I EXAMINATION
M.TECH.(CS) I YEAR

PROBABILITY AND STOCHASTIC PROCESSES
(BACK PAPER)
Date: 09/02/2017 Maximum marks: 100 Duration: 3 hours

Answer all questions. Each question carries 10 marks.

1. Let A, B, C be three arbitrary events. Find expressions for the following events.
   (a) At least one of A, B, C occurs.
   (b) None of the events A, B, C occur.
   (c) Not more than two of the events occur.

2. Let X_1 and X_2 be independent random variables with Poisson distributions $p(k; \lambda_1)$ and $p(k; \lambda_2)$. Show that $X_1 + X_2$ has the Poisson distribution $p(k; \lambda_1 + \lambda_2)$.

3. There are three persons and you want to pick one such that the probability of any one of them getting chosen is $1/3$. You are given a fair coin. Describe a method for doing this.

4. State the weak law of large numbers for a sequence of i.i.d. random variables having a common distribution. Assuming that the variance exists, prove the law.

5. Suppose a Markov chain has two states 0 and 1. With probability p_i (with $0 < p_i < 1$) the chain moves from state i to state $(1 - i)$ and with probability $(1 - p_i)$ it remains in state i. Find the stationary distribution of this Markov chain.

6. Suppose that in an election, a candidate P get p votes and another candidate Q gets q votes, with $p > q$. Show that the probability that throughout the counting there are always more votes for P than for Q equals $(p - q)/(p + q)$.

7. Let $\mathcal{E}$ be a recurrent event in a Markov chain and

$$\begin{aligned} u_n &= \mathsf{P}[\mathcal{E} \text{ occurs at the } n\text{th trial }]; \\ f_n &= \mathsf{P}[\mathcal{E} \text{ occurs for the first time at the } n\text{th trial }]. \end{aligned}$$

   Let $U(s)$ and $F(s)$ be the generating functions for $\{u_n\}$ and $\{f_n\}$. Show that

$$U(s) = 1/(1 - F(s)).$$

8. Define mean deviation from a measure of central tendency. Show that the mean deviation is minimum when taken about the median.

9. The frequency distribution of family-size for 250 families is given below.

family-size	frequency
1	4
2	22
3	25
4	45
5	52
6	41
7	36
8	15
9	7
10	3

Find the mean, median and the mode.

10. When is a statistic called sufficient for a parameter? Let $X_1, \ldots, X_n$ be i.i.d. Geometric(p) with common p.m.f. $f(x;p) = p(1-p)^x$, $x = 0, 1, 2, \ldots$. Show that $\sum_{i=1}^{n} X_i$ is sufficient for p.

Indian Statistical Institute
Mid-Semester Examination: 2017
Course Name: M. Tech in Computer Science -I Year
Subject Name: Computer Networks

Date: 20-02-2017 Maximum Marks: 60 Duration: 3 hours

Instructions:

You **may** attempt **all** questions which cary a total of **65** marks. However, the maximum marks you can score is only **60**.

1. (a) For the bit stream 0 1 0 0 1 1 0 0 0 1 1, sketch the waveforms for each of the following encoding schemes. Assume that the signal level for the most recent preceding 1 bit for Bipolar-AMI has a negative voltage.
   i. Bipolar-AMI.
   ii. Differential Manchester. [2.5+2.5=5]

   (b) State three factors that determine how successful a receiver will be in interpreting an incoming signal. State the relative advantages and disadvantages of Biphase techniques over NRZ techniques. [3+3=6]

   (c) Consider a Multiple FSK scheme with carrier frequency $f_c = 250\ kHz$, difference frequency $f_d = 25\ kHz$ and number of different signal elements $M = 8$ ($L = 3$ bits). Make a frequency assignment for each of the 8 possible 3-bit data combinations. [3]

   (d) What is the bandwidth efficiency for PSK for a bit error rate (BER) of 10^{-7} on a channel with SNR of 12 dB? Assume that E_b/N_o corresponding to the $BER = 10^{-7}$ is 11.2 dB for PSK. [3]

   (e) Consider an audio signal with frequency components in the range 300 to 3000 Hz. Suppose we generate a PCM signal with 6000 samples/second. What is the number of uniform quantization levels needed to achieve a data rate of 30000 bps? [3]

2. (a) Describe the functions of frequency-hopping spread spectrum. State the difference between slow and fast frequency hopping. [5+3=8]

   (b) What is meant by spread spectrum? Describe the basic principle of direct-sequence spread spectrum. [3+5=8]

   (c) What is a pseudo noise sequence? Briefly describe the main properties of a maximum length pseudo noise sequence? [3+6=9]

3. (a) Consider the framing method *starting and ending flag bytes with bit stuffing* where 0 1 1 1 1 1 1 0 is used as the flag byte. Given an original data size of 64 bits, what is the largest number of bits that may need to be transmitted? What is the smallest number of bits that may need to be transmitted? Remember to include both begin and end flag bytes and the stuffed bits. [2+2=4]

P.T.O

(b) Consider a simple block code in which each codeword consists of 3 data bits and one parity bit. List all the codewords of this code. What is the Hamming distance of this code? Can this code detect any single bit error? Justify. [2+2+2=6]

(c) Recall that a code with a minimum Hamming distance of D can correct any error pattern of $\lfloor \frac{D-1}{2} \rfloor$ or fewer errors. Show an error pattern with $\lfloor \frac{D-1}{2} \rfloor + 1$ errors that cannot be corrected by the code. [4]

(d) Consider the use of CRC with generator polynomial $G(x) = x^4 + x^3 + 1$ for error detection. Show the transmitted string for the message string 1 1 0 0 1 1. Construct a burst error of length 5 on the transmitted string in such a way that the error cannot be detected by the CRC with the given $G(x)$. [3+3=6]

INDIAN STATISTICAL INSTITUTE

Mid Semestral Examination

M. Tech (CS) - I Year, 2016-2017 (Semester - II)

Design and Analysis of Algorithms

Date : 21.02.2017 Maximum Marks : 60 Duration : 3.0 Hours

Note: This is a two page question paper and it is of 75 marks. Answer as much as you can, but the maximum you can score is 60.

(Q1) Solve the recurrence relation

$$T(n) = \begin{cases} 0 & \text{if } n = 2; \\ 2T(\lfloor\sqrt{n}\rfloor) + 1 & \text{if } n > 2. \end{cases}$$

[10]

(Q2) We want to use a binary counter to count from 0 to n. You can think of a binary counter as an array X storing values 0 and 1; and any number is represented as a string of 0s and 1s. As an example, 11 (eleven) is represented as $\boxed{1011}$; to count 12 (twelve) using this counter, we have to change the configuration of the binary counter from $\boxed{1011}$ to $\boxed{1100}$. So in this example, we have to change 3 bit positions.

Now, the size or number of bits k of the binary counter to count numbers upto n is $\lfloor \log_2 n \rfloor + 1$. To increment the counter by 1, we can use the following routine (the routine is given for your help):

Method *Increment Counter*
Input: A counter $X[0, \ldots, k-1]$ of size $k = (\lfloor \log_2 n \rfloor + 1)$ to count from 0 to n with any number x_i between 0 to $n-1$.
Output: The number $x_i + 1$ stored in the counter X.

1. $j \leftarrow 0$;
2. **while** ($X[j] = 1$)
3. $\quad X[j] \leftarrow 0; j \leftarrow j + 1;$
4. **endwhile**
5. $X[j] \leftarrow 1;$

So, in the worst case we have to increment $\Theta(k) = \Theta(\log n)$ bits for incrementing by one. As, we have n numbers in all, the time complexity is $\Theta(nk) = \Theta(n \log n)$.

Do you think this is a tight time complexity analysis of the method? If not, then try improving the time complexity by doing a tighter analysis of the same method. [5]

[Hints: Write the binary representation of consecutive numbers and then see.]

(Q3) Let $A[1 \ldots n]$ be an array of n distinct numbers. A is unimodal, i.e., for some i, $1 \leq i \leq n$, $A[1] < \ldots < A[i]$ and $A[i] > A[i+1] > \ldots > A[n]$. Design and analyze an efficient algorithm to find i. [5]

(Q4) A *binary heap* is an *almost-complete* binary tree with each node satisfying the *heap property:* If v and par(v) are a node and its parent, respectively, then the key of the item stored in par(v) is not greater than the key of the item stored in v. A binary heap supports the following operations: (i) deleting the minimum element in $O(\log_2 n)$ time, and (ii) inserting/shifting an element up the heap in $O(\log_2 n)$ time. A d-ary heap is a generalization of a binary heap in which each internal node in the almost-complete d-ary rooted tree has at most d children instead of 2, where $d > 2$ can be arbitrary.

Design and analyze efficient algorithms for these operations in a d-ary heap. Explicitly mention the time complexity.

- deleting the minimum element
- inserting/shifting an element up the heap

[(5+5=10)]

(Q5) Let $\mathcal{A}$ be an array of n integers. Each $a_i \in \mathcal{A}$ lies in the range $[0, n^3 - 1]$. Design an efficient algorithm to sort $\mathcal{A}$. [10]

[Hints: Sorting in $O(n \log n)$ time using any known algorithm will not fetch any credit. Try using any linear time sorting!]

(Q6) Consider the problem of constructing a binary search tree from a list $\mathcal{L}$ of n unsorted elements. Find out a non-trivial lower bound of this problem. [10]

(Q7) Let $G = (V, E)$ be a directed acyclic graph with weight $w(u, v)$ on edge $(u, v) \in E$. Design and analyze an efficient algorithm to find the average path length from a source vertex $s \in V$ to a destination vertex $t \in V$. The average path length is defined as the total weight of all paths from s to t divided by the total number of distinct paths. [10]

(Q8) Let $G = (V, E)$ be a directed graph in which $E = E_1 \cup E_2$ and $E_1 \cap E_2 = \phi$. The edges in E_1 have weights greater than or equal to zero and the edges in E_2 have weights less than zero. Now to find out the single source shortest path, we do the following. Let w_e be the minimum weight of all edges in E_2. Surely $w_e < 0$. Now we add $|w_e|$ to all the edge weights to make them non-negative. Now we run Dijkstra's shortest path algorithm on this graph with the new edge weights (i.e., all edge weights increased by $|w_e|$) to get the desired shortest path.

Can we do so? Prove or disprove the above statement. [5]

(Q9) (a) Let $G = (V, E)$ be a connected graph with the weight on each edge being a real number. The maximum spanning tree is a spanning tree whose sum of edge weights is the maximum. Design and analyze an efficient algorithm to compute the maximum spanning tree. Prove that your algorithm is correct.

(b) Show that if all the edge weights of the graph are unique, the maximum spanning tree is also unique.

[5+5=10]

INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2016-17

Course Name: M. Tech. (CS) I Year

Subject Name: Database Management Systems

Date: 22/02/2017 **Maximum Marks: 60** **Duration: 3hours**

Note: Answer all questions

1. Design a generalization – specialization hierarchy for a motor vehicle sales company. The company sells motorcycles, autos, passenger cars, buses and trucks. Justify your placement of attributes at each level of the hierarchy. Explain why they should not be placed higher or lower levels.

20

2. Consider the following relations:

   Suppliers(supplier_no: int, supplier_name: char(50), status: int, city: char(40))

   Parts(part_no: int, part_name: char(100), color: char(10), weight: real)

   Projetcs(project_no: int, project_name: char(20), city: char(40))

   Shipment(supplier_no: int, part_no: int, project_no: int, quantity: int)

   A. Draw the Directed Graph and show graphically some example data

   B. Draw the Hierarchical Organizations and show graphically some example data

   C. Give a relational algebra or tuple / domain relational calculus expression for the following queries:

      I. Find the maximum value of part_no

      II. All parts with same name, color and weight.

      III. All the projects for which there are no shipments.

      IV. Name of suppliers who supply only Red and Blue color parts.

10 + 10 + 20 = 40

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination 2016-17
M.Tech I Year (CS)
Automata, Languages and Computations.

Date:23-02-16 Maximum Marks:80 Duration:2hours 30 mins.

Note: Answer as many as you can. Maximum score is 80
Unless otherwise stated, notation used is as defined in the class.

1. Define a non-deterministic finite automaton(NFA).
   Construct an NFA that accepts all string over $\{0,1\}$ containing an even number of occurences of 01. Explain(without proof) your construction. Find its equivalent DFA. [3+8]

2. Construct a DFA that accepts all binary strings which are binary representation of (non-negative) integers which are multiples of 4.(No proof) [6]

3. Let $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ be 3 DFAs accepting $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ respectively over the same alphabet. Construct a single DFA that accepts $(\mathcal{L}_1 - \mathcal{L}_2) - \mathcal{L}_3$. Explain, in brief, your construction [8]

4. State Kleene's Theorem.
   Let r and s be two regular expsessions. Prove the following identity

   $$(\mathbf{r}+\mathbf{s})^* = (\mathbf{r}^*.\mathbf{s}^*)^*.$$

   Consider the following equation in X

   $$X = \mathbf{r} + X.\mathbf{s},$$

   where r and s are regular expressions. Show that the equation has a solution. [2+7]

5. Write an algorithm to test, given a DFA $\mathcal{M}$, whether $\mathcal{L}(\mathcal{M})$is finite or not. Prove its correctness. [10]

6. Two states p and q of a DFA are said to be equivalent if:

$$\text{for all input strings w, } \delta^*p, w) \text{ is an accepting state}$$
$$\text{iff } \delta^*(q, w) \text{ is an accepting state.}$$

Show that this is an equivalence relation. [6]

7. Find a CFG that generates the language $\mathcal{L} = \{a^n b^n c^m : n, m > 0\}$.
Is the Language $\mathcal{L} = \{a^p :$ p is prime$\}$ regular? Justify. [6+7].

8. When is a grammar said to be in Chomsky Normal Form(CNF)? Convert the following grammar into an equivalent grammar in CNF.

$$S \rightarrow aB/Ab;\ A \rightarrow aAB/a$$
$$B \rightarrow b/ABb$$

[2+6]

9. Given a regular language $\mathcal{L}$ write down a regular grammar that generates $\mathcal{L}$ (No proof required) [6]

10. State Bar-Hillel's Pumping Lemma.
Use it to show that the language over $\{a, b, c\}$ with equal occurrences of a, b, c is not context-free. [3+7]

INDIAN STATISTICAL INSTITUTE
Mid Semestral Examination 2016-17
M.Tech I Year (CS)
Automata, Languages and Computations.

Date:23-02-16 Maximum Marks:80 Duration:2hours 30 mins.

Note: Answer as many as you can. Maximum score is 80
Unless otherwise stated, notation used is as defined in the class.

1. Define a non-deterministic finite automaton(NFA).
   Construct an NFA that accepts all string over $\{0,1\}$ containing an even number of occurences of 01. Explain(without proof) your construction. Find its equivalent DFA. [3+8]

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Indian Statistical Institute

Semester-II 2016-2017

M.Tech.(CS) - First Year

Mid-term Examination (24 February, 2017)

Subject: Operating Systems

Total: 35 marks Maximum marks: 30 Duration: 2 hrs.

Please keep your answers brief and to the point.

1. (a) List the types of events that cause a process to switch from user mode to kernel mode. For each type of event,
   - explain in 1-2 lines when the event occurs;
   - state whether the event is synchronous / asynchronous;
   - state whether the event is handled by the kernel in user context or system context.

   (b) The following (pseudo-)code fragment contains the outline of a very basic shell. Fill in the blank portions so that it works as intended. Assume that all arrays are large enough. For full credit, (i) the syntax for all necessary system calls should be correct; and (ii) your code should check for errors returned by system calls.

```
char linebuf[BUF_SIZE];
char **argv[MAX_ARGS];
int argc;
/* Put any declarations here */
while (1) {
    /* write code to print the shell prompt (e.g., "$ ") */
    readline(linebuf); // contains the command entered by the user
    argc = parseline(linebuf); // puts the command (program name) in argv[0],
                               // and the remaining arguments (if any) in
                               // argv[1], argv[2] ...
    /* write code to create child process */
    /* execute the command entered by the user in the child */
}
```

   (c) When does the kernel reclaim the space that is allocated to hold the *proc* structure of user-created processes? Clearly explain your answer considering all possible scenarios.

[6 + 6 + 6 = 18]

P.T.O.

2. Clearly explain if and how the Linux 2.4 scheduler uses ideas from each of the following standa scheduling methods: (i) FCFS; (ii) priority-based scheduling; (iii) round-robin; (iv) multi-le queue; (v) multi-level queue with feedback. Provide excerpts from the scheduler code if appropri to justify your answer. [2 × 5 =

3. (a) Recall that a semaphore S is an integer variable that can be accessed only through two operation *wait*(S) and *signal*(S) which behave as follows:

```
wait(S) {
   while (S <= 0)
     /* do nothing */ ;
   S--;
}
```

```
signal(S)
{
   S++;
}
```

A proper implementation of these functions ensures the following:

- two processes should never change S concurrently;
- two processes should never concurrently execute the two statements in the body of the wa function.

Show how you would implement the *wait* and *signal* functions without using any special hardwa instructions (atomic test-and-set, interrupt disabling, etc.). You may assume, however, that bas machine instructions (load, store, test) are executed atomically. You may also assume that a var able declared as `shared` (e.g. `shared int x = 0;`) is available to a set of co-operating processe If you use the entry and exit sections from one of the standard solutions to the Critical Sectio Problem, clearly explain which solution you would use, and how you would use it.

Indian Statistical Institute

Second Semestral Examination: 2017

Course: M. Tech. in Computer Science

Subject: Computer Networks

Date: 17-04-2017 Total Marks: 110 Duration: 3 hours 30 minutes

Instructions: You **may** attempt **all** questions. But the maximum marks you can score is only **100**.

1. What is the window size in Stop-and-Wait ARQ? How many unique sequence numbers does Stop-and-Wait need? How many bits are needed to represent Stop-and-Wait's unique sequence numbers? [2+2+2=6]

2. State the difference between Go-Back-N ARQ and Selective-Repeat ARQ in terms of buffer requirements and retransmissions. [6]

3. Suppose that a sender and a receiver are using ARQ to perform reliable data delivery over a 1 Mbps channel with 144 milliseconds propagation delay. Assume that the user data frame size is 1500 bytes. Assume that the transmission time for ACK frame and the processing time at nodes are negligible. You may also assume that the sender always has frames to send and that no frames are lost.

   (a) What is the channel utilization if Stop-and-Wait ARQ is used? [5]

   (b) Propose a solution to increase the channel utilization to 100%. [5]

   (c) State the additional complexity of your solution compared to Stop-and-Wait ARQ in terms of number of bits required to represent the sequence numbers and memory space requirements. [5]

4. Link-state routing requires routing updates to be flooded to all participating routers. Besides the actual routing update, a link-state packet carries the *node id*, a *sequence number*, and a *time-to-live*. Why do you think these extra informations are necessary? [6]

5. Assume A and B can hear each other, B and C can hear each other, and C and D can hear each other. No other nodes can hear each other. Also assume that RTS and CTS are not being used.

   (a) If C wants to send to B while A is sending to B, will a collision occur? Why or why not? Will this be considered hidden or exposed terminal? [1+3+2=6]

   (b) If B wants to send to A while C is sending to D, will a collision occur? Why or why not? Will this be considered hidden or exposed terminal? [1+3+2=6]

6. Consider a shared medium with N nodes running the slotted Aloha MAC protocol without any backoffs. An idle slot is one in which no node sends data.

P.T.O

(a) If each node has a sending probability of p, what is the probability that a slot is idle? [4]

(b) Assume N is infinitely large. If the value of p for each node is picked so as to maximize the utilization, what will be the corresponding probability of an idle slot? [6]

7. Binary exponential backoff is a mechanism used in some MAC protocols. Which of the following statements is correct and why? [6]

   (a) It ensures that two or more nodes that experience a collision in a time slot will *never collide* with each other when they each retry that packet.

   (b) It ensures that two or more nodes that experience a collision in a time slot will experience a *lower probability of colliding* with each other when they each retry that packet.

8. Consider building a CSMA/CD network running at 1 Gbps over a 1-KM cable with no repeaters. The signal speed in the cable is 200,000 KM/sec. What is the minimum frame size? [6]

9. State the difference between 1-persistent CSMA and non-persistent CSMA in terms of delay and throughput. [6]

10. Computer A has 19.5 MBytes to send on a network and transmits the data in burst at the rate 6 Mbps. The maximum transmission rate across routers in the network is 4 Mbps. Suppose computer A uses a leaky bucket to shape its transmission. How much capacity must the queue in the bucket hold so that no data is discarded? [6]

11. A CIDR block for a single network extends from 112.45.152.0 to 112.45.159.255.

    (a) How many bits are in the network id? [2]

    (b) How many bits are in the host id? [2]

    (c) What is the mask in binary, dotted decimal and slash notation? [2+2+2=6]

12. A process on host 1 has been assigned port p and a process on host 2 has been assigned port q. Is it possible for there to be two or more TCP connections between these two ports at the same time? Justify. [5]

13. Describe Nagle algorithm in brief. State a potential disadvantage when this algorithm is used in a badly congested network? [6+4=10]

14. Suppose that the TCP congestion window is set to 18 KBytes and a timeout occurs. How big will the window be if the next 4 transmission bursts are all successful? Assume that the maximum segment size is 1 KByte. [6]

INDIAN STATISTICAL INSTITUTE

Second Semester Examination: 2016-17

Course Name: M. Tech. (CS) I Year

Subject Name: Database Management Systems

Date: 19/04/2017 **Maximum Marks: 100** **Duration: 3 hours**

1. Consider the following information for a University database:

   a. Every teacher has a unique teacher no, a name, a designation and an area of specialisation.

   b. Each teacher is associated with only one department.

   c. A department has a unique name and address consisting of a unique building name and floor no.

   d. One of the teachers belonging to a department serves as its Head.

   e. University runs various projects sponsored by different funding agencies.

   f. Every project has a unique project no, sponsor name, a starting date, a completion date and a budget.

   g. Each project has a principal investigator (PI) who is one of the teachers of the university who mainly manages the project along with the project members.

   h. Few other teachers may also be associated with each project as member.

   i. Teachers from other University may also be associated with each project as external member.

   j. External members name along with their affiliation is unique.

   k. A teacher can be PI and / or associated with multiple projects.

   l. Each project is assigned to the department where its principal investigator works. The other members of the project may, however, belong to different departments.

   m. Each research scholar of the university is either a teaching assistant or a research assistant.

   n. A teaching assistant is associated with the department of his / her supervisor.

   o. A research assistant may, however, be associated with a project which may not belong to the department where his / her supervisor is working.

   p. Research assistant is allowed to work in only one project.

   q. Each scholar has a unique roll no, name and a year of enrolment.

   From the above description draw an ER/EER diagram. 20

2. Define all the variations of the join operation. Express them in terms of cross product, selection and projection, if possible. Why is the join operation given special attention for query optimization?

20

3. What is two-phase locking protocol? How does it guarantee serializability? Compare binary locks to exclusive / shared locks. Which one is preferable and why?

15

4. Proof or disprove: Any two-attribute (atomic) relation may or may not be in BCNF.

5

5. Which of the following schedules is (conflict) serializable? For each serializable schedule determine the equivalent serial schedules.
   a. $r_1(X); r_3(X); w_1(X); r_2(X); w_3(X);$
   b. $r_1(X); r_3(X); w_3(X); w_1(X); r_2(X);$
   c. $r_3(X); r_2(X); w_3(X); r_1(X); w_1(X);$
   d. $r_3(X); r_2(X); r_1(X); w_3(X); w_1(X);$

10

6. Give a set of FDs for the relation schema R(W, X, Y, Z) with primary key {W, X} under which R is in 2NF, 3NF but not in BCNF. Justify your answer.

10

7. Consider the following relations:

   Sailors(sid: integer, sname: string, rating: integer, age: real);

   Boats(bid: integer, bname: string, color: string);

   Reserves(sid: integer, bid: integer, day: date).

   Solve the queries below using any one of the following: Relational Algebra, Domain Relation Calculus, Tuple Relation Calculus, SQL.

   a) Find the names of sailors who have reserved at least two different boats on the same day.
   b) Find the ids of sailors who either have only reserved all Red coloured boat or all Green coloured boat.
   c) Find the name and the age of the youngest sailor.
   d) Find the names and ratings of sailor whose rating is better than some sailor called Horatio.
   e) Find the names of sailors who have reserved all boats.

20

INDIAN STATISTICAL INSTITUTE

End Semestral Examination

M. Tech (CS) - I Year, 2016-2017 (Semester - II)

Design and Analysis of Algorithms

Date: 22.04.2017 Maximum Marks: 100 Duration : 3.5 Hours

Note: This is a three page question paper and it is of 132 marks, 11 questions each of 12 marks. Answer as much as you can, but the maximum you can score is 100.

(Q1) Let X and Y be two arrays of n distinct integers each. X and Y are sorted in an increasing order. Now, consider $Z = X \cup Y$; you can assume that Z has $2n$ distinct elements. Find the median of Z efficiently. [12]

[Hints: Doing it in linear time is too trivial!]

(Q2) Let A be an unsorted array on n integers. We need to search for a value a in A. Consider the following randomized strategy: choose a random index $i \in [1, n]$ and check if $A[i] = a$. If $A[i] = a$, then we terminate; otherwise, we continue the search by choosing a new random index into A. We continue choosing random indices into A until we find an index j such that $A[j] = a$, or until we have checked every element of A. Notice that, we choose from the whole set of indices every time, so that we may examine a given element more than once.

(a) Suppose, there is only one index i such that $A[i] = a$. Calculate the expected number of indices that needs to be tried before a is found and the algorithm terminates?

(b) Suppose, there is no index i for which $A[i] = a$. Calculate the expected number of indices into A that must be tried before all elements of A have been checked and the algorithm terminates?

[5+7=12]

(Q3) Let $G = (V, E)$ be a graph with $V = \{v_1, v_2, \ldots, v_n\}$. (v_i, v_j) is an edge in G if and only if the numbers i and j differ by exactly 1, i.e., $E = \{(v_1, v_2), (v_2, v_3), \ldots, (v_{n-1}, v_n)\}$. Each node $v_i \in V$ has a positive integer weight w_i. A subset of the nodes in V is called an independent set if no two of them are joined by an edge.

(a) Design and analyse an algorithm that finds an independent set of maximum weight.

(b) Prove the correctness of your algorithm.

[9+3=12]

[Hints: We are not trying to find the maximum cardinality independent set but the one with the maximum weight. Can you locate overlapping subproblems?]

(Q4) Let $G = (V, E)$ be a connected weighted graph with distinct weights on edges. A particular edge $e \in E$ is specified. Design an algorithm with running time $O(|V| + |E|)$ to decide whether e is contained in the *minimum spanning tree* of G. Prove the correctness of your algorithm. [12]

(Q5) Let $G = (V, E)$ be a strongly connected directed graph with positive edge weights, and $v_0 \in V$ be a particular node.

(a) Design and analyse an efficient algorithm to find shortest paths between all pairs of vertices in V that go through v_0.

(b) Prove the correctness of your algorithm.

[9+3=12]

(Q6) The *intersection detection problem* for a set S of n line segments is to determine whether there exists a pair of segments in S that intersect. Design an $O(n \log n)$ time algorithm that solves the *intersection detection problem.* [12]

(Q7) The problem is about finding a schedule for n workers during m vacation periods where each vacation period consists of a certain number of consecutive days. Let D_j be the set of days included in the vacation period j, $1 \leq j \leq m$. So, $\bigcup_j D_j$ is the union of all these vacation days. Design an algorithm to prepare the work schedule keeping the following constraints under consideration:

- A worker i, $1 \leq i \leq n$, has a set of vacation days S_i when he is available for work. S_i's will obviously be spread across D_j's.
- Each worker should be assigned to work at most c vacation days in total among the days in which he is available.
- For each vacation period j, each worker should be assigned to work at most one of the days in the set D_j.

[12]

[Hints: Do you see a network flow formulation here?]

(Q8) (a) The Knuth-Morris-Pratt algorithm constructs a finite state machine as a *failure function* from the *pattern string* to test if the *text string* contains the pattern. Construct such a failure function for the following string: ABAABAABAABA.

(b) Design and analyze an algorithm to determine if a text string S is a reverse cyclic rotation of another string S'. sore and rose are reverse cyclic rotations of each other.

[6+6=12]

(Q9) Let $\mathcal{H}$ be a 2-universal hash family. A random hash function $h : U \to [m]$ is chosen from $\mathcal{H}$. $S \subset U$ is stored in a hash table using h. Fix $u \in U$, and let X_u be the random variable which denotes the look up time for u in the hash table. Then, prove that $\mathbb{E}(X_u) \leq 1 + \frac{|S|-1}{m}$. [12]

(Q10) (a) Prove or disprove the following statement. For an NP-Complete problem Π, if $\Pi \in$ co-NP, then NP = co-NP.

(b) Consider the following problem which is known as *Set Cover*. Given a set $\mathcal{U}$ of n elements, a collection $\mathcal{S} = S_1, \ldots, S_m$ of subsets of $\mathcal{U}$, and a number k, does there exist a collection of at most k of these sets whose union is equal to all of $\mathcal{U}$. Show that *Set Cover* is NP-Complete.

[Hints: You can do it by a reduction from vertex cover.]

[6+6=12]

(Q11) Consider the following problem known as *minimum makespan scheduling*. Given k (k is a positive integer) identical machines and processing times $p_1, \ldots, p_n$ for n jobs, find an assignment of the jobs to the machines so that the makespan (i.e., the completion time) is minimized. Design a 2-factor approximation algorithm for this problem. [12]

[Hints: Order the jobs arbitrarily and assign it to a machine that has the least amount of work so far.]

INDIAN STATISTICAL INSTITUTE
Semestral Examination 2016-17
M.Tech. (Computer Science) I Year
Automata, Languages and Computations.

Date:25.-04-17 Maximum Marks:100 Duration: 3 hours

Note: Answer as many as you can. Maximum score is 100
Unless otherwise stated, notation used is as defined in the class.

1. Given a DFA $\mathcal{M}$, write an algorithm for testing whether the language it accepts is empty. Justify your answer. [7]

2. Is the intersection of two context-free languages context-free? Justify your answer. [6]

3. (a) Show that the intersection of a context-free language with a regular language is context-free. Give only an outline of your proof.
   (b) Use this fact to show that the language $\mathcal{L}$ over $\{a, b, c\}$ with equal occurrences of a, b, c is not context-free.
   (c) Construct a Turing machine(TM) that accepts $\mathcal{L}$. What is the time complexity ? Can you construct an $O(\log n)$-space bounded TM accepting it. Justify.

   [8+7+10]

4. (a) Define a pushdown automaton(PDA). When is a string said to be accepted by a PDA? When is the PDA said to be deterministic?
   (b) Show that if $\mathcal{L}$ is accepted by a deterministic PDA(DPDA) $\mathcal{M}$ by final states and $\mathcal{L}$ has the prefix property, then $\mathcal{L}$ is also accepted by a DPDA $\mathcal{M}'$ by empty stack.

   [(4+3+3)+10]

5. (a) Construct a TM that uses only $O(\lceil \log n \rceil)$ cells (not counting cells on the input tape) that accepts all binary strings in which the number of 0's is twice the number of 1's.
   (b) When is a language $\mathcal{L}$ said to be recursive? Show that $\mathcal{L}$ is recursive iff there is a TM accepting $\mathcal{L}$ that halts on all inputs.

   [7+8]

KB

6. (a) When is a function $f : \mathbb{N}^k \to \mathbb{N}$ said to be recursive? What is its relation with (Turing) computable functions?

(b) Show that the following functions and predicates are recursive.

(a) $f(n,m) = [n/m] = \begin{cases} 0 & \text{if } m = 0 \\ \text{the largest integer} \leq n/m & \text{if } m > 0 \end{cases}$

(b) $Prime(n) \longleftrightarrow n$ is prime.

(c) $p(n) =$ the nth prime number.

(d) $lcm(n,m) = \begin{cases} 0 & \text{if } n = 0 \text{ or } m = 0 \\ \text{the least common multiple of } n,m & \text{otherwise} \end{cases}$

(e) $gcd(n,m) = \begin{cases} 0 & \text{if } n = 0 \text{ or } m = 0 \\ \text{the greatest common divisor of } n,m & \text{otherwise} \end{cases}$

[5+(5× 4=20)]

7. (a) When is a language said to be $\mathcal{NP}$-complete?

(b) Show that 3-SAT is in $\mathcal{NP}$.

(c) Assuming that 3-SAT is $\mathcal{NP}$- complete, consider the following reduction of 3-SAT to CHROMATIC NUMBER problem.
Given an instance $\phi := C_1 \wedge C_2 \wedge \cdots \wedge C_m$ in 3-CNF over the atoms $\{x_1, \cdots, x_n\}$, consider the graph $G(\phi)$ whose set of vertices is

$$V = \{v_i : 1 \leq i \leq n\} \bigcup \{x_i, \bar{x}_i : 1 \leq i \leq n\} \bigcup \{C_k : 1 \leq k \leq m\}$$

and whose edges are as follows.

i. For every $i \neq j$, add an edge $\{v_i, v_j\}$.

ii. For every i add an edge $\{x_i, \bar{x}_i\}$.

iii. For every $i \neq j$ add edges $\{v_i, x_j\}, \{v_i, \bar{x}_j\}$.

iv. For every i, j add an edge $\{C_i, x_j\}$ if x_j does NOT appear in C_i and add an edge $\{C_i, \bar{x}_j\}$ if $\bar{x}_j$ does NOT appear in C_i.

Show that ϕ is satisfiable iff $G(\phi)$ is $n+1$ colourable.
(Hint: Think of colour $n+1$ as **false** colour.)

[4+4+8]

Indian Statistical Institute

Semester-II 2016-2017

M.Tech.(CS) - First Year

Semestral Examination (28 April, 2017)

Subject: Operating Systems

There are 5 questions in this paper, carrying a total of 55 marks. You may answer as many questions as you like. The maximum marks that you can score is 50.

Duration: 3.5 hrs.

Please keep your answers brief and to the point.

1. Suppose that you have to write a deadlock-free C program to simulate the Dining Philosophers Problem. Each philosopher is modelled using a process, and the processes are synchronised using semaphores. Write a procedure `philosopher()` to simulate the functioning of each philosopher.

   You may use either SysV style or POSIX style system calls, but your code should clearly show how the semaphores are (i) created / initialised; and (ii) used to synchronize the philosophers. [6]

2. (a) Consider the following analogy:

   A user-level thread library manages user-level threads in the same way that the kernel manages multiple processes.

   Recall that the kernel takes over the processor when one of the following events occurs: (i) system calls (ii) exceptions (iii) interrupts. For each of the above events, explain whether an analogous event exists that permits a user-level thread library to take control over the threads that it is managing.

   (b) An operating system may be designed so that, when a multi-threaded process forks, the child process gets (i) only one thread (corresponding to the thread that called `fork()`), or (ii) a copy of each thread in the parent process. State one advantage and one disadvantage of design option (ii) above.

   (c) Explain what the following program will do.

   NOTE: This is a valid program that can be compiled and run.

```
int main(int ac, char *av[]) {
    static unsigned int i = 0;
    pthread_t tid;

    pthread_create(&tid, NULL, main, NULL);
    while (1) {
        printf("Hello %u\n", i);
        i++;
    }
    return 0;
}
```

[(2+1+2) + (2+2) + 4 = 13]

P.T.O

3. (a) Consider an operating system that uses a paging-based memory management scheme. Logical addres are 28 bits long. Each page table entry occupies 4 bytes. Calculate the minimum and maximum p sizes for which exactly 2 levels of paging will be required.

NOTE: In a 2-level paging scheme, the second-level page table must fit within a single page.

(b) Recall that the page size in the i386 architecture is 4KB. Compute the (i) total size of the segme tables, and (ii) total size of the page tables. [7 + 4 =

4. Consider a newly created SVR2 filesystem (i.e., no files or directories have been created on this filesyst so far). Each block in this filesystem is 4096 bytes long. The total number of data blocks in the filesyste is 2^{32}, and block numbers are stored using 4-byte integers.

(a) Calculate the number of disk blocks required initially to store the list of free disk blocks. Clearl **state any assumptions that you make.**

(b) The filesystem is now mounted at `/mnt/new`, a directory in an existing filesystem that is in use. Ne the super-user runs the following commands.

```
# cd /mnt/new
# ls > out1
# df > out2
# cp out1 ls-output
# rm out1
# mv out2 df-output
# ln -s ls-output file-list
# ln df-output used-and-free-space
```

Draw a diagram showing the state of the directory data block for the root directory of the mount filesystem after the above commands have been executed.

(c) For this filesystem, calculate the range of file offsets covered by each entry in the inode table of conten

(d) If a process sequentially reads a file of size 276520, how many blocks will be read in all?

[2 + 10 + 6 + 2 =

5. Consider the following Resource Allocation State involving 4 processes and 4 resources. $Total[i]$ specifies th total number of instances of resource i that exist in the system (including both allocated and free instances $Alloc[i,j]$, and $Req[i,j]$ denote, respectively:

- the number of instances of resource j currently allocated to process i, and
- the number of instances of resource j that process i is currently requesting.

$$Total = [\ 4\ \ 6\ \ 4\ \ 4\ ] \quad Alloc = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad Req = \begin{bmatrix} 3 & 2 & 2 & 0 \\ 1 & 4 & 3 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 4 & 4 & 1 \end{bmatrix}$$

Show that the system is currently in a deadlocked state. Also, list the processes involved in the deadlo

INDIAN STATISTICAL INSTITUTE

Back Paper Examination

M. Tech (CS) - I Year, 2016-2017 (Semester - II)

Design and Analysis of Algorithms

Date: 11.07.2017 Maximum Marks: 100 Duration : 3.0 Hours

Note: This is a 2 page question paper and it is of 110 marks, 11 questions each of 10 marks. Try to answer as much as you can.

(Q1) Consider a sequence of elements $S = \langle s_1, \ldots, s_i, \ldots, s_n \rangle$, where each element is unique. The *span* of any s_i, $i > 1$, is an index $j < i$, such that $s_j > s_i$ and all s_k's between s_j and s_i are less than s_i. If no such j exists for an i, then the span of s_i is set to 0. As an example, let $S = \{10, 9, 1, 0, 2, 4, 5, 8, 7\}$. Here $s_7 = 5$ and its span is 2, as $s_2 = 9 > s_7$, and s_3, s_4, s_5 and s_6 are all less than s_7.

Design and analyse an $O(n)$ time algorithm to find the span of each $s_i \in S$, $i > 1$. [10]

(Q2) (a) Let $A[1, \ldots, n]$ be an array of n integers, and x be an integer. Design an efficient algorithm to determine whether there are two elements in A, if any, whose sum is exactly x. Analyze the time complexity of your algorithm.

(b) Given an array X of positive integers such that any $x_i \in X$ lies in the range $[0, k]$, where k is a positive integer. Also, $k = O(n)$. Design an efficient algorithm to find the median of X. Note that the linear time median finding algorithm is not very efficient.

[5+5=10]

(Q3) In the *SELECTION* (median finding) algorithm studied in class, we divided the input elements into groups of 5; and found out by our analysis that the algorithm runs in linear time. Deduce what happens to the time complexity of the *SELECTION* algorithm if the input was divided into groups of 3. [10]

(Q4) We have a set of n requests — the i-th request corresponds to an interval of time starting at $s(i)$ and finishing at $f(i)$. A subset of requests is *compatible* if no two of them overlap in time. Design and analyze an efficient algorithm to find a compatible subset that has the maximum number of elements. [10]

(Q5) Let G be a directed acyclic graph with positive weights on edges of G. Design and analyze an efficient algorithm to find the longest path in G. [10]

[Hints: Can you notice overlapping subproblems?]

(Q6) (a) Define a strongly connected component in a directed graph.

(b) Design and analyze an algorithm to find the strongly connected components in a directed graph. Prove your results.

[2+8=10]

(Q7) A *cut-set* is a minimal set of edges in a graph such that the removal of the set of edges will increase the number of connected components in the remaining subgraph. Now, prove the following statements.

(i) A cut-set and any spanning tree of a graph must have at least one edge in common.

(ii) In a graph, every cycle has an even number of edges in common with every cut set.

[4+6=10]

(Q8) (a) Define the maximum flow and the minimum cut problem in a network.

(b) Describe the Ford-Fulkerson algorithm for obtaining the maximum flow in the network.

(c) Deduce how a minimum cut is obtained from the Ford-Fulkerson algorithm.

[2+5+3=10]

(Q9) Let P be a simple polygon of n vertices in $\mathbb{R}^2$. [10]

(a) A *diagonal* of P is a line segment that joins two vertices of P and stays completely inside P Prove that at least one diagonal always exists for a simple polygon.

(b) Design and analyze an efficient algorithm to compute the area of P.

[3+7=10]

(Q10) (a) Prove or disprove the following statement. If Π_1 and Π_2 are two NP-Complete problems, then $\Pi_1 \leq_P \Pi_2$; where $\leq_P$ denotes polynomial reducibility.

(b) Prove or disprove the following statement. If any problem in NP is not polynomially solvable, then no NP-Complete problem is polynomially solvable.

[5+5=10]

(Q11) Given an undirected graph $G = (V, E)$, we define a *cut* of G to be a *partition* of V into two non-empty sets A and B. For a cut (A, B) in G, the size of the cut is the number of edges with one end in A and the other in B. A *global minimum cut* is a cut of minimum size. Design and analyze an algorithm to solve this problem. Your algorithm can be randomized. [10]

Examination: Back Paper Exam (2016-17)
M.Tech(CS) I Year
Automata, Languages and Computations.

Date: 14/7/..2017 Maximum Marks:100 Duration:3hours

Note: Answer as many as you can.
Unless otherwise stated, notation used is as defined in the class.

1. State Kleene's Theorem.
   Define regular expressions over an alphabet Σ.
   Let $\mathbf{r}$ and $\mathbf{s}$ be two regular expsessions. Prove the following identity

$$(\mathbf{r}+\mathbf{s})^* = (\mathbf{r}^* + \mathbf{s}^*)^*.$$

   [3+3+5]

2. Write an algorithm to test, given DFAs $\mathcal{M}_1$ and $\mathcal{M}_2$, whether $\mathcal{L}(\mathcal{M}_1) \subseteq \mathcal{L}(\mathcal{M}_2)$. Prove its correctness. [9]

3. Find a CFG that generates the set of all palindromes over the alphabet $\{a, b\}$. [6].

4. When is a grammar said to be in Chomsky Normal Form(CNF)? Convert the following grammar into an equivalent grammar in CNF.

$$S \to aB/Ab;\ A \to aAB/a$$
$$B \to b/ABb$$

   [2+6=8]

5. State Bar-Hillel's Pumping Lemma.
   Use it to show that $\{a^p : p \text{ is prime}\}$ is not context-free. [3+7=10]

6. (a) Explain in detail when a pushdown automata (PDA) accepts a string by empty stack.
   (b) Show that if $\mathcal{L}$ is a context-free language then there is a PDA that accepts $\mathcal{L}$.
   (c) Show that if $\mathcal{L}$ is a language accepted by a deterministic PDA by empty stack, then $\mathcal{L}$ has the prefix property *i.e.* no proper prefix of a string in $\mathcal{L}$ is in $\mathcal{L}$.

(d) Show that

$$\mathcal{L} = \{ww : w \in (0+1)^*\}$$

is not context-free

[5+7+5+8=25]

7. Consider the language
$\{w : w \in \{a,b,c\}^*$, w contains equal numbers of a's, b's and c's $\}$
Construct a Turing Machine which accepts this language.

[6]

8. (a) Design (i) single-tape (ii) multitape Turing machines(TMs) that accept the set of all strings over $\{a,b\}$ containing an equal numbers of a's and b's. Compare the number of moves made by the respective TMs on accepting a string of length n.
Also, construct a TM that uses only $\lceil \log n \rceil$ cells (not counting cells on the input tape)

(b) A language $\mathcal{L}$ over Σ is said to be recursive if both $\mathcal{L}$ and $\Sigma^* - \mathcal{L}$ are recursively enumerable. Show that $\mathcal{L}$ is recursive iff there is a TM accepting $\mathcal{L}$ that halts on all inputs. [(5+5+2+5)+8=25]

9. (a) When is a function $f : \mathbb{N}^k \to \mathbb{N}$ said to be (Turing) computable?

(b) Consider the following functions on $\mathbb{N} \times \mathbb{N}$.

$$A(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ or } y = 0 \\ \mu z \leq x(\exists w \leq x(w.y + z = x)) & \text{otherwise} \end{cases}$$

$$B(x,y) = \begin{cases} 0 & \text{if } x = 0 \text{ or } y = 0 \\ x \dot{-} (\mu z \leq x(\exists w \leq x(w.(x \dot{-} z) = x) & \\ \quad \text{and } \exists u \leq y(u.(x \dot{-} z) = y))) & \text{otherwise} \end{cases}$$

Here "'$\mu z \leq xP(x)$"' means "'the smallest $z \leq x$ such that $P(x)$ holds"'. Describe, in simple English, the functions A and B.

[5+6=11]

10. (a) Define the classes $\mathcal{P}$ and $\mathcal{NP}$. When is a language said to be $\mathcal{NP}$-complete?

(b) Show that SAT is in $\mathcal{NP}$.

(c) Describe the COMPLETE SUBGRAPH problem. Assuming that SAT is $\mathcal{NP}$-complete, prove that COMPLETE SUBGRAPH is also $\mathcal{NP}$-complete. [7+5+8=20]